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Mathematical modelling of the epidemiology of COVID-19 infection in Ghana

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ABSTRACT

In this paper, Covid-19 patients with self-immunity is incorporated in the Susceptible-Exposed-Infected-Quarantined-Recovered (*SEIQR*) model is applied to describe the epidemiology of Covid-19 infection in Ghana. Based on data on the epidemiology of the Covid-19 infection in Ghana, we observed that, on an average, three persons contract the Covid-19 infection from an infected person daily based using the basic reproductive number (R_o) derived from the SEIQR model. In addition, the threshold condition for the long term stability of the Covid-19 infection in Ghana is derived from this model. Based on the Dulac criterion, it was observed that for a long period of time the epidemiology of Covid-19 in Ghana will be under control. Again, we observed that both the transmission rate natural death rate of a person in the various classes mostly influence the spread of Covid-19 infection followed by the exposed rate from exposure class to the infected class, then the rate at which an infected person is quarantined and finally, the rate at an exposed person is quarantined. On the other hand, the rate at which an exposed person recovers from his/her have least influence on the spread of Covid-19 infection in the country. Nevertheless, the rates of birth, transmission of Covid-19 infection to a susceptible person, exposure to Covid-19 infection and Covid-19 patient who is quarantined by the facilities provided by the Ghana Health Service (*GHS*) are in direct relationship with R_o. However, the rates at which a quarantiner dies from a Covid-19 infection, an infected person dies from a Covid-19 infection, natural death from each class and the recoveries from an infected class, exposed class and quarantined class are in relationship with R_o.

Introduction

The new virus was unknown until the outbreak began in Wuhan, China on 31st December, 2019. Covid-19 is an infectious disease caused by the most recently discovered coronavirus SARS-Cov-2. Covid-19 has the potential to endanger

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people's lives and livelihoods all across the world. The disease is currently a major health issue in Ghana and beyond. Despite the fact that human-to-human transmission has been shown, little research works have been done on the dynamics of the virus in the environment. The SARS-Cov 2 virus is transferred mostly if a susceptible person comes in contact with a Covid-19 patient through droplets in air space by either coughing or sneezing [9]. Rather than traveling over vast distances in air, the droplet normally falls to the ground or onto objects. People can get infected by contacting a contaminated surface with SARS-Cov 2 virus and then touching their faces, albeit this is a rare occurrence. It is most contagious in the first three days after symptoms start, but it can also spread before symptoms begin or from those who do not show symptoms. While certain western, traditional, or home remedies may provide relief from Covid-19 symptoms.

According to the World Health Organization (WHO) no drug intake can prevent the disease. However, a number of clinical trials including both orthodox and traditional medicines are currently underway to reduce the severity of the persons with Covid-19 infections. Ghana discovered her first two cases of Covid-19 infection on 12th March, 2020 and since then, the number of Covid-19 cases has increased geometrically with new cases being recorded almost everyday. The author in [8] observed boundaries of Covid 19 importation risk scenarios in the sub-Saharan Africa. In their work, the circumstances of SARS Cov-2 virus importation into a sub-Saharan Africa as a low risk of new coronavirus transmission into Africa. However, the model by these researchers did not take into account asymptomatic cases to help characterized the current coronavirus pandemic. Several methods such as statistical techniques have been consistently applied to describe the epidemiology of covid−19 in Ghana, for example, see [10], but using mathematical tools uncover the threshold condition, R_o, to ensue the spread of SARS virus in the country. Nevertheless, a mathematical model unveals key underlying parameters in describing the epidemiology of this disease and more importantly gives reasoned estimates for these parameters with minimal variations as compared with statistical estimations of the parameters. Mathematical models (analysis) are used to influence public health policy by suggesting the main underlying mechanisms in the Covid-19 epidemiology. By using these mathematical tools the strategies for controlling this disease is revealed with a very high degree of accuracy.

The authors in [5] introduced SEIQR model for modelling SARS epidemiology and suggested some measures that must be put in place in curbing the disease. The authors in [2] applied a Susceptible-Exposed-Infected-Recovered (SEIR) model for describing coronavirus phase-based transmissivity. They used a bats-hosts-reservoir-people transmission network to simulate the spread of infection from infected bats to human beings. They observed that, on average, a bat transmits Covid-19 virus to three people whereas a Covid-19 patient transmits infection to four persons daily. Notwithstanding, authors in [3] optimized the interplay of the vaccination and the social distancing measures. On the contrary, the relationship between disease dynamics and individual adherence to protection strategies were not captured in their model. Authors in [6] made used of a data-driven model and forecasted incidence of the Covid-19 infections in India and correlation analysis of the virus transmission with socio-economic factors. The connection between the spread of the new coronavirus and the socio-economic factors of various states in India was obtained with minimal precision. In [7], authors observed the population variations in the Covid-19 dynamics in Hubei, Lombardy and New York City. Authors in [1] used statistical tools to estimate Covid-19 infections. They analyzed the total cases and cummulative death toll of countries in the northern hemisphere using both statistical author and cubic models of natural logarithm. They indicated that, if we stick to the safety measures in place, the prediction of the spread of the new coronavirus could be halted. Nevertheless, the *SEIR* model was applied by the authors in [11] for epidemiology of the Covid-19 epidemics in China. Data on recoverends, death cases, susceptibles, the covid−19 patients were used to simulate the various components of the model.

By and large, the mathematical models describing the Covid-19 epidemiology have not included quarantiners with self-immunity. Also, the number of expected number of persons with Covid-19 infection cannot be overemphasized as it play a key role in understanding the dynamics of Covid-19 infection in the country. This paper investigates scientific inquiry into the forescasting of the epidemiology of Covid-19 infection by incorporating the quarantiners with self-immunity in the classical SEIQR model. In addition, the effects of the underlying mechanisms (parameters) in determining the spread of this disease in the susceptible population of Ghana are uncovered. The threshold conditions for the long term behaviour of the equilibria of the modified SEIQR model is provided.

Main result

In this paper, Covid-19 patients with self-immunity is introduced in a *SEIQR* model to describe the epidemilogy of covid−19 in Ghana. The total population size of Ghana, N, is divided into five compartments namely; Susceptible class S(t), Exposed class E(t), Infectious class I(t), Quarantine class Q(t) and Recovered class $R_1(t)$. This mathematical model involves first-order differential operator with solution in the subspace of the Hilbert space.

The assumptions of the *SEIQR* model for describing the epidemiology of Covid-19 in Ghana is as follows.

1. The rate at which an individual becomes infected is proportional to the number of infectives present at that time.
2. Exposure to the virus comes as a results of an interaction with an individual from the infectious and quarantined class.
3. New borns are recruited to the susceptible class.
4. The travellers with the covid-19-virus into the country are not included in this model.

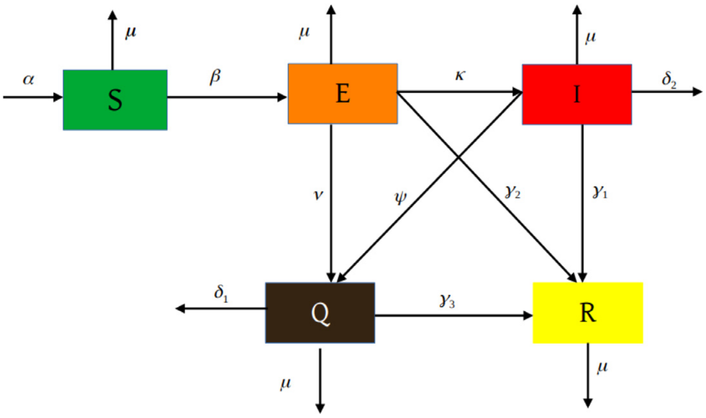

Fig. 1. Fig. 1 shows a mathematical model for describing an epidemiology of Covid-19 infection in Ghana.

Based on the Fig. 1, the following system of ODEs is obtained to describe the epidemiology of Covid-19 in Ghana.

$$\begin{aligned}
\frac{dS}{dt} &= \alpha N - \frac{\beta S(I+Q)}{N} - \mu S \\
\frac{dE}{dt} &= \frac{\beta S(I+Q)}{N} - (\mu + \kappa + \gamma_2 + \nu)E \\
\frac{dI}{dt} &= \kappa E - (\mu + \delta_2 + \gamma_1 + \psi)I \\
\frac{dQ}{dt} &= \nu E + \psi I - (\mu + \delta_1 + \gamma_3)Q \\
\frac{dR}{dt} &= \gamma_1 I + \gamma_2 E + \gamma_3 Q - \mu R \\
N(t) &= S(t) + E(t) + I(t) + Q(t) + R(t),
\end{aligned} \tag{1}$$

where α is the rate at which a baby is born into a susceptible class, β is the rate at which a person contracts Covid-19 virus, κ is the rate at which exposed person becomes infectious as he/she develops weak immune system, ν is the rate at which an exposed person is quarantined, ψ is the rate at which an infected person is quarantined, γ_1 is the rate at which an infected person recovers from his/her illness. The rate at which an exposed person recovers from Covid-19 infection is denoted by γ_2, the rate at which a quarantiner recovers from Covid-19 infection is γ_3. μ is the rate at which individuals die naturally from the compartments, δ_1 is the rate at which quarantined person dies from the covid-19 infection and δ_2 is the rate at which infected person dies from the Covid-19 infection.

