LQR Controller for Two Link Rigid Manipulator

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Abstract- **This paper discusses a multiple input and multiple output (MIMO) model to simulate a two link rigid manipulator. The mathematical modeling of two link rigid manipulator is obtained by using Euler's Lagranges method.The model is non linear and the response is unbounded. Proportional Integral Derivative (PID) and Linear quadratic regulator(LQR) are designed and the responses are compared using Matlab and simulink model. It is found that LQR delivers the best response and is well suited for two link rigid manipulator.**
Keywords: **Two link manipulator, PID and LQR.**

I. INTRODUCTION

The industry is moving from current state of automation to robotization to increase the productivity for delivering uniform quality. Robots are now commonly employed in hostile environment such as at various place in an atomic plant for handling radioactive materials. One type of robot commonly used in the industry is robotic manipulator or simple a robotic arm. Manipulator is a machine that have functions similar to human upper limbs and move the objects spatially.

Review of the research work reveals that much work has been done on various aspects of control of manipulators using PID controllers, different algorithms etc. but the results are not generalized fit well to everyone. Thambirajah et al presented the work on numerical optimization techniques of two link rigid manipulator [1]. But they can't make it to study the vector optimization for generating the entire set of optimal solutions . Reza Fotouhi et al, modeled a two link rigid manipulator based on the specific velocity to be followed [2]. J.W.S. Chong et al, studied the augmented reality (AR) environment [3]. Alessandro Gasparetto et al, studied on a technique for optimal trajectory planning of robotic manipulator [4].

Figure1demonstrates the mechanical structure of a manipulator consists of rigid links connected by means of joints are segmented into an arm that ensures mobility, wrist that confers orientation and an end effector that performs the specified task. To attain a desired position, a manipulator is required to

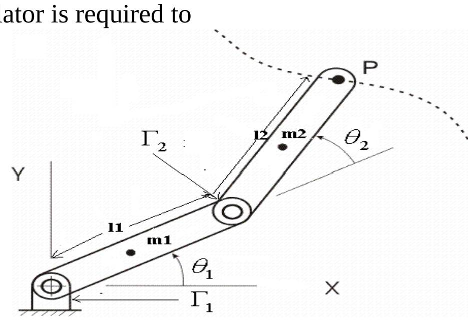

Figure 1. Two link Rigid Manipulator

accelerate from rest, travel at specified path and finally decelerate to stop. To accomplish the task, controlling torques are applied by the actuators at the manipulator joints. This torque is computed from the dynamic model for the mechanical design of manipulator. The dynamic model and generated trajectory constitute the inputs to the sytem design. The problem of manipulator control is to find the time behavior of the forces and torques delivered by the actuators for executing the desired task. Bothe the manipulator motion control and its force interaction with the environment are monitored by the control algorithms. The tasks to be performed by the manipulator are to move the end effector along the desired trajectory and to extract a force on the environment to carry out the desired task. The former is called the position control and the latter force control.

The organization of the work deals with the dynamic model of a manipulator followed by controlling algorithms.

II. DYNAMIC MODELING

The dynamics of two link manipulator is formulated using Lagranges Euler method. The mass of each link is assumed to be a point mass located at the centre of mass of each link. The Lagrange requires kinetic and potential enegies of the manipulator. The kinetic energy can be expressed as:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad (1)$$

Where I is the moment of inertia, m is the mass of the link , v is the linear velocity and w is the angular velocity

Thus the kinetic energy of the link 1 with linear velocity $v_1 = \frac{1}{2} L_1 \theta_1$, and angular velocity $\omega_1 = \dot{\theta}_1$, moment of inertia $I_1 = \frac{1}{2} m_1 L_1^2$ is

$$k_1 = 1/2 m_1 v_1^2 + 1/2 I_1 \omega_1^2 + 1/8 m_1 L_1^2 \dot{\theta}_1 + 1/24 m_1 L_1^2 \dot{\theta}^2{}_1 \quad (2)$$

And its potential energy is

$P_1 = 1/2 m_1 g L_1 \theta_1$ where g is the magnitude of acceleration due to gravity in the negative y-axis direction.

