

**LO.a: Define a probability distribution and distinguish between discrete and continuous random variables and their probability functions.**

1. X is a discrete random variable with four possible outcomes:  $X = \{1, 2, 3, 4\}$ . Which of the following *best* represent the probability function  $f(x)$  for the discrete variable X?
  - A.  $f(X): f(1) = -0.25 \ f(2) = -0.25 \ f(3) = 0.5 \ f(4) = -0.05$ .
  - B.  $f(X): f(1) = 0.25 \ f(2) = 0.25 \ f(3) = 0.4 \ f(4) = 0.1$ .
  - C.  $f(X): f(1) = 0.2 \ f(2) = 0.2 \ f(3) = 0.6 \ f(4) = 0.01$ .
2. The probability function for a discrete random variable is denoted by  $g(y) = g(Y = y)$ . Which of the following is *most likely* true?
  - A. The probability that a random variable Y takes on the value y.
  - B. Sum of the probabilities  $g(y)$  over all values of Y equals 0.
  - C. The probability  $g(y)$  is a number between -1 and 1.

**LO.b: Describe the set of possible outcomes of a specified discrete random variable.**

3. A discrete uniform distribution consists of the following 12 values:  
2.0, 6.2, -1.5, 2.4, 9.0, 4.1, -3.2, -1.0, 5.5, 8.2, 4.1 and 0.8  
The probability of a value lying between -3.0 and 1.0 in a single draw from the distribution is *closest* to:
  - A. 16.67%.
  - B. 25.00%.
  - C. 33.33%.
4. A six sided biased dice has the probability of landing on its edge twice out of 50 throws. The probability of any number showing up is equal. When the dice is rolled, the prize is equal to the number it lands on, i.e. \$1 for showing 1, \$2 for showing 2 and so on. The prize of landing on its edge is \$10. What is the expected value of the prize on a single roll of dice?
  - A. 3.76.
  - B. 3.91.
  - C. 3.81.
5. The outcomes of rolling a dice can be *best* represented by which of the following types of probability distributions?
  - A. A continuous probability distribution.
  - B. A discrete probability distribution.
  - C. A normal distribution.

**LO.c: Interpret a cumulative distribution function.**

6. The notation ' $F(x) = P(X \leq x)$ ' *best* describes which of the following?
  - A. Cumulative distribution function.
  - B. Probability density function.

C. Probability function.

**LO.d: Calculate and interpret probabilities for a random variable, given its cumulative distribution function.**

7. Discrete uniform probability distribution of net profits for a currency option on EURO (€) is as follows:

Net Profit (€)	Probability
Net profit of 0	0.25
Net profit of 2 or less	0.50
Net profit of 4 or less	0.75
Net profit of 6 or less	1.00

The probability of a net profit greater than €2 and less than or equal to €6 is *closest* to:

- A. 1.00.  
B. 0.75.  
C. 0.50.
8. A box contains 10 labeled pieces of paper. Each piece of paper has one integer ranging between 1 and 10 written on it, and the numbers are not repeated. If you draw a piece of paper at random, then the probability that the number is greater than 3 and less than or equal to 7 is *closest* to:
- A. 0.40.  
B. 0.50.  
C. 0.60.

**LO.e: Define a discrete uniform random variable, a Bernoulli random variable, and a binomial random variable.**

9. Which of the following *best* describes the binomial distribution? The binomial distribution:
- A. has an infinite number of specified outcomes.  
B. has an infinite number of unspecified outcomes.  
C. is based on the Bernoulli random variable.
10. Which of the following is *least likely* an assumption of the binomial distribution?
- A. The probability,  $p$ , of success is constant for all trials.  
B. The trials are independent.  
C. The probability of failure is the reciprocal of the probability of success.

**LO.f: Calculate and interpret probabilities given the discrete uniform and the binomial distribution functions.**

11. Karachi Footy Club, an emerging football team, had a tough last season with a win to loss record of 1 to 5. In order to contest and increase its chances of winning, the team signed 2 new players and it is estimated that the chances of winning a game in the next season are

60%. Assuming that winning a single game is independent of other games, the probability that the team will win 2 out of next 5 games is *closest* to:

- A. 0.2304.
- B. 0.3456.
- C. 0.5184.