The system of Eq. (1) is scaled into proportions by setting

$$\begin{aligned}
s(t) &= \frac{S(t)}{N}, \qquad e(t) = \frac{E(t)}{N}, \qquad i(t) = \frac{I(t)}{N}, \\
q(t) &= \frac{Q(t)}{N}, \qquad e(t) = \frac{R(t)}{N},
\end{aligned}$$

which in turn, yields

$$\begin{aligned}
\frac{ds}{dt} &= \alpha - \beta s(i+q) - \mu s \\
\frac{de}{dt} &= \beta s(i+q) - (\mu + \kappa + \gamma_2 + \nu)e \\
\frac{di}{dt} &= \kappa e - (\mu + \delta_2 + \gamma_1 + \psi)i \\
\frac{dq}{dt} &= \nu e + \psi i - (\mu + \delta_1 + \gamma_3)q \\
\frac{dr}{dt} &= \gamma_1 i + \gamma_2 e + \gamma_3 q - \mu r,
\end{aligned} \tag{2}$$

where,

$$s(t) + e(t) + i(t) + q(t) + r(t) = 1.$$

The fixed points and their stability analysis

Putting all the derivatives on the right-hand side of system of Eq. (2) and solving the resulting equations simultaneously yields

$$(s^*, e^*, i^*, q^*, r^*) = \left(\frac{\alpha}{\mu}, 0, 0, 0, 0\right)$$

as a disease-free fixed point and

$$(s^*, e^*, i^*, q^*, r^*) = (s_2^*, e_2^*, e_2^*, i_2^*, q_2^*, r_2^*),$$

where,

$$\begin{aligned}
s_2^* &= \frac{m_1 m_2 m_3}{\beta\kappa\psi + \beta\kappa m_3 + \beta\nu m_2},\\
e_2^* &= \frac{\alpha\beta\kappa(m_3+\psi) + m_2(\alpha\beta\nu - \mu m_1 m_3)}{\beta m_1(\kappa(m_3+\psi)+\nu m_2)},\\
i_2^* &= \frac{\kappa(\alpha\beta\kappa(m_3+\psi) + m_2(\alpha\beta\nu - \mu m_1 m_3))}{\beta m_1 m_2(\kappa(m_3+\psi)+\nu m_2)},\\
q_2^* &= (\kappa\psi + \nu m_2)\left(\frac{\alpha}{m_1 m_2 m_3} - \frac{\mu}{\beta\kappa\psi + \beta\kappa m_3 + \beta\nu m_2}\right), \ \textit{and}\\
r_2^* &= \frac{(\gamma_3(\kappa\psi+\nu m_2) + m_3(\gamma_1\kappa + \gamma_2 m_2))(\alpha\beta\kappa(m_3+\psi) + m_2(\alpha\beta\nu - \mu m_1 m_3))}{\beta\mu m_1 m_2 m_3(\kappa(m_3+\psi)+\nu m_2)}.
\end{aligned}$$

as an endemic fixed point. Linearizing system of Eq. (2), we obtain

$$J(s,e,i,q,r) = \begin{bmatrix} -(\beta(i+q)+\mu) & 0 & -\beta s & -\beta s & 0 \\ \beta(i+q) & -m_1 & \beta s & \beta s & 0 \\ 0 & \kappa & -m_2 & 0 & 0 \\ 0 & \nu & \psi & -m_3 & 0 \\ 0 & \gamma_2 & \gamma_1 & \gamma_3 & -\mu \end{bmatrix}. \tag{3}$$

Using the Diekmann and Heersterbeek method for deriving the basic reproductive number R_o, we observed that matrices of whose entries are new infections and other infections are

$$F = \begin{bmatrix} 0 & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\textit{and } V = \begin{bmatrix} m_1 & 0 & 0 \\ -k & m_2 & 0 \\ -\nu & -\psi & m_3 \end{bmatrix},$$

respectively.

The corresponding

$$V^{-1} = \begin{bmatrix} \frac{1}{m_1} & 0 & 0 \\ \frac{\kappa}{m_1 m_2} & \frac{1}{m_2} & 0 \\ \frac{\kappa\psi + \nu m_2}{m_1 m_2 m_3} & \frac{\psi}{m_2 m_3} & \frac{1}{m_3} \end{bmatrix}$$

$$\textit{and } FV^{-1} = \begin{bmatrix} \frac{\beta\alpha(\kappa\psi+\nu m_2)}{\mu m_1 m_2 m_3} + \frac{\beta\alpha\kappa}{\mu m_1 m_2} & \frac{\beta\alpha\psi}{\mu m_2 m_3} + \frac{\beta\alpha}{\mu m_2} & \frac{\beta\alpha}{\mu m_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

At disease-free fixed point we observed that,

$$R_0 = \varphi(FV^{-1}) = \frac{\beta\alpha(\kappa m_3 + k\psi + \nu m_2)}{\mu m_1 m_2 m_3}$$

$$\Rightarrow R_0 = \frac{\alpha\beta(\kappa(\gamma_3+\delta_1+\mu) + \nu(\gamma_1+\delta_2+\mu+\psi) + \kappa\psi)}{\mu(\gamma_3+\delta_1+\mu)(\gamma_1+\delta_2+\mu+\psi)(\gamma_2+\kappa+\mu+\nu)}. \tag{4}$$

Substituting $(s^*, e^*, i^*, q^*, r^*) = (s_2^*, e_2^*, i_2^*, q_2^*, r_2^*)$ into the Jacobian matrix in Eq. (3) and finding the eigenvalues yields a characteristic polynomial

$$\lambda^4 + T\lambda^3 + U\lambda^2 + V\lambda + W = 0, \qquad (30)$$

where,

$$\begin{aligned}
T &= \beta(i+q) + \mu + m_1 + m_2 + m_3,\\
U &= \beta m_1(i+q) + \beta m_2(i+q) + \beta m_3(i+q) + \mu m_1 + \mu m_2 + \mu m_3 + m_1 m_2 + m_1 m_3\\
&\quad + m_2 m_3 - \beta\kappa s - \beta\nu s,\\
V &= \beta m_1 m_2(i+q) + \beta m_1 m_3(i+q) + \beta m_2 m_3(i+q) + \mu m_1 m_2 + \mu m_1 m_3 + \mu m_2 m_3 - \beta\kappa m_3 s\\
&= -\beta\nu m_2 s + m_1 m_2 m_3 + \beta - \kappa\mu s - \beta\kappa s\psi - \beta\mu\nu s,\\
W &= \beta m_1 m_2 m_3(i+q) + \mu m_1 m_2 m_3 - \beta\kappa\mu m_3 s - \beta\mu\nu m_2 s + \beta - \kappa\mu s\psi.
\end{aligned}$$

Using the Routh-Hurwitz conditions for determining the negative sign of the eigenvalues in Eq. (5) yields

$$T > 0,\;\; TU - V > 0,\;\; TUV - V^2 - T^2 W > 0,\;\text{ and } W > 0. \qquad (4)$$

Sensitivity analysis

The sensitivity analysis on R_0 uncovers effects of each parameter in this threshold condition. In order to obtain the effects of parameters in the model, we use an algorithm which provides the sensitivity index of each parameter in R_0 in Eq. (7). Sensitivity index enables us to quantify the relative change in the R_0 when the parameter changes. The normalized forward sensitivity index used provides a better result in estimating sensitivity index of the parameter with regards to R_0 as compared with other methods. The normalized forward sensitivity index of R_0 that depends differentiably on a parameter ρ is defined by

$$\phi_\rho^{R_0} = \frac{\partial R_0}{\partial \rho}\cdot\frac{\rho}{R_0}. \qquad (5)$$

