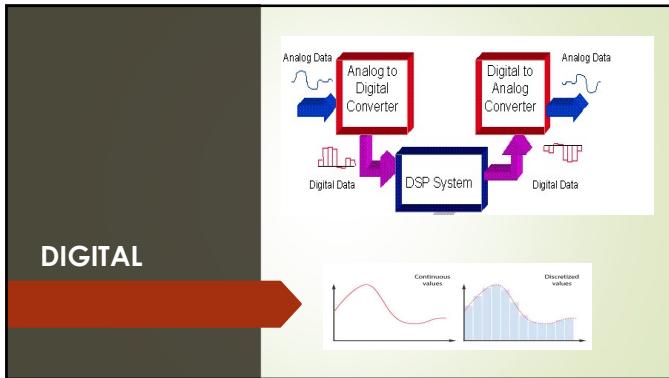


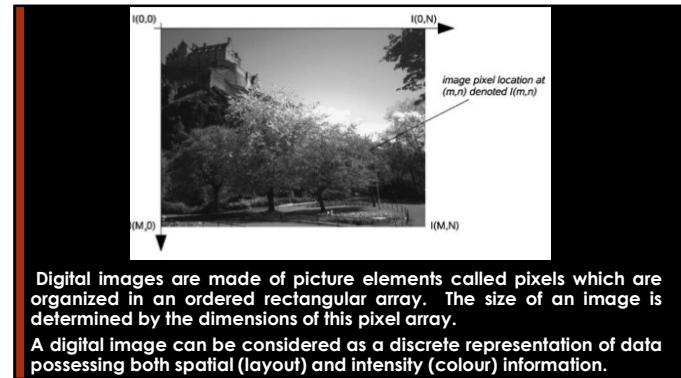
1



2



3

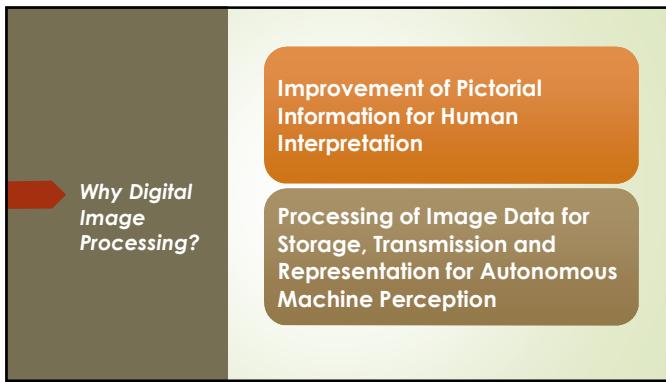


4

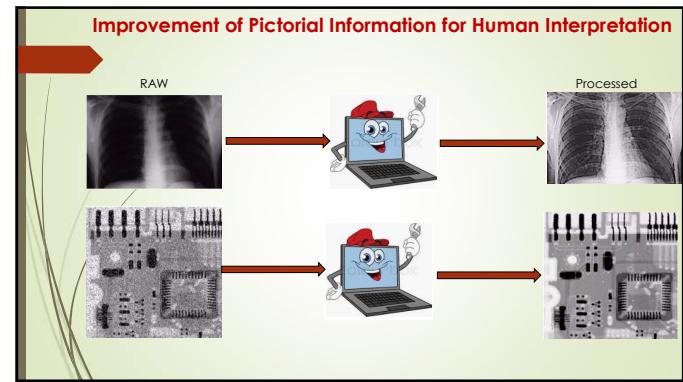


5

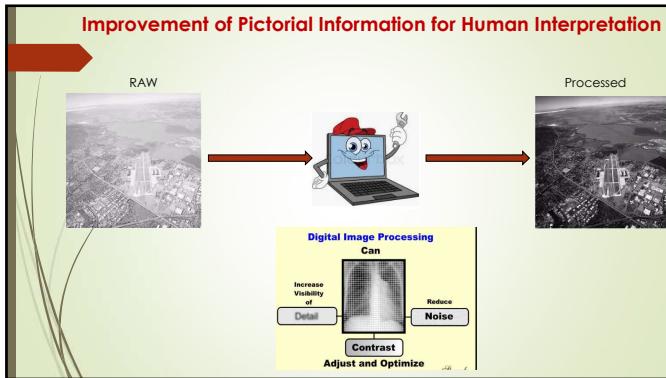




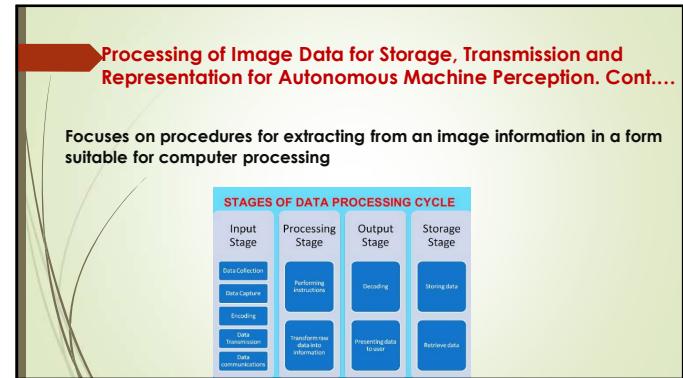
7



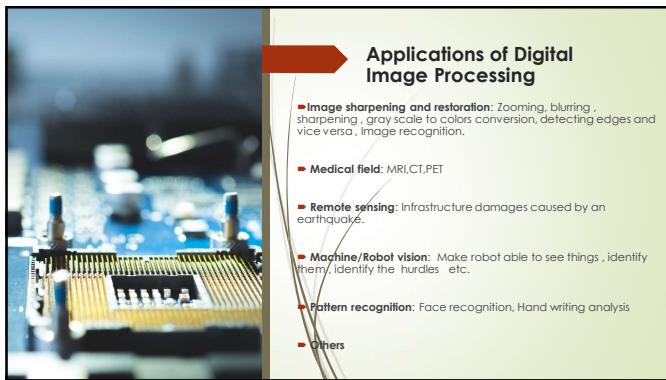
8



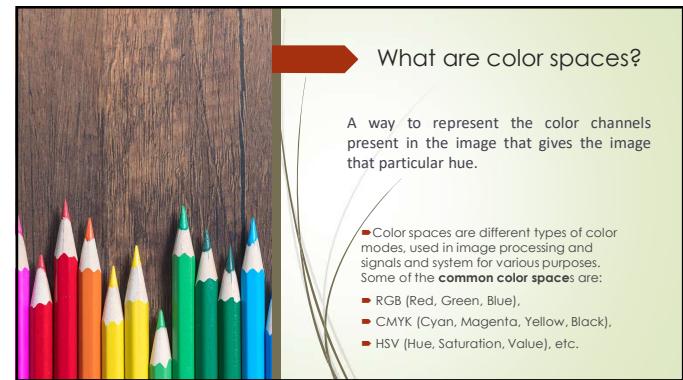
9



10



11



12

Color spaces : RGB Images

Each pixel coordinate (x, y) contains 3 values ranging for intensities of 0 to 255 (8-bit)
- Red - Green - Blue

Mixing different intensities of each color gives us the full color spectrum.

One Pixel Consist With 3 Channels / Layers

R channel Range: 0 ~ 255
G channel Range: 0 ~ 255
B channel Range: 0 ~ 255

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Color spaces : RGB Images

The RGB color model is additive: red, green, and blue light are added together in varying proportions to produce an extensive range of colors.

