Homework #2: Futures and credit risk

FE-620 Fall 22

Due 14 Oct 2022

Problem 2.1
A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is
(a) 48.20 cents per pound
(b) 51.30 cents per pound

Solution.
(a) The trader makes a gain of

$$50,000 \times (50c - 48.2c) = \$900.00$$

(b) The trader makes a loss of

$$50,000 \times (51.3c - 50.0c) = \$650.00$$

Problem 2.2: Arbitrage with commodity futures
The spot price of oil is \$80.0 per barrel and the cost of storing a barrel of oil for one year is \$2, payable at the end of the year. The risk-free interest rate is 4.0% per annum continuously compounded. What is the estimated one-year futures price of oil?

Solution. We require that the following arbitrage strategy does not make any profit.

At time $t = 0$, borrow \$80 from the bank, purchase one barrel of oil and put it in storage. Take the short side of a forward contract maturing in one year.

At maturity $t = 1Y$, pay the storage fee c, deliver the oil and receive the forward price $F(T)$. Pay the bank $S_0 e^{rT}$.

The net profit of the arbitrage strategy is

$$F(T) - c - S_0 e^{rT}$$

This profit must vanish if the futures contract is priced correctly, so the futures price is

$$F(T) = S_0 e^{rT} + c = \$85.2649$$

Problem 2.3: Equity index (variation on problem 5.27 in Hull)
An equity index is 1,700. The three-month risk-free rate is 1.0% per annum and the dividend yield over the next three months is 1.5% per annum. The six-month risk-free rate is 1.5% per annum and the dividend yield over the next six months is also 1.5% per annum.

Estimate the futures price of the index for three-month and six-month contracts. All interest rates and dividend yield are continuously compounded.

Solution. Use the formula $F(T) = S_0 e^{(r-q)T}$ to get

$$F(3M) = \$1,697.88 \tag{1}$$

$$F(6M) = \$1,700.00 \tag{2}$$

(3)

Problem 2.4: Bootstrapping the hazard rates from CDS spreads
Consider a Credit Default Swap with maturity 2 years, paying a premium with semi-annual frequency.

Assume that defaults can occur only at times 0.25 years, 0.75 years, 1.25 years and 1.75 years, as in the example discussed in class. (This is similar to the simplifying assumption made in the example discussed in Chapter 25.2

in Hull; in real applications the defaults are allowed to take place any day, but this would complicate the computation.)

The CDS spread is 255 basis points. Assume that the risk-free interest rate is 2.5% (with continuous compounding) and the recovery rate is $R = 40\%$.

What is the hazard rate of the reference name? Assume a constant hazard rate for the entire maturity of the CDS.

Solution. Modify the spreadsheet for CDS pricing by keeping only 4 cash flows, modifying the cash flow times, and most importantly, adding a factor of 0.5 to the premium and accrual legs because of the semi-annual frequency of the CDS considered.

We take the coupon as 2.55% and change the hazard rate until the CDS price vanishes. This gives the hazard rate

$$\lambda = 4.224\%$$

See Figure 2 for the details of the Excel spreadsheet.

As a check on the result, we verify that this is close to the simple estimate for the hazard rate given by the formula

$$\lambda = \frac{sp}{1-R} = \frac{2.55\%}{1-0.4} = 4.25\% \tag{4}$$

However the result from the CDS is more precise, as it takes into account also the risk free rate, which does not appear in the simpler estimate above.

Problem 2.5

The spread between the yield on a 2-year corporate bond and the yield on a similar risk-free bond is 250 basis points. The recovery rate is 40%.

i) Estimate the average hazard rate over the 2-year period.

ii) Compute the probability that the company issuing the bond will default in 2 years.

Solution.

Risk free rate	2.50%		**Coupon (s)**	2.55%
Hazard rate	4.2240%			

Premium Leg

n	Time	D(T)	V(T)	PV
1	0.5	0.9875778	0.97910169	0.96693909
2	1	0.97530991	0.95864011	0.9349712
3	1.5	0.96319442	0.93860615	0.9040602
4	2	0.95122942	0.91899086	0.87417115
			PV(Premium)	3.68014164

Accrual Leg

n	Time	D(T)	Prob(default)	ExpectedPay	PV
1	0.25	0.99376949	2.09%	0.01044916	0.01038405
2	0.75	0.98142469	2.05%	0.01023079	0.01004075
3	1.25	0.96923323	2.00%	0.01001698	0.00970879
4	1.75	0.95719323	1.96%	0.00980764	0.00938781
				PV(Accrual)	0.0395214

RecoveryRate	40%

Default Leg

		(each period)	(1-R)*Pr(def)		
Default Time	**V(T)**	**Prob(default)**	**ExpectedPay**	**DiscFactor**	**PV**
0.25	0.97910169	2.09%	0.0125	0.99376949	0.01246086
0.75	0.95864011	2.05%	0.0123	0.98142469	0.0120489
1.25	0.93860615	2.00%	0.0120	0.96923323	0.01165055
1.75	0.91899086	1.96%	0.0118	0.95719323	0.01126537
				PV(Default)	0.04742568

Leg	PV
Premium	0.046921806
Accrual	0.000503898
Default	-0.047425682
Total PV	2.1852E-08

Figure 1: Solution for Problem 2.4 in Excel.

i) Use the triangle rule

$$\lambda = \frac{sp}{1-R} = 4.167\%$$

ii) The default probability is

$$p_d(2Y) = 1 - e^{-\lambda \cdot 2} = 7.996\%$$