

Deep Reinforcement Learning

Building Blocks

Arjun Chandra
Research Scientist

Telenor Research / Telenor-NTNU AI Lab

arjun.chandra@telenor.com

 @boelger

8 November 2017

<https://join.slack.com/t/deep-rl-tutorial/signup>

The Plan

- The Problem
- (deep) RL Concepts by Example
- Problem Decomposition
- Solution Methods
 - Value Based
 - Policy Based
 - Actor-Critic

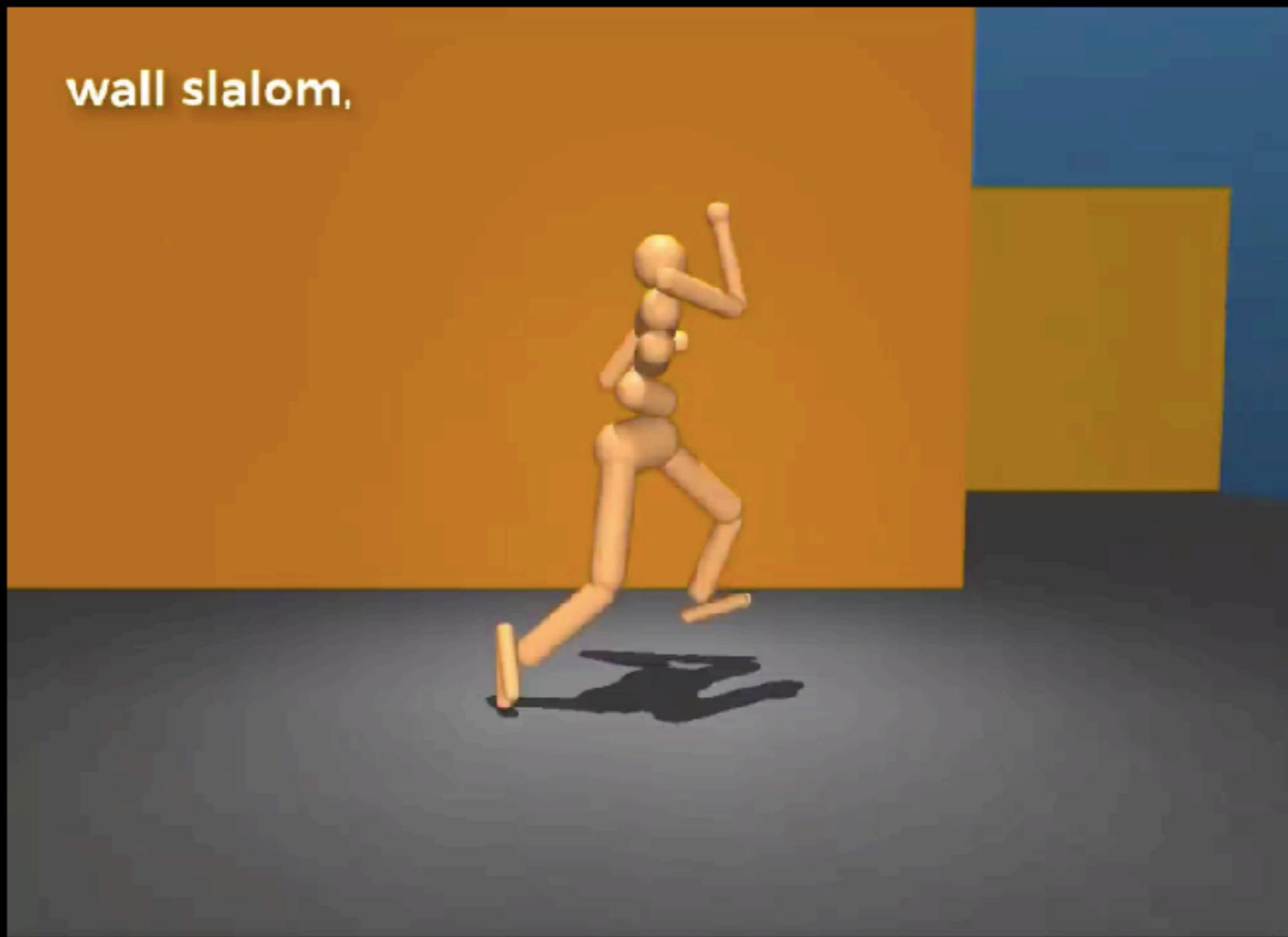
how to make
decisions over time
to maximise my
return / “long term reward”?



<http://cs.stanford.edu/groups/littledog/>



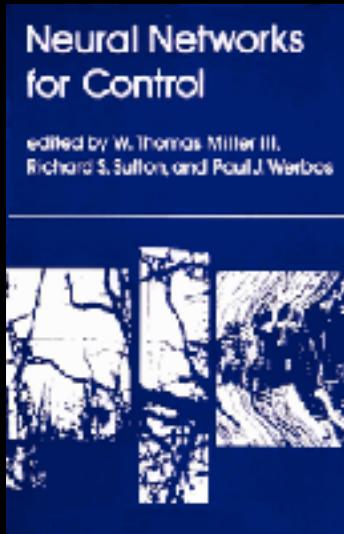
emergence of locomotion



<https://deepmind.com/blog/producing-flexible-behaviours-simulated-environments/>

https://www.youtube.com/watch?v=hx_bgoTF7bs

<https://arxiv.org/abs/1707.02286>



late
1980s

RL for robots using
NNs, L-J Lin. **PhD**
1993, CMU

Gerald Tesauro



1995

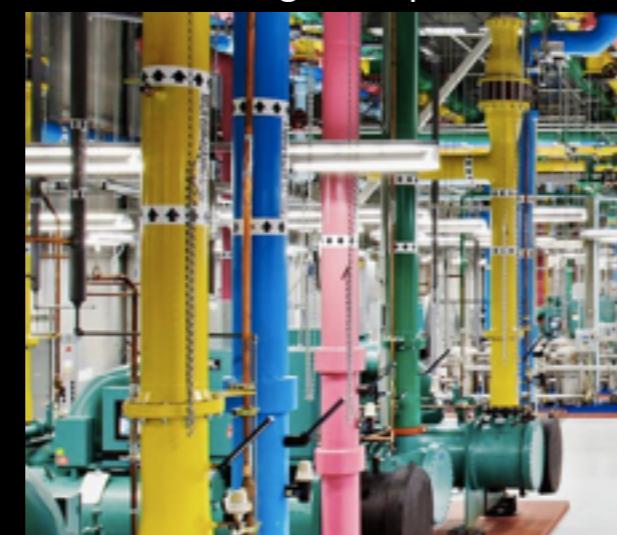
Stanford



<http://heli.stanford.edu/>

2004

Google DeepMind

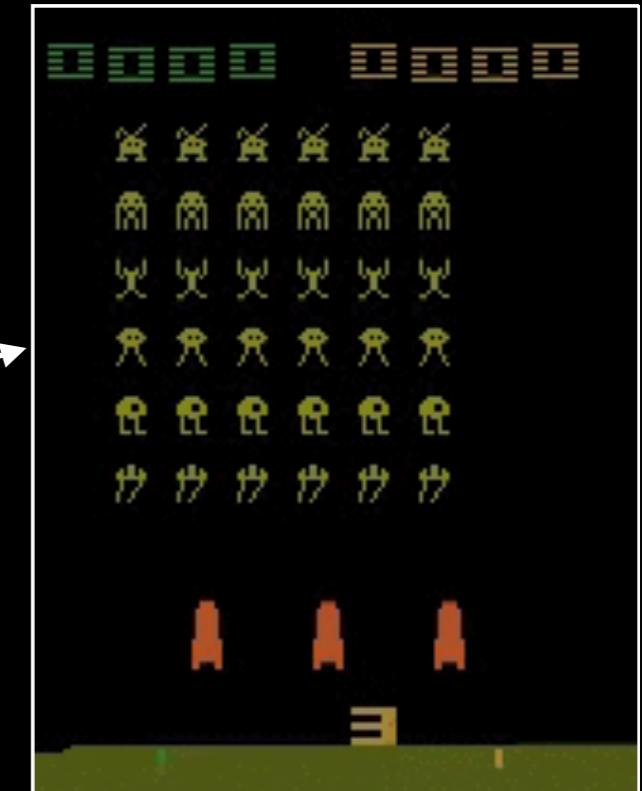


David Silver et. al.



2015 —

Vlad Mnih et. al.



2013 —

Problem Characteristics

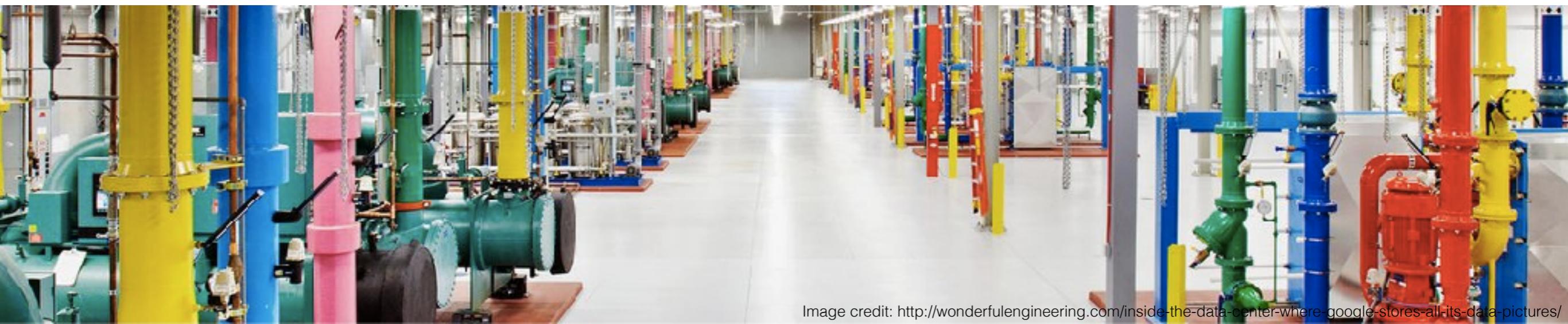
dynamic

uncertainty/volatility

uncharted/**unimagined**/
exception laden

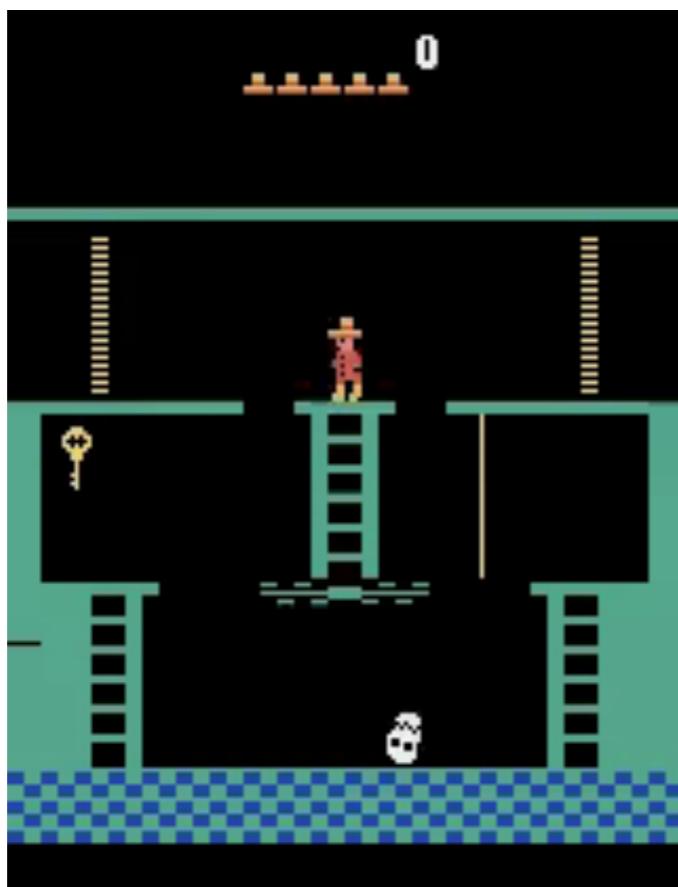
delayed consequences

requires **strategy**



Solution

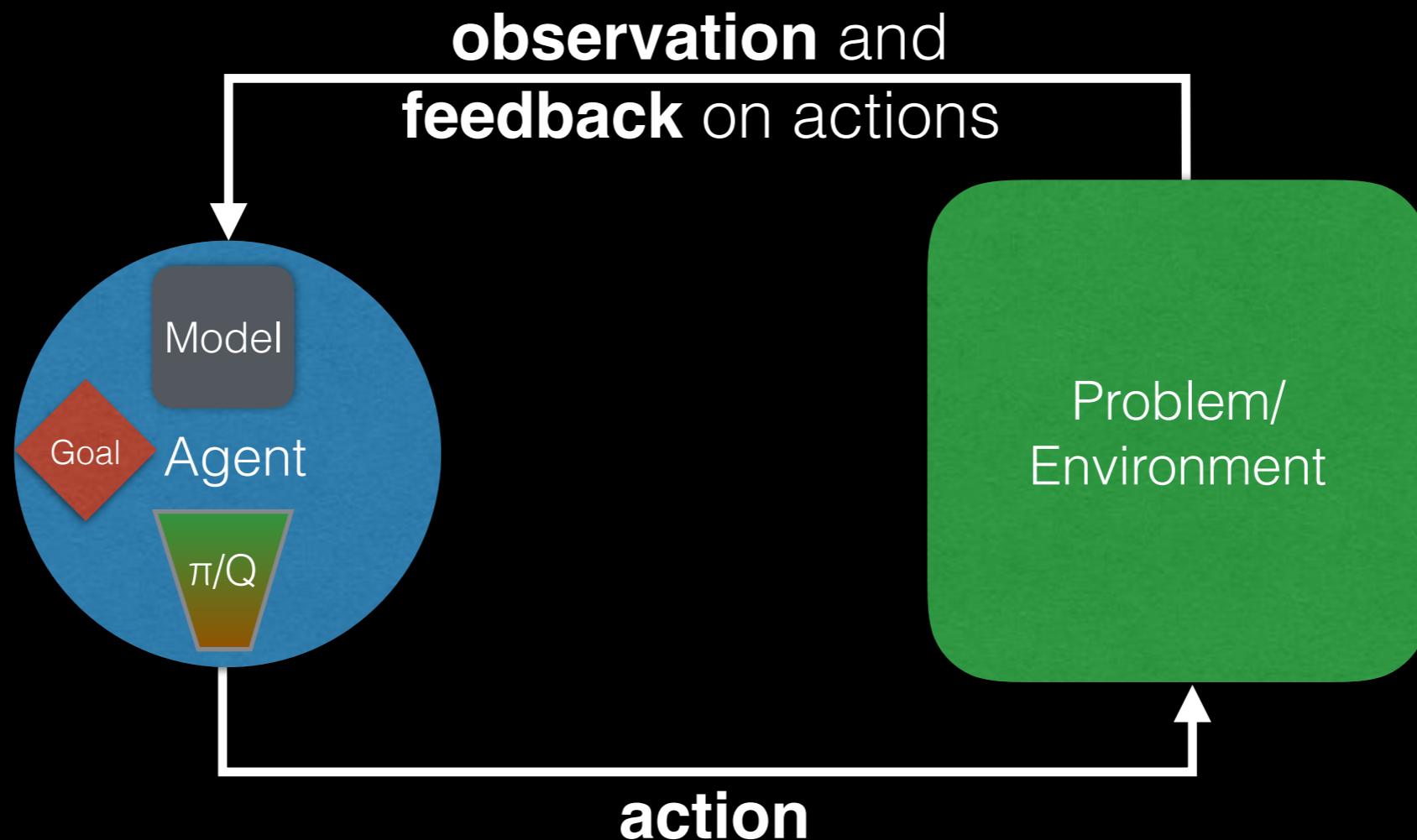
machine with **agency** which **learn**, **plan**, and **act** to find a strategy for solving the problem



autonomous to some extent
probe and **learn from feedback**
focus on the **long-term objective**
explore and **exploit**

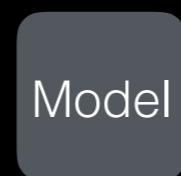
what is the
sequence of actions
I could take to maximise my
return / “long term reward”?

Reinforcement Learning



Goal

maximise return $E\{R\}$



Model

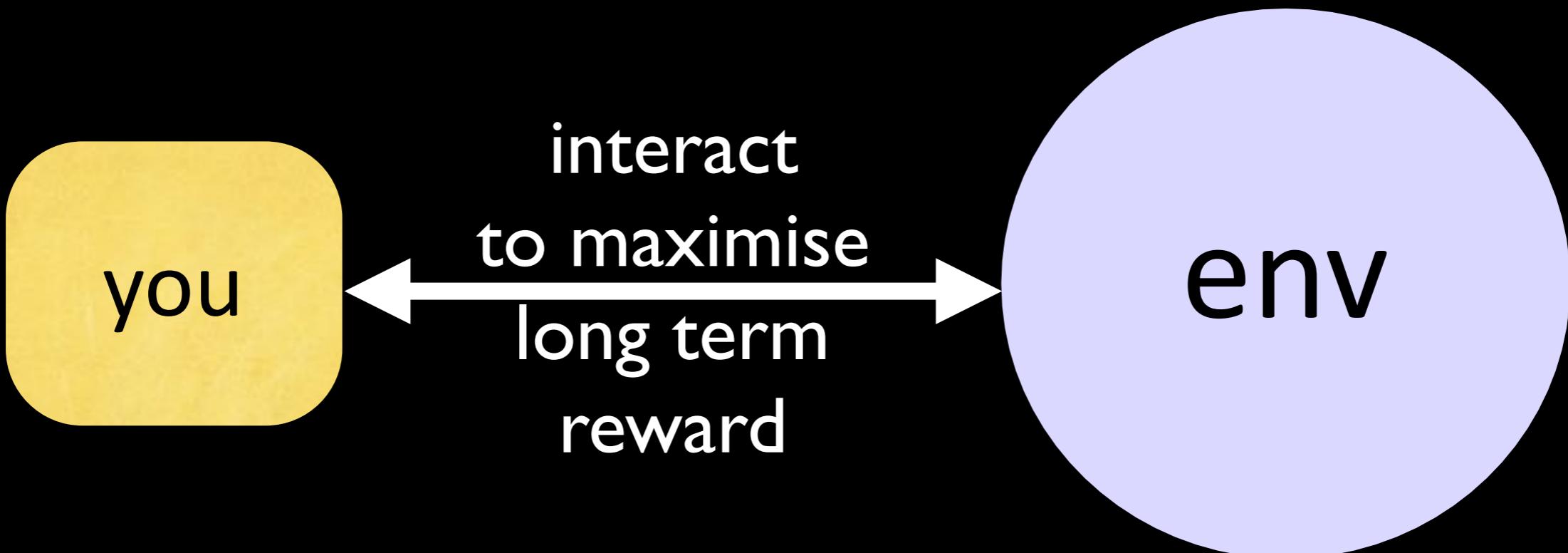
dynamics model



π/Q

policy/value function

the excruciatingly awesome MDP game!

