Homework #4 - Lectures 8 and 9: Option pricing in continuous time, BSM model, Greeks

FE-620

October 28, 2022

Problem 4.1 (14.16 in Hull)
Assume that the stock price S_t follows the geometric Brownian motion process

$$\frac{dS_t}{S_t} = \mu dt + \sigma dz_t \,. \tag{1}$$

What is the process followed by:

(a) $y = 2S$

(b) $y = S^2$

(c) $y = \sin(S)$

(d) $y = \frac{1}{S}e^{r(T-t)}$

Note: This is to get some practice applying Itô's lemma. Quant job/internship interviews often contain such a problem.

Problem 4.2 (Evaluating the Black-Scholes formula)
Consider an European option on a non-dividend-paying stock. Currently the stock price is \$34, the option exercise price is \$33, the risk-free interest rate is 5.5%, the volatility is 25% per annum, and the time to maturity of the option is 6 months.

Using the Black-Scholes European option price formula, evaluate the price of this option, assuming that:

(a) it is an European call option

(b) it is an European put option

(c) Verify that put-call parity holds

Problem 4.3(variation on 15.28 in Hull)
The closing prices of Tesla stock for the month of February 2022 are given in the table below.

Estimate the annualized stock price volatility. What is the standard error of your estimate?

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> head(price,30)
           TSLA.Close
2022-02-01     931.25
2022-02-02     905.66
2022-02-03     891.14
2022-02-04     923.32
2022-02-07     907.34
2022-02-08     922.00
2022-02-09     932.00
2022-02-10     904.55
2022-02-11     860.00
2022-02-14     875.76
2022-02-15     922.43
2022-02-16     923.39
2022-02-17     876.35
2022-02-18     856.98
2022-02-22     821.53
2022-02-23     764.04
2022-02-24     800.77
2022-02-25     809.87
2022-02-28     870.43
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Problem 4.4(Variation on 19.26 in Hull)
Consider a 1-year European call option on a stock when the stock price is \$28, the strike price is \$28, the risk-free rate is 2.0%, and the volatility is 25% per annum. Compute the price, delta, gamma and vega of the option under the Black-Scholes-Merton model.

i) Verify that delta is correct by changing the stock price to \$28.1 and recomputing the option price.

ii) Verify that gamma is correct by recomputing the delta for the situation where the stock price is \$28.1.

Problem 4.5 (Delta hedging an option position)
A trader sells a 1-year maturity European call option on TSLA stock on 1-Feb-2022, with strike \$932.00, and Delta hedges it by taking a long position in the stock. The trader adjusts the hedge dynamically each day of the month of February, assuming the Black-Scholes model with implied volatility $\sigma = 60\%$ for the computation of Delta.

The trader unwinds the hedge (sells the stock position) after the market close on 28-Feb-2022. What is the profit/loss of the hedging strategy? Assume zero interest rate.

Hint. You can modify the Delta hedging spreadsheet in Canvas for this problem.