The Boosting Approach to Machine Learning

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Boosting

- General method for improving the accuracy of any given learning algorithm.

- Works by creating a series of challenge datasets s.t. even modest performance on these can be used to produce an overall high-accuracy predictor.

    - Works well in practice --- Adaboost and its variations one of the top ML algorithms.

    - Backed up by solid foundations.

Readings:

- The Boosting Approach to Machine Learning: An Overview. Rob Schapire, 2001
- Theory and Applications of Boosting. NIPS tutorial.

  http://www.cs.princeton.edu/~schapire/talks/nips-tutorial.pdf

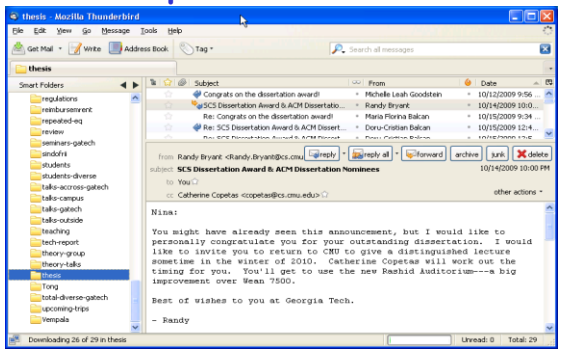
Plan for today:

- Motivation.
- A bit of history.
- Adaboost: algo, guarantees, discussion.
- Focus on supervised classification.

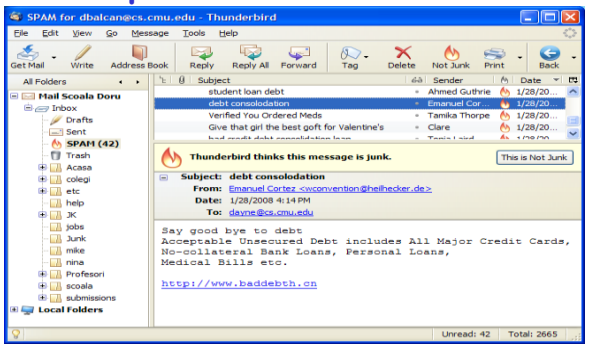
An Example: Spam Detection

- E.g., classify which emails are spam and which are important.

Not spam



spam

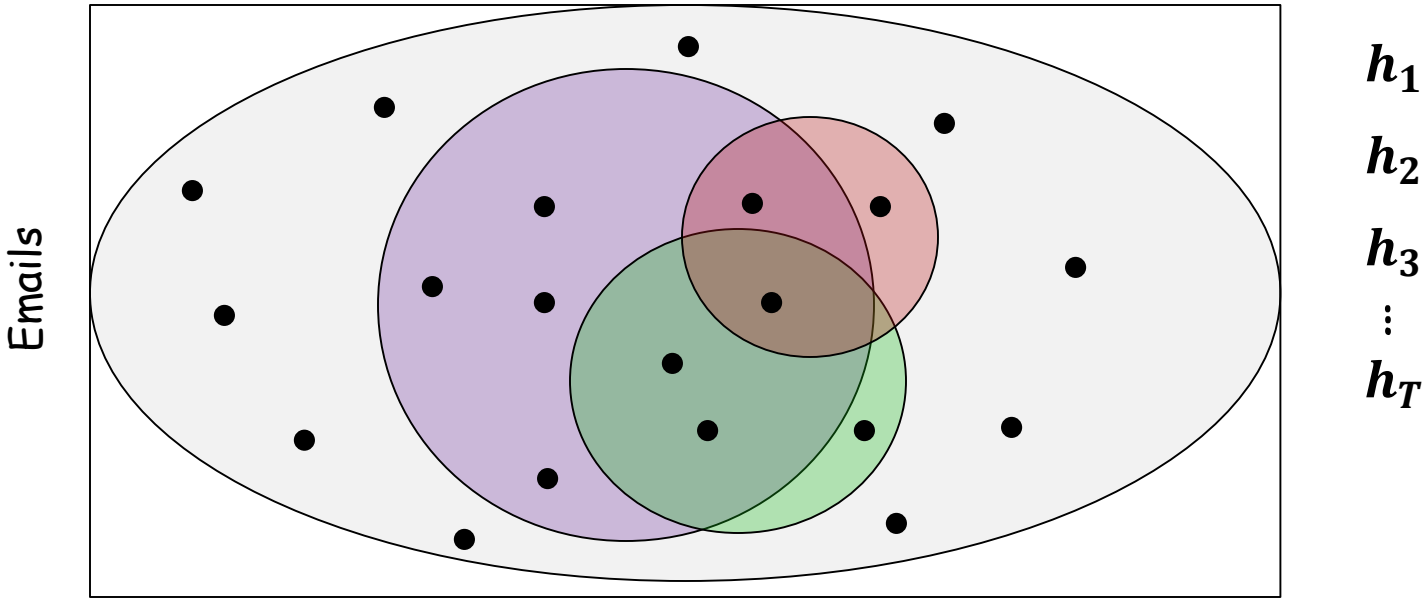


Key observation/motivation:

