Ma 410 - Differential Geometry **Hw-9** **Due: November 16, 2022 by 11:59 pm**

Name (Printed):

Pledge and Sign:

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. [10 pts.] Use the results of Example 1.3 from Section 5.1 or class notes to find principal vectors of

(a) The cylinder $x^2 + y^2 = 1$ at every point.

(b) The saddle surface $z = xy$ at the origin $(0, 0, 0)$.

[Hint: you need to show that the candidate vectors are eigenvectors of S.]

2. [5 pts.] Show that there are no umbilics on a surface with Gaussian curvature $K < 0$, and then when $K \leq 0$, umbilic points are planar.

3. Let $\mathbf{x}$ be the patch $\mathbf{x}(u, v) = (u, v, f(u, v))$ for a smooth function $z = f(x, y)$.

(a) [10 pts.] Find E, F, G, L, M and N. Then find formulas for K and H.

(b) [5 pts.] Show that the image of $\mathbf{x}$ is flat, that is $K = 0$, if and only if

$$f_{uu}f_{vv} - f_{uv}^2 = 0$$

(c) [5 pts.] Show that the image of $\mathbf{x}$ is minimal, that is $H = 0$, if and only if,

$$(1 + f_u^2)f_{vv} - 2f_uf_vf_{uv} + (1 + f_v^2)f_{uu} = 0.$$