
Homework 8:

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1. For this problem we are going to try and show that

$$\int_0^t u dW(u) = tW(t) - \int_0^t W(u) du$$

- (a) First determine what your process $\Delta_n(u)$ should be such that

$$\int_0^t u dW(u) = \lim_{n \rightarrow \infty} \sum_{j=0}^{n-1} \Delta_n\left(\frac{j}{n}\right) (W_{j+1} - W_j)$$

where $W_j = W\left(\frac{j}{n}\right)$

- (b) Show that this can be simplified to

$$\lim_{n \rightarrow \infty} \frac{t}{n} \left((n-1)W_n - \sum_{j=1}^{n-1} W_j \right)$$

- (c) Take the above limit to show that (note that the second term is just a Riemann integral)

$$\int_0^t u dW(u) = tW(t) - \int_0^t W(u) du$$

From your textbook (Shreve volume 2):

2. Problem 4.1
3. Problem 4.2
4. Problem 4.3