

1 Homework 03

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. Use the results that we learned in class to express the following using exact expressions (not just decimal approximations)
 - $\Gamma(1.5)$
 - $\Gamma(2.5)$
 - $\Gamma(3.5)$

Find a general formula for $\Gamma(\frac{n}{2})$ where n is a positive integer?

2. Random variables X and Y are each exponential random variables with parameter $\lambda = 2$. They have joint density function given by:

$$f_{X,Y}(x,y) = 4e^{-2(x+y)}$$

- (a) Show that X and Y are independent.
- (b) Use their independence to calculate $\mathbb{E}[e^{-\frac{1}{2}(X+Y)}]$.

3. Let V be a random variable with density function:

$$f_V(v) = \begin{cases} \frac{1}{v^2}, & v \geq 1 \\ 0, & \text{else} \end{cases}$$

Find the density functions for the following random variables:

$$U = \log(V)$$

(note that in this class, \log will by default be base e). Make sure you include the bounds of the density function as well.

4. For the expression

$$f_{X,Y}(x,y) = x + y; 0 \leq x \leq y \leq b$$

For what value of b is this a valid joint density function for random variables X and Y ?

5. For independent random variables $X \sim \text{Exp}(\lambda_1)$ and $Y \sim \text{Exp}(\lambda_2)$, find the density function for the convolution of these two random variables.