RBE 500 Homework #4

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Problem 4.6

Given $R = R_{x,\theta}R_{y,\phi}$, compute $\frac{\partial R}{\partial \phi}$. Evaluate $\frac{\partial R}{\partial \phi}$ at $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$.

Solution

$$\frac{\partial}{\partial \phi}\left(R_{x,\theta}R_{y,\phi}\right) = R_{x,\theta}\frac{\partial}{\partial \phi}\left(R_{y,\phi}\right)$$

Using the the fact that $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(R_{y,\theta}\right) = S(j)R_{y,\theta}$,

$$R_{x,\theta}\frac{\partial}{\partial \phi}\left(R_{y,\phi}\right) = R_{x,\theta}S(j)R_{y,\phi} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{bmatrix}$$
$$= \begin{bmatrix}-\sin\left(\phi\right) & 0 & \cos\left(\phi\right)\\ \cos\left(\phi\right)\sin\left(\theta\right) & 0 & \sin\left(\phi\right)\sin\left(\theta\right)\\ -\cos\left(\phi\right)\cos\left(\theta\right) & 0 & -\cos\left(\theta\right)\sin\left(\phi\right)\end{bmatrix}$$

Now, plugging in the values $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$, we get

$$\begin{bmatrix}-1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

We performed the above computations using the following MATLAB code.

```
% Calculation code for problem 4.6 of the RBE500 textbook (HW 4)

clear; close all; clc;

syms theta phi

% Define the matrices in our problem
rotx_theta = [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
Sj = [0 0 1; 0 0 0; -1 0 0];
roty_phi = [cos(phi) 0 sin(phi); 0 1 0; -sin(phi) 0 cos(phi)];

% Multiply the matrices
product = rotx_theta*Sj*roty_phi;

% Get latex output
latex(product)

phi_val = pi/2;
theta_val = pi/2;

% Now plug in values
product_val = [ -sin(phi_val), 0, cos(phi_val);
cos(phi_val)*sin(theta_val), 0, sin(phi_val)*sin(theta_val);
-cos(phi_val)*cos(theta_val), 0, -cos(theta_val)*sin(phi_val)]
```