RBE 500 Homework #4

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Problem 4.6

Given $R = R_{x,\theta} R_{y,\phi}$, compute $\frac{\partial R}{\partial \phi}$. Evaluate $\frac{\partial R}{\partial \phi}$ at $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$. First parametrically compute, then evaluate by plugging the values in.

Solution

$$\frac{\partial}{\partial \phi}\left(R_{x,\theta} R_{y,\phi}\right) = R_{x,\theta} \frac{\partial}{\partial \phi}\left(R_{y,\phi}\right)$$

Using the the fact that $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(R_{y,\theta}\right) = S(j) R_{y,\theta}$,

$$\begin{aligned}
R_{x,\theta} \frac{\partial}{\partial \phi}\left(R_{y,\phi}\right) = R_{x,\theta} S(j) R_{y,\phi} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \\
&= \begin{bmatrix} -\sin(\phi) & 0 & \cos(\phi) \\ \cos(\phi)\sin(\theta) & 0 & \sin(\phi)\sin(\theta) \\ -\cos(\phi)\cos(\theta) & 0 & -\cos(\theta)\sin(\phi) \end{bmatrix}
\end{aligned}$$

Now, plugging in the values $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$, we get

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

We performed the above computations using the following MATLAB code.

```
% Calculation code for problem 4.6 of the RBE500 textbook (HW 4)

clear; close all; clc;

syms theta phi

% Define the matrices in our problem
rotx_theta = [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
Sj = [0 0 1; 0 0 0; -1 0 0];
roty_phi = [cos(phi) 0 sin(phi); 0 1 0; -sin(phi) 0 cos(phi)];

% Multiply the matrices
product = rotx_theta*Sj*roty_phi;

% Get latex output
latex(product)

phi_val = pi/2;
theta_val = pi/2;

% Now plug in values
product_val = [ -sin(phi_val), 0, cos(phi_val);
cos(phi_val)*sin(theta_val), 0, sin(phi_val)*sin(theta_val);
-cos(phi_val)*cos(theta_val), 0, -cos(theta_val)*sin(phi_val)]
```

Problem 4.10

Two frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogeneous transformation

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_1(t) = (3, 1, 0)$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

Solution

The given H is the homogeneous transformation H_1^0. Therefore, $R_1^0 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $o_1^0 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

At any given point in time, the position of the particle with respect to frame $o_1x_1y_1z_1$ is given as $p^1(t)$. We know that

$$p^0(t) = R_1^0 p^1(t) + o_1^0$$

Taking the derivative of both sides and using the product rule, we get

$$v^0(t) = \dot{R}_1^0 p^1(t) + R_1^0 v^1(t) + 0$$

But, $\dot{R}_1^0 = 0$ since R_1^0 is a constant in time. Therefore,

$$v^0(t) = R_1^0 v^1(t)$$

$$v^0(t) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

Problem 4.15

Find the 6×3 Jacobian for the three links of the cylindrical manipulator of Figure 3.7. Find the singular configurations for this arm.