Ma 410 - Differential Geomtery **Hw-1** **Due: September 14, 2021 by 11:59 pm**

Name (Printed): ______________________________

Pledge and Sign: ______________________________

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Find $\partial f/\partial x$ if $h = x^2 - yz$, and

(a) [3 pts.] $f = h(x + y, y^2, x + z)$

(b) [3 pts.] $f = h(e^z, e^{x+y}, e^x)$

2. Let $V_1 = U_1 - xU_3$, $V_2 = U_2$, and $V_3 = xU_1 + U_3$.

(a) [5 pts.] Prove that vectors $V_1(p)$, $V_2(p)$ and $V_3(p)$ are linearly independent at every point p.

(b) [5 pts.] Express the vector field $xU_1 + yU_2 + zU_3$ as a linear combination of V_1, V_2, V_3.

3. Let $V = y^2U_1 - xU_3$, and let $f = xy, g = z^3$. Compute the directional derivatives.

(a) [5 pts.] $V[g]$

(b) [5 pts.] $fV[g] - gV[f]$

4. [4 pts.] If $V[f] = W[f]$ for every function f on $\mathbb{R}^3$, prove that $V = W$.