Memo

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Discontinuities and differential equations

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1 Introduction

Simple model for dissolution of solid matter

2 Differential equation

Basic:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \tag{1}$$

with initial condition:

$$C(t=0) = \begin{cases} 0 & \text{if} \quad x < 0 \\ C_0 & \text{if} \quad x \geq 0 \end{cases} \tag{2}$$

Applying the Laplace transformation (transforming C to Γ) gives:

$$\begin{cases} s\Gamma - C_0 & = D\frac{d^2\Gamma}{dx^2} \quad \text{if} \quad x < 0 \\ s\Gamma & = D\frac{d^2\Gamma}{dx^2} \quad \text{if} \quad x \geq 0 \end{cases} \tag{3}$$

This leads – together with the boundary and continuity conditions – to the solution:

$$\begin{cases} C = \frac{1}{2}C_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) & \text{if} \quad x < 0 \\ C = C_0\left(1 - \frac{1}{2}\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)\right) & \text{if} \quad x \geq 0 \end{cases} \tag{4}$$