RBE 500 Homework #2

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Problem 3.5

Consider the three-link articulated robot of Figure 3.16. Derive the forward kinematic equations using the DH convention.

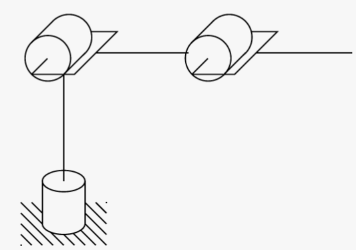
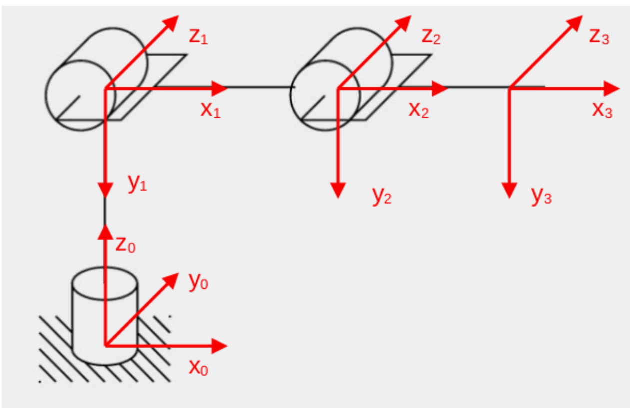


Figure 3.16: Three-link articulated robot.

Solution

First we assign coordinate frames 0 through 3 (links 0 through 3). This is done as per the following figure.



Now, we create a table for quantities $\alpha_i, a_i, \theta_i, d_i$ for links 1 through 3.

Link	α_i	a_i	θ_i	d_i
1	-90°	0	θ_1	d_1
2	0	a_2	θ_2	0
3	0	a_3	θ_3	0

Next, we use the matrix obtained from equation 3.10 of the textbook to calculate A_1, A_2, A_3.

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos(-90°) & \sin\theta_1\sin(-90°) & 0\cdot\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos(-90°) & -\cos\theta_1\sin(-90°) & 0\cdot\sin\theta_1 \\ 0 & \sin(-90°) & \cos(-90°) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where $s_1 = \sin\theta_1$ and $c_1 = \cos\theta_1$. Similarly,

Problem 3.5 continued on next page. . .

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we can find $T_3^0 = A_1A_2A_3$. We use the following MATLAB code to compute this.

```
% Calculation code for problem 3.5 of the RBE500 textbook (HW 2)

clear; close all; clc;

syms c1 s1 d1 c2 s2 a2 c3 s3 a3;
A1 = [c1 0 -s1 d1; s1 0 c1 0; 0 -1 0 d1; 0 -1 0 d1; 0 0 0 1];
A2 = [c2 -s2 0 a2*c2; s2 c2 0 a2*s2; 0 0 1 0; 0 0 0 1];
A3 = [c3 -s3 0 a3*c3; s3 c3 0 a3*s3; 0 0 1 0; 0 0 0 1];

T = A1*A2*A3;

% Generate LaTex code
latex(T)
```

Therefore,

$$T_3^0 = \begin{bmatrix} c_1\,c_2\,c_3 - c_1\,s_2\,s_3 & -c_1\,c_2\,s_3 - c_1\,c_3\,s_2 & -s_1 & d_1 + a_2\,c_1\,c_2 - a_3\,c_1\,s_2\,s_3 + a_3\,c_1\,c_2\,c_3 \\ c_2\,c_3\,s_1 - s_1\,s_2\,s_3 & -c_2\,s_1\,s_3 - c_3\,s_1\,s_2 & c_1 & a_2\,c_2\,s_1 - a_3\,s_1\,s_2\,s_3 + a_3\,c_2\,c_3\,s_1 \\ -c_2\,s_3 - c_3\,s_2 & s_2\,s_3 - c_2\,c_3 & 0 & d_1 - a_2\,s_2 - a_3\,c_2\,s_3 - a_3\,c_3\,s_2 \\ -c_2\,s_3 - c_3\,s_2 & s_2\,s_3 - c_2\,c_3 & 0 & d_1 - a_2\,s_2 - a_3\,c_2\,s_3 - a_3\,c_3\,s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This gives the configuration of frame 3 with respect to the base frame (frame 0).

