Stevens Institute of Technology **Homework 1**
QF 202: Intro to Time Series *Due Date: February 14, 2022, 11:55pm*

Please finish the following questions in Rmarkdown notebook. And submit both rmd and pdf files.

Problem 1. (15 points) You are asked to invest in one of 3 equities. Suppose we know the future distribution of returns is:

	Return	**Probability**
Stock A	6%	0.6
	-3%	0.4
Stock B	3%	0.7
	-1%	0.3
Stock C	45.43%	0.5
	-50%	0.5

(a) Calculate the expected return for each stock.

(b) Compute the variance of the returns for each stock.

(c) Which choice you would make and argue why.

Problem 2. (30 points)
Let the stochastic processes $X = \{X_t : t \in \{0, 1, 2, 3, 4\}\}$ represent the price of stock through years 1 to 4. ω here represents possible paths of the processes. Specifically, the table below gives the probability distribution for the 4 variables.

	$X_0(\omega)$	$X_1(\omega)$	$X_2(\omega)$	$X_3(\omega)$	$X_4(\omega)$	$\mathbb{P}(\omega)$
ω_1	11	13	15	16	16	0.06
ω_2	11	13	15	16	14	0.065
ω_3	11	13	15	15	16	0.06
ω_4	11	13	15	15	14	0.065
ω_5	11	13	15	14	16	0.06
ω_6	11	13	15	14	14	0.065
ω_7	11	13	12	13	14	0.0625
ω_8	11	13	12	13	18	0.08
ω_9	11	13	12	13	12	0.04
ω_{10}	11	13	12	13	11	0.0675
ω_{11}	11	11	12	13	14	0.0625
ω_{12}	11	11	12	13	12	0.0625
ω_{13}	11	11	12	12	12	0.0625
ω_{14}	11	11	11	14	20	0.0625
ω_{15}	11	11	11	14	12	0.0625
ω_{16}	11	11	11	12	12	0.0625

(a) Give the conditional distribution of X_4, knowing X_2.

(b) Calculate the conditional expectation of X_4, knowing X_2.

Problem 3. (40 points) Please recall the example question from lecture 2. We roll a die Y times to get the first number 6. ($Y \sim Geometric(\frac{1}{6})$). Let X denote the number of occurrences of the number 1 in the Y rolls. Recall that fixing the number of trials $Y = y$, there are no 6 in the first $y-1$ trails, and the last roll is a 6. Thus X has a binomial distribution. ($X \mid Y = y \sim Binomial\left(y-1, \frac{1}{5}\right)$)

(a) Simulate $X \mid Y = y$ using a sample size of 100 when $y = 10$

(b) Calculate the mean of the sample. Also, calculate the expectation of $X \mid Y = y$. Compare your numerical result with the theoretical result and interpret your finding.

(c) Calculate the variance of the sample. Also, calculate the variance of $X \mid Y = y$. Compare your numerical result with the theoretical result and interpret your finding.

(d) Increase the sample size in simulation (a) from 100 to $10,000$ with an increment of 100. (That is generate a sequence of sample with size 100, 200, ..., 10000) Calculate the expectation of each sample. And plot the expectation over sample size. (Expectation on the Y-axis and sample size on the X-axis) Add a horizontal line in the plot indicating the theoretical value of expectation. Please interpret your finding.

Problem 4. (15 points) A 90% confidence interval for a population mean is (25, 32).

(a) Can you reject the null hypothesis that $\mu = 24$ against the two-sided alternative at the 10% significance level? Why?

(b) Can you reject the null hypothesis that $\mu = 30$ against the two-sided alternative at the 10% significance level? Why?

(c) Can you reject the null hypothesis that $\mu = 24$ against the two- sided alternative at the 5% significance level? Explain.