RGB image is simply a composite of three independent grayscale images that correspond to the intensity of red, green, and blue light.

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The CMY Color Model

- any color is written as a sum of the primary colors R(ed), G(reen) and B(lue):
 $Color = rR + gG + bB, \quad r, g, b \in [0, 1]$ (1)
- any color is written as a sum of the primary colors C(yan), M(agenta) and Y(ellow):
 $Color = cC + mM + yY, \quad c, m, y \in [0, 1]$ (2)
- subtractive model

Primary colors: red, green and blue

$$\begin{pmatrix} C \\ M \\ Y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

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The HSV Color Model

Hue is the color portion of the model, expressed as a number from 0 to 360 degrees:
Saturation describes the amount of gray in a particular color, from 0 to 100 percent. Reducing this component toward zero introduces more gray and produces a faded effect. Sometimes, saturation appears as a range from 0 to 1, where 0 is gray, and 1 is a primary color
Value works in conjunction with saturation and describes the brightness or intensity of the color, from 0 to 100 percent, where 0 is completely black, and 100 is the brightest and reveals the most color.

Figure 1. (a) HSV coordinates system (b) HSV color model

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Important terms

- Resolution.
- Pixel Depth
- Dynamic Range

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Resolution and Bit depth

Resolution is the ability of the imaging system to produce the smallest discernible details.

Spatial Resolution depends on two parameters: **Number of Pixels** and **bit depth** (number of bits necessary for adequate intensity resolution)

BIT DEPTH is determined by the number of bits used to define each pixel. The greater the bit depth, the greater the number of tones (grayscale or color) that can be represented.

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Measuring Spatial Resolution

A bitonal image: It is represented by pixels consisting of 1 bit each, which can represent two tones (typically black and white), using the values 0 for black and 1 for white or vice versa. To represent two values one bit is sufficient.

A grayscale image: is composed of pixels represented by multiple bits of information, typically ranging from 2 to 8 bits or more. (bit depth of 2 and 8 bit image is 2 and 8 respectively)

Binary calculations for the number of tones represented by common bit depths:

1 bit (2^1)	= 2 tones
2 bits (2^2)	= 4 tones
3 bits (2^3)	= 8 tones
4 bits (2^4)	= 16 tones
8 bits (2^8)	= 256 tones
16 bits (2^{16})	= 65,536 tones
24 bits (2^{24})	= 16.7 million tones

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Measuring Spatial Resolution

Total number of bits necessary to represent the image is :
Number of Rows X Number of Columns X Bit depth

As spatial resolution refers to clarity, so for different devices, different measure has been made to measure it.

For example:

- **Dots per inch:** DPI is usually used in monitors.
- **Lines per inch:** LPI is usually used in laser printers
- **Pixels per inch:** PPI is measure for different devices such as tablets , Mobile phones etc..

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QUIZ 1
<https://quizizz.com/admin/quiz/5f2a296b6e2cf5001b6a8b9f/image-processing>

21

Using Python, perform following task on a given image:

- I. Reading and Displaying the Image
- II. Know the dimension of an image
- III. Know the pixel intensities inside a ROI and at specific locations
- IV. Cropping the Image and changing the size of an image

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Introduction to Image Enhancement

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Image Enhancement

Need of Image Enhancement: Provides better contrast and a more detailed image

Spatial Domain:
Manipulation with pixels

Frequency Domain:
Modifying the frequency transform of an image

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Image Enhancement

The transformation function has been given below:

$$\mathbf{s = T(r)}$$

Or

$$\mathbf{g(x,y) = T(f(x,y))}$$

where $r: f(x,y)$: the pixels of the input image and $s: g(x,y)$: the pixels of the output image.

T is a transformation function that maps each value of r to each value of s .

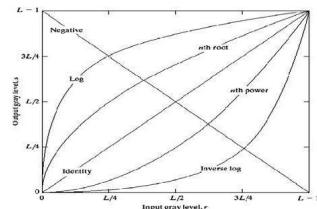
Image enhancement can be done through gray level transformations

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Gray Level Transformations

There are three basic gray level transformation.

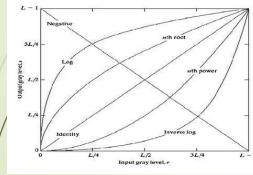
- **Linear:** The Identity and Negative curves
- **Logarithmic:** The Log and Inverse-Log curves
- **Power - law:** n th root and n th power transformations



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Linear Gray Level Transformations

Also called the Identity and Negative curves



The negative of an image with grey levels in the range $[0, L-1]$ is obtained by the negative transformation shown in figure, which is given by the expression, $s = L - 1 - r$ or $g(x,y) = L - 1 - f(x,y)$

For Binary image: changes 0 to 1 and 1 to 0

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Logarithmic Gray level Transformations

The Log and Inverse-Log curves

The log transformation curve shown in fig, is given by the expression, $s = c \log(1 + r)$ where c is a constant and it is assumed that $r \geq 0$.

- Maps
- a narrow range of low-level grey scale intensities into a wider range of output values.
 - the wide range of high-level grey scale intensities into a narrow range of high-level output values.

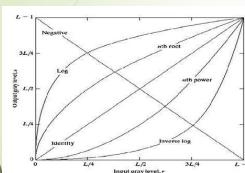
Expand values of dark pixels (increases the dynamic range of dark regions) and compress values of bright pixels (reduces the dynamic range of lighter regions)

The opposite is true for inverse-log transform.

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Power Law Transformations /Gamma correction

n th power and n th root curves

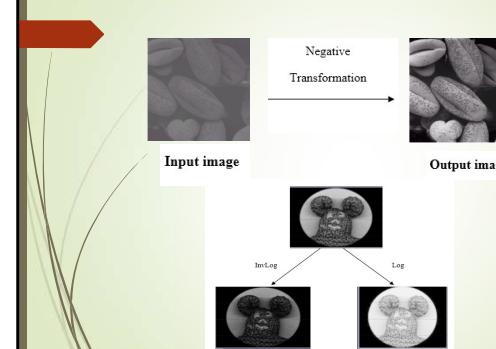


The gamma transformation curve shown in fig, is given by the expression, $s = c r^\gamma$

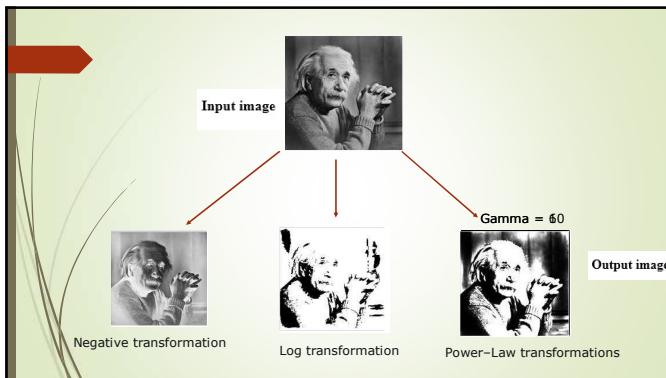
For various values of γ different levels of enhancements can be obtained

Different display monitors display images at different intensities and clarity. That means, every monitor has built-in gamma correction in it with certain gamma ranges

