

# **RBE 500 Homework #3**

**Arjan Gupta**

## Problem 4.2

Verify Equation (4.7) by direct calculation.

$$S(a)p = a \times p \quad (4.7)$$

### Solution

Suppose the vectors  $a$  and  $p$  are given as

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

By the definition of the cross-product, we know

$$a \times p = \begin{bmatrix} a_2p_3 - a_3p_2 \\ a_3p_1 - a_1p_3 \\ a_1p_2 - a_2p_1 \end{bmatrix} \quad (1)$$

Also, by the definition of skew-symmetric matrices, we know the form of  $S(a)$ , where  $a$  is the vector we have already defined,

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Hence, by normal matrix multiplication,

$$S(a)p = \begin{bmatrix} 0 - a_3p_2 + a_2p_3 \\ a_3p_1 + 0 - a_1p_3 \\ -a_2p_1 + a_1p_2 + 0 \end{bmatrix} = \begin{bmatrix} a_2p_3 - a_3p_2 \\ a_3p_1 - a_1p_3 \\ a_1p_2 - a_2p_1 \end{bmatrix}$$

Which is the same as (1). Therefore, Equation (4.7) is proved.

## Problem 4.3

Prove the assertion given in Equation (4.9) that  $R(a \times b) = Ra \times Rb$  for  $R \in SO(3)$ .

### Solution

Let  $v = R(a \times b)$ , and  $u = Ra \times Rb$ . If  $v$  and  $u$  are to be proved as the same vector, then it must be shown that they have the exact same magnitude and direction. Let us consider magnitude and direction of  $v$  and  $u$  separately.

### Magnitude

Since  $R \in SO(3)$ ,  $\det R = 1$ , which means that the linear transformation  $R$  does not change the length (norm) of any vector that it transforms. Hence we can say

$$\|R(a \times b)\| = \|a \times b\| \quad (1)$$

$$\|Ra\| = \|a\| \quad (2)$$

$$\|Rb\| = \|b\| \quad (3)$$

Using the definition of the cross product along with (1), we can state that

$$\|v\| = \|R(a \times b)\| = \|a \times b\| = \|a\|\|b\| \sin \theta \quad (4)$$

Again using the definition of the cross product along with (2) and (3), we can state that

$$\|u\| = \|Ra \times Rb\| = \|Ra\|\|Rb\| \sin \theta = \|a\|\|b\| \sin \theta \quad (5)$$

We can see that (4) and (5) are equal. Therefore,

$$\|v\| = \|u\|$$

Which means that the magnitude of  $v$  and  $u$  are the same.

### Direction