IE 5318 Final Project

Optimizing Maine Congressional Districts with Integer Programming

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Executive Summary Letter

In alignment with the decennial census, U.S. states are responsible for establishing congressional districts for the purpose of electing representatives, every ten years. While the redistricting responsibility (legislature, commission, hybrid, etc.) may differ from state to state, the process must satisfy the complex requirements set forth in both Federal and state statutes. The task is furthered complicated by numerous political subdivision types within each state, rural/urban landscapes, and wide-ranging racial demographics. As the demographics of each congressional district can directly influence a party's or individual candidates' election chances, the task of redistricting is highly politicized and is a topic of intense debate, taking up valuable Federal and state resources. As an example, according to the Loyola Law School, in the 2011 cycle, courts struck all or part of congressional plans in five states and drew congressional districts in twelve states [1].

This project proposes that operations research principles, specifically, integer programming (IP), can be applied to the Congressional redistricting process to objectively provide redistricting solutions that satisfy Federal and state legal requirements. To demonstrate the application of integer programming to redistricting, the project selected the state of Maine as a subject and utilized counties as the smallest political subdivision. An IP model was built using the Federal and state requirements to define the model's constraints and set an objective function of minimizing the number of shared county border's (referred to as *cut* edges) between the proposed districts.

The IP model was constructed in Python and employed Gurobi Optimizer to produce two feasible solutions that met all criteria. The optimal solution (see Figure 1) created two Congressional districts with populations within 0.5% of the ideal Congressional district population and that shared five county borders. While the IP produced an acceptable result, the integer program did not directly incorporate a constraint to produce contiguous districts nor minimize the population difference between districts to the greatest extent practical. These limitations could be addressed with a more complex IP and/or changing the smallest political subdivision to municipalities. Ultimately, this project demonstrates that integer programming provides an objective method to solve a highly politicized and complex process to empower U.S. voters.

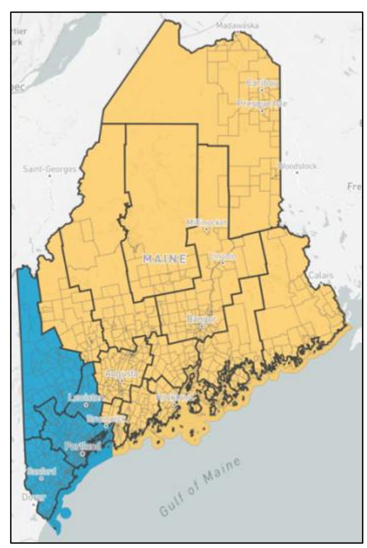

Figure 1

Introduction

According to the U.S. Census Bureau [2], in the United States, in order to proportionally distribute the 435 seats in the House of Representatives:

> "After the apportionment of congressional seats among the states, which is based on decennial census population counts, each state with multiple seats is responsible for establishing congressional districts for the purpose of electing representatives. Each congressional district is to be as equal in population to all other congressional districts in a state as practicable. The boundaries and numbers shown for the congressional districts are those specified in the state laws or court orders establishing the districts within each state."

By definition, the task of congressional redistricting is complicated by federal requirements and each state's legal requirements as well as the numerous political subdivision types within each state. Furthermore, as the demographics of each congressional district can directly influence a party's or individual candidates' election chances, the task of redistricting is highly politicized and is a topic of intense debate, taking up valuable resources within the state and Federal legislatures [3] and court systems [4].

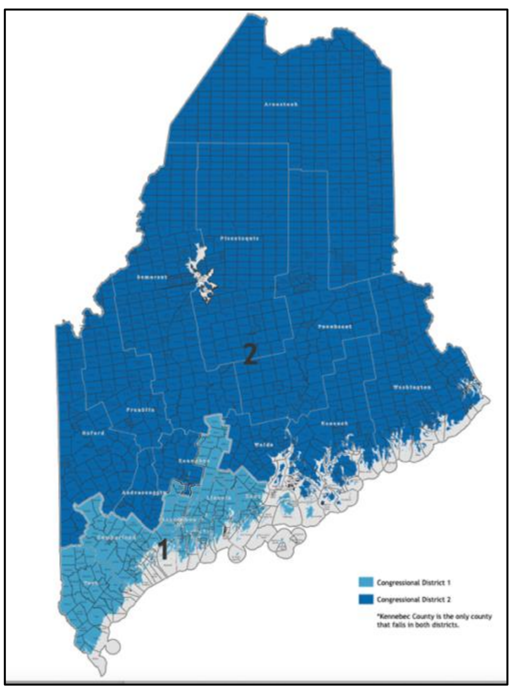

Figure 2: 2021 Maine Congressional Districts Map

Like the redistricting criteria, each state also has its unique process for approving new district plans. In this project's subject state of Maine, advisory commissions recommend district plans to the legislature, which need a supermajority (2/3) to approve any new redistricting plans, and in the instance the state legislature cannot agree on a plan, the responsibility falls to the state Supreme Court. Most recently, "on Sept. 29, 2021, the Maine state legislature adopted final maps for Congress (LD 1739), state House (LD 1738), and state Senate (LD 1741) districts. Governor Janet Mills signed all three bills into law that same day." [5] According to the Secretary of State of Maine, when the boundaries were drawn, the ideal population for each district was 681, 179.5 members. Figure 2 depicts the Congressional district boundaries passed in LD 1739 which created two districts of 681, 179 and 681, 180 members, respectively. [6] This project's goal is to create a congressional district map, using integer programming, that equals or improves upon the commission and legislatures adherence to the Federal and state criteria in 2021.