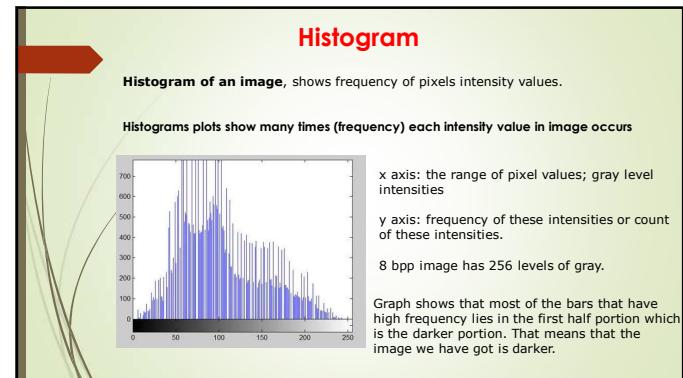
29



30



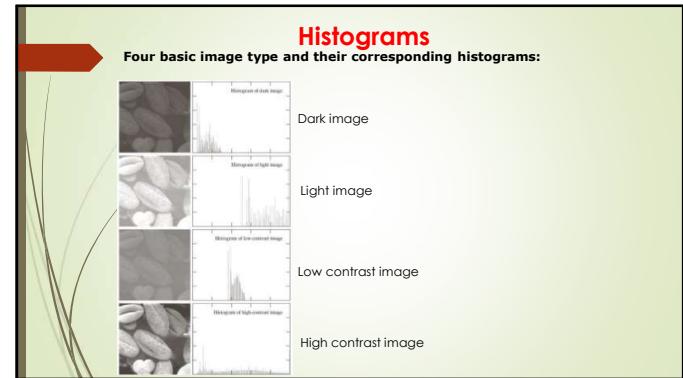
31



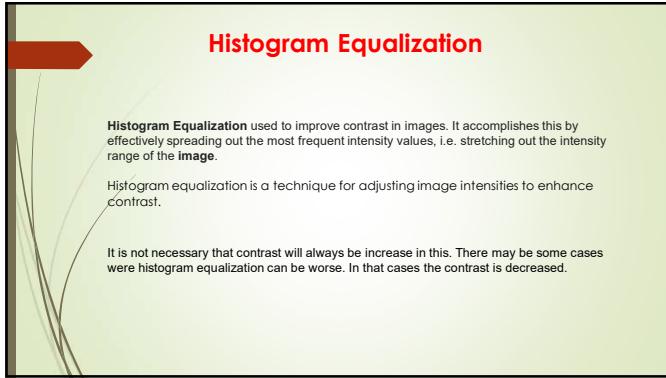
32



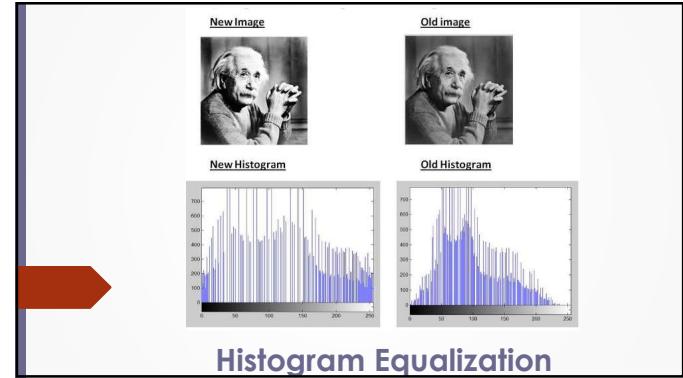
33



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35



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PYTHON

Using Python, perform following task on an image:

1. Histogram Equalization
2. Gray level Transformation

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QUIZ 2

<https://quizizz.com/admin/quiz/5f3614dd2f75f7001c09ce1a/image-processing-part>

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Arithmetic operations

- Arithmetic operations between images are array operations means that arithmetic operations are carried out between corresponding pixel pairs.
- **Addition**
- **Subtraction**
- **Division**
- **Averaging**
- **Multiplication**

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Addition

Useful for combining information between two images

$$Q(i, j) = P_1(i, j) + P_2(i, j)$$

Or if it is simply desired to add a constant value C to a single image then:

$$Q(i, j) = P_1(i, j) + C$$

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Subtraction

Useful for "change" detection., background elimination, Brightness reduction

The difference between two images $f(x, y)$ and $h(x, y)$, expressed as

$$g(x, y) = f(x, y) - h(x, y),$$

$$O(r, c) = |I_1(r, c) - I_2(r, c)|$$

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Subtraction

Useful for "change" detection.

Medical application: Iodine medium injected into the bloodstream

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Multiplication

Use to adjust the brightness of the image, scaling by a constant.

multiplication by 2

Given a scaling factor greater than one, scaling will brighten an image. Given a factor less than one, it will darken the image.

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DIVISION: Example 1

Images are of the same scene except two objects have been slightly moved between the exposure

New position of the moved part in the contrast-stretched image

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Logical operations

AND **OR** **NOT** **XOR**

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Logical operations

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Logical operations

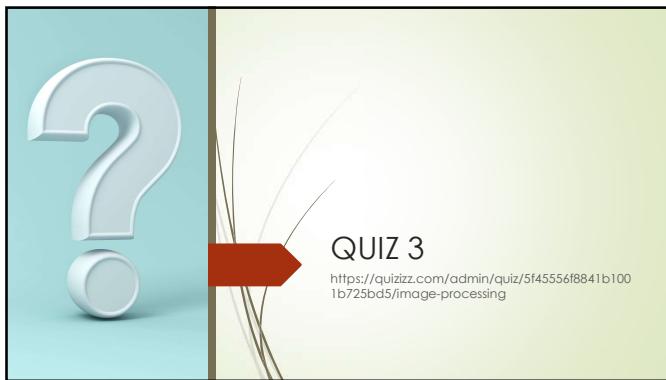
47

PYTHON

- Using Python, perform following task on a given image:

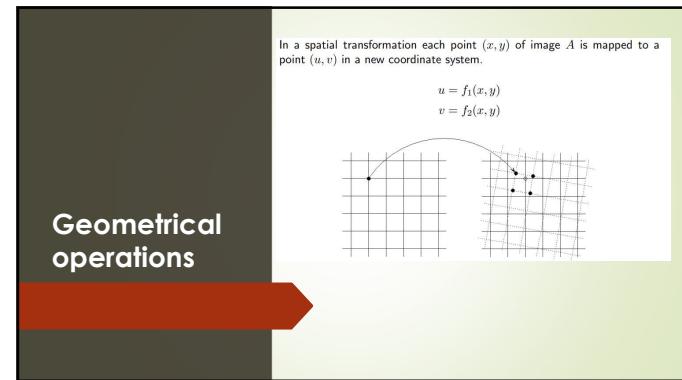
Use given images to generate the required output.

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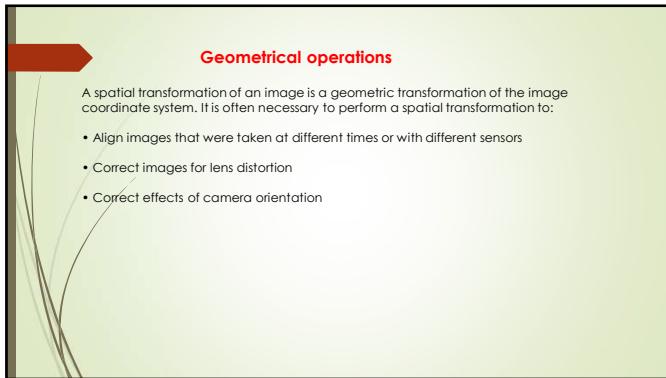
A slide titled "QUIZ 3" with a large question mark icon on the left and a red arrow pointing right. Below the arrow is the URL: <https://quizizz.com/admin/quiz/5f45556f8841b1001b725bd5/image-processing>.

