# RBE 500 Homework #3

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# Problem 4.2

Verify Equation (4.7) by direct calculation.

$$S(a)p = a \times p \tag{4.7}$$

**Solution**
Suppose the vectors $a$ and $p$ are given as

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

By the definition of the cross-product, we know

$$a \times p = \begin{bmatrix} a_2p_3 - a_3p_2 \\ a_3p_1 - a_1p_3 \\ a_1p_2 - a_2p_1 \end{bmatrix} \tag{1}$$

Also, by the definition of skew-symmetric matrices, we know the form of $S(a)$, where $a$ is the vector we have already defined,

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Hence, by normal matrix multiplication,

$$S(a)p = \begin{bmatrix} 0 - a_3p_2 + a_2p_3 \\ a_3p_1 + 0 - a_1p_3 \\ -a_2p_1 + a_1p_2 + 0 \end{bmatrix} = \begin{bmatrix} a_2p_3 - a_3p_2 \\ a_3p_1 - a_1p_3 \\ a_1p_2 - a_2p_1 \end{bmatrix}$$

Which is the same as (1). Therefore, Equation (4.7) is proved.