1 Homework 02

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. For two fair six-sided dice, roll the two dice and observe the results.
   (a) What is Ω for this experiment?
   (b) Let X be the smallest of the two die and let Y be the value of the largest (for a tie, $X = Y$). Are X and Y independent? Hint: create the joint distribution and use it to get the marginal mass functions and check independence.

2. For a joint mass function of X and Y given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{70}x(x+y), & x = 1,2,3; y = 3,4 \\ 0, & \text{else} \end{cases}$$

   (a) Determine the marginals $f_X(x)$ and $f_Y(y)$.
   (b) Find $\mathbb{E}[X]$ and $\mathbb{E}[Y]$.

3. In the lecture, we discussed Buffon's needle problem with the lines spaced 2 units apart and a needle of length 1. Resolve this problem (as in what is the probability that the needle crosses a line), but this time set the lines to be L units apart and the needle to be of length ℓ. You can assume that $L > 0$, $\ell > 0$, and $\ell < \frac{L}{2}$.

4. Jeff and Scott will arrive independently at a coffeehouse (of course socially distanced) randomly between noon and 1pm. What is the probability that they arrive within 10 minutes of each other? Hint: To approach this, you can either split the region into integrable regions or to look to proportions.

5. For a joint density function:

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 < x < y, 0 < y < 1 \\ 0, & \text{else} \end{cases}$$

   Are X and Y independent?

