RBE 500 Homework #6

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Question 1

Consider the following robot joint model

$$J\ddot{\theta}(t) + B\dot{\theta}(t) = u(t) + d(t)$$

where J is the inertia of the link, B is the effective damping on the link, θ is the joint angle, u is the actuator torque (input), and d is the disturbance acting on the system.

First, assume that disturbance is zero and take $J = 2$, $B = 0.5$. Design a PD controller such that the closed loop system is critically damped, and settling time is 2 second. Do not do this by tuning the gains; calculate the K_p and K_d gains using natural frequency and damping ratio.

Solution

Since $d(t) = 0$, $J = 2$, $B = 0.5$, we have

$$2\ddot{\theta}(t) + 0.5\dot{\theta}(t) = u(t)$$

Transform to Laplace domain,

$$\begin{aligned} 2\Theta(s)s^2 + 0.5\Theta(s)s &= U(s) \\ \Theta(s)[2s^2 + 0.5s] &= U(s) \\ \frac{\Theta(s)}{U(s)} &= \frac{1}{2s^2 + 0.5s} \end{aligned}$$

So our charateristic equation is,

$$\begin{aligned} 2s^2 + 0.5s = 0 \\ s^2 + 0.25s = 0 \end{aligned}$$

The general form of the charateristic equation is

$$s^2 + (2\xi\omega_n)s + {\omega_n}^2 = 0$$

Where ξ is the damping ratio and ω_n is the natural frequency.

Hence, we have,

$${\omega_n}^2 = 0 \tag{1}$$

and

$$2\xi\omega_n = \frac{3.5 + K_d}{10} \tag{2}$$

Question 2

Find the coordinates of point p expressed in frame 1 (i.e. p^1) given the following.

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0.9553 & 0.2955 & -0.9553 \\ 0 & -0.2955 & 0.9553 & 0.2955 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ p^2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

Solution

From our knowledge of homogeneous transformations, we know that

$$P^2 = H_1^2 P^1$$

Where $P^2 = \begin{bmatrix} p^2 \\ 1 \end{bmatrix}$ and $P^1 = \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$.

However, we want to find P^1, so we apply the inverse of H to both sides,

$$\begin{aligned} P^2 &= H_1^2 P^1 \\ (H_1^2)^{-1} P^2 &= P^1 \end{aligned}$$

We know that $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$, where $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9553 & 0.2955 \\ 0 & -0.2955 & 0.9553 \end{bmatrix}$ and $d_1^2 = \begin{bmatrix} -1 \\ -0.9553 \\ 0.2955 \end{bmatrix}$.

For accuracy while computing the inverse of H_1^2, we use equation 2.67 of the book (page 63). Therefore,

$$(H_1^2)^{-1} = \begin{bmatrix} (R_1^2)^T & -(R_1^2)^T d_1^2 \\ 0 & 1 \end{bmatrix}$$

. We use the following MATLAB code for this computation.
Which gives us the answer,

$$\boxed{p^1 = \begin{bmatrix} 3.0000 \\ 5.7764 \\ 1.4775 \end{bmatrix}}$$