Homework #1: Interest rates, bonds and credit risk

FE-620 Fall 22

Due 30 September 2022

Problem 1.1
Assume that an interest rate is quoted as 3.75% with annual compounding. What is the rate when expressed with:

i) semi-annual compounding
ii) quarterly compounding
iii) continuous compounding?

Problem 1.2

Credit card companies quote the APR on the outstanding balance. APR means Annual Percentage Rate and is the interest rate with annualized compounding. Typical APRs are shown in the Table below. However, credit card interest rate is compounded *daily*, for 365 days a year.

Suppose the balance on a credit card is $1000. What is the total balance including interest after 30 days, if the customer has excellent credit?

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Table 1: Current Credit Card Interest Rates. As of Sep 2022, from `https://wallethub.com/edu/cc/credit-card-interest-rates/52541`.

Category	Interest Rate
Excellent Credit	14.98%
Good Credit	21.04%
Fair Credit	23.87%
Store Credit Cards	26.57%
Introductory APR	19.86%

Problem 1.3

This is an application of the bootstrapping procedure discussed in class for determining the zero rates from bond prices. We are given the prices of four bonds with maturities and coupons shown in Table 2. Determine the zero rates $R(T)$ from the prices of these bonds. All bonds pay coupons every six months.

Recall that for this type of problem we assume that the zero rate $R(T)$ is piece-wise constant on the time interval between the maturities of the bonds used for bootstrapping. For example, $R(T)$ has the same value for all $T : [0, 1Y]$, another value for $T : (1Y, 2Y]$, and so on.

Table 2: Bond data for Problem 1.3. All bonds pay coupons every six months.

Bond principal	Maturity	Coupon	Bond price
\$100	1Y	1.25%	100.196
\$100	2Y	2.25%	100.190
\$100	5Y	2.50%	101.142
\$100	10Y	3.15%	101.422

Problem 1.4

The spot price of copper is \$7,060 per tonne. The forward price for 1 year maturity is \$7,511. The risk-free interest rate is $r = 5.0\%$.

i) Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage. Neglect the storage cost.

ii) Consider now the case when the forward price is \$7,415. Is there an arbitrage opportunity?

Problem 1.5

A US company is due to make a payment of 1.5m Euros in 6 months. The spot exchange rate for EUR-USD is 1.00207, and the 6 month forward exchange rate is 1.01546[1]. This exchange rate is expressed as the number of USD for 1 EUR.

i) What is the present value of the payment expressed in USD? Assume that the risk-free rate in USD is $r_{USD} = 1.0\%$ for all maturities.

ii) What is the interest rate differential between the USD and EUR risk-free rates implied by the observed FX forward rate?

[1]From `https://www.fxempire.com/currencies/eur-usd/forward-rates`

Solution 1.1 We convert from annualized rates r_1 to rates compounding frequency m using the formula

$$1 + r_1 = \left(1 + \frac{r_m}{m}\right)^m$$

Conversion to continuous compounding is done with the following formula

$$1 + r_1 = e^{r_c}$$

We get, with $r_1 = 0.0375$:
i) semi-annual compounding $m = 2$

$$r_2 = 2[(1 + r_1)^{1/2} - 1] = 3.715\%$$

ii) quarterly compounding $m = 4$

$$r_4 = 4[(1 + r_1)^{1/4} - 1] = 3.689\%$$

iii) continuous compounding

$$r_c = \log(1 + r_1) = 3.681\%$$

Solution 1.2 The total balance after 30 days is obtained by compounding the initial amount \$1000 with daily compounding at the daily rate

$$r_{365} = 365[(1 + 0.1498)^{1/365} - 1] = 0.1396 \tag{1}$$

which is thus 13.96%.
The total balance is

$$1000(1 + r_{365}/365)^{30} = 1011.54 \tag{2}$$

Solution 1.3 The problem can be solved e.g. using the Excel spreadsheet *BondPricing.xls*. The solutions for $R(T)$ are shown in Table 3.

Solution 1.4

Table 3: Solutions for $R(T)$ for Problem 1.3.

T	$R(T)$
1Y	1.05%
2Y	2.15%
5Y	2.25%
10Y	3.00%

The fair forward price is

$$F(T) = S_0 e^{rT} = 7421.97$$

i) We have $F = 7,511 > F(T)$. We can realize the arbitrage by going short the forward contract, borrowing cash and purchasing the asset at $t = 0$.

ii) For this case $F < F(T)$. We can realize the arbitrage by going long the forward contract, short the asset and invest the cash.

Problem 1.5.

i) The price in USD of the payment of $N_{EUR} = 1.5m$ EUR can be expressed as

(3)
$$PV_{USD} = N_{EUR} \times X_{fwd}(T) \times 4^{-r_{USD}T} = 1.5m \times 1.01546 \times e^{-0.01 \cdot 0.5} = 1.5156m \text{ USD}$$

ii) The interest rate differential can be extracted from the formula for the forward FX rate

$$X_{fwd}(T) = X_0 e^{(r_{USD} - r_{EUR})T} \tag{4}$$

We get

$$r_{USD} - r_{EUR} = \frac{1}{0.5} \ln \frac{1.01546}{1.00207} = 0.0265 \tag{5}$$

The interest rate differential USD - EUR is 2.65%.