PRACTICAL NO 1

Topic: Limits and Continuity

1] $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \right]$$

$$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

2] $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$= \lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$

$= \lim_{y\to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$

$= \frac{1}{\sqrt{a+0}\left(\sqrt{a+0}+\sqrt{a}\right)}$

$= \frac{1}{\sqrt{a}\left(\sqrt{a}+\sqrt{a}\right)}$

$= \frac{1}{\sqrt{a}\left(2\sqrt{a}\right)} = \frac{1}{2a}$

3] $\lim_{x\to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h\, \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h\to 0} \frac{\left(\cos h \frac{\sqrt{3}}{2} + \frac{\sin h}{2}\right) - \sqrt{3}\left(\frac{1}{2}\cos h - \frac{\sqrt{3}}{2}\sin h\right)}{6h}$$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3}\cos h}{2} + \frac{\sin h}{2} - \frac{\sqrt{3}\cos h}{2} + \frac{3\sin h}{2}}{6h}$$

$$\lim_{h\to 0} \frac{4\sin h}{2(6h)}$$

$$\lim_{h\to 0} \frac{4 \cdot \sin h}{12h}$$

$$= \frac{1}{3}$$

4] $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing numerator and denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8}{2}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit
we get,

$= 4$

5](i) $f(x)=\dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0<x\le\dfrac{\pi}{2}$

$=\dfrac{\cos x}{\pi-2x}$, for $\dfrac{\pi}{2}<x<\pi$

at $x=\dfrac{\pi}{2}$

$$f(\pi/2)=\frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}}$$

$\therefore f\left(\dfrac{\pi}{2}\right)=0$

f at $x=\dfrac{\pi}{2}$ defined.

(ii) $\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{\pi - 2x}$

By substituting method

$$x - \frac{\pi}{2} = h$$

$$x = h + \frac{\pi}{2}$$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2}$$

6]
$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cos x}{\sqrt{2} \sin x}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{2 \cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \frac{\pi}{2}^-} \cos x$$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

5(ii)
$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & 0 < x < 3 \\ x+3 & 3 \leq x \leq 6 \\ \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{cases}$$
at $x = 3$ and $x = 6$

at $x = 3$

(i) $f(3) = \dfrac{x^2 - 9}{x - 3} = 0$

f at $x = 3$ defined

(ii) $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} x + 3$

$f(3) = x + 3 = 3 + 3 = 6$

f is defined at $x = 3$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (x + 3) = 6$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^-} \dfrac{x^2 - 9}{x - 3} = \dfrac{(x-3)(x+3)}{x - 3}$

$\therefore$ LHL = RHL

f is continuous at $x = 3$

for $x = 6$

$f(6) = \dfrac{x^2 - 9}{x + 3} = \dfrac{36 - 9}{6 + 3} = \dfrac{27}{9} = 3$

2] $\lim\limits_{x \to 6^+} \dfrac{x^2 - 9}{x + 3}$

$\lim\limits_{x \to 6^+} \dfrac{(x-3)(x+3)}{(x+3)}$

$\lim\limits_{x \to 6^+} (x - 3) = 6 - 3 = 3$

$\lim_{x \to 6^+} x + 3 = 3 + 6 = 9$

$\therefore$ LHL $\neq$ RHL

Function is not continuous

6.(i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ = k & x = 0 \end{cases}$ at $x = 0$

Soln: f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

(ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x = 0 \end{cases}$ at $x = 0$

Soln:

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

We know that,

$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$

$\therefore e$

$\therefore k = e$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \dfrac{\pi}{3}$
$\quad\;\; = k \quad x = \dfrac{\pi}{3}$ } at $x = \dfrac{\pi}{3}$

$x - \dfrac{\pi}{3} = h$

$x = h + \dfrac{\pi}{3}$

where $h \to 0$

$f\left(\dfrac{\pi}{3} + h\right) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$$

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$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - \tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3h\left(1 - \sqrt{3}\tan h\right)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h\,(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

7](i) $$f(x) = \left.\begin{array}{ll} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{array}\right\} \text{at } x = 0$$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\,x}{x\tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$$

$$= 2\lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= 2 \times \frac{9}{4}$$

$$= \frac{9}{2}$$

$\lim_{x \to 0} f(x) = \frac{9}{2}$ $\quad 9 = f(0)$

$\therefore$ f is not continuous at $x = 0$

Redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x \tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

7 (ii) $$f(x) = \begin{cases} \frac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases} \text{ at } x = 0$$

$$\lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x \to 0} \frac{3 \cdot e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is continuous at $x = 0$

8] $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

∴ Given

f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \cancel{\frac{1 - \cos x}{x^2}} \frac{2 \sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin^2 x/2}{x} \right)^2$$

Multiply with 2 in numerator and denominator

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9.] $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \dfrac{\pi}{2}$

$f(0)$ is continuous at $x = \dfrac{\pi}{2}$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x \left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x \left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{1 + \sin x}{(1-\sin x)(1+\sin x)\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$= \frac{1}{2\left(\sqrt{2} + \sqrt{2}\right)}$$

$$= \frac{1}{2\left(2\sqrt{2}\right)} = \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

5/12/19

PRACTICAL NO 2

Topic: Application of Derivatives

~~Q.1]~~ ~~(i)~~ ~~$\cot x$~~

~~$\frac{d}{dx}$~~

Q.2] $f(x) = 4x + 1 \quad , \quad x \leq 2$

$= x^2 + 5 \quad , \quad x > 2 \quad$ at $x = 2$

At $x = 2$

$f(2) = 4(2) + 1 = 9$

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$

$\lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$

$\lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$

$\lim_{x \to 2^+} \frac{(x-2)(x+2)}{\cancel{x-2}}$

$\lim_{x \to 2^+} (x + 2)$

$= 4$

$f(2^-)$

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$\lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$\lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$\lim_{x \to 2^-} \frac{4(x-2)}{x-2}$$