Maine Congressional Redistricting Criteria

Like the Maine state legislature, this project must adhere to the criteria outlined by the Federal government and state law. Federal law requires that each district is substantially equal and prohibits racial discrimination. The Maine revised statutes require districts are compact, contiguous, and preserve political subdivisions to the greatest extent possible. These terms are described in greater detail in the following sections.

Federal

Substantially equal: According to the Apportionment Clause of Article I, Section 2, of the U.S. Constitution, all districts be as nearly equal in population as practicable. [7]

Racial discrimination: Section 2 of the Voting Rights Act of 1965 prohibits plans that intentionally or inadvertently discriminate on the basis of race, which could dilute the minority vote. [7]

State

Compactness: Having the minimum distance between all the parts of a constituency (a circle, square or a hexagon is the most compact district). [7]

Contiguous: All parts of a district being connected at some point with the rest of the district. [7]

Preserve political subdivisions: refers to not crossing county, city, or town, boundaries when drawing districts (NCSL). Title 21A, Ch. 15 §§ 1206 of the Maine Revised Statutes states that the Legislative Apportionment Commission "shall ensure that each congressional district…crosses political subdivisions the least number of times necessary to establish districts as equally populated as possible." [7]

Problem Statement

U.S. history [8] has shown that, when left to individuals or groups, congressional district maps can be drawn in a way that gives political parties or politicians unfair advantages and disenfranchises voters. To avoid unfair political influence and increase the voting power of each individual constituent within the congressional districts of Maine, this project will employ integer programming to optimize Maine's Congressional district's adherence to Federal and state criteria.

Operations Research Model

The concept of using IP model's to create Congressional district maps is not novel; the project referenced the other researchers' models [9] to design a effective IP, specifically for the state of Maine.

This project's first step to optimize Maine's Congressional districts was to determine which political subdivisions would be used in the model. Counties were chosen over municipalities to adhere to the state's criteria for preserving political boundaries to the maximum extent possible, but, theoretically, this same model could employ municipalities to determine Congressional districts but would mostly likely require the splitting of counties (as see in Figure 1). After determining the model's unit of measure, the project graphed each county in Maine as a node with population p_c and every shared county border as an edge E (defined by the end nodes $\{u, v\}$) connecting neighboring counties (see Figure 3). The project defined the integer program's (IP's) objective function (1) as minimizing the number of county boundaries shared between each district, or *cut edges*, to construct Congressional districts as compactly as possible.

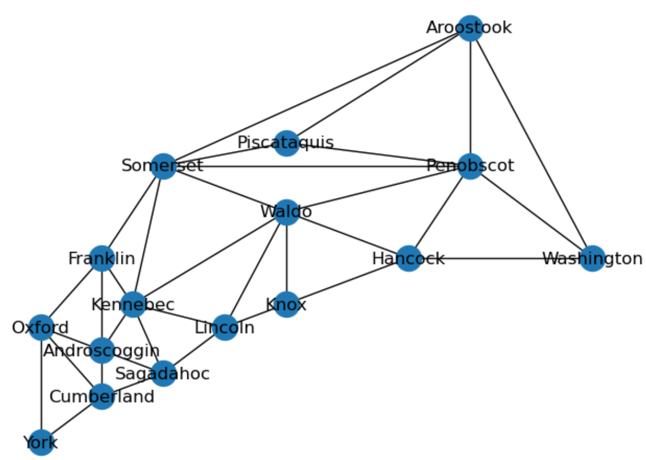

Figure 3: Graph depicting Maine county layout

The project defined the IP's variables (6) as y_e and x_{cd}. y_e is a binary variable equal to 1 when the edge *e* is *cut* or shared between districts. x_{cd} is also defined as a binary variable equal to 1 when the county *c* falls within district *d.* Lists of counties, county populations, and districts were referenced from Eugene Lykhovyd's 2020 census data [10] and included as data sets within the LP (see attached Gurobi code).

Lastly, the project wrote constraints to define a cut edge as an edge with nodes in both Congressional districts (2), to assign each county (node) c to only one district *d* (3), and to bound the populations of each district to within +/- 0.5% of the ideal district population of 681,180 (total population of Maine divided by two districts) (4,5).

