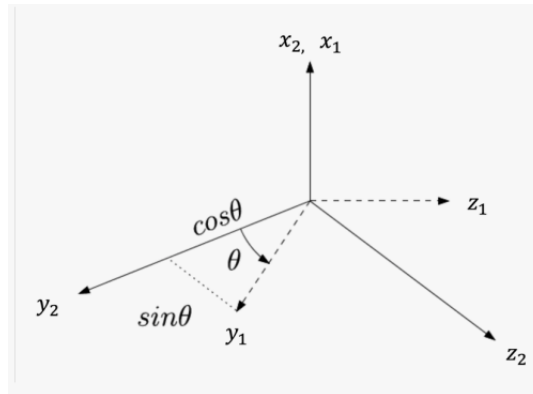


# **RBE 500 Midterm**

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## Question 1

Derive the rotation matrix  $R_2^1$  (you can leave sines and cosines as is).



## Solution

Since the x-axis remains the same in the rotation, we know this is a basic 3D rotation matrix representing a rotation about the x-axis. However, using the right-hand screw rule, we see that the angle  $\theta$  here is negative. So, by using equation 2.7 (page 43) of our main textbook, the rotation matrix is given by,

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) \\ 0 & \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

## Question 2

Find the coordinates of point  $p$  expressed in frame 1 (i.e.  $p^1$ ) given the following.

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0.9553 & 0.2955 & -0.9553 \\ 0 & -0.2955 & 0.9553 & 0.2955 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad p^2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

### Solution

From our knowledge of homogeneous transformations, we know that

$$P^2 = H_1^2 P^1$$

Where  $P^2 = \begin{bmatrix} p^2 \\ 1 \end{bmatrix}$  and  $P^1 = \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$ .

However, we want to find  $P^1$ , so we apply the inverse of  $H$  to both sides,

$$P^2 = H_1^2 P^1 \\ (H_1^2)^{-1} P^2 = P^1$$

We know that  $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$ , where  $R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9553 & 0.2955 \\ 0 & -0.2955 & 0.9553 \end{bmatrix}$  and  $d_1^2 = \begin{bmatrix} -1 \\ -0.9553 \\ 0.2955 \end{bmatrix}$ .

For accuracy while computing the inverse of  $H_1^2$ , we use equation 2.67 of the book (page 63). Therefore,

$$(H_1^2)^{-1} = \begin{bmatrix} (R_1^2)^T & -(R_1^2)^T d_1^2 \\ 0 & 1 \end{bmatrix}$$

. We use the following MATLAB code for this computation.

```
1 % Calculation code for problem 2 of the RBE500 Midterm
2
3 clear; close all; clc;
4
5 P2 = [2;5;0;1];
6
7 R2_1 = [1 0 0; 0 0.9553 0.2955; 0 -0.2955 0.9553];
8 d2_1 = [-1; -0.9553; 0.2955];
9 H_inv = [R2_1' (-R2_1'*d2_1); zeros(1,3) 1];
10
11 P1 = H_inv*P2
```

Which gives us the answer,

$$P^1 = \begin{bmatrix} 3.0000 \\ 5.7764 \\ 1.4775 \\ 1.0000 \end{bmatrix}$$

### Question 3

If  $R_1^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$ ,  $R_2^0 = \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}$ , and  $R_3^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$ , calculate  $R_2^1$ .

### Solution

Knowing the composition law for rotational transformations, we can write

$$\begin{aligned}
 R_2^1 &= R_0^1 R_2^0 \\
 R_2^1 &= (R_1^0)^T R_2^0 \\
 R_2^1 &= \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}^T \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix} \\
 R_2^1 &= \begin{bmatrix} 0.7071 & 0 & -0.7071 \\ 0 & 1 & 0 \\ 0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

We use the following MATLAB script to compute this multiplication.

```

1 % Calculation code for problem 2 of the RBE500 Midterm
2
3 clear; close all; clc;
4
5 R01 = [0.7071 0 0.7071; 0 1 0; -0.7071 0 0.7071];
6 R02 = [0 0.866 0.5; 0 0.5 -0.866; -1 0 0];
7
8 R12 = R01'*R02

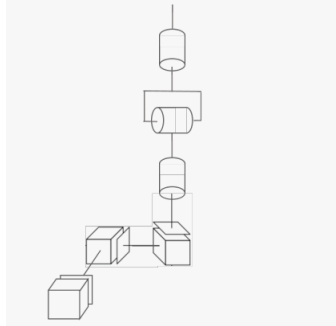
```

Therefore,

$$R_2^1 = \begin{bmatrix} 0.7071 & 0.6123 & 0.3535 \\ 0 & 0.5000 & -0.8660 \\ -0.7071 & 0.6123 & 0.3535 \end{bmatrix}$$