$= 4$

f is diffrentiable at $x=2$

3] $f(x) = 4x+7 \quad , \quad x < 3$
$\quad = x^2 + 3x + 1 \quad , \quad x \geq 3$

$f(3) = 3^2 + 3(3) + 1$
$\quad = 19$

$$f(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x-3}$$

$$\lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$\lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$\lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$\lim_{x\to 3^+} \frac{(x-3)(x+6)}{x-3}$$

$$\lim_{x\to 3^+} (x+6)$$

$$= 9$$

$f(3^-)$

$$\lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$\lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$\lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$\lim_{x\to 3^-} \frac{4(x-3)}{x-3}$$

$= 4$ $\therefore$ Not diffrentiable

4] $f(x) = 8x-5, \quad x \le 2$

$= 3x^2-4x+7, \quad x > 2$

$$f(2) = 8(2)-5$$

$$= 16-5$$

$$= 11$$

$$f(2^+) = \frac{f(x)-f(2)}{x-2}$$

$$\lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$\lim_{x \to 2^+} \frac{(3x+2)(x-2)}{x - 2}$$

$$= 3(2) + 2$$

$$= 8$$

$$f(2^-) = \frac{f(x) - f(2)}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8(x-2)}{x - 2}$$

$= 8$

f is diffrentiable at $x = 2$

1.]

(i) $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} = \frac{\cot(x+h) - \cot x}{h}$$

$$\lim_{h\to 0} \frac{\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}}{h}$$

$$\lim_{h\to 0} \frac{\sin(x)\cos(x+h) - \cos(x)\sin(x+h)}{h.\sin(x)\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin(x-(x+h))}{h.\sin x\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin h}{h\sin x\sin(x+h)}$$

$$\lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{\sin x\cdot\sin(x+h)}$$

$$= -1\cdot\frac{1}{\sin x\cdot\sin x}$$

$$= -\operatorname{cosec}^2 x$$

(ii) $\operatorname{cosec} x$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\operatorname{cosec}(x+h) - \operatorname{cosec}(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h}$$

$$\lim_{h\to 0} \frac{\sin(x) - \sin(x+h)}{h \cdot \sin x \sin(x+h)}$$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x+h}{2}\right)}{h \cdot \sin x \sin(x+h)}$$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{h \cdot \sin x \cdot \sin(x+h)}$$

$$\lim_{h\to 0} 2\cos(x+h) \lim_{h\to 0} \frac{\sin\left(\frac{-h}{2}\right)}{\frac{h}{2}} \frac{1}{2} \lim_{h\to 0} \frac{1}{\sin x \sin(x+h)}$$

$$= 2\cos x \cdot \frac{1}{2} \cdot \frac{1}{\sin x \cdot \sin x}$$

$$= \frac{-1}{\sin x} \frac{\cos x}{\sin x}$$

$$= \frac{-1}{\sin x} \cdot \cot x$$

$$= -\operatorname{cosec} x \cdot \cot x.$$

Topic : Application of derivatives

Q.1] Find the intervals in which function is increasing or decreasing

a] $f(x) = x^3 - 5x - 11$

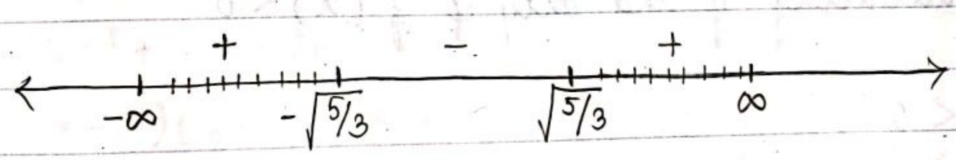
Solution : f is increasing if and only if $f'(x) > 0$

$\because f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$



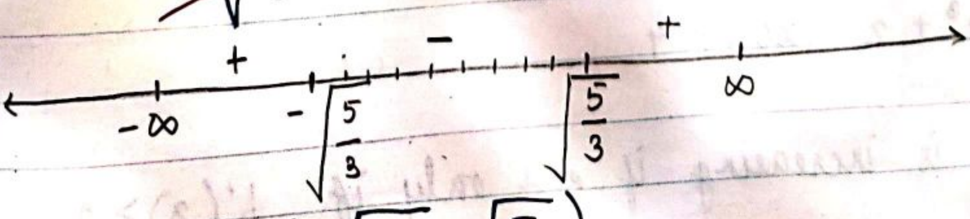
$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if and only if $f'(x) < 0$

$f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore \cancel{x = \pm\sqrt{\frac{5}{3}}}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

Solution: f is increasing if and only if $f'(x) > 0$

$\because f(x) = x^2 - 4x$

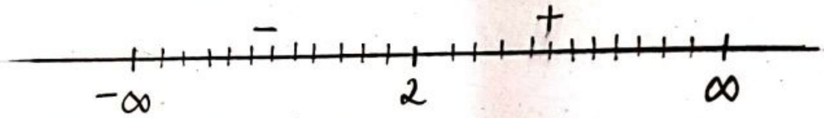
$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) \geq 0$

$\therefore x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

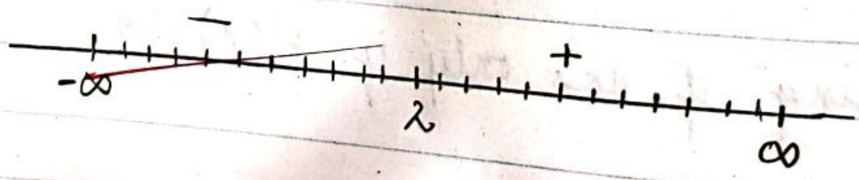
Now if is decreasing if and only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$



$\therefore x \in (-\infty, 2)$

c] $f(x) = 2x^3 + x^2 - 20x + 4$

Solution: f is increasing if and only if $f'(x) > 0$

$\because f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$\therefore x = -2, \frac{5}{3}$

