1 Homework 09

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. What would be the best description of each of the random walks from the last two problems from Homework 8 (problems 5 and 6)? Would it be martingale, submartingale, supermartingale, or none of those? Show your work to justify your answer.

2. For a random walk given by:

$$Y(n) = \sum_{i=1}^{n} X_i, \text{ where } X_i = \begin{cases} 5, & \omega_i = H \\ -4, & \omega_i = T \end{cases}$$

   for what values of $\mathbb{P}(\omega = H) = p$ would you have the following:

   (a) $Y(n)$ is a martingale

   (b) $Y(n)$ is a submartingale

   (c) $Y(n)$ is a supermartingale

   (you need to give me all possible values of p that would satisfy each condition)

3. We have a simple symmetric random walk, $M(n)$, and a stopping time, τ, which describes the time that you get your second head (overall, not in a row). Assume that your walk only last until time $N = 5$.

   (a) Express the random variable τ (i.e. in terms of ω's)

   (b) What is $\mathbb{E}[\tau]$?

   (c) Express the process $Y(5) = M(5 \wedge \tau)$

   (d) What is $\mathbb{P}(Y(5) > 1)$?

   (e) What is $\mathbb{P}(Y(5) < 1)$?

   (f) What is $\mathbb{E}[Y(5)]$?

   (g) What is $\mathbb{E}[Y(5)|\mathcal{F}(1)]$?

4. We have a simple symmetric random walk, $M(n)$, and a stopping time, τ, which describes the first time that you get your second head in a row. Assume that your walk only last until time $N = 5$.

   (a) Express the random variable τ (i.e. in terms of ω's)

   (b) What is $\mathbb{E}[\tau]$?

   (c) Express the process $Y(5) = M(5 \wedge \tau)$

   (d) What is $\mathbb{P}(Y(5) > 1)$?

   (e) What is $\mathbb{P}(Y(5) < 1)$?

   (f) What is $\mathbb{E}[Y(5)]$?

   (g) What is $\mathbb{E}[Y(5)|\mathcal{F}(1)]$?