From the R_0 in Eq. (4), the threshold depends on the parameters. Based on the definition in Eq. (11), the forward sensitivity index of the R_0 with respect to β and α are obtained as:

$$\phi_\beta^{R_0} = \frac{\partial R_0}{\partial \beta}\frac{\beta}{R_0} = 1.$$

Similarly, the sensitivity indices with respect to $\kappa, \nu, \psi, \mu, \delta_1, \delta_2, \gamma_1$ and γ_2 are obtained as follows

$$\begin{aligned}
\phi_\alpha^{R_0} &= 1,\\
\phi_\kappa^{R_0} &= \frac{\kappa(\gamma_3+\delta_1+\mu+\psi)}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)} - \frac{\kappa}{\gamma_2+\kappa+\mu+\nu},\\
\phi_\nu^{R_0} &= -\frac{\kappa(\gamma_3+\delta_1+\mu+\psi)}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)} - \frac{\nu}{\gamma_2+\kappa+\mu+\nu} + 1,\\
\phi_\psi^{R_0} &= \frac{\psi(\kappa+\nu)}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)} - \frac{\psi}{\gamma_1+\delta_2+\mu+\psi}\\
\phi_\mu^{R_0} &= \frac{\mu(\kappa+\nu)}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)}\\
&\quad - \frac{\mu}{\gamma_1+\delta_2+\mu+\psi} - \frac{\mu}{\gamma_3+\delta_1+\mu} - \frac{\mu}{\gamma_2+\kappa+\mu+\nu} - 1,\\
\phi_{\delta_1}^{R_0} &= \frac{\kappa\delta_1}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)} - \frac{\delta_1}{\gamma_3+\delta_1+\mu},\\
\phi_{\delta_2}^{R_0} &= -\frac{\delta_2\kappa(\gamma_3+\delta_1+\mu+\psi)}{(\gamma_1+\delta_2+\mu+\psi)(\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi))},\\
\phi_{\gamma_1}^{R_0} &= -\frac{\gamma_1\kappa(\gamma_3+\delta_1+\mu+\psi)}{(\gamma_1+\delta_2+\mu+\psi)(\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi))},\\
\phi_{\gamma_2}^{R_0} &= -\frac{\gamma_2}{\gamma_2+\kappa+\mu+\nu}\\
\textit{and } \phi_{\gamma_3}^{R_0} &= \frac{\kappa\gamma_3}{\kappa(\gamma_3+\delta_1)+\nu(\gamma_1+\delta_2)+(\kappa+\nu)(\mu+\psi)} - \frac{\gamma_3}{\gamma_3+\delta_1+\mu}.
\end{aligned}$$

Data for the coronavirus disease were obtained from the [4], which aided in the estimation of the parameters in the model as summarized in the Table 1 below.

Table 1
List of parameters for the model.

Parameter	Description of Parameter	Value per day	Source
α	rate at which a baby is born into a susceptible class	0.00007	Estimated
β	exrate at which a person is exposed to the Covid-19 infection	0.02500	Estimated
κ	rate at which an exposed persone becomes infected with Covid-19 virus	0.07142	Estimated
δ_1	rate at which quarantined person dies from his or her Covid-19 infection	0.00081	Estimated
δ_2	rate at which an infected person dies from his or her Covid-19 infection	0.00729	Estimated
ν	rate at which an exposed person is quarantined	0.01430	Estimated
γ_1	rate at which an infected person recovers from his or her Covid-19 infection	0.00025	Estimated
γ_2	rate at which an exposed person recovers from his or her Covid-19 infection	0.00051	Estimated
γ_3	rate at which a quarantined person recovers from his or her Covid-19 infection	0.00178	Estimated
ψ	rate at which an infected person is quarantined	0.00288	Estimated
μ	rate at which an individual dies naturally from each compartment	0.00002	Estimated

Table 2
Table showing choice of initial condition.

Compartment	Value	Proportion
Susceptible	29091025	0.95631246
Exposed	1108763	3.6448×10^{-02}
Infected	92740	3.0487×10^{-03}
Quarantined	37096	1.2195×10^{-03}
Recovered	90376	2.9709×10^{-03}

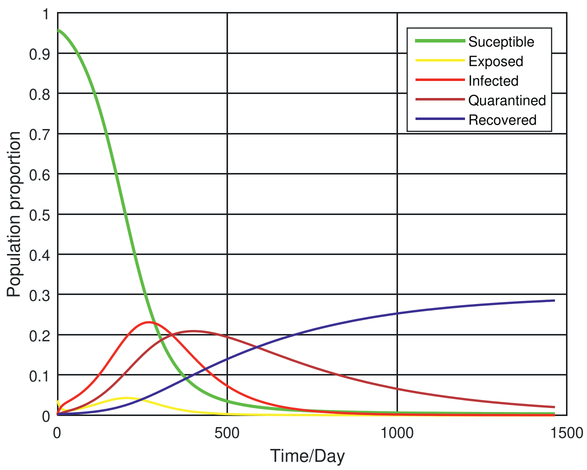

Fig. 2. Fig. 2 shows the dynamics of the novel coronavirus in Ghana.

In Table 2 below contains the initial values of the susceptible, exposed, infected, quarantined and recoverends.

All the quantitative solutions (figures) below were plotted using matlab software. The qualitative solution of the systems of (ODEs) in equation (3) is shown in the Fig. 2 below.

The Fig. 2 shows the susceptible persons, those in the waiting period (exposed class), infected persons, quarantined individuals and recoverends change as the day go by. The susceptible curve decreases as the time increases implies that, the sum of the transmission rate and persons who die naturally from the susceptible class is more than babies who are born everyday. The curve for the exposed persons rises to the peak and falls asymptotically to the t-axis as time increases, implying the number of patients infected with the coronavirus goes up to a carrying capacity of approximately 0.22 per the population size of the country, and this number will be the bearest minimum over a period of time. Similar trends were observed for the infected patients and quarantined persons in the various quarantined facilities in the country. On the other hand, the persons who recover from the Covid-19 grow exponentially day by day. The disease-free and fixed points were obtained as

$$y_1^* = (3.95745, 0, 0, 0, 0)$$
$$\textit{and } y_2^* = (0.173875, 0.000872949, 0.00596746, 0.0113552, 1.11733),$$

respectively.

Numerical results of stability of fixed points

The basic reproductive number R_o is the average number of secondary infections caused by a single covid−19 patient. In this work, the new infections are recruited from the infectious and quarantined groups as susceptibles come in contact with them. Substituting the values of the parameters in Table 1 into Eq. (7) yields

$$R_o = 2.27604$$
$$R_o \approx 3.$$

This implies that, on the average, a Covid−19 person spreads the disease to three susceptible individuals daily in the country.

In order to determine the sign of the roots of an endemic fixed point we use the conditions in Eq. (9) yields

$$T = 0.0997753 > 0, \quad TU - V = 0.0249178 > 0,$$
$$V(TU - V) - T^2W = -0.00087183 < 0, \quad and \quad W = 0.025 > 0.$$

We see from above inequalities that the Routh-Hurwitz conditions are not met, which implies that at least, one of the eigenvalues in the characteristic polynomial in Eq. (8) is positive. Hence, the endemic fixed point is unstable. The covid−19 is still spreading in Ghana.

Numerical results of sensitivity index of the parameters in R_o

The effect of changes in parameters on the basic reproductive number R_o by finding the sensitivity index of each parameter in the expression for R_o. Findings on the sensitive indices are summarized in the table below.

Parameter(ρ)	Sensitivity Index($\gamma_\rho^{R_0}$)
α	1.00000
β	1.00000
κ	0.72442
ψ	0.17998
μ	-1.00660
δ_1	-0.20304
δ_2	-0.50541
ν	0.10879
γ_1	-0.01767
γ_2	-0.00591
γ_3	-0.44747

The research further carried out a sensitivity analysis on the parameters used in the SEIQR model. The sensitivity index of a parameter on the basic reproductive number, R_0, determine the effect of the parameter, either directly or inversely, in determining the spread of COVID-19 in Ghana. In assessing the effect of the parameter on R_0, the high the value of the magnitude of the parameter, the higher its effect on this threshold for the epidemiology to ensue. It was observed that, the rate at which people are exposed to the COVID-19 virus, β, recorded a sensitivity index of 1, that is, 100%. This indicates that the parameter β has a direct effect on the spread of the COVID-19 virus in Ghana. Hence, as the parameter β is increased at 100%, it results in a 100% increase in the spread of the COVID-19 virus in Ghana. Similarly, the rate at which a baby is born into the susceptible class yields same results as recorded for the parameter, β. Also, it was observed that, a 100% increase in the rate at which exposed individuals become results in a 72% increase in the spread of the covid-19 virus in Ghana. In addition, a 100% increase in both the rate at which infected individuals are quarantined and the rate at which exposed individuals are quarantined yields 17.2% and 10.8% increase respectively. It was further observed that, an increase in the rate at which individuals die naturally from the subgroups in the SEIQR model, μ, yields an inverse effect on the spread of the COVID-19 virus. So when μ is increased 100%, there occurs a 100% reduction in the spread of the COVID-19 virus in Ghana. It was also seen that a 100% increase in both the rate at which infected people die from the disease and the rate at which quarantined people recover from the disease resulted in 50.5% and 44.7% reduction in the spread of the COVID-19 virus in Ghana. Subsequently, the rate at which exposed individuals, γ, recover from the disease recorded a minimum influence on the spread of COVID-19, as γ is increased by 100%, the spread rate of covid-19 in Ghana decreases by 0.591%.

We visualize the sensitivity indices of each of the parameters and their corresponding effects on the R_0. In Figs. 3 and 4 show positive relationships between the natural birth rate and R_o, and the exposed rate and R_o. Figures 4 and 5 show positive relationship between the natural birth rate and R_o. A similar observation was made on the exposed rate and R_o. Also, in Figs. 5 and 6, negative relationships exist between the rate at which quarantees die naturally from Covid-19 and R_o, and the rate at which infected people die naturally from Covid-19 and R_0.