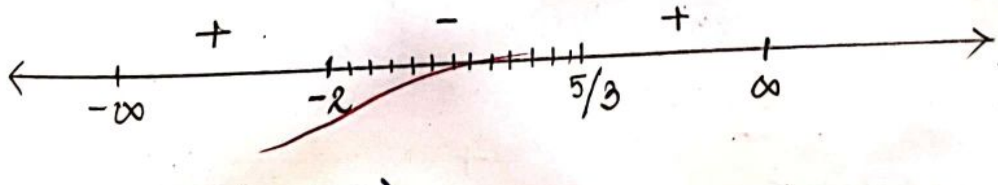
$\therefore x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$\therefore x = -2, 5/3$



$\therefore x \in \left(-2, \frac{5}{3}\right)$

d] $f(x) = x^3 - 27x + 5$

Solution: f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

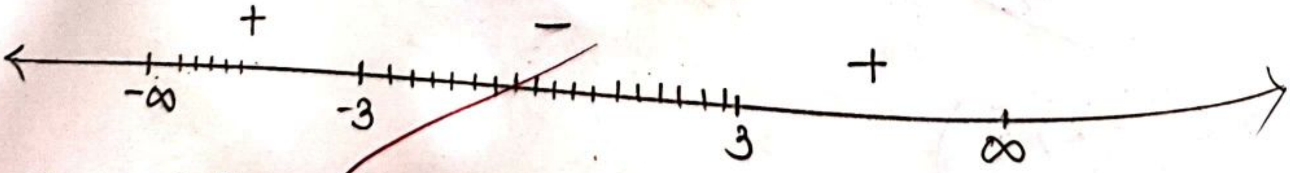
Now f is decreasing if and only if $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$



$\therefore x \in (-3, 3)$

e] $f(x) = 69 - 24x - 9x^2 + 2x^3$

Solution: f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24 - 18x + 6x^2 > 0$

$\therefore 6(-4 - 3x + x^2) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore \cancel{x(x-} \; x^2 - 4x + x - 4 > 0$

$\therefore x(x-4)\,1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x = 4, -1$

$x \in (-\infty, -1) \cup (4, \infty)$

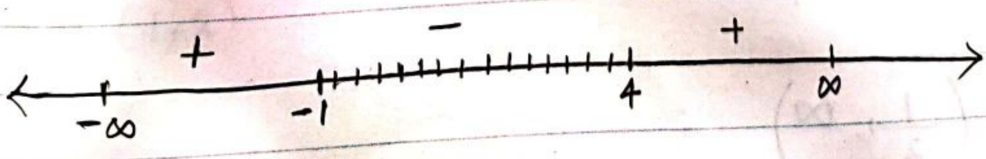
Now f is decreasing if and only if $f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$\therefore (x-4)(x+1) < 0$

$\therefore x = 4, -1$



$x \in (-1, 4)$

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Q.2] Find the intervals in which function is concave up] concave downwards.

a] $y = 3x^2 - 2x^3$

Solution: $\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

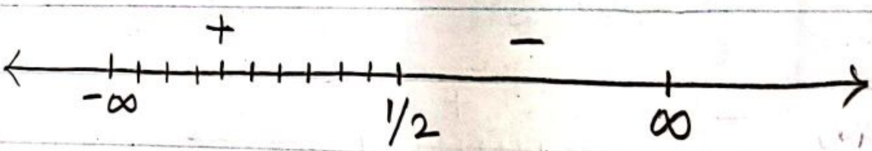
$\because f$ is concave upward if and only if $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1-2x) > 0$

$\therefore 1 - 2x > 0$

$\therefore -(2x-1) > 0$

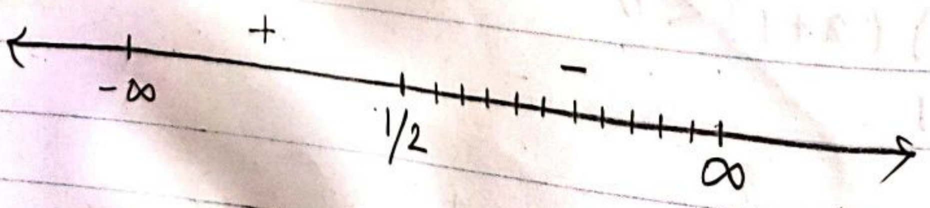


$x \in \left(-\infty, \frac{1}{2}\right)$

$\because f$ is concave downward if and only if $f''(x) < 0$

$\therefore 6(1-2x) < 0$

$\therefore -(2x-1) < 0$



$x \in \left(\frac{1}{2}, \infty\right)$

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b] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Solution: $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore f$ is concave upward if and only if $f''(x) > 0$

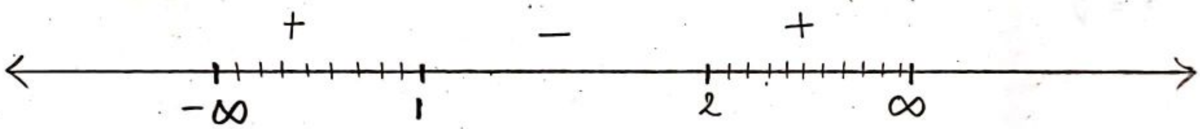
$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$\therefore x = 2, 1$



$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

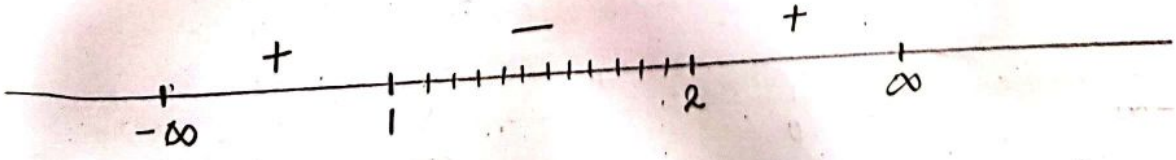
$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

