RBE 500 Midterm

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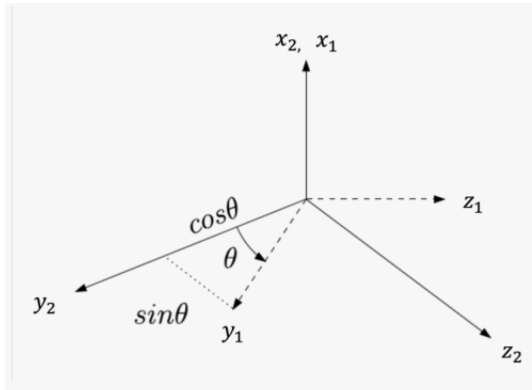
Problem 1

Derive the rotation matrix R_2^1 (you can leave sines and cosines as is).



Solution

Since the x-axis remains the same in the rotation, we know this is a basic 3D rotation matrix representing a rotation about the x-axis. However, using the right-hand screw rule, we see that the angle θ here is negative. So, by using equation 2.7 (page 43) of our main textbook, the rotation matrix is given by,

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) \\ 0 & \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\boxed{R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}}$$

Problem 2

Find the coordinates of point p expressed in frame 1 (i.e. p^1) given the following.

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0.9553 & 0.2955 & -0.9553 \\ 0 & -0.2955 & 0.9553 & 0.2955 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ p^2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

Solution

From our knowledge of homogeneous transformations, we know that

$$P^2 = H_1^2 P^1$$

Where $P^2 = \begin{bmatrix} p^2 \\ 1 \end{bmatrix}$ and $P^1 = \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$.

However, we want to find P^1, so we apply the inverse of H to both sides,

$$P^2 = H_1^2 P^1$$
$$(H_1^2)^{-1} P^2 = P^1$$

We know that $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$, where $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9553 & 0.2955 \\ 0 & -0.2955 & 0.9553 \end{bmatrix}$ and $d_1^2 = \begin{bmatrix} -1 \\ -0.9553 \\ 0.2955 \end{bmatrix}$.

For accuracy while computing the inverse of H_1^2, we use equation 2.67 of the book (page 63). Therefore,

$$(H_1^2)^{-1} = \begin{bmatrix} (R_1^2)^T & -(R_1^2)^T d_1^2 \\ 0 & 1 \end{bmatrix}$$

. We use the following MATLAB code for this computation.

```
% Calculation code for problem 2 of the RBE500 Midterm

clear; close all; clc;

P2 = [2;5;0;1];

R2_1 = [1 0 0; 0 0.9553 0.2955; 0 -0.2955 0.9553];
d2_1 = [-1; -0.9553; 0.2955];
H_inv = [R2_1' (-R2_1'*d2_1); zeros(1,3) 1];

P1 = H_inv*P2
```

Which gives us the answer,

$$\boxed{P^1 = \begin{bmatrix} 3.0000 \\ 5.7764 \\ 1.4775 \\ 1.0000 \end{bmatrix}}$$

Problem 3

If $R_1^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$, $R_2^0 = \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}$, and $R_3^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, calculate R_2^1.

Solution

Knowing the composition law for rotational transformations, we can write

$$
\begin{aligned}
R_2^1 &= R_0^1 R_2^0 \\
R_2^1 &= (R_1^0)^T R_2^0 \\
R_2^1 &= \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}^T \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix} \\
R_2^1 &= \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & 1 & 0 \\ 0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

We use the following MATLAB script to compute this multiplication.

```
% Calculation code for problem 2 of the RBE500 Midterm

clear; close all; clc;

R01 = [0.7071 0 0.7071; 0 1 0; -0.7071 0 0.7071];
R02 = [0 0.866 0.5; 0 0.5 -0.866; -1 0 0];

R12 = R01'*R02
```

Therefore,

$$
\boxed{R_2^1 = \begin{bmatrix} 0.7071 & 0.6123 & 0.3535 \\ 0 & 0.5000 & -0.8660 \\ -0.7071 & 0.6123 & 0.3535 \end{bmatrix}}
$$