- Easy to find rules of thumb that are often correct.
  - E.g., "If buy now in the message, then predict spam."
  - E.g., "If say good-bye to debt in the message, then predict spam."
- Harder to find single rule that is very highly accurate.

An Example: Spam Detection

- Boosting: meta-procedure that takes in an algo for finding rules of thumb (weak learner). Produces a highly accurate rule, by calling the weak learner repeatedly on cleverly chosen datasets.



- apply weak learner to a subset of emails, obtain rule of thumb
- apply to 2nd subset of emails, obtain 2nd rule of thumb
- apply to 3rd subset of emails, obtain 3rd rule of thumb
- repeat T times; combine weak rules into a single highly accurate rule.

Boosting: Important Aspects

How to choose examples on each round?

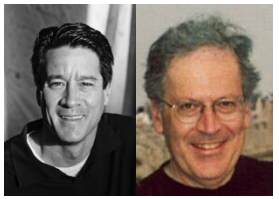
- Typically, concentrate on "hardest" examples (those most often misclassified by previous rules of thumb)

How to combine rules of thumb into single prediction rule?

- take (weighted) majority vote of rules of thumb

Historically….

Weak Learning vs Strong Learning



- [Kearns & Valiant '88]: defined weak learning: being able to predict better than random guessing (error $\leq \frac{1}{2} - \gamma$) , consistently.

- Posed an open pb: "Does there exist a boosting algo that turns a weak learner into a strong learner (that can produce arbitrarily accurate hypotheses)?"

- Informally, given "weak" learning algo that can consistently find classifiers of error $\leq \frac{1}{2} - \gamma$, a boosting algo would provably construct a single classifier with error $\leq \epsilon$.

Surprisingly….

Weak Learning =Strong Learning

Original Construction [Schapire '89]:

- poly-time boosting algo, exploits that we can learn a little on every distribution.

  - A modest booster obtained via calling the weak learning algorithm on 3 distributions.

$$\text{Error} = \beta < \frac{1}{2} - \gamma \rightarrow \text{error } 3\beta^2 - 2\beta^3$$

  - Then amplifies the modest boost of accuracy by running this somehow recursively.

- Cool conceptually and technically, not very practical.

An explosion of subsequent work

Background (cont.)

- [Freund & Schapire '95]:
  - introduced "AdaBoost" algorithm
  - strong practical advantages over previous boosting algorithms
- experiments and applications using AdaBoost:

[Drucker & Cortes '96]
[Jackson & Craven '96]
[Freund & Schapire '96]
[Quinlan '96]
[Breiman '96]
[Maclin & Opitz '97]
[Bauer & Kohavi '97]
[Schwenk & Bengio '98]
[Schapire, Singer & Singhal '98]
[Abney, Schapire & Singer '99]
[Haruno, Shirai & Ooyama '99]
[Cohen & Singer' 99]
[Dietterich '00]
[Schapire & Singer '00]
[Collins '00]
[Escudero, Màrquez & Rigau '00]
[Iyer, Lewis, Schapire, Singer & Singhal '00]
[Onoda, Rätsch & Müller '00]
[Tieu & Viola '00]
[Walker, Rambow & Rogati '01]
[Rochery, Schapire, Rahim & Gupta '01]
[Merler, Furlanello, Larcher & Sboner '01]
⋮

- continuing development of theory and algorithms:

