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Results on a class of analytic functions with finitely many fixed coefficients related to a generalised multiplier transformation

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ABSTRACT

A new class of univalent functions denoted by $T_m(\xi, \varpi, \sigma, e_n)$ is defined and studied in this work. The class of functions $T_m(\xi, \varpi, \sigma, e_n)$ generalises certain previously introduced classes of functions in the literature. Geometric properties of the class of functions in the class such as coefficient estimate, radius of starlikenes and radius convexity were studied.

Introduction and preliminaries

Let $\mathbb{U}$ be the unit disk $z \in \mathbb{C} : |z| < 1$, A be the class of analytic functions in $\mathbb{U}$, satisfying the conditions $f(0) = 0$ and $f'(0) = 1$, and of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

We denote T the subclass of A of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \quad a_k \geq 0. \tag{1.2}$$

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Amourah and Darus in [2] defined a differential operator

$$A^m_{\mu,\lambda,\delta}(\alpha,\beta)f(z) = z - \sum_{k=2}^{\infty}\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m a_k z^k \tag{1.3}$$

for $f \in T$, $\alpha, \beta, \delta, \lambda, \epsilon, \omega \geq 0$; $\mu > 0$ *and* $\mu \neq \lambda$; $m \in \mathbb{N}_0 = \mathbb{N}\cup\{0\}$. The operator generalized the differential operator in [3]. Other authors such as [1] and [10] have also successfully used generalized differential operators.

In paper [13], the following class of univalent functions was introduced.

Definition 1.1. [13] For $\alpha, \delta \geq 0$, $\beta, \lambda, \mu > 0$, $\alpha \neq \lambda$ and $m \in \mathbb{N}_0$, let $0 < \xi \leq 1$, $\sigma \leq 1$ and $\varpi \in \mathbb{C} - \{0\}$. Then, the function $f \in T$ is said to be in the class $T_m(\xi, \varpi, \sigma)$ if

$$\left|\frac{1}{\varpi}\left(\frac{z(A^m_{\mu,\lambda,\delta}(\alpha,\beta)f(z))'}{A^m_{\mu,\lambda,\delta}(\alpha,\beta)f(z)} - \sigma\right)\right| < \xi, \quad z \in U. \tag{1.4}$$

The following Lemma will be required to prove the main results in the work.

Lemma 1.1. *[13] Let f be defined by (1.2), then $f \in T_m(\xi, \varpi, \sigma)$ if and only if*

$$\sum_{k=2}^{\infty}[k-\sigma+\xi|\varpi|]\left(1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right)^m a_k \leq [1-\sigma+\xi|\varpi|] \tag{1.5}$$

See [13] for the proof.

Remark 1.1. The result is sharp for the function

$$f(z) = z - \frac{\xi|\varpi|+\sigma-1}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m} z^k.$$

Let $f \in T_m(\xi, \varpi, \sigma)$. Then,

$$a_k \leq \frac{\xi|\varpi|+\sigma-1}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}. \tag{1.6}$$

Class $T_m(\xi, \varpi, 1) \equiv TS^n_1(\beta, \gamma)$ and

$$a_k \leq \frac{\xi|\varpi|}{(k-1+\varpi+\xi|\varpi|)k^m} z^k, \quad k \geq 2,$$

with equality only for functions of the form

$$f(z) = z - \frac{\xi|\varpi|}{(k-1+\xi|\varpi|)k^m} z^k,$$

which affirms that $T_m(\xi, \varpi, 1) \equiv TS^n_1(\beta, \gamma)$ in [1].

Investigating geometric properties of univalent functions is one of the interests in geometric function theory. Studies in [1,2], [3,11–15] are some examples. The object of this study is to introduce a new class of univalent functions and study some of its properties.

Definition 1.2. We introduce the class $T_m(\xi, \varpi, \sigma, e_n)$ as a class of functions of the form

$$f(z) = z - \sum_{n=2}^{t}\frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} z^n - \sum_{k=t+1}^{\infty} a_k z^k, \tag{1.7}$$

where

$$e_n = \frac{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]} a_n$$

and

$$a_k = \frac{[\xi|\varpi|+\sigma-1](1-\sum_{n=2}^{t} e_n)}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}. \tag{1.8}$$

Remark 1.2. Class $T_m(\xi, \varpi, \sigma, e_n) \subset T_m(\xi, \varpi, \sigma)$. In particular, $T_m(\xi, \varpi, \sigma, 1) \equiv T_m(\xi, \varpi, \sigma)$.

Example of functions in the class is a function of the form

$$f(z)=z-\sum_{n=2}^{t}\frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}z^n-\sum_{k=t+1}^{\infty}a_kz^k.$$

The parameter

$$-\sum_{n=2}^{t}\frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}z^n$$

of the above equation is the fixed finitely many negative coefficients.

A function $f(z)$ in the class $T_m(\xi,\varpi,\sigma,0)$ is given by (1.2).

Similar classes of functions with fixed coefficients were also studied in [4–9,11,12,16–18].

Main result

In this section we state and prove the main results of this paper.

We begin by proving the necessary and sufficient condition for a function to belong to the class $T_m(\xi,\varpi,\sigma,e_n)$.

