

# RBE 500 Homework #4

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## Problem 4.6

Given  $R = R_{x,\theta}R_{y,\phi}$ , compute  $\frac{\partial R}{\partial \phi}$ . Evaluate  $\frac{\partial R}{\partial \phi}$  at  $\theta = \frac{\pi}{2}$ ,  $\phi = \frac{\pi}{2}$ . First parametrically compute, then evaluate by plugging the values in.

### Solution

$$\frac{\partial}{\partial \phi} (R_{x,\theta}R_{y,\phi}) = R_{x,\theta} \frac{\partial}{\partial \phi} (R_{y,\phi})$$

Using the the fact that  $\frac{d}{d\theta} (R_{y,\theta}) = S(j)R_{y,\theta}$ ,

$$\begin{aligned} R_{x,\theta} \frac{\partial}{\partial \phi} (R_{y,\phi}) &= R_{x,\theta} S(j) R_{y,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{bmatrix} \\ &= \begin{bmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ \cos(\phi) \sin(\theta) & 0 & \sin(\phi) \sin(\theta) \\ -\cos(\phi) \cos(\theta) & 0 & -\cos(\theta) \sin(\phi) \end{bmatrix} \end{aligned}$$

Now, plugging in the values  $\theta = \frac{\pi}{2}$ ,  $\phi = \frac{\pi}{2}$ , we get

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

We performed the above computations using the following MATLAB code.

```

1  % Calculation code for problem 4.6 of the RBE500 textbook (HW 4)
2
3  clear; close all; clc;
4
5  syms theta phi
6
7  % Define the matrices in our problem
8  rotx_theta = [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
9  Sj = [0 0 1; 0 0 0; -1 0 0];
10 roty_phi = [cos(phi) 0 sin(phi); 0 1 0; -sin(phi) 0 cos(phi)];
11
12 % Multiply the matrices
13 product = rotx_theta*Sj*roty_phi;
14
15 % Get latex output
16 latex(product)
17
18 phi_val = pi/2;
19 theta_val = pi/2;
20
21 % Now plug in values
22 product_val = [ -sin(phi_val), 0, cos(phi_val);
23 cos(phi_val)*sin(theta_val), 0, sin(phi_val)*sin(theta_val);
24 -cos(phi_val)*cos(theta_val), 0, -cos(theta_val)*sin(phi_val)]

```

## Problem 4.10

Two frames  $o_0x_0y_0z_0$  and  $o_1x_1y_1z_1$  are related by the homogeneous transformation

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity  $v_1(t) = (3, 1, 0)$  relative to frame  $o_1x_1y_1z_1$ . What is the velocity of the particle in frame  $o_0x_0y_0z_0$ ?

### Solution

The given H is the homogeneous transformation  $H_1^0$ . Therefore,  $R_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $o_1^0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ .

At any given point in time, the position of the particle with respect to frame  $o_1x_1y_1z_1$  is given as  $p^1(t)$ . We know that

$$p^0(t) = R_1^0 p^1(t) + o_1^0$$

Taking the derivative of both sides and using the product rule, we get

$$\dot{p}^0(t) = \dot{R}_1^0 p^1(t) + R_1^0 \dot{p}^1(t) + 0$$

But,  $\dot{R}_1^0 = 0$  since  $R_1^0$  is a constant in time. Therefore,

$$\begin{aligned} v^0(t) &= R_1^0 v^1(t) \\ v^0(t) &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \\ v^0(t) &= \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \end{aligned}$$

