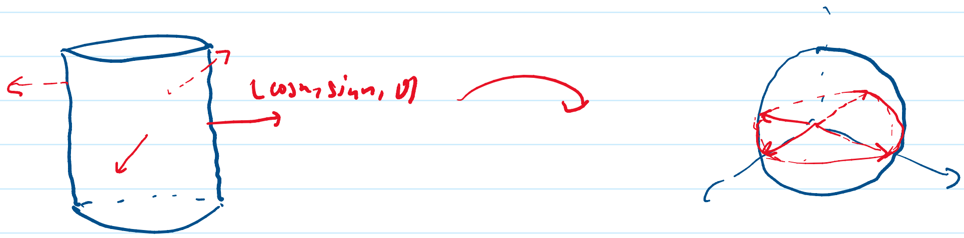
1

(a) we take unit normal $\vec{U}=(\cos u, \sin u, 0)$, $0 \le u \le 2\pi$

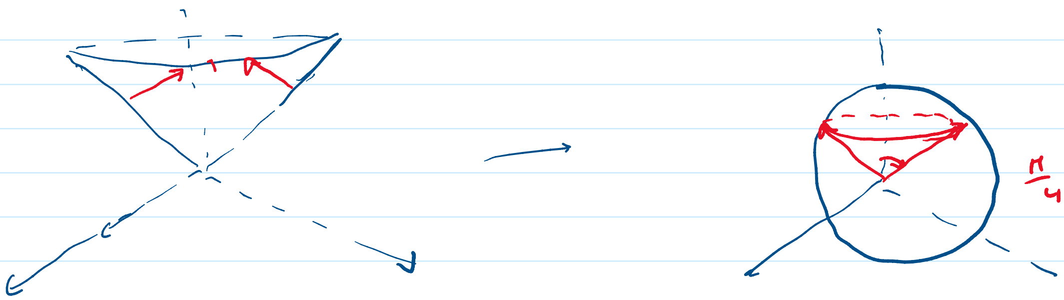
Drawing it from the origin rather than at the point

$\vec{x}(u,v) = (r\cos u, r\sin u, v)$ we get the unit circle

$x^2+y^2=z^2$



(b)



circle $x^2+y^2=\frac{1}{2}$

at $z=\frac{1}{\sqrt{2}}$

$x(u,v) = (u, v, \sqrt{u^2+v^2})$, $\sqrt{u^2+v^2} \neq 0$

$x_u \times x_v = \left(\frac{-u}{\sqrt{u^2+v^2}}, \frac{-v}{\sqrt{u^2+v^2}}, 1\right)$

$\|x_u \times x_v\| = \sqrt{2}$

$\Rightarrow$ upward unit normal

$$\vec{u} = \frac{1}{\sqrt{2}}\left(\frac{-u}{\sqrt{u^2+v^2}}, \frac{-v}{\sqrt{u^2+v^2}}, 1\right)$$

$\Rightarrow x^2+y^2=\frac{1}{2}$ at height $z=\frac{1}{\sqrt{2}}$

or

or

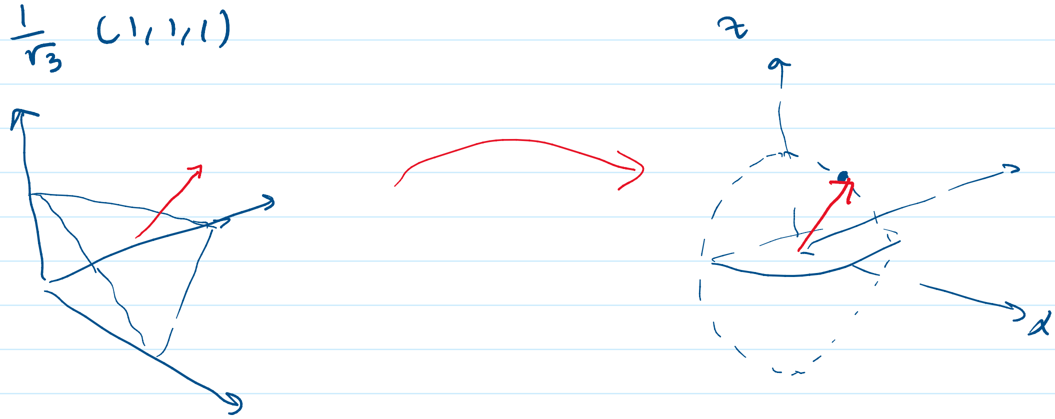
$$x(u,v) = (u\cos v,\ u\sin v,\ u), \quad u>0$$