[Breiman '98, '99]
[Schapire, Freund, Bartlett & Lee '98]
[Grove & Schuurmans '98]
[Mason, Bartlett & Baxter '98]
[Schapire & Singer '99]
[Cohen & Singer '99]
[Freund & Mason '99]
[Domingo & Watanabe '99]
[Mason, Baxter, Bartlett & Frean '99, '00]
[Duffy & Helmbold '99, '02]
[Freund & Mason '99]
[Ridgeway, Madigan & Richardson '99]
[Kivinen & Warmuth '99]
[Friedman, Hastie & Tibshirani '00]
[Rätsch, Onoda & Müller '00]
[Rätsch, Warmuth, Mika, Onoda, Lemm & Müller '00]
[Allwein, Schapire & Singer '00]
[Friedman '01]
[Koltchinskii, Panchenko & Lozano '01]
[Collins, Schapire & Singer '02]
[Demiriz, Bennett & Shawe-Taylor '02]
[Lebanon & Lafferty '02]
⋮

Adaboost (Adaptive Boosting)

"A Decision-Theoretic Generalization of On-Line Learning and an Application to Boosting"

[Freund-Schapire, JCSS'97]

Godel Prize winner 2003

Informal Description Adaboost

- Boosting: turns a weak algo into a strong learner.

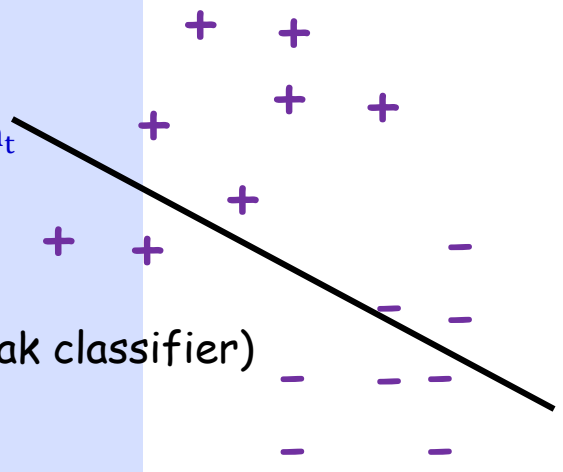
Input: S={(x_1, y_1), …,(x_m, y_m)}; $x_i \in X, y_i \in Y = \{-1,1\}$

weak learning algo *A* (e.g., Naïve Bayes, decision stumps)

- For t=1,2, … ,T
  - Construct D_t on $\{x_1, \ldots, x_m\}$
  - Run *A* on D_t producing $h_t: X \to \{-1,1\}$ (weak classifier)

$\epsilon_t = P_{x_i \sim D_t}(h_t(x_i) \neq y_i)$ error of h_t over D_t

- Output $H_{final}(x) = \text{sign}(\sum_{t=1} \alpha_t h_t(x))$



Roughly speaking D_{t+1} increases weight on x_i if h_t incorrect on x_i ; decreases it on x_i if h_t correct.

Adaboost (Adaptive Boosting)

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- For t=1,2, … ,T
  - **Construct D_t on $\{x_1, \ldots, x_m\}$**
  - Run *A* on D_t producing h_t

Constructing D_t

- D_1 uniform on $\{x_1, \ldots, x_m\}$ [i.e., $D_1(i) = \frac{1}{m}$]
- Given D_t and h_t set

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t\}} \text{ if } y_i = h_t(x_i)$$

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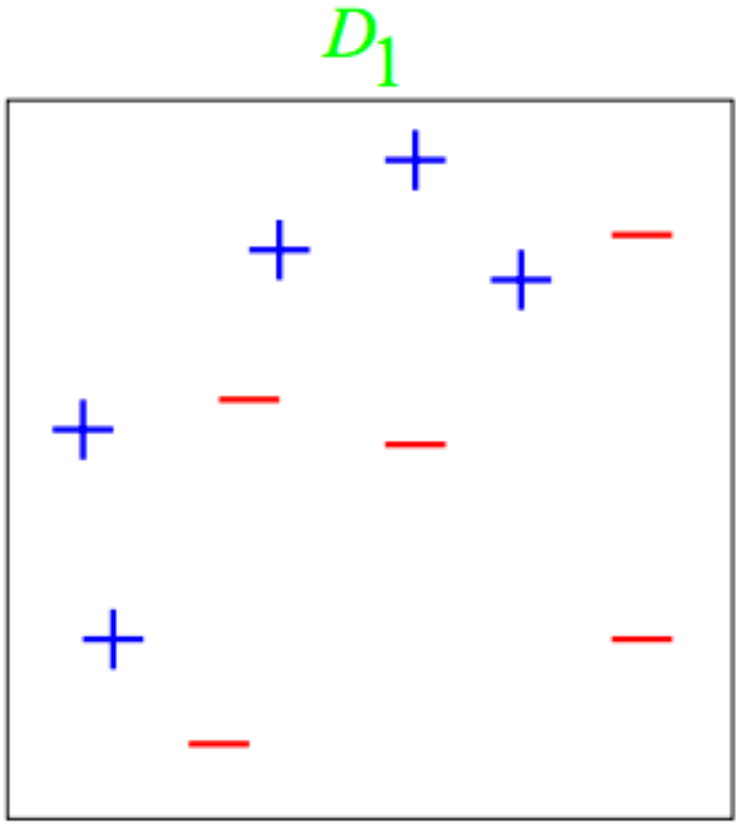
$$\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

