

Problem 1. ARIMA seasonal model (50 points)

Simulate 500 observations for an ARIMA seasonal model. The model includes a non-seasonal $AR(1)$ term, a seasonal $AR(1)$ term, no differencing, no MA terms and the seasonal period is $S = 12$. Specifically the model is:

$$(1 - \phi_1 B)(1 - \Phi_1 B^{12})(x_t - \mu) = w_t, \quad (1)$$

where,

$$\begin{cases} \mu = 0 \\ \phi_1 = 0.3 \\ \Phi_1 = 0.3 \end{cases} \quad (2)$$

- (a) Please display the ACF and PACF plots obtained using the generated data
- (b) Perform a unit root test on the data and explain your finding.
- (c) Using the AIC criterion to find the best order of a model fitting the data. Assume you try the airplane model as well as the model below and to have multiple S values in the loop. That is please try bimonthly, quarterly, semiannual, annual, and bi-annual effects.
- (d) Please use the optimal order you find in question 1.(c) to fit the model. Obtain diagnostics plots. Then compare the estimates with the original parameters used to generate the model.

Problem 2. Regression with time series and ARIMAX (50 points)

Download daily equity trade data for Coca Cola (KO) and Pepsi (PEP) from January 2017 to Dec 2019 and calculate the log returns (using the adjusted close price).

- (a) We want to use the return of Pepsi to explain the return of Coca. Please perform regression with time series errors on these two time series. To this end, fit the data using the Cochran-Orcutt procedure.
- (b) Fit an ARIMAX model for KO with PEP as explanatory variable.
- (c) Please comment on your findings.