12. You toss a coin 14 times. The probability of getting exactly 6 tails is:

- A. 0.016.
- B. 0.183.
- C. 0.428

**LO.g: Construct a binomial tree to describe stock price movement.**

13. An investor charts the movement of a stock's price over the next three years as follows:

<b>T = 0</b>	<b>T = 1</b>	<b>T = 2</b>
$S_0 = 45$	$S_u = 49.5$	$S_{uu} = 54.45$
	$S_d = 40.5$	$S_{ud,du} = 44.55$
		$S_{dd} = 36.45$

The initial price of the stock is \$45. The probability of the price increasing at any given point is 55% and that of the price decreasing is 45%. The probability, using the binomial model, that the stock's price would be \$36.45 two years later is *closest* to:

- A. 20.25%.
- B. 24.75%.
- C. 30.25%.

14. Suppose National Refinery Limited's (NRL) expected price over the next two periods is as shown below.

<b>Time = 0</b>	<b>Time = 1</b>	<b>Time = 2</b>
$NRL_0 = 100$	$NRL_u = 110$	$NRL_{uu} = 121$
	$NRL_d = 90$	$NRL_{ud,du} = 99$
		$NRL_{dd} = 81$

NRL's current price is \$100. The probability of an up move in any given period is 65% and the probability of a down move in any given period is 35%. Using the binomial model, the probability that the NRL's price will be \$99 at the end of two periods is *closest* to:

- A. 42.25%.
- B. 22.75%.
- C. 45.50%.

15. Assume that a stock's price over the next two periods is as shown below:

Time = 0	Time = 1	Time = 2
$S_0 = 60$	$S_u = 69$	$S_{uu} = 79.35$
	$S_d = 51$	$S_{ud,du} = 58.65$
		$S_{dd} = 43.35$

The initial value of the stock is \$60. The probability of an up move in any given period is 60% and the probability of a down move in any given period is 40%. Using the binomial model, the probability that the stock's price will be \$58.65 at the end of two periods is *closest* to:

- A. 24%.
- B. 32%.
- C. 48%.

16. A stock's price over the next two periods is as shown below.

Time = 0	Time = 1	Time = 2
$S_0 = 100$	$S_u = 105$	$S_{uu} = 110$
	$S_d = 95$	$S_{ud,du} = 102$
		$S_{dd} = 90$

The initial value of the stock is 100. From historical data, it has been observed that the probability of an up move in any given period is 30% and the probability of a down move in any given period is 70%. Using the above data, the probability that the stock's price will be equal to 102 at the end of period 2 is *closest* to:

- A. 21%.
- B. 49%.
- C. 42%.

17. Using the same data as in the previous question, the expected prices of the stock at the end of period 1 and period 2 are *closest* to:

	Time 1	Time 2
A.	98	96.84
B.	102	83.42
C.	98	84.84

18. A bank has issued loans to 60 customers. Based on past experience, the bank expects 10% of the customers to default. Which of the following is *most likely* the expected number of defaults and the standard deviation of the number of defaults?

	Expected number of defaults	Standard deviation
A.	5.40	6
B.	6	5.40
C.	6	2.32

**LO.h: Calculate and interpret tracking error.**

19. Alex, a fund manager at Morgan Investment Bank manages Anil's portfolio. At the beginning of the year, the portfolio had a value of \$50,000 and at the end it was \$45,000. Alex's performance is measured against an index which declined by 8% in that year due to below average economic conditions. The tracking error of this portfolio is *closest* to:
- A. 2%.
  - B. -2%.
  - C. -18%.

**LO.i: Define the continuous uniform distribution and calculate and interpret probabilities, given a continuous uniform distribution.**

20. At a restaurant, the service time for a single order is uniformly distributed between 10 to 18 minutes. If a customer places an order at 5:30 PM, what is the probability that the order will be served after 5:45 PM?
- A. 0.375.
  - B. 0.833.
  - C. 0.660.

**LO.j: Explain the key properties of the normal distribution.**

21. Which of the following statements about a normal distribution is *most* accurate? A normal distribution:
- A. has an excess kurtosis of 3.
  - B. is partially described by two parameters.
  - C. can be the linear combination of two or more normal random variables.
22. Which of the following is the *most likely* characteristic of the normal probability distribution? The normal probability distribution:
- A. has an excess kurtosis of 3.0.
  - B. has a mode higher than mean and median.
  - C. is more suitable as a model for returns than for asset prices.
23. Which of the following is *least likely* a characteristic of the normal distribution?
- A. Skewness = 0.
  - B. Kurtosis = 3.