D_{t+1} puts half of weight on examples x_i where h_t is incorrect & half on examples where h_t is correct

Final hyp: $H_{final}(x) = \text{sign}(\sum_t \alpha_t h_t(x))$

Adaboost: A toy example

Weak classifiers: vertical or horizontal half-planes (a.k.a. decision stumps)



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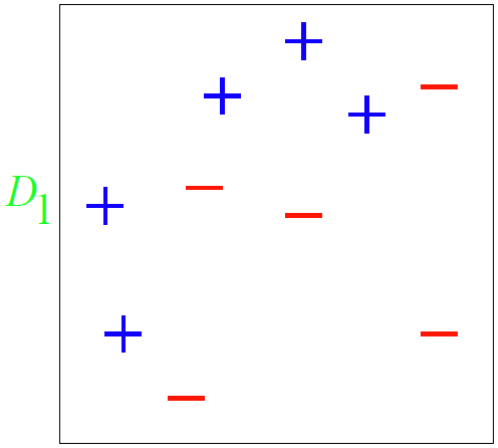
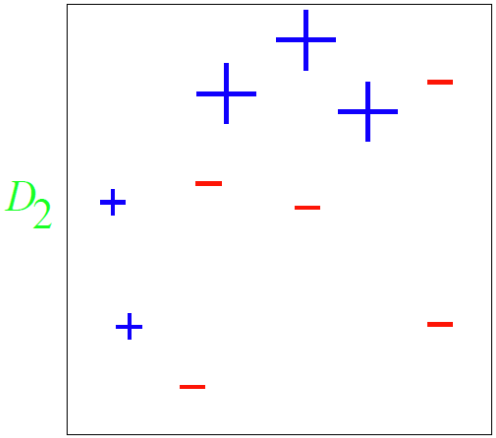
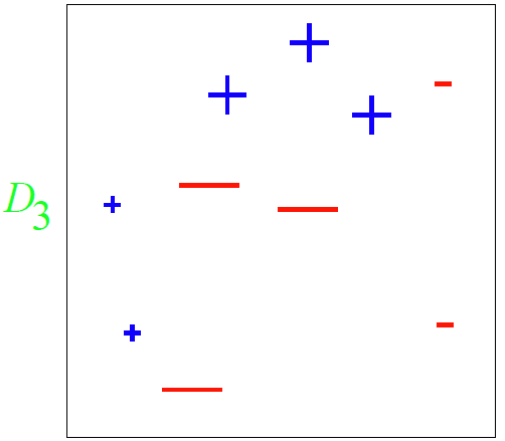
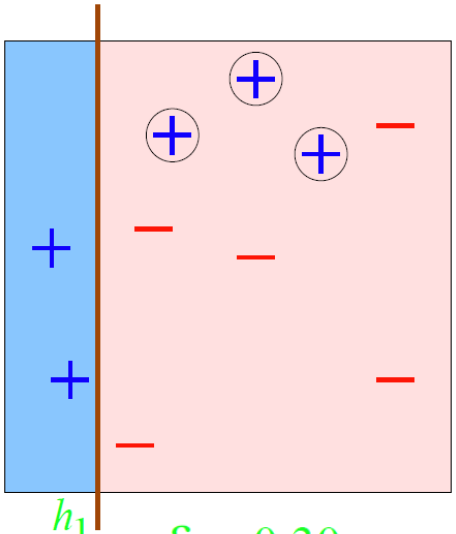


