Q1

$x(u,v) = (u^2, uv, v^2)$, $u>0$, $v>0$

$x(u_1,v_1) = x(u_2,v_2) \Rightarrow (u_1^2, u_1v_1, v_1^2) = (u_2^2, u_2v_2, v_2^2)$

$\Rightarrow u_1^2 = u_2^2 \Rightarrow |u_1| = |u_2| \overset{u>0}{\Longrightarrow} u_1 = u_2$

. The same for $v_1 = v_2$

$\Rightarrow$ x is 1-1.

$x(u,v) = (u^2, uv, v^2)$

$x^{-1}(u_1, u_2, u_3) = (\sqrt{u_1}, \sqrt{u_3})$

$x \circ x^{-1} : (u_1, u_2, u_3) \xmapsto{x^{-1}} (\sqrt{u_1}, \sqrt{u_2}) \xmapsto{x} (u_1, \sqrt{u_1u_3}, u_3)$

when $u_2 = \sqrt{u_1u_3}$

$x^{-1} \circ x : (u, v) \xmapsto{x} (u^2, uv, v^2) \xmapsto{x^{-1}} (u, v)$

$u>0, v>0$

. note that x^{-1} is smooth on $x(D)$

since $\sqrt{u}$ is smooth if $u>0$.

. $x_u \times x_v = \begin{vmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix} = 2v^2\vec{u}_1 - 4uv\vec{u}_2 + 2u^2\vec{u}_3 \neq \vec{0}$

since $u>0$, $v>0$

so x is regular.

$\Rightarrow$ x is a proper patch.

Q2

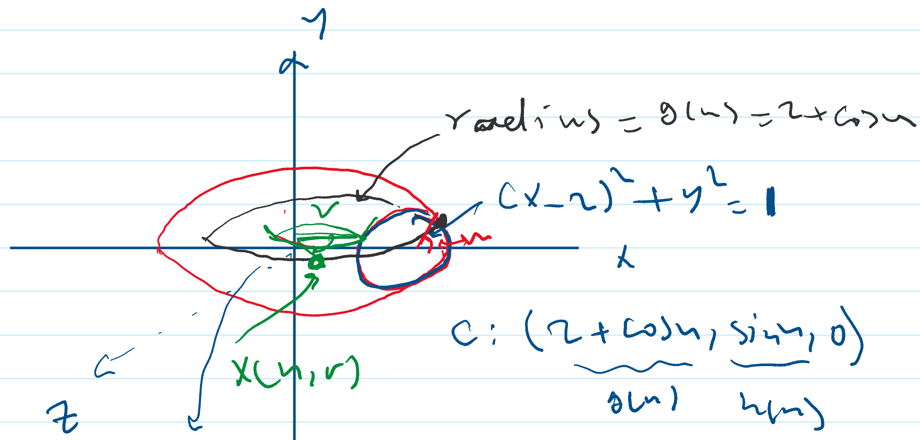
(a) C is parameterized as

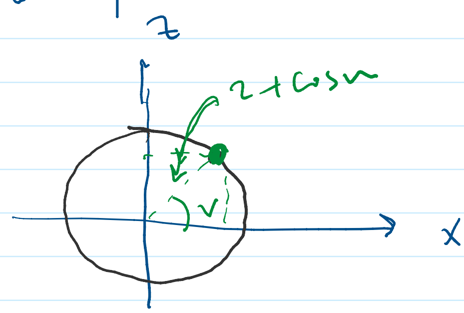
$$\alpha(u) = (\underbrace{u}_{g(u)}, \underbrace{\cosh u}_{h(u)}, 0)$$

$$\Rightarrow \quad x(u,v) = (u, \cosh u \cos v, \cosh u \sin v)$$

from the discussions in class

(b)





$$\Rightarrow \quad x(u,v) = ((2+\cos u)\cos v, \sin u, (2+\cos u)\sin v)$$

Q3

x is regular $\iff x_u \times x_v \neq \vec{0}$ at all points

$\iff \underbrace{(v\,\delta'(u))}_{x_u} \times \underbrace{\delta(u)}_{x_v} \neq \vec{0}$

$\iff v\,(\delta'(u) \times \delta(u)) \neq \vec{0}$

$\iff v \neq 0$ and $\delta'(u) \times \delta(u) \neq \vec{0}$