For the program in this analysis, we implement the build_heap and heap_sort functions to construct a max-heap and sort the items in a heap respectively. The values used in implementing these functions are random sets of integers from the input file.

For both methods, the value for n is taken to be the input size, that is, number of elements in the array. We use the same input for both methods as we are trying to analyse the two algorithms.

For the empirical analysis we measure the number of basic operations performed to construct a heap (build heap) for a given input size.

To analyze the build_heap function, we chose another function, percolate_down, on line 153 as our basic operation.

To analyze the heap_sort function, we chose the two functions, build_heap, and percolate_down in lines 108 and 115 as our basic operations.

An empirical analysis of the build_heap running the algorithm for multiple values of n produces the results shown below. Standard functions f(n) = n is also shown.

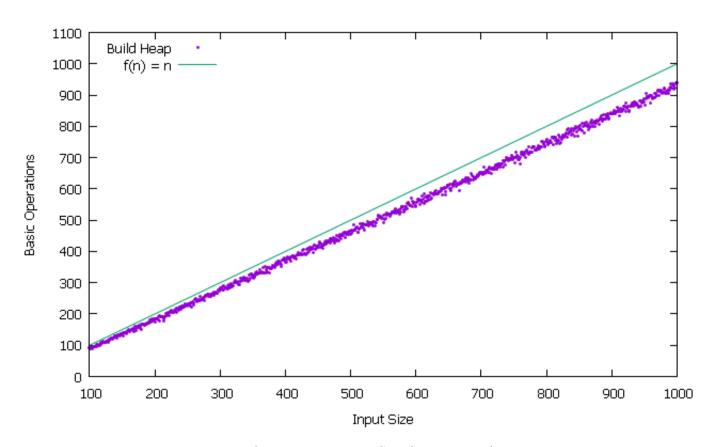


Figure: Basic Operation vs Input Size (build_heap)

An examination of the code itself explains the empirical results when we observe that the build_heap shows a linear nature when plotted for multiple values of n. Here, the standard function f(n) = n asymtotically upper bounds the running time curve for this function.

Therefore, we conclude that the build_heap function in this program is described by

$$T(n) \in O(n)$$

Secondly, an empirical analysis of the heap_sort function running the algorithm for multiple values of n produces the results shown below. Standard functions $f(n) = n \lg(n)$ is also shown.

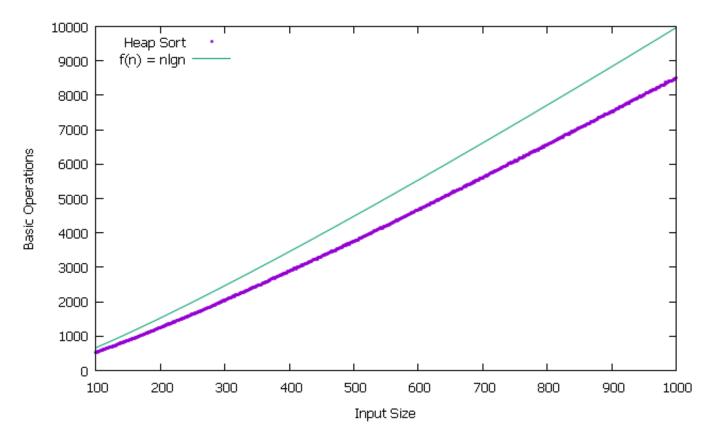


Figure: Basic Operation vs Input Size (heap_sort)

For the analysis of the heap_sort function, the above figure shows that the standard function $n \lg(n)$ upperbounds the number of basic operation as the input size increases. We can also know from the code and our build_heap analysis, that since build_heap takes O(n) time and each percolate_down takes $O(\lg n)$ the array is sorted in $O(n \lg n)$ time.

With the help of the graph above we can conclude that the heap_sort has an analysis of:

$$T(n) \in O(n \lg n)$$