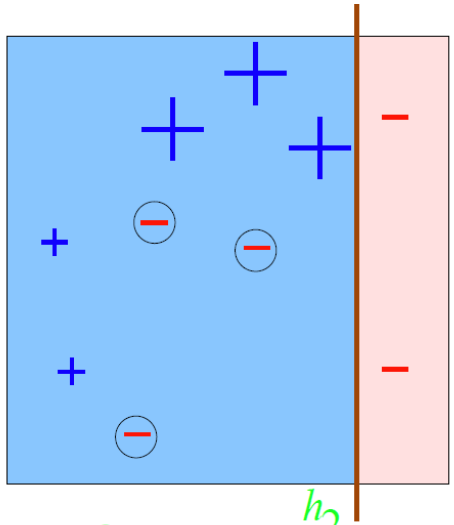






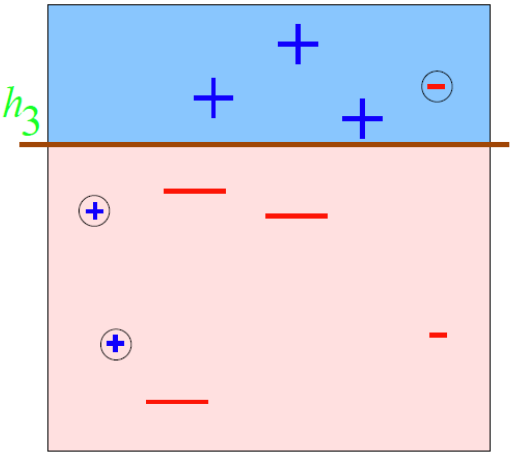
$\varepsilon_1 = 0.30$

$\alpha_1 = 0.42$



$\varepsilon_2 = 0.21$

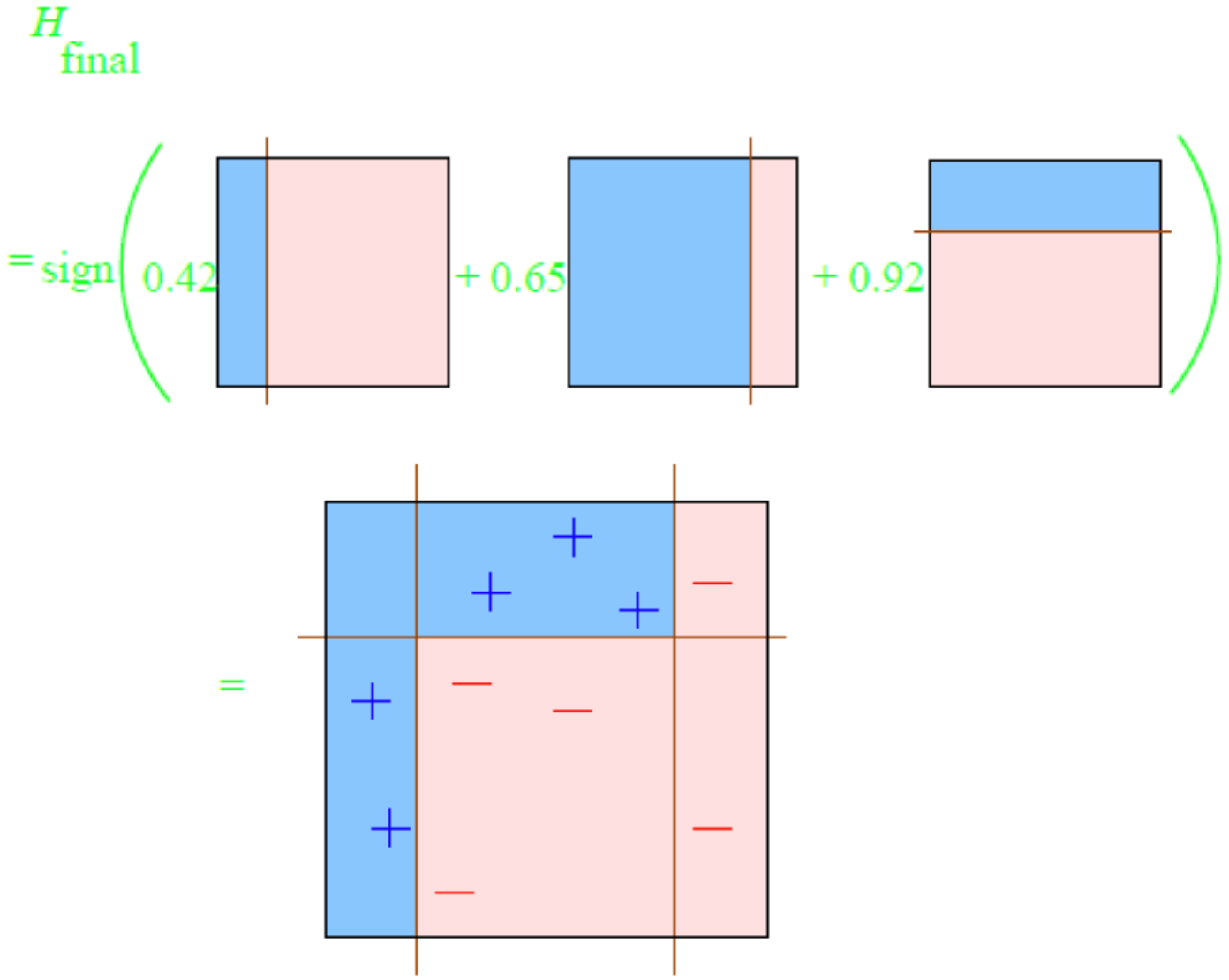
$\alpha_2 = 0.65$



$\varepsilon_3 = 0.14$

$\alpha_3 = 0.92$

Adaboost: A toy example



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Nice Features of Adaboost

- Very general: a meta-procedure, it can use any weak learning algorithm!!! (e.g., Naïve Bayes, decision stumps)
- Very fast (single pass through data each round) & simple to code, no parameters to tune.
- Shift in mindset: goal is now just to find classifiers a bit better than random guessing.
- Grounded in rich theory.
- Relevant for big data age: quickly focuses on "core difficulties", well-suited to distributed settings, where data must be communicated efficiently [Balcan-Blum-Fine-Mansour COLT'12].

Analyzing Training Error

Theorem $\epsilon_t = 1/2 - \gamma_t$ (error of h_t over D_t)

$$err_S(H_{final}) \leq \exp\left[-2\sum_t \gamma_t^2\right]$$

So, if $\forall t, \gamma_t \geq \gamma > 0$, then $err_S(H_{final}) \leq \exp[-2\gamma^2 T]$

The training error drops exponentially in T!!!

To get $err_S(H_{final}) \leq \epsilon$, need only $T = O\left(\frac{1}{\gamma^2}\log\left(\frac{1}{\epsilon}\right)\right)$ rounds

Adaboost is adaptive

- Does not need to know γ or T a priori
- Can exploit $\gamma_t \gg \gamma$

Understanding the Updates & Normalization