Theorem 2.1. *Let f be defined by (1.2), then $f\in T_m(\xi,\varpi,\sigma,e_n)$ if*

$$\sum_{k=t+1}^{\infty}\frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]}a_k<1-\sum_{n=2}^{t}e_n. \tag{2.1}$$

Proof. Let

$$a_n=\frac{[\xi|\varpi|+\sigma-1](1-\sum_{n=2}^{t}e_n)}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}. \tag{2.2}$$

Then, $f\in T_m(\xi,\varpi,\sigma,e_n)\subset T_m(\xi,\varpi,\sigma)$ if and only if

$$\sum_{n=2}^{t}\frac{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]}a_n+$$

$$\sum_{k=t+1}^{\infty}\frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]}a_k<1.$$

That is,

$$\sum_{k=t+1}^{\infty}\frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]}a_k<1-\sum_{n=2}^{t}e_n.$$

□

Corollary 2.2. *Let $f\in T_m(\xi,\varpi,\sigma,e_n)$, then*

$$a_k\le\frac{[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t}e_n\right)}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}, \tag{2.3}$$

with equality only for functions of the form

$$f(z)=z-\frac{\xi|\varpi|}{(k-1+\xi|\varpi|)k^m}z^k,$$

which affirms that $T_m(\xi,\varpi,1)\equiv TS_1^n(\beta,\gamma)$ in [1].

Theorem 2.3. *Let $f_j,\quad(j=1,2,3\ldots\in\mathbb{N})$ defined by*

$$f_j(z)=z-\sum_{n=2}^{t}\frac{[\xi|\varpi|+\sigma-1](1-\sum_{n=2}^{t}e_n)}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}z^n-\sum_{k=t+1}^{\infty}a_{k,j}z^k \tag{2.4}$$

be in the class $T_m(\xi,\varpi,\sigma)$. Then

$$H(z)=\sum_{j=1}^{i}\lambda_jf_j\text{ and }\sum_{j=1}^{i}\lambda_j=1,\quad 0\le\sum_{n=2}^{t}e_n\le1\quad 0\le e_n\le1$$

also belongs to the class.

Proof. Let $f_j(z) \in T_m(\xi, \varpi, \sigma)$. Then from Theorem 2.1,

$$\sum_{k=t+1}^{\infty} \frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]} a_{k,j} < 1-\sum_{n=2}^{t} e_n,$$

for every $j = 1, 2, 3 \ldots i$. So that

$$H(z) = \sum_{j=1}^{i} \lambda_j f_j =$$

$$z - \sum_{n=2}^{t} \frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} - \sum_{k=t+1}^{\infty}\left(\sum_{j=1}^{i} \lambda_j a_{k,j}\right) z^k$$

and

$$\sum_{k=t+1}^{\infty} \frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]}\left(\sum_{j=1}^{i}\lambda_j a_{k,j}\right)$$

$$= \sum_{j=1}^{i}\sum_{k=t+1}^{\infty}\left(\frac{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}{[\xi|\varpi|+\sigma-1]} a_{k,j}\right)\lambda_j$$

$$< \sum_{j=1}^{i}\left(1-\sum_{n=2}^{t} e_n\right)\lambda_j = 1-\sum_{n=2}^{t} e_n.$$

□

Theorem 2.4. *The function f defined by (1.2) in the class $T_m(\xi, \varpi, \sigma, e_n)$ is a starlike function of order ϑ $(0 \le \vartheta \le 1)$ in $|z| < r_1$, r_1 is the largest value that satisfies*

$$\sum_{k=t+1}^{\infty} \frac{[(2-n)-\vartheta][\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} +$$
$$\frac{[(2-k)-\vartheta][\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m} r^{k-1} \le \vartheta. \tag{2.5}$$

Proof. Let f be a starlike function of order ϑ. Then, we need to show that

$$\left|\frac{zf'(z)}{f(z)}-1\right| \le \frac{\sum_{n=2}^{t} \frac{(n-1)[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} - \sum_{k=t+1}^{\infty}(k-1)a_k r^{k-1}}{1-\sum_{n=2}^{t} \frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} - \sum_{k=t+1}^{\infty} a_k r^{k-1}}$$

$$\le 1-\vartheta \qquad (|z| \le r).$$

Thus,

$$\sum_{n=2}^{t} \frac{(2-n)-\vartheta[\xi|\varpi|+\sigma-1]}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} e_n r^{n-1} + \sum_{k=t+1}^{\infty}[(2-k)-\vartheta]a_k r^{k-1} \le 1-\vartheta. \tag{2.6}$$

Following (2.3) of Corollary 2.2, we set

$$a_k \le \frac{[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}\lambda_k; \quad k \ge t+1, \tag{2.7}$$

such that $\lambda_k \ge 0$ and $\sum_{k=t+1}^{\infty} \lambda_k \le 1$.

