Homework - Basic numerical methods and Risk measurement

FE-620 - Fall 2022

November 11, 2022

Problem 5.1
Apply the control variate method to the two American options on futures in Problem 3.5, using an European option as control variate, in order to obtained an improved estimate for the American option prices.

Use the same option and market parameters as in Problem 3.5.

Problem 5.2
Explain why the Monte Carlo simulation approach cannot be used easily for American-style derivatives.

Hint. Recall that when pricing American options, we have to compare the *continuation value* with the *exercise value* at any intermediate time. Which do you expect to be more difficult to compute?

Problem 5.3
Build a Monte Carlo simulation of two normally distributed random variables $\varepsilon_1, \varepsilon_2$ with mean zero and unit variance, with correlation $\rho = +0.7$. Using 1000 MC samples, compute the expectation

$$M = \mathbb{E}[max(\varepsilon_1, \varepsilon_2)] \tag{1}$$

Hint. You can construct the random variables from two independent N(0,1) random variables, as explained in class. The simulation can be performed in Excel.

Problem 5.4
Consider a portfolio which can have a loss over 1 year of 1m with probability 98%, or a loss of 10m with probability 2%. Compute the value-at-risk and expected shortfall for this portfolio at confidence level 97.5%.

Problem 5.5

Use the historical simulation method described in class (Ch. 22.2 in Hull, page 496) to compute the daily 95% Confidence Level Value-at-Risk of a portfolio consisting of 1000 shares of TSLA stock on 17-Oct-2022. Denote $PnL_t = V_{t+1} - V_t$ the portfolio PnL (Price-and-Loss) for day t.

At the close of the day on 17-Oct-2022 we do not know yet the PnL for the next day, and we would like to estimate its probability distribution, in order to compute the VaR.

Use the past 1 year of data (17-Oct-2021 to 17-Oct-2022) as inputs to the computation.

You can organize the computation using Excel, by modifying the spreadsheet used in class.