C. Mean > Mode.

24. The normal density with  $\mu = 0$  and  $\sigma = 1$  is called a:

- A. standard normal distribution.
- B. lognormal distribution.
- C. binomial distribution.

**LO.k: Distinguish between a univariate and a multivariate distribution, and explain the role of correlation in the multivariate normal distribution.**

25. A multivariate distribution for the returns on  $y$  stocks is *most likely* defined by which of the following parameters?

- A. mean returns on  $(y-1)$  securities.
- B. variances of returns of  $(y-1)$  securities.
- C.  $\frac{y(y-1)}{2}$  pairwise return correlations.

26. Which of the following is an example of a multivariate distribution?

- A. Distribution of returns on each asset in a group of assets.
- B. Distribution of returns on the assets as a group.
- C. Distribution considering the means and variances of the assets in the group.

**LO.l: Determine the probability that a normally distributed random variable lies inside a given interval.**

27. In a normal distribution, approximately what percent of all observations fall in the interval  $\mu \pm \sigma$ ?

- A. 68 percent.
- B. 50 percent.
- C. 99 percent.

28. In a normal distribution, approximately 99 percent of all observations fall within which of the following intervals?

- A.  $\mu \pm 2\sigma$ .
- B.  $\mu \pm 3\sigma$ .
- C.  $\mu \pm \sigma$ .

29. Two students, Miley and Mariah, make the following statements:

- Miley: Approximately 5% of the observations lie outside the range of  $\mu \pm 2\sigma$ .
- Mariah: Approximately 68% of the observations lie in the interval  $\mu \pm \sigma$ .

Which of the above statements is *most likely* correct?

- A. Only Miley's statement.
- B. Only Mariah's statement.
- C. Both Miley and Mariah's statements.

**LO.m: Define the standard normal distribution, explain how to standardize a random variable, and calculate and interpret probabilities using the standard normal distribution.**

30. Given that  $X$  follows a normal distribution with a mean of 4.5 and standard deviation of 1.5, the standardized value corresponding to  $X=8.9$  would be *closest* to:
- A. 1.64.
  - B. 2.93.
  - C. 5.93.
31. Consider a variable that is normally distributed with a mean of 10 and a variance of 16. To find the probability of observing a value of -1 or less, the calculated  $Z$  value is *closest* to :
- A. -2.7500.
  - B. -0.6875.
  - C. 0.0035.
32. In order to standardize a random variable  $X$ , the steps that *most accurately* describe the process are :
- A. subtract the mean of  $X$  from  $X$ , and then divide that result by the standard deviation of  $X$ .
  - B. subtract the mean of  $X$  from  $X$ , and then divide that result by the standard deviation of the standard normal distribution.
  - C. divide  $X$  by the difference between the standard deviation of  $X$  and the standard deviation of the standard normal distribution.
33. A portfolio has a mean return of 15% and a standard deviation of return of 20% per year. Given the following information, the probability that the portfolio return will be below 18% is *closest* to:
- $P(Z < 0.15) = 0.5596$ ,  $P(Z > 0.15) = 0.4404$ ,  $P(Z < 0.18) = 0.5714$ ,  $P(Z > 0.18) = 0.4286$ .
- A. 0.5596.
  - B. 0.4404.
  - C. 0.5714.
34. A portfolio has a mean return of 15% and a standard deviation of return of 20% per year. Given the following information, the probability that the portfolio return will be between 16% and 20% is *closest* to:
- $P(Z < 0.16) = 0.5636$ ,  $P(Z < 0.20) = 0.5793$ ,  $P(Z < 0.05) = 0.5199$ ,  $P(Z < 0.25) = 0.5987$
- A. 0.0157.
  - B. 0.0788.
  - C. 0.1186.