Claim: D_{t+1} puts half of the weight on x_i where h_t was incorrect and half of the weight on x_i where h_t was correct.

Recall $D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t y_i h_t(x_i)\}}$

Probabilities are equal!

$$\Pr_{D_{t+1}}[y_i \neq h_t(x_i)] = \sum_{i:y_i \neq h_t(x_i)} \frac{D_t(i)}{Z_t} e^{\alpha_t} = \epsilon_t \frac{1}{Z_t} e^{\alpha_t} = \frac{\epsilon_t}{Z_t} \sqrt{\frac{1-\epsilon_t}{\epsilon_t}} = \frac{\sqrt{\epsilon_t(1-\epsilon_t)}}{Z_t}$$

$$\Pr_{D_{t+1}}[y_i = h_t(x_i)] = \sum_{i:y_i = h_t(x_i)} \frac{D_t(i)}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t} e^{-\alpha_t} = \frac{1-\epsilon_t}{Z_t} \sqrt{\frac{\epsilon_t}{1-\epsilon_t}} = \frac{\sqrt{(1-\epsilon_t)\epsilon_t}}{Z_t}$$

$$Z_t = (1-\epsilon_t)e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$$

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Think about generalization aspects!!!!

What you should know

- The difference between weak and strong learners.
- AdaBoost algorithm and intuition for the distribution update step.
- The training error bound for Adaboost

Advanced (Not required) Material

Analyzing Training Error: Proof Math

Step 1: unwrapping recurrence: $D_{T+1}(i) = \frac{1}{m}\left(\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}\right)$

where $f(x_i) = \sum_t \alpha_t h_t(x_i)$. [Unthresholded weighted vote of h_i on x_i]

Step 2: $\mathrm{err}_S(H_{final}) \leq \prod_t Z_t$.