The project's integer program was constructed as follows:

objective function: min $\sum_{e \in E} y_e$ (1)

s.t. $x_{ud} - x_{vd} \leq y_e \quad \forall e = \{u, v\} \in E, \ \forall d \in D$ (2)

$\sum_{d \in D} x_{cd} = 1 \quad \forall c \in C$ (3)

$\sum_{c \in C} p_c x_{cd} \geq 677774 \quad \forall d \in D$ (4)

$\sum_{c \in C} p_c x_{cd} \leq 684586 \quad \forall d \in D$ (5)

$x \in \{0,1\}, y \in \{0,1\}$ (6)

C: list of counties

D: list of districts

E: list of edges

Experiments

The IP was solved with Jupyter Notebook and Gurobi Optimizer version 11.0.0. The MacBook used had 8 GB RAM and featured a 3.1 GHz Dual-Core Intel Core i5 processor. Gurobi Optimizer solved the LP with 214 simplex iterations in 0.19 seconds and produced two feasible solutions with the smallest objective value of 5.0 *cut edges*.

Plans and Map

Table 1 details the IP's optimal solution, listing the counties assigned to each proposed district and total District populations. Figure 4 graphically represents the same results.

Optimal Maine Congressional Districts			
Proposed District 1		Proposed District 2	
County	Population	County	Population
Kennebec	123642	York	211972
Piscataquis	16800	Oxford	57777
Lincoln	35237	Androscoggin	111139
Hancock	55478	Cumberland	303069
Knox	40607		
Franklin	29456		
Aroostook	67105		
Somerset	50477		
Sagadahoc	36699		
Washington	31095		
Waldo	39607		
Penobscot	152199		
Total	**678402**		**683957**

Table 1: County distribution amongst proposed districts

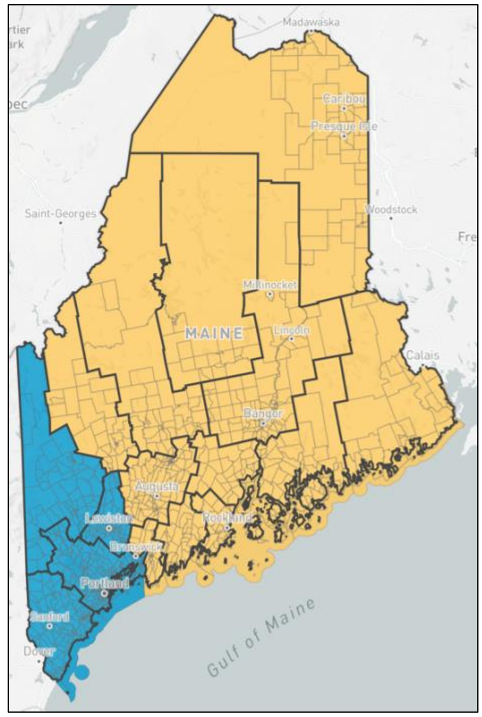

Figure 4: Proposed Congressional district map [11]

Evaluation of Plan

Each criterion was independently considered when analyzing the proposed district map. Regarding Federal redistricting criteria, the methodology did not consider race as a factor, eliminating the possibility for intentional racial description. Furthermore, the proposed district racial demographics are proportional to the statewide demographics [11], satisfying the Federal criteria prohibiting inadvertent racial discrimination. Additionally, the district populations are within +/- 0.5% of the ideal district population of 681,180, meeting Federal requirements for substantially equal. In regard criteria outlined by the state of Maine, the proposed districts are comprised of adjacent counties (contiguous), feature compact geometry (minimal *cut edges)*, and preserve political subdivisions to the maximum extent possible. Analyzing the solution based on the proposed map's adherence to Federal and state criteria, the IP has produced a feasible and practical result. However, the integer program has some limitations as it did not directly incorporate a constraint to produce contiguous districts nor minimize the population difference between districts to the greatest extent practical. While it is possible for the IP to produce non-contiguous districts, it is unlikely as the IP's objective is to minimize the number of *cut edges* which encourages contiguous maps. A solution that creates more substantially equal district populations is feasible but would require the use of municipalities, instead of counties, to build districts. This is an acceptable methodology; however, it requires the IP designer to prioritize the criteria of substantially equal over preservation of political subdivisions.

Conclusions

This project successfully employed operations research principles to model Congressional district mapping procedures. Through careful research, the project was able to define all requirements for Congressional map drawing in the state of Maine and convert those requirements (prohibition on racial discrimination, substantially equal, contiguous, compact, and preservation of political subdivisions) into a mathematical model. The model was solved using Gurobi Optimizer (Simplex method) by minimizing the number of *cut edges* within the feasible solutions. While the model could be improved with additional constraints for contiguous districts and the initial assumptions (political subdivision selection) could be modified to create greater parity between district populations, ultimately, the integer program produced a feasible solution that met all constraints. The project demonstrates that operations research principles could be employed to objectively solve the challenges presented by the U.S. congressional redistricting requirements.

References

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