

Decentralized Deep Learning with Inexact Consensus

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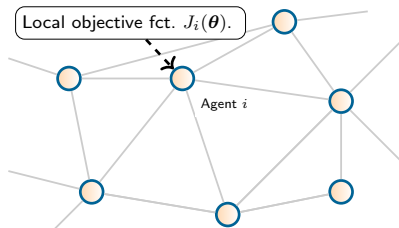
Problem Description: Decentralized Consensus

Optimization Problem

- Consider a **finite sum unconstrained** optimization of a d -dimensional variable θ :

$$\min_{\theta \in \mathbb{R}^d} J(\theta) := \frac{1}{N} \sum_{i=1}^N J_i(\theta). \quad (1)$$

- Where $d \in \mathbb{N}$ is the problem dimension
- $J_i : \mathbb{R}^d \rightarrow \mathbb{R}$ is a continuous, differential private objective function of worker i
- $G = (V, E)$ is an **undirected communication graph**; $V = [N] = \{1, \dots, N\}$ represents the set of N workers and $(i, i) \in E \ \forall \ i \in V$



Problem Description: Decentralized Consensus Optimization Problem

- Consider a **finite sum unconstrained** optimization of a d -dimensional variable θ :

$$\min_{\theta \in \mathbb{R}^d} J(\theta) := \frac{1}{N} \sum_{i=1}^N J_i(\theta). \quad (1)$$

- Equation (1) can be written as the decentralized consensus optimization problem:

$$\min_{\theta_i \in \mathbb{R}^d, i \in V} \sum_{i=1}^N J_i(\theta_i) \quad \text{s.t.} \quad \theta_i = \theta_j, \quad \forall (i, j) \in E \quad (2)$$

- $\theta_i \in \mathbb{R}^d$ is a private/local variable held by the i th worker.

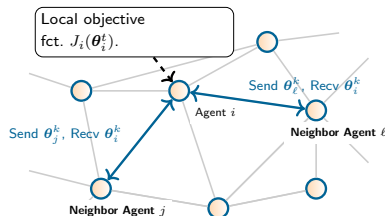
Background: Decentralized Deep Learning

Our problem —
$$\min_{\theta_i \in \mathbb{R}^d, i \in V} \sum_{i=1}^N J_i(\theta_i) \quad \text{s.t.} \quad \theta_i = \theta_j, \forall (i, j) \in E.$$

- We are interested in training a large neural network (NN) over N workers. For a supervised classification problem, $J_i(\theta)$ takes the form of empirical risk:

$$J_i(\theta) = \frac{1}{|\mathcal{D}_i|} \sum_{j=1}^{|\mathcal{D}_i|} \text{loss}(f(\mathbf{x}_j; \theta); y_j) \quad (3)$$

- The N workers must learn a common model θ^* given only a subset of the training data $D = \cup_{i=1}^M D_i$.
- **Solution: consensus + optimize strategy** where workers communicate with neighbors to optimize their θ_i .



Decentralized Gradient Descent (DGD) Method

1. Agent i holds local parameter copy θ_i^t on iteration t .

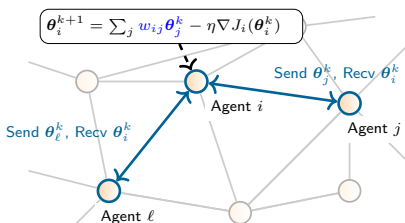
2. Calculate local gradient $\nabla J_i(\theta_i^k)$

3. receive θ_j from neighbors

$$\forall j \in V, W_{i,j} > 0$$

$$\theta_i^{k+\frac{1}{2}} = \underbrace{\sum_j w_{ij} \theta_j^k}_{\text{Gossip Averaging}}$$

4. Update $\theta_i^{k+1} = \theta_i^{k+\frac{1}{2}} - \eta \nabla J_i(\theta_i^k)$



Improvement: D-PSGD Method: Local Stochastic Gradient and Gossip Averaging Run in Parallel