**LO.n: Define shortfall risk, calculate the safety-first ratio, and select an optimal portfolio using Roy's safety-first criterion.**

35. Information about three possible asset allocations is given below:

Allocation	Expected Annual Return	Standard deviation of return
I	8%	3%
II	19%	11%
III	24%	16%

Assuming a minimum acceptable return of 6%, based on Roy's safety-first criterion, the *most appropriate* allocation is:

- A. I.
- B. II.
- C. III.

36. The table below shows data on three portfolios:

Portfolio	Expected Return	Standard Deviation
1	15%	33%
2	19%	40%
3	22%	48%

Assuming the minimum acceptable rate of return is 6%, under Roy's safety-first criterion, which of the following portfolios is the *most appropriate* choice?

- A. Portfolio 1.
- B. Portfolio 2.
- C. Portfolio 3.

37. An investor has a portfolio of \$100,000. His investment objective is long term growth but he will need \$2,000 for his medical insurance and another \$2,000 for his rent expenses by the end of the year. The investor is considering investing in one of these three available portfolios:

Portfolio	Expected Return	Standard Deviation
A	5%	10%
B	8%	13%
C	14%	22%

Using Roy's safety-first criterion ratio, which one of these portfolios will *minimize* the probability of the investor's portfolio falling below \$100,000?

- A. Portfolio A.
- B. Portfolio B.
- C. Portfolio C.

38. Which of the following risks is evaluated by the safety-first rules?

- A. Downside risk.
- B. Default risk.
- C. Upside risk.



39. An investor has a portfolio worth \$750,000. At the end of the year, the investor wishes to liquidate \$33,750, without using the initial capital. According to the safety-first criterion, which of the following alternatives is the *best* approach?

Alternative	Expected annual return	Standard deviation of return
A.	30%	32%
B.	15%	12%
C.	20%	25%

- A. Allocation A.
- B. Allocation B.
- C. Allocation C.

**LO.o: Explain the relationship between normal and lognormal distributions and why the lognormal distribution is used to model asset prices.**

40. Which of the following statements regarding the distributions used for asset pricing is *most* accurate?
- A. Normal distribution returns will not let returns fall below zero in the case of asset pricing.
  - B. Lognormal distribution returns will never fall in value below zero.
  - C. Binomial distribution will allow the asset's value to stay positive and realistic as per the current market.

**LO.p: Distinguish between discretely and continuously compounded rates of return, and calculate and interpret a continuously compounded rate of return, given a specific holding period return.**

41. The continuously compounded daily returns for ICI shares are normally distributed. The probability distribution for ICI share prices *most likely* follow a:
- A. Normal distribution.
  - B. Lognormal distribution.
  - C. Neither of the above.
42. The price per share of RBook is \$45. Exactly, one year later, the stock is trading at \$55. The continuously compounded return over the one year period is closest to:
- A. 18.18%.
  - B. 20.06%.
  - C. 22.22%.

**LO.q: Explain Monte Carlo simulation and describe its applications and limitations.**

43. Which of the following models can be *efficiently used* to value a call option?
- A. Black-Scholes-Merton model.

- B. Monte Carlo simulation.
- C. Historical simulation.

44. Which of the following is *most likely* a weakness of the Monte Carlo simulation?
- A. It provides only estimates, and not exact results.
  - B. It is based only on past data, and is not fully reflective of all risks.
  - C. It is possible only through the use of supercomputers.

**LO.r: Compare Monte Carlo simulation and historical simulation.**

45. Which of the following statements about simulation models is *least* accurate?
- A. Historical simulation models use the historical data to analyze “what-if” scenarios.
  - B. Historical simulation analyzes risks from the events that occurred during the sample period.
  - C. Monte Carlo simulation provides only statistical estimates, and not exact results.
46. Which of the following methods addresses the ‘what-if’ question?
- A. Historical simulation.
  - B. Monte Carlo simulation.
  - C. Value at risk approach.

