4 pages

ARJUN
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HW-1

Problem 1

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$:→

In order for this to hold; the following two properties must also hold:→

(I) $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

(II) $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

(I) Let $x \in A \cap (B \cup C) \Rightarrow x \in A \wedge x\, (B \cup C)$

$\Rightarrow x \in (A \cap B) \vee x \in (A \cap C)$

$\Rightarrow x \in \cap (B \cup C) \Rightarrow x \in (A \cap B) \cup A \cap C)$

$\therefore \underline{A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)}$

(II) Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \vee x \in (A \cap C)$

$\Rightarrow x \in A \wedge \{B \cup C\}$

$\Rightarrow x \in A \cap (B \cup C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B$

(b) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

since $x \cap y^c = x \setminus y$: we can rewrite this as:→

$A \cap (B \cup C)^c = (A \cap B^c) \cup (A \cap C^c)$

LHS:

Using demorgan's law:→

$A \cap (B \cup C)^c = A \cap (B^c \cap C^c)$

RHS:

Using distributive property:→

$(A \cap B^c) \cup (A \cap C^c) = A \cap (B^c \cup C^c$

Hence : LHS $\neq$ RHS

$$\boxed{A \cap (B^c \cap C^c) \neq A \cap (B^c \cup C^c)}$$

equality is incorrect