RBE 500 Homework #5

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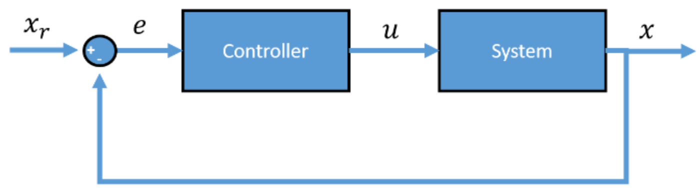
Problem 1

Consider the following block diagram:



The dynamics of the system is given in the following differential equation

$$m\ddot{x} + b\dot{x} = u$$

The controller is designed as

$$k_p e + k_d \dot{e} = u$$

Convert the system model and the controller to the Laplace domain.

Solution

For the system model, take the Laplacian on both sides,

$$\begin{aligned} \mathcal{L}\{m\ddot{x} + b\dot{x}\} &= \mathcal{L}\{u\} \\ m\mathcal{L}\{\ddot{x}\} + b\mathcal{L}\{\dot{x}\} &= U(s) \\ \boxed{ms^2X(s) + bsX(s) = U(s)} \end{aligned}$$

Similarly, take the Laplacian on both sides of the controller model,

$$\begin{aligned} \mathcal{L}\{k_p e + k_d \dot{e}\} &= \mathcal{L}\{u\} \\ k_p\mathcal{L}\{e\} + k_d\mathcal{L}\{\dot{e}\} &= U(s) \\ \boxed{k_pE(s) + k_dsE(s) = U(s)} \end{aligned}$$

Problem 2

Find the transfer functions for $\frac{U(s)}{E(s)}$, $\frac{X(s)}{U(s)}$, and $\frac{X(s)}{E(s)}$.

Solution

Re-arranging the Laplace-domain controller model from Problem 1,

$$E(s)\,(k_p + k_d s) = U(s) \tag{1}$$

$$\boxed{\frac{U(s)}{E(s)} = (k_p + k_d s)}$$

Re-arranging the Laplace-domain system model from Problem 1,

$$X(s)\left(ms^2 + bs\right) = U(s) \tag{2}$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{1}{(ms^2 + bs)}}$$

Equating (1) and (2) as we have found above, we get

$$E(s)\,(k_p + k_d s) = X(s)\left(ms^2 + bs\right)$$
$$\frac{(k_p + k_d s)}{(ms^2 + bs)} = \frac{X(s)}{E(s)}$$

$$\boxed{\frac{X(s)}{E(s)} = \frac{(k_p + k_d s)}{(ms^2 + bs)}}$$