► $g^k(\theta_i^k; \xi_i^k) := \sum_j \nabla J_i(\theta_i^k; \xi_i^k) \xrightarrow{\text{avg}} \left[\theta_{k+\frac{1}{2}}^1, \theta_{k+\frac{1}{2}}^d, \dots, \theta_{k+\frac{1}{2}}^n \right] = [\theta_k^1, \theta_k^2, \dots, \theta_k^n] W_k$

► **Drawback:** Limited Communication Bandwidth; Increases with dimensionality d

CHOCO-SGD [Koloskova et al., 2019a]

- **Solution:** Communication compression of θ_i with a compression operator $\mathcal{Q} : \mathbb{R}^d \rightarrow \mathbb{R}^d$
- **Assumption 1:** $\mathbb{E}_{\Omega} [\|\mathcal{Q}(\theta; \Omega) - \theta\|^2] \leq (1 - \delta)\|\theta\|^2, \quad \forall \theta \in \mathbb{R}^d$
 - ω is the randomness of compression operator; $\delta \in (0, 1]$ denotes compression error
- **Assumption 2:** $\mathbb{E}[\mathbf{g}_i^{(t)} | \mathcal{F}_t] = \nabla J_i(\theta_i^{(t)}) \quad \mathbb{E}[\|\mathbf{g}_i^{(t)} - \nabla J_i(\theta_i^{(t)})\|^2 | \mathcal{F}_t] \leq \sigma^2.$
- **Assumption 3:** Lipschitz-Smooth Gradient $\nabla J_i(\theta)$

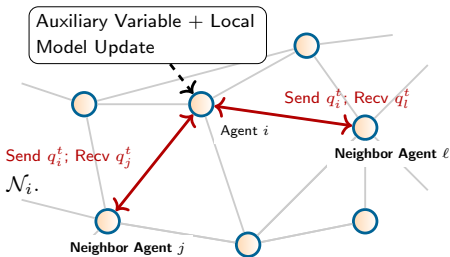
1. Local SGD: $\theta_i^{t+1/2} = \theta_i^t - \eta_t \mathbf{g}_i^t$
2. Agent i : **Send** a difference vector $q_i^t = \mathcal{Q}(\theta_i^{(t+\frac{1}{2})}) - \hat{\theta}_{i,i}^{(t)}$, **receive** q_j^t from neighbors $\forall j \in V, W_{i,j} > 0$

3. Update an auxiliary variable:

$$\hat{\theta}_{i,j}^{(t+1)} = \hat{\theta}_{i,j}^{(t)} + \mathcal{Q}(\theta_j^{(t+\frac{1}{2})}) - \hat{\theta}_{j,j}^{(t)}, \quad \forall j \in \mathcal{N}_i.$$

4. Update Local Model:

$$\theta_i^{(t+1)} = \theta_i^{(t+\frac{1}{2})} + \gamma \sum_{j \in \mathcal{N}_i} W_{ij} \{\hat{\theta}_{i,j}^{(t+1)} - \hat{\theta}_{i,i}^{(t+1)}\}.$$



Convergence of CHOCO-SGD

Theorem — Convergence of CHOCO-SG [Koloskova et al., 2019a, Koloskova et al., 2019b]

Under Assumptions 1, 2, and 3, There exists $\eta, \gamma > 0$ such that if we consider a constant step size with $\eta_t \equiv \eta$, then for any $T \geq 1, \eta, \gamma > 0$

$$\mathbb{E}[\|\nabla J(\bar{\theta}^{(T)})\|^2] = \mathcal{O}\left(\sqrt{\frac{L\sigma^2 J_0}{NT}} + \left(\frac{LGJ_0}{\rho^2 \delta T}\right)^{\frac{2}{3}}\right)$$

- ▶ $\delta \in (0, 1]$ is the compression error $\rho \in (0, 1]$ is the spectral gap of W
- ▶ $\bar{\theta}^{(t)} = \frac{1}{N} \sum_{i=1}^N \theta_i^{(t)}$ $J_0 = J(\bar{\theta}^{(0)}) - \min_{\theta} J(\theta)$

Question: How well does CHOCO-SGD converge for $d \gg 1$?

- ▶ when $\delta = \frac{k}{d}$, we apply Theorem 1 to get...