Which was calculated by the following MATLAB script

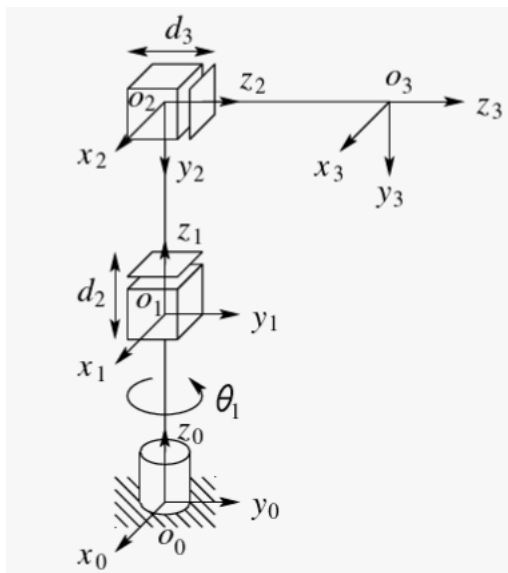
```
1 % Calculation code for problem 4.10 of the RBE500 textbook (HW 4)
2
3 clear; close all; clc;
4
5 R = [0 -1 0; 1 0 0; 0 0 1];
6 v1 = [3; 1; 0];
7
8 v0 = R*v1
```

## Problem 4.15

Find the  $6 \times 3$  Jacobian for the three links of the cylindrical manipulator of Figure 3.7. Find the singular configurations for this arm.

### Solution

Figure 3.7 of our book is the following.



We can see that we have 3 joints, so  $n = 3$ . Let us also form the table given in the lecture videos:

	Linear component	Angular component
Revolute joint	$J_{v_i} = z_{i-1}^0 \times (o_n^0 - o_{i-1}^0)$	$J_{v_i} = z_{i-1}^0$
Prismatic joint	$J_{\omega_i} = z_{i-1}^0$	$J_{\omega_i} = 0$

Using this table, and the fact that the upper half of the Jacobian contains linear components while the bottom half contains angular components, we have

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 & z_2 \\ z_0 & 0 & 0 \end{bmatrix}$$

In this Jacobian matrix, we know that  $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  and  $o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ . To find  $z_1$ ,  $z_2$ , and  $o_3$ , we need to find

$T_n^0 = A_1 \dots A_n$  for  $n = 1, 2, 3$ . By following the DH convention (which Figure 3.7 already abides by), we have the following table for quantities  $\alpha_i, a_i, \theta_i, d_i$ .

Link	$\alpha_i$	$a_i$	$\theta_i$	$d_i$
1	0	0	$\theta_1$	$d_1$
2	0	0	0	$d_2$
3	0	0	0	$d_3$

Which gives us the following  $A_i$  matrices.

$$A_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we can compute our  $T$  matrices using the following MATLAB code,

```

1  % Calculation code for problem 4.15 (T matrices) of the RBE500 textbook (HW 4)
2
3  clear; close all; clc;
4
5  syms theta1 d1 d2 d3;
6
7  % Form the A matrices
8  A1 = [cos(theta1) -sin(theta1) 0 0; sin(theta1) cos(theta1) 0 0; 0 0 1 d1; 0 0 0 1];
9  A2 = [1 0 0 0; 0 1 0 0; 0 0 1 d2; 0 0 0 1];
10 A3 = [1 0 0 0; 0 1 0 0; 0 0 1 d3; 0 0 0 1];
11
12 % Compute T matrices
13 T2 = A1*A2;
14 T3 = A1*A2*A3;
15
16 % Output to LaTeX
17 latex(T2)
18 latex(T3)

```

$$T_1^0 = A_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 + d_2 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From these  $T$  matrices, we get

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, o_3 = \begin{bmatrix} 0 \\ 0 \\ d_1 + d_2 + d_3 \end{bmatrix}$$

Now we are ready to compute our Jacobian matrix. We do this using the following MATLAB code.

```
1 % Calculation code for problem 4.15 (Jacobian) of the RBE500 textbook (HW 4)
2
3 clear; close all; clc;
4
5 syms d1 d2 d3;
6
7 % Declare vectors
8 z0 = [0; 0; 1];
9 z1 = z0;
10 z2 = z0;
11 o0 = [0; 0; 0];
12 o3 = [0; 0; d1 + d2 + d3];
13
14 % Compute Jacobian
15 J = [cross(z0, (o3 - o0)) z1 z2; z0 zeros(3,1) zeros(3,1)]
16
17 % Output LaTeX
18 latex(J)
```

Which gives us the following Jacobian,

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$