Similarly, Figs. 7 and 8 show that, rates at which an infected person recovers from Covid-19 and an exposed person recovers from Covid-19 respectively, with R_o.

Figures 11 and 12 show a negative relationship between the rate infected persons and the rate at which individuals die naturally from a class and R_o. Increase in these rates decrease the average spread of covid-19 in Ghana.

Figure 13 shows a positive relationship between the rate at which exposed people are quarantined and R_o. Thus, increasing this rate decreases R_o

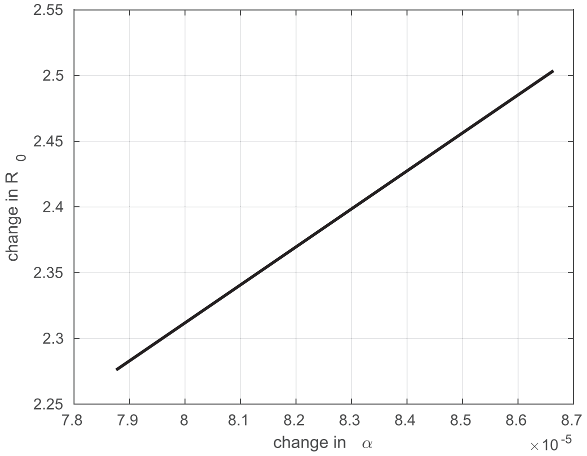

Fig. 3. Fig 3 shows the positive relationship between the natuaral birth rate and the basic reproductive number as the natural birth rate is being varied.

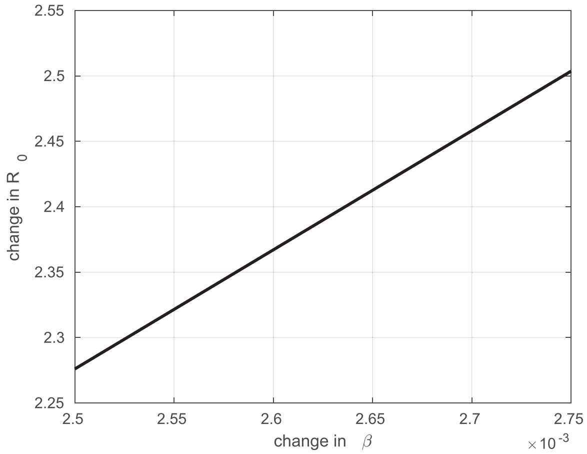

Fig. 4. Fig. 4 shows a positive relationship between the transmission rate and the basic reproductive number as β is varied.

It can be deduced from the sensitivity analysis that the parameters $\{\mu, \delta_1, \delta_2, \gamma_1, \gamma_2, \gamma_3\}$ are inversely related to the R_0 and thus increasing these parameters will cause significant reduction in the basic reproductive number and vice versa. The parameters $\{\nu, \kappa, \alpha, \beta, \psi\}$ on the otherhand are directly related to the reproductive number implying that an increase in these parameters will cause significant increase in the reproductive number and vice versa.

Numerical simulation

Figures 14 and 15 below display the number of susceptibles, exposed persons, infected persons, quarantined persons and recoverends change over time as α is been varied and other parameters in the model are kept at equilibrium.

Varying the value of α and keeping β, κ, ν, ψ, γ_1, γ_2, γ_3, μ, δ_1 and δ_2 constant.

When the value of the parameter α is decreased from 0.00078 to 0.00050, there is a slight decrease of the people in the awaiting period and the other classes. The slow decrease in the susceptible indicates that, although people in this class are contracting Covid-19 virus, and at the same time people are dying naturally, these numbers are insignificant.

Varying the value of β and keeping α, κ, ν, ψ, γ_1, γ_2, γ_3, μ, δ_1 and δ_2 constant.

Increasing β from 0.025 to 0.09 shows a sharp decrease in the number of susceptibles and a sharp increase in the infected population from over 22% to over 50% of the total population of Ghana with the infected period varied from over 340days to about 100days.There is an increased in the number of quarantined and exposed as well. From the graph, there is a clear indication that, an increase in the value of the parameter β reduces the susceptible population and increases the other subgroups within a short period of time, while decreasing the value of β decreases the susceptible population gradually and reduces the population of the other subgroups over long period of time as indicated in the figures above.

Varying the value of $\delta 1$ and keeping β, κ, ν, ψ, γ_1, γ_2, γ_3, μ, α and δ_2 constant.

Figures 18 and 19 also explain how the susceptible, exposed, infected, quarantined and recovered population change over time as the rate at which quarantines die from the Covid-19 (δ_1) is increased from 0.00081 to 0.00095 whiles holding the

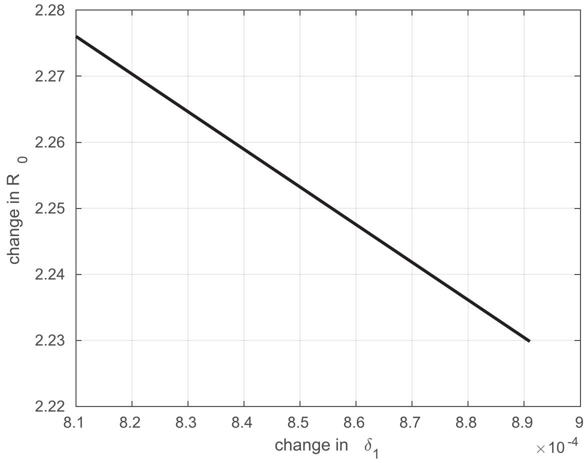

Fig. 5. Fig. 5 depicts the negative relationship between the rate of quarantined person who dies from the Covid-19 and the basic reproductive number.

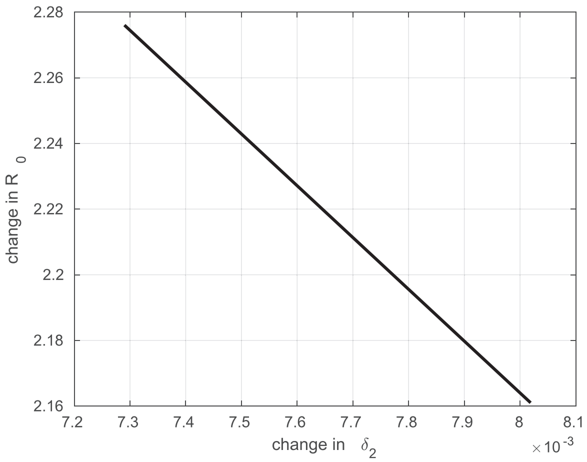

Fig. 6. Fig. 6 show a negative relationship of the rate at which a Covid-19 person dies due to illness and the basic reproductive number.

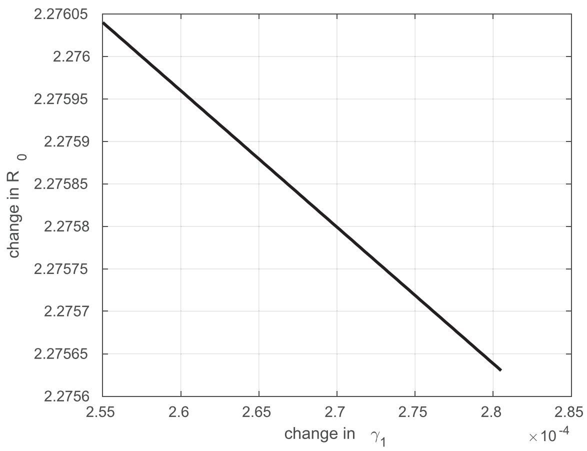

Fig. 7. Fig. 7 depicts a negative relationship between the rate at which a Covid-19 patient recovers from his or her illness and the basic reproductive number.

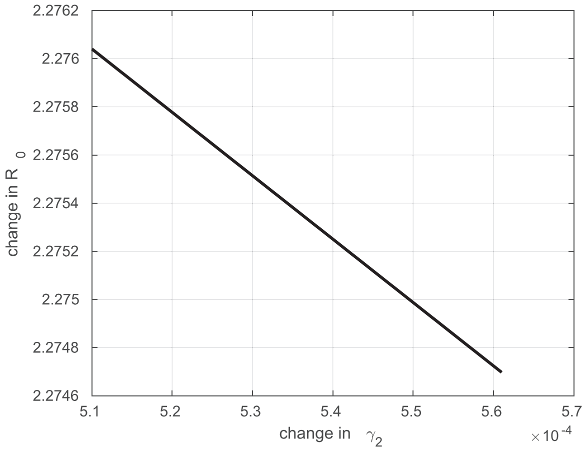

Fig. 8. Fig 8 depicts a negative relationship between the rate at which an exposed person recovers from his or her illness and the basic reproductiv number

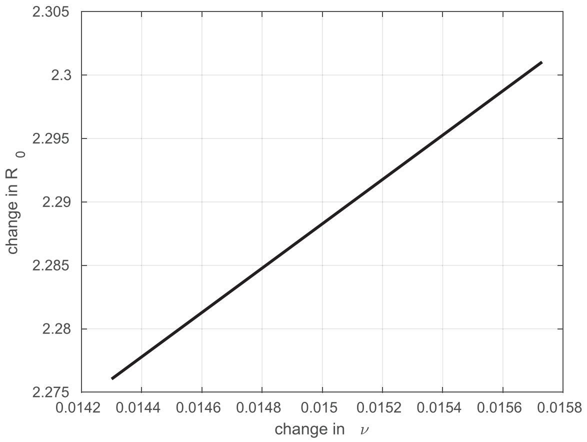

Fig. 9. Fig 9 shows that a positive relationship exists between the rate of exposed person in quarantine and the basic reproductive number.

other parameters constant. From the graph, an increase in δ_1 reduces both the quarantined and the recovery population while a decrease in δ_1 0.00081 to 0.0005 increases the quarantines and the recovery population.