Problem 3.6

Consider the three-link Cartesian manipulator of Figure 3.17. Derive the forward kinematic equations using the DH convention.

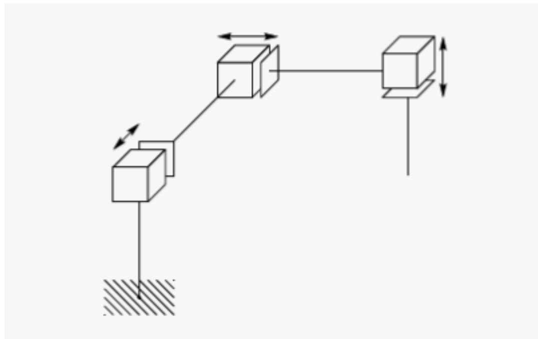
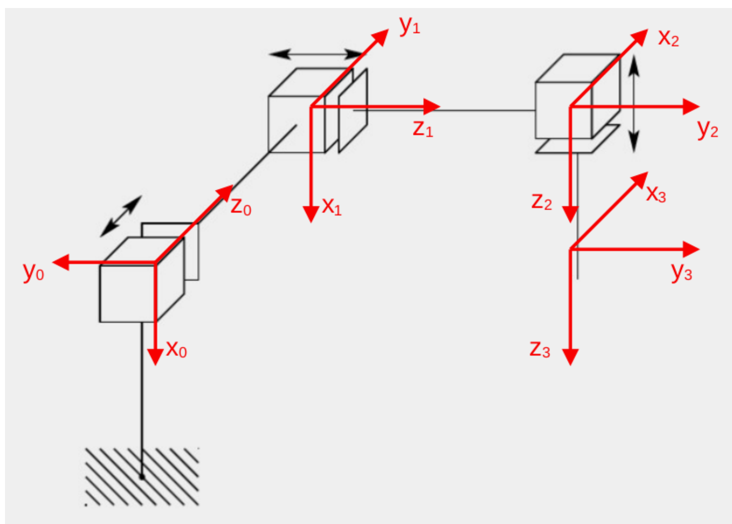


Figure 3.17: Three-link Cartesian robot.

Solution

First we assign coordinate frames 0 through 3 (links 0 through 3). This is done as per the following figure.



Now, we create a table for quantities $\alpha_i, a_i, \theta_i, d_i$ for links 1 through 3.

Link	α_i	a_i	θ_i	d_i
1	$90°$	0	0	d_1
2	$90°$	0	$90°$	d_2
3	0	0	0	d_3

Next, we use the matrix obtained from equation 3.10 of the textbook to calculate A_1, A_2, A_3.

Problem 3.6 continued on next page. . .

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we can find $T_3^0 = A_1 A_2 A_3$. We use the following MATLAB code to compute this.

```
% Calculation code for problem 3.6 of the RBE500 textbook (HW 2)

clear; close all; clc;

syms d1 d2 d3;
A1 = [1 0 0 0; 0 0 -1 0; 0 1 0 d1; 0 0 0 1];
A2 = [0 0 1 0; 1 0 0 0; 0 1 0 d2; 0 0 0 1];
A3 = [1 0 0 0; 0 1 0 0; 0 0 1 d3; 0 0 0 1];

T = A1*A2*A3;

% Generate LaTex code
latex(T)
```

Therefore,

$$T_3^0 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & -d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This gives the configuration of frame 3 with respect to the base frame (frame 0).

Problem 5.3

Solve the inverse position kinematics for the cylindrical manipulator of Figure 5.15.

