Ma 410 - Differential Geometry Hw-10 Due: November 30, 2022 by 11:59 pm

Name (Printed):

Pledge and Sign:

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. [10 pts.][Prove Lemma 5.1 of Section 5.5]. Let Z be a nonvaishing normal vector field on M. If V and W are tangent vector fields such that $V \times W = Z$, then

$$K = \frac{Z \cdot (\nabla_V Z \times \nabla_W Z)}{\|Z\|^4},$$

$$H = -Z \cdot \frac{\nabla_V Z \times W + V \times \nabla_W Z}{2\|Z\|^3}$$

2. [10 pts.] Use the formulas from Q1 to find Gaussian curvature K and mean curvature H for the saddle surface $z = xy$. [Hint: Take $g(x, y, z) = xy - z$ and let $Z = \nabla g$.]

3. Which of the following curves on a torus with axis of symmetry the z-axis is a geodesic? Why? Assume the curves have unit speed.

(a) [5 pts.] The top circle.

(b) [5 pts.] The outer equator.