Ma 410 - Differential Geomtery Hw-3 Due: September 28, 2022 by 11:59 pm

Name (Printed): ____________________

Pledge and Sign: ____________________

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. [10 pts.] Consider the curve

$$\beta(s) = \left(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}} \right)$$

defined on $I : -1 < s < 1$. Show that β has unit speed, and compute its Frenet apparatus.

2. (a) [3 pts.] Show that the curve $\alpha(t) = (t\cos t, t\sin t, t)$ lies on a double cone and passes through the vertex at $t = 0$.

(b) [6 pts.] Find the Frenet apparatus of α at $t = 0$ (**only at** $t = 0$).

(c) [1 pts.] Describe the curve near $t = 0$ or draw it.

3. [10 pts.] Let $\alpha : I \to \mathbb{R}^3$ be a cylindrical helix with unit vector $\mathbf{u}$. For $t_0 \in I$, the curve

$$\gamma(t) = \alpha(t) - [(\alpha(t) - \alpha(t_0))\cdot\mathbf{u}]\mathbf{u}$$

is called a *cross-sectional curve* of the cylinder on which α lies. Prove:

(a) γ lies in the plane $\alpha(t_0)$ orthogonal to $\mathbf{u}$

(b) The curvature of γ is $\kappa/\sin^2\theta$, where κ is the curvature of α.