RBE 501 Week 1 Assignment

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***Abstract*—This document provides an in-depth solution for Problem 2–37 described in Robot Modeling and Control. This is the assignment for the first week in RBE 501 (Robot Dynamics), Spring 2023 at Worcester Polytechnic Institute.**

***Index Terms*—robotics, homogeneous transformation, frames**

I. Introduction

We are asked to solve Problem 2–37 of the main textbook [1]. Here, a robot is near a table, with a camera placed over the table to supervise the robot. A cube is also placed in the center of the table-top. The objective of the problem is to express all the frames in the environment (frame for the edge of the table, frame for the cube, and frame for the overhead camera) in terms of the homogeneous transformations with respect to the base frame, situated at the base link of the robot. An additional objective of the problem is to find the homogeneous transformation relating the center of the cube's frame to the camera's frame. The graphic below, given in the textbook, gives a pictorial representation of the problem.

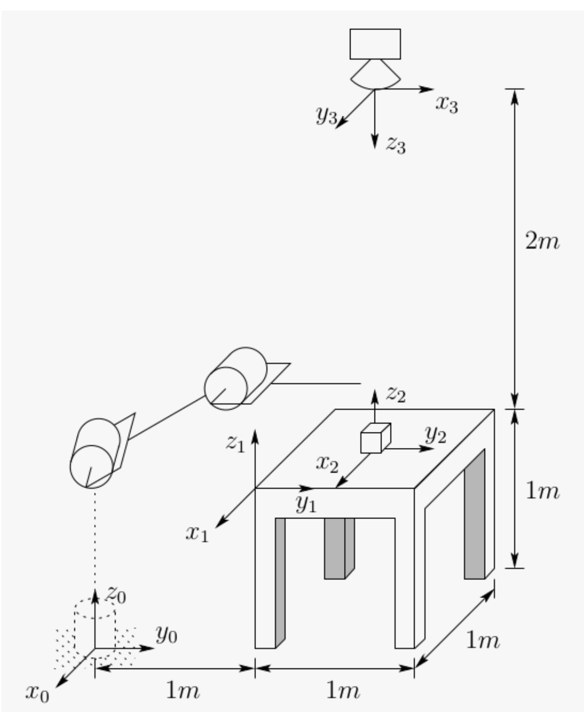

Fig. 1. Figure for Problem 2–37

II. Materials and Methods

A. Restate objective in technical terms

We know from the textbook [1] that given a rotational matrix R, and a rigid translation d, we have the following definition of a homogeneous transformation,

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}; R \in SO(3), d \in \mathbb{R}^3$$

Also, the homogeneous transformation H_n^m describes the transformation of frame n with respect to m. Therefore, given our objectives, and the frames as numbered in Figure 1, we need to find H_1^0, H_2^0, H_3^0, and H_2^3.

B. Compute all rotation matrices

As shown in the figure, there are no rotational differences between frames 0, 1, and 2. Therefore, the corresponding rotational matrices are the 3×3 identity matrix. So,

$$R_1^0 = R_2^0 = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By visually inspecting the figure, we also determine that the rotation for frame 3 can be obtained by a $-180°$ rotation around the x-axis, and a $-90°$ rotation around the z-axis. Using our MATLAB Live Script, we can determine this as,

$$R_3^0 = R_{x,-\pi} R_{z,-\frac{\pi}{2}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Similarly, the rotation for frame 2 with respect to frame 3 can be obtained by a $180°$ rotation around the y-axis, and a $90°$ rotation around the z-axis. This can be stated as,

$$R_2^3 = R_{y,\pi} R_{z,\frac{\pi}{2}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

C. Compute all the translations

By visually inspecting the figure, it is easy to see that,

$$d_1^0 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

In order to find d_2^0 and d_3^0, we need to consider the geometric distance from frame 0 to the very center of the cube. In the XY plane, we consider the table top as a square ABCD of side 1, as shown in Figure 2. Using the geometric properties of a square, we determine that $AE = EB = 0.5$ because $\triangle AEO \cong \triangle EOB$.

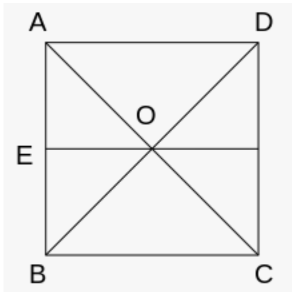

Fig. 2. Table-top square for Problem 2–37

For the Z component of the translations, we need to consider half of the height of the cube, which is 10 cm, or 0.1 m. Therefore, we can write,

$$d_2^0 = \begin{bmatrix} -0.5 \\ 1.5 \\ 1.1 \end{bmatrix}, d_3^0 = \begin{bmatrix} -0.5 \\ 1.5 \\ 3 \end{bmatrix}$$

Also, since the cube is directly under the camera, we can state,

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

III. Results

Given the fact that we have found all of the rotation matrices and rigid transformations, we can now form the final results of our objective. These are as follows,

$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} R_2^0 & d_2^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = \begin{bmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Furthermore, a MATLAB Live Script has been submitted alongside this report to show the code used to compute these results.

IV. Discussion

In the opinion of the author, this problem is very practical in real-world robotics. If a 3-DOF robotic arm is being operated (which seems to be the case in Figure 1), then one of the modern ways to monitor the arm would be to use computer vision via a camera. An overhead camera provides a good perspective in such a case, where the robot perhaps needs to perform tasks on the table-top. Some examples of table-top tasks could be moving chess pieces, manufacturing assembly, and other pick-and-place operations. In the case of this problem, the robot is doing something with a cube.

Generally, in such a setup, it is imperative to know the relative transformations between the frames of the robot, table, and the camera in order for a computer to properly command the controllers of the robot to perform accurate and precise tasks. For a simple example, if the computer wants to command the robot to go to a location on the table, first the computer needs to compute that location using H_1^3. Next, the computer needs to use H_1^0 to help the robot know about that location.

Calibration and frame transformations are very important in order for a robot to perform any real-world tasks. As for students of robotics, our spatial and geometric understanding of robotics is tested by this type of problem.

References

[1] M. W. Spong, S. Hutchinson, and M. Vidyasagar, "Robot modeling and control," 2006.