Homework 4:

You must provide your rational for your answer. Just giving the solution will earn 0 points.

1. Assume you have a sample of random variables drawn from a distribution
$$f_X(x) = \frac{x}{\beta^2} e^{-\frac{x}{\beta}}, x > 0$$
   (a) Estimate β using MLE
   (b) Estimate β using MOM
   (c) How do these estimates compare?

2. The gamma distribution with parameters α, λ follows the distribution
$$f_X(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, x > 0$$
$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt$$
   A gamma random variable will have mean $\frac{\alpha}{\lambda}$ and variance $\frac{\alpha}{\lambda^2}$.
   (a) Use MOM to come up with a formula for the estimates of the parameters α and λ.
   (b) If your sample consists of these numbers:
   0.3390468802, 0.0822332406, 0.0937637497, 0.3282473405, 0.1931273521, 2.9364388997, 0.1145126764, 0.0071946593, 0.0006511787, 0.0280782986
   What are your estimates?

3. A sample of n elements is created by recording the first time that a person gets a "heads" on a coin after flipping it. Using MLE, determine an estimate of the probability of getting a "heads" on the coin.