Homework #1

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September 30, 2022

Problem 1.1

Assume that the limit order book is as shown in the table below.

1) What is the bid-ask spread in cents?

2) If you buy 100 shares with a market order, and immediately sell them also with a market order, what is your P&L (profit/loss)?

3) A limit order to buy 250 shares at \$34.51 arrives in the market. What transactions occur, and what is the resulting bid-ask spread in cents?

4) A limit order buy order for 1900 shares at \$34.54 arrives. List all transactions, and determine the bid-ask spread after the order is executed.

Orders	Shares	Price
offer	1900	34.56
offer	1700	34.55
offer	1200	34.54
offer	400	34.53
offer	300	34.52
bid	1000	34.50
bid	1100	34.49
bid	1400	34.48
bid	1500	34.47
bid	2200	34.46

Solution:

1) The bid-ask spread is defined to be the difference between the ask (offer) and bid (buy) of a security. The bid price is the highest price that a buyer is willing to pay for an asset and the ask price is the lowest price that a seller is willing to accept. In this scenario, the bid-ask spread in cents is \$34.52 - \$34.50 = \$0.02.

2) If we buy 100 shares immediately, we want to buy at the lowest possible price and when we sell, we want to sell at the highest possible price. Since there is an offer to purchase a maximum of 300 shares at $34.52 per share, which is the lowest possible price, it will cost us $3452 for 100 shares. If we sell immediately, then there are buyers willing to pay the highest price of $34.50 per share for a maximum of 1000 shares. The trade will then be executed at the price of $34.50 per share and we will receive $3450 for 100 shares. This transaction will result in a loss of $2.

3) Since the buyer placed a limit order and no one is offering to sell at the price of $34.51 or lower, the trade is not executed.

4) A limit order buy of 1900 shares at $34.54 will result in the following transactions. First, 300 shares will be purchased at $34.52 per share. Then 400 shares will be purchased at $34.53 per share and lastly 1200 shares will be purchased at $34.54 per share. The highest price the buyer is willing to pay is $34.50 for 1000 shares and $34.49 for the other 200 shares. The bid-ask spread then becomes $34.54 - $34.49 = $0.05.

Problem 1.2

Using the R code SylizedFacts.R from Canvas download 2 years' daily prices for a stock of your choice, and investigate the validity of the following stylized facts of the financial markets:

i) Absence of autocorrelation of the log-returns.

ii) The heavy tails in the log-returns distribution. Hint: compute the excess kurtosis of the daily log-returns.

iii) Volatility clustering.

Solution:

We investigate the validity of the following stylized facts of financial markets using Amazon stock data from the period January 1, 2019 to December 31, 2021.

i) ACF of Daily Log-Returns for Amazon Stock:

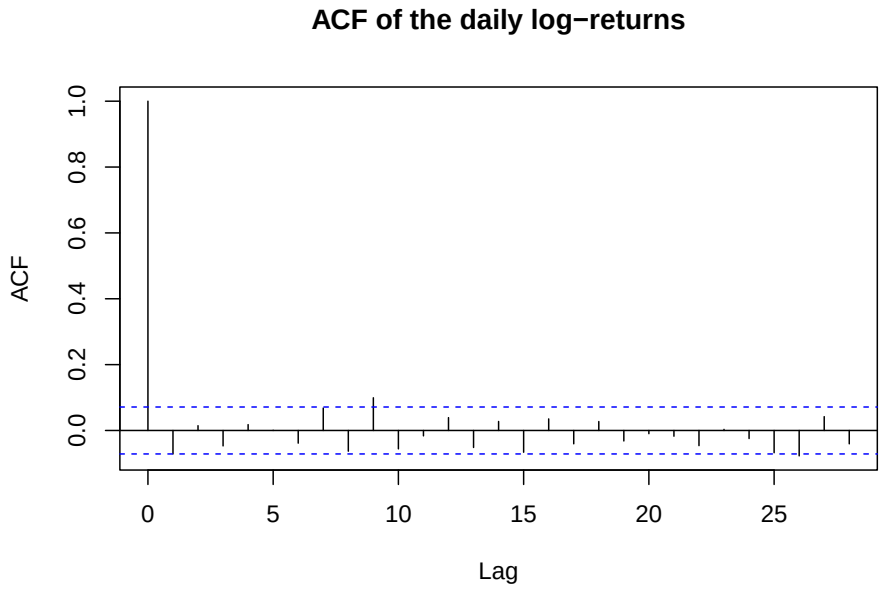

Figure 1: The autocorrelation of the daily log-returns of Amazon are almost zero except at lag 9 and lag 26, which is in agreement with the stylized fact #1

ii) Histogram of Daily Log-Returns of Closing Prices for Amazon Stock:

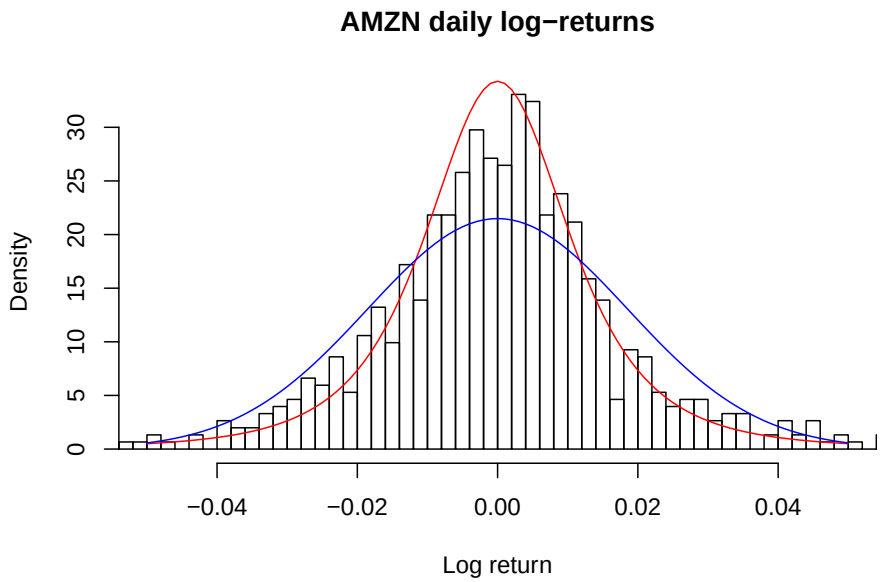

Figure 2: The histogram of the daily log-returns for Amazon, comparing with a normal distribution and a Student-t distribution with 3 degrees of freedom.

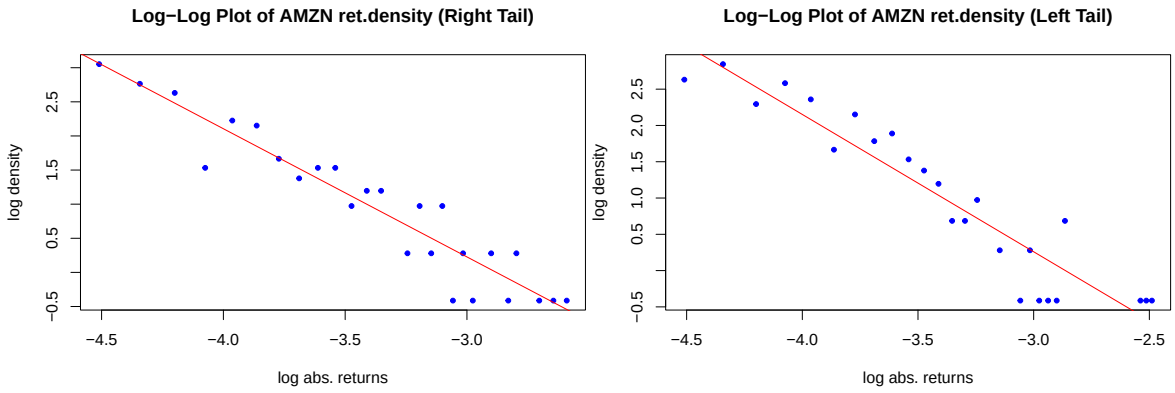

Figure 3: Log-log plots for the density of the daily log-returns.