CHOCO-SGD in the **Overparameterized** Regime

Convergence of CHOCO-SGD with $m \gg 1$

Consider a rand_k or top_k sparsifier with fixed co-ordinate retention k . Fix number of training iterations at T . From Theorem 1, we have:

$$\mathbb{E}[\|\nabla J(\bar{\theta}^{(T)})\|^2] = \mathcal{O}\left(\sqrt{\frac{L\sigma^2 J_0}{NT}} + d^{\frac{2}{3}} \left(\frac{LGJ_0}{\rho^2 k T}\right)^{\frac{2}{3}}\right)$$

For $\mathbb{E}[\|\nabla J(\bar{\theta}^{(T)})\|^2] \leq \epsilon$, Minimum iterations required T is of the order:

$$T = \Omega\left(LJ_0 \cdot \max\left\{\frac{\sigma^2}{N\epsilon^2}, \frac{d}{k} \frac{G}{\rho^2 \epsilon^{1.5}}\right\}\right)$$

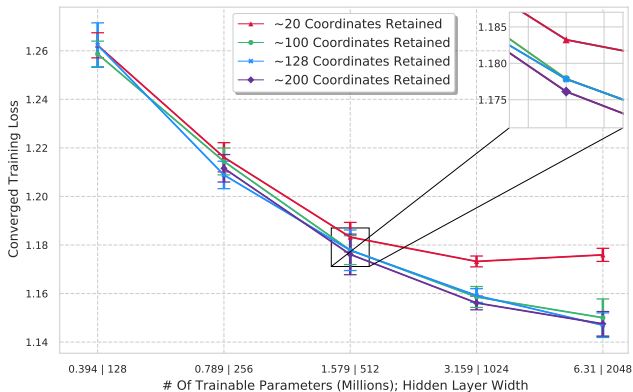
- **Implication:** Communication cost/iteration reduced, but Compressed DSG algorithms require more iterations to converge
- Pitfall in existing theory! Need to observe implications for practical performance.

Numerical Experiments – Two-Layer ReLU Network

Goal: Empirically investigate convergence of CHOCO-SGD with Overparameterized NNs

- ▶ Decentralized graph simulated by an MPI network environment with a fixed communication graph W .
 - ▶ Independent CPU process assigned to each worker.
- ▶ Train Dataset: CIFAR-10 – 10 classes, 50K datapoints as a $32 \times 32 \times 3$ RGB image divided in an **i.i.d** fashion among N workers; reshuffled every epoch.
- ▶ Test Dataset: To test generalization ability, CIFAR-10.1 [Recht et al., 2018]
- ▶ Model: ReLU Linear NNs with increasing layer widths $m = [128, 256, 512, 1024, 2048]$
 $\underbrace{\hspace{10em}}_{0.3 \text{ to } 6.31 \times 10^6 \text{ parameters}}$
- ▶ Constant consensus (γ) and SGD (η) step size run over a constant number of training iterations (T). top_k and rand_k used with constant number of co-ordinates retained k

Converged Training Loss vs Model Dimensionality – CIFAR10: top_k sparsification



- Setting: $N = 8$ workers on a ring topology, CIFAR-10, 300 epochs, 2-layer ReLU network with increasing m and constant k (#bits transmitted is constant)
- Overparameterized models exhibit better convergence and training loss decreases with increase in d .

Are Overparameterized Models in Consensus?

- Consensus Distance captures expected disagreement between averaged model $\bar{\theta}^T$ and each node θ_i :

$$\Upsilon = \frac{1}{N} \sum_{i=1}^N \frac{\|\theta_i^T - \bar{\theta}^T\|^2}{\|\bar{\theta}^T\|^2}$$

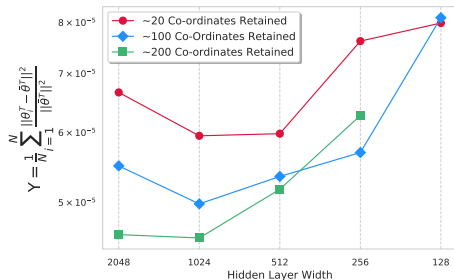
- If Υ satisfies the following bound [Kong et al., 2021]

$$\Upsilon_t^2 \leq \left(\frac{1}{Ln} \gamma \sigma^2 + \frac{1}{8L^2} \|\nabla J(\bar{\theta}^T)\|^2 \right)$$

we can recover centralized SGD's convergence rate with a larger stepsize

$$\gamma \leq \gamma_{max}$$

- **Overparameterized models enjoy greater consensus among workers with only marginal dependence on k**