$\therefore x = 2, 1$



$x \in (1, 2)$

c] $y = x^3 - 27x + 5$

Solution:

$\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

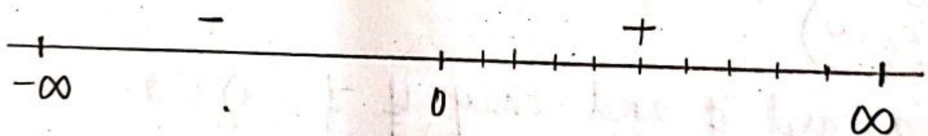
$\therefore f''(x) = 6x$

$\therefore f$ is concave upward if and only if $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$



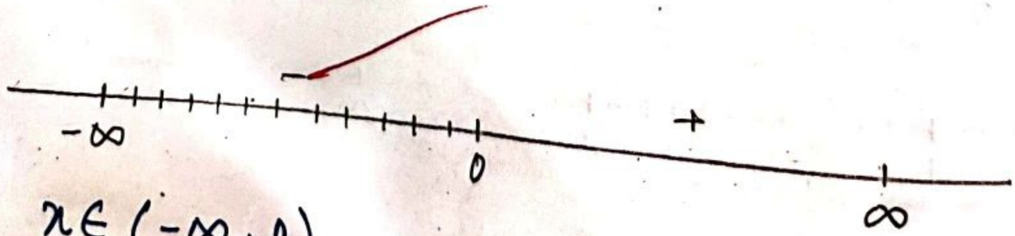
$\therefore x \in (0, \infty)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$



$\therefore x \in (-\infty, 0)$

d] $y = 69 - 24x - 9x^2 + 2x^3$

Solution: $\therefore$ $y = f(x)$

$\therefore$ $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore$ $f'(x) = -24 - 18x + 6x^2$

$\therefore$ $f''(x) = -18 + 12x$

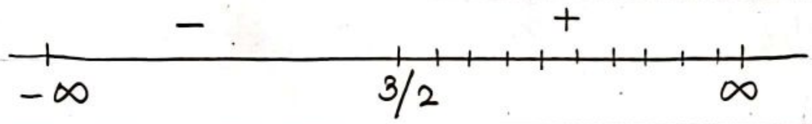
$\therefore$ f is concave upward if and only if $f''(x) > 0$

$\therefore$ $-18 + 12x > 0$

$\therefore$ $6(2x - 3) > 0$

$\therefore$ $2x - 3 > 0$

$\therefore$ $x = 3/2$



$\therefore$ $x \in \left(\frac{3}{2}, \infty\right)$

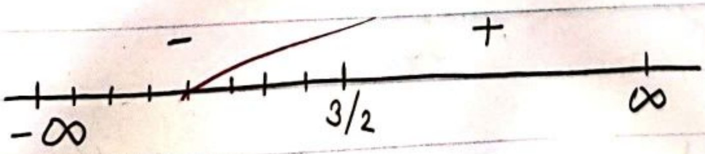
$\therefore$ f is concave downwards if and only if $f''(x) < 0$

$\therefore$ $-18 + 12x < 0$

$\therefore$ $6(2x - 3) < 0$

$\therefore$ $2x - 3 < 0$

$\therefore$ $x = 3/2$



$\therefore$ $x \in \left(-\infty, \frac{3}{2}\right)$

e] $y = 2x^3 + x^2 - 20x + 4$

Solution: $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

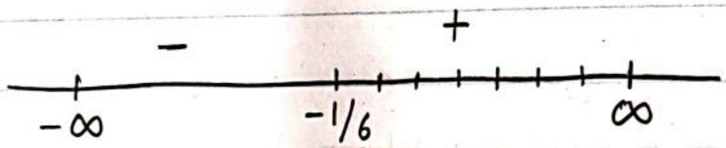
$\therefore f$ is concave upwards if and only if $f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x+1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$



$\therefore x \in \left(\frac{-1}{6}, \infty\right)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x+1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

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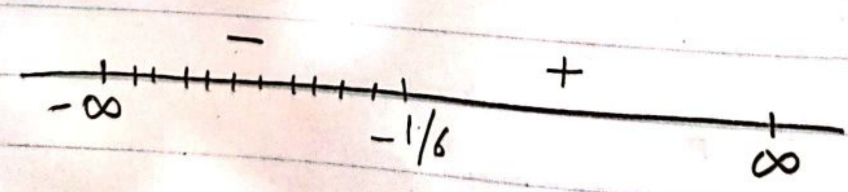

$\therefore x \in \left(-\infty, \frac{-1}{6}\right)$

PRACTICAL NO 4

Aim: Application of derivatives and newton's method

Q.1] find maximum and minimum value of following:

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Q.2] find the root of the following equation by Newton's (Take 4 iteration only correct upto 4 decimal)

(i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$

Solution:

Q.1]

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(x) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has maximum value at $x = 2$

$f(2) = 2^2 + 16/22$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has maximum value at $x = 2$

function reaches minimum value at $x = 2$ and $x = -2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore 15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ is Maximum

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 = 1$

$f''(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and has minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$
$f'(x) = 3x^2 - 6x$
consider, $f'(x) = 0$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$3x = 0$ or $x - 2 = 0$.
$x = 0$ or $x - 2 = 0$. $x = 2$
$\therefore$ $f''(x) = 6x - 6$
$\therefore$ $f''(0) = 6(0) - 6$
$= -6 < 0$ $\therefore$ f has maximum value at $x = 2$
$\therefore$ $f(0) = (0)^3 - 3(0)^2 + 1 = 1$
$f''(2) = 6(2) - 6$
$= 12 - 6$
$= 6 > 0$
$\therefore$ f has minimum value at $x = 2$
$f(2) = (2)^3 - (3)(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3.$
$\therefore$ f has maximum value 1 at $x = 0$ and
f has minimum value -3 at $x = 2$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum value at $x = +2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has Maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value at $x = -1$