Varying the value of $\delta 2$ and keeping β, κ, ν, ψ, γ_1, γ_2, γ_3, μ, δ_1 and α constant.

Figure 20 is obtained by perturbing the parameter value δ_2 which is the rate at which infected people die from Covid-19. A slight increase in the value of δ_2 from 0.00729 to 0.0085 while holding the other parameters constant decreases the number of infected and the quarantined population while Fig. 21 is obtained by decreasing the value of δ_2 from 0.00729 to 0.003. A decrease in δ_2 increases the number of infected, quarantines, exposed and the recoveries of the total population of Ghana.

Varying the value of $\gamma 1$ and keeping α, β, κ, ν, ψ, γ_2, γ_3, μ, δ_1 and δ_2 constant.

Increasing the parameter value of the rate at which infected individuals recover from Covid-19 (γ_1) from 0.000255 to 0.0004 and holding the other parameters constant increases the number of recoveries while decreasing γ_1 from 0.000255 to 0.00015 and holding the other parameters constant does not change the population of the subgroups.

Varying the value of $\gamma 2$ and keeping α, β, κ, ν, ψ, γ_1, γ_3, μ, δ_1 and α constant.

Increasing the parameter value of the rate at which exposed individuals recover from Covid-19 (γ_2) from 0.00051 to 0.0008 and holding the other parameters constant increases the number of recoveries while decreasing the parameter value from 0.000255 to 0.0003 does not change the population of the subgroups as seen in Figs. 24 and 25.

Varying the value of $\gamma 3$ and keeping α, β, κ, ν, ψ, γ_1, γ_2, μ, δ_1 and δ_2 constant.

Increasing the parameter value of the rate at which quarantined individuals recover from Covid-19 (γ_3) from 0.001785 to 0.005 and holding the other parameters constant increases the number of recoveries and decreases the population of the infectives and quarantines while decreasing the parameter γ_3 from 0.001785 to 0.0009 increases the number of quarantees and reduces the number of recoveries as seen in Figs. 26 and 27.

Varying the value of ν and keeping α, β, κ, ψ, γ_1, γ_2, γ_3, μ, δ_1 and δ_2 constant.

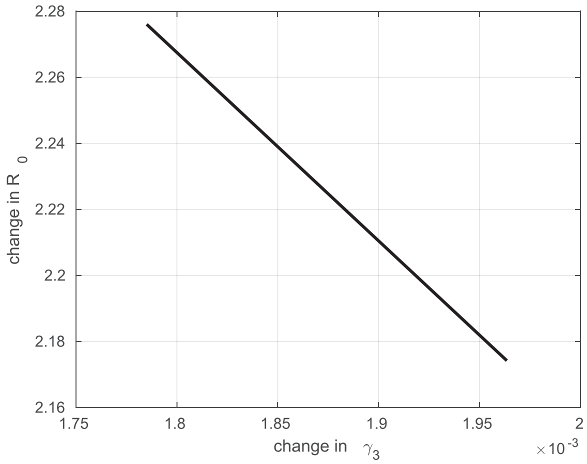

Fig. 10. Fig. 10 depicts a negative relationship between a quarantined person who recovers from Covid-19 and the basic reproductive number.

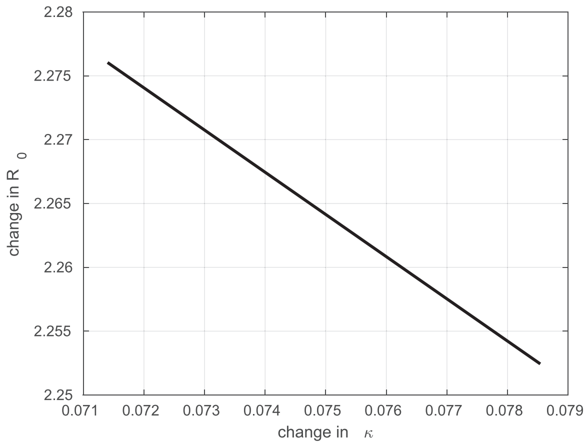

Fig. 11. Fig. 11 depicts a negative relationship between an exposed rate and the basic reproductive number.

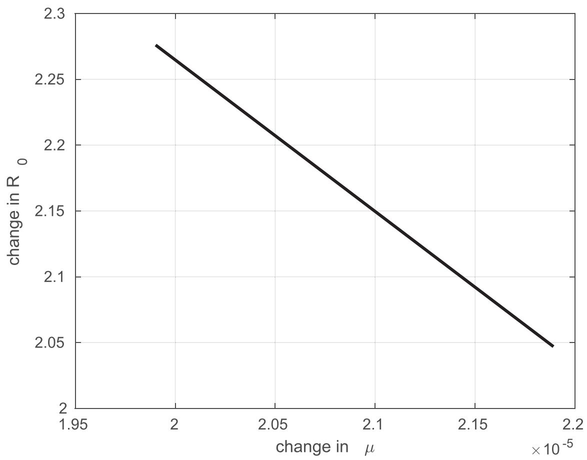

Fig. 12. Fig. 12 depicts a negative relationship between natural death rate from each progression of the Covid-19 infection and the basic reproductive number.

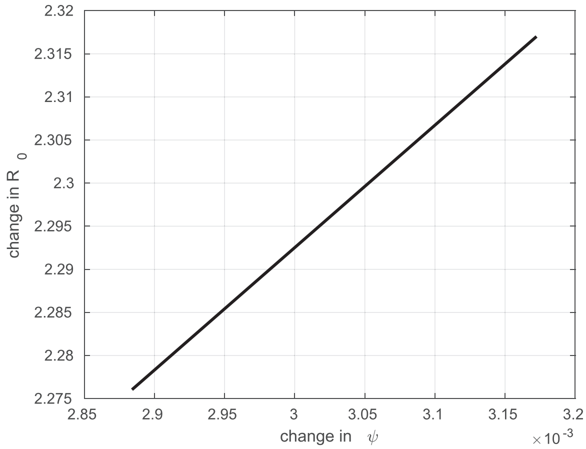

Fig. 13. Fig. 13 shows a positive relationship between the progression rate from the infectious class to the quarantined class and the basic reproductive number.

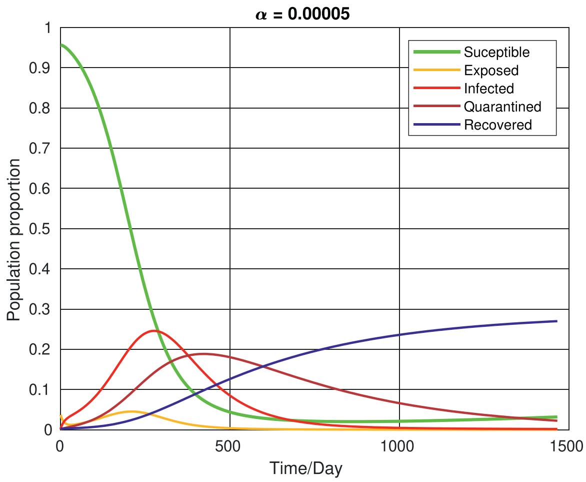

Fig. 14. Fig. 14 depicts the changes in the various compartments as birth rate is being varied and holding other parameters in the model fixed.

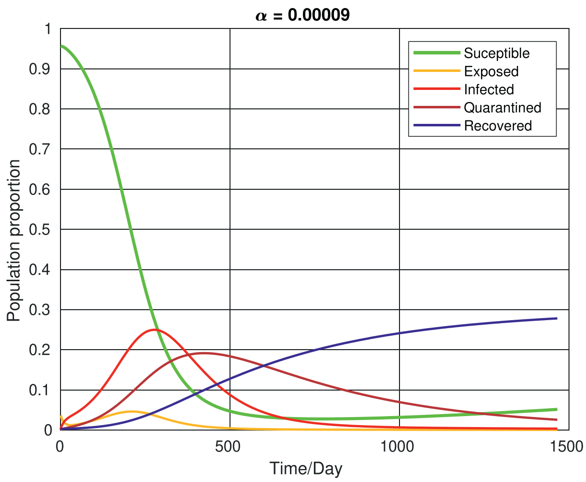

Fig. 15. Fig. 15 depicts the changes in the various compartments as birth rate is being varied and holding other parameters in the model fixed..

