# Homework 1

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**Theorem.** *For any natural number $n$, the number $n^2 - n$ is even.*

*Proof.* We have two cases: either $n$ is even or $n$ is odd.

Case 1: $n$ is even.
If $n$ is even, then we can represent $n = 2k$ for some integer $k$. Then, $n^2 - n = n(n-1)$ by factoring out $n$. Therefore, $n(n-1) = 2k(2k-1)$, which has a factor of 2, and is even.

Case 2: $n$ is odd.
If $n$ is odd, then we can represent $n = 2k+1$ for some integer $k$. Then, $n^2 - n = n(n-1)$ by factoring out $n$. Therefore, $n(n-1) = (2k+1)(2k) = 4k^2+2k = 2k(2k+1)$, which has a factor of 2, and is even.

Since both the cases are true, for any natural number $n$, the number $n^2 - n$ must be even. $\square$