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A slide titled "Geometrical operations". It shows a diagram illustrating a spatial transformation where points (x, y) in image A are mapped to points (u, v) in a new coordinate system. The equations $u = f_1(x, y)$ and $v = f_2(x, y)$ are given. The diagram shows a grid being transformed into a curved shape.

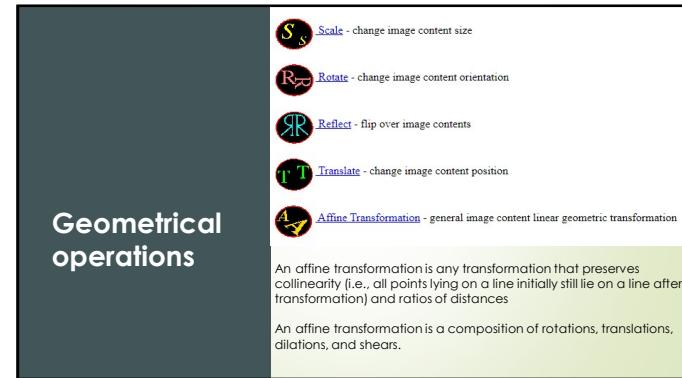
50



A slide titled "Geometrical operations". It states that a spatial transformation of an image is a geometric transformation of the image coordinate system. It is often necessary to perform a spatial transformation to:

- Align images that were taken at different times or with different sensors
- Correct images for lens distortion
- Correct effects of camera orientation

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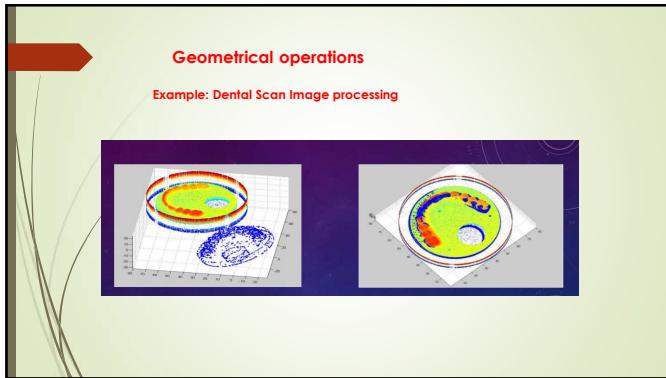


A slide titled "Geometrical operations". It lists five types of transformations with icons:

- S** Scale - change image content size
- R** Rotate - change image content orientation
- R** Reflect - flip over image contents
- T** Translate - change image content position
- A** Affine Transformation - general image content linear geometric transformation

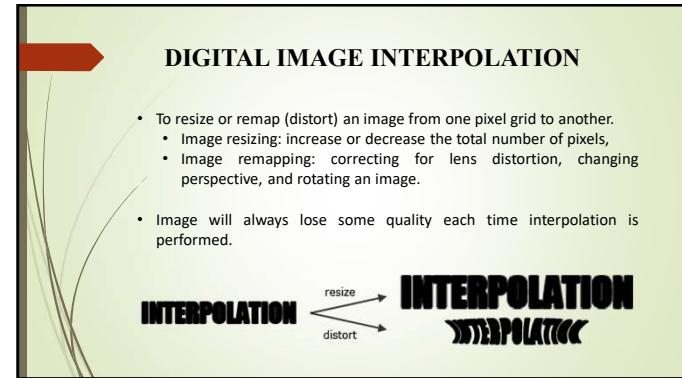
 An affine transformation is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances. An affine transformation is a composition of rotations, translations, dilations, and shears.

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A slide titled "Geometrical operations". It shows an example of dental scan image processing, displaying two 3D point cloud models of teeth.

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A slide titled "DIGITAL IMAGE INTERPOLATION". It lists the following points:

- To resize or remap (distort) an image from one pixel grid to another.
 - Image resizing: increase or decrease the total number of pixels,
 - Image remapping: correcting for lens distortion, changing perspective, and rotating an image.
- Image will always lose some quality each time interpolation is performed.

 A diagram shows the word "INTERPOLATION" with arrows pointing to "resize" and "distort", which then point to a stylized "INTERPOLATION" logo.

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DIGITAL IMAGE INTERPOLATION

- Interpolation works using known data to estimate values at unknown points.
- For example:
 - To know the temperature at noon, but only measured it at 11AM and 1PM, you could estimate its value by performing a linear interpolation.
 - If we have an additional measurement of 11:30AM, we could see that the bulk of the temperature rise occurred before noon, and could use this additional data point to perform a quadratic interpolation.

The more temperature measurements you have which are close to noon, the more sophisticated (and hopefully more accurate) your interpolation algorithm can be

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DIGITAL IMAGE INTERPOLATION

Tries to achieve a best approximation of a pixel's color and intensity based on the values at surrounding pixels

45° Rotation 90° Rotation

Detail is lost in just the first rotation, although the image continues to deteriorate with successive rotations

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TYPES OF INTERPOLATION

- Nearest neighbour Interpolation
- Bilinear Interpolation
- Bicubic interpolation

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Nearest neighbour Interpolation

- Most basic method
- Requires the least processing time
- Only considers one pixel: the closest one to the interpolated point
- Has the effect of simply making each pixel bigger

$w_1 - 4 \text{ pixels}$

$w_2 - 8 \text{ pixels}$

complete

$h_1 - 4 \text{ pixels}$

$h_2 - 8 \text{ pixels}$

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Bilinear Interpolation

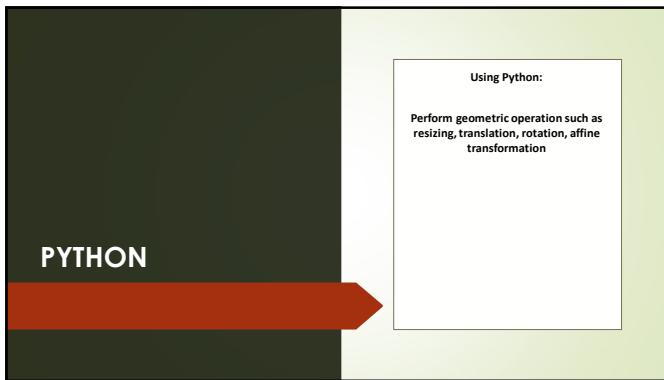
- Considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixels
- Takes a weighted average of these 4 pixels to arrive at the final interpolated values
- Results in smoother looking images than nearest neighborhood
- Needs more processing time

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Bicubic Interpolation

- One step beyond bilinear by considering the closest 4x4 neighborhood of known pixels, for a total of 16 pixels
- Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation
- Produces sharper images than the previous two methods.
- Good compromise between processing time and output quality