Step 3: $\prod_t Z_t = \prod_t 2\sqrt{\epsilon_t(1-\epsilon_t)} = \prod_t \sqrt{1-4\gamma_t^2} \leq e^{-2\sum_t \gamma_t^2}$

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Recall $D_1(i) = \frac{1}{m}$ and $D_{t+1}(i) = D_t(i)\,\frac{\exp(-y_i \alpha_t h_t(x_i))}{Z_t}$

$$D_{T+1}(i) = \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times D_T(i)$$

$$= \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times \frac{\exp(-y_i \alpha_{T-1} h_{T-1}(x_i))}{Z_{T-1}} \times D_{T-1}(i)$$

……

$$= \frac{\exp(-y_i \alpha_T h_T(x_i))}{Z_T} \times \cdots \times \frac{\exp(-y_i \alpha_1 h_1(x_i))}{Z_1}\,\frac{1}{m}$$

$$= \frac{1}{m}\,\frac{\exp(-y_i(\alpha_1 h_1(x_i) + \cdots + \alpha_T h_T(x_T)))}{Z_1 \cdots Z_T} \quad = \frac{1}{m}\,\frac{\exp(-y_i f(x_i))}{\prod_t Z_t}$$

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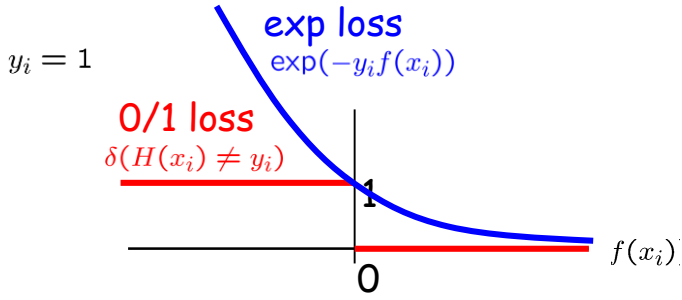
$$\mathrm{err}_S(H_{final}) = \frac{1}{m}\sum_i 1_{y_i \neq H_{final}(x_i)}$$

$$= \frac{1}{m}\sum_i 1_{y_i f(x_i) \leq 0}$$

$$\leq \frac{1}{m}\sum_i \exp(-y_i f(x_i))$$

$$= \sum_i D_{T+1}(i) \prod_t Z_t = \prod_t Z_t.$$



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Note: recall $Z_t = (1-\epsilon_t)e^{-\alpha_t} + \epsilon_t e^{\alpha_t} = 2\sqrt{\epsilon_t(1-\epsilon_t)}$

α_t minimizer of $\alpha \to (1-\epsilon_t)e^{-\alpha} + \epsilon_t e^{\alpha}$

Generalization Guarantees (informal)

Theorem $err_S(H_{final}) \leq \exp\left[-2\sum_t \gamma_t^2\right]$ where $\epsilon_t = 1/2 - \gamma_t$

How about generalization guarantees?

Original analysis [Freund&Schapire'97]

- Let H be the set of rules that the weak learner can use
- Let G be the set of weighted majority rules over T elements of H (i.e., the things that AdaBoost might output)

Theorem [Freund&Schapire'97]

$\forall\, g \in G, err(g) \leq err_S(g) + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$