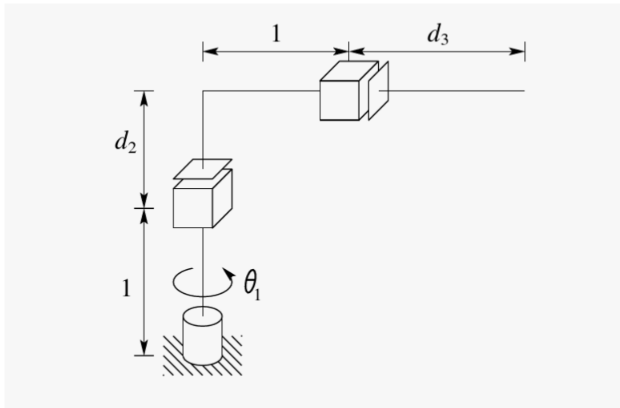
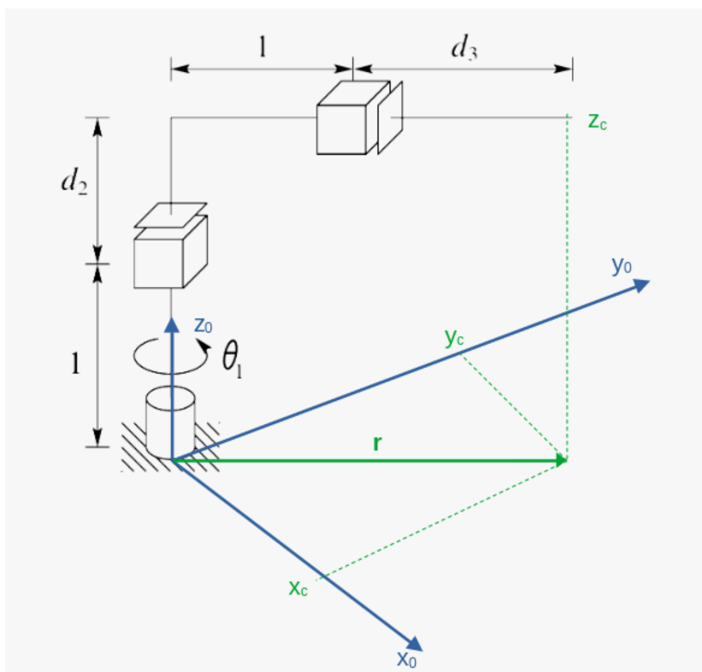


Figure 5.15: Cylindrical configuration.

Solution

Let us draw the base frame's axes $x_0y_0z_0$ as shown in the figure below. Also, let us select a point (x_c, y_c, z_c) as the wrist center at the far end of the second prismatic joint, as shown.



To solve the inverse position kinematics problem for this configuration, we need to find q_1, q_2, q_3, or more precisely, θ_1, d_2, d_3.

Using the $Atan2()$ algorithmic function as described in the appendix of the textbook, we determine from the figure that,

$$\boxed{\theta_1 = Atan2(x_c, y_c)}$$

or, alternatively,

$$\boxed{\theta_1 = \pi + Atan2(x_c, y_c)}$$

Furthermore, it is apparent that

$$z_c = 1 + d_2$$
$$\boxed{d_2 = z_c - 1}$$

We also see from the figure that

$$r = \sqrt{{x_c}^2 + {y_c}^2}$$

But,

$$r = 1 + d_3$$

So,

$$\boxed{d_3 = \sqrt{{x_c}^2 + {y_c}^2} - 1}$$

This solves the inverse position kinematics problem for the given cylindrical configuration.