Problem: Consensus is Expensive in The Overparameterized Regime

Layer Width	Normalized Consensus Distance		
	Epoch = 200	Epoch = 100	Epoch = 50
2048	5.499×10^{-5}	9.8206×10^{-3}	1.3977×10^{-2}
1024	4.980×10^{-5}	1.0346×10^{-2}	1.5307×10^{-2}
512	5.349×10^{-5}	1.0026×10^{-3}	1.3478×10^{-2}
256	5.694×10^{-5}	8.7639×10^{-3}	1.2423×10^{-2}
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Average consensus is expensive for overparameterized models. Can DSGD algorithms with overparameterized models converge with [inexact consensus](#)?

From Parameter Estimation to Function Estimation

- ▶ Consider the objective of learning regressors $\tilde{f} \in \mathcal{H}$ for hypothesized function class \mathcal{H}
- ▶ (x_n, y_n) are drawn i.i.d over $(x, y) \in \mathcal{X} \times \mathcal{Y}$ s.t $\mathcal{X} \subset \mathbb{R}^p$ (feature vector) and $\mathcal{Y} \subset \mathbb{R}$ (label)
- ▶ Now, consider empirical risk formulation to find optimal function $f^* \in \mathcal{H}$

$$\operatorname{argmin}_{\tilde{f} \in \mathcal{H}} J_i(\tilde{f}) = \frac{1}{|\mathcal{D}_i|} \sum_{j=1}^{|\mathcal{D}_i|} \operatorname{loss}(\tilde{f}(\mathbf{x}_j); y_j) + \underbrace{\frac{\lambda}{2} \|\tilde{f}\|_{\mathcal{H}}^2}_{\text{Hilbert Norm Penalty}} \quad (4)$$

- ▶ where loss is a strictly convex loss function used to penalize the deviation of regressor f from the output label y given by $l : \mathcal{H} \times \mathcal{X} \times \mathcal{Y} \rightarrow \mathcal{R}$

This problem is intractable!

From Parameter Estimation to Function Estimation

- For decentralized learning, impose functional consensus constraints [Koppel et al., 2018]:

$$J^T = \operatorname{argmin}_{f_i \in \mathcal{H}} \left(\sum_{i \in V} (\mathbb{E}_{x_i, y_i} [l_i f_i(x), y_i]) + \frac{\lambda}{2} \|f_i\|_{\mathcal{H}}^2 \right) \quad (5)$$

$$\text{such that } f_i = f_j \quad \forall (i, j) \in E \quad (6)$$

- To solve 5, equip the hypothesized function class \mathcal{H} with a kernel function over the feature vector space $\kappa : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ that satisfies:

$$\langle f, \kappa(x_i, \cdot) \rangle_{\mathcal{H}} = f(x_i) \quad \mathcal{H} = \overline{\operatorname{span}(\kappa(x_i, \cdot))} \quad (7)$$

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- Given 7 is satisfied, \mathcal{H} is an RKHS. Note that from 7, we also get:

$$\tilde{f}(\mathbf{x}_i) = \sum_N w_{i,n} \kappa(\mathbf{x}_{i,n}, \mathbf{x}_i) \quad (6)$$

$$\Rightarrow J^T = \underbrace{\underset{w \in \mathbb{R}^n}{\text{argmin}}}_{\text{Kernel Trick!}} \frac{1}{N} \sum_{i=1}^N \left(\text{loss} \left(\sum_{j=1}^N w_j \kappa(\mathbf{x}_j, \mathbf{x}_i), y_i \right) \right) \quad (7)$$

$$+ \frac{\lambda}{2} \left\| \sum_{i=1}^N \sum_{j=1}^N w_i w_j \kappa(\mathbf{x}_j, \mathbf{x}_i) \right\|_{\mathcal{H}}^2 \quad (8)$$