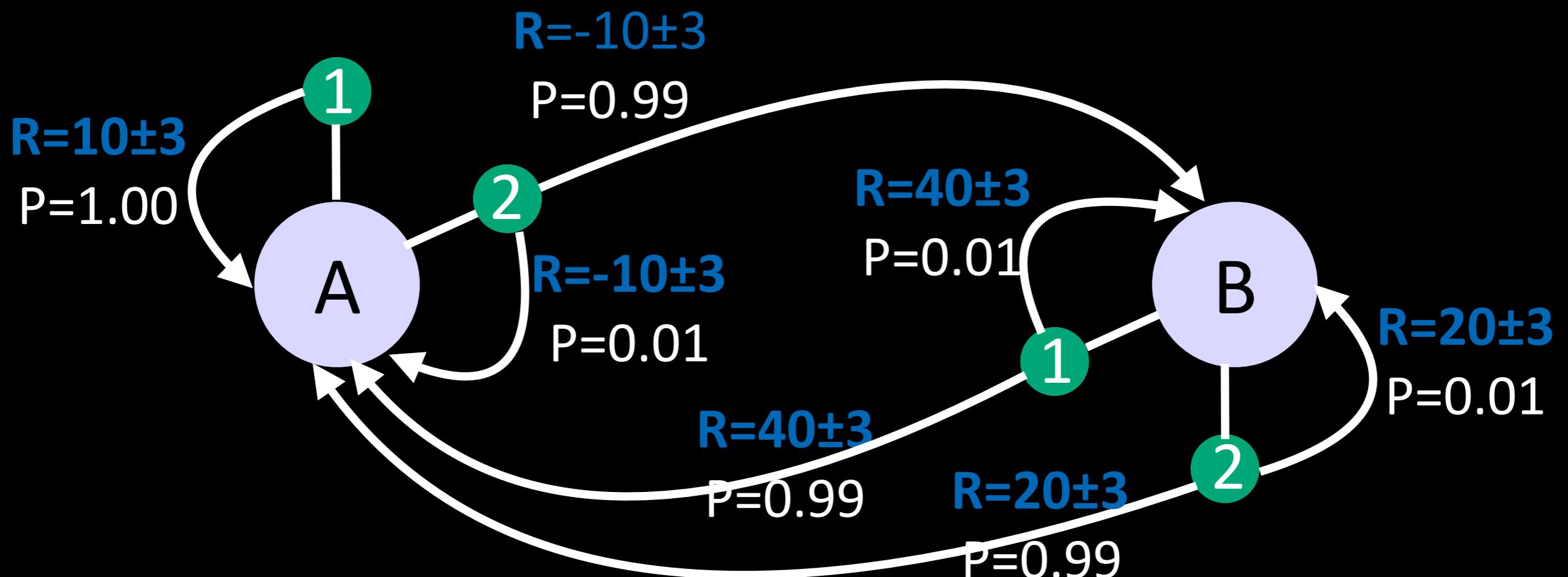


Inspired by Rich Sutton's tutorial:
<https://www.youtube.com/watch?v=ggqnxyjaKe4>

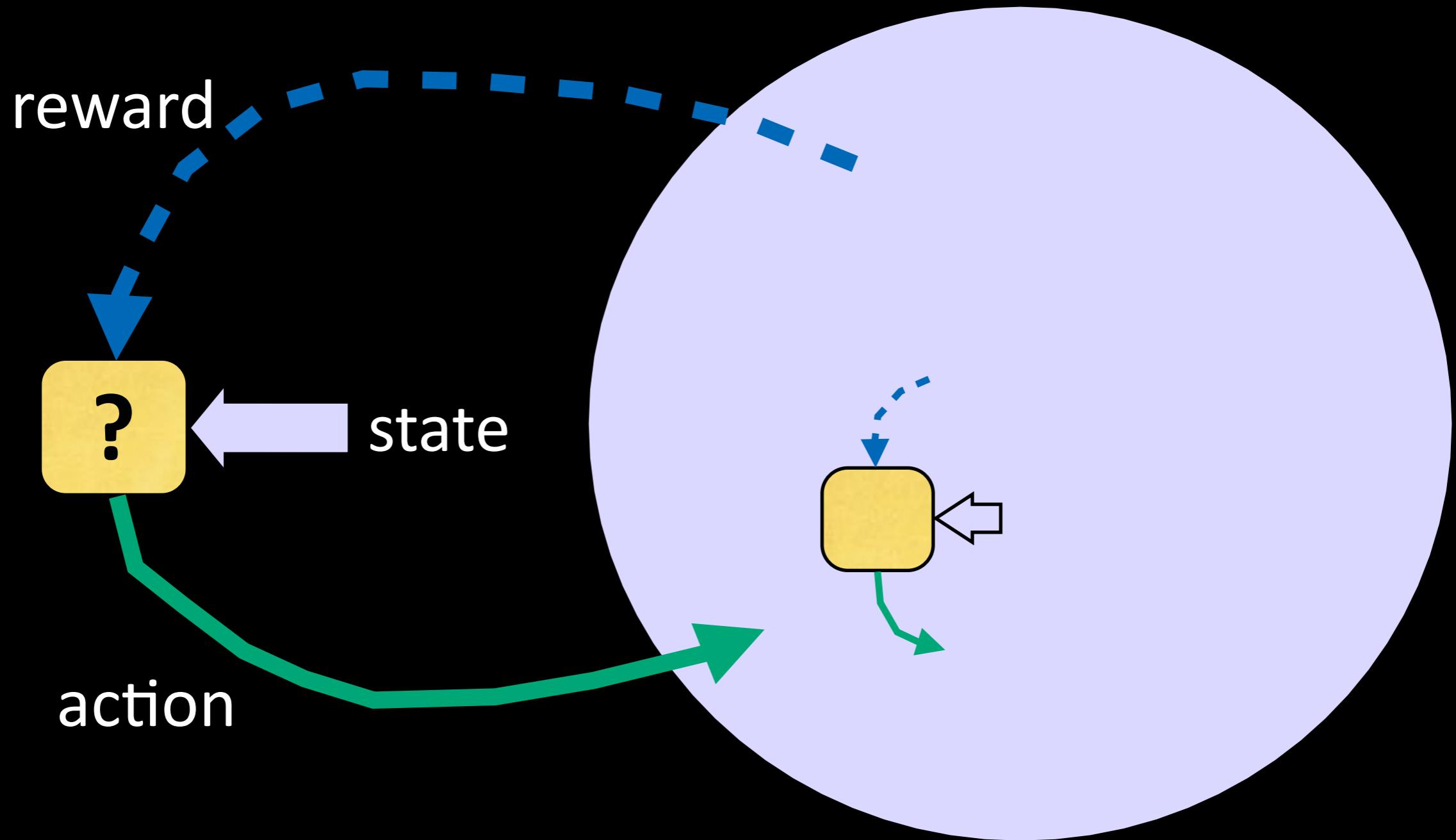
the MDP (S, A, P, R, γ)

R: immediate reward function $R(s, a)$

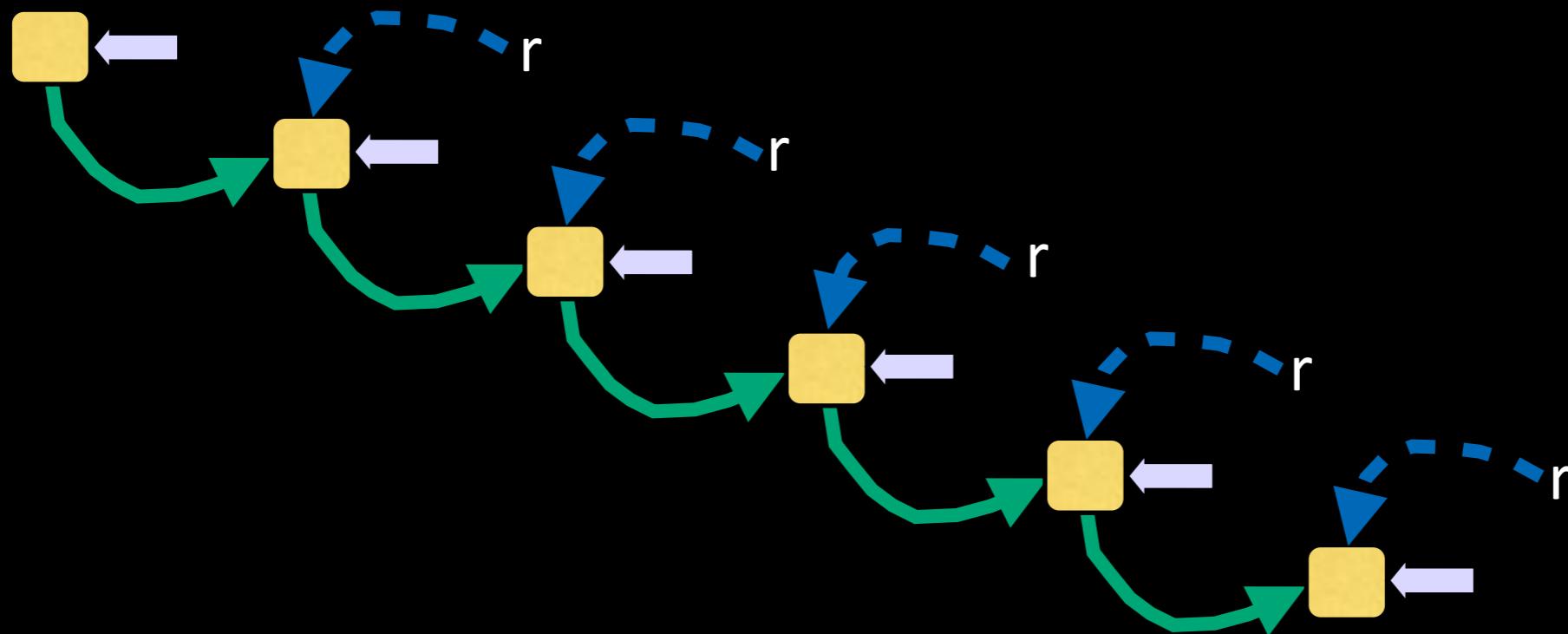
P: state transition probability $P(s' | s, a)$



the problem (cartoon of an MPD)

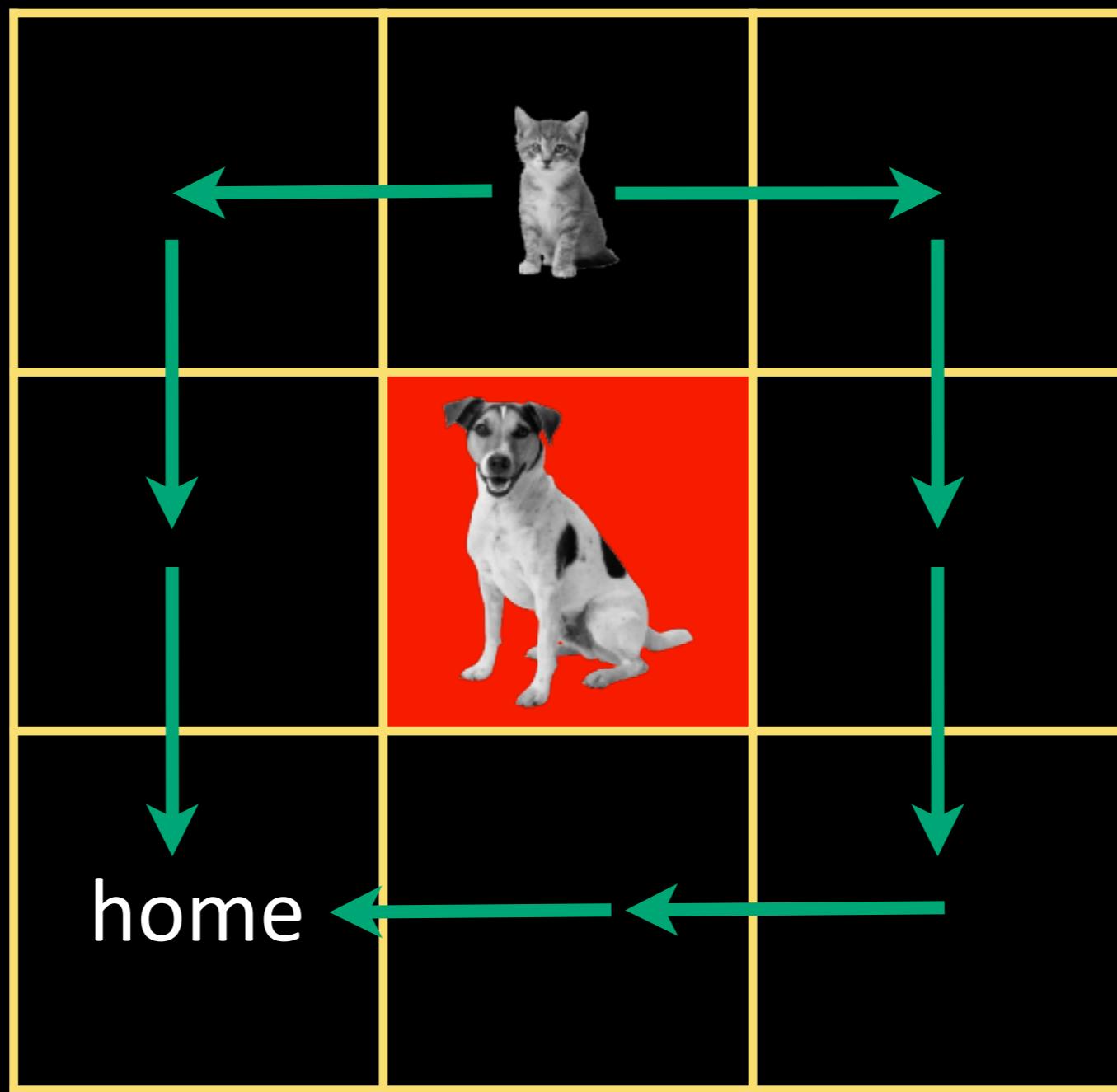


agent's job/goal?



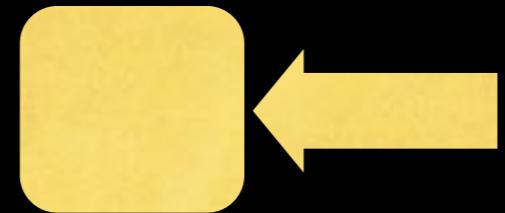
maximise expected
cumulative reward/return

toy problem



state and action spaces

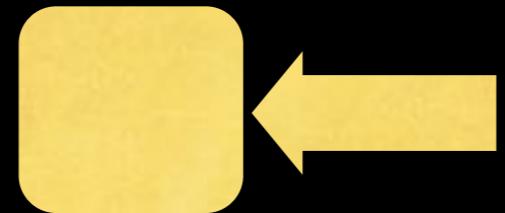
- size of these spaces can be quite large
- specifying the spaces is crucial in designing a good learning agent



5 integer values between
1 and 100: {22,44,12,67,9}

size of state space = $100 \times 100 \times 100 \times 100 \times 100$

can quantise state space differently



5 values belonging
to 2 classes: {1, 2, 1, 2, 1}

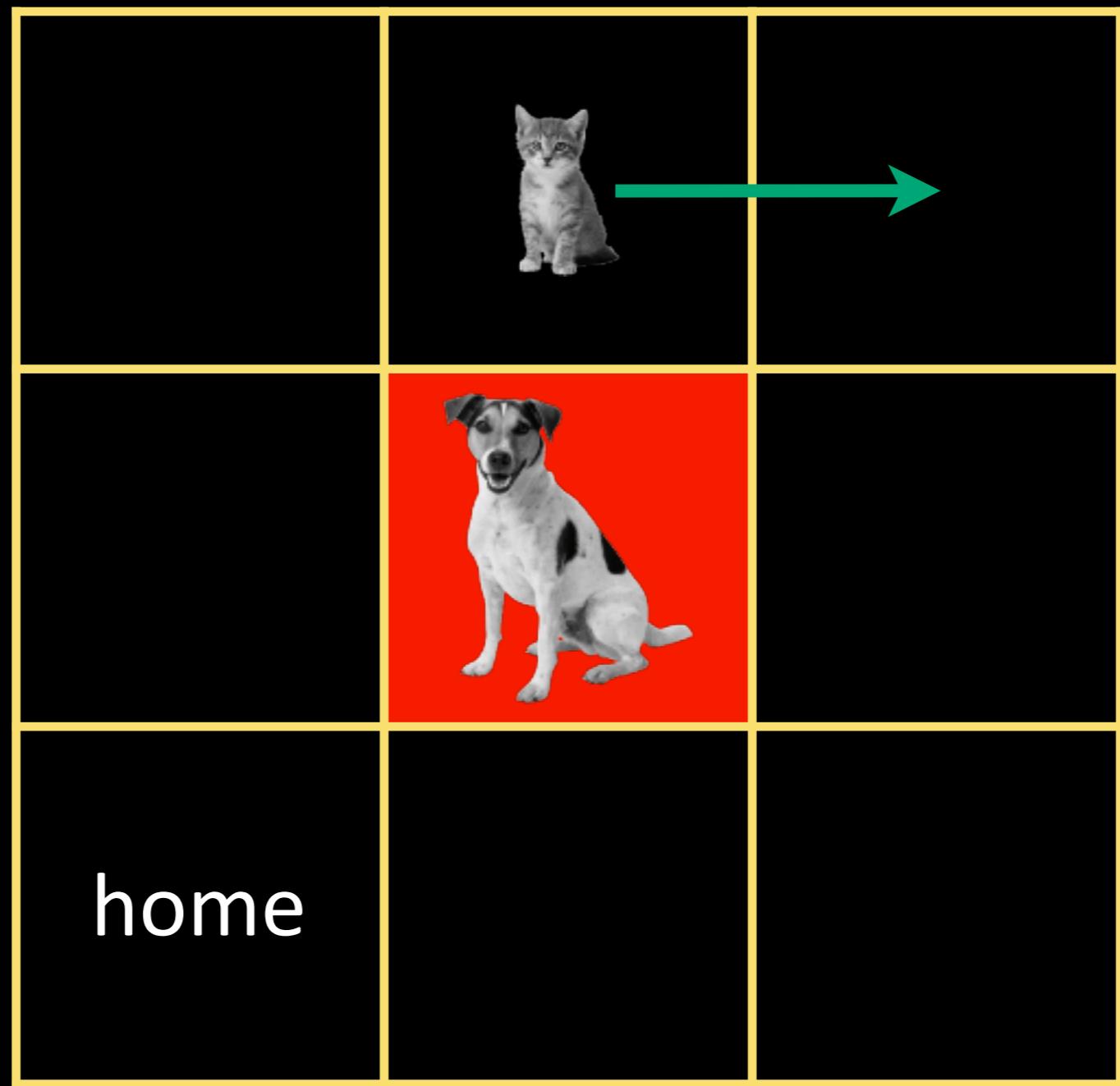
size of state space = $2 \times 2 \times 2 \times 2 \times 2$

