MA 232 - Linear Algebra
Homework 6 (due November 27)

Problem 1 [15 pts]
Find the parabola $C + Dt + Et^2$ that fits best the following set of data: $b = 0, 0, 1, 0, 0$, at the times $t = -2, -1, 0, 1, 2$.

Problem 2 [15 pts]
Find orthonormal vectors q_1, q_2, q_3 such that q_1, q_2 span the column space of
$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$

Problem 3 [5 pts]
Find the determinants of U, U^{-1} (when it exists), U^2 for:

$$U = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}, U = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

Problem 4 [5 pts]
Show that if A is not invertible, then AB is not invertible.

Problem 5 [15 pts]
Find whether the following matrix is diagonalizable.

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

Problem 6 [15 pts]
Orthogonally diagonalize the matrix A, i.e. $A = PDP^T$, where P is orthogonal.

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Problem 7 [15 pts]

Consider the matrix $A = \begin{bmatrix} 2 & \mathrm{b} \\ 1 & 0 \end{bmatrix}$. Find a value of b that makes:

- $A = QDQ^T$ possible, i.e. orthogonal diagonalization possible.
- $A = SDS^{-1}$ impossible.
- A^{-1} impossible.

Problem 8 [15 pts]
Find the Cholesky factor of A (Recall the Cholesky factor C must be upper triangular with positive diagonal entries and such that $A = C^T C$).

$$A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 8 \end{bmatrix}$$