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RBE 500 – Midterm

First of all, good luck!

Please read the following instructions carefully before you start (even if you read the related announcement earlier as some instructions might have slightly changed):

- The midterm starts at Oct. 27th 10am and ends at Oct. 28th 10am. You have 24 hours. There are 8 questions (1 bonus question).
- While you are given 24 hours, this exam would not require that much time. I estimate it to take around 3 hours if you are comfortable with the material.
- It is a take home exam. It will be submitted online.
- All the topics are included until the end of Week 7.
- This is an open books exam. You can use the resources available on your textbook, on the internet or any other. However, you need to solve the problems individually. Any communication with other people is strictly prohibited (online or in person). Plagiarism policies apply.
- Please explicitly show your derivations. You can use software to check the correctness of your results yourself, but your grades will be determined based on your own hand-written (or computer-typed) derivations. When you need some mathematical operation, e.g. matrix inverse, I assume that you know using a software to do that.
- There is **no** ROS coding assignment in this exam.
- I will respond all the questions in the discussion board called "Midterm exam". I will not be answering any questions 1-on-1 via email. The logic behind it is that every clarification regarding the exam needs to be heard by everyone. Do not give away answers. "I am following this approach, am I in the right trajectory?" is not a valid question. Your questions should only be related to clarify the questions, not to check your answers. If such questions are posted in which a portion of the solutions is shared, it will result in grade penalties.
- Please do not answer each other's questions in the Midterm discussion board. The questions are strictly for me to address.
- I recommend you read all the questions carefully as you receive the exam, and if you need any clarification, please post it to the discussion board. I will keep checking the discussion boards frequently throughout the exam, but as you can imagine, I will not be available for 24 hours.
- In our regular office hour, i.e. October 27th 11.30am, we will have an online Q/A session (zoom link is the same). This session will be recorded and published in Canvas right after the session.

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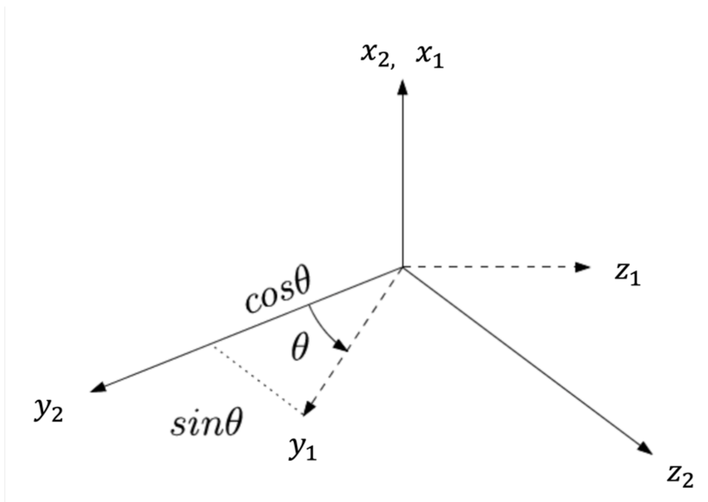
- Late submissions: Every 1-hour delay will be considered as 5 pts penalty. Please do not leave your submission to the last minute. There can be technical difficulties, but you need to incorporate them to your submission. Any delay will start from 5 points penalty. You can upload your HW multiple times, but the latest submission will be recorded and considered as the final submission time.

Question 1 (10 pts)

Derive the rotation matrix R_2^1 (you can leave sines and cosines as is)



Question 2 (5 pts)

Find the coordinates of point p expressed in frame 1 (i.e. p^1) given the following.

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0.9553 & 0.2955 & -0.9553 \\ 0 & -0.2955 & 0.9553 & 0.2955 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad p^2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

Question 3 (5 pts)

If $R_1^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$, $R_2^0 = \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}$, and $R_3^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$,

Calculate R_2^1.

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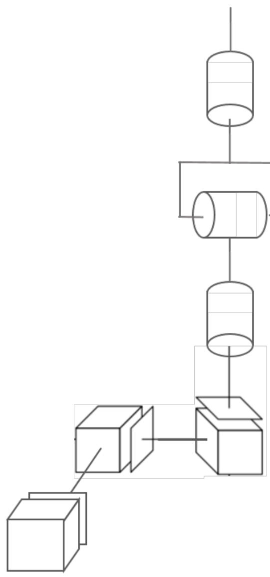
Question 4 (20 pts)

(a) Calculate Denavit Hertanberg parameters for the given manipulator (just filling out the Denavit Hertanberg table would suffice). For this question, you are expected to solve it parametrically, i.e. you can leave sines, cosines, joint values, and link lengths as parameters.



(b) Derive H_2^1. You can leave sines, cosines, joint values, and link lengths as parameters.

Question 5 (20 pts)

Calculate inverse kinematics for the manipulator in Question 4. Assume that all the forward kinematics information is available (i.e. all homogenous transformation matrices). Since there are no values given, you will be deriving your expressions parametrically, but please be sure to explicitly show, which homogeneous transformation matrix is required for the corresponding information, and which segment of the matrix is used to obtain that information. (e.g. in your derivations you can say something like: “to calculate this expression, I would need z_3^0, which is available to me at the 3rd column of H_3^0”).

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Question 6 (20 pts)

Calculate Jacobian matrix for the manipulator in Question 4. Assume that all the forward kinematics information is available (i.e. all homogenous transformation matrices). Since there are no values given, you will be deriving your Jacobian matrix parametrically, but please be sure to explicitly show which homogeneous transformation matrix is required for the corresponding information, and which segment of the matrix is used to obtain that information. (e.g. in your derivations you can say something like: "to calculate this element of the Jacobian matrix, I would need z_3^0, which is available to me at the 3rd column of H_3^0").

Question 7 (20 pts)

The dynamic system below

$$a\ddot{x} + b\dot{x} + cx = u$$

Here u is the force applied to the system, x is the position of the system. a, b, and c are model parameters all of which are constant. There parameters are

$$a = 10, \quad b = 3.5, \quad c = 0.6$$

a) (5 pts) Find the open loop transfer function for this system, i.e. $\frac{X(s)}{U(s)}$ in laplace domain.
b) (3 pts) Draw a block diagram of a closed loop system with a PD controller for controlling the position of the system. Explicitly write the transfer functions (Laplace domain) of the system and the PD controller inside the blocks (leave K_p and K_d as parameters).
c) (5 pts) Derive the closed loop transfer function i.e. $\frac{x}{x_r}$, where x_r is the position reference signal.
d) (7pts) Find the values of the K_p and K_d gains for a critically damped system with 2 seconds settling time.

Show all your steps explicitly.

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Question 8 (Bonus Question) (15 pts) Consider your derivation in question 6 in which you solved the Jacobian matrix parametrically. In this question you are expected to give the numerical value of that Jacobian matrix when joint values are $q = [0,0,0.2,0,0,0]$. You can assume link lengths as 1.

Tip: For achieving this, you will need to calculate the forward kinematics numerically as well. Use a tool like Matlab (or any other) to obtain such numerical values.