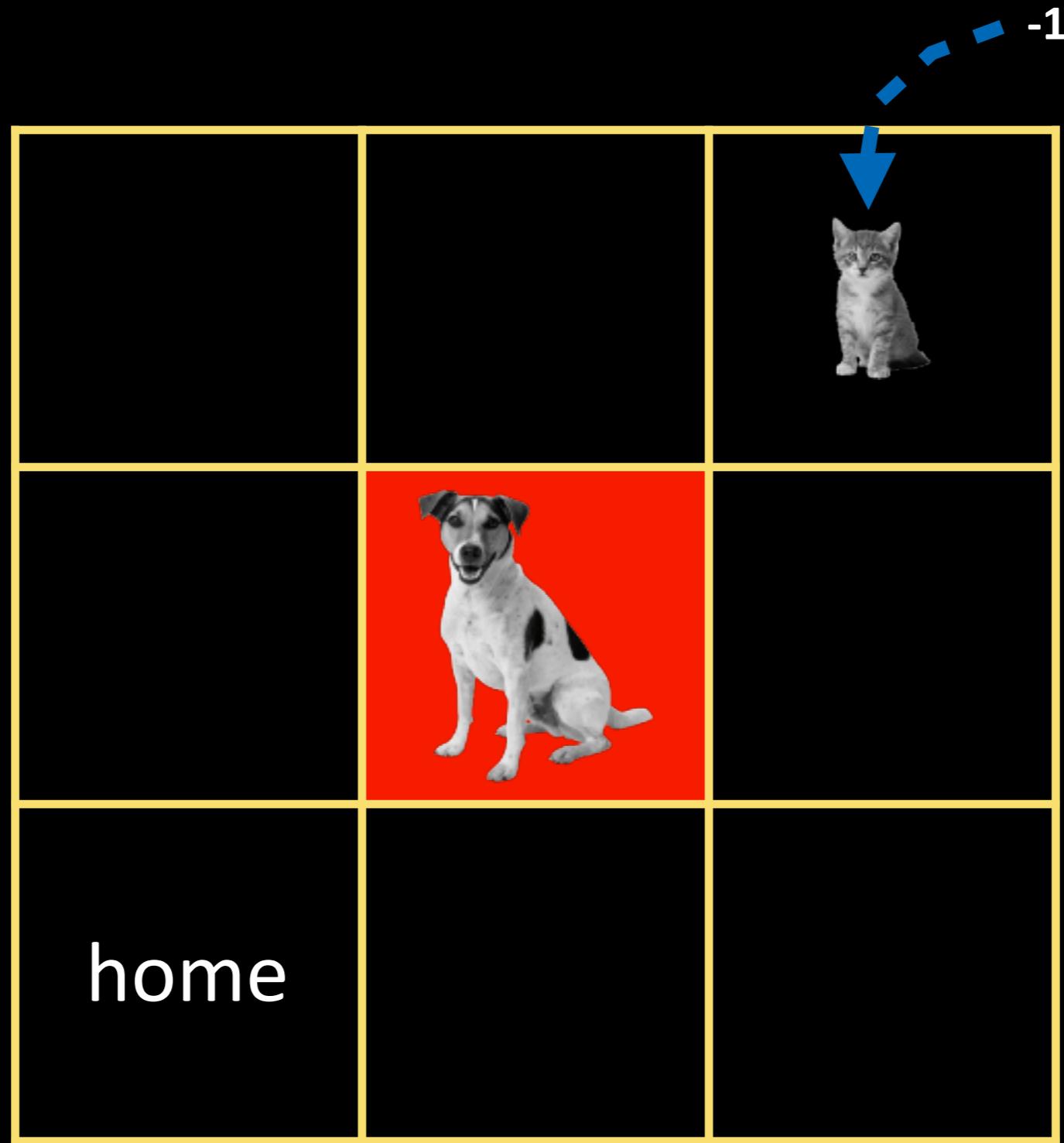
in the toy problem? 9

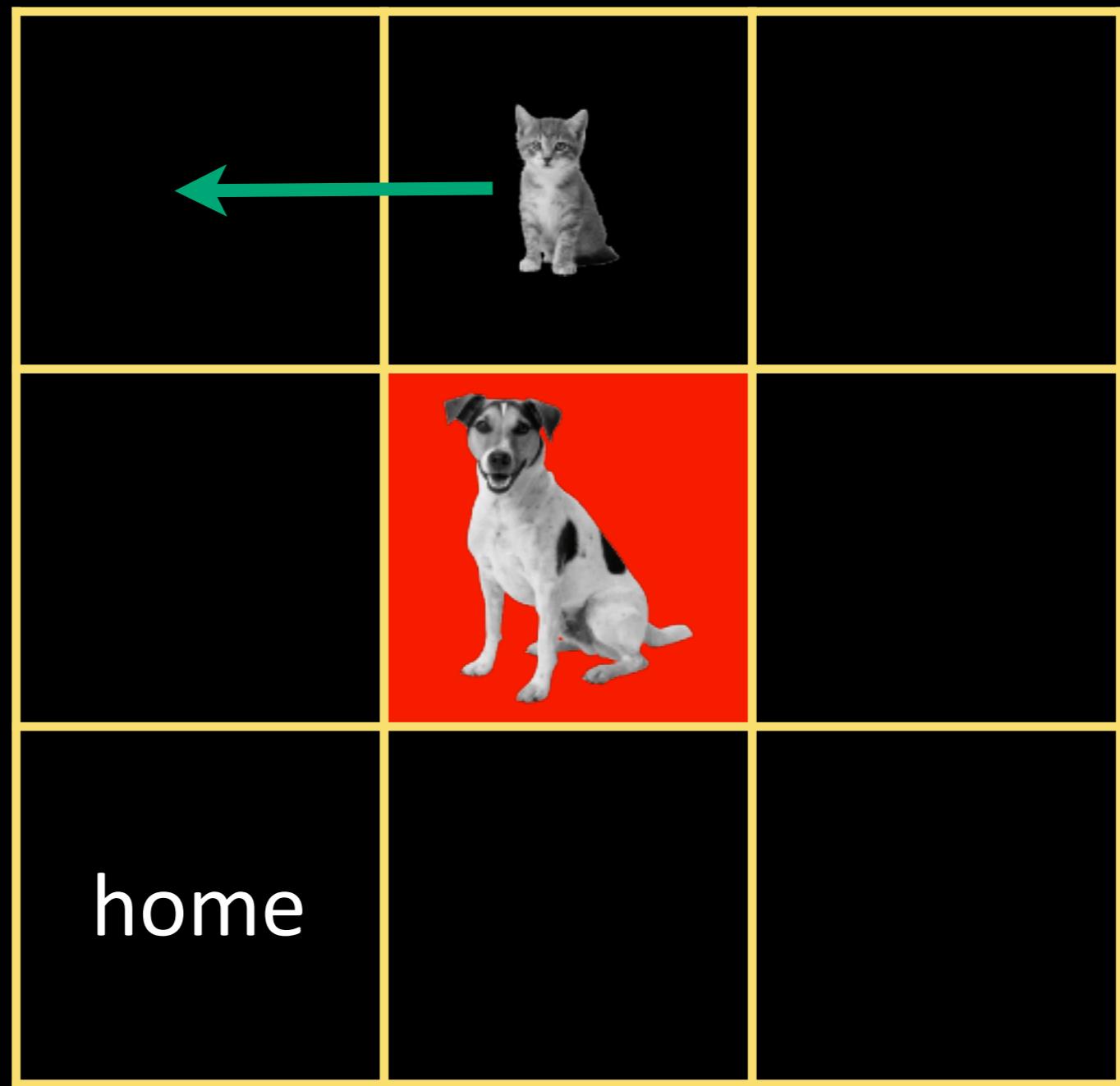


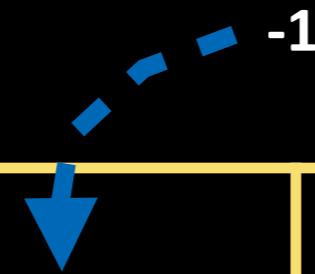
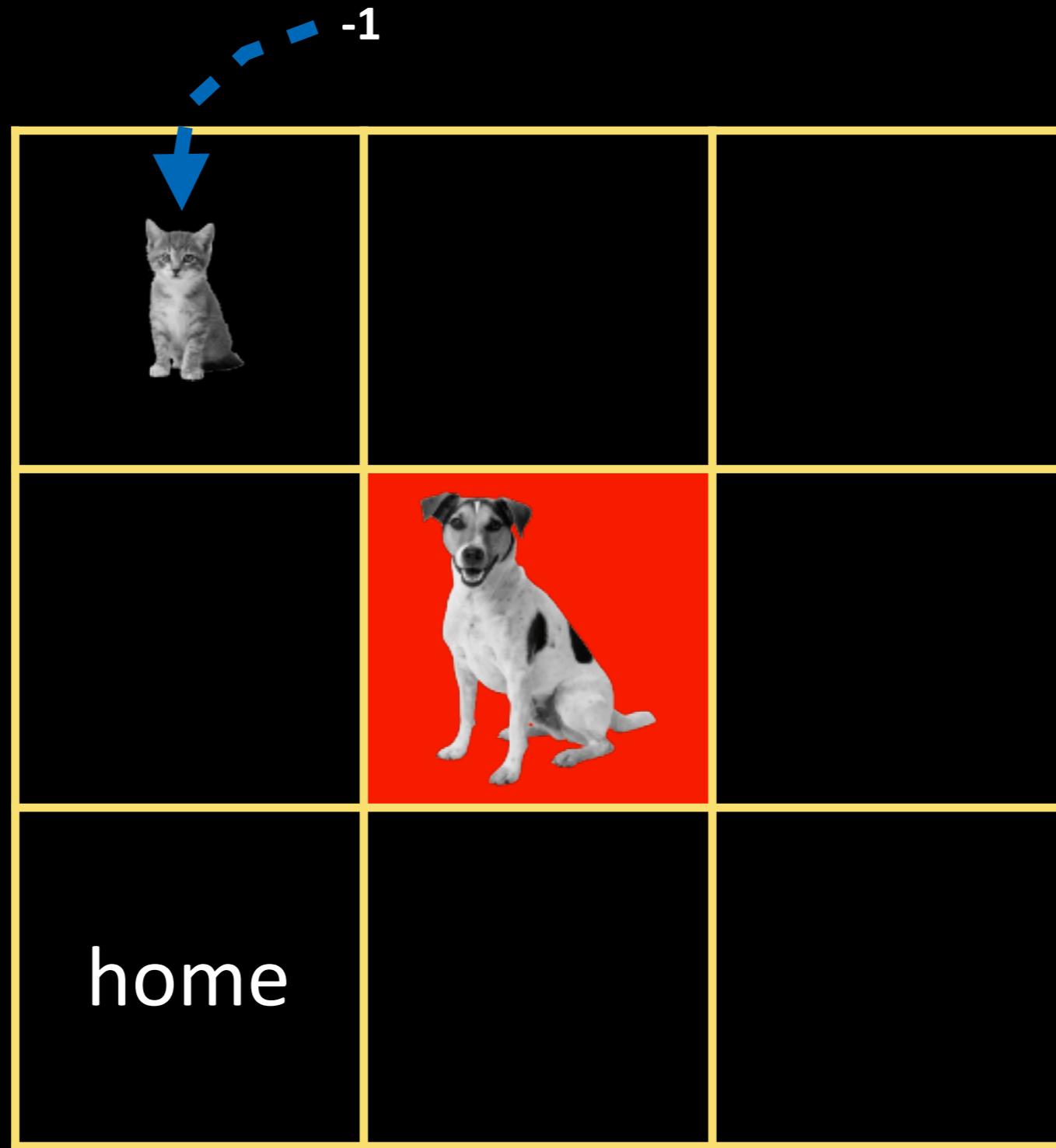
reward

taking an action in
some state results in an
immediate reward
(can be negative)

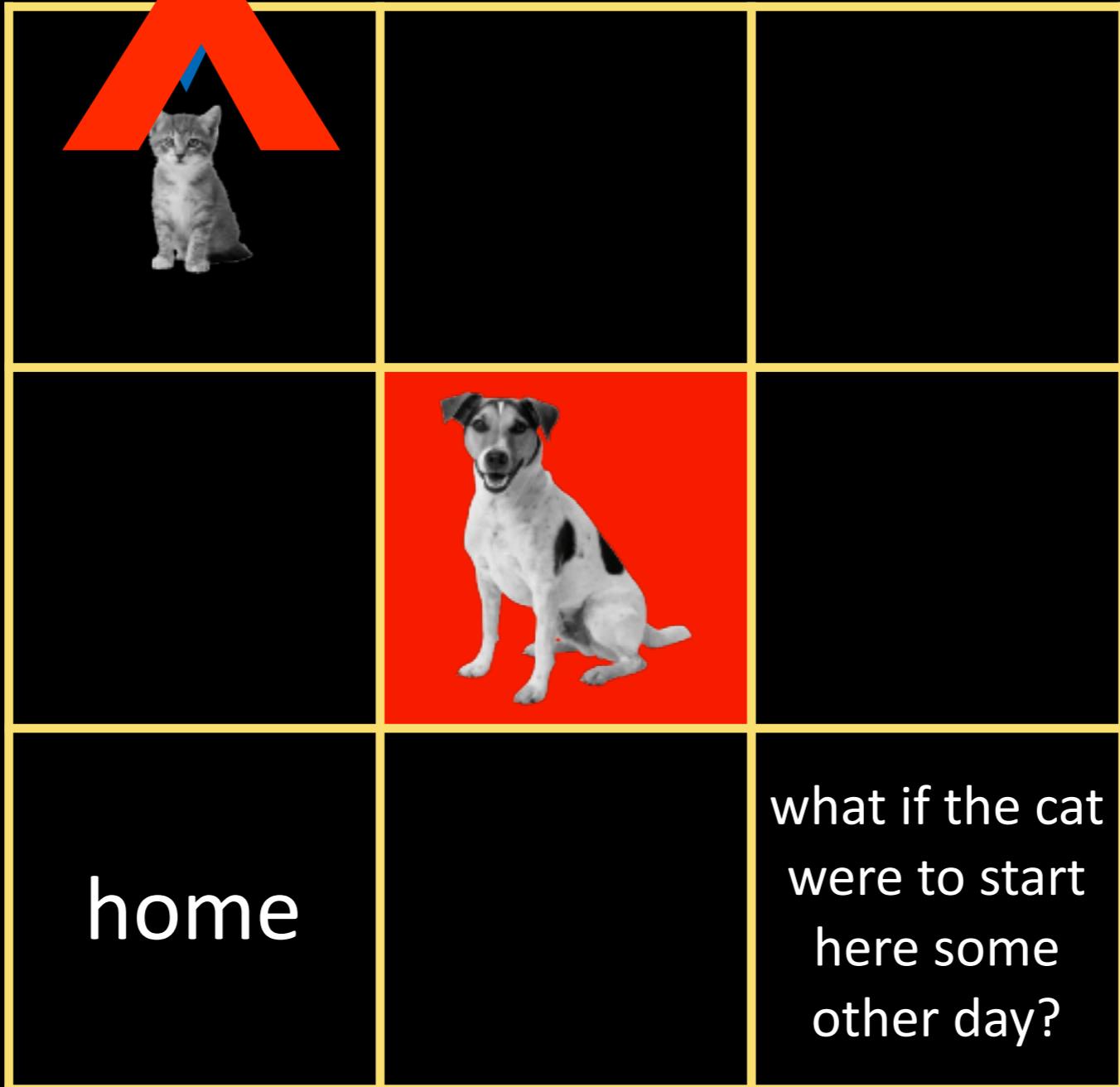








X $\rightarrow -1$



reward system should tell
the agent:

what to achieve
rather than how to achieve

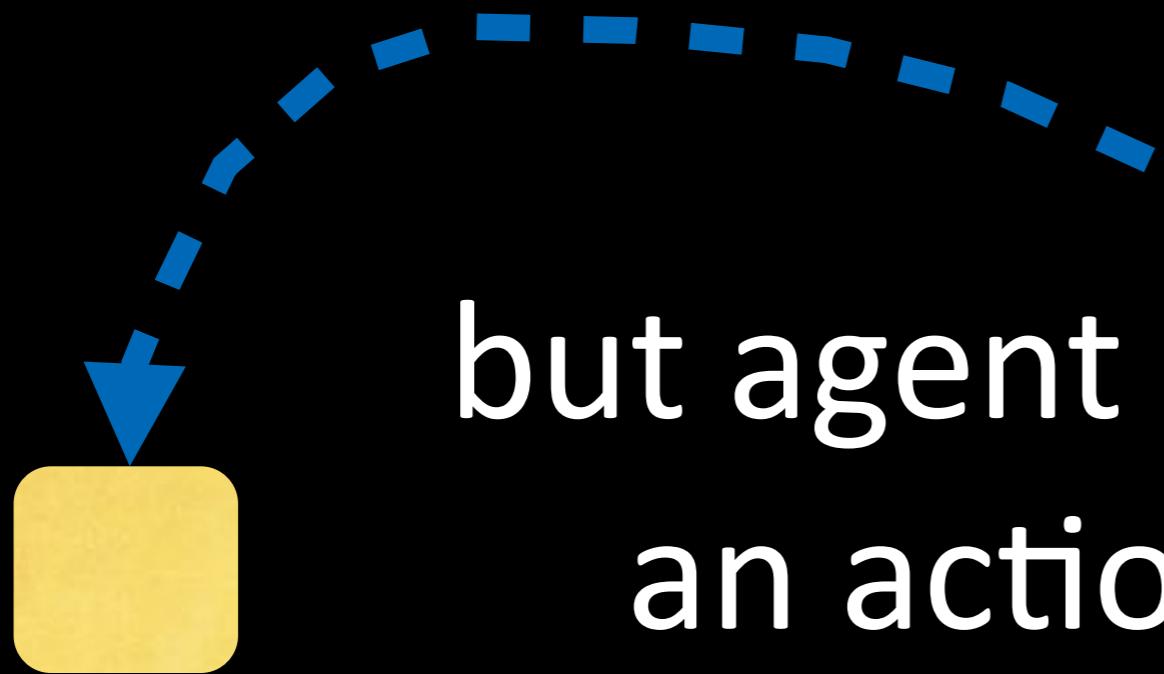
reward



this is all the
feedback an agent gets!

immediate!

reward

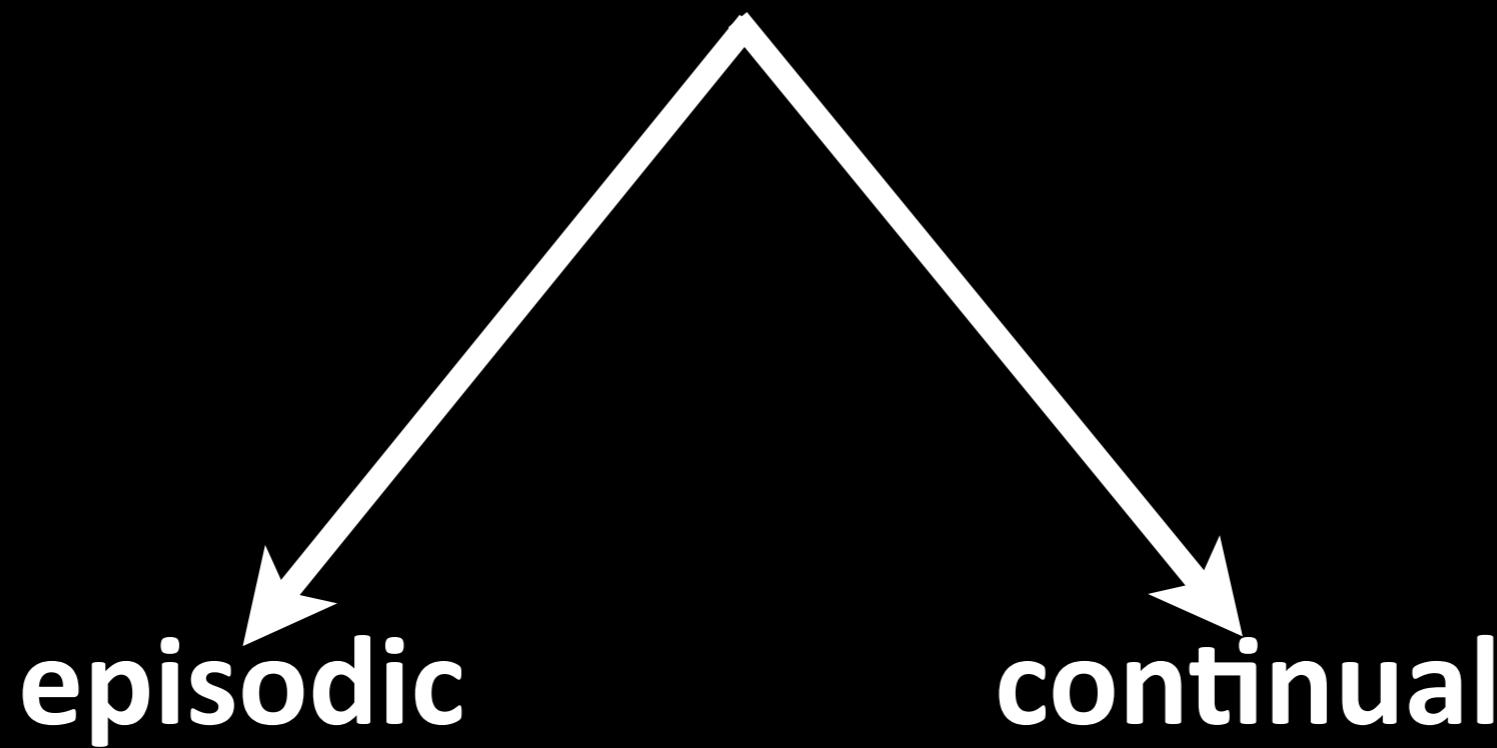


but agent has to choose
an action based on
expected return

expected return



task



episodic

(there is an **end**)

continual

(there is no **end**)

episodic

(there is an **end**)

agent taking **finite** (say 5) **steps** till the end...

should act based on the
e.g. average of the following

$$R_0 = r_1 + r_2 + r_3 + r_4 + r_5$$

continual

(there is no **end**)

agent can continue acting for **infinite steps in time...**

should **discount** future rewards and act based on
e.g. **average of the following**

$$R_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \gamma^4 r_5 + \dots$$

discount

future reward is probably more
uncertain than **immediate reward**

shortsighted?

$$\gamma = 0$$

$$0 \leq \gamma \leq 1$$

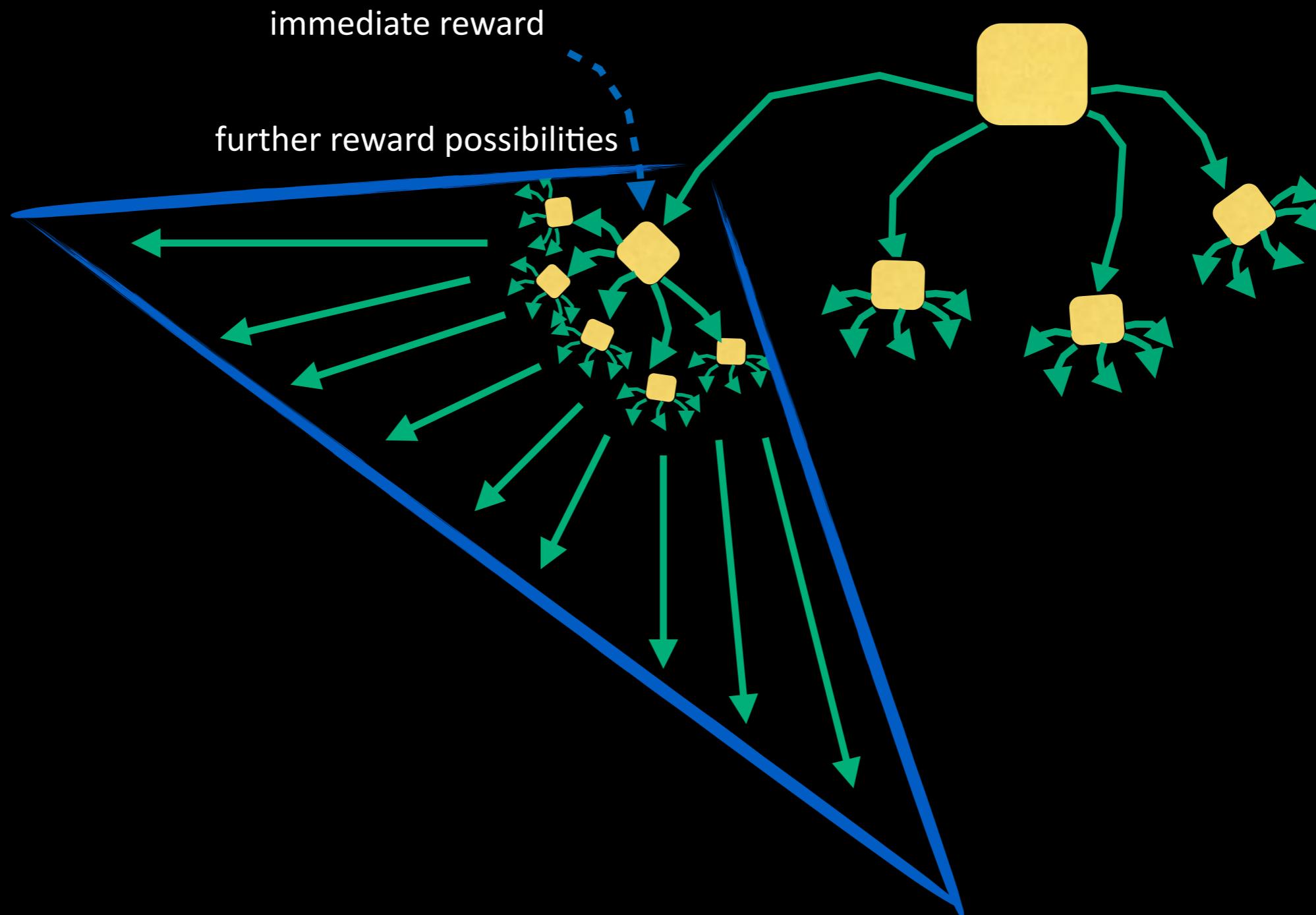
farsighted?

