Ma 410 - Differential Geometry Hw-8 Due: November 9, 2022 by 11:59 pm

Name (Printed):

Pledge and Sign:

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Let ϕ be a 1-form on a surface M.

(a) [5 pts.] We say that ϕ is *closed* if the exterior derivative of ϕ is zero, that is $d\phi = 0$. Show that if ϕ is closed, then $\int_{\partial \mathbf{x}} \phi = 0$ for every 2-segment $\mathbf{x}$ on M.

(b) [5 pts.] We say that ϕ is *exact* if, there exists a smooth function f on M, such that $\phi = df$. Show that if ϕ is exact, then $\int_{\alpha} \phi = 0$, for any closed piecewise smooth curve α.

2. The 1-form

$$\psi = \frac{udv - vdu}{u^2 + v^2}$$

is well-defined on the plane $\mathbb{R}^2$ with the origin removed. Show:

(a) [5 pts.] ψ is closed but not exact on $\mathbb{R}^2 - 0$. [Hint: Integrate the unit circle and use #1 above.]

(b) [5 pts.] The restriction of ψ, to say, the right half-plane $u > 0$ is exact.

3. [10 pts.] Use Exercise #2, of Section 5.1 or your notes from class to express the value $S_{\mathbf{0}}(a\mathbf{u}_1 + b\mathbf{u}_2)$ of the shape operator of each surface only at the origin $\mathbf{0} = (0, 0, 0)$ in terms of $\mathbf{u}_1 = (1, 0, 0)$ and $\mathbf{u}_2 = (0, 1, 0)$. Find a matrix representation for $S_{\mathbf{0}}$ in each case. Determine the rank (number of pivots) of the matrix.

(a) $z = xy$

(b) $z = (x + y)^2$