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- As training points $n \rightarrow \infty$, infinite memory requirement

RKHS stochastic saddle-point problems in the Decentralized Setting

- Formulate functional consensus constraint $f_i = f_j \ \forall (i,j) \in \mathcal{E}$ as a penalty function [Koppel et al., 2018]:

$$\min \sum_{i \in \mathcal{V}} (\mathbb{E}_{(x_i, y_i)} [l_i(f_i(x_i, y_i))] + \frac{\lambda}{2} \|f_i\|_{\mathcal{H}}^2 + \underbrace{\frac{c}{2} \sum_{j \in n_i} \mathbb{E}_{x_i} ([f_i(x_i) - f_j(x_i)]^2)}_{\text{Inexact consensus Penalty}} \quad (8)$$

$$:= \min \sum_{i \in \mathcal{V}} (l_i(f_i(x_{i,t}), y_{i,t}) + \frac{\lambda}{2} \|f_i\|_{\mathcal{H}}^2 + \underbrace{\frac{c}{2} \sum_{j \in n_j} (f_i(x_{i,t}) - f_j(x_{i,t}))^2}_{\text{Inexact consensus Penalty}}) \quad (9)$$

(i.i.d samples $(x_{i,t}, y_{i,t})$ are revealed to each worker f_i) (10)

(11)

- Where the representer theorem implies that at time t, the regressor f can be expanded as:

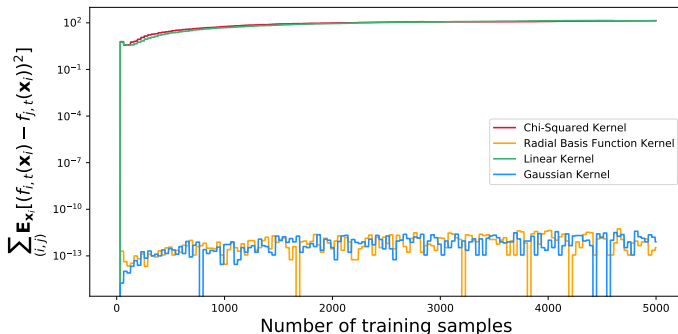
$$f_{i,t}(x) = \sum_{n=1}^{t-1} w_{i,n} \kappa(x_{i,n}, x) = w_{i,t}^T \kappa_{x_{i,t}}(x)$$

Numerical Experiments II

Question: What is the effect of kernel choice $\kappa(\cdot)$ on consensus term

$$\frac{c}{2} \sum_{j \in n_i} \mathbb{E}_{x_i} [(f_i(x_i) - f_j(x_i))^2].$$

- Implement Gaussian and Radial basis kernel $\kappa(x, x') = \exp\left(-\frac{\|x - x'\|^2}{2\sigma^2}\right)$ and compare consensus error with polynomial kernel, chi-square kernel.

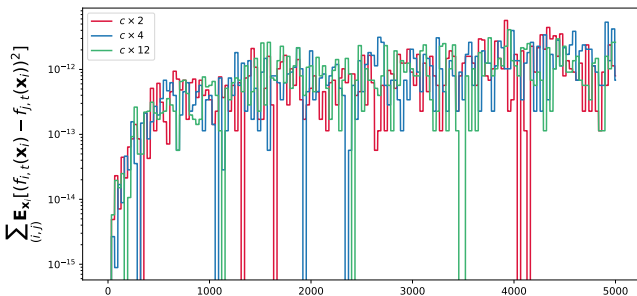


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- Implement Gaussian and Radial basis kernel $\kappa(x, x') = \exp\left(-\frac{\|x - x'\|^2}{2\sigma^2}\right)$ and compare consensus error with polynomial kernel, chi-square kernel.