## Question 4

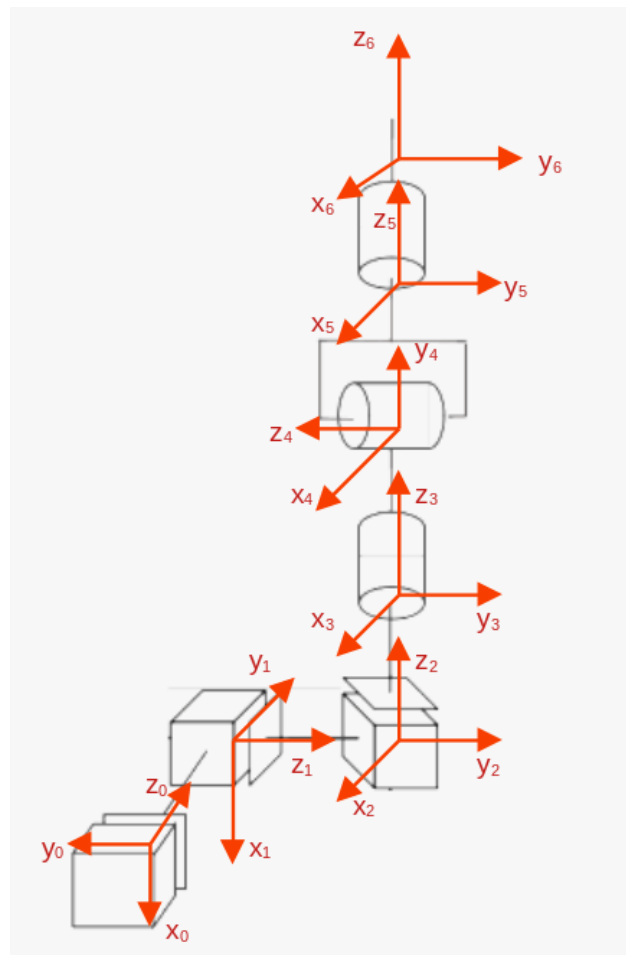
(a) Calculate Denavit Hertenberg parameters for the given manipulator (just filling out the Denavit Hertenberg table would suffice). For this question, you are expected to solve it parametrically, i.e. you can leave sines, cosines, joint values, and link lengths as parameters.



(b) Derive  $H_2^1$ . You can leave sines, cosines, joint values, and link lengths as parameters.

### Solution for 4(a)

First we assign coordinate frames 0 through 5 (links 0 through 5). This is done as per the following figure.



Now, we create a table for quantities  $\alpha_i, a_i, \theta_i, d_i$  for links 1 through 6. In this table,  $d_1, d_2, d_3, \theta_4, \theta_5, \theta_6$  are variable. However,  $l_4, l_5, l_6$  are fixed (constants).

Link	$\alpha_i$	$a_i$	$\theta_i$	$d_i$
1	$90^\circ$	0	0	$d_1$
2	$90^\circ$	0	$-90^\circ$	$d_2$
3	0	0	0	$d_3$
4	$90^\circ$	0	$\theta_4$	$l_4$
5	$-90^\circ$	0	$\theta_5$	$l_5$
6	0	0	$\theta_6$	$l_6$

### Solution for 4(b)

We know that  $H_2^1 = A_2$ , where  $A_2$  is the DH matrix  $A_i$  with  $i = 2$ .

$$\begin{aligned}
 H_2^1 = A_2 &= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \cos \alpha_2 & \sin \theta_2 \sin \alpha_2 & a_i \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 \cos \alpha_2 & -\cos \theta_2 \sin \alpha_2 & a_i \sin \theta_2 \\ 0 & \sin \alpha_2 & \cos \alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) \cos(90^\circ) & \sin(-90^\circ) \sin(90^\circ) & a_i \cos(-90^\circ) \\ \sin(-90^\circ) & \cos(-90^\circ) \cos(90^\circ) & -\cos(-90^\circ) \sin(90^\circ) & a_i \sin(-90^\circ) \\ 0 & \sin(90^\circ) & \cos(90^\circ) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \boxed{\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
 \end{aligned}$$

## Question 5

Calculate inverse kinematics for the manipulator in Question 4. Assume that all the forward kinematics information is available (i.e. all homogenous transformation matrices). Since there are no values given, you will be deriving your expressions parametrically, but please be sure to explicitly show, which homogeneous transformation matrix is required for the corresponding information, and which segment of the matrix is used to obtain that information. (e.g. in your derivations you can say something like: “to calculate this expression, I would need  $z_3^0$ , which is available to me at the  $3^{rd}$  column of  $H_3^0$ ”).