## Solutions

1. B is correct because the sum of  $f(x)$  over all values of  $X$  must equal 1 and  $0 \leq p(x) \leq 1$ .
2. A is correct. The sum of the probabilities  $g(y)$  over all values of  $Y$  equals 1. The probability  $g(y)$  is a number between 0 and 1.
3. B is correct. First order the values from smallest to largest. Then note that three of the twelve values are between -3.0 and 1.0. Thus, the probability of a draw from the distribution being between -3.0 and 1.0 is  $3/12 = 0.25$ .
4. A is correct. The probability of the dice landing on its edge is  $2/50 = 0.04$ . Since the dice is biased, the probability of showing up any number won't be  $1/6$ . It has been stated that the probability of showing any number is equal, i.e.  $\frac{1-0.04}{6} = 0.16$ . The distribution is as follows:

Outcome	Probabilities	Prizes	Expected Value
Edge	0.04	\$10	0.4
1	0.16	\$1	0.16
2	0.16	\$2	0.32
3	0.16	\$3	0.48
4	0.16	\$4	0.64
5	0.16	\$5	0.80
6	0.16	\$6	0.96
			<b>3.76</b>

5. B is correct. When a dice is rolled, since there are a finite number of outcomes, it is an example of a discrete probability distribution.
6. A is correct. The cumulative distribution function gives the probability that a random variable  $X$  is less than or equal to a particular value  $x$ ,  $P(X \leq x)$ .
7. C is correct.  $P(X \leq 6) = 1.0$  and  $P(X \leq 2) = 0.50$ . Therefore,  $P(2 \leq X \leq 6) = 1.0 - 0.50 = 0.50$ .
8. A is correct. The probability of each piece of paper being drawn will be  $1/10 = 0.10$ .  
 $P(3 < x \leq 7) = P(x \leq 7) - P(x \leq 3)$   
 $P(3 < x \leq 7) = P(x \leq 1, 2, 3, 4, 5, 6, 7) - P(x \leq 1, 2, 3)$   
 $P(3 < x \leq 7) = (0.1 * 7) - (0.1 * 3)$   
 $P(3 < x \leq 7) = 0.40$ .
9. C is correct. A binomial distribution is based on the Bernoulli random variable.
10. C is correct. Probability of failure = 1 – probability of success. Hence, it is not the reciprocal.

In addition, this statement is not an assumption of the binomial distribution. The binomial distributions make only two assumptions, which are the first two choices listed in this example.

11. A is correct. Using Binomial to calculate the probability of 2 wins out of 5 games:  
 $P(x) = P(X = x) = [\text{number of ways to choose } x \text{ from } n] \times p^x \times (1 - p)^{n-x}$   
 $P(\text{Win}) = 0.6$   
 $P(\text{Lose}) = 0.4$   
 $P(2 \text{ out of } 5) = {}^5C_2 \times 0.6^2 \times (1-0.6)^3 = 0.2304$
12. B is correct.  
The probability of getting 6 tails =  ${}^nC_r (p)^r (1 - p)^{n-r} = {}^{14}C_6 (0.5)^6 (0.5)^8 = 0.183$ .
13. A is correct. Only two consecutive downward movements would lead to the price being \$36.45 two years later. The probability of two consecutive downward movements is  $0.45 * 0.45 = 0.2025 = 20.25\%$ .
14. C is correct. The probability of an up move followed by a down move is  $0.65 * 0.35 = 0.2275$ . The probability of a down move followed by an up move is  $0.35 * 0.65$  also = 0.2275. Both of these sequences result in an end value of \$99. Therefore, the probability of an end value of \$99 is  $(0.2275 + 0.2275) = 45.5\%$ .
15. C is correct. The probability of an up move followed by a down move is 0.60 times 0.40 equals 0.24. The probability of a down move followed by an up move is 0.40 times 0.60 also equals 0.24. Both of these sequences result in an end value of \$58.65. Therefore, the probability of an end value of \$58.65 is 48%.
16. C is correct. There are four possibilities that stock price can take. It can move up-up, up-down, down-up and down-down. In order to reach 102, a stock's price can either go up first and then down or the other way. Probability for the stock going up first and then down is  $0.3 * 0.7 = 0.21$ . This probability will be the same for the stock going down first and then up which is,  $0.7 * 0.3 = 0.21$ . Therefore, the probability of the stock's price reaching 102 is 42% (21% + 21%)
17. A is correct.