Similarly for link 2

$$k_2 = 1/2 m_2 v_2^2 + 1/2 I_2 \omega_2^2 + 1/8 m_1 L_1^2 \dot{\theta}_1 + 1/24 m_2 L_2^2 \quad (3)$$

And the potential energy of link 2 is

$$P_2 = 1/2 m_2 g L_2 \theta_1 + 1/2 m_2 g L_2 \theta_{12} \quad (4)$$

The lagrangain L = K - P. Hence, it is calculated for both the links which give the respective torque from the formulation as :

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1}$$
$$\tau_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} \quad (5)$$

Hence the resultant torques after simplification is

$$\tau_1 = M_{11} \ddot{\theta}_1 + M_{21} \ddot{\theta}_2 + H_1 + G_1$$
$$\tau_2 = M_{21} \ddot{\theta}_1 + M_{22} \ddot{\theta}_2 + H_2 + G_2 \quad (6)$$

Where H and G are the respective centrifugal and coiolis acceleration forces

A state space representation is a mathematical model of a physical system as asset of input, output and state variables related by first order differential equations. To abstract from the number of inputs for the equation 6, they are analyzed using matrix form.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\left[M^{-1}H\right]_{2*1} - \left[M^{-1}G\right]_{2*1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \left[M^{-1}\right]_{2*2} & \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix}$$

III. CONTROL DESIGN

PID Controller design

The PID controller brings together the best characteristics of the P, I and D controllers. The proportional component corresponds to a manipulating device whenever the deviation occurs. The derivative component counteracts changes in the variable and increases the stability of the control loop. The parameter deviation is removed by the integral component.

Since the PID controller is applicable for linear models, hence it is assumed the link as a rigid manipulator. The transfer function thus obtained as:

$$\frac{\theta}{\tau} = \frac{k_p + k_{ds}}{Js^2 + (B + k_d)s + k_p} \quad (7)$$

And for a single link rigid manipulator

$$\frac{\theta}{\tau} = \frac{1}{Js^2 + Bs} \quad (8)$$

Where

$J = \frac{ml^2}{3}$ and m= mass, l =length

Where B is the mean friction coefficient

The closed loop transfer function is as follows:

$$Closedloop T.F. = \frac{s^2 k_d + s k_p + k_i}{Js^3 + (B + k_d)s^2 + k_i} \quad (9)$$

For a system to be stable, Routh's stability criteria is implemented. It is obvious that for stability $\frac{k_p(B + k_d) - Jk_i}{B + k_d} > 0$ and $k_i > 0$

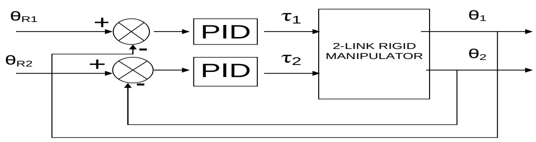

Figure2. Model of PID Controller

By selecting the values in the range of 0 to 100, the values of the P, D could be calculated.

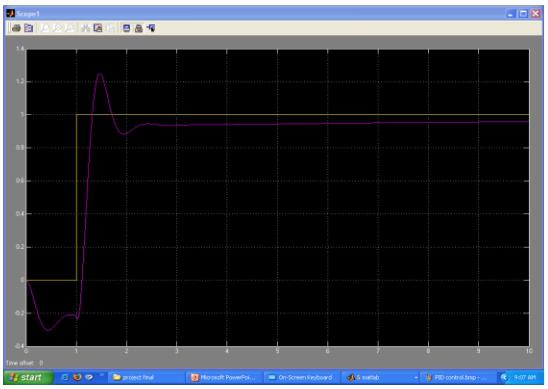

Figure 3 Response of θ_1 for two link rigid manipulator

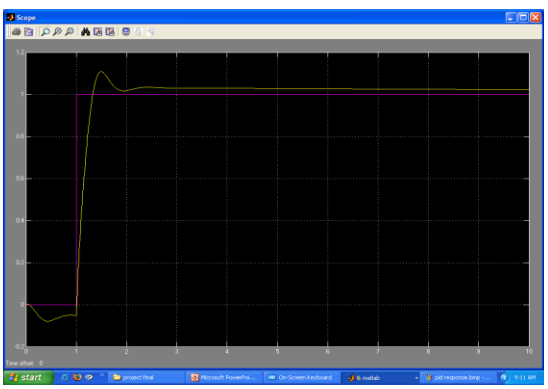

Figure 4 Response of θ_2 for two link rigid manipulator

By substituting the values of PID parameters, the above results were obtained.

