Homework #3 - Lectures 6 and 7

FE-620 Fall 2022

Due 28-Oct-2022

Problem 3.1
Assume that a stock price is $S_0 = \$100$, the stock does not pay any dividends, and the risk-free interest rate is 5% per annum (with continuous compounding).

1. Compute the lower bound for the price of a 3-month European call option with strike price $K = \$92$

2. Compute the lower bound on the price of a 3-month European put option with strike price $K = \$105$.

3. Assume that the price of a 3-month European call option with strike price \$95 is \$8.05. What is the price of the European put option on the same stock with the same strike and maturity?

Problem 3.2
Suppose that c_1, c_2 and c_3 are the prices of European call options with strike prices K_1, K_2 and K_3, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All option have the same maturity. Show that

$$c_2 \leq 0.5(c_1 + c_3)$$

This condition must be satisfied by all European option prices with the same maturity, and is known as the *butterfly arbitrage condition*.

Hint: Consider a portfolio that is long one option with strike K_1, long one option with strike K_3, and short two options with strike price K_2. Plot the payoff of this portfolio as a function of the stock price at maturity $S(T)$, and convince yourself that it is always positive.

Problem 3.3
The current price of a non-dividend paying pharma stock is \$100 and its volatility is 55%. The risk-free rate is 5.20%.

Using a one-time step binomial tree, compute the price of a call option on this stock with strike $K = \$100$ and maturity $T = 3M$.

Problem 3.4
A stock price is currently \$60. It is known that at the end of 6 months it will be either \$55 or \$62. The risk-free rate of interest with continuous compounding is 5% per annum. Calculate the value of a 6-month European call option on the stock with an exercise price $K = \$60$.

Problem 3.5
The price of a non-dividend paying stock is $100. Use a three-step tree to value:

(a) a 6-month American call option with strike price $100

(b) a 6-month American put option with strike price $100.

The volatility is 35% and the risk-free rate for all maturities is 5% with continuous compounding.