$$\gamma = 1$$

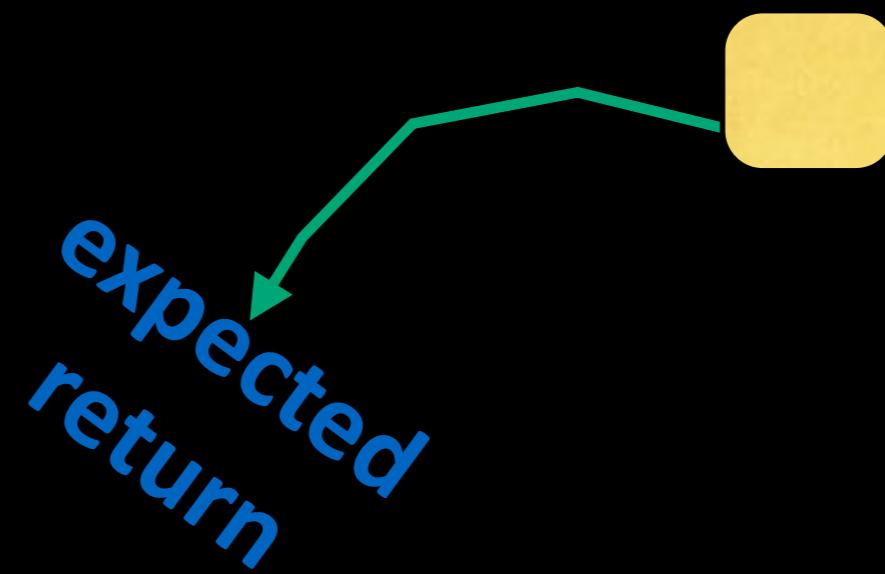
$$R_0 = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \gamma^4 r_5 + \dots$$

$$R_0 = \sum_{k=0}^T \gamma^k r_{k+1}$$

$$\mathbb{E}\left\{ R_t = \sum_{k=0}^T \gamma^k r_{t+k+1} \right\}$$



$$\mathbb{E}\left\{ R_t = \sum_{k=0}^T \gamma^k r_{t+k+1} \right\}$$

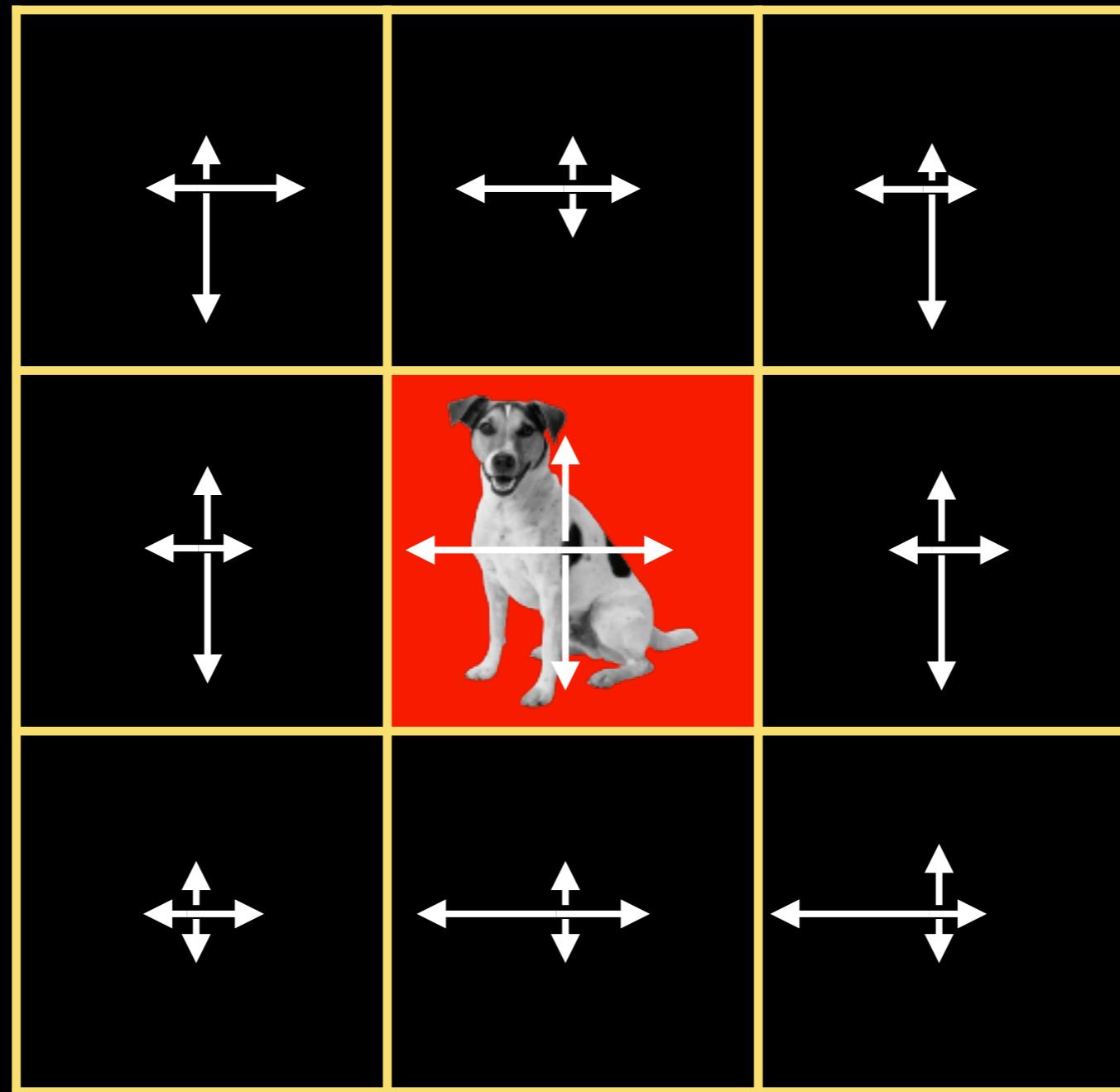


but these expected
returns are
not known to agent
beforehand!

what knowledge might
the agent try to acquire
to behave properly?



ranking/probability of an action in some state
bringing max expected return (long term value)?



expected long term value of
being in each state,
under some action selection scheme?



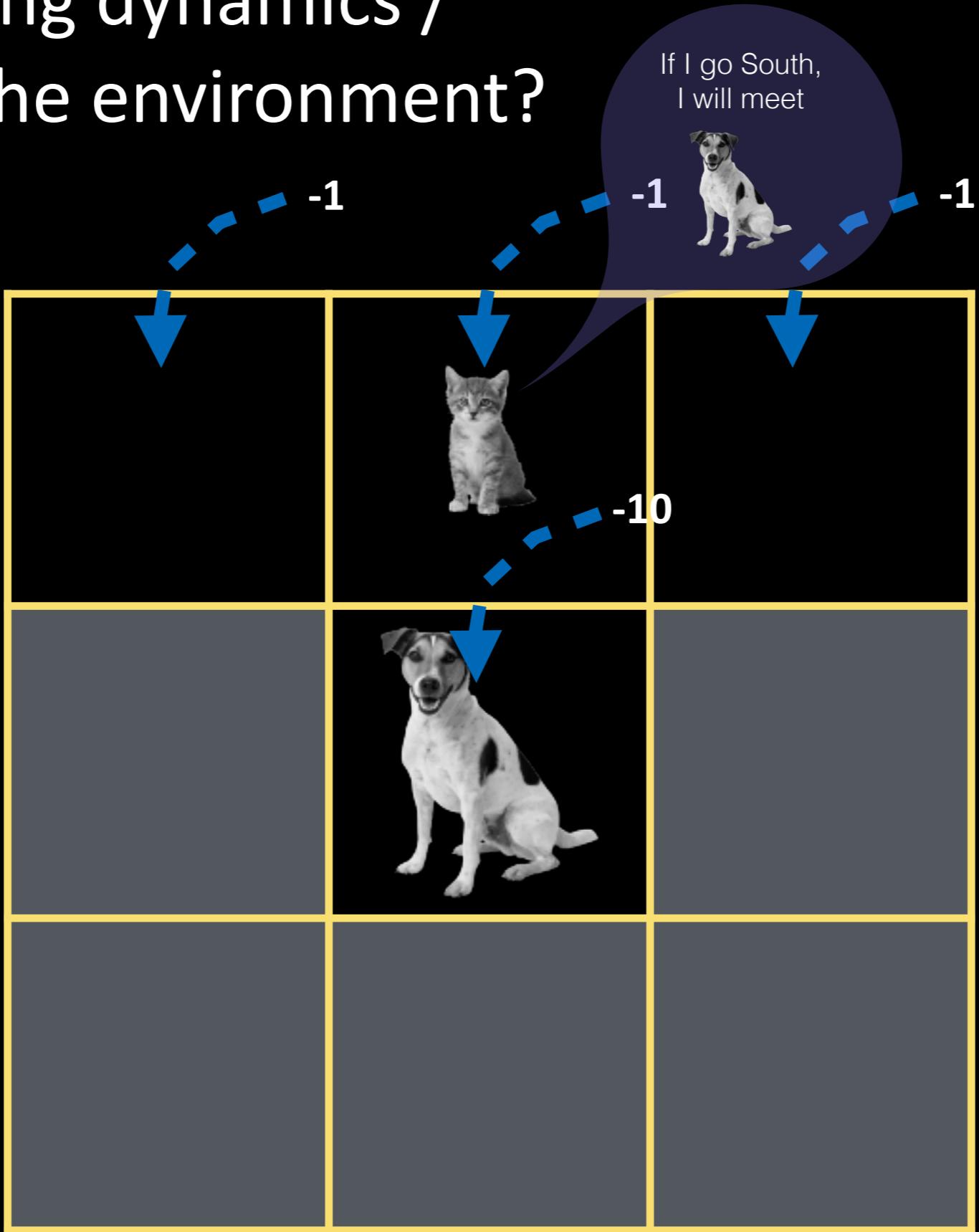
$E\{R\}$	$E\{R\}$	$E\{R\}$
$E\{R\}$	 $E\{R\}$	$E\{R\}$
$E\{R\}$ home	$E\{R\}$	$E\{R\}$

expected long term value of taking some action in each state, then behaving using some action selection scheme?

$E\{R\}$		$E\{R\}$		$E\{R\}$	
$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$
	$E\{R\}$		$E\{R\}$		$E\{R\}$
$E\{R\}$		$E\{R\}$	$E\{R\}$		$E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$
	$E\{R\}$		$E\{R\}$		$E\{R\}$
$E\{R\}$		$E\{R\}$	$E\{R\}$		$E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$
	$E\{R\}$		$E\{R\}$		$E\{R\}$
$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$	$E\{R\}$



modelling dynamics / mapping the environment?



prediction problem

learn to predict expected long
term reward/value

control problem

learn to find the optimal action
selection scheme/policy



policy: action selection

value: how good is an action/state

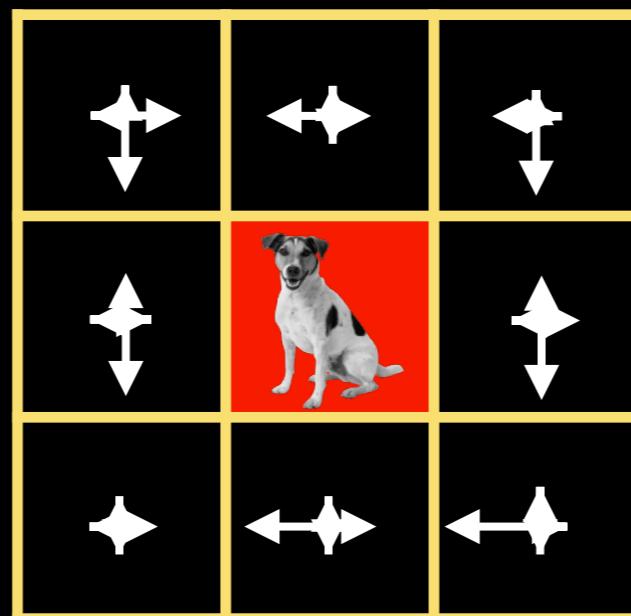
model: predict next state/reward
to look ahead/plan

types of RL agents?

value
based

E{R}	E{R}	E{R}	E{R}	E{R}	E{R}
E{R}		E{R}	E{R}		E{R}
E{R}		E{R}	E{R}	E{R}	
E{R}		E{R}	E{R}	E{R}	E{R}
E{R}		E{R}	E{R}	E{R}	E{R}
E{R}	h	E{R}	E{R}	E{R}	E{R}
E{R}		E{R}		E{R}	

policy
based



both value and policy

model
based

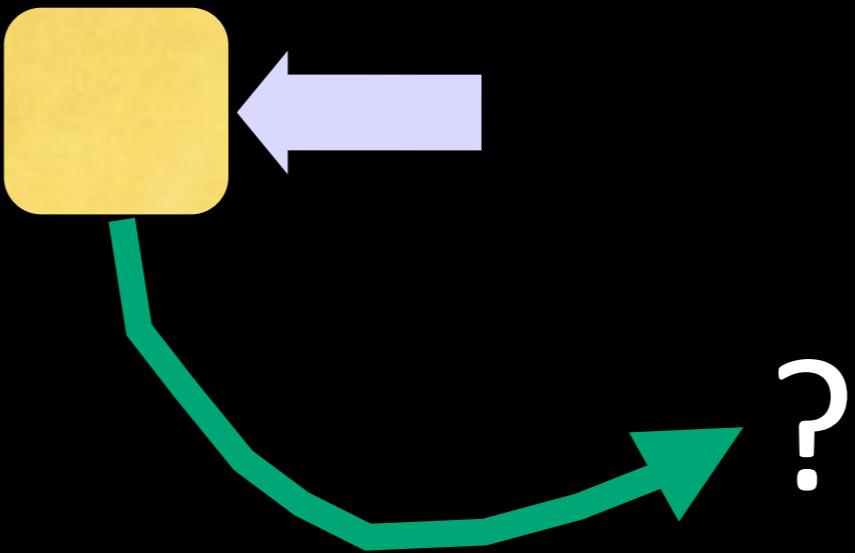
value/policy

+

model of
dynamics

we will focus on value
based RL in the first half

action selection?



expected return for carrying out an
action is its **value**

values of each possible action in the
current state helps select actions!

policy can be
derived from value
(e.g. act greedily)

but what are
these values?

<<expected returns **unknown**>>
<<actions based on **unknowns**>>

value can be **estimated** by
sampling environment
while acting using some policy

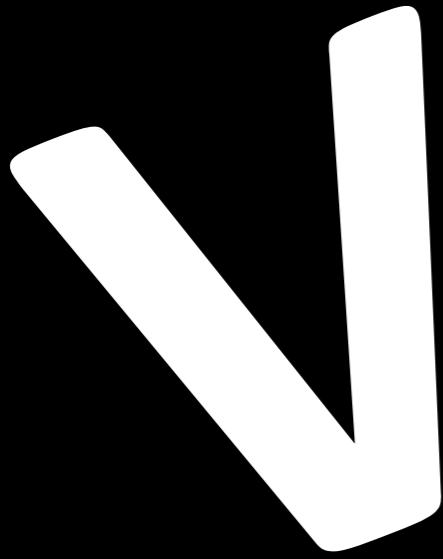
e.g. act, accumulate new
reward (ground truth), and update



	a	b	c
1	2	0	1
2	3	0	-1
3	-5	6	2
4	2	3	1
.	.	.	.
.	.	.	.
.	.	.	.
n	7	8	7

agent maintains **values**
for actions within each state

selects actions using these values under some
“policy”

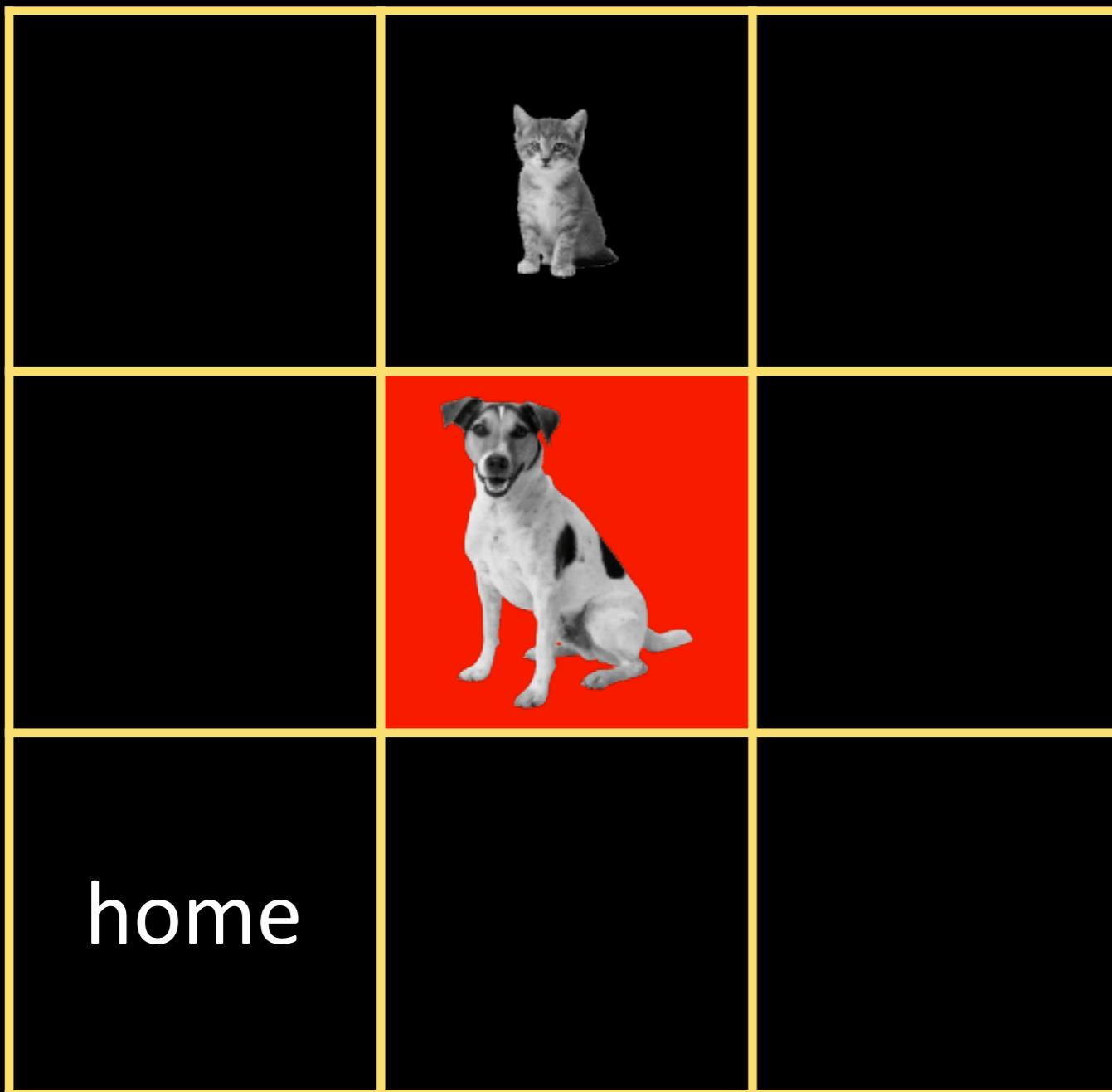


1	2
2	3
3	-5
4	2
.	.
.	.
.	.
n	7

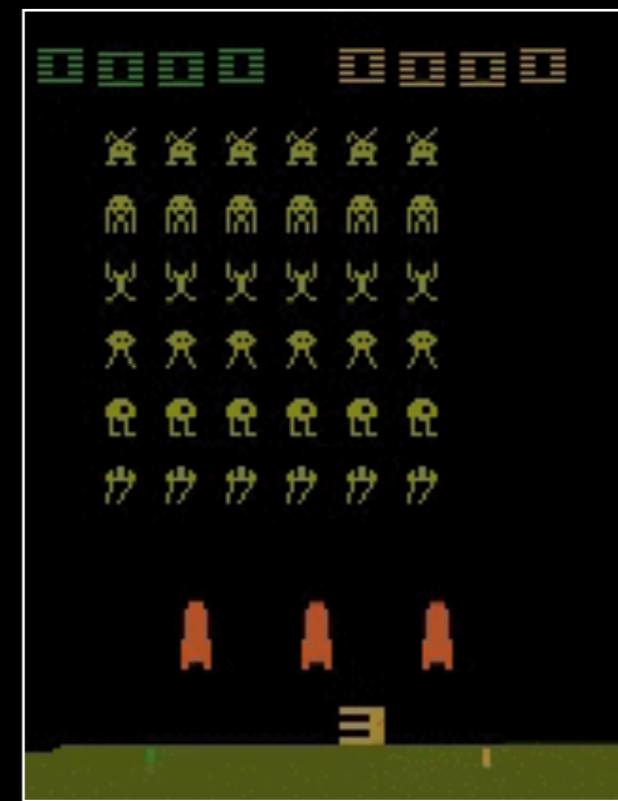
agent maintains state values

selects actions using these values under some
“policy”

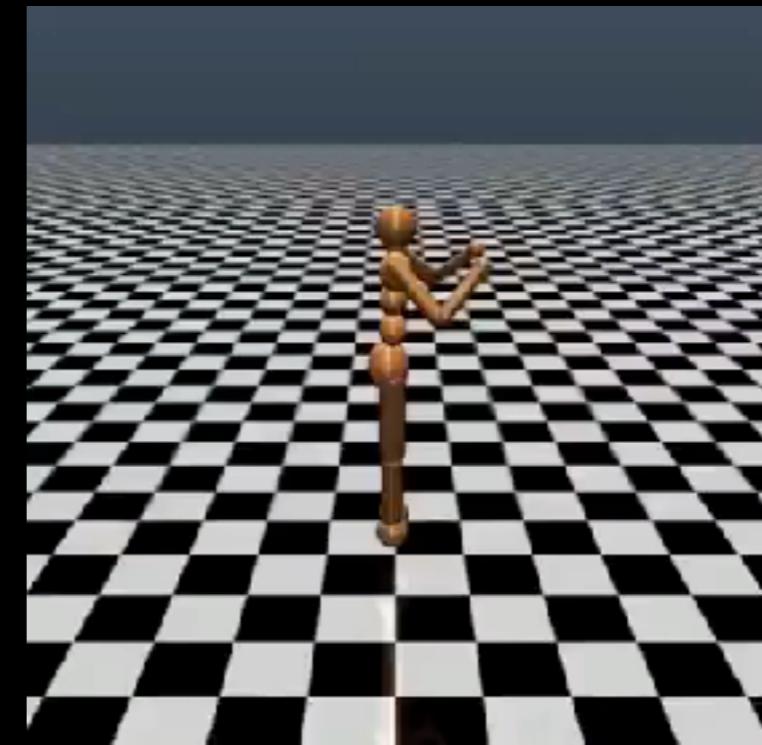
but... agent needs a model of the environment!



9 states

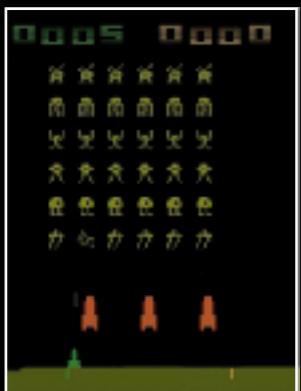


10^{16992} (pixels)
 10^{308} (ram)

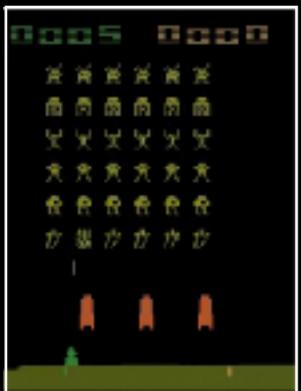


continuous!

