

1 Homework 04

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. For variables X and Y which have joint density function given by:

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}xy, & 0 \leq x \leq y \leq 2 \\ 0, & \text{else} \end{cases}$$

Find the correlation coefficient for X and Y ($\rho_{X,Y}$).

2. For a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, the definition of expected value for continuous random variables is given by:

$$\mathbb{E}[X(\omega)] = \int_{\Omega} X(\omega) d\mathbb{P}(\omega)$$

Using this result prove the following:

- For random variables X and Y and scalar constants a and b , we have $\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$,
- the random variable 1, which is a degenerate random variable and is a constant, has expectation $\mathbb{E}[1] = 1$.

3. For variables X and Y which have joint density function given by:

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}xy, & 0 \leq x \leq y \leq 2 \\ 0, & \text{else} \end{cases}$$

(a) What is $\mathbb{P}(X \leq \frac{1}{4} | Y = 1)$?

(b) What is $\mathbb{P}(Y \leq \frac{3}{2} | X = 1)$?

4. For a random variable $X \sim N(\mu, \sigma^2)$,

- show that the third central moment is 0.
- show that the fourth central moment is $3\sigma^4$.

- what are the fifth and sixth standardized moments?

5. For a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, define $\Omega = \{a, b, c, d\}$ and let $\mathbb{P}(a) = \frac{1}{12}$, $\mathbb{P}(b) = \frac{1}{4}$, $\mathbb{P}(c) = \frac{1}{6}$, and $\mathbb{P}(d) = \frac{1}{2}$. Define the random variables:

$$X(\omega) = \begin{cases} 1, & \omega \in \{a, b\} \\ 0, & \omega \in \{c, d\} \end{cases}$$

$$Y(\omega) = \begin{cases} -2, & \omega \in \{a, c\} \\ -1, & \omega \in \{b, d\} \end{cases}$$

- (a) What is $\mathbb{E}[X|Y]$ and $\mathbb{E}[Y|X]$?
- (b) Define $Z = 2X + 4Y$. What is $\mathbb{E}[Z|Y]$ and $\mathbb{E}[Z|X]$?