DESIGN AND ANALYSIS OF ALGORITHMS (DAA.)

ASSIGNMENT:-3

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Section :- DS1 Class Roll no :- 23

Q1) What do you mean by Minimum Spanning Tree? What are the applications of MST.

Ans. A tree which spans all the vertices of a graph having minimum total weight of edges is known as a minimum Spanning Tree. It is a connected graph which has no cycles.

Applications of MST :-

→ Games Development → in generating procedural maps, creating paths between points of interests in gaming world.
→ Image Processings → used in image segmentations and edge detection tasks.
→ Network Designing → helps in designing of networks with least possible costs of designing purposes.
→ Civil Engineering and Transportations → To minimise the cost of purpose while ensuring connectivity between different locations as per purposes.
→ Clusterings :→ In data analysis used for clustering data points.

Ans 2) * **Prim's Algorithm:-**

→ Time Complexity:-

(using binary heap i.e. priority queue):-

$O((V+E).\log V)$

Vertices

Edges

(using fibonacci heap):-

$O(E+V.\log V)$

→ Space Complexity:-

$O(V+E)$

* **Kruskal's Algorithm:-**

→ Time Complexity:-

$O(E\log E)$

→ Space Complexity:-

$O(V+E)$

* **Dijkstra's Algorithm:-**

→ Time Complexity:-

$O((V+E).\log V)$

(using adjacency lists and binary heaps).