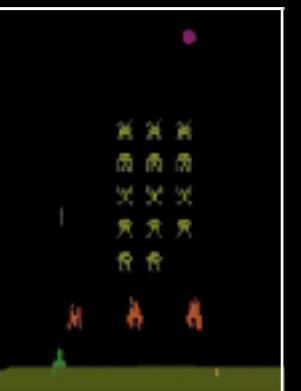
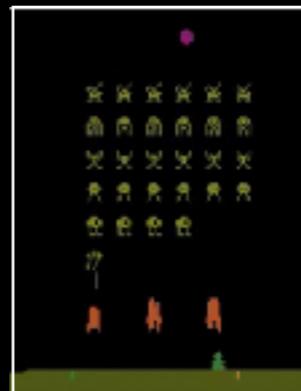
extract features that help generalise across states



⋮

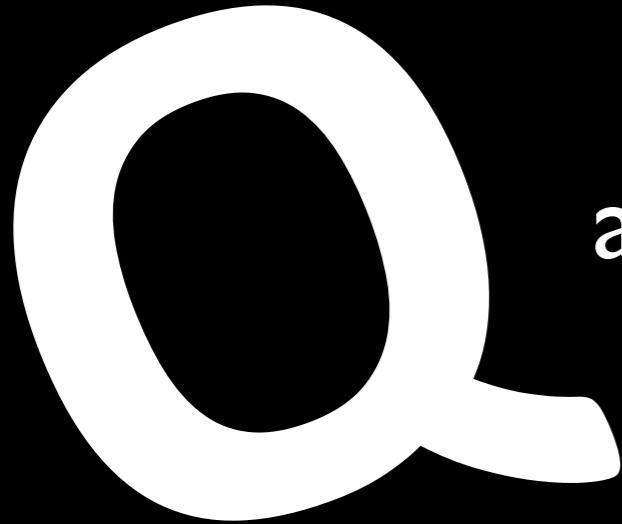


⋮

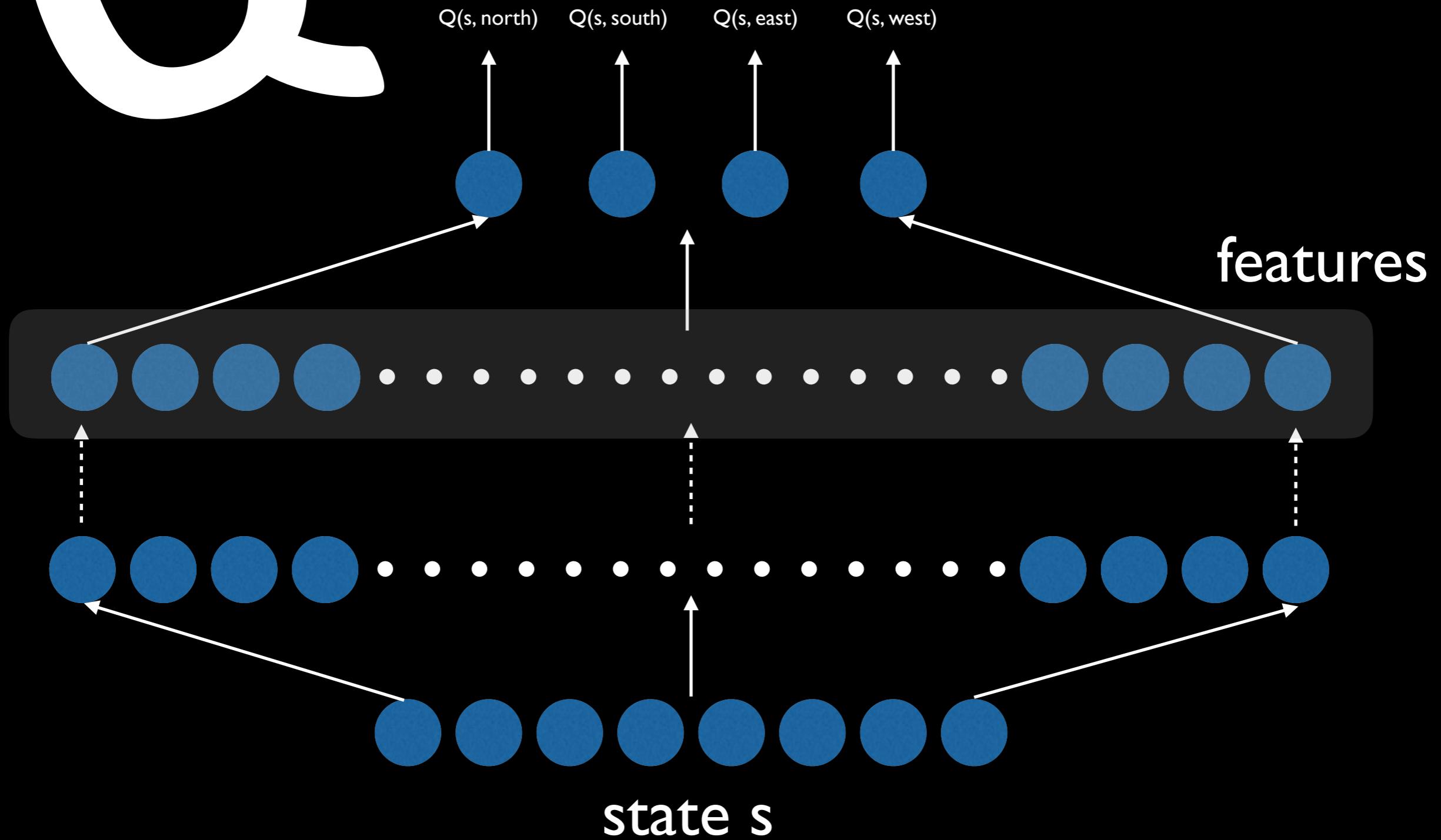


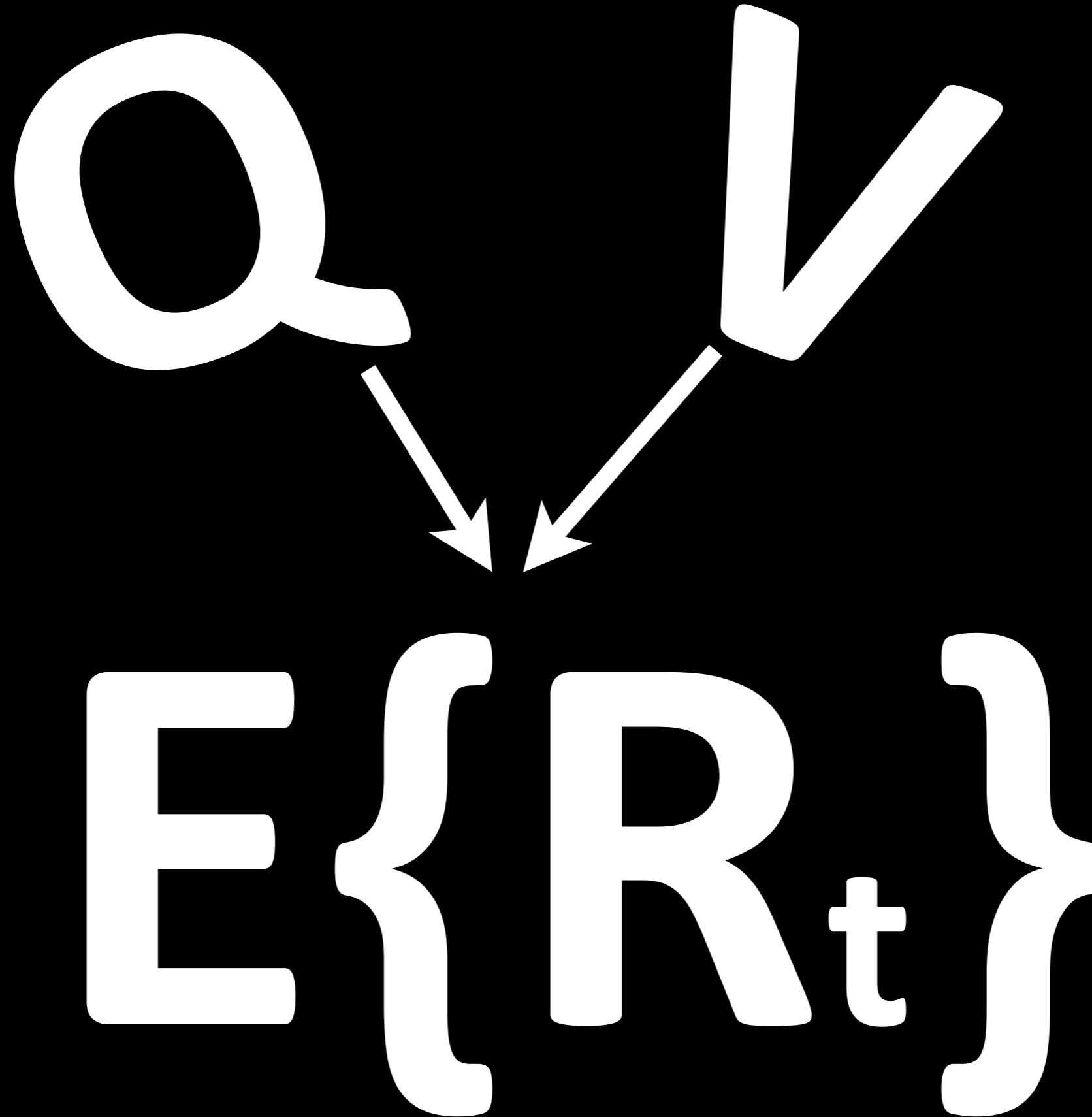
⋮





action values given state



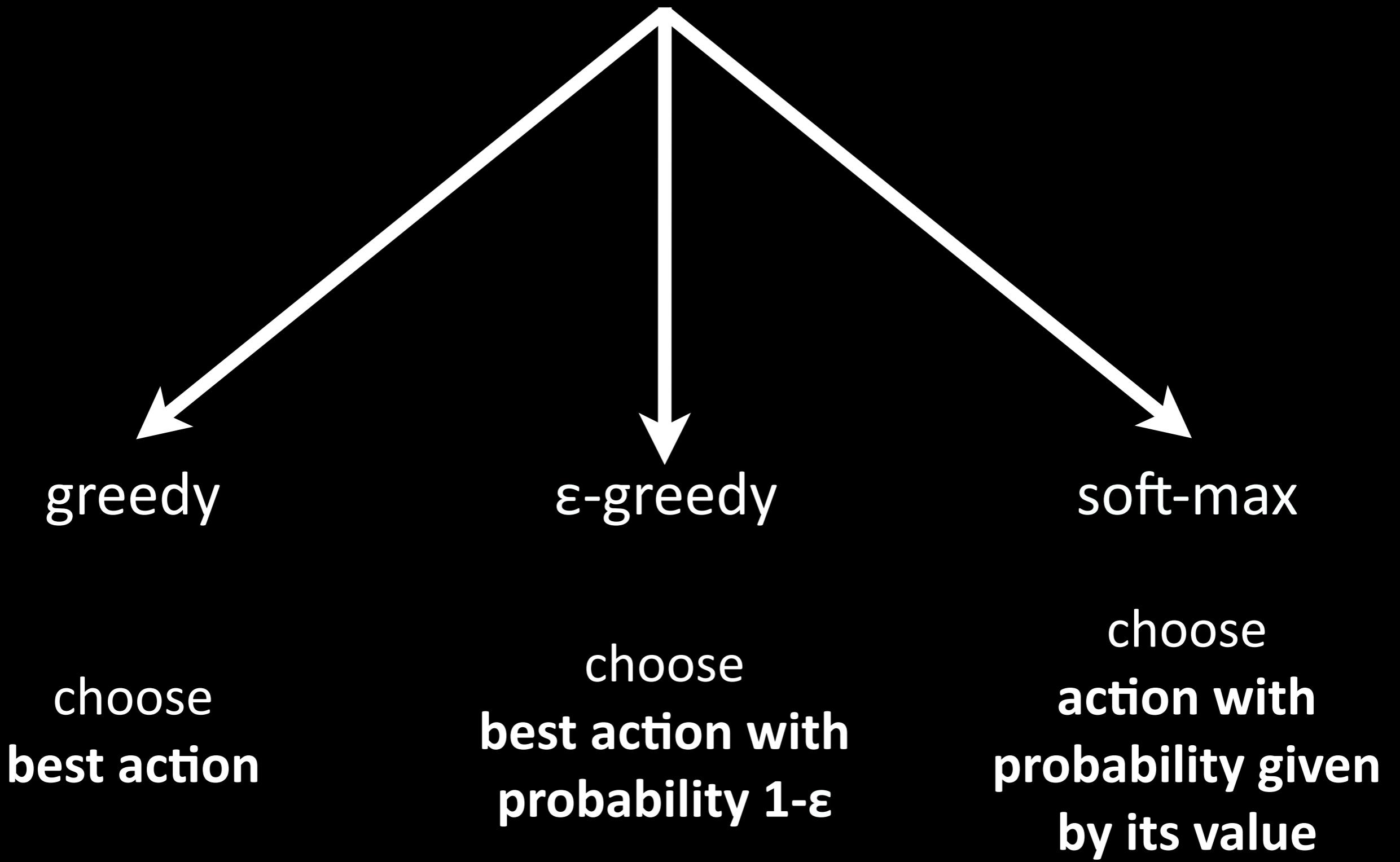


policy?

probability of choosing
an action in state/feature
representation thereof

$$\pi \quad Q^\pi(s,a)$$
$$V^\pi(s)$$

usual policies



exploration vs. exploitation trial and error

game play: try new moves

ads: try new ads

a/b testing:

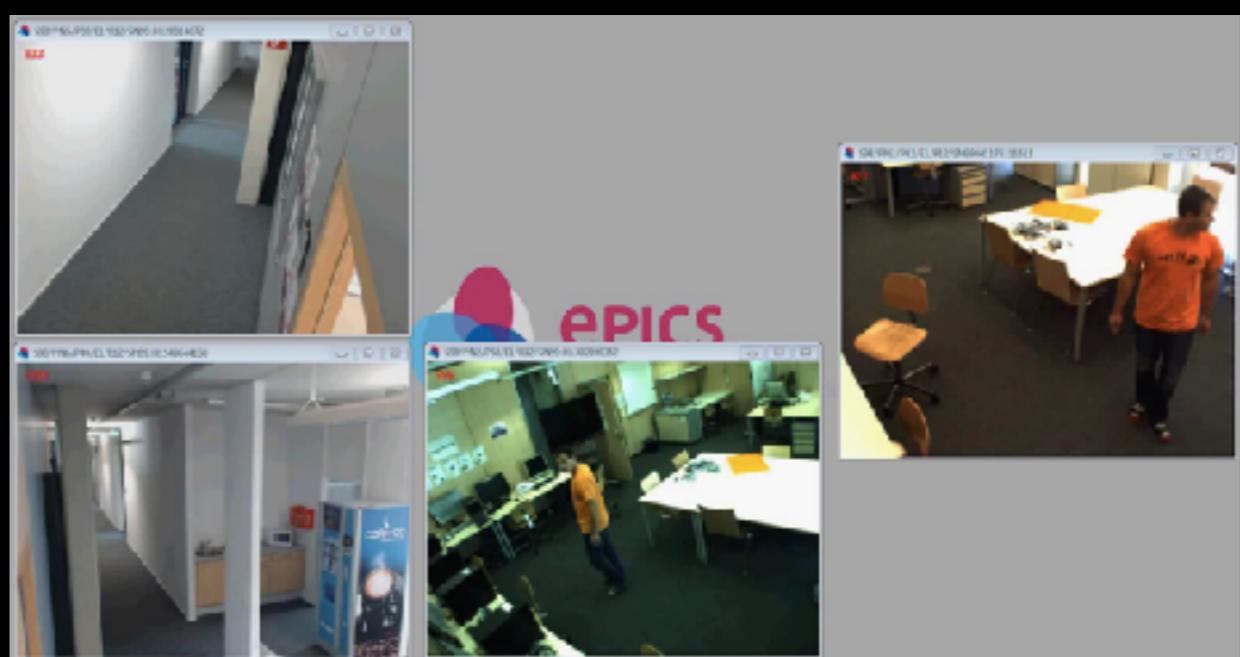
try new website feature

smart camera networks:
try new comm. protocol

Static, dynamic and adaptive heterogeneity in socio-economic distributed smart camera networks, P. R. Lewis, L. Esterle, A. Chandra, B. Rinner, J. Torresen, and X. Yao, ACM Transactions on Autonomous and Adaptive Systems (TAAS), ACM, 2015.



Yamaguchi先生, http://en.wikipedia.org/wiki/File:Las_Vegas_slot_machines.jpg



V*

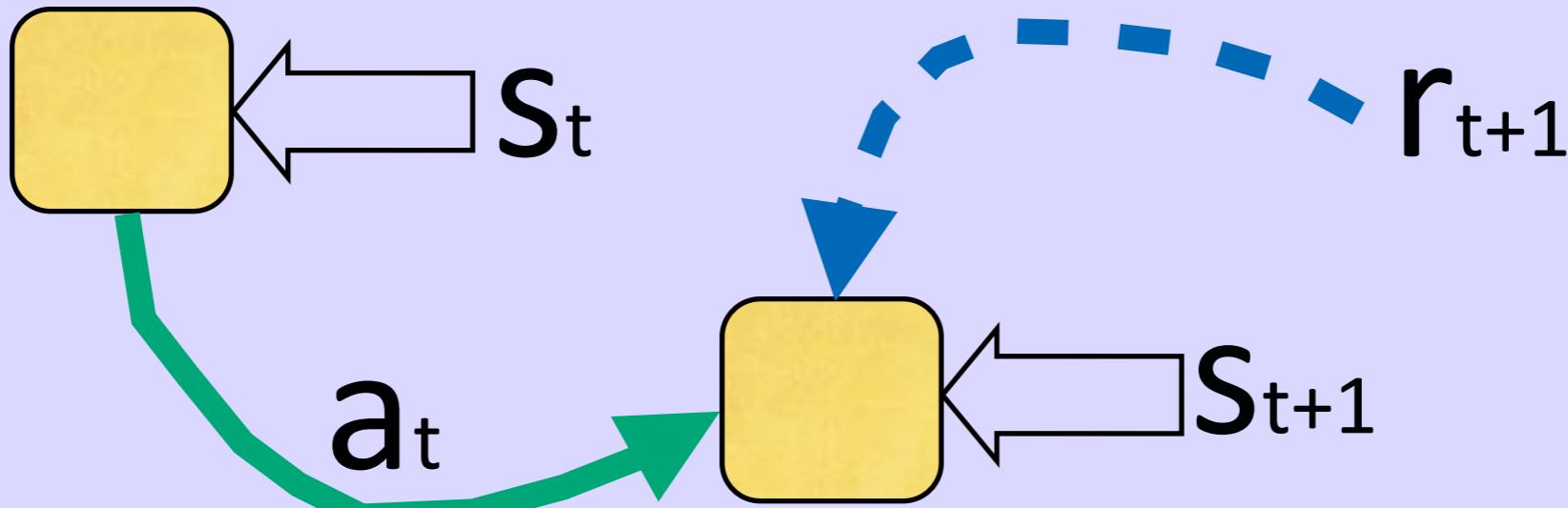
$$V^*(s) = \max_{\pi} V^\pi(s)$$

Q*

$$Q^*(s,a) = \max_{\pi} Q^\pi(s,a)$$

π*

$$\pi^*(a|s) = \begin{cases} 1 & \text{if } a = \arg \max_a Q^*(s,a) \\ 0 & \text{otherwise} \end{cases}$$



estimation?

<<use currently visible returns to update values of
where you are coming from>>

the current state (or state-action pair) has an
estimated value (say zero/random initially),

which can be used **together with r_{t+1}** to update
value of previous state (or state-action pair)

i.e.

fraction of (currently visible returns - old value)

+

old value



new value

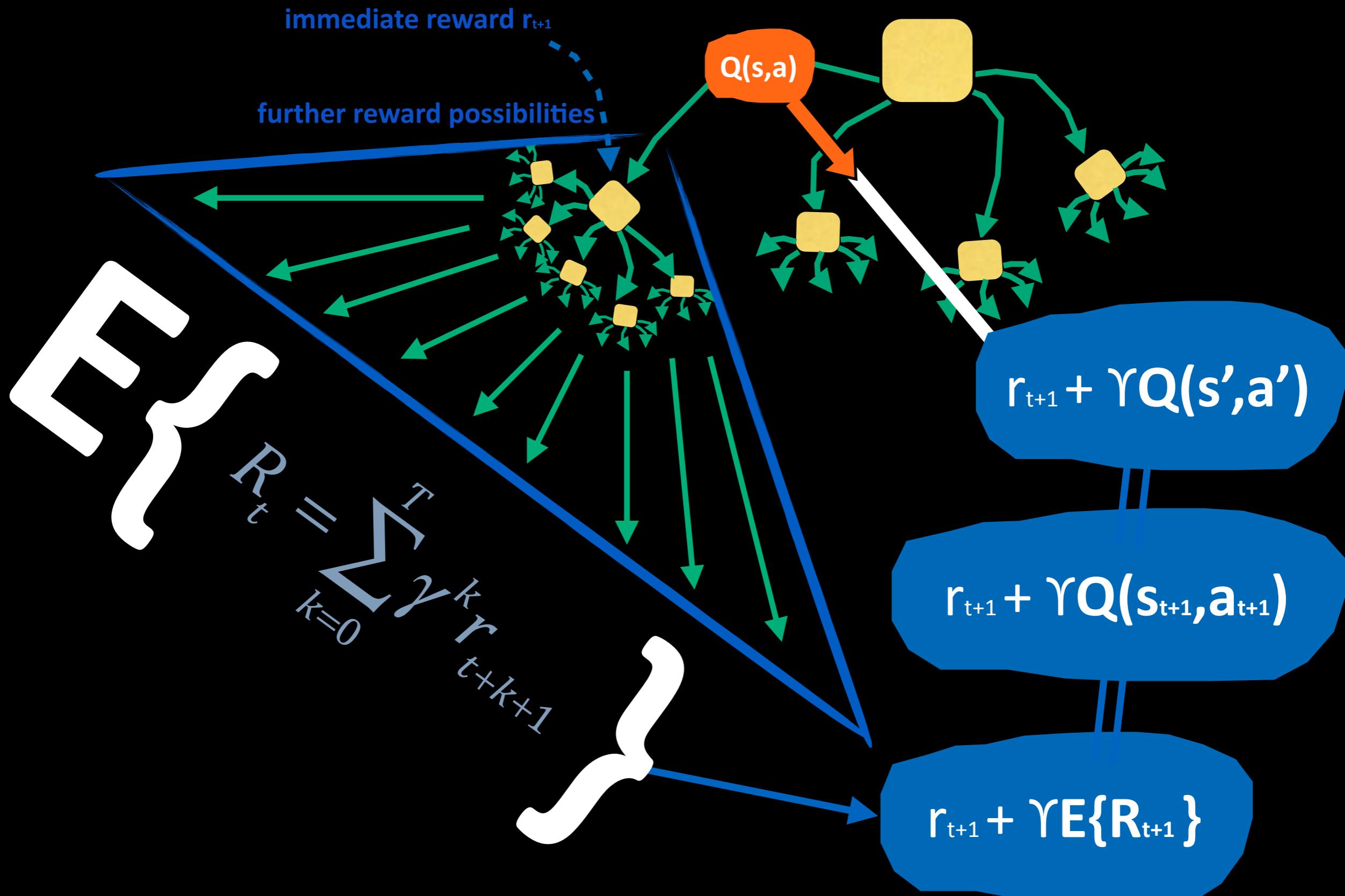
(1-fraction)

old value

+ fraction

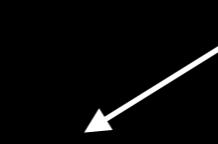
curr. vis. returns





$$V(s) \leftarrow V(s) + \alpha(r_s^a + \gamma V(s') - V(s))$$

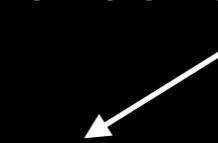
under some policy $\pi(a|s)$



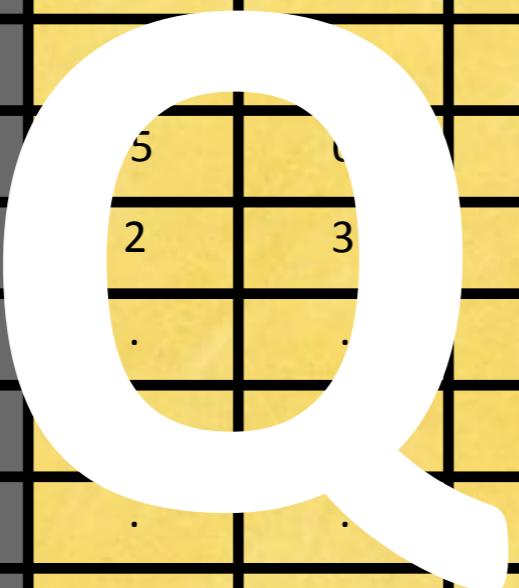
e.g.