Problem 5.5

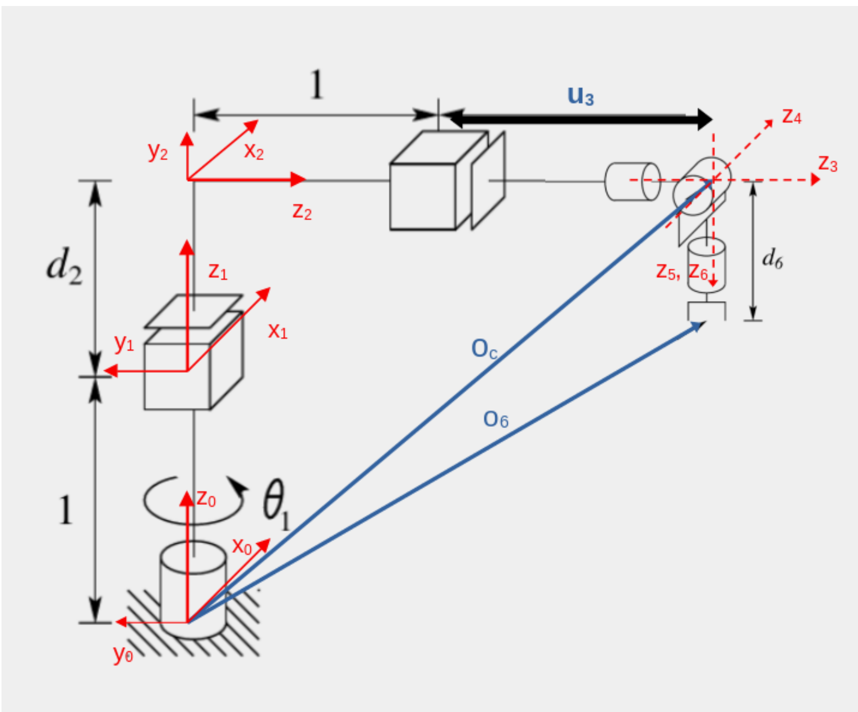
Add a spherical wrist to the three-link cylindrical arm of Problem 5–3 and write the complete inverse kinematics solution.

Solution

Let us consider a spherical wrist identical to the one used in the textbook. We attach this spherical wrist such that the wrist center, now denoted by vector o_c, coincides with the point (x_c, y_c, z_c) as we found in Problem 5–3. We have concluded that the wrist center lies at this point because axes z_3, z_4, z_5 intersect at this point. This point is also where the origins o_3, o_4, o_5 lie as per the frame assignment by DH conventions. We also know that the position of o_c does not change despite $\theta_4, \theta_5, \theta_6$ being variables.

Also, for the sake of clearly denoting d_3 as joint variable q_3, we have now used u_3 in the figure. It is still the same distance found in Problem 5–3, i.e. $u_3 = \sqrt{{x_c}^2 + {y_c}^2} - 1$. Given our placement of frame 2, we now have $d_3 = u_3 + 1$. Therefore, $q_3 = d_3 = \sqrt{{x_c}^2 + {y_c}^2}$.

An additional thing to note is that although z_6 is along the same direction as z_5, the coordinates of o_6 lie on the point shown by the vector o_6.



Before we proceed further, let us make a brief list of steps we need to take to solve the complete inverse kinematics problem for our particular manipulator's configuration.

1. Find wrist center o_c.
2. Find $q1, q2, q3$.
3. Perform forward kinematics to arrive at $R_3^0 = (R_0^3)^T$.
4. Get $R_6^3 = R_0^3 R_6^0$.

Problem 5.5 continued on next page...

5. Use R_6^3 to find ϕ, θ, ψ of Euler configuration to find q_4, q_5, q_6.

In essence, once we have found all joint variables given the end-effector's homogeneous transformation, we have solved the inverse kinematics problem.

Step 1

The end-effector's homogeneous transformation is known to us as the 4×4 matrix

$$H_6^0 = \begin{bmatrix} R_6^0 & o_6^0 \\ 0 & 1 \end{bmatrix}$$

where

$$R_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, o_6^0 = \begin{bmatrix} x_6 \\ y_6 \\ z_6 \end{bmatrix}$$

Where o_6^0 is o_6 as shown in the diagram. As shown in the figure, we can establish a relationship between o_6 and o_c as

$$o_c = o_6 - d_6 R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} x_6 - d_6 r_{13} \\ y_6 - d_6 r_{23} \\ z_6 - d_6 r_{33} \end{bmatrix}$$

Where d_6 is a scalar.