By computing the excess kurtosis of the daily log-returns, we get 2.561216. This is high, which indicates a sign of heavy tails and does not indicate normality according to stylized fact #2.

iii) Volatility Clustering of Daily Log-Returns for Amazon Stock:

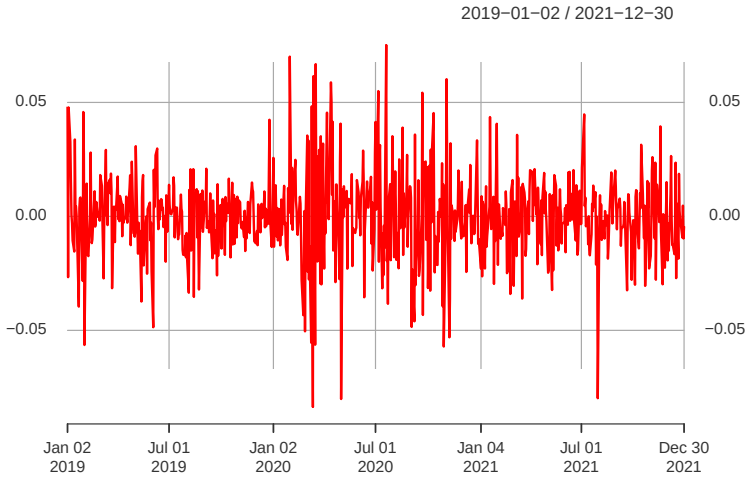

Figure 4: The volatility of Amazon Daily-Log Returns from January 01, 2019 to December 31, 2021

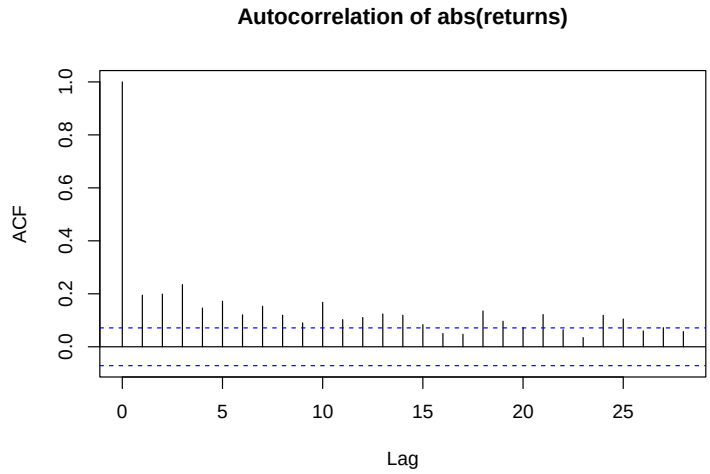

Figure 5: The autocorrelation of the absolute value daily log-returns of Amazon. They are non-vanishing, in agreement with the stylized fact #8.

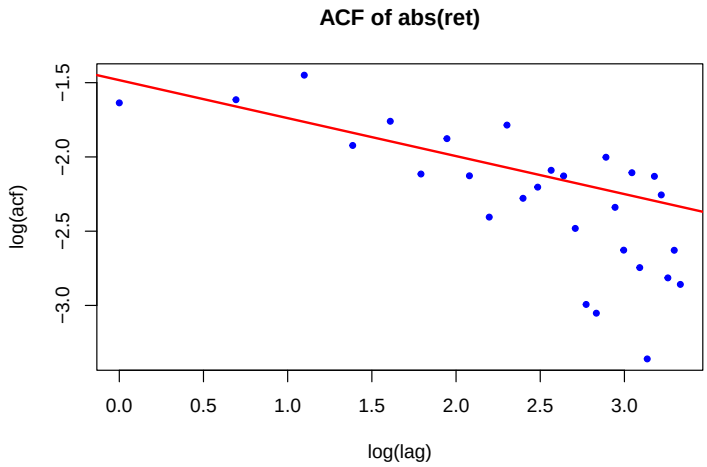

Figure 6: Log-log plot of the autocorrelation of the absolute value daily log-returns of Amazon. A linear regression shows a slow decay

We would like to investigate their decay. For this we represent the ACF as a log-log plot in Fig. 6. A linear regression gives

$$y(x) = -1.48 - 0.26x \tag{1}$$

which shows that the autocorrelations decay like $\Delta^{-0.26}$.