PH-EP-Tech-Note-2010-013
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22/07/2010

Calculation of radiation length in materials

Abstract

We introduce an analytical way of finding the radiation length of elements and composites. The values are compared to experimentally measured ones for a selected number of them. Since the values match well, we conclude that the formulas are quite reliable when experimental data is not available. We also note that the bigger the radiation length of a material, the more "transparent" to radiations it is.

1. Introduction

- **Definition:** The radiation length of a material is the mean length (in cm) to reduce the energy of an electron by the factor 1/e.
- **Physical explanation:** An electron arriving in the vicinity of an atom will be affected by the electromagnetic field produced by the electrons of this atom. Because of this interaction, the electron will emit photons which will reduce its energy. This is called the Bremsstrahlung radiation. It is clear that this interaction will depend on the number of electrons of the atom (atomic number Z) but also the size of the atom, represented by its atomic weight A.

2. Calculation of radiation length for an element

The radiation length can be approximated by the following analytical formula.

$$Xo = \frac{716.4}{Z(Z+1)\ln\frac{287}{\sqrt{Z}}}\ g.cm^{-3}$$

Often the radiation length is expressed in $g.cm^{-2}$. We can obtain the result in cm by dividing through by the density.

Since this formula is only an approximation, it should only be used when data is not available. Values of radiation lengths can be found from the Particle Data Group website, where most of the data is measured experimentally. (http://pdg.lbl.gov/2009/AtomicNuclearProperties/). Refer to Table 1 of the Appendix for a comparison of the two values.

3. Radiation length in a composite material

If we have a component with a number of different materials, we can also estimate the combined radiation length of the sample. Results comparing analytical values and values from the website can be found in Table 2 of the Appendix. The general formula for the radiation length of composite materials is:

$$\frac{Wo}{Xo} = \sum \frac{Wi}{Xi}$$

Wo is the total mass of the sample in g.

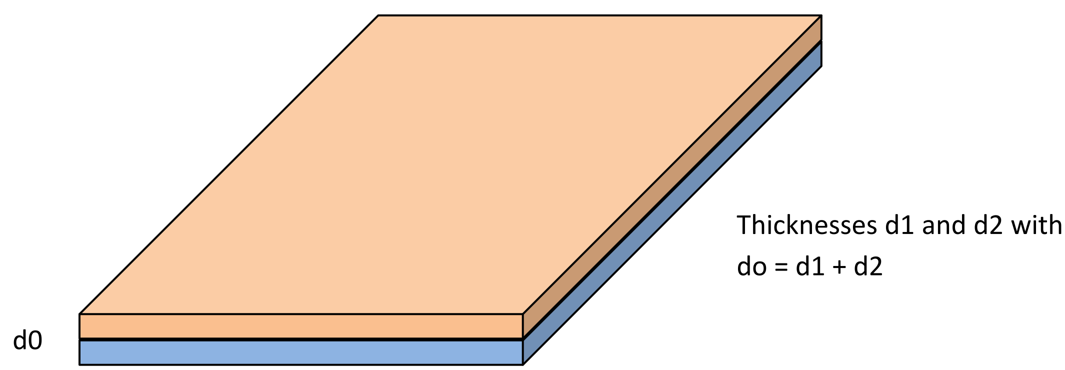
Xo is the combined radiation length of the sample in g.cm^{-2}.

Wi is the mass of the individual component in g.

Xi is the radiation length of the individual component in g.cm^{-2}.

Example 1:

We consider a slab made of two different materials.



The equation becomes:

$$\frac{do\rho o}{Xo} = \frac{d1\rho 1}{X1} + \frac{d2\rho 2}{X2}$$

With $$do\rho o = d1\rho 1 + d2\rho 2$$

We can calculate Xo. We can also apply the same method if we know the chemical composition of a certain material.

Example 2

Consider the epoxy of composition $C_{60}H_{79}N_2O_3$ and of density 1.33 g.cm^{-3}.

1. We want to find the radiation length of the epoxy.

2. We add diamond powder to the epoxy in weight ratio 75% diamond 25% epoxy. Diamond has radiation length 42 g.cm^{-2} and density 3.52 g.cm^{-3}. We want to find the new radiation length.

1. The equation becomes:

$$\frac{AoNo}{Xo} = \sum \frac{AiNi}{Xi}$$

Ao is the average atomic mass in g/mol.

No is the total number of moles.

Ai is the atomic mass of each constituent atom in g/mol.

