RBE 500 Midterm

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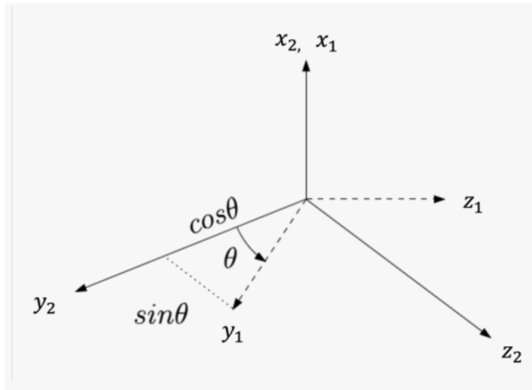
Problem 1

Derive the rotation matrix R_2^1 (you can leave sines and cosines as is).



Solution

Since the x-axis remains the same in the rotation, we know this is a basic 3D rotation matrix representing a rotation about the x-axis. However, using the right-hand screw rule, we see that the angle θ here is negative. So, by using equation 2.7 (page 43) of our main textbook, the rotation matrix is given by,

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) \\ 0 & \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\boxed{R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}}$$

Problem 2

Find the coordinates of point p expressed in frame 1 (i.e. p^1) given the following.

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0.9553 & 0.2955 & -0.9553 \\ 0 & -0.2955 & 0.9553 & 0.2955 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ p^2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

Solution

From our knowledge of homogeneous transformations, we know that

$$P^2 = H_1^2 P^1$$

Where $P^2 = \begin{bmatrix} p^2 \\ 1 \end{bmatrix}$ and $P^1 = \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$.

However, we want to find P^1, so we apply the inverse of H to both sides,

$$P^2 = H_1^2 P^1$$
$$(H_1^2)^{-1} P^2 = P^1$$

We know that $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$, where $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9553 & 0.2955 \\ 0 & -0.2955 & 0.9553 \end{bmatrix}$ and $d_1^2 = \begin{bmatrix} -1 \\ -0.9553 \\ 0.2955 \end{bmatrix}$.

For accuracy while computing the inverse of H_1^2, we use equation 2.67 of the book (page 63). Therefore,

$$(H_1^2)^{-1} = \begin{bmatrix} (R_1^2)^T & -(R_1^2)^T d_1^2 \\ 0 & 1 \end{bmatrix}$$

. We use the following MATLAB code for this computation.

```
% Calculation code for problem 2 of the RBE500 Midterm

clear; close all; clc;

P2 = [2;5;0;1];

R2_1 = [1 0 0; 0 0.9553 0.2955; 0 -0.2955 0.9553];
d2_1 = [-1; -0.9553; 0.2955];
H_inv = [R2_1' (-R2_1'*d2_1); zeros(1,3) 1];

P1 = H_inv*P2
```

Which gives us the answer,

$$\boxed{P^1 = \begin{bmatrix} 3.0000 \\ 5.7764 \\ 1.4775 \\ 1.0000 \end{bmatrix}}$$

Problem 3

If $R_1^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$, $R_2^0 = \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}$, and $R_3^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, calculate R_2^1.

Solution

Knowing the composition law for rotational transformations, we can write

$$
\begin{aligned}
R_2^1 &= R_0^1 R_2^0 \\
R_2^1 &= (R_1^0)^T R_2^0 \\
R_2^1 &= \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}^T \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix} \\
R_2^1 &= \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & 1 & 0 \\ 0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

We use the following MATLAB script to compute this multiplication.