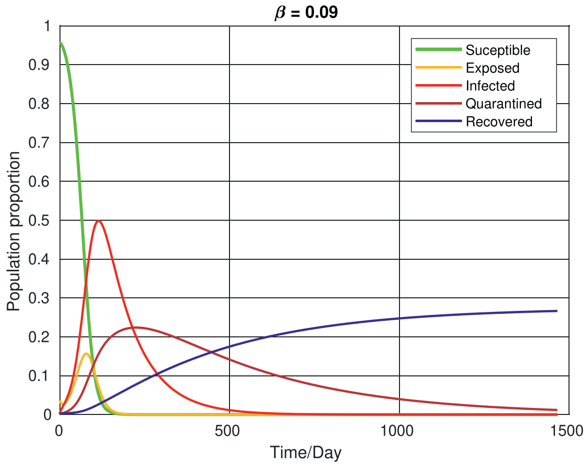

Fig. 16. Fig 16 shows the changes in the various compartments as exposed rate is being varied and holding other parameters in the model fixed.

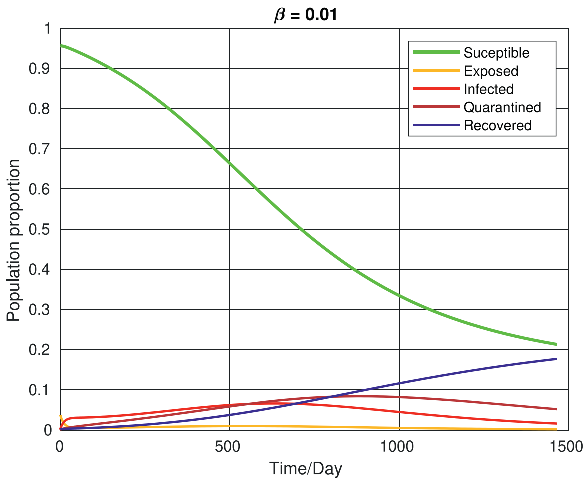

Fig. 17. Fig. 17 shows the changes in the various compartments as exposed rate is being varied and holding other parameters in the model fixed.

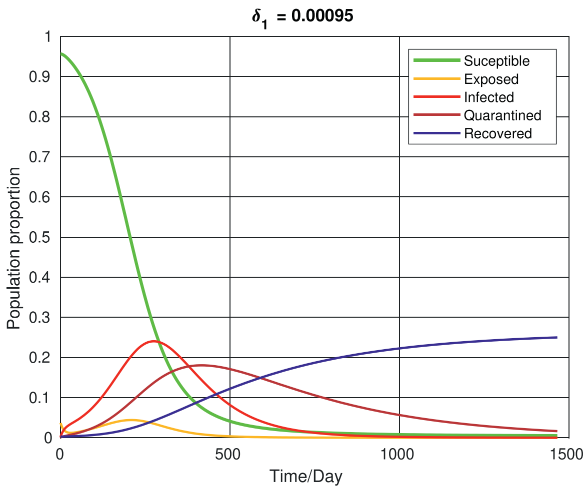

Fig. 18. Fig. 18 shows the changes in the various compartments as the rate at which a quarantined person dies from his or her Covid-19 infection is being varied whiles holding other parameters in the constant.

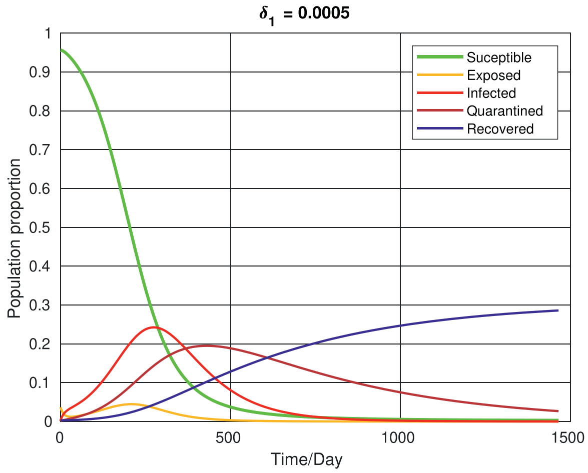

Fig. 19. Fig. 19 shows the changes in the various compartments as the rate at which a quarantined person dies from Covid-19 infection is being varied whiles holding other parameters in the constant.

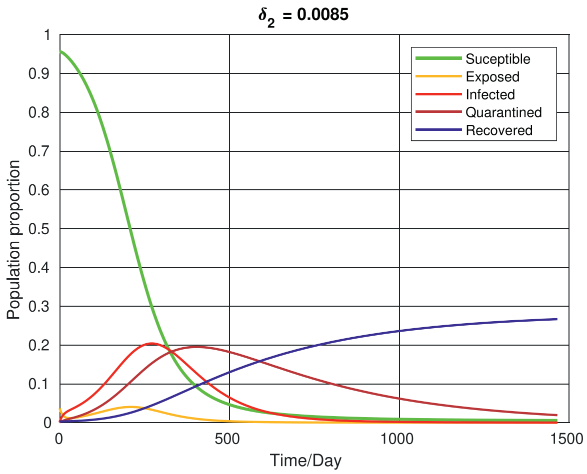

Fig. 20. Fig. 20 depicts the changes in the various compartments as rate at which an infected person dies from Covid-19 infection is being varied and holding other parameters fixed.

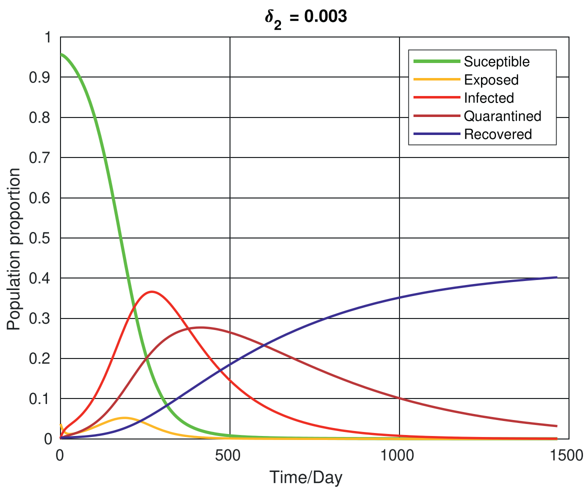

Fig. 21. Fig. 21 depicts the changes in the various compartments as rate at which an infected person dies from Covid-19 infection is being varied and holding other parameters fixed.

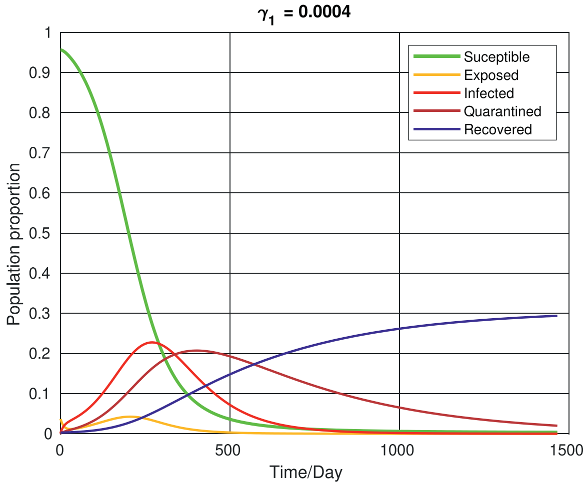

Fig. 22. Fig. 22 indicates the changes in the various compartments as the rate at which an infected person recovers from Covid-19 infection is being varied and other parameters kept fixed.

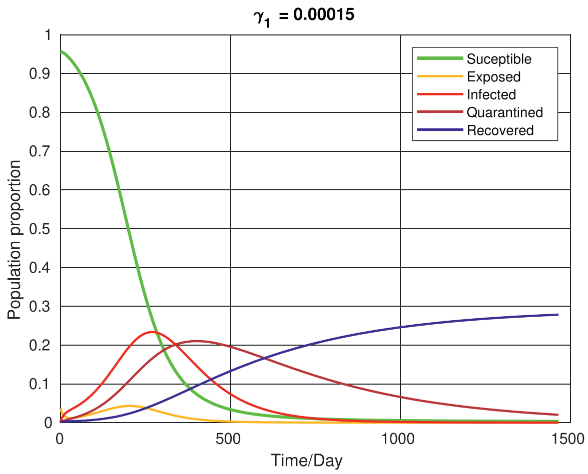

Fig. 23. Fig. 23 indicates the changes in the various compartments as the rate at which an infected person recovers from Covid-19 infection is being varied and keeping other parameters fixed.

