Q2

we'll use theorem 1.4 of sec. 4.1

(a) $g = (x^2+y^2)^2 + 3z^2 = 1$

$\Rightarrow \nabla g = (g_x, \; g_y, \; g_z)$

where

$g_x = 2(x^2+y^2)(2x)$

$g_y = 2(x^2+y^2)(2y)$

$g_z = 6z$

so $\nabla g = \vec{0} \Rightarrow g_x = g_y = g_z = 0$

$g_z = 0 \Rightarrow z = 0$

$g_x = 0 \Rightarrow \begin{cases} \text{either } x^2+y^2=0 & \text{not possible } x^2+y^2=0, z=0 \text{ not on } g=1 \\ \text{or } x=0 \end{cases}$

$g_y = 0 \Rightarrow \begin{cases} \text{either } x^2+y^2=0 & \text{not possible as above} \\ \text{or } y=0 \end{cases}$

$\Rightarrow$ so the only possibility is $x=y=z=0$

but $x=y=z=0$ is not on $g=1$

$\Rightarrow \nabla g \neq \vec{0}$ for all points $g=1$

$\Rightarrow g=1$ defines a surface.

b)

let $g = z(z-2) + xy = c$

$\Rightarrow g_x = y$

$g_y = x$

$g_z = 2z - 2$

so $\nabla g = \vec{0} \iff x = 0$ and $y = 0$ and $z = 1$

$\iff g(x,y,z) = -1$

so for any $c \neq -1$, $g = c$ defines

a surface.