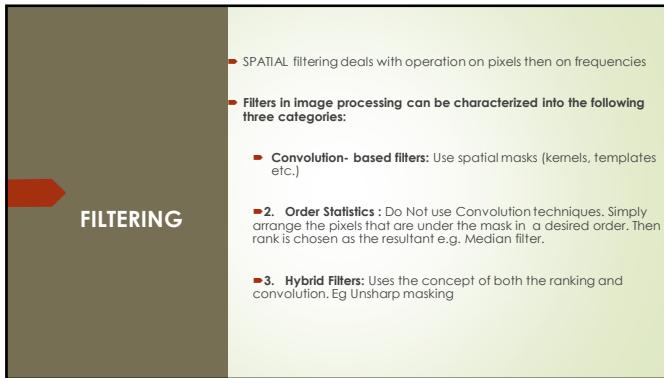
60



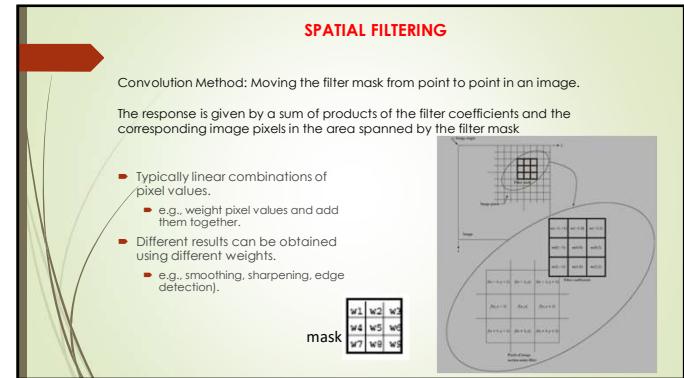
61



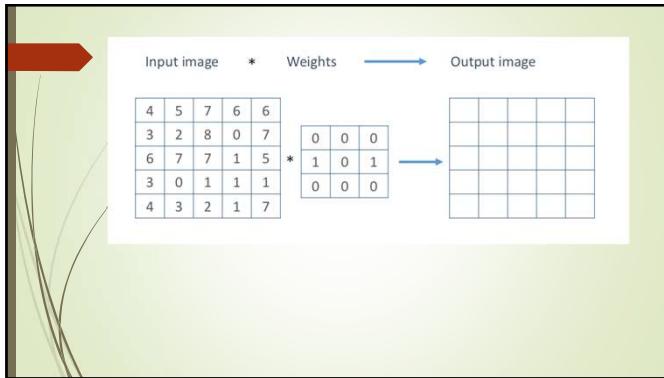
62



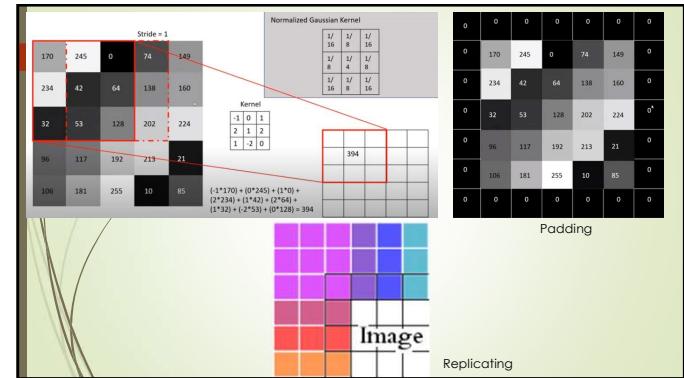
63



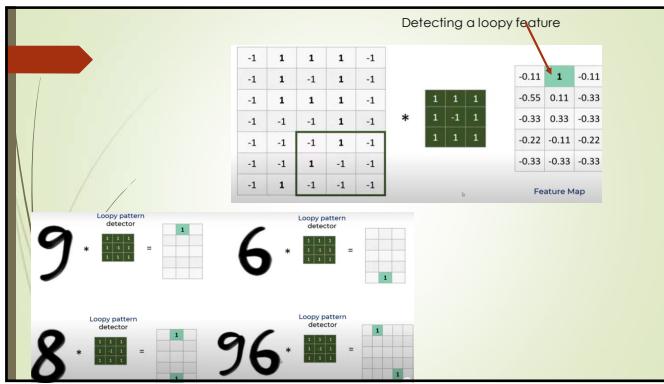
64



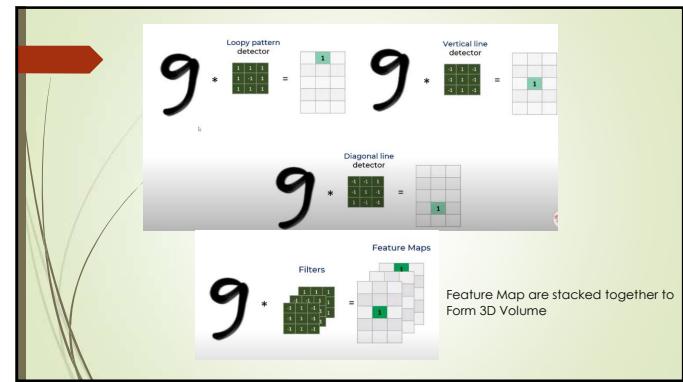
65



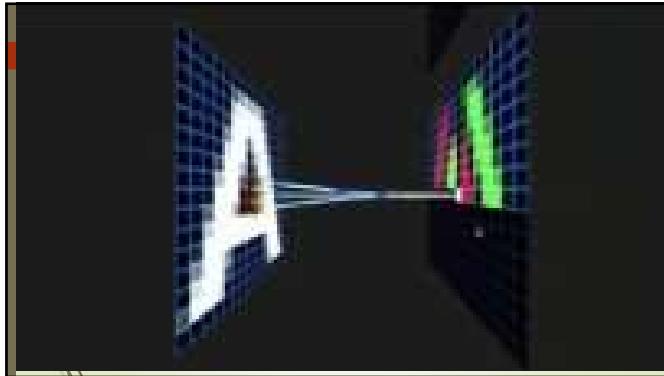
66



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SMOOTHING SPATIAL FILTERING

One of the simplest spatial filtering operations we can perform is a smoothing operation

- Simply **average** all of the pixels in a neighbourhood around a central value
- Especially useful in removing noise from images
- Also useful for highlighting gross detail

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

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SMOOTHING SPATIAL FILTERING:

Box filtering: all coefficients in filter are same

Origin

Simple 3x3 Neighbourhood

3x3 Smoothing Filter

Original Image Pixels

Filter

$e = \frac{1}{9} * 104 + \frac{1}{9} * 100 + \frac{1}{9} * 108 + \frac{1}{9} * 99 + \frac{1}{9} * 98 + \frac{1}{9} * 95 + \frac{1}{9} * 90 + \frac{1}{9} * 85 = 98.3333$

The above is repeated for every pixel in the original image to generate the smoothed image.

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SMOOTHING SPATIAL LINEAR FILTERING

Original

3x3

- The image at the top left is an original image of size 500*500 pixels
- The subsequent images show the image after filtering with an averaging filter of increasing sizes
 - 3, 5, 9, 15 and 35
- Notice how detail begins to disappear

5x5

9x9

35x35

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WEIGHTED SMOOTHING FILTERING

More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function

- Pixels closer to the central pixel are more important
- Often referred to as a *weighted averaging*

$1/16$	$2/16$	$1/16$
$2/16$	$4/16$	$2/16$
$1/16$	$2/16$	$1/16$

Weighing the centre point the highest and reducing the value of the coefficient as a function of increasing distance from origin is to reduce blurring in smoothing process.

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WEIGHTED SMOOTHING FILTERING

Gaussian

7 × 7 Gaussian mask

1	1	2	2	2	1	1
1	2	2	4	2	2	1
2	2	4	8	4	2	2
2	4	8	16	8	4	2
2	4	8	4	2	2	1
1	2	2	4	2	2	1
1	1	2	2	1	1	1

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SMOOTHING SPATIAL Non LINEAR FILTERING

Median Filter

1	7	4	10	9
8	16	21	7	9
18	16	12	9	8
7	10	11	12	13

Ascending order Descending order Median Value

Median Filter

1	4	7	9	10
21	18	16	12	9
8	16	21	7	9
7	10	11	12	13

Ascending order Descending order Median Value

The **Median Filter** is a non-linear digital filtering technique, often used to remove noise from an image or signal.