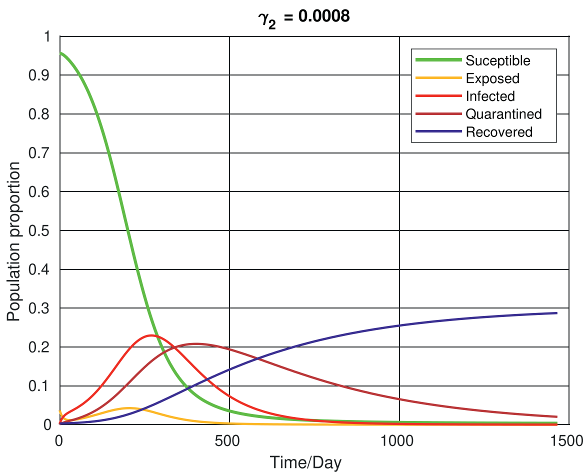

Fig. 24. Fig. 24 depicts the changes in the various compartments as the rate at which an exposed person recovers from Covid-19 infection is being varied and maintaining other parameters fixed.

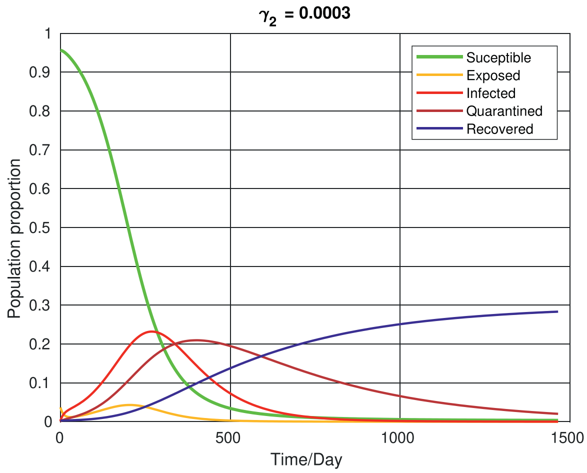

Fig. 25. Fig. 25 shows the changes in the various compartments as the rate at which an exposed person recovers from Covid-19 infection is being varied and maintaining other parameters fixed.

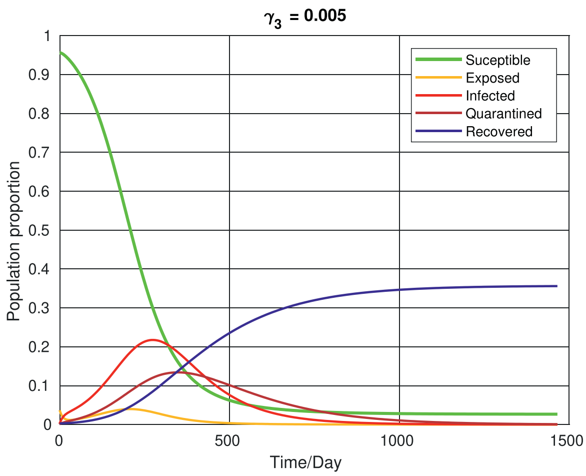

Fig. 26. Fig. 26 depicts the changes in the various compartment as the recovery rate in the quarantined compartment is being varied and holding all other parameters constant.

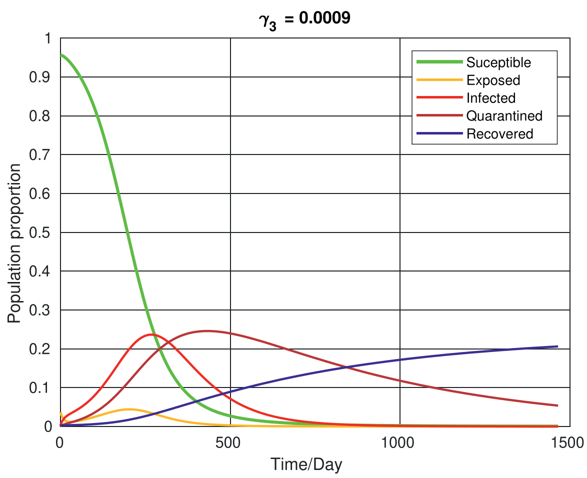

Fig. 27. Fig. 27 depicts the changes in the various compartment as the recovery rate in the quarantined compartment is being varied and keeping other parameters fixed.

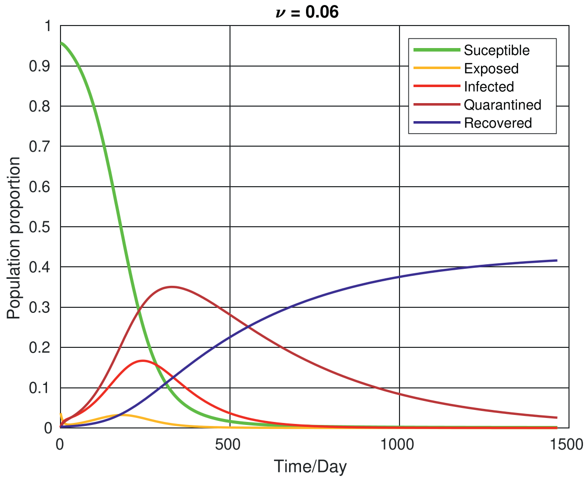

Fig. 28. Fig. 28 shows the changes in the compartments as the rate at which an exposed person is quarantined is being varied and holding other parameters fixed.

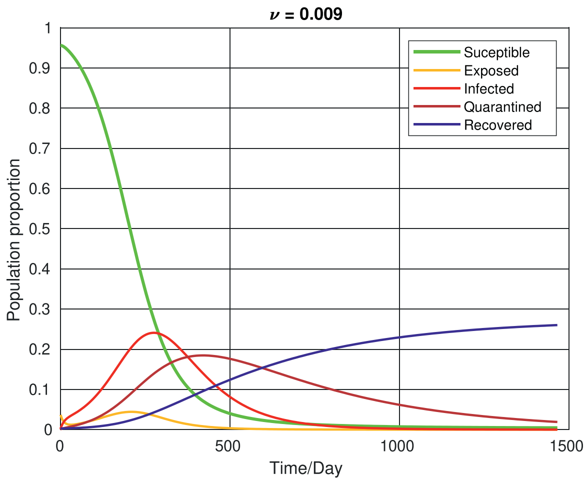

Fig. 29. Fig. 29 shows the changes in other compartments as the rate at which an exposed person is quarantined is being varied and holding other parameters fixed.

Increasing the parameter value of the rate at which exposed individuals are quarantined (ν) from 0.0143 to 0.06 and holding the other parameters constant decreases the number of infectives and increases the number of quarantines and the recovery population while decreasing the parameter value of ν from 0.0143 to 0.009 reduces both the number of quarantines and the recoveries as shown in Figs. 28 and 29 above.

Varying the value of κ and keeping α, β, ν, ψ, γ_1, γ_2, γ_3, μ, δ_1 and δ_2 constant.

Figures 30 and 31 are obtained by increasing and decreasing respectively the parameter value of the rate at which exposed individuals become infected (κ) while holding the other parameters constant. From the graph, a slight perturbation of the value of κ from 0.0714286 to 0.08 decreases the number of quarantines and recoveries while decreasing the parameter value κ from 0.0714286 to 0.04 reduces the number of infected and increases the number of recoveries and exposed.

Varying the value of ψ and keeping α, β, κ, ν, γ_1, γ_2, γ_3, μ, δ_1 and δ_2 constant.

Decreasing the parameter value of the rate at which infected individuals are quarantined (ψ) from 0.002884 to 0.0005 while holding the other parameters constant decreases the number of people quarantined and that of the recoveries and increases the number of infected persons while increasing the parameter value of ψ 0.002884 to 0.007 decreases the number of people infected and increases the number of people quarantined and the that of recoveries.

Varying the value of μ and keeping α, β, κ, ν, ψ, γ_1, γ_2, γ_3, δ_1 and δ_2 fixed

decreasing the parameter value of the rate at which an individual dies naturally from the various compartments; (μ), from 0.0000199 to 0.000008 whereas holding the other parameters fixed does not change the population of the subgroups, and increasing the parameter value of μ from 0.0000199 to 0.000004 reduces slightly the quarantines and the recovery population.

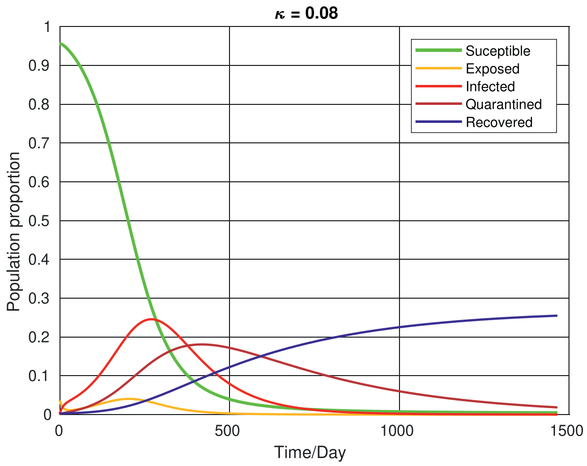

Fig. 30. Fig. 30 indicates the changes in the various compartments as the rate at which an exposed persone becomes infected with Covid-19 virus is varied and keeping other parameters fixed.