$\therefore f$ has minimum value at $x = -19$

Q.2] (i)

$f(x) = x^3 - 3x^2 - 55x + 9.5$, $x_0 = 0 \rightarrow$ given

$f(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5 / 55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 0.1727 - 0.0829 / 55.9467$

$= 0.1712$

$\therefore f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f''(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.\cancel{9393}$

$x^3 = x_2 - f(x_2) / f'(x_2)$

$= 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

$\therefore$ The root of the eqn is 0.1712

(ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(13) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_n = 3$ be the initial approximation by

$\therefore$ By Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 f(x_0) / f'(x_0)$

$= 3 - 6 (23)$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 6.546 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9857 - 4$

$= 17.9851$

$2.7071 - \frac{0.0102}{17.9857}$

$= 2.7071 - 0.0056 = \cancel{2.77}015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7)58 - 10.806 - 9 = -0.0901$

$f(3) = 3(2.7015)^2 - 4 = 2.18943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/\cancel{1}7.8943 = 2.7015 + 0.0050$

$= 2.7065$

3] $f(x) = x^3 - 18x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(\cancel{2})^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation by Newton's method

$x_n + 1 = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_2) / f'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 44.764 - 15.77 + 17$

$= 0.6755$

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$3(1.577)^2 - 3.6(1.577) - 10$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5647 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x^3 = x^2 - f(x_2) / f'(x_2)$

$= 1.6592 / 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= 1.6618$

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PRACTICAL NO 5

Topic : Integration

a] Solve the following integration

(i) $\frac{dx}{\sqrt{x^2+2x-3}}$

(ii) $\int (4e^{3x+1})\,dx$

(iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

(v) $\int t^7 \sin(2t^4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$

(vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

(ix) $\int e^{\cos^2 x} \cdot \sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

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(i) $\int \frac{1}{x^2+2x-3} dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}} dx$

$= a^2 + 2ab + b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substitute put $x+1 = t$

$dx = \frac{1}{t} dt$ where $t = 1$, $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

Using

$\int \frac{1}{\sqrt{x^2-9^2}} dx = \ln(|x + \sqrt{x^2-9^2}|)$

$\ln(|t + \sqrt{t^2-4}|)$

$\ln(|x+1 \sqrt{(x+1)^2-4}|)$

$\ln(|x+1 + \sqrt{x^2+2x-3}|)$

$\ln(|x+1 + \sqrt{x^2+2x-3}|) + C$

2] $\int \left(4e^{3x+1}\right) dx$

$= \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + \int 1\, dx$ # $\int e^{dx}\, dx = 1/x \times e^{dx}$

$= \dfrac{4e^{3x}}{3} + x$

$= \dfrac{4e^{3x}}{3} + x + c$

3] $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \dfrac{2x^3}{3} + 3\cos x + \dfrac{\cos\sqrt{x}}{3} + c$

$= \dfrac{2x^3 + \cos\sqrt{x}}{3} + 3\cos x + c$

4] $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \dfrac{x^3 + 3x + 4}{x^{1/2}}\, dx$

split the denominator

$= \int \dfrac{x^3}{x^{1/2}} + \dfrac{3x}{x^{1/2}} + \dfrac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \dfrac{4}{x^{1/2}}\, dx$

$= \int x^{5/2}\, dx + \int 3x^{1/2}\, dx + \int \dfrac{4}{x^{1/2}}\, dx$

$$= \frac{x^{5/2+1}}{5/2+1}$$

$$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$$

5] $\int t^7 \times \sin(2t^4)\,dt$

Put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\,du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$$

Substitute t^4 with $u/2$

$$= \int \frac{\frac{u}{2} * \sin(u)}{8}\,du$$

$$= \int \frac{\frac{u \times \sin(u)}{2}}{8}\,du$$

$$= \int \frac{u \times \sin(u)}{16}\,du$$

$$= \frac{1}{16}\int u \times \sin(u)\,du$$

$\int u\,dv = uv - \int v\,du$

Where $u = 4$

$dv = \sin(u) \times du$

$du = 1\, du \qquad v = -\cos(u)$

$= \frac{1}{16} \left(u \times (-\cos(u)) \right) - \int -\cos(u)\, dx$

$= \frac{1}{16} \times \left(u \times (-\cos u) \right) + \int \cos(u)\, dx$

\# $\int \cos x\, dx = \sin(x)$

$= \frac{1}{16} \left(u \times (-\cos(u)) + \sin(u) \right)$

Return the substitution $u = 2t^4$

$= \frac{1}{16} \times \left(2t^4 \times (-\cos(2 \times u)) + \sin(2t^4) \right)$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

(vi) $\int \sqrt{x}\,(x^2 - 1)\, dx$

$= \int \sqrt{x} \times x^2 \cancel{- \sqrt{x}\, dx}$

$= \int \left(x^{1/2} \times x^2 - x^{1/2} \right) dx$

$= \int \left(x^{5/2} - x^{1/2} \right) dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

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$$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$$

$$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$$

(iii) $$\int \frac{\cos x}{3\sqrt{\sin(x)^2}}\, dx$$

$$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$$

Put $t = \sin(x)$

$d^t t = \cos x$

$$= \int \frac{\cos x}{\sin(x)^{3/2}} \times \frac{1}{\cos x}\, dx$$

$$= \frac{1}{\sin x^{3/2}}\, dx$$

$$= \frac{1}{t^{2/3}}\, dt$$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3^{-1})\, t^{2/3-1}} = \frac{-1}{-1/3\, t^{2/3-1}}$$

$$= \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + c$$

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(x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

Put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dx$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3\cancel{x}(x^2-2x)}\,dx$

$= \int \frac{\cancel{x^2-2x}\ 1}{x^3-3x^2+1} \times \frac{1}{3}\,dx$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dx$