LQR Design

The dynamic model of robotic manipulator is nonlinear in nature. Firstly, the model is converted into linear using one input port and two output ports.

For a continuous time system, the state equations are a set of first order differential equation as given below:

$$\dot{x}(t) = f(x,u,t) \quad \text{for } t \in [t_0, t_1] \qquad (10)$$

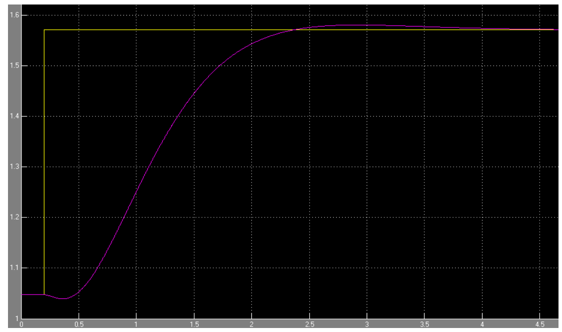

Figure 5 Response of θ_1 using LQR Controller

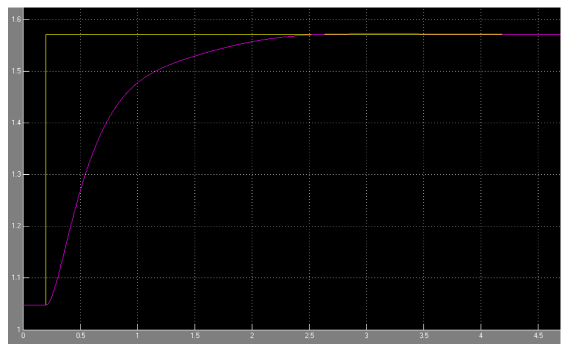

Figure 6. Response of θ_2 using LQR Controller

Where $\dot{x}(t)$ is $n \times 1$ is the state vector, u(t) is $p \times 1$ the input vector, f is the vector valued function and $[t_0, t_1]$ is the control interval. The target is to achieve the desired point while satisfying the operational constraints. The cost functional in question is the time integral of a quadratic form in the state vector x under the reasonable measure of the system transient response from time t_0 to t_1. In general, it is stated as :

$$J = \int_{t_0}^{t_1} x^t(t) Q x(t) dt \qquad (11)$$

Where Q is a real, symmetric , positive semi-definite, constraint matrix. Also, to minimize the deviation of the final state of the system from the desired state, a possible performance measure is

$$J = x^T(t_1)Hx(t_1) \qquad (12)$$

Where H is a positive semi-definite, real constant matrix. To make the solution realistic, the performance index is modified by adding a penalty term on the input vector u.

$$J = \int_{t_0}^{t_1} u^T(t)Ru(t)dt \qquad (13)$$

Where R is a positive definite, real, symmetric, constant matrix. By giving sufficient weight to control terms, the amplitude of control signals may be kept within bounded limits. Hence , a useful performance index is therefore

$$J = \tfrac{1}{2}[x^T(t_1)Hx\ (t_1)] + \int_{t_0}^{t_1} x^T(t_1)Qx(t) + u^T(t)Ru(t)dt \ (14$$

IV. CONCLUSION

The response of PID and LQR were compared. It is found that LQR gives a better performance for a two link rigid manipulator. The obtained results are tabulated to compare the performance of two controllers.

Step Input	Theta 1		Theta 2	
	PID	LQR	PID	LQR
Rise Time	1.5 sec	1.3 sec	1.4 sec	0.6 sec
Settling Time	4.9 sec	4.1 sec	4.5 sec	2.55 sec
Peak Overshoot	0.03	0.01	0.003	0.001

Table1. Comparison of PID and LQR control

V. FUTURE SCOPE

For future scope, the conventional PID controller could be replaced with fuzzy PID controller. It will be more convenient to find the gain matrix and for better accuracy.

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