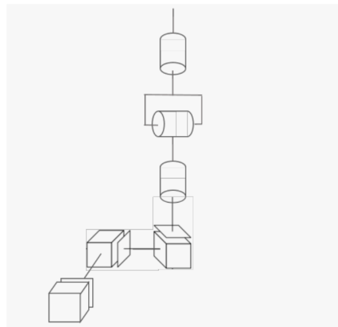
```
% Calculation code for problem 2 of the RBE500 Midterm

clear; close all; clc;

R01 = [0.7071 0 0.7071; 0 1 0; -0.7071 0 0.7071];
R02 = [0 0.866 0.5; 0 0.5 -0.866; -1 0 0];

R12 = R01'*R02
```

Therefore,

$$
\boxed{R_2^1 = \begin{bmatrix} 0.7071 & 0.6123 & 0.3535 \\ 0 & 0.5000 & -0.8660 \\ -0.7071 & 0.6123 & 0.3535 \end{bmatrix}}
$$

Problem 4

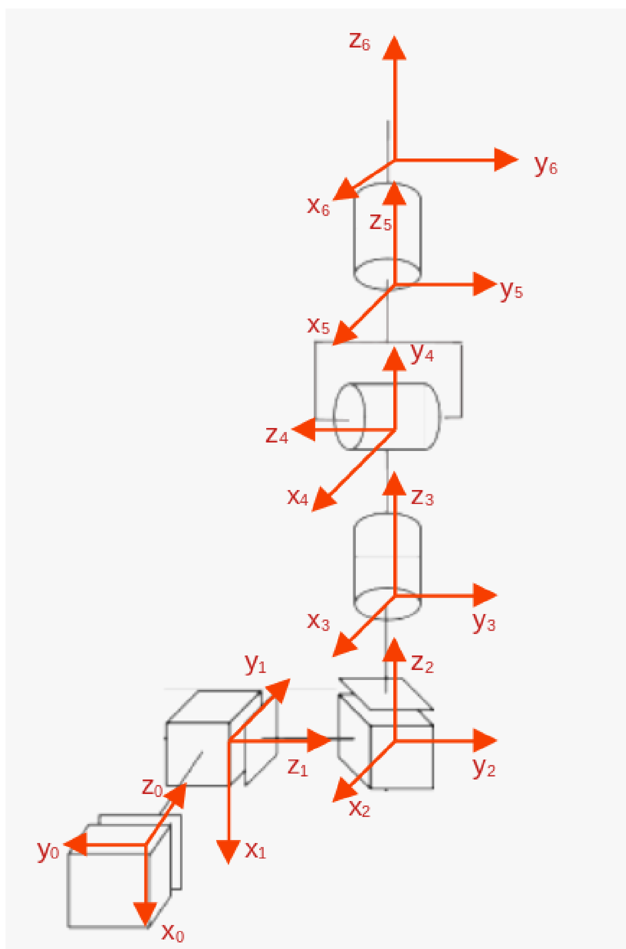
(a) Calculate Denavit Hertanberg parameters for the given manipulator (just filling out the Denavit Hertanberg table would suffice). For this question, you are expected to solve it parametrically, i.e. you can leave sines, cosines, joint values, and link lengths as parameters.



(b) Derive H_2^1. You can leave sines, cosines, joint values, and link lengths as parameters.

Solution for part (a)

First we assign coordinate frames 0 through 5 (links 0 through 5). This is done as per the following figure.



Now, we create a table for quantities $\alpha_i, a_i, \theta_i, d_i$ for links 1 through 6. In this table, d_1, d_2, d_3, θ_4, θ_5, θ_6 are variable. However, l_4, l_5, l_6 are fixed (constants).

Link	α_i	a_i	θ_i	d_i
1	$90°$	0	0	d_1
2	$90°$	0	$-90°$	d_2
3	0	0	0	d_3
4	$90°$	0	θ_4	l_4
5	$-90°$	0	θ_5	l_5
6	0	0	θ_6	l_6

Solution for part (b)

We know that $H_2^1 = A_2$, where A_2 is the DH matrix A_i with $i = 2$.

$$H_2^1 = A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\alpha_2 & \sin\theta_2\sin\alpha_2 & a_i\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos\alpha_2 & -\cos\theta_2\sin\alpha_2 & a_i\sin\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-90°) & -\sin(-90°)\cos(90°) & \sin(-90°)\sin(90°) & a_i\cos(-90°) \\ \sin(-90°) & \cos(-90°)\cos(90°) & -\cos(-90°)\sin(90°) & a_i\sin(-90°) \\ 0 & \sin(90°) & \cos(90°) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{H_2^1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$