25. C is correct. The three parameters that define a multivariate distribution for returns on  $y$  stocks include mean returns on  $y$  securities, variances of returns on  $y$  securities, and  $\frac{y(y-1)}{2}$  pairwise return correlations.
26. B is correct. C is incorrect because it did not include correlations between the assets. A is incorrect because it is not a distribution of a single asset (or a random variable), but as a group.
27. A is correct. Approximately 50 percent of all observations fall in the interval  $\mu \pm (2/3)\sigma$ .
28. B is correct. Approximately 68 percent of all observations fall in the interval  $\mu \pm \sigma$ . Approximately 95 percent of all observations fall in the interval  $\mu \pm 2\sigma$ .
29. C is correct. Miley and Mariah are both correct.
30. B is correct. If  $X$  follows a normal distribution with parameters  $\mu$  and  $\sigma$ ,  $X$  can be standardized using the formula:  

$$Z = (X - \mu) \div \sigma$$

$$= (8.9 - 4.5) \div 1.5$$

$$= 2.93$$
31. A is correct. First, standardize the value of interest,  $-1$ , for the given normal distribution:  

$$Z = (X - \mu) / \sigma = (-1 - 10) / 4 = -2.75.$$
32. A is correct. There are two steps in standardizing a random variable  $X$ : Subtract the mean of  $X$  from  $X$ , and then divide that result by the standard deviation of  $X$ .
33. A is correct.  

$$P\left(Z < \frac{X - \mu}{\sigma}\right) = P\left(Z < \frac{0.18 - 0.15}{0.20}\right) = P(Z < 0.15)$$
 If the values from the table are not given, then using the cdf table for standard normal variable, find the entry corresponding to 0.10 in the row and 0.05 in the column, which is 0.5596.  
 This implies the probability that the portfolio return will be less than 18% is 55.96 percent.
34. B is correct.  

$$P\left(\frac{X - \mu}{\sigma} < Z < \frac{X - \mu}{\sigma}\right)$$

$$P\left(\frac{0.16 - 0.15}{0.20} < Z < \frac{0.20 - 0.15}{0.20}\right)$$

$$P(0.05 < Z < 0.25) = P(Z < 0.25) - P(Z < 0.05) = 0.5987 - 0.5199 = 0.0788$$

35. B is correct. Roy's safety-first ratio =  $[E(RP) - RL] / \text{Standard deviation of } P$  with the optimal portfolio having the highest ratio. The safety-first ratios for the three allocations are:

Allocation	Safety-first ratio
I	0.67
II	1.18
III	1.125

36. C is correct. The portfolio with the highest SF-Ratio is preferred. The SF-Ratio is calculated by subtracting the target return from the expected return and dividing by the standard deviation.

Portfolio 1: 0.270

Portfolio 2: 0.325

Portfolio 3: 0.333.

37. C is correct. The investor requires a minimum return of  $\frac{\$4,000}{\$100,000}$  or 4 percent. Roy's safety-first model uses the excess of each portfolio's expected return over the minimum return and divides that excess by the standard deviation for that portfolio. The highest safety-first ratio is associated with Portfolio 3:  $(14\% - 4\%) / 22\% = 0.45$ .

38. A is correct. The safety-first rules is an approach to evaluate the downside risk or shortfall risk.

39. B is correct.

$$\text{Shortfall Level} = \frac{33750}{750000} = 4.5\%$$

The formula for SF ratio is  $\frac{[E(R_p) - R_L]}{\sigma_p}$

Calculate the safety first ratios for each allocation:

$$\text{A: } \frac{30 - 4.5}{32} = 0.7968$$

$$\text{B: } \frac{15 - 4.5}{12} = 0.8750$$

$$\text{C: } \frac{20 - 4.5}{25} = 0.6200$$

Since allocation B has the highest ratio, it's the best one.

40. B is correct. Only lognormal distributions are used for asset pricing models because their value is above zero, i.e. they always stay positive and asset pricing cannot be negative. The normal distribution will allow prices to be negative.

41. B is correct. If the continuously compounded returns are normally distributed, then the future stock price is log-normally distributed.
42. B is correct. Continuously compounded return =  $\ln(1 + \text{holding period return}) = \ln(55/45) = 0.2006 = 20.06\%$ .
43. A is correct. BSM model takes into account the sensitivity of call values to changes in a stock price and other variables which the others do not.
44. A is correct. Monte Carlo simulation only provides statistical estimates and not exact results.
45. A is correct. Historical simulation does not analyze what-if scenarios.
46. B is correct. The Monte Carlo simulation addresses the what-if question.