Ma 410 - Differential Geometry **Hw-11** **Due: December 7, 2022 by 11:59 pm**

Name (Printed):

Pledge and Sign:

Upload solutions to Grade Scope by the due date. Assign solution pages to corresponding problems. You need to pledge and sign on the cover page of your solutions. You may use this page as the cover page.

Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.

1. Let $G : M \to S^2$ be the Gauss (spherical) map of the orientable surface M. In each case draw the image of M using its upward unit normal.

(a) [4 pts.] M is the cylinder: $x^2 + y^2 = r^2$.

(b) [4 pts.] M is the cone $z = \sqrt{x^2 + y^2}$.

(c) [4 pts.] M is the plane $x + y + z = 0$.

(d) [3 pts.] Use your answers to the above to explain why in all cases above Gaussian curvature $K(p) = 0$. Use the following definition of K: If p is a point on M in the interior of a shrinking region with area ΔA, and $\Delta \tilde{A}$ is the area of Gauss (spherical) image of the above region under G, then

$$K(p) = \lim_{\Delta A \to 0} \frac{\Delta \bar{A}}{\Delta A}$$

2. [15 pts.] On the sphere of radius R, suppose a particle travels at unit speed curve α along the circle of (constant) latitude ϕ.

(a) Find the acceleration of the particle. Draw it on a latitude circle. [Don't forget to find the unit speed parameterization for α.

(b) Sketch the projection of this acceleration onto the tangent of the sphere and deduce that the geodesic curvature $\kappa_g = \dfrac{\tan\phi}{R}$

(c) Verify that this formula for κ_g yields the geometrically correct answer when $\phi \to 0$ and $\phi \to \pi/2$.

(d) Show that the component of the acceleration directed towards the center of the sphere has magnitude $1/R$, independent of ϕ. Explain this geometrically.