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SMOOTHING SPATIAL Non LINEAR FILTERING

Descending order

8	9	10	11	12
12	8	9	10	11
13	12	8	9	10
14	13	12	8	9
15	14	13	12	8

Median Value

Ascending order

0	0	0	0	0
0	8	9	10	11
13	12	8	9	10
14	13	12	8	9
15	14	13	12	8

Median Value

Descending order

0	0	0	0	0
0	12	9	10	11
13	12	8	9	10
14	13	12	8	9
15	14	13	12	8

Median Value

Ascending order

0	0	0	0	0
0	8	9	10	11
13	12	8	9	10
14	13	12	8	9
15	14	13	12	8

Median Value

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SMOOTHING SPATIAL Non LINEAR FILTERING

► The Median Filter is 50th percentile of a ranked set of numbers

The Other Filter:

- Max filter is 100th percentile filter: Useful in finding brightest points in image
- Min filter is 0 percentile filter: Useful in finding dark points in image

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BILATERAL FILTERING

Tonal weighting makes that the bilateral filter is capable of preserving edges (large differences in tonal value) while smoothing in the more flat regions (small tonal differences).

Based on :

- **Space parameter:** Weighting (based on a Gaussian distribution) intensity based on Euclidean distance of pixels,
- **Range parameter:** But also, on the radiometric differences (e.g., range differences, such as color intensity, depth distance, etc.). This preserves sharp edges.

The diagram shows two side-by-side images of a person's face. The left image is labeled "original" and the right image is labeled "bilateral filtering". The bilateral filtering image appears smoother than the original, while still retaining the sharpness of the person's features.

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Gaussian Vs Bilateral Filtering

Gaussian

The diagram illustrates the Gaussian filtering process. It shows an "input" image, a "kernel" (a small square with a central bright spot), and an "output" image. A blue arrow points from the input to the kernel, followed by a multiplication symbol (*). Another blue arrow points from the result of the multiplication to an "average" block, which then leads to the final "output".

Space parameter: size of the window

The diagram illustrates the Bilateral Filtering process. It shows an "input" image, three different "kernel" shapes (each with a central bright spot), and an "output" image. Orange arrows point from the input to each of the three kernels. Below the kernels, the text "Same Gaussian kernel everywhere." is written.

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Gaussian Vs Bilateral Filtering

Bilateral Filtering

The diagram illustrates the Bilateral Filtering process. It shows an "input" image, three different "kernel" shapes (each with a central bright spot), and an "output" image. Orange arrows point from the input to each of the three kernels. Below the kernels, the text "The kernel shape depends on the image content. Avoids averaging across edges" is written.

Based on :

- **Space parameter:** Weighting (based on a Gaussian distribution) intensity based on Euclidean distance of pixels,
- **Range parameter:** But also, on the radiometric differences (e.g., range differences, such as color intensity, depth distance, etc.). This preserves sharp edges.

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PYTHON

Using Python:

Perform Image smoothing using different smoothing filters.

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?

QUIZ 4

<https://quizizz.com/admin/quiz/5f60410777d3bf001b2e541>
//image-processing

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SHARPENING SPATIAL FILTER

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SHARPENING SPATIAL FILTERING

Previously we have noticed that: The image blurring is accomplished by pixel averaging in a neighborhood. Averaging is analogous to integration.

Blurring vs. Sharpening

- Blurring/smooth is done in spatial domain by pixel averaging in a neighborhood, it is a process of integration
- Sharpening is an inverse process, to find the difference by the neighborhood, done by spatial differentiation.

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SHARPENING SPATIAL FILTERING

Derivative operator

- The strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
- Image differentiation
 - enhances edges and other discontinuities (noise)
 - deemphasizes area with slowly varying gray-level values.

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SHARPENING SPATIAL FILTERING

First and Second-order derivative of 2D

- when we consider an image function of two variables, $f(x, y)$, at which time we will dealing with partial derivatives along the two spatial axes.

$$\text{Gradient operator } \nabla f = \frac{\partial f(x, y)}{\partial x} \hat{i} + \frac{\partial f(x, y)}{\partial y} \hat{j}$$

(linear operator)

$$\text{Laplacian operator } \nabla^2 f = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

(non-linear)

Gradient refers to the difference between the pixels of an image.

If neighboring pixels have the same intensity, the difference is zero therefore **no edge** is present.

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Image Gradient

Gradient in x only  $\nabla f = \left[\frac{\partial f}{\partial x}, 0 \right]$	Gradient in y only  $\nabla f = \left[0, \frac{\partial f}{\partial y} \right]$	Gradient in both x and y  $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$
---	---	---

Gradient direction

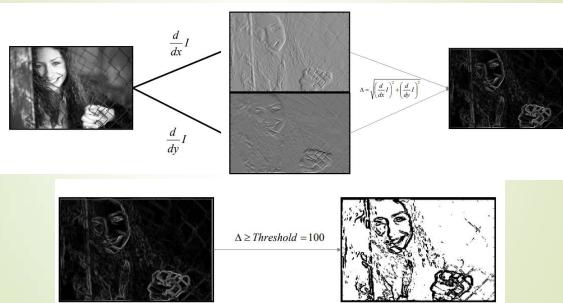
$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

Gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

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Edge Detection Using the Gradient



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Gradient

How do you compute the image gradient?

Choose a derivative filter $S_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$ $S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Run filter over image $\frac{\partial f}{\partial x} = S_x \otimes f$ $\frac{\partial f}{\partial y} = S_y \otimes f$

Image gradient $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

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SHARPENING SPATIAL FILTERING: Laplacian Operator

- Laplacian Operator is
 - a derivative operator
 - highlights regions of rapid intensity change
 - used to find edges in an image.
- The major difference between Laplacian and other operators like Prewitt, Sobel, Robinson and Kirsch is that these all are first order derivative masks but Laplacian is a second order derivative mask.
- Derivatives are strongly affected by noise, thus image is first smoothed such as with Gaussian smoothing filter in order to reduce its sensitivity to noise.

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SHARPENING SPATIAL FILTERING: Laplacian Operator

- In Laplacian mask we have two classifications
- Positive Laplacian Operator and
 - Negative Laplacian Operator.

Positive Laplacian Operator

In Positive Laplacian we have standard mask in which center element of the mask should be negative and corner elements of mask should be zero.

0	1	0
1	-4	1
0	1	0



Negative Laplacian Operator

In negative Laplacian operator we also have a standard mask, in which center element should be positive. All the elements in the corner should be zero and rest of all the elements in the mask should be -1.

0	-1	0
-1	4	-1
0	-1	0



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SHARPENING SPATIAL FILTERING: Laplacian Operator

Effect of Laplacian Operator

- as it is a derivative operator,
 - it highlights gray-level discontinuities in an image
 - it deemphasizes regions with slowly varying gray levels
- tends to produce images that have
 - grayish edge lines and other discontinuities, all superimposed on a dark,
 - featureless background.

While applying Laplacian highlights fine detail, it de-emphasizes smooth regions (e.g., background features). It results in featureless background with grayish fine details.