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma Q(s',a') - Q(s,a))$$

under some policy $\pi(a|s)$



e.g. update
a lookup table maintaining
expected returns



	a	b	c
1	2	0	1
2			-1
3	5	3	2
4	2	3	1
.	.	.	.
.	.	.	.
n	7	8	7



1	2
2	3
3	5
4	2
.	.
.	.
n	7

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma Q(s',a') - Q(s,a))$$

let's play with a version of the
above update rule:

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

indicates a' to be the action
with maximum value in next
state s'

let's play with a version of the
above update rule:

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

our toy problem

lookup table

		N	S	E	W
1	0	0	0	0	
2	0	0	0	0	
3	0	0	0	0	
4	0	0	0	0	
5	0	0	0	0	
6	0	0	0	0	
7	0	0	0	0	
8	0	0	0	0	
9	0	0	0	0	

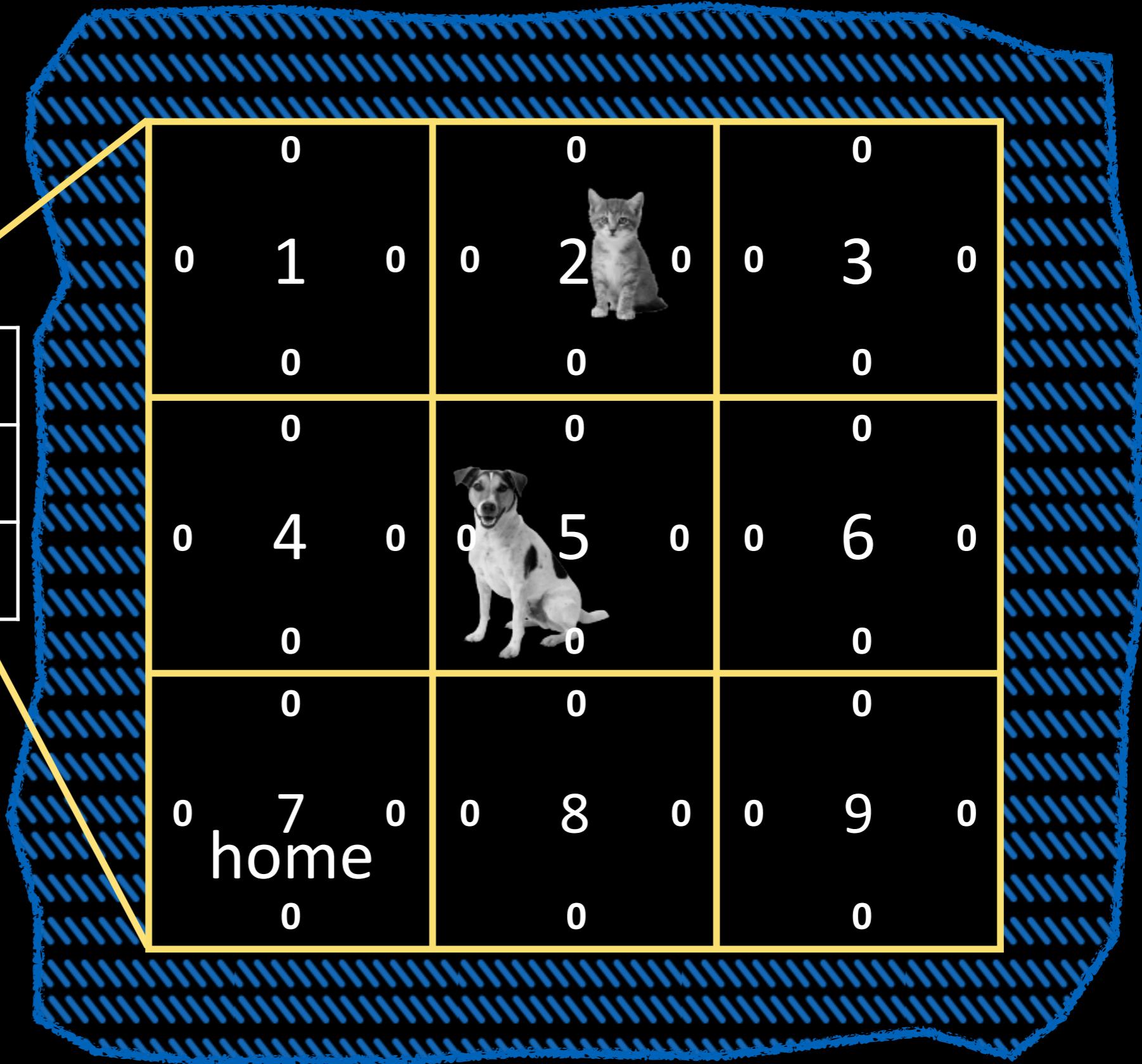
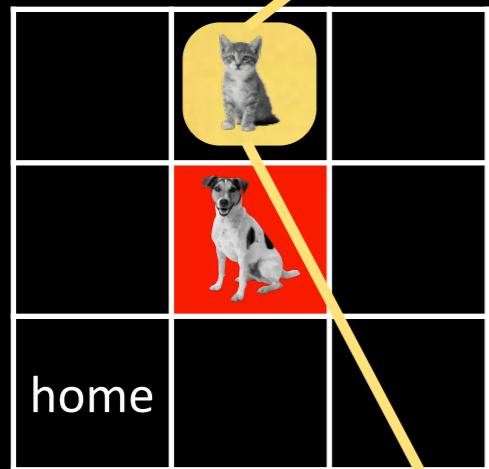
A diagram illustrating a toy problem lookup table. On the left, a 3x3 grid represents a state space. The bottom-left cell is labeled "home". The middle-right cell contains a red dog icon. The top-middle cell contains a yellow cat icon, which is highlighted by a yellow oval. A yellow diagonal line connects the "home" cell to the "N" column of the lookup table on the right. The lookup table has columns labeled N, S, E, and W, and rows labeled 1 through 9. All entries in the table are currently 0.

our toy problem

lookup table

		0	0	0
	0	1	0	0
	0	0	2	0
	0	0	0	3
	0	0	0	0
home	0	4	0	0
	0	0	5	0
	0	0	0	6
	0	0	0	0
home	0	7	0	0
	0	0	8	0
	0	0	0	9
	0	0	0	0

reward
structure?



move...

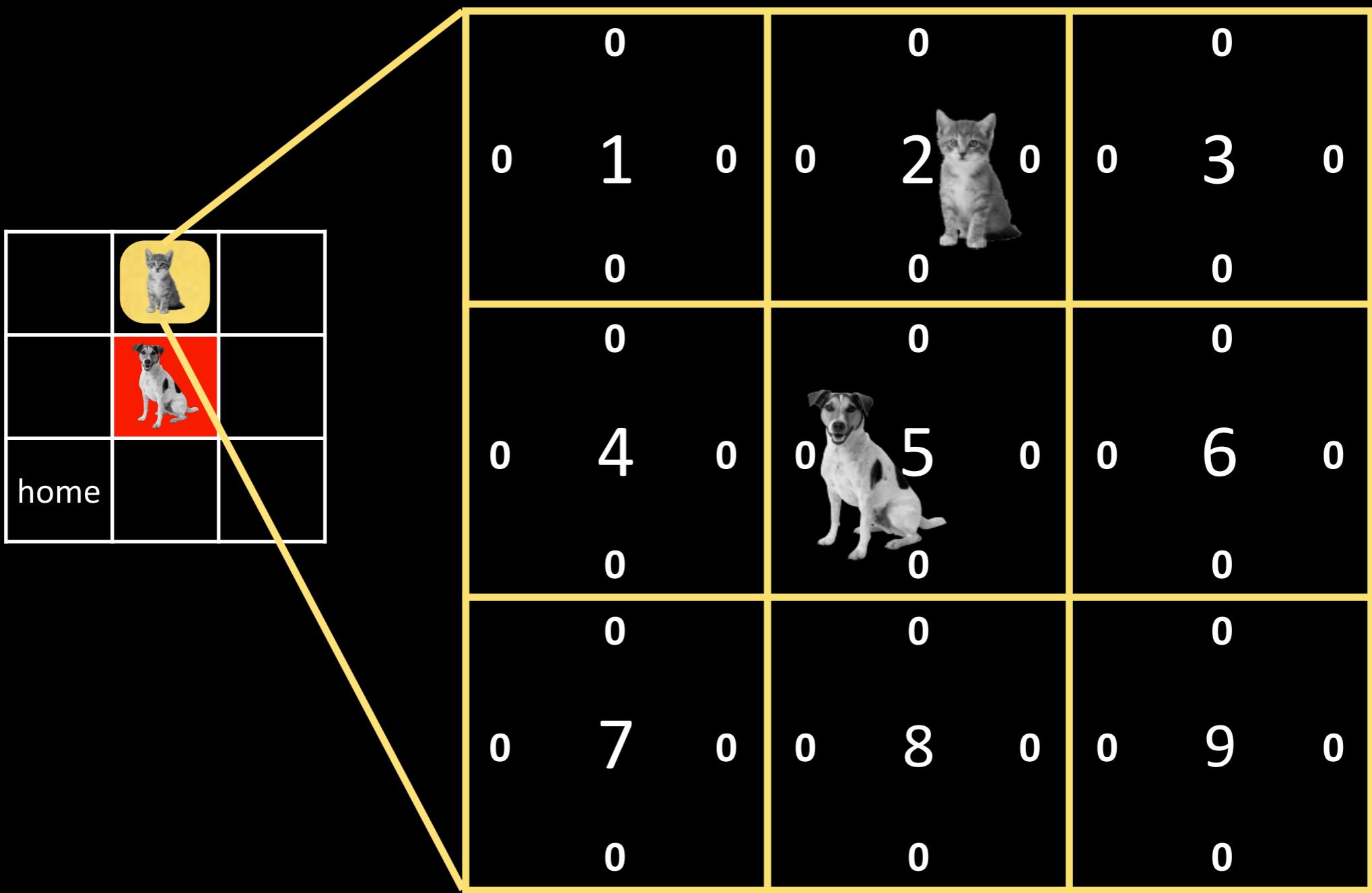
to any cell except 5 and 7:
-1

out of bounds:
-5

to 5:
-10

to 7/home:
10

let's fix $\alpha = 0.1, \gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0	0	0	0	0	0	0	0	0
0	1	0	0	2	0	0	3	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$
$$\gamma = 0.5$$

say ϵ -greedy policy...
episode 1 begins...

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

		-1
0 0 0	1 ?	0 2 0 0 3 0
0 0 0	4 0 0	0 5 0 0 6 0
0 0 0	7 home 0	0 8 0 0 9 0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
0		0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

0	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

		-5	
	?		
0	1	0	-0.1 2 0 0 3 0
0			0 0 0 0 0 0
0	4 0		0 5 0 0 6 0
0			0 0 0 0 0 0
0	7 0		0 8 0 0 9 0
0			0 0 0 0 0 0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0
0	1	0
0		0
0	-0.1	2
0	0	0
0	0	3
0	0	0
0	0	0
0	0	0
0	4	0
0	0	5
0	0	0
0	0	6
0	0	0
0	0	0
0	7	0
home	0	0
0	0	0
0	0	0
0	0	0
0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3
0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9
home	0	0	0	0	0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
?	0	-1	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0		0		0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	0	0	5	0	0	6	0
0		0		0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	-10	0	0	0
0	4	?	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	?	0	-1	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	0	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$

$$\gamma = 0.5$$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	-0.1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	?	8	0	0	9	0
home	0	0	10	0	0	0	0	0

$\alpha = 0.1$

$\gamma = 0.5$



$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_s^a + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0	0	0	0	-0.1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	7	0	1	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$$\alpha = 0.1$$
$$\gamma = 0.5$$

episode 1 ends.

let's work out the next
episode, starting at state 4

-0.5	0	0	0	0	0	0	0	0
0	1	0	-0.1	2	0	0	3	0
-0.1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	4	-1	0	5	0	0	6	0
0				-0.1				
0	0	0	0	0	0	0	0	0
0	7	0	1	8	0	0	9	0
home	0	0	0	0	0	0	0	0

$\alpha = 0.1$
 $\gamma = 0.5$

go WEST and then SOUTH

how does the table change?

-0.5		0	0					
0	1	0	-0.1	2	0	0	3	0
	-0.1			0			0	
	0			0			0	
-0.5	4	-1	0	5	0	0	6	0
	1			-0.1			0	
	0			0			0	
0	7	0	1	8	0	0	9	0
	0			0			0	

$\alpha = 0.1$

$\gamma = 0.5$



and the next episode,
starting at state 3

go WEST -> SOUTH -> WEST -> SOUTH

-0.5		0		0					
0	1	0	-0.1	2	0	-0.1	3	0	
	-0.1			-1			0		
	0			0			0		
-0.5	4	-1	-0.05	5	0	0	6	0	
	1.9			-0.1			0		
	0			0			0		
0	7	0	1	8	0	0	9	0	
	0			0			0		

$\alpha = 0.1$

$\gamma = 0.5$



over time, values will converge to optimal!

what we just saw was
some episodes of
Q-learning

values update towards value of **optimal policy**:
target comes from value of
assumed next best action

off-policy learning

SARSA-learning?

values update towards value of **current policy**:
target comes from value of
the actual next action

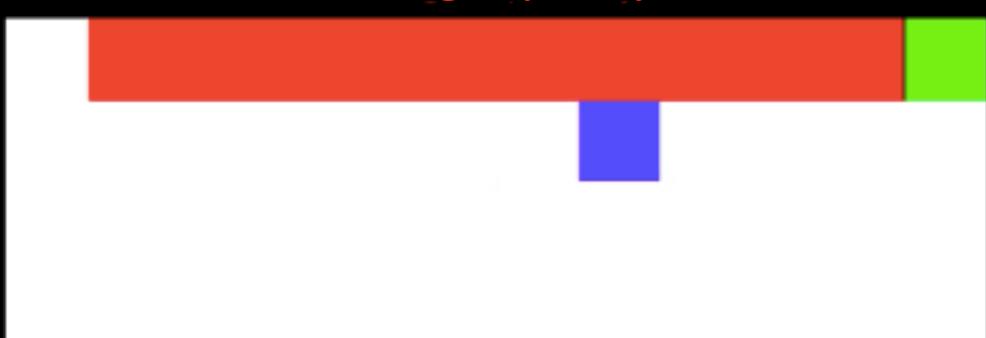
on-policy learning



By Andreas Tille (Own work) [GFDL (www.gnu.org/copyleft/fdl.html) or CC-BY-SA-3.0-2.5-2.0-1.0 ([www.creativecommons.org/licenses/by-sa/3.0](http://creativecommons.org/licenses/by-sa/3.0))], via Wikimedia Commons

data **not generated** by

target policy



Q

$\varepsilon: 0.1$

$\gamma: 1.0$

data **generated** by

target policy



SARSA

Example credit **Travis DeWolf**: <https://studywolf.wordpress.com/> and <https://git.io/vFBvv>

Problem Decomposition

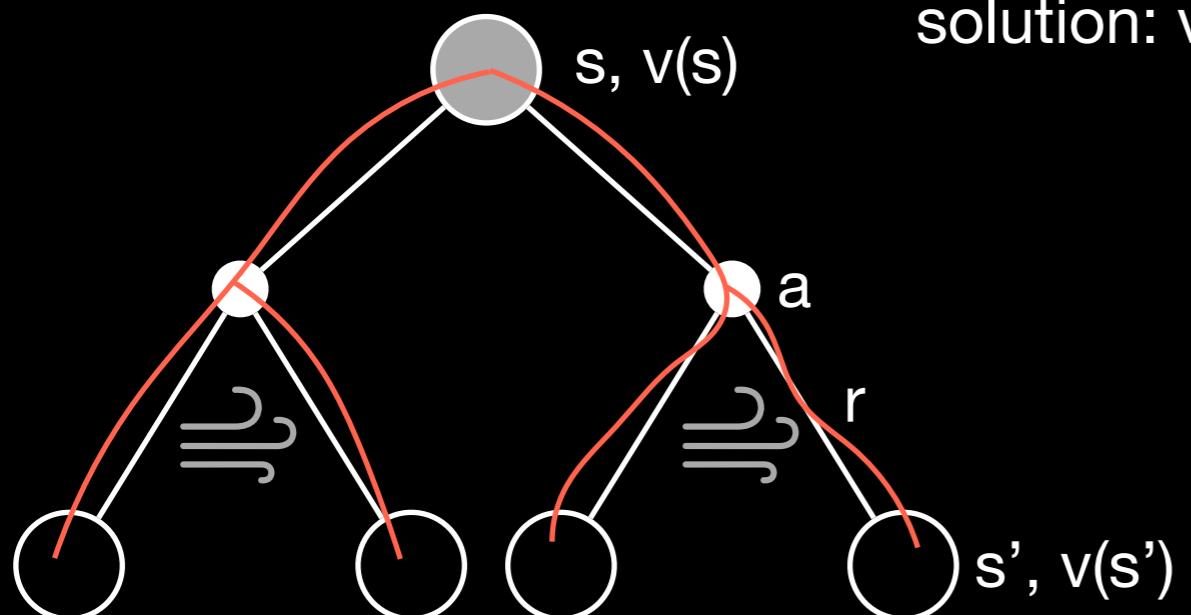
nested
sub-problems

solution to sub-problem
informs
solution to whole problem

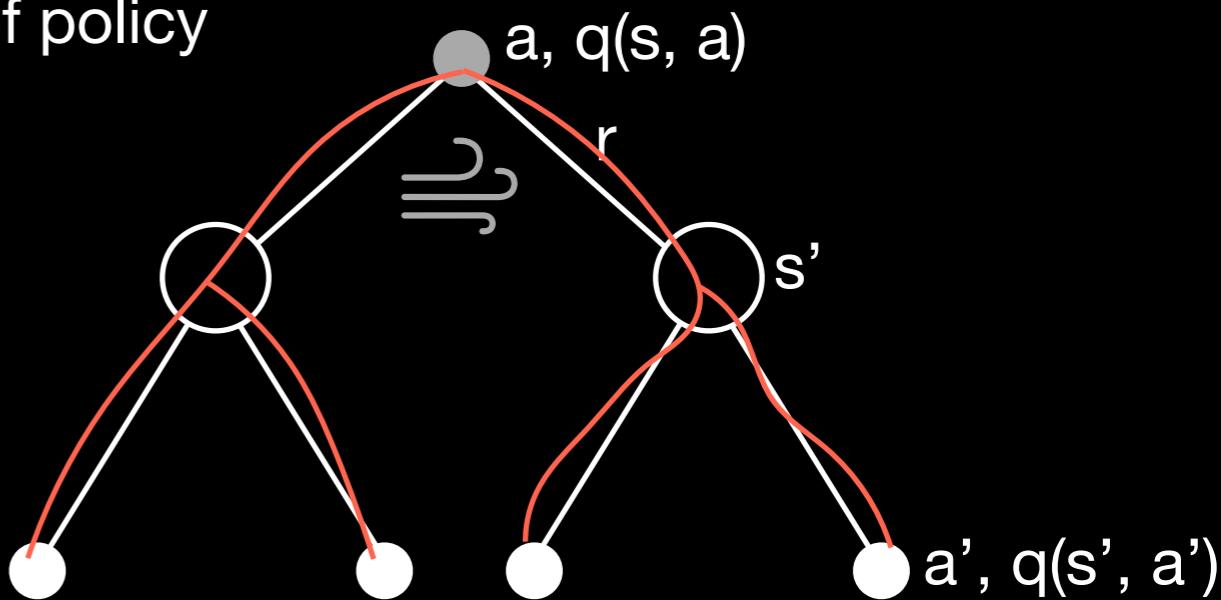


Bellman Expectation Backup

system of linear equations
solution: value of policy



Value of = $P(\text{path}) * \text{Value}(\text{path})$



Value of = $P(\text{path}) * \text{Value}(\text{path})$

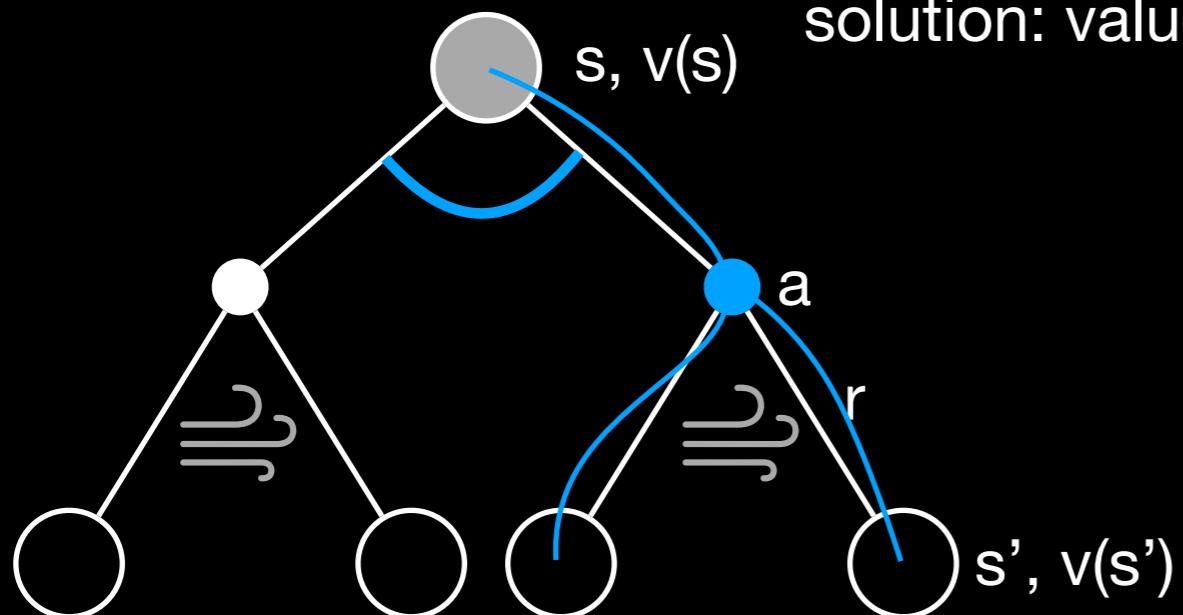
$$v_{\pi}(s) = \sum_a \pi(a|s) \left(r_s^a + \gamma \sum_{s'} P_{ss'}^a v_{\pi}(s') \right)$$

$$q_{\pi}(s, a) = r_s^a + \gamma \sum_{s'} \sum_{a'} P_{ss'}^a \pi(a'|s') q_{\pi}(s', a')$$

