1   Homework 11

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. For times s and t such that $s < t$, answer the following:

   (a) For the process $X(t) = 3W(t) - 2W(s)$, what is $\mathbb{E}[X(t)]$?

   (b) What is the variance of $X(t)$?

2. For a Brownian motion, let τ_m be the first passage time to level m. Please answer the following:

   (a) $\mathbb{P}(\tau_3) < 5$

   (b) $\mathbb{P}(\tau_{-4} < 8$

   (c) $\mathbb{P}(\tau_2) > 6$

3. Assume that we have a probability space created by a repeatable experiment with one of three possible outcomes $\Omega_i = \{a, b, c\}$, where each time is independent of every other. We can define a new type of random walk as

$$L(n) = \sum_{i=1}^{n} Y_i$$

   where

$$Y_i(\omega_i) = \begin{cases} 1, & \omega_i = a \\ 0, & \omega_i = b \\ -1, & \omega_i = c \end{cases}$$

   (a) For any set of probabilities, prove this process is Markov (hint: you might want to look up either the multinomial distribution or more specifically the trinomial distribution to help you with this)

   (b) Under what set of probabilities is this process a Martingale?

   (c) For probabilities $p_a = \mathbb{P}(\omega = a)$ and similarly for p_b and p_c, express the random variable $V_L^2(0, n)$ and determine its mean and variance. (Hint for this, you are better off starting with the fact that this can be modeled as a binomial random variable).