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SHARPENING SPATIAL FILTERING: Laplacian Operator

Correct the effect of featureless background

- easily by adding the original and Laplacian image.
- be careful with the Laplacian filter used

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive} \end{cases}$$

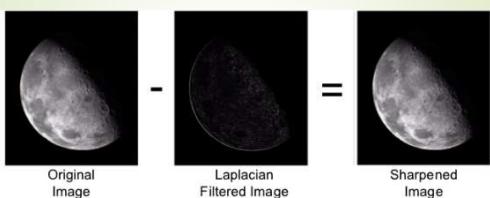
Important :

Subtract the resultant image from the original image to get the sharpened image if positive Laplacian operator is used

Add the resultant image onto original image to get the sharpened image if negative Laplacian operator is used.

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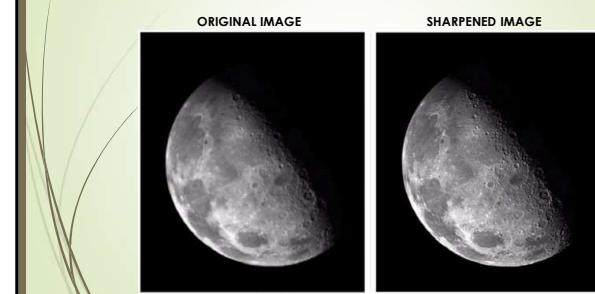
SHARPENING SPATIAL FILTERING: Laplacian Operator



In the final sharpened image edges and fine detail are much more obvious

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SHARPENING SPATIAL FILTERING: Laplacian Operator



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Unsharp Masking

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$

sharpened image = original image – blurred image

- to subtract a blurred version of an image produces sharpening output image.

Where $f_s(x, y)$ denotes the sharpened image obtained by unsharp masking, an $\bar{f}(x, y)$ is a blurred version of $f(x, y)$

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Unsharp Masking and Highboost filtering

How we can sharpen an image or perform edge enhancement using a smoothing

- First, blur the image: Smoothing an image suppress most of the high-frequency components.
- Then, subtract this smoothed image from the original image (the resulting difference is known as a mask). The output image will have most of the high-frequency components that are blocked by the smoothing filter.
- Adding this mask back to the original will enhance the high-frequency components.

As blurred or unsharp image is used to create a mask this technique is known as Unsharp Masking.

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Unsharp Masking and Highboost filtering

Thus, unsharp masking first produces a mask $m(x,y)$ as

$$m(x, y) = f(x, y) - f_b(x, y)$$

$f(x, y)$ is original image and $f_b(x, y)$ is blurred image

The this mask is added back to the original image which results in enhancing the high-frequency components.

$$g(x, y) = f(x, y) + k * m(x, y)$$

where k specifies what portion of the mask to be added. When $k=1$ this is known as Unsharp masking. For $k>1$ we call this as high-boost filtering because we are boosting the high-frequency components by giving more weight to the masked (edge) image.

$$g(x, y) = (k+1) * f(x, y) - k * f_b(x, y)$$

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PYTHON

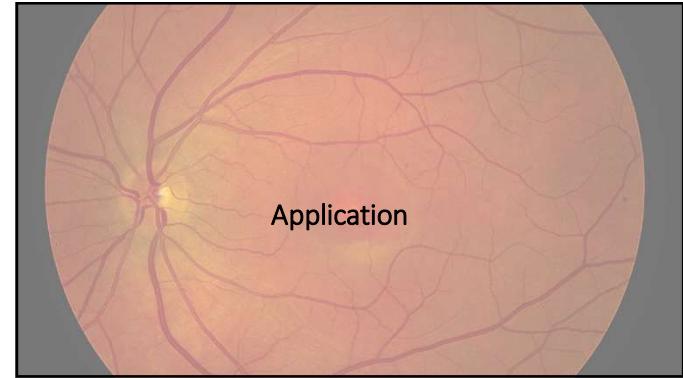
Using Python:
Perform Image sharpening using different filters.

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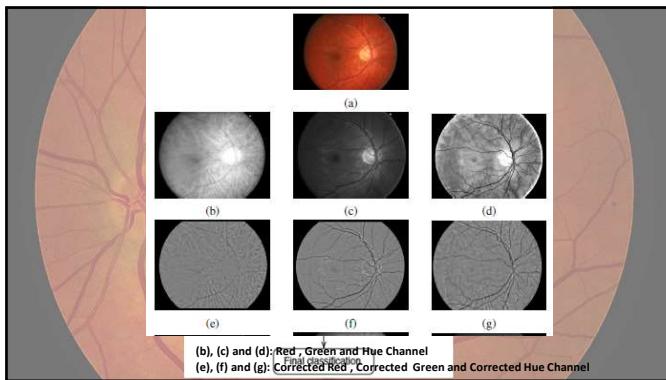
QUIZ 5

<https://quizizz.com/admin/quiz/5f4d05443478a8001b8624f7/image-processing>

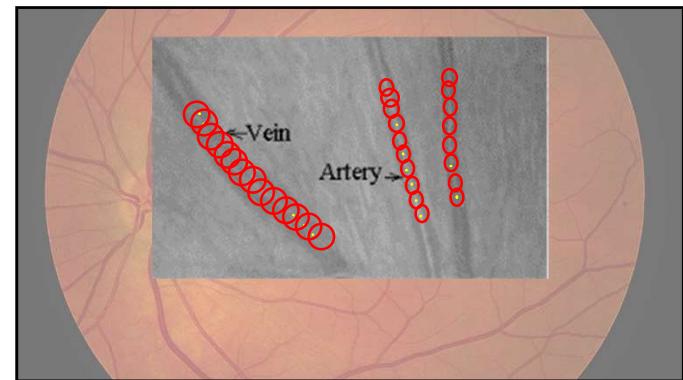
101



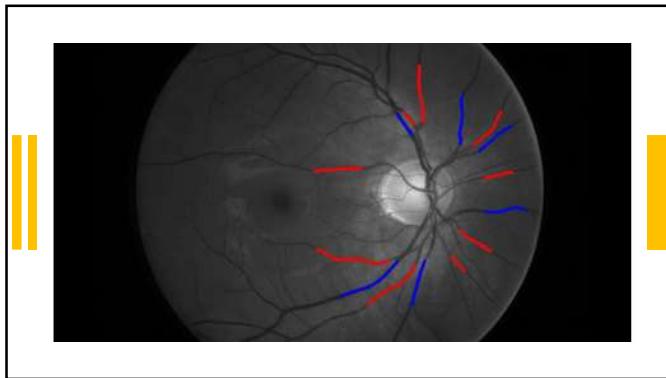
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Frequency Domain FILTERING

- Based on Frequency content
- Fourier transform such as discrete cosine transform, Fourier transforms, Hartley transforms etc. are Frequency domain analysis methods.
- Example :
 - Edges and sharp transitions (example Noise) in gray values in an image contributes significantly to **high frequency content of its Fourier Transform**.
 - Regions of relatively uniform gray values in an image contribute to **low-frequency content of its Fourier Transform**. In other words **Low frequency contents in the Fourier Transform** are responsible to the general appearance of the image over smooth area

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Fourier transform (FT)

One Dimensional FT

The Fourier transform, $F(u)$, of a single variable, continuous function, $f(x)$, is defined by the equation

$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux} dx \quad (4.2-1)$$

inverse Fourier transform

$$f(x) = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux} du.$$

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Fourier transform (FT)

FT/Fourier series using two variable

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy \quad (4.2-3)$$

and, similarly for the inverse transform,

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv. \quad (4.2-4)$$

The Fourier transform simply states that the non periodic signals whose area under the curve is finite can also be represented into integrals of the sines and cosines after being multiplied by a certain weight.