$= \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt$

$\int \frac{1}{x}\,dx = \ln(x)$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + c$

PRACTICAL NO 6

1) $y = \sqrt{4-x^2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$= 2\left[\cancel{\sin^{-1}}(x/2)\right]^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= 2\pi$$

3] $y = x^{3/2} \qquad x \in [0,4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\left[\frac{[4+9x]^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]$$

$$= \frac{-1}{27}\left[(4+0)^{3/2} - (4+31)^{7/5}\right]$$

$$= \frac{1}{27}\left(40^{3/2} - 8\right) \text{ units}$$

4] $x = 3\sin t$ $\qquad$ $y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$ $\qquad$ $\frac{dy}{dt} = -3\sin t$

$$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3\,[x]_0^{2\pi}$$

$$= 3\,[2\pi - 0]$$

$I = 6\pi$ units

5] $x = \frac{1}{6} y^3 + \frac{1}{2y}$

$\therefore \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$I = \int_0^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\int_0^2 \sqrt{1 + \frac{(y^4-1)^2}{4y^4}}\, dy$

$\int_0^2 \sqrt{\frac{(y^4-1) + 4 \times y^4 \times 1}{4y^4}}\, dy$

$\int_0^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$\int_0^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2} \int_1^2 y^2 dy + \frac{1}{2} \int_1^2 y^{-2} dy$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-1}}{1} \right]_1^2$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$= \frac{17}{12}$ units

Q.2 (ii)

$$\int_0^2 e^{x^2}\,dx \quad \text{with} \quad n = 4$$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2}\,dx = \frac{2}{3}\left[(Y_0 + Y_4) + 4(Y_1 + Y_3) + 2(Y_2)\right]$$

$$= \frac{0.5}{3}\left[(1+54.5982) + 5(1.284 + 9.487) + 2.23885\right]$$

$$= \frac{0.5}{3}\left[55.5982 + 43.0866 + 5.436\right]$$

$$\int_0^2 e^{x^2}\,dx = 17.3535$$

(ii) $\int_0^4 x^2\,dx \qquad n = 4$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

$$= \frac{1}{3}\left[0 + 16 + 4(1+9) + 2\times 4\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$= \int_0^4 x^2\,dx = 21.3333$$

(iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \qquad n = 6$

$h = \dfrac{\frac{\pi}{3} - 0}{6} = \dfrac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}\left(\frac{\pi}{3}\right)$
y	0	1.4167	0.4585	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[Y_0 + Y_6 + 4(Y_1 + Y_3 + Y_5) + 2(Y_2 + Y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.9752) + 2(0.5848 + 0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 7.996 + 2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

PRACTICAL NO 7

Topic: Diffrential Equation

Q.1] I]

$$x\frac{dy}{dx} + y = e^x$$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$P(x) = \frac{1}{x} \qquad Q(x) = \frac{e^x}{x}$$

$$I.F = e^{\int P(x)\,dx}$$

$$= e^{\int 1/x\,dx}$$

$$= e^{\ln x}$$

$$I.F = x$$

$$y(I.F) = \int Q(x)\,(I.F)\,dx + C$$

$$= \int \frac{e^x}{x} \cdot x \cdot dx + C$$

$$= \int\int e^x\,dx + 1$$

$$xy = e^x + C.$$

Q.2]

$$e^x \frac{dy}{dx} + 2e^x y = 1.$$

$$\frac{dy}{dx} + 2\frac{e^x}{e^x} y = \frac{1}{e^x} \quad (\div \text{ by } e^x)$$

$$\frac{dy}{dx} + 2y = \frac{1}{e^x}$$

$$\frac{dy}{dx} + 2y = e^{-x}$$

$$P(x) = 2 \qquad Q(x) = e^{-x}$$

$$\int P(x)\, dx$$

$$IF = e^{\int 2\, dx}$$

$$= e^{2x}$$

$$y(I\cdot F) = \int Q(x)(I\cdot F)\, dx + C$$

$$y\cdot e^{2x} \int e^{-x} + 2x\, dx + C$$

$$= \int e^x\, dx + C$$

$$y\cdot e^{2x} = e^x + C.$$

(iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2/x$ $Q(x) = \frac{\cos x}{x^2}$

$I.F = e\int P(x)\,dx$

$= e\int 2/x\,dx$

$= e^{2/x}\ x$

$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{\cos x}{x^2} - x^2\,dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

(iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides)

$P(x) = 3/x$ $Q(x) = \sin x/x^3$

$= e\int P(x)\,dx$

$= e\int 3/x\,dx$

$= e^{3/x}\ x$

$= e^{\ln x^3}$

$I.F = x^3$

$$y(I\cdot F) = \int g(x)(I\cdot F)\,dx + c$$

$$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$$

$$= \int \sin x\,dx + c$$

$$x^3 y = -\cos x + c$$

(v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$IF = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$$y(IF) = \int Q(x)(I\cdot F)\,dx + c$$

$$= \int 2xe^{-2x}e^{2x}\,dx + c$$

$$= \int 2x\,dx + c$$

$$ye^{2x} = x^2 + c$$

(vi) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$$

$$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

$$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$$

$$\therefore \log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x - \tan y| = c$$

$$\tan x \cdot \tan y = e^c$$

(vii) $\dfrac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

Differentiating on both sides

$$x \cancel{\mp} y + 1 = v$$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\cancel{1 - \frac{dv}{dx} = \frac{dy}{dx}}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x + y - 1) = x + c$$

(viii) $$\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$$

Put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v + 2}$$

$= \frac{3v + 3}{v + 2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$

$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$

$v + \log|x| = 3x + C$

$2x + 3y + \log|2x + 3y + 1| = 3x + C$

$3y = x - \log|2x + 3y + 1| + c$

AV
09/01/2020

PRACTICAL NO 8

TOPIC : Euler's Method

1] $\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2) = ?$

Soln:

$f(x) = y + e^x - 2 \quad x_0 = 0 \quad y(0) = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2] $\frac{dy}{dx} = 1 + y^2, \ y(0) = 1 \quad h = 0.2$ find $y(1) = ?$

$y_0 = 0, \ y_0 = 0, \ h = 0.2$

n	x_n	Y_n	$f(x_n, y_n)$	Y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

3] $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3573
4	0.8	1.3573	0.7696	1.5057
5	1	1.5057		

$\therefore y(1) = 1.5057$

4] $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

$h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	7.754	4
1	~~1.5~~	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

4] $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$Y(2) = 299.9960$

5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$Y(1.2) = 3.6$

23/01/2020

PRACTICAL NO 9

Topic: Limits and Partial Order Derivatives

1] evaluate the following limits

(i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

Applying limit

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Applying limit

$$= \frac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)}$$

$$= \frac{1(4-8)}{2} = \frac{-4}{2} = -2$$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x)^2-(yz)^2}{x^2(x-yz)}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)(x-yz)}{x^2(x-yz)} \quad [\because \{a\}^2-b^2=(a+b)(a-b)]$$

ca

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

$$= \frac{1+(1)(1)}{(1)^2} = 2$$

Q.7 Find f_x, f_y for each of the following

(i) $f(x,y) = xye^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial(xye^{x^2-y^2})}{\partial x}$$

$$= \frac{y\,\partial(x.e^{x^2+y^2})}{\partial x}$$

$$= y\left[x\cdot\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\cdot\frac{d}{dx}(x)\right]$$

$$\left[\because \frac{d}{dx}(UV) = U\cdot V' + VU'\right]$$

$$= y[x\cdot e^{x^2+y^2}\cdot 2x + e^{x^2+y^2}(1)]$$

$$= y\cdot e^{x^2+y^2}[2x+1]$$

Now, $f(y) = \frac{\partial f}{\partial y}$

$$= \frac{\partial(xy\,e^{x^2+y^2})}{\partial y}$$

$$= x \cdot \frac{\partial}{\partial y} \left(y \cdot e^{x^2+y^2} \right)$$

$$= x \left[y \cdot \frac{d}{dx} \left(e^{x^2+y^2} \right) + e^{x^2+y^2} \cdot \frac{d}{dy} (y) \right]$$

$$= x \cdot \left[2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2} \right] \quad \left[\frac{d}{dx}(uv) = u \cdot v' + v \cdot u' \right]$$

$$= x \cdot e^{x^2+y^2} \left[2y^2 + 1 \right]$$

(ii) $f(x, y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$$f(y) = e^x \frac{\partial}{\partial y} (\cos y)$$

$$= e^x \cdot (-\sin y)$$

$$= -e^x \sin y$$

(iii) $f(x, y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial \left(x^3 y^2 - 3x^2 y + y^3 + 1 \right)}{\partial x}$$

$$= 3x^2 y^2 - 3(2x) y$$

$$= 3x^2 y^2 - 6xy$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial \left(x^3 y^2 - 3x^2 y + y^3 + 1 \right)}{\partial y}$$

$$= x^3 (2y) - 3(1) x^2 + 3y^2$$

$$= 2x^3 y - 3x^2 + 3y^2$$

Q.3] Using defination find values of f_x, f_y at $(0,0)$ for

$$f(x,y) = \frac{2x}{1+y^2}$$

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

where $(a,b) = (0,0)$

$$\therefore f\{x\}(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h - 0}{2} = 2$$

Similarly

$$f_y(a,b) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$\therefore f_x = 2, \quad f_y = 0$$

Q.4] Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

(i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial(y^2 - xy)}{x^2}$$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\cdot\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V \cdot U' + U \cdot V'}{V^2} \right]$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$$

$$f(x) = \frac{xy - 2y^2}{x^3}$$

$$f(y) = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right)$$

$$= \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$$

$$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$$

$$= \frac{1}{x^2} \cdot 2y - \frac{1}{x}$$

rify

$$f(y) = \frac{2y - x}{x^2}$$

$$f(x, y) = \frac{\partial\left(\cancel{\frac{xy - 2y^2}{x^2}}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy - 2y^2)(3x^2)}{x6}$$

$$= \frac{x^3 y - 3x^3 y + 6x^2 y^2}{x6}$$

$$= \frac{6x^2y^2 - 2x^3 y}{x^6} = \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$f(yy) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial y}$$

$$= \frac{\frac{\partial}{x^2}(2y - x)}{\partial y} = \frac{1}{x^2}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{1}{x^2} - \frac{1}{x^3}\, 2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4yx^2}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= -\frac{4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$$

Hence Verified

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$$

$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$

$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial y}$

$= 0 + 3(2y)(x^2) + 0$

$f(y) = 6x^2y$

$f(xx) = \frac{\partial f_x}{\partial x} = \frac{\partial}{\partial x}\left(3x^2 \pm 6xy^2 + \frac{2x}{x^2+1}\right)$

$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$

$\left[\frac{d}{dx} U/V = \frac{V.U' - U.V'}{V^2}\right]$

$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$

$f(yy) = \frac{\partial f_y}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$

$= 6x^2(1) = 6x^2$

$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

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$$f(yx) = \frac{\partial}{\partial x} f_y$$

$$= \frac{\partial (6x^2 y)}{\partial x}$$

$$= 12xy$$

$$\therefore f(xy) = f(yx) = 12xy$$

Hence verified

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (\sin(xy) + e^{x+y})}{\partial x}$$

$$= \cos(xy)(y) + e^{x+y}(1)$$

$$= y\cos xy + e^{x+y}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (\sin(xy) + e^{x+y})}{\partial y}$$

$$= \cos(xy)(x) + e^{x+y}(1)$$

$$= \cos xy + e^{x+y}$$

$$f(xx) = \frac{\partial}{\partial x} f_x = \frac{\partial (y\cos xy + e^{x+y})}{\partial x}$$

$$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$$

$$\left[\because \frac{d}{dx}(UV) = U \cdot V' + V \cdot U'\right]$$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$f(yx) = \frac{\partial f}{\partial x} y = \frac{\partial (x\cos xy + e^{x+y})}{\partial x}$