Step 2

We have already found q_1, q_2, q_3 in Problem 5.3. We summarize our findings here,

$$q_1 = \theta_1 = Atan2(x_c, y_c)$$
$$q_2 = d_2 = z_c - 1$$
$$q_3 = d_3 = \sqrt{{x_c}^2 + {y_c}^2}$$

We have discarded the second possibility of q_1 as our choice.

Step 3

We perform forward kinematics for the first three joint variables. Here is our table,

Link	α_i	a_i	θ_i	d_i
1	0	0	θ_1	1
2	90°	0	0	d_2
3	0	0	0	d_3

Next, we use the matrix obtained from equation 3.10 of the textbook to calculate A_1, A_2, A_3.

$$A_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We obtain T_3^0 using the following MATLAB code.

```
% Calculation code for the forward kinematics portion of problem 5.5
% of the RBE500 textbook (HW 2)

clear; close all; clc;

syms c1 s1 d2 d3;
A1 = [c1 -s1 0 0; s1 c1 0 0; 0 0 1 1; 0 0 0 1];
A2 = [1 0 0 0; 0 0 -1 0; 0 1 0 d2; 0 0 0 1];
A3 = [1 0 0 0; 0 1 0 0; 0 0 1 d3; 0 0 0 1];

T = A1*A2*A3;

% Generate LaTex code
latex(T)
```

Therefore,

$$T_3^0 = \begin{bmatrix} c_1 & 0 & s_1 & d_3\, s_1 \\ s_1 & 0 & -c_1 & -c_1\, d_3 \\ 0 & 1 & 0 & d_2 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From here, it is clear that

$$R_3^0 = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

Step 4

We know that $R_0^3 = (R_3^0)^T$. Given this fact, we use the following MATLAB code to calculate R_6^3.

```
% Calculation code for step 4 of problem 5.5
% of the RBE500 textbook (HW 2)

clear; close all; clc;

syms c1 s1 r11 r12 r13 r21 r22 r23 r31 r32 r33;

R30 = [c1 s1 0; 0 0 1; s1 -c1 0];
R06 = [r11 r12 r13; r21 r22 r23; r31 r32 r33];

R36 = R30*R06;

latex(R36)
```

$$R_6^3 = \begin{bmatrix} c_1\, r_{11} + r_{21}\, s_1 & c_1\, r_{12} + r_{22}\, s_1 & c_1\, r_{13} + r_{23}\, s_1 \\ r_{31} & r_{32} & r_{33} \\ r_{11}\, s_1 - c_1\, r_{21} & r_{12}\, s_1 - c_1\, r_{22} & r_{13}\, s_1 - c_1\, r_{23} \end{bmatrix}$$

Step 5

For the final step, we make use of the Euler angles matrix, where $q_4 = \phi, q_5 = \theta, q_6 = \psi$. The matrix for this, as given in the textbook, is

Problem 5.5 continued on next page. . .

$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

Now, if we equate this with our matrix from Step 4, we get

$$\begin{aligned} c_4s_5 &= c_1\, r_{13} + r_{23}\, s_1 \\ s_4s_5 &= r_{33} \\ c_5 &= r_{13}\, s_1 - c_1\, r_{23} \end{aligned}$$

So, finally,

$$\begin{aligned} \theta_4 &= \\ \theta_5 &= \\ \theta_6 &= \end{aligned}$$

Report for ROS2 Portion

For our ROS part of this assignment, we created a subscriber just like how we did in the last homework assignment. We did not create a publisher, however instead we tested the subscriber's conversion of data by using the "ros2 topic pub ..." command. A sample of this has been submitted under publish.sh.
The subscriber listens for a Float32MultiArray, and when it is received, it checks if the array length is 3. If it is not 3, then the subscriber notifies that no action will be taken. When the array length is exactly 3, the subscriber passes the array to a helper function to convert the incoming euler angle data to quaternions. The euler angle order is assumed to be given as [yaw, pitch, roll]. We use the formula given on the Wikipedia page to convert the Euler angles to quaternions. The converted data is then printed to the terminal. A screenshot of this output has been submitted as well.