Now we choose an integer $k_0 = k_0(r)$ for which $\frac{(2-k)-\vartheta[\xi|\varpi|+\sigma-1]r^{k-1}}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m}$ is maximum for each fixed r so that

$$\sum_{k=t+1}^{\infty}(k-\vartheta)a_k r^{k-1} \le \frac{[(2-k_0)-\vartheta][\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m} r^{k_0-1}, \tag{2.8}$$

which implies that f is starlike of order ϑ in $|z| \leq r_1$ provided

$$\sum_{n=2}^{t} \frac{[(2-n)-\vartheta][\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} + \frac{[(2-k_0)-\vartheta][\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m} r^{k_0-1} \leq 1-\vartheta \tag{2.9}$$

We now estimate the value of k_0 and the equivalent $k_0(r_0)$

$$\sum_{n=2}^{t} \frac{[(2-n)-\vartheta][\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} + \frac{[(2-k_0)-\vartheta][\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m} r^{k_0-1} \leq 1-\vartheta. \tag{2.10}$$

The above expression yields the radius of starlikeness of order ϑ for functions in the class $T_m(\xi, \varpi, \sigma, e_n)$ which completes the prove. □

Theorem 2.5. *The function f defined by (1.2) in the class $T_m(\xi, \varpi, \sigma, e_n)$ is convex of order ϑ $(0 \leq \vartheta \leq 1)$ in $|z| < r_2$, r_2 is the largest value that satisfies*

$$\sum_{k=t+1}^{\infty} \left\{ \frac{n(n-\vartheta)[\xi|\varpi|+\sigma-1]}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} \right\} r^{n-1} + \frac{k(k-\vartheta)[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k-\sigma)+\xi|\varpi|]\left\{1+\frac{(k-1)[(\lambda-\alpha)\beta+k\delta]}{\mu+\lambda}\right\}^m} r^{k-1} \leq \vartheta. \tag{2.11}$$

Proof. Let f be a convex function of order ϑ. Then, we need to show that

$$\left|\frac{zf''(z)}{f'(z)}\right| \leq \frac{\sum_{n=2}^{t} \frac{n(n-1)[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{n(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} + \sum_{k=t+1}^{\infty} k(k-1)a_k r^{k-1}}{1-\sum_{n=2}^{t} k\frac{n[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} - \sum_{k=t+1}^{\infty} a_k r^{k-1}}$$

$$\leq 1-\vartheta \qquad (|z| \leq r).$$

So that

$$\sum_{n=2}^{t} \frac{n(n-\vartheta)[\xi|\varpi|+\sigma-1]}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} e_n r^{n-1} + \sum_{k=t+1}^{\infty} k(k-\vartheta)a_k r^{k-1} \leq 1-\vartheta. \tag{2.12}$$

From Corollary 2.2, we choose an integer $k_0 = k_0(r)$ for which $\frac{k_0(k_0-\vartheta)[\xi|\varpi|+\sigma-1]r^{k-1}}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m}$ is maximum for each fixed r so that

$$\sum_{k=t+1}^{\infty} (k-\vartheta)a_k r^{k-1} \leq \frac{k_0(k_0-\vartheta)[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m} r^{k_0-1}, \tag{2.13}$$

which implies that f is convex of order ϑ in $|z| \leq r_1$ provided

$$\sum_{n=2}^{t} \frac{n(n-\vartheta)[\xi|\varpi|+\sigma-1]}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} + \frac{k_0(k_0-\vartheta)[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t} e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m} r^{k_0-1} z \leq 1-\vartheta. \tag{2.14}$$

We now estimate the value of k_0 and the equivalent $k_0(r_0)$

$$\sum_{n=2}^{t} \frac{n(n-\vartheta)[\xi|\varpi|+\sigma-1]}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m} r^{n-1} +$$

$$\frac{(k_0-\vartheta)[\xi|\varpi|+\sigma-1]\left(1-\sum_{n=2}^{t}e_n\right)}{[(k_0-\sigma)+\xi|\varpi|]\left\{1+\frac{(k_0-1)[(\lambda-\alpha)\beta+k_0\delta]}{\mu+\lambda}\right\}^m}r^{k_0-1}\le 1-\vartheta. \tag{2.15}$$

The above expression yields the radius of convex of order ϑ for functions in the class $T_m(\xi,\varpi,\sigma,e_n)$. □

Conclusion

The study introduced a class of univalent functions with finitely many fixed coefficients denoted $T_m(\xi,\varpi,\sigma,e_n)$. Functions belonging to the new class is sharp with the function

$$f(z)=z-\sum_{n=2}^{t}\frac{[\xi|\varpi|+\sigma-1]e_n}{[(n-\sigma)+\xi|\varpi|]\left\{1+\frac{(n-1)[(\lambda-\alpha)\beta+n\delta]}{\mu+\lambda}\right\}^m}z^n-\sum_{k=t+1}^{\infty}a_kz^k.$$

Also, $T_m(\xi,\varpi,\sigma,e_n)\subset T_m(\xi,\varpi,\sigma)$ introduced in [13]. The new class of functions does not only generalize some existing results in literature but also add to the body knowledge as relating to class of functions in this direction. The study uses an existing differential operator $A^m_{\mu,\lambda,\delta}(\alpha,\beta)$ as a tool to develop and establish new class of functions. Some geometric properties of the new class of functions were also studied.

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