Ni is the number of moles of each component.

No = 60+79+2+3 = 144 moles

Ao = (60*12 + 79*1 + 14*2 + 16*3)/144 = 6.076 g/mol

From the equation we get:

$$\frac{6.076 * 144}{Xo} = \frac{60 * 12}{42.7} + \frac{79 * 1}{61.28} + \frac{14 * 2}{37.99} + \frac{16 * 3}{34.24}$$

Xo = 43.25 g.cm^{-2} or **Xo = 32.5 cm**

2. If we take 1g of this new glue, we have: m1 = 0.75g (diamond) and m2 = 0.25g (epoxy)

$$\frac{1}{\mathrm{Xo}} = \frac{0.75}{42.7} + \frac{0.25}{43.25}$$

Xo = 42.8 g.cm^{-2}

$$mo = m1 + m2$$

$$Vo\rho o = V1\rho 1 + V2\rho 2$$

$$with\ Vo = V1 + V2$$

$$V1 = \frac{0.75}{3.52} = 0.188\ cm^3 \text{ and V2 } = \frac{0.25}{1.32} = 0.213\ \text{cm}^3$$

$$\rho o = 2.49\ g.cm^{-3}$$

Xo = 17.6 cm

Here we assumed that the density of the mix is the average of the densities of the two components. This is true if there is no chemical reaction between the two elements. However, if there are some chemical reactions with some new molecules or polymers that are formed, the density may vary. For example if the reactions are exothermic and water evaporates, there is mass that escapes. Therefore, the density of the mixed sample should be measured experimentally for better accuracy.

4. Number of radiation length in a component

If we consider a slab of material, we could be interested in calculating the number of radiation lengths N through the thickness of the slab.

$$N = \frac{do}{Xo}$$

<u>Note</u>: The number of radiation lengths is also additive. For the case of **Example 1**, we would have

N = N1 +N2.

Appendix

Table 1: Comparison of values of radiation length for elements obtained experimentally and analytically

Element	Z	A [g/mol]	Rad. Length (expt.) [g.cm^{-2}]	Rad. Length (analyt.) [g.cm^{-2}]	Error [%]
H	1	1.00794	63.04	63.79	1.2
He	2	4.0026	94.32	89.95	4.63
C	6	12.0108	42.7	43.01	0.72
N	7	14.0067	37.99	38.23	0.64
O	8	15.9994	34.24	34.46	0.64
F	9	18.9984	32.93	33.16	0.69
Ne	10	20.1797	28.93	29.15	0.77
Na	11	22.9897	27.74	27.97	0.84
Mg	12	24.305	25.03	25.27	0.96
Al	13	26.9815	24.01	24.26	1.06
Si	14	28.0855	21.82	22.08	1.18
P	15	30.9737	21.21	21.47	1.25
S	16	32.065	19.5	19.76	1.35
Cl	17	35.453	19.28	19.56	1.47
Ar	18	39.948	19.55	19.86	1.57
K	19	39.0983	17.32	17.6	1.64
Ca	20	40.078	16.14	16.43	1.78
Ti	22	47.867	16.16	16.47	1.94
Cr	24	51.9961	14.94	15.25	2.09
Fe	26	55.845	13.84	14.14	2.17
Ni	28	58.6934	12.68	12.97	2.27
Cu	29	63.546	12.86	13.16	2.34
Zn	30	65.38	12.43	12.72	2.35
Ag	47	107.868	8.97	9.17	2.26
Pt	78	195.084	6.54	6.52	0.38
Au	79	196.967	6.46	6.43	0.53
Pb	82	207.2	6.37	6.31	0.93

Table 2: Comparison of values of radiation length for molecules obtained experimentally and analytically

Molecule	**Rad. Length (expt.) [g.cm^{-2}]**	**Rad. Length (analyt.) [g.cm^{-2}]**	**Error [%]**	**Density [g.cm^{-3}]**
H20	36.08	36.04	0.1	1
Al2O3	27.94	27.94	0	3.97
$CaCO_3$	24.03	24.02	0.04	2.8
CO2	36.2	36.2	0	1.84E-03
Freon-12 (CF2Cl2)	23.65	23.65	0	1.12
Cellulose acetate (C6H7O2(OCOCH3)3)n	38.75	39.11	0.93	1.42
Cellulose acetate butyrate (C15H22O8)n	39.75	39.76	0.03	1.2
Polyethylene ($[CH_2CH_2]_n$)	44.77	44.78	0.02	0.89