RBE 595 — FAIR-AV
Week 6 Homework #1

Arjan Gupta

Problem 1

Let's define the time to accelerate a vehicle from 0 speed to 100 km/h as τ_{100}, which is constantly used by battery-electric car (BEV) manufacturers to brag about their technological advancement. Provide the formulas to compute the minimal τ_{100} for

(a) a four-wheel drive car (with an electric motor installed at each of the axles)

(b) a front-wheel drive (with an electric motor installed at the front axle)

(c) rear-wheel drive (with an electric motor installed at the rear axle)

Solution

The minimum time to accelerate a vehicle from 0 speed to 100 km/h can be found using the definite integral of the maximum acceleration that the vehicle can achieve. This would be given by the velocity-time relationship for the vehicle, which in this case would be

$$100\,\mathrm{km/h} = \int_0^{\tau_{100}} a(t)\,dt \tag{1}$$

assuming that the vehicle starts from rest. The maximum acceleration that the vehicle can achieve is dependent on the configuration of the vehicle, as well as the road gradient.

Four-Wheel Drive

As per the lecture slides, the maximum acceleration that a vehicle can achieve in 4WD configuration is given by

$$a_{\max} = g\left(\mu\cos(\alpha) - \sin(\alpha)\right) \tag{2}$$

where g is the acceleration due to gravity, μ is the coefficient of friction, and α is the road gradient. Substituting this into the integral in equation (1), we get

$$100 = \int_0^{\tau_{100}} g\left(\mu\cos(\alpha) - \sin(\alpha)\right)\,dt \tag{3}$$

Since everything in the integrand is constant, we can pull it out of the integral to get

$$100 = g\left(\mu\cos(\alpha) - \sin(\alpha)\right)\int_0^{\tau_{100}} dt \tag{4}$$

Therefore, the minimum time to accelerate a 4WD vehicle from 0 speed to 100 km/h is

$$\tau_{100} = \frac{100}{g\left(\mu\cos(\alpha) - \sin(\alpha)\right)} \tag{5}$$

Front-Wheel Drive

The maximum acceleration that a vehicle can achieve in front-wheel drive configuration is given by

$$a_{\max} = g\left(\frac{\mu}{1+\mu\frac{h}{a_1+a_2}}\frac{a_2}{a_1+a_2}\right) \tag{6}$$

where h is the height of the center of gravity of the vehicle, a_1 is the distance from the front axle to the center of gravity, and a_2 is the distance from the rear axle to the center of gravity.

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Substituting this into the integral in equation (1), we get

$$100 = \int_0^{\tau_{100}} g \left(\frac{\mu}{1 + \mu \frac{h}{a_1 + a_2}} \frac{a_2}{a_1 + a_2} \right) dt \tag{7}$$

Since everything in the integrand is constant, we can pull it out of the integral to get

$$100 = g \left(\frac{\mu}{1 + \mu \frac{h}{a_1 + a_2}} \frac{a_2}{a_1 + a_2} \right) \int_0^{\tau_{100}} dt \tag{8}$$

Therefore, the minimum time to accelerate a FWD vehicle from 0 speed to 100 km/h is

$$\tau_{100} = \frac{100}{g \left(\frac{\mu}{1 + \mu \frac{h}{a_1 + a_2}} \frac{a_2}{a_1 + a_2} \right)} \tag{9}$$

Rear-Wheel Drive

The maximum acceleration that a vehicle can achieve in rear-wheel drive configuration is given by

$$a_{\max} = g \left(\frac{\mu}{1 - \mu \frac{h}{a_1 + a_2}} \frac{a_1}{a_1 + a_2} \right) \tag{10}$$

Following similar steps as above, we get the minimum time to accelerate a RWD vehicle from 0 speed to 100 km/h as

$$\tau_{100} = \frac{100}{g \left(\frac{\mu}{1 - \mu \frac{h}{a_1 + a_2}} \frac{a_1}{a_1 + a_2} \right)} \tag{11}$$