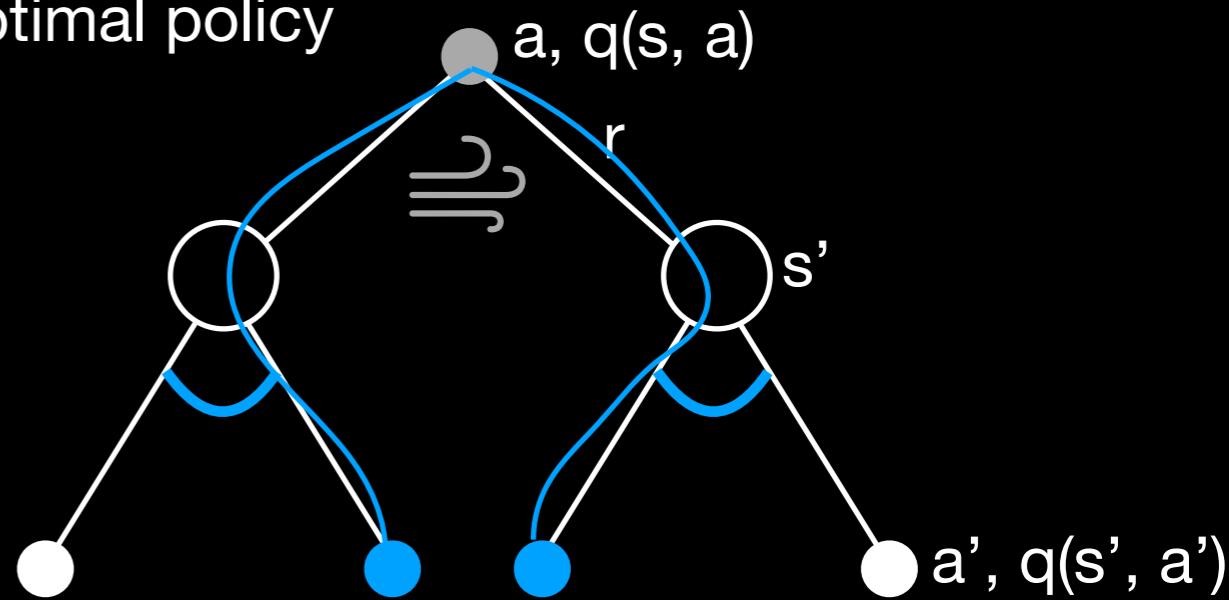
Bellman expectation equations
under a **given policy**

Bellman Optimality Backup

system of non-linear equations
solution: value of optimal policy



Value of = $P(\text{path}) * \text{Value}(\text{path})$



Value of = $P(\text{path}) * \text{Value}(\text{path})$

$$v_*(s) = \max_a \left(r_s^a + \gamma \sum_{s'} P_{ss'}^a v_*(s') \right)$$

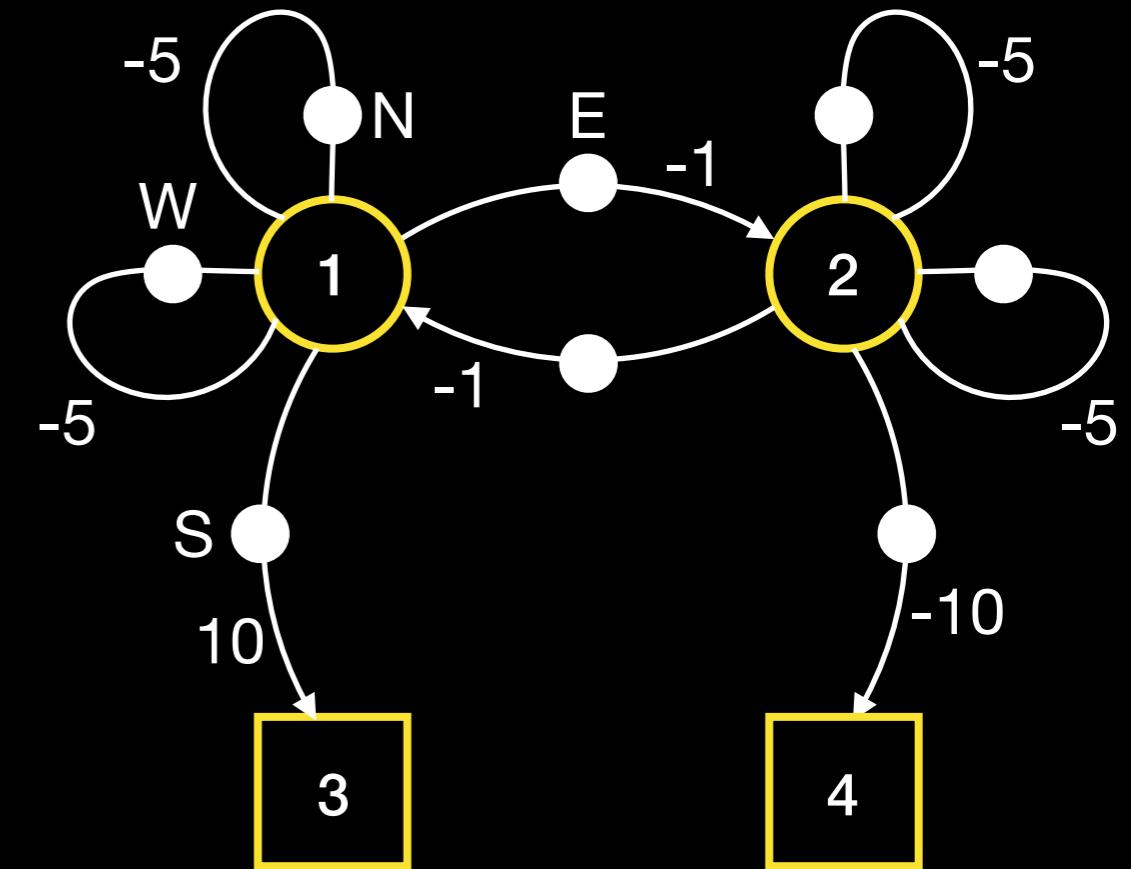
$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$

Bellman optimality equations
under **optimal policy**

Value Based

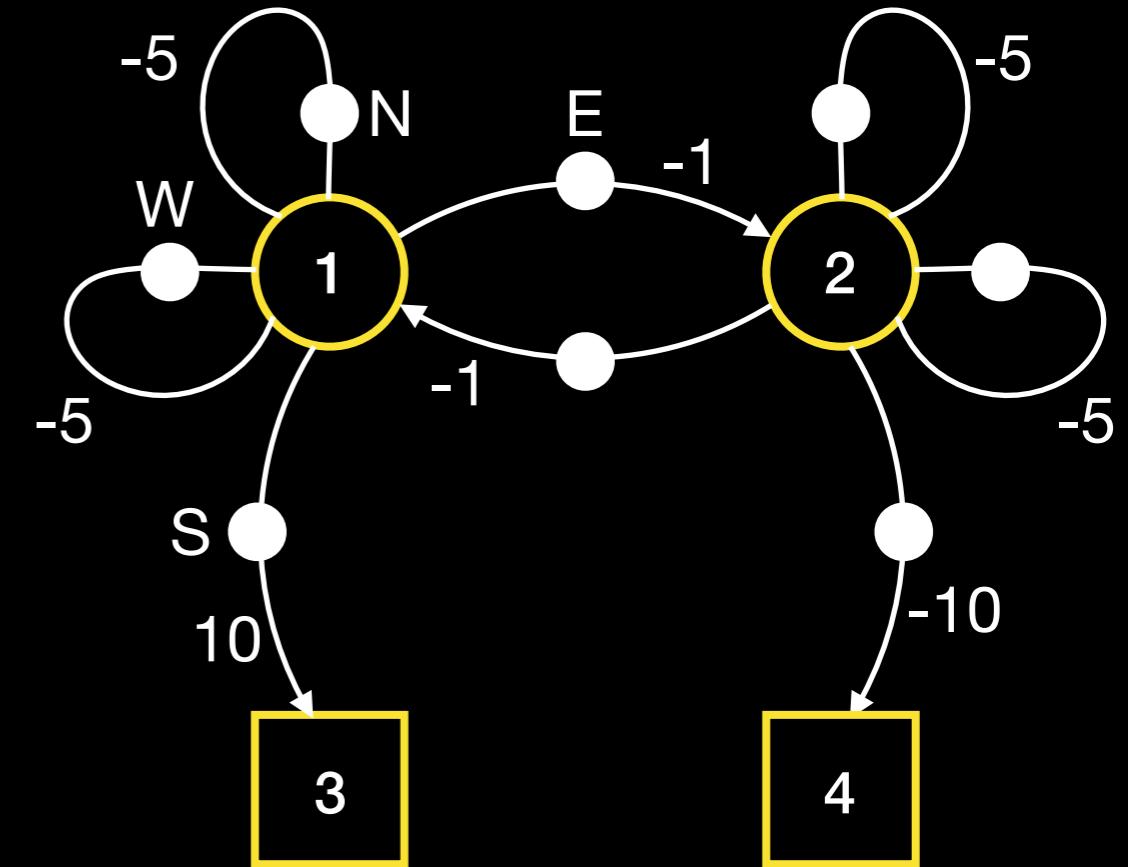
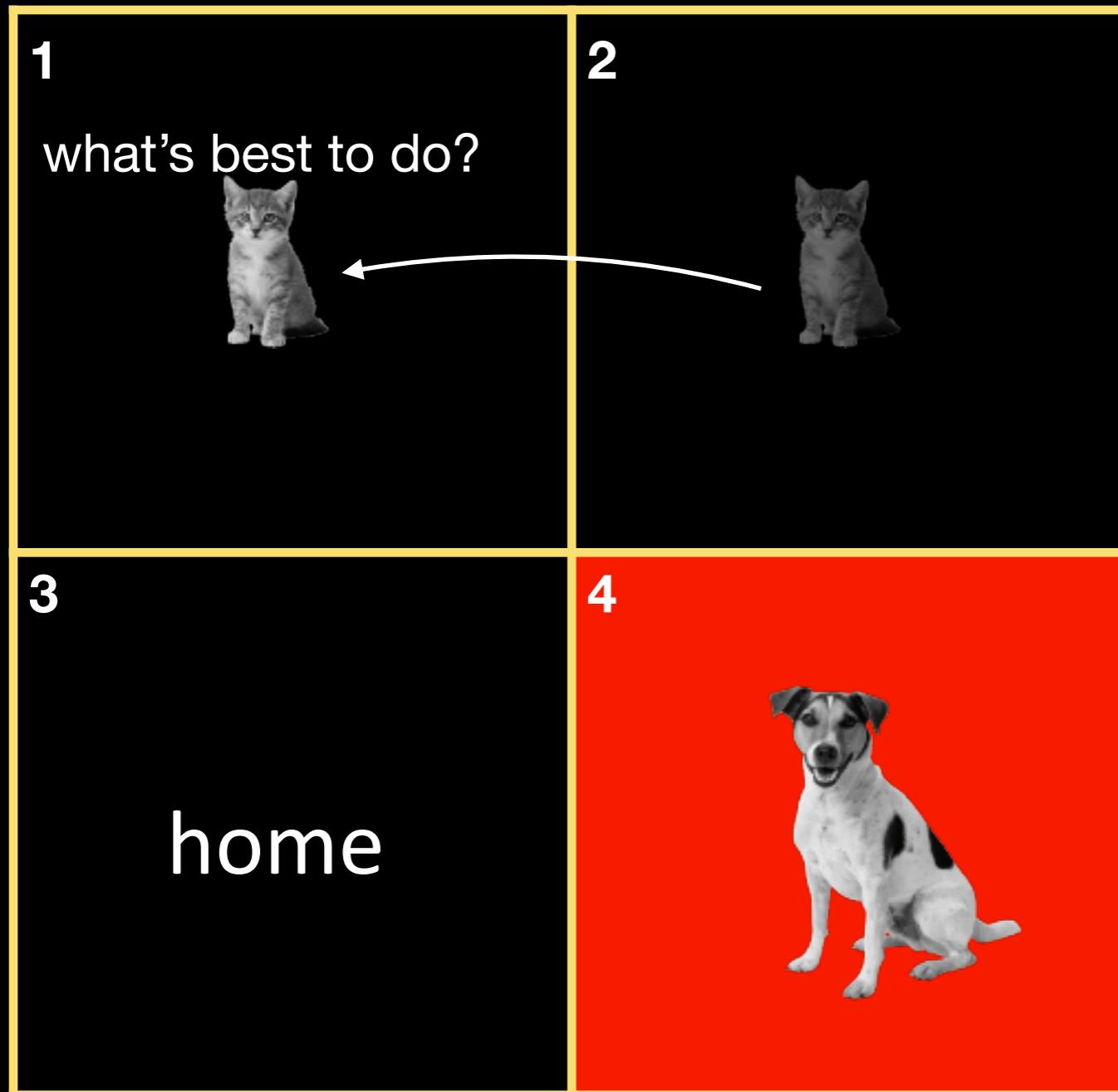
Dynamic Programming

...using Bellman equations as iterative updates

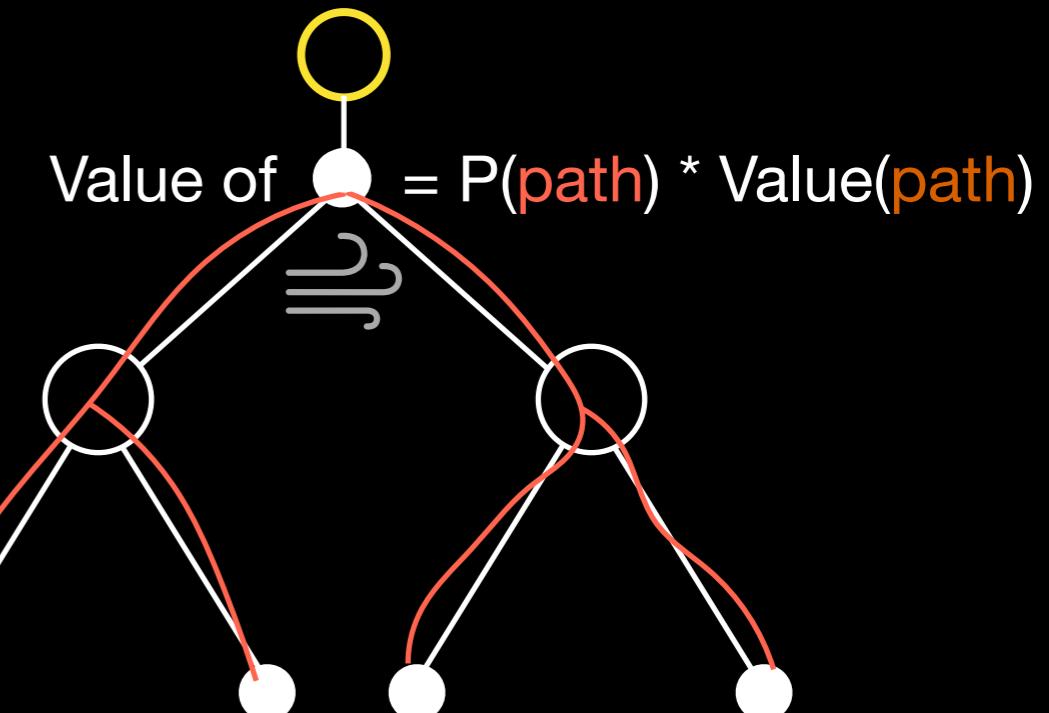
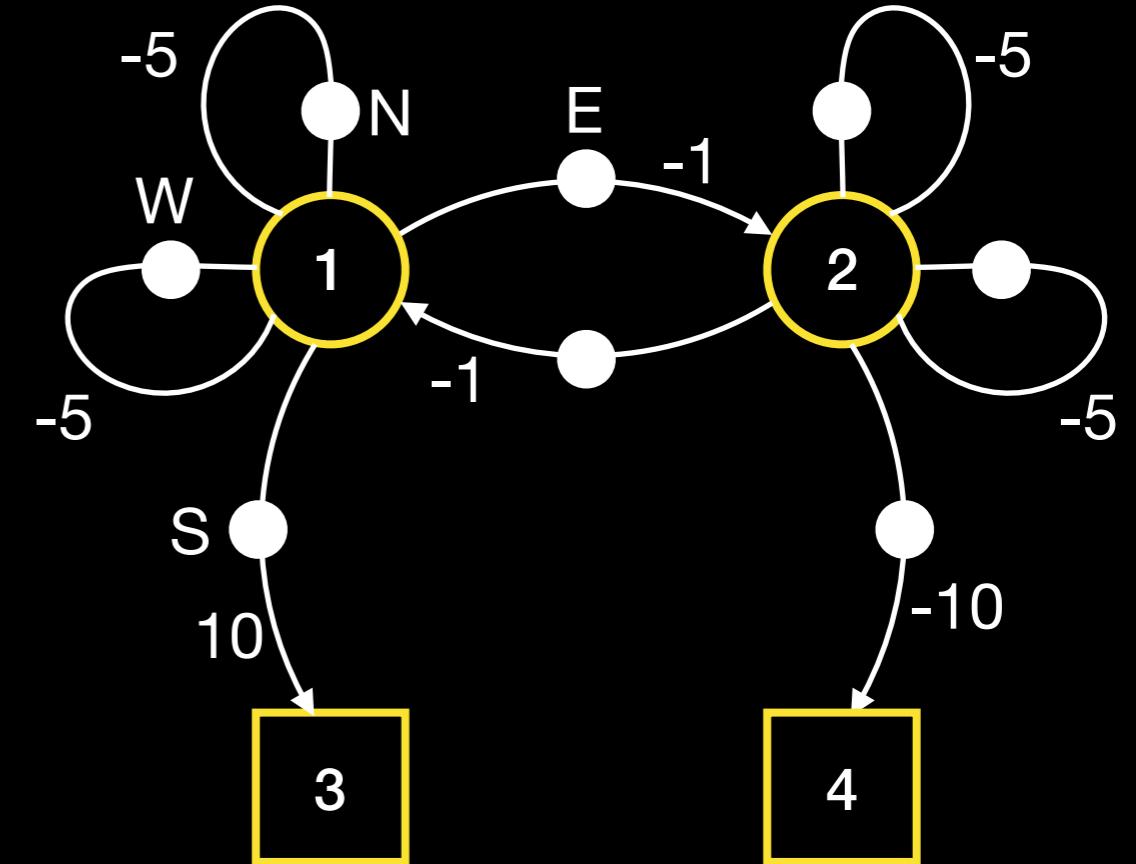
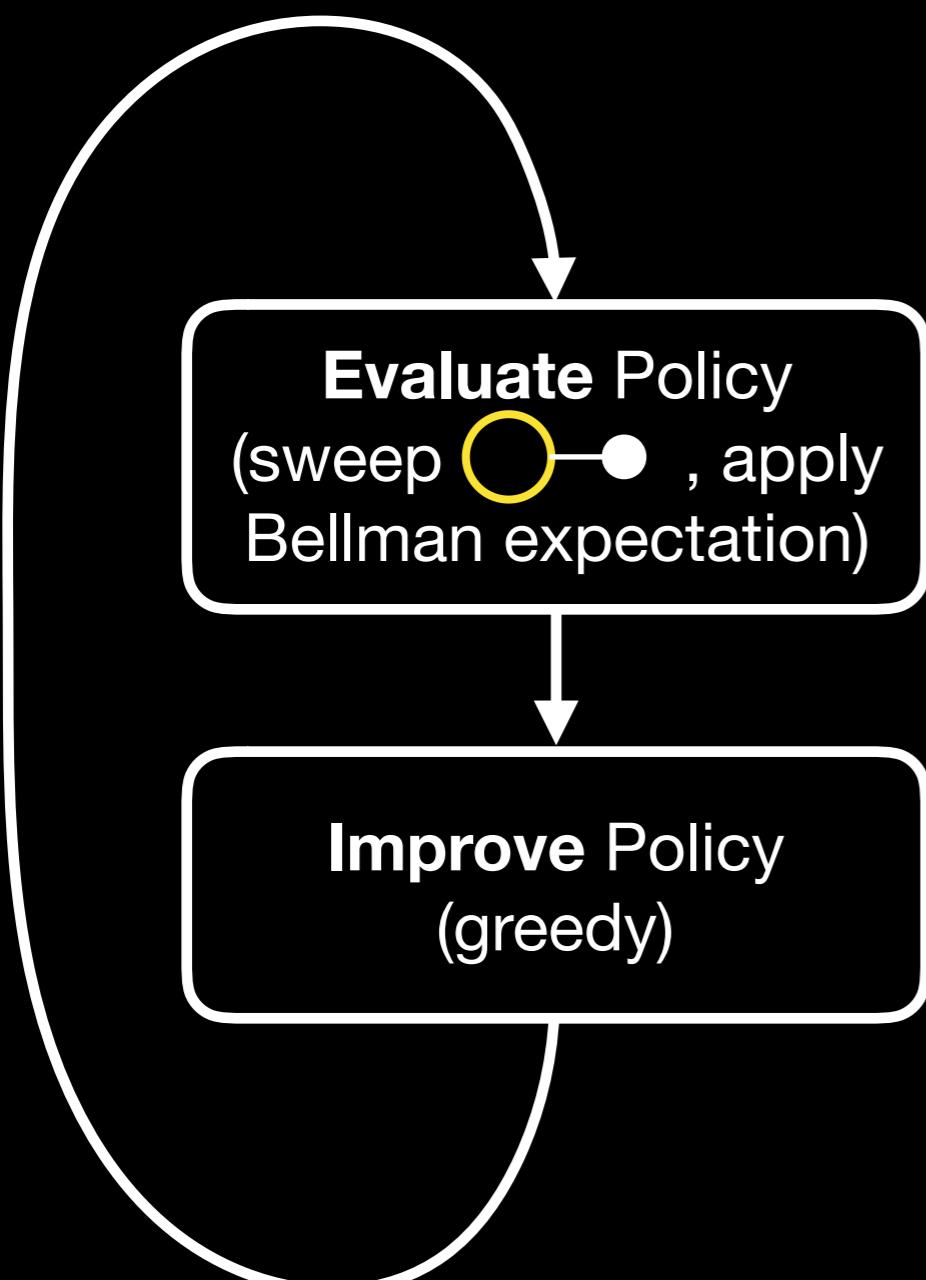