→ Space Complexity:-

$O(V+E)$

* Bellman's - Ford Algorithms :-

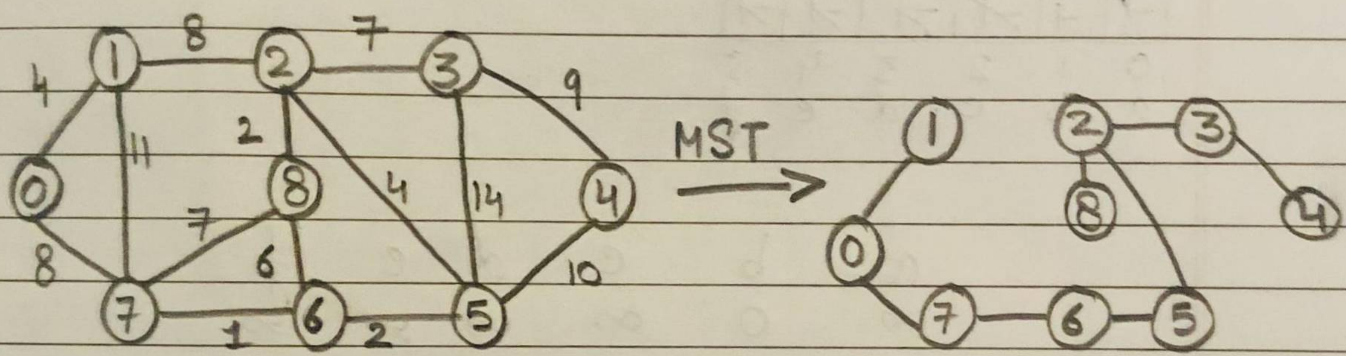
→ Time Complexity :-

$O(VE)$

→ Space Complexity :-

$O(V+E)$

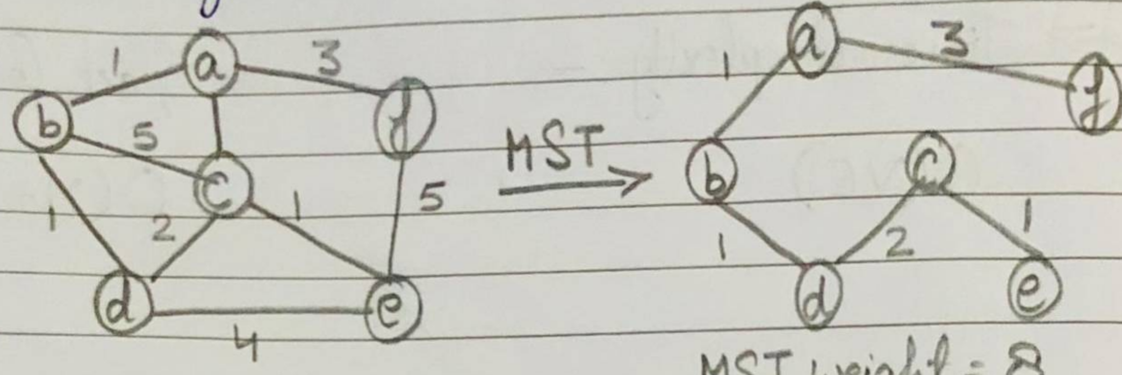
Ans3) * Kruskals Algorithms :-



MST weight = 37

u	v	w	
7	6	1	✓
2	8	2	✓
6	5	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	×
7	8	7	×
2	3	7	✓
0	7	8	✓
1	2	8	×
3	4	9	✓
5	4	10	×
1	7	11	×
3	5	14	×

* Prim's Algorithms :-



MST weight = 8

b	d	~~A~~b	c	a
~~1~~	~~3~~	~~1~~	~~2~~	~~0~~

~~-1~~	-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~
0	1	2	3	4	5
a	b	c	d	e	f

a	b	c	d	e	f
~~∞~~	0	~~∞~~	~~∞~~	~~∞~~	~~∞~~
1		2	1	1	3

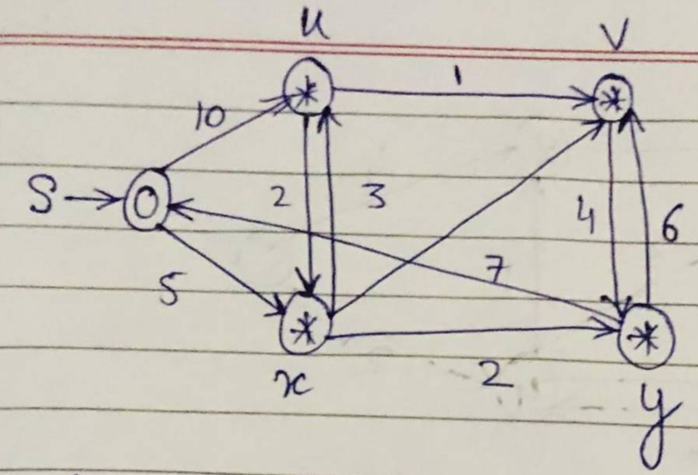
Ans 4 * Increasing every edge by 10 units :-

-> It is an addictive transformation.
-> Since the absolute difference between the weights of paths remain the same, the relative ordering of path does not changes.
-> Thus, no change in shortest paths.

* Multiplying every edge weight by 10 units :-

-> It is a multiplicative transformation.
-> As the changes are uniform in all edges, therefore, no change in shortest paths.

Ans 5)

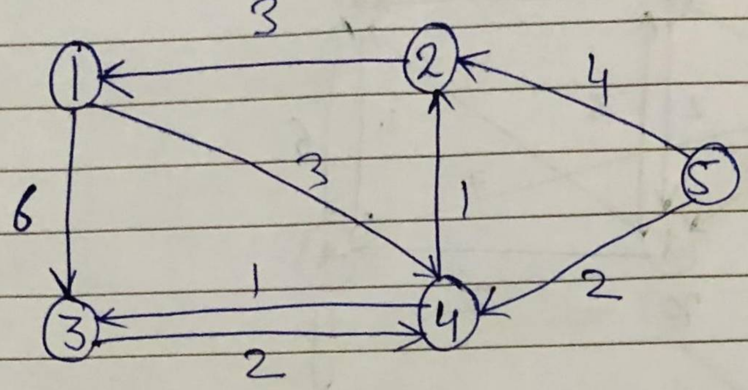


* Dijkstra's Algorithms :-

selected vertexes	S	u	v	x	y
S	$\boxed{0}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$
x		$\cancel{10}$	∞	$\boxed{5}$	$\cancel{\infty}$
y		$\cancel{8}$	$\cancel{14}$		$\boxed{7}$
u		$\boxed{8}$	$\cancel{13}$		
v			$\boxed{9}$		

Nodes	Distance
S	0
x	5
y	7
u	8
v	9

Ans 6



* FLOYD WARSHALL'S ALGORITHMS :-

$$D_0 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D_1 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D_2 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$D_4 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

(Final Resultant matrices obtained.).

→ Time Complexity :-

$O(n^3)$

→ Space Complexity :-

$O(n^2)$

(where n is no. of vertices of graphs.).

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