RBE 501 Week 1 Assignment

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Abstract—This document provides an in-depth solution for Problem 2–37 described in Robot Modeling and Control. This is the assignment for the first week in RBE 501 (Robot Dynamics), Spring 2023 at Worcester Polytechnic Institute.

Index Terms—robotics, homogeneous transformation, frames

I. INTRODUCTION

We are asked to solve Problem 2–37 of the main textbook [1]. Here, a robot is near a table, with a camera placed over the table to supervise the robot. A cube is also placed in the center of the table-top. The objective of the problem is to express all the frames in the environment (frame for the edge of the table, frame for the cube, and frame for the overhead camera) in terms of the homogeneous transformations with respect to the base frame, situated at the base link of the robot. An additional objective of the problem is to find the homogeneous transformation relating the center of the cube's frame to the camera's frame. The graphic below, given in the textbook, gives a pictorial representation of the problem.
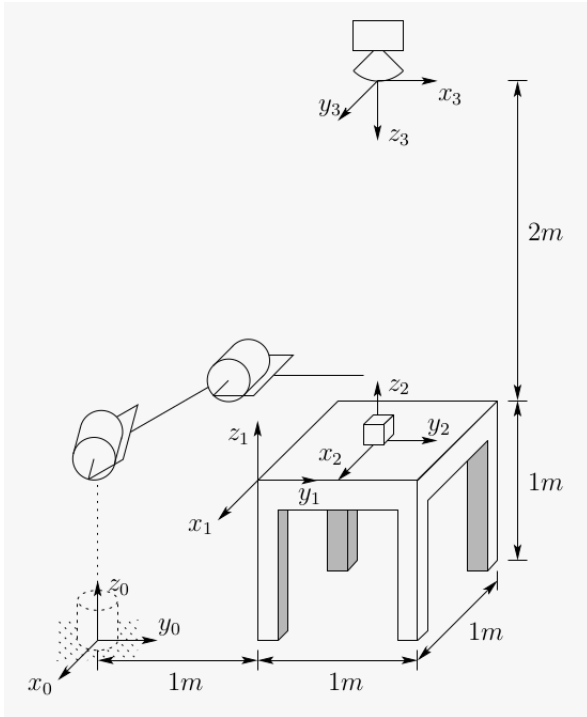

Fig. 1. Figure for Problem 2–37

II. MATERIALS AND METHODS

A. Restate objective in technical terms

We know from the textbook [1] that given a rotational matrix R, and a rigid translation d, we have the following definition of a homogeneous transformation,

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}; R \in SO(3), d \in \mathbb{R}^3$$

Also, the homogeneous transformation H_n^m describes the transformation of frame n with respect to m. Therefore, given our objectives, and the frames as numbered in Figure 1, we need to find H_1^0, H_2^0, H_3^0, and H_2^3.

B. Compute all rotation matrices

As shown in the figure, there are no rotational differences between frames 0, 1, and 2. Therefore, the corresponding rotational matrices are the 3×3 identity matrix. So,

$$R_1^0 = R_2^0 = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By visually inspecting the figure, we also determine that the rotation for frame 3 can be obtained by a $-180°$ rotation around the x-axis, and a $-90°$ rotation around the z-axis. Using our MATLAB Live Script, we can determine this as,

$$R_3^0 = R_{x,-\pi} R_{z,-\frac{\pi}{2}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Similarly, the rotation for frame 2 with respect to frame 3 can be obtained by a $180°$ rotation around the y-axis, and a $90°$ rotation around the z-axis. This can be stated as,

$$R_2^3 = R_{y,\pi} R_{z,\frac{\pi}{2}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

C. Compute all the translations

By visually inspecting the figure, it is easy to see that,

$$d_1^0 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

In order to find d_2^0 and d_3^0, we need to consider the geometric distance from frame 0 to the very center of the cube. In the XY plane, we consider the table top as a square ABCD of

side 1, as shown in Figure 2. Using the geometric properties of a square, we determine that $AE = EB = 0.5$ because $\triangle AEO \cong \triangle EOB$.
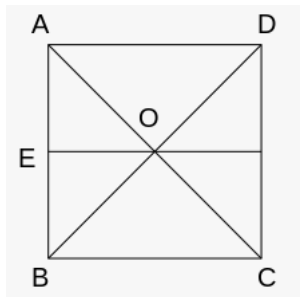

Fig. 2. Table-top square for Problem 2–37

For the Z component of the translations, we need to consider half of the height of the cube, which is 10 cm, or 0.1 m. Therefore, we can write,

$$d_2^0 = \begin{bmatrix} -0.5 \\ 1.5 \\ 1.1 \end{bmatrix}, d_3^0 = \begin{bmatrix} -0.5 \\ 1.5 \\ 2 \end{bmatrix}$$

Also, since the cube is directly under the camera, we can state,

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

REFERENCES

[1] M. W. Spong, S. Hutchinson, and M. Vidyasagar, "Robot modeling and control," 2006.