- **Problem:** High dependence on choice of kernel, consensus reaches machine precision zero, and inefficient as $N \rightarrow \infty$ (overparameterized models use $N \gg 1$ training samples)

Inex-SGD for Inexact Consensus Deep Learning

- Consider a dense feedforward NN model on the i th worker $f_i(x_i; \theta_i)$. We are interested in the following optimization problem:

$$\min_{f_i} \sum_{i=1}^N \left(\left[\mathbb{E}[l_i(f_i(x_i, y_i))] + \frac{\lambda}{2} \|f_i\|^2 \right] + \frac{c}{2} \sum_{j \in \mathcal{N}_i} \mathbb{E}_{x_i} [|f_i(x_i) - f_j(x_i)|^2] \right) \quad (12)$$

For the NN model parameterized by θ , we have: (13)

$$\min_{\theta_i \forall i=1, \dots, N} \sum_{i=1}^N \left(\mathbb{E}_{x_i} [l_i(f(x_i; \theta_i), y_i)] + \frac{\lambda}{2} \|\theta_i\|^2 \right) \quad (14)$$

$$+ \sum_{j \in \mathcal{N}_i} \mathbb{E}_{x_i} \left[\frac{c}{2} |f(x_i; \theta_i) - f(x_i; \theta_j)|^2 \right] \quad (15)$$

Inex-SGD for Inexact Consensus Deep Learning

1. Agent i holds local parameter copy θ_i^t on iteration t , and mini-batch sample $\xi_{i,k} = [\xi_i^{k,1}, \xi_i^{k,2}, \dots, \xi_i^{k,M}]$
2. Evaluate model on batch $\sum_{j=1}^M J(\theta_i^k, \xi_i^{k,j})$
3. Receive $\left(\sum_{p=1}^M J(\theta_j^k, \xi_j^{k,p}), \xi_{j,k} \right) \forall j \text{ in } \mathcal{N}_i$
4. Calculate Stochastic gradient on worker i :

$$\begin{aligned}
 g_i^k &= \nabla_{\theta_i} l_i(J(\xi_{i,k})) + \lambda \theta_i^k \\
 &\quad + c \sum_{j \in \mathcal{N}_i} \left(J(\xi_i^k; \theta_i^k) - J(\xi_i^k; \theta_j^k) \right) \nabla J(\xi_{i,k}, \theta_i^k) \\
 &\quad + c \sum_{j \in \mathcal{N}_i} \left(J(\xi_j^k; \theta_i^k) - J(\xi_j^k; \theta_j^k) \right) \nabla J(\xi_{j,k}, \theta_i^k)
 \end{aligned} \tag{12}$$

5. Perform SGD Update: $\theta_i^{k+1} = \theta_i^k - \eta^k g_i^k$

Inex-SGD for Inexact Consensus Deep Learning

- Stochastic gradient on worker i on iteration k is given by:

$$\begin{aligned} g_i^k &= \nabla_{\theta_i} l_i(J(\xi_{i,k})) + \lambda \theta_i^k \\ &\quad + c \sum_{j \in \mathcal{N}_i} \left(J(\xi_i^k; \theta_i^k) - J(\xi_i^k; \theta_j^k) \right) \nabla J(\xi_{i,k}, \theta_i^k) \\ &\quad + c \sum_{j \in \mathcal{N}_i} \left(J(\xi_j^k; \theta_i^k) - J(\xi_j^k; \theta_j^k) \right) \nabla J(\xi_{j,k}, \theta_i^k) \end{aligned} \tag{12}$$

- **Problem:** To calculate $\nabla J(\xi_{i,k}, \theta_i^k)$ and $\nabla J(\xi_{j,k}, \theta_i^k)$, each worker i must send data points/ mini-batches $\xi_{i,k}, \xi_{j,k}$ to neighbors $j \in \mathcal{N}_i$. Not recommended for sensitive data.

Summary of Findings

How do compressed DSGD algorithms perform in the **overparameterized** regime?

- ▶ Utilizing overparameterized NNs in the decentralized setting is **practical** and **beneficial**
- ▶ However, overparameterized models reach consensus with an increased cost compared to smaller NN models.

Proposed Solution

- ▶ **Inexact Consensus** with RKHS-valued functional SGD reformulated without dependence on kernel choice.

Next Steps

- ▶ Fix bugs in algorithm development; more rigorous analysis of proposed solution
- ▶ Propose a more secure alternative

Thank you! Questions?

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