Assignment 1.

There are total 22 points in this assignment. 18 points is considered 100%. If you go over 18 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers section 1.1, 1.3 in Bartle–Sherbert.

(1) [1pt] (1.1.10) Let $f(x) = 1/x^2$, $x \neq 0$, $x \in \mathbb{R}$. Determine the image $f(E)$ where $E = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$.

(2) [5pt] (1.1.22+) Let $f : A \to B$ and $g : B \to C$.
  (a) Show that if $g \circ f$ is injective, then f is injective. Give an example that shows that g need not be injective.
  (b) Show that if $g \circ f$ is surjective, then g is surjective. Give an example that shows that f need not be surjective.

(3) [2pt] (1.3.5) Exhibit (define explicitly) a bijection from N to the set of all odd integers greater than 2016.

(4) Exhibit (define explicitly) a bijection between
  (a) [2pt] $\mathbb{Z}$ and $\mathbb{Z} \setminus \{0\}$,
  (b) [3pt, optional[1]] $\mathbb{Q}$ and $\mathbb{Q} \setminus \{0\}$.

(5) [3pt] (1.3.12) Show that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.

(6) [3pt] Show that the collection $\mathcal{I}(\mathbb{N})$ of all *infinite* subsets of $\mathbb{N}$ is uncountable. (*Hint:* Use the previous problem and fact that union of two countable sets is countale..)

(7) [3pt] A number is called *algebraic* if it is a root of a non-constant polynomial with integer coefficients. (For example, $\sqrt{3}$ is algebraic because it's a root of $x^2 - 3$.) Show that the set of all algebraic numbers is countable.
(*Hint:* You can take for granted that a polynomial of degree N has not more than N roots..)

[1] That is, not included in the denominator of the grade.