Ma 410 - Differential Geometry **Hw-6** **Due: October 19, 2022 by 11:59 pm**

Name (Printed):

Pledge and Sign:

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. [8 pts.] Let $\mathbf{x} : \mathbb{R}^2 \to \mathbb{R}^3$ be the mapping:

$$\mathbf{x}(u, v) = (u^2, uv, v^2)$$

on the first quadrant $D : u > 0, v > 0$. Show that $\mathbf{x}$ is 1-1 and find a formula for its inverse $\mathbf{x}^{-1} : \mathbf{x}(D) \to D$. Then prove that $\mathbf{x}$ is a proper patch.

2. Find a parameterization for each surface of revolution, obtained by revolving:

(a) [8 pts.] $C : y = \cosh x = \dfrac{e^x + e^{-x}}{2}$ about the x-axis (catenoid).

(b) [8 pts.] $C : (x - 2)^2 + y^2 = 1$ about the y-axis (torus).

3. [6 pts.] A generalized cone is a ruled surface with a parameterization of the form:

$$\mathbf{x}(u, v) = \mathbf{p} + v\delta(u)$$

Thus all rulings pass through the vertex $\mathbf{p}$. Show that $\mathbf{x}$ is regular if and only if v and $\delta \times \delta'$ are never zero.