Discrete Fourier transform (DFT)

DFT using one variable

The Fourier transform of a discrete function of one variable, $f(x), x = 0, 1, 2, \dots, M - 1$, is given by the equation

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad \text{for } u = 0, 1, 2, \dots, M - 1. \quad (4.2-5)$$

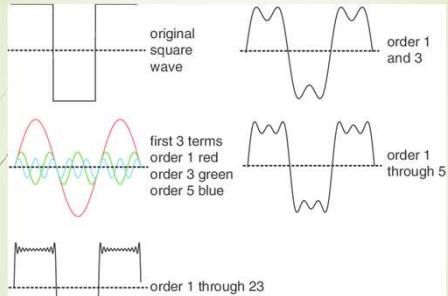
Similarly, given $F(u)$, we can obtain the original function back using the inverse DFT:

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad \text{for } x = 0, 1, 2, \dots, M - 1. \quad (4.2-6)$$

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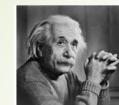
Fourier transform



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Discrete Fourier transform (DFT)



In frequency domain →



The relationship between the spatial domain and the frequency domain can be established by convolution theorem.

Frequency Domain FILTERING

- Image in frequency domain has two components :
 - Magnitude (Consists of the frequencies content of images) and
 - Phase (Used to restore the image back to the spatial domain)

The magnitude of the sinusoid directly relates with the contrast.

Phase contains the color information.

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The spatial function $f(x, y)$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

is decomposed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.

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Frequency Domain filtering: Basic operation

To filter an image in the frequency domain:

- Compute $F(u,v)$ the DFT of the image
- Multiply $F(u,v)$ by a filter function $H(u,v)$
- Compute the inverse DFT of the result

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FILTERS

Filter function $H(u, v)$ can:

- amplifies some frequencies and
- suppresses certain frequency components in an image.

Application: blurring/smoothing, sharpening and edge detection in an image.

Based on the property of using the frequency domain the image filters are broadly classified into two categories:

1. Low-pass filters / Smoothing filters.
2. High-pass filters / Sharpening filters.

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Low-pass filters / Smoothing filters VS High-pass filters / Sharpening filters.

Low Pass Filter

Scanning electron microscope image of an integrated circuit magnified ~2500 times

High Pass Filter

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Image Smoothing

uses Low-pass Frequency Domain Filters

A low-pass filter **attenuates (suppresses) high frequencies** while **passing the low frequencies** which results in creating a blurred (smoothed) image.

In other words: It leaves the low frequencies of the Fourier transform relatively unchanged and ignores the high frequency noise components.

Three main low-pass filters are:

- Ideal low-pass filter (ILPF) (very sharp)
- Butterworth low-pass filter (tunable)
- Gaussian low-pass filter (very smooth)

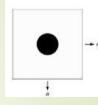
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Ideal Low-pass filters

Original image	Result of filtering with ideal low pass filter of radius 5	Result of filtering with ideal low pass filter of radius 15	Result of filtering with ideal low pass filter of radius 30

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Image Sharpening uses High-pass Frequency Domain Filters



High pass filtering process attenuates (suppress) low frequency components without disturbing high frequency information in the Fourier transform of the image.

The high-pass filter H_{hp} is often represented by its relationship to the low-pass filter (H_p) as: $H_{hp}(u, v) = 1 - H_p(u, v)$ (reverse of each other)

Three main low-pass filters are:

- Ideal high-pass filter (ILPF)
- Butterworth high-pass filter
- Gaussian high-pass filter

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Ideal High-pass filters



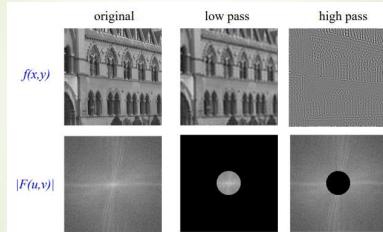
Results of ideal high pass filtering with $D_0 = 15$

Results of ideal high pass filtering with $D_0 = 30$

Results of ideal high pass filtering with $D_0 = 60$

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Action of filters on a real image



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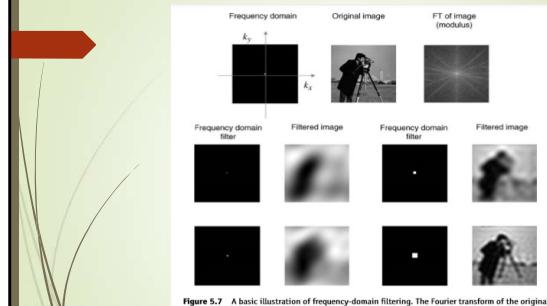


Figure 5.7 A basic illustration of frequency-domain filtering. The Fourier transform of the original image is multiplied by the filter in the frequency domain (white indicates spatial frequency parts which are preserved and black indicates total removal). This modified Fourier transform then returns us to the spatial domain and the filtered image is displayed to the right. (The Matlab code for this figure can be found at <http://www.fundamentals.com/materials/>.)

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The convolution theorem

$$\mathcal{F}\{f(x, y) * h(x, y)\} = \mathcal{F}(k_x, k_y) \mathcal{H}(k_x, k_y) \quad (5.14)$$

The Fourier transform of the convolution of the two functions is equal to the product of the individual transforms.

$$\mathcal{F}\{f(x, y)h(x, y)\} = \mathcal{F}(k_x, k_y) * \mathcal{H}(k_x, k_y) \quad (5.15)$$

The Fourier transform of the product of the two functions is equal to the convolution of their individual transforms.

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Homomorphic filtering

- Removing multiplicative noise. $\tilde{I}(x, y) = I(x, y) n(x, y)$
- Most commonly used for correcting non-uniform illumination in image
- According to illumination-reflectance model : Intensity at any pixel $I(x, y)$ (which is the amount of light reflected by a point on the object) is the product of the illumination of the scene $L(x, y)$ and the reflectance of the object(s) $R(x, y)$ in the scene, i.e.,

$$I(x, y) = L(x, y) R(x, y)$$

R arises from the properties of the scene and L results from the lighting conditions at the time of image capture.

Homomorphic filtering

- To compensate for the non-uniform illumination: remove the L component and keep only the R component.
 - If we consider illumination as the noise signal (which we want to remove), this model is similar to the multiplicative noise model shown earlier.
 - Illumination typically varies slowly across the image and reflectance which can change quite abruptly at object edges.
- This difference is the key to separating out the illumination component from the reflectance component.

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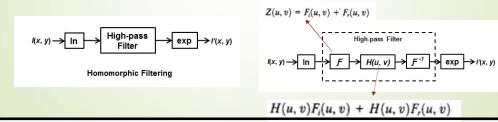
Homomorphic filtering

- In homomorphic filtering we first transform the multiplicative components to additive components by moving to the log domain:

$$\ln(I(x, y)) = \ln(L(x, y) R(x, y))$$

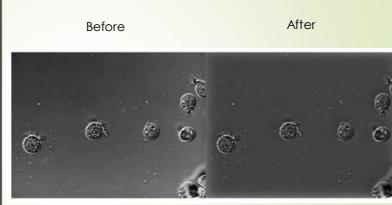
$$\ln(I(x, y)) = \ln(L(x, y)) + \ln(R(x, y))$$

- Use a high-pass filter in the log domain to remove the low-frequency illumination component while preserving the high-frequency reflectance component.



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Homomorphic filtering



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THANK YOU