$= \cos\{xy\}(1) + x(-\sin xy)(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$

Q.5] Find the linerization of $f(x,y)$ at given point

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}}(2y) = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$$

$$= \frac{2 + x + y - 2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

(ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0\left(\sin\left(\frac{\pi}{2}\right)\right)$$

$$= 1 - \frac{\pi}{2}$$

$$f(x) = -1 + y\cos x$$

$$f(y) = 1$$

$$f_x\left(\frac{\pi}{2}, 0\right) = -1 + 0\cos\left(\frac{\pi}{2}\right)$$

$$= -1$$

$$f_y\left(\frac{\pi}{2}, 0\right) = 1$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$$

$$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$$

$$= y - x + 1$$

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(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0 + 0 = 0$

$f(x) = \frac{1}{x}$ $\quad$ $f(y) = \frac{1}{y}$

$f_x(1,1) = 1$ $\quad$ $f_y(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a-b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

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PRACTICAL NO 10

Q.1] Find the directional derivative of the following function at given points & in the direction of given Vector

(i) $f(x,y) = x + 2y - 3 \qquad a = (1,-1) \qquad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Unit Vector along u is $\dfrac{u}{|u|} = \dfrac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 \mp \frac{h}{\sqrt{10}}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

(ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3, 4)$ $\quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f = \left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_uf(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$\therefore D_uf(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

(iii) $2x + 3y \cdot \quad a = (1, 2)\ ,\ v = (3i + 4j)$

Here $v = 3i + 4j$ is not a unit vector

$|v| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Unit vector along v is $\frac{v}{|v|} = \frac{1}{5}(3, 4)$.

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$f(a + hv) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

$f(a + hv) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$Duf(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

Q.2] Find gradient vectors for the following function at given point

(i) $f(x,y) = x^y + y^x \Rightarrow a(1,1)$

$fx = y \cdot x^{y-1} + y^x \log y$

$fy = x^y \log x + xy^{x-1}$

$f(x,y) = (fx, fy)$

$= (yx^{y-1} + y^2 \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) y^2 \quad a = (1,-1)$

$fx = \frac{1}{1+x^2} \cdot y^2$

$fy = 2y \cdot \tan^{-1} x$

$f(x,y) = (fx, fy)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4_2}(-\cancel{2})\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

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(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, \; xz - e^{x+y+z}, \; xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, \; 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Find the equation of tangent & normal to each of the following using curves at given points

$x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \; 2x + e^{xy} . y$

$f_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1,0) = \quad \therefore x_0 = 1, \; y_0 = 0$

eqn of tangent

$fx(x - x_0) + fy(y - y_0) = 0$

$fx(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$fy(x_0, y_0) = (1)^2(-\sin 0) + e^0 . 1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

It is the required eqn of tangent

Eqn of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

(i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$fx = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad \therefore x_0 = 2, \; y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \longrightarrow$ It is required eqn of tangent

Equation of Normal

$ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$= -x + 2y + d = 0$ at $(2, -2)$

$= -2 + 2(-2) + d = 0$

$= -2 - 4 + d = 0$

$= -6 + d = 0$

$\therefore d = 6$

Q.4] Find the eqn of tangent & normal line to each of the following surface.

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 \neq 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eqn of tangent

Equation of Normal

$ax + by + c = 0$

$bx + ay + d = 0$

$-1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q.4.] Find eqn of tangent & normal line to each of the following surface

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$f_y = 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

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$(x_0, y_0, z_0) = (2, 1, 0) \quad \therefore \quad x_0 = 2, \; y_0 = 1, \; z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Eqn of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 + 0 = 0$

$4x + 3y - 11 = 0$ → Required eqn of tangent

Eqn of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2) \therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Eqn of tangent:

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ This is reqd eqn of tangent

Eqn of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q. 5] Find the local maxima & minima for the following function

(i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$fy = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$fx = 0$
$6x - 3y + 6 = 0$
$3(2x + 2 - y) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ — (1)

$fy = 0$
$2y - 3x - 4 = 0$
$2y - 3x = 4$ — (2)

Multiply eqn 1 with 2
$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$
Substitute value of x in eqn (1)
$2(0) - y = -2$
$-y = -2$
$\therefore y = 2$
Critical points are $(0,2)$

$r = fxx = 6$
$t = fyy = 2$
$s = fxy = -3$
Here $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$3x^2+y^2-3xy+6x-4y$ at $(0,2)$

$3(0)^2+(2)^2-3(0)(2)+6(0)-4(2)$

$0+4-0+0-8$

$=-4$

(ii) $f(x,y)=2x^4+3x^2y-y^2$

$f_x = 8x^3+6xy$

$f_y = 3x^2-2y$

$f_x = 0$

$\therefore 8x^3+6xy=0$

$2x(4x^2+3y)=0$

$4x^2+3y=0 \rightarrow (1)$

$f_y=0$

$3x^2-2y=0 \rightarrow (2)$

Multiply eqn (1) with 3

(2) with 4

$12x^2+9y=0$

$-12x^2-8y=0$

$11y=0$

$y=0$

Substitute value of y in eqn (1)

$4x^2+3(0)=0$

$4x^2=0$

$x=0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2+6x$

$t=f_{yy}=0-2=-2$

$s=f_{xy}=6x-0=6x=6(0)=0$

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r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - 0$

$= 0 + 0 - 0$

$= 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2}$

$\therefore x = -1$

$f_y = 0 \quad = -2y + 8 = 0$

$y = \frac{-8}{2}$

$\therefore y = 4$

$\therefore$ critical point is $(-1, 4)$

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$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 37 - 70$

$= -33$

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