

1 Homework 05

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. A geometric random variable X is a discrete variable taking values in $\{1, 2, \dots\}$ with the parameter p which models the probability of getting the first success on the n^{th} trial. This variable has density function $f_X(x) = p(1 - p)^{x-1}$. Find the moment generating function of X and use it to find the mean and variance of X .
2. For a normal random variable $X \sim N(\mu, \sigma^2)$, we defined a variable $Y = e^X$ as a log-normal random variable. Use the moment generating function of X to find the mean and variance of Y .
3. A random variable Y has moment generating function $M_Y(t) = e^{2t + \frac{t^2}{2}}$. Find $\mathbb{P}(1 < Y < 3)$.
4. For a variable X with

$$f_X(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{else} \end{cases}$$

- (a) Find $M_X(t)$
 - (b) Use the moment generating function to get the first moment and the second central moment
5. Given a random variable $Y \sim \text{Pois}(\lambda)$ (a Poisson random variable with parameter $\lambda > 0$),
 - (a) Find $M_Y(t)$
 - (b) Use this result to get the mean and variance of Y .
 6. For independent variables $X \sim \text{Exp}(\lambda_1)$ and $Y \sim \text{Exp}(\lambda_2)$ where λ_1 and λ_2 are positive constants, use moment generating functions to investigate:
 - (a) $Z = 3X$

(b) $W = 2Y$

(c) $R = Z + W$