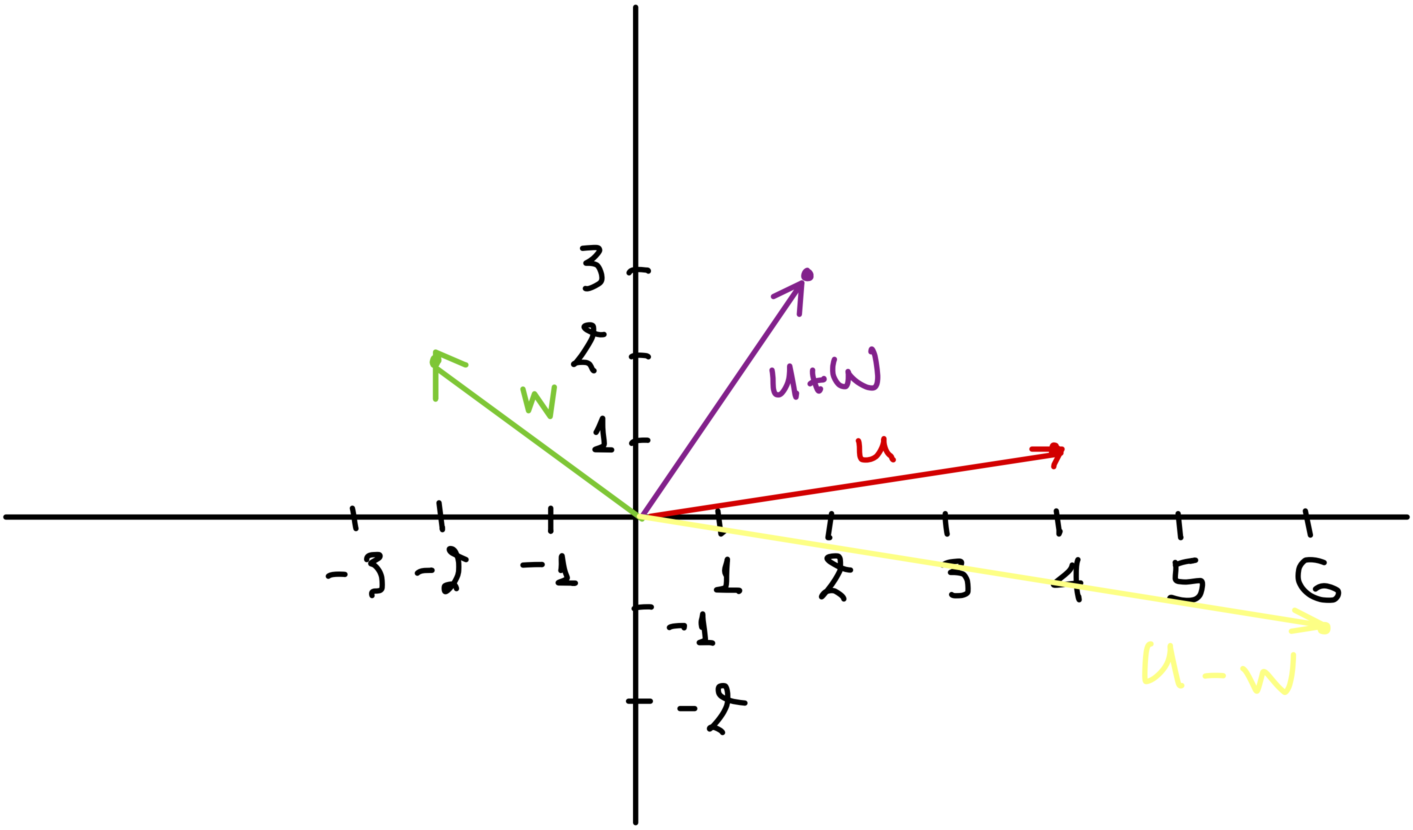
MA 232 - Linear Algebra

Homework 1 (Solutions)

Problem 1 [20pts] Draw $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $(u + w)$, $(u - w)$ in the plane.



Problem 2 [20pts] Find vectors u and w such that $u + w = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ and $u - w = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$.

$$(u+w) + (u-w) = \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix}$$

$$\Rightarrow \quad 2u = \begin{bmatrix} 6 \\ 10 \\ 14 \end{bmatrix} \Rightarrow u = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$$w = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Problem 3 [20pts] Find two vectors u and w which are perpendicular to $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and to each other.

Let $u = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $w = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$$u \cdot \begin{bmatrix}1\\0\\1\end{bmatrix} = 0 \Rightarrow a_1 + a_3 = 0$$

$$w \cdot \begin{bmatrix}1\\0\\1\end{bmatrix} = 0 \Rightarrow b_1 + b_3 = 0$$

$$u \cdot w = 0 \Rightarrow a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$$

Choose randomly $a_1 = a_3 = 0$

Then $b_1 + b_3 = 0$ & $a_2 b_2 = 0$

Since we don't want the trivial vector we choose $a_2 = 1$, $b_2 = 0$

and $b_1 = 1$, $b_3 = -1$ $u = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $w = \begin{bmatrix}1\\0\\-1\end{bmatrix}$

Problem 4 [20pts] How long is the vector $u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$?

$$\|u\| = \sqrt{u \cdot u} = \sqrt{1^2 + 1^2 + 1^2 + 1^2 + 1^2}$$

$$= \sqrt{5}$$

Problem 5 [20 pts] Consider the following system of equations: $\begin{cases} 2x + 3y + z = 8 \\ 4x + 7y + 5z = 20 \\ -2y + 2z = 0 \end{cases}$

(i) Apply Gauss Elimination in order to solve it;

(ii) Transform the above system of equations in matrix form and apply the Gauss Elimination in matrix form (indicate all matrices you used in the process).

(i)
$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{array}\right] \sim \begin{aligned} 2x + 3y + z &= 8 \\ y + 3z &= 4 \\ 8z &= 8 \end{aligned}$$

$$z = 1, \; y = 1, \; x = 2$$

(ii)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right] \right)$$