Lecture 2 — Formula Sheet

Vector Spaces & Subspaces (over R)
- Vector space: closed under + and scalar multiplication; addition has identity (0) and inverses.
- Subspace U of V: $U \neq \emptyset$, closed under + and scalar multiplication.

Linear Combination, Independence, Span, Basis, Dimension
- Linear combo: $\sum(\lambda_i x_i)$
- Independence: $\sum \lambda_i x_i = 0 \Rightarrow$ all $\lambda_i = 0$
- $\mathrm{Span}[x1,\ldots,xk] = \{ \sum \lambda_i x_i \}$
- Basis: independent set that spans the space
- Dimension: number of vectors in any basis

Determinant Essentials
- Row swap ⇒ det changes sign
- Scale a row by λ ⇒ det scales by λ
- Row replacement ($R_i \leftarrow R_i + \lambda R_j$) ⇒ det unchanged
- 2×2: det[[a,b],[c,d]] = a*d − b*c
- 2×2 inverse (if det ≠ 0): (1/(ad−bc)) * [[d, −b], [−c, a]]

Sarrus' Rule (3×3 only)
For A = [[a,b,c],[d,e,f],[g,h,i]]:
det(A) = (a*e*i + b*f*g + c*d*h) − (c*e*g + a*f*h + b*d*i)

Nullspace (kernel)
- $N(A) = \{ x : A x = 0 \}$ is a subspace
- Solve via Gaussian elimination; express in terms of free variables to get a spanning set