$$x_u \times x_v = \begin{vmatrix} u_1 & u_2 & u_3 \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (-u\cos v, -u\sin v, u)$$

$$\|x_u \times x_v\| = \sqrt{2u^2} = \sqrt{2}\,u$$

$$\Rightarrow \vec{u} = \frac{1}{\sqrt{2}}(-\cos v, -\sin v, 1) \Rightarrow \boxed{\begin{array}{c} x^2+y^2 = \frac{1}{2} \\ z = \frac{1}{\sqrt{2}} \end{array}}$$

(c) $u = \frac{1}{\sqrt{3}}(1,1,1)$



(d) in all cases $\Delta\tilde{A}$ is the area of a curve or a point, so $\Delta\tilde{A} = 0$ in all cases

$$\Rightarrow \frac{\Delta\tilde{A}}{\Delta A} = 0 \Rightarrow K(p) = \lim_{\Delta A \to 0} \frac{\Delta\tilde{A}}{\Delta A} = 0$$

constant

$\alpha(t) = R(\cos\phi \cos t, \cos\phi \sin t, \sin\phi)$

$\alpha'(t) = R(-\cos\phi \sin t, \cos\phi \cos t, 0)$

$\Rightarrow |\alpha'(t)| = R\cos\phi = C \Rightarrow \beta(s) = \alpha\left(\frac{t}{R\cos\phi}\right)$ is the unit speed parameterization

$\Rightarrow \beta(s) = R\left(\cos\phi \cos\frac{s}{c}, \cos\phi \sin\frac{s}{c}, \sin\phi\right)$

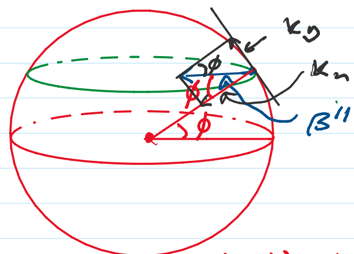
$\beta'(s) = \left(-\sin\frac{s}{c}, \cos\frac{s}{c}, 0\right)$

$\beta''(s) = \frac{1}{c}\left(-\cos\frac{s}{c}, -\sin\frac{s}{c}, 0\right) \leftarrow$ Pointing towards the z-axis horizontally

(b) projection of β'' onto the tangent plane

is $\|\beta''(s)\| \sin\phi = \frac{1}{c}\sin\phi$

$= \frac{1}{R\cos\phi}\sin\phi = \frac{\tan\phi}{R}$



(c) As $\phi \to \frac{\pi}{2}$ we expect $K_g \to \infty \leftarrow$ the smaller the circle, the bigger the curvature

and $\phi \to 0$ " " $K_g \to 0 \leftarrow$ since equator is a geodesic

And as $\phi \to \frac{\pi}{2}$, $\tan\phi \to +\infty \Rightarrow K_g \to \infty$ ✓

$\phi \to 0$, $\tan\phi \to 0 \Rightarrow K_g \to 0$ ✓

(d) K_n = comp. of β'' projected onto the unit normal

$= \frac{1}{c} \cdot \cos\phi = \frac{1}{R}$, which makes sense

since K_n is the normal curvature, that is the curvature of the great circle through that point, which is $\frac{1}{R}$.