

## 1 Homework 10

You will find all the problems for this homework in this document. You are responsible for uploading a pdf document with all of your results and the necessary work to the Canvas shell for the class. Please make sure that your homework pdf is legible, clear, and pledged.

1. We have a random walk with probability of getting a heads on any toss being  $\frac{1}{3}$ , where the random walk is defined as

$$L(n) = \sum_{i=1}^n Y_i, \text{ where } Y_i = \begin{cases} 3, & \omega_i = H \\ -1, & \omega_i = T \end{cases}$$

with  $L(0) = 0$ , and will stop at time  $N = 5$ . In addition, you have a stopping time,  $\tau$ , which describes the first time that you have at least one head and one tail (in the path up to that point). Define the process  $Y(n) = L(n \wedge \tau)$ .

- (a) Express the random variable  $\alpha = V_Y^1(0, 4)$
  - (b) What is  $\mathbb{E}[\alpha]$ ?
2. For a function  $f(x) = pe^x + qe^{-x}$  where  $p$  and  $q$  are non-negative real numbers such that  $p + q = 1$ , for what value of  $x$  does this function take its minimum value?
  3. For a Brownian motion  $W(t)$ , answer the following:
    - (a) Find  $\mathbb{P}(W(t) > 4)$
    - (b) Find  $\mathbb{P}(W(5) > 2)$
  4. For a Brownian motion  $W(t)$ , and times  $0 < s < t < u$  answer the following:
    - (a) Find  $\mathbb{E}[W^2(t)W(s)|\mathcal{F}(s)]$
    - (b) Find  $\mathbb{E}[W^3(u) + W^2(t) + W(s)]$
    - (c) Find  $\mathbb{E}[W^4(u)] - \mathbb{E}[W^2(u)]^2$