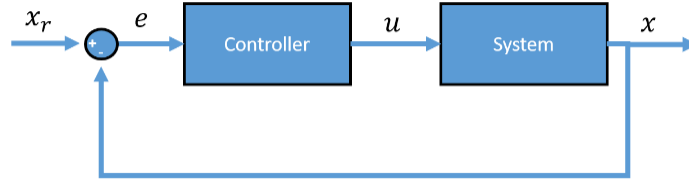


RBE 500 Homework #5

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Problem 1

Consider the following block diagram:



The dynamics of the system is given in the following differential equation

$$m\ddot{x} + b\dot{x} = u$$

The controller is designed as

$$k_p e + k_d \dot{e} = u$$

Convert the system model and the controller to the Laplace domain.

Solution

For the system model, take the Laplacian on both sides,

$$\mathcal{L}\{m\ddot{x} + b\dot{x}\} = \mathcal{L}\{u\}$$

$$m\mathcal{L}\{\ddot{x}\} + b\mathcal{L}\{\dot{x}\} = U(s)$$

$$\boxed{ms^2 X(s) + bsX(s) = U(s)}$$

Similarly, take the Laplacian on both sides of the controller model,

$$\mathcal{L}\{k_p e + k_d \dot{e}\} = \mathcal{L}\{u\}$$

$$k_p \mathcal{L}\{e\} + k_d \mathcal{L}\{\dot{e}\} = U(s)$$

$$\boxed{k_p E(s) + k_d s E(s) = U(s)}$$

Problem 2

Find the transfer functions for $\frac{U(s)}{E(s)}$, $\frac{X(s)}{U(s)}$, and $\frac{X(s)}{E(s)}$.

Solution

Re-arranging the Laplace-domain controller model from Problem 1,

$$E(s) (k_p + k_d s) = U(s)$$

$$\boxed{\frac{U(s)}{E(s)} = (k_p + k_d s)} \quad (1)$$

Re-arranging the Laplace-domain system model from Problem 1,

$$X(s) (ms^2 + bs) = U(s)$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{1}{(ms^2 + bs)}} \quad (2)$$