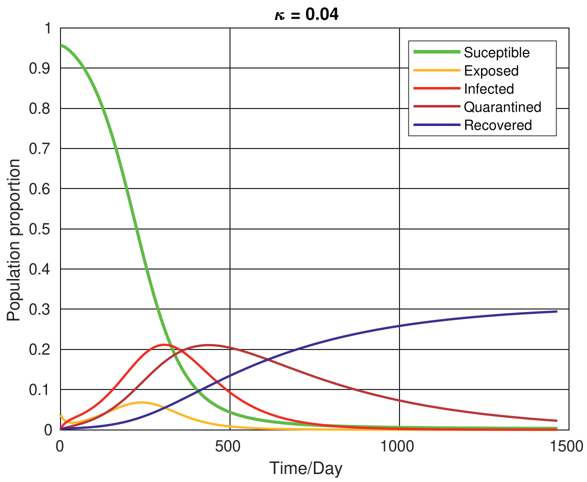

Fig. 31. Fig. 31 indicates the changes in the various compartments as the rate at which an exposed persone becomes infected with Covid-19 virus is varied and holding other parameters fixed.

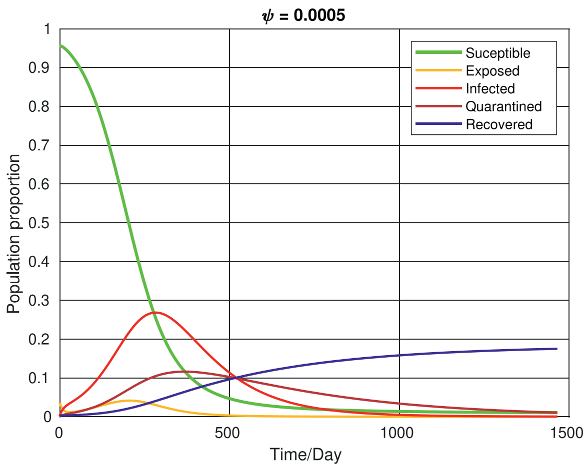

Fig. 32. Fig. 32 depicts the changes in the various compartments as the rate at which an infected person is quarantined is being varied and holding other parameters fixed.

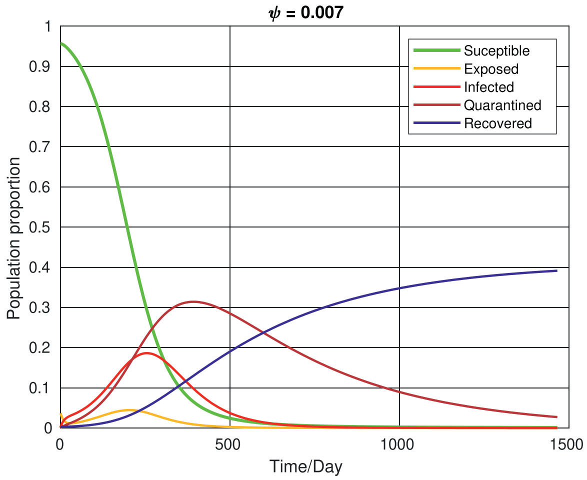

Fig. 33. Fig. 33 depicts the changes in the various compartments as the rate at which an infected person is quarantined is being varied and holding other parameters fixed.

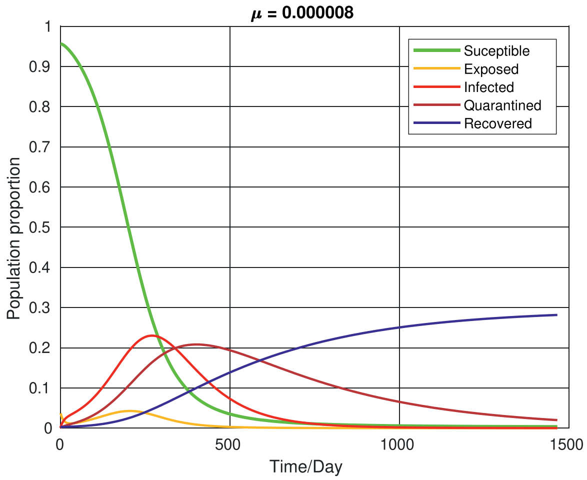

Fig. 34. Fig .34 shows the changes in the various compartments as death rate is being varied and holding other parameters fixed.

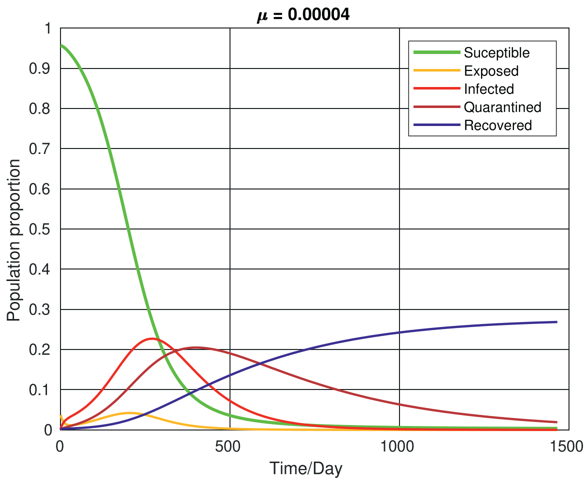

Fig. 35. Fig .35 shows the changes in the various compartments as death rate is being varied and keeping other parameters fixed.

Global solution of the system of ODEs

In this section, the global solution for the system of ODEs, Eq. (2), provided using the Dulac criterion. Thus, the right-hand sides of the system of Eq. (2) are relabelled as:

$$\frac{ds}{dt} = F(s,e,i,q,r),$$
$$\frac{de}{dt} = G(s,e,i,q,r),$$
$$\frac{di}{dt} = H(s,e,i,q,r),$$
$$\frac{dq}{dt} = L(s,e,i,q,r), \text{ and } \frac{dr}{dt} = N(s,e,i,q,r).$$

Using the Dulac criterion for determining global stability of solution, we have:

$\triangledown .g(\dot{s}) = \frac{\partial}{\partial s}.gF(s,e,i,q,r) + \frac{\partial}{\partial e}.gG(s,e,i,q,r) + \frac{\partial}{\partial i}.gH(s,e,i,q,r) + \frac{\partial}{\partial q}.gL(s,e,i,q,r) + \frac{\partial}{\partial r}.gN(s,e,i,q,r)$ where $g(s,e,i,q,r) = 1.$ yields $\triangledown .g(\dot{s}) = -\beta(i+q) - 2\mu - (\mu+\kappa+\gamma_2+\nu) - (\mu+\delta_2+\gamma_1+\psi) - (\mu+\delta_1+\gamma_3)$

$\triangledown .g(\dot{s}) = -\beta(i+q) - 5\mu - \kappa - \nu - \psi - \delta_1 - \delta_2 - \gamma_1 - \gamma_2 - \gamma_3 < 0$

For all parameters greater than zero, thus $\forall(\mu,\beta,\kappa,\nu,\psi,\delta_1,\delta_2,\gamma_1,\gamma_2,\gamma_3) > 0.$

$-\beta(i+q) - 5\mu - \kappa - \nu - \psi - \delta_1 - \delta_2 - \gamma_1 - \gamma_2 - \gamma_3 < 0.$ is strictly negative in a simply connected region D, therefore there are no periodic orbits of

$$\frac{ds}{dt} = F(s,e,i,q,r)$$
$$\frac{de}{dt} = G(s,e,i,q,r).$$
$$\frac{di}{dt} = H(s,e,i,q,r).$$
$$\frac{dq}{dt} = L(s,e,i,q,r). \text{ and } \frac{dr}{dt} = N(s,e,i,q,r). \text{ in D}$$

The above result indicates that, for a long period of time, the epidemiology of Covid-19 in Ghana will be under control.

Conclusion

We have developed a mathematical model for the system of ODEs to describe the epidemiology of Covid-19 in Ghana. We observed that in Ghana, on the average, a Covid-19 patient transmits SARS virus to three persons daily in the country. Also, the presence of Covid-19 in our society will be asymptotically stable over a long period of time. Based on the sensitivity index of the parameters on the basic reproductive number, it revealed that, the natural death rate mostly influence the value of R_o followed by the rate at which new borns enter the susceptible class (α) and the rate at which the susceptible become exposed (β) whereas the rate at which exposed people recover from the disease (γ_2) has the least influence on the value of the basic reproductive number followed by the rate at which the infected people recover from the disease (γ_1) and the rate at which the exposed people are quarantined (ν). Some parameters such as α, β, κ, ψ and ν are in direct relationship with R_0, whereas μ, δ_1, δ_2, γ_1, γ_2, and γ_3 are in inverse relationship with R_o.

Declaration of Competing Interest

We hereby declare that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere. We know of no conflicts of interest associated with this publication, and there has been no significant financial support for this work that could have influenced its outcome. As corresponding Author, I confirm that the manuscript has been read and approved for submission by all the named authors.

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