Homework 5:

1. For this problem, you'll be using a Ho-Lee model with $N = 4, a_0 = .03, a_1 = .025, a_2 = .02, a_3 = .015, b_1 = b_2 = b_3 = .005$, and $\tilde{p} = \tilde{q} = \frac{1}{2}$. Answer the following:

   (a) Plot the yield curve for all maturities from 0 to 4

   (b) Determine the value of a coupon bearing bond that pays 10 dollars at times 1,2 and 3 and a hundred dollars at time 4.

2. For this problem, we'll be using a stock model where the up and down factors are not constant. Assume that we have a stock process where $S_0 = 100$ and $u_0 = 1.09$ and $d_0 = .97$ give the up and down factors for the stock as it evolves from time 0 to time 1. For the up and down factors from time 1 to time 2 we have:

$$u_1 = \begin{cases} 1.0625 & \omega_1 = H \\ 1.0575 & \omega_1 = T \end{cases}, d_1 = \begin{cases} 1.0025 & \omega_1 = H \\ .9975 & \omega_1 = T \end{cases}$$

Further assume that the interest rate process follows a Ho-Lee model with $a_0 = .03, a_1 = .0275, b_1 = .005$ and with $\tilde{p} = \tilde{q} = \frac{1}{2}$.

   (a) Confirm that the discounted stock price is a martingale.

   (b) Determine the 2-forward price.

   (c) What is the value of a forward where $K = 100$?

3. Using a Ho-Lee model with parameters

n	a_n	b_n
0	.035	
1	.034	.002
2	.033	.002
3	.032	.002

and $\tilde{p} = \tilde{q} = \frac{1}{2}$, answer the following:

   (a) For $K = .033$, determine the time zero value of a 4-period interest rate swap

   (b) Determine the 4-period swap rate.