T= # of rounds

d= VC dimension of H

Generalization Guarantees

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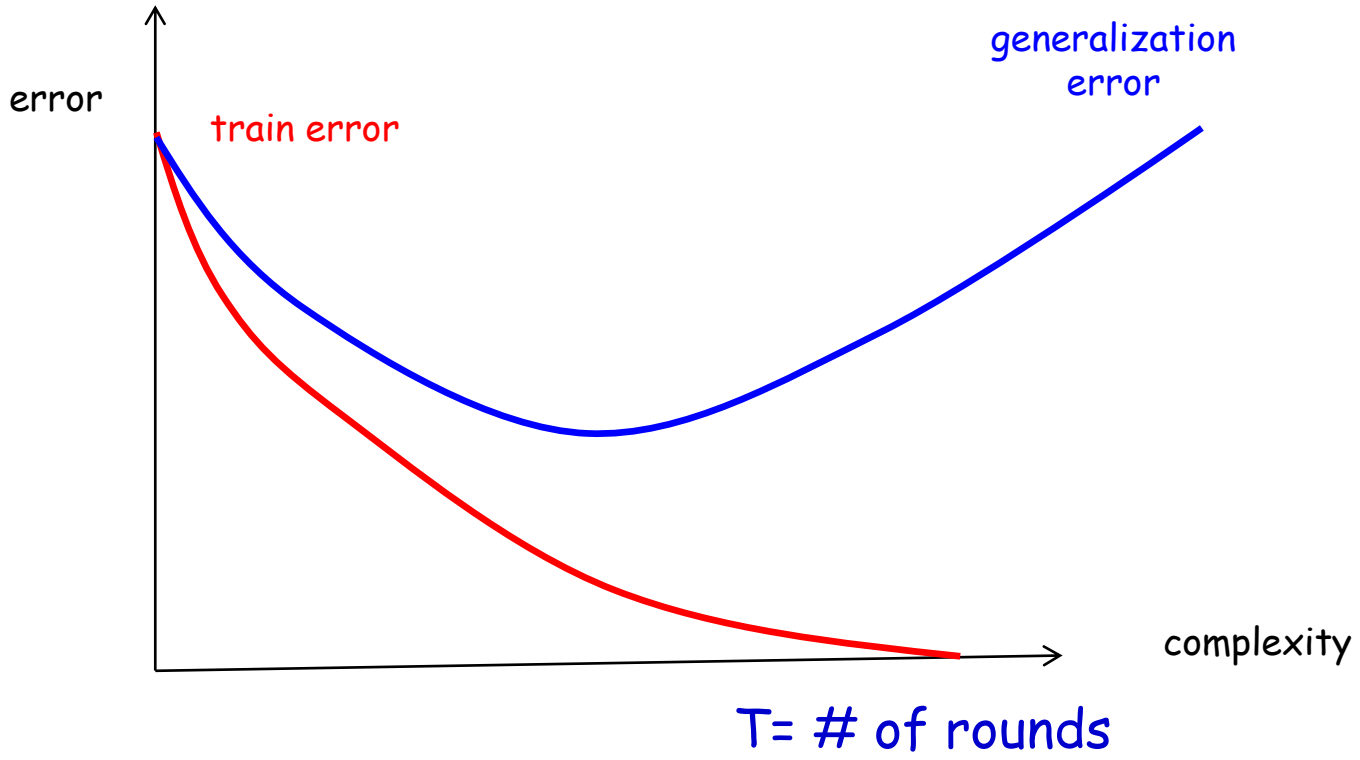
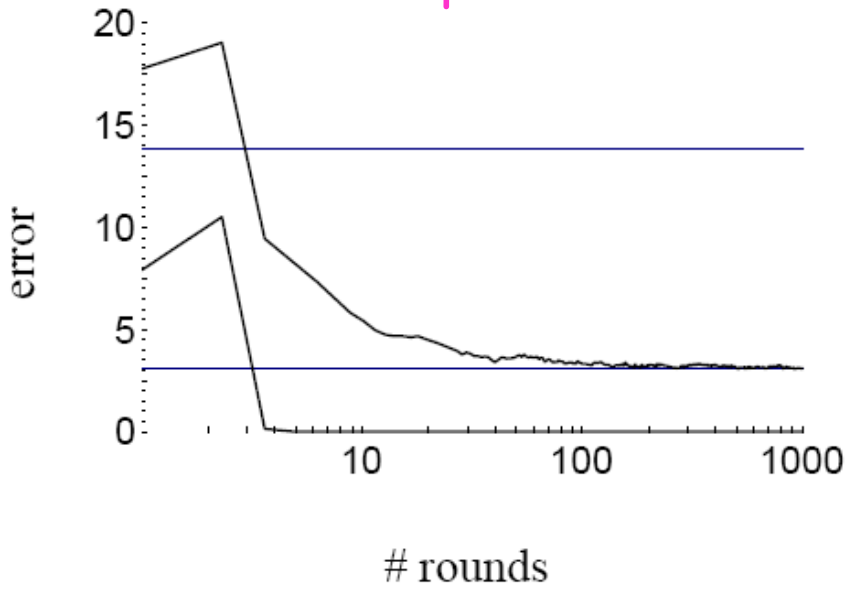


Generalization Guarantees

- Experiments with boosting showed that the test error of the generated classifier usually does not increase as its size becomes very large.

- Experiments showed that continuing to add new weak learners after correct classification of the training set had been achieved could further improve test set performance!!!



Generalization Guarantees

- Experiments with boosting showed that the test error of the generated classifier usually does not increase as its size becomes very large.
- Experiments showed that continuing to add weak learners after correct classification of the training set had been achieved could further improve test set performance!!!
- These results seem to contradict FS'87 bound and Occam's razor (in order to achieve good test error the classifier should be as simple as possible):




How can we explain the experiments?

R. Schapire, Y. Freund, P. Bartlett, W. S. Lee. present in "*Boosting the margin: A new explanation for the effectiveness of voting methods*" a nice theoretical explanation.

Key Idea:

Training error does not tell the whole story.

We need also to consider the classification confidence!!