Dynamic Programming

...using Bellman equations as iterative updates



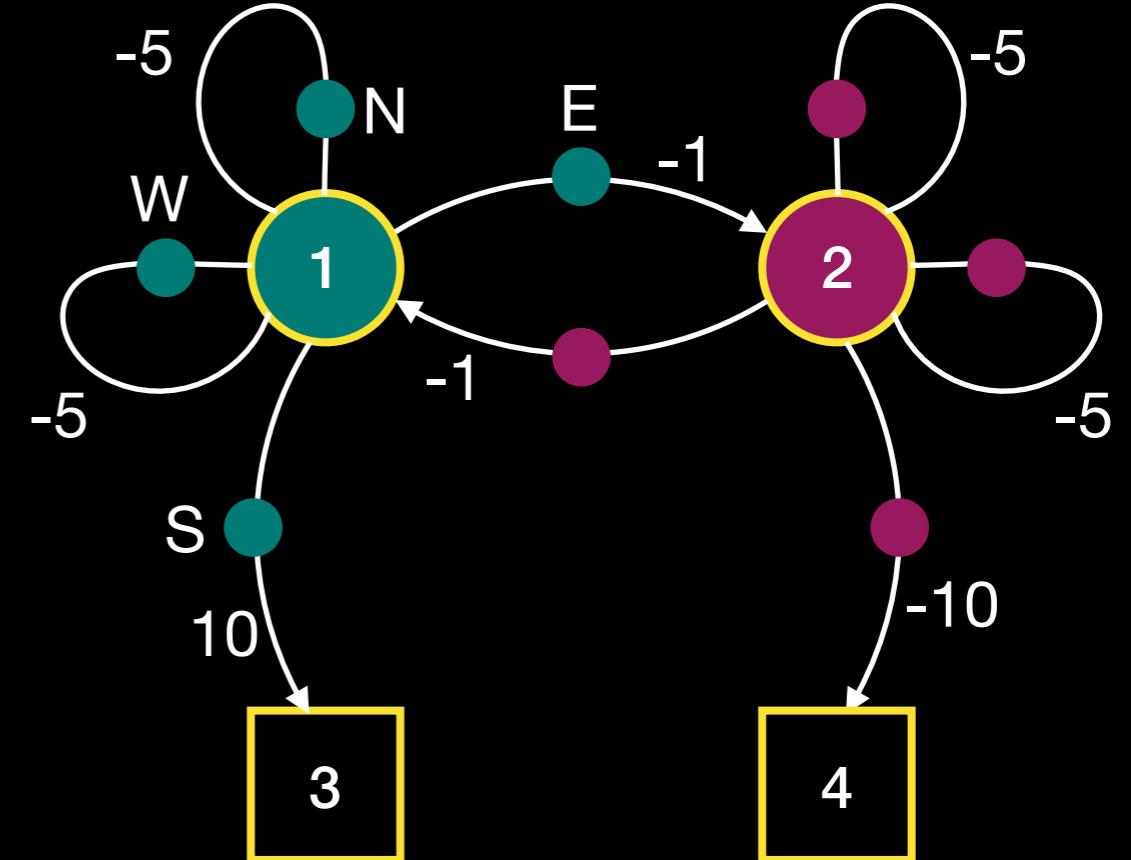
Policy Iteration



$$q_{\pi}(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a'|s') q_{\pi}(s', a')$$

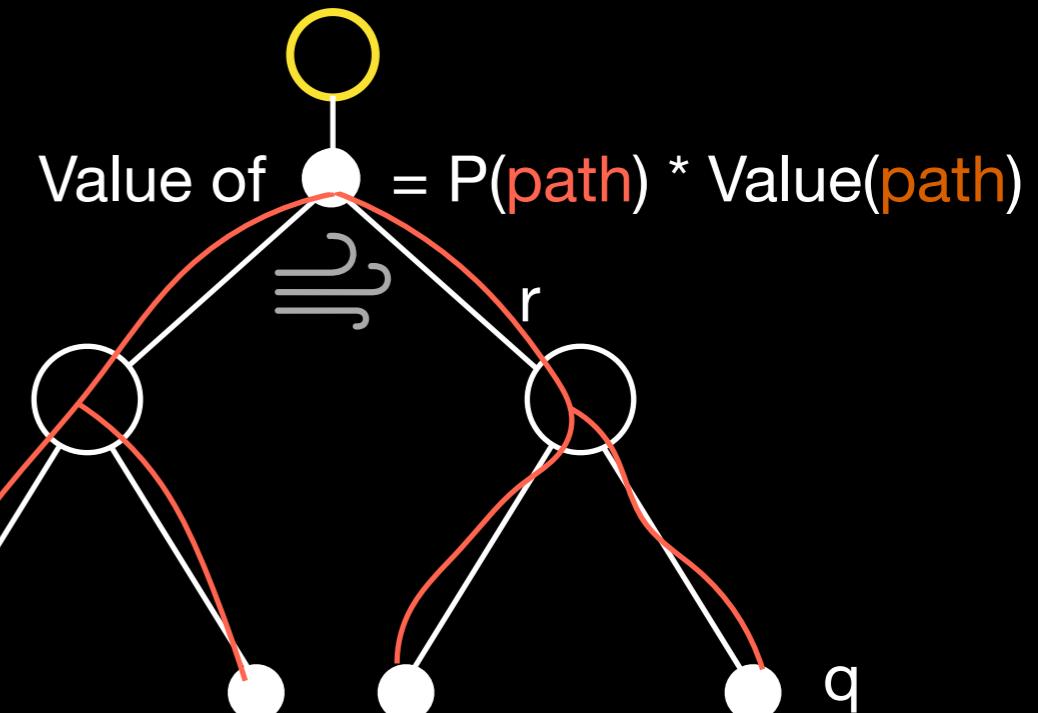
$$\begin{array}{ll}
 N: -5 + 0.9*0 & N: -5 + 0.9*0 \\
 E: -1 + 0.9*0 & E: -5 + 0.9*0 \\
 S: 10 + 0.9*0 & S: -10 + 0.9*0 \\
 W: -5 + 0.9*0 & W: -1 + 0.9*0
 \end{array}$$

iteratively apply Bellman expectation equations in inner loop until values do not change much



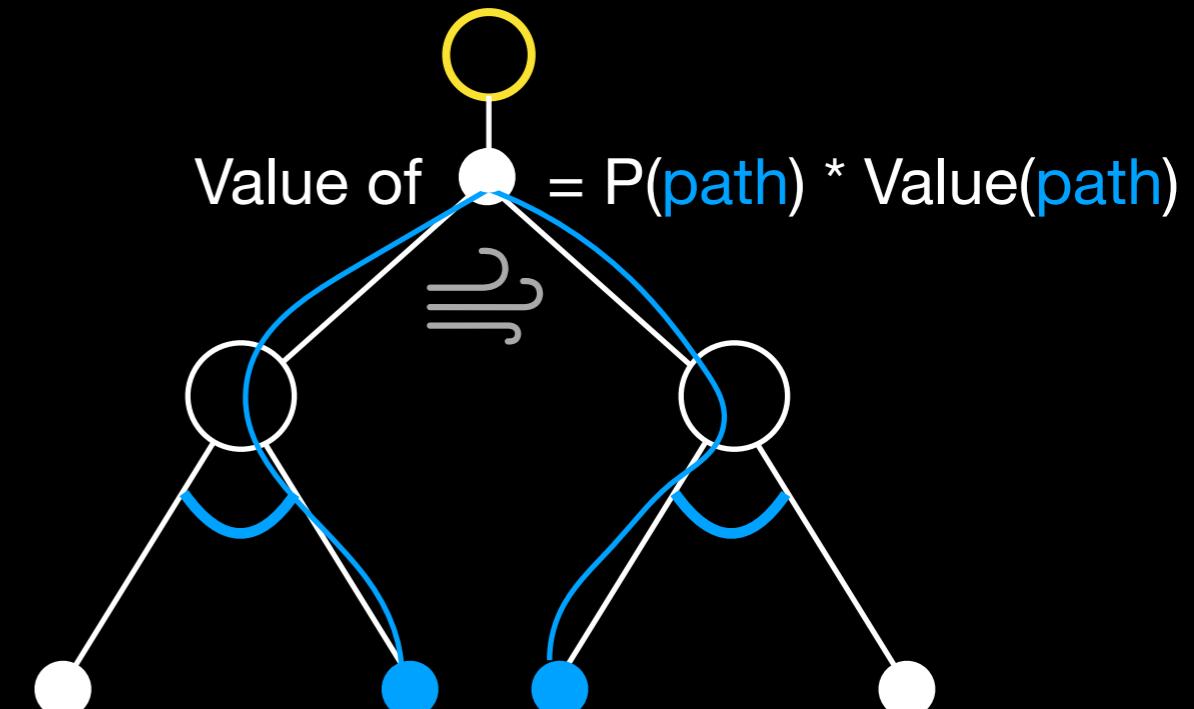
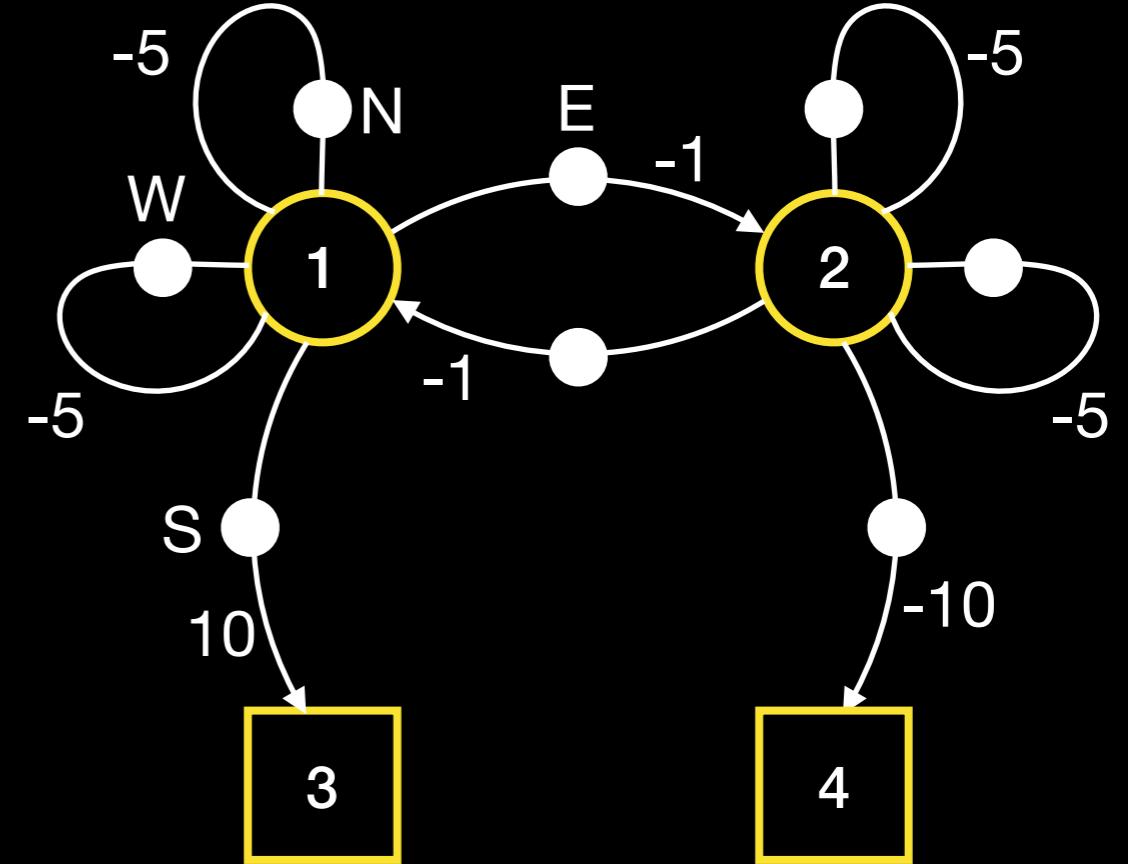
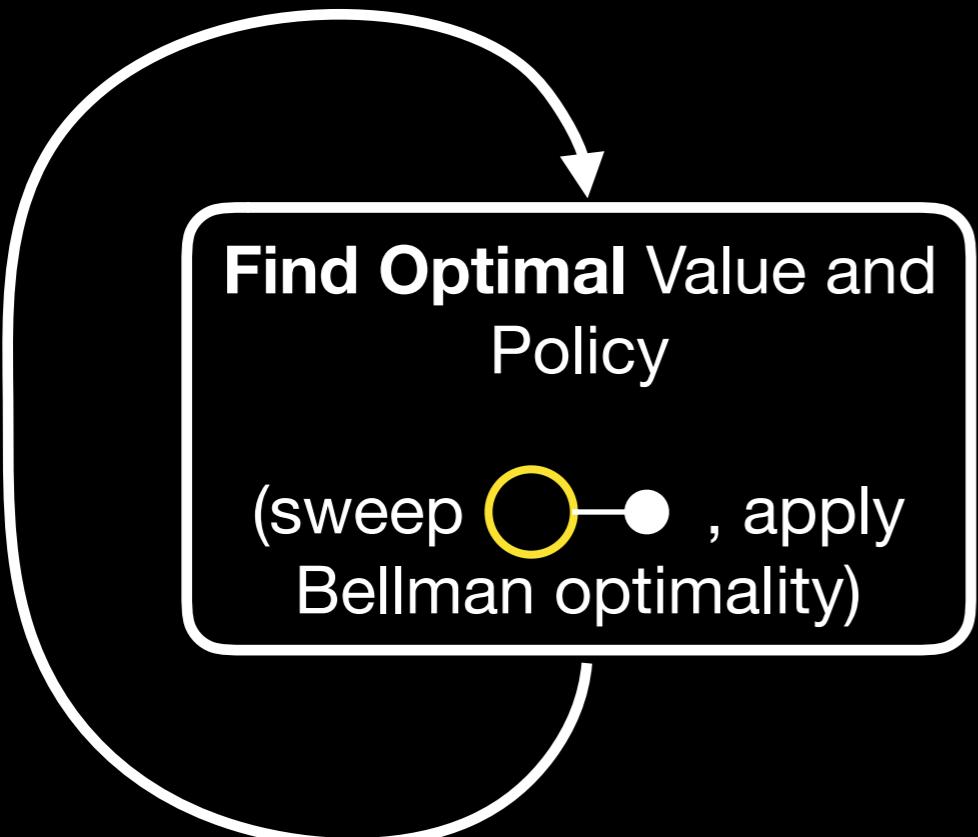
use greedy policy, given new values

$\pi(S|1): 1.0$ (greedy)
 $\pi(W|2): 1.0$ (greedy)



$$q_{\pi}(s,a) = r_s^a + \gamma \sum_{s'} \sum_{a'} P_{ss'}^a \pi(a'|s') q_{\pi}(s',a')$$

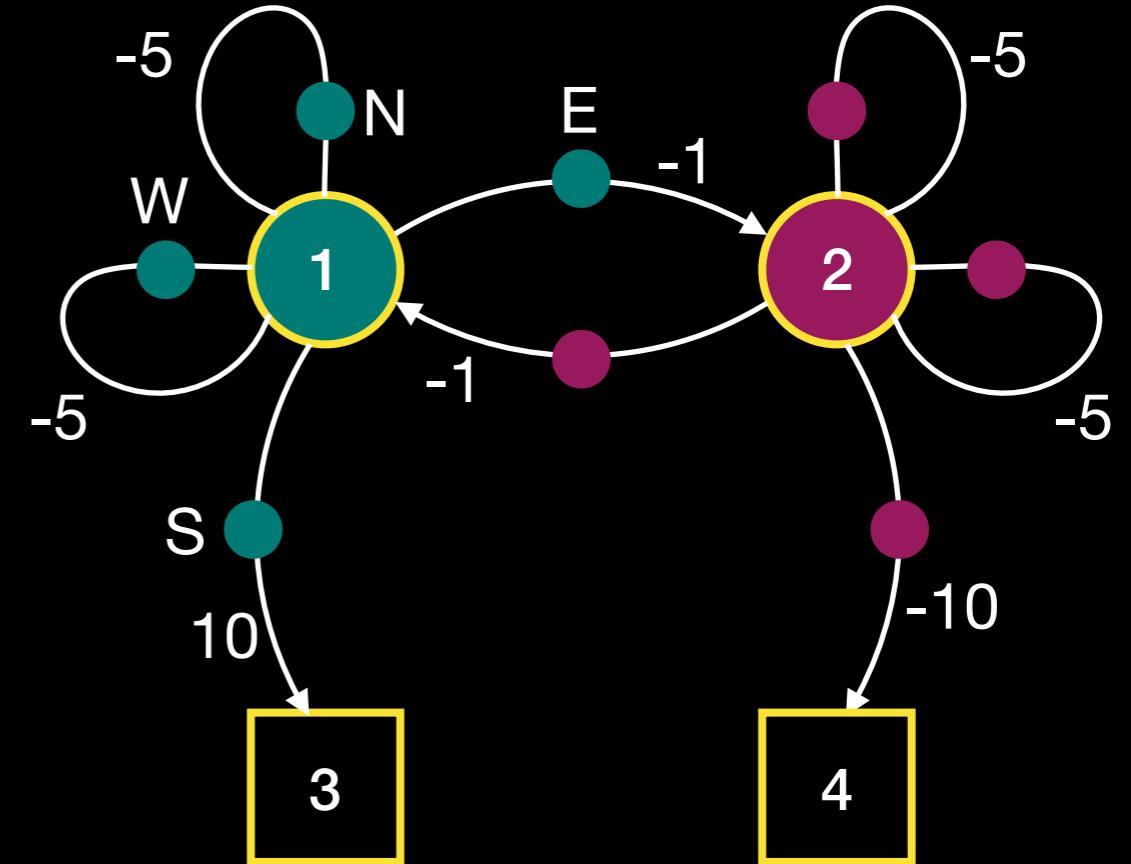
Value Iteration



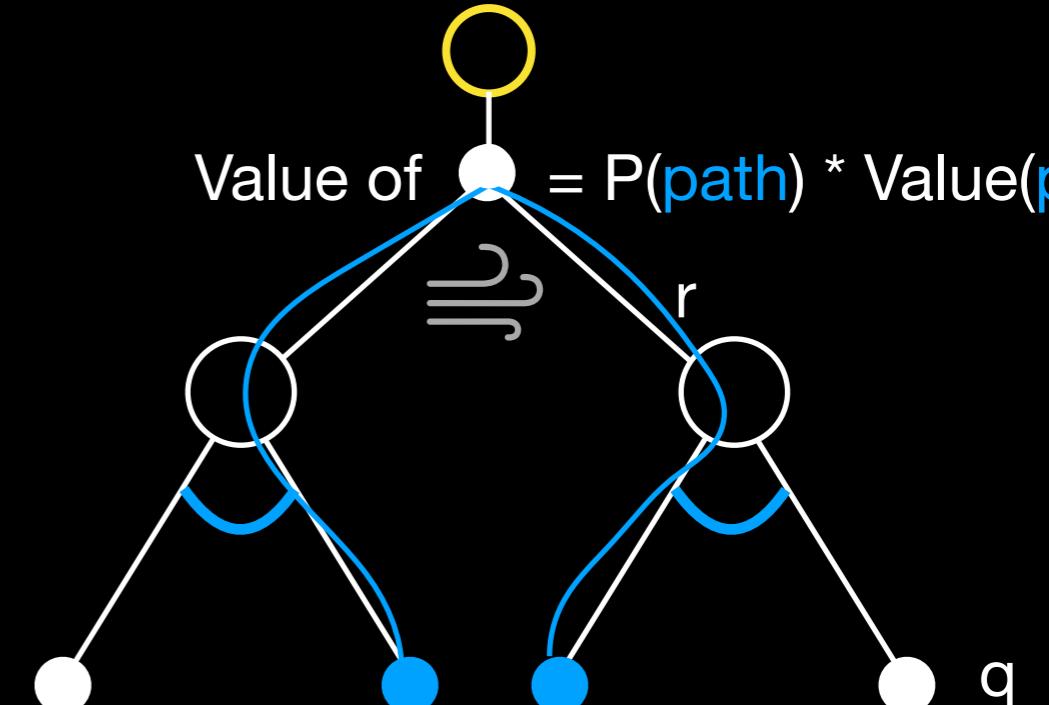
$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$

N: $-5 + 0.9*0$ N: $-5 + 0.9*0$
 E: $-1 + 0.9*0$ E: $-5 + 0.9*0$
 S: $10 + 0.9*0$ S: $-10 + 0.9*0$
 W: $-5 + 0.9*0$ W: $-1 + 0.9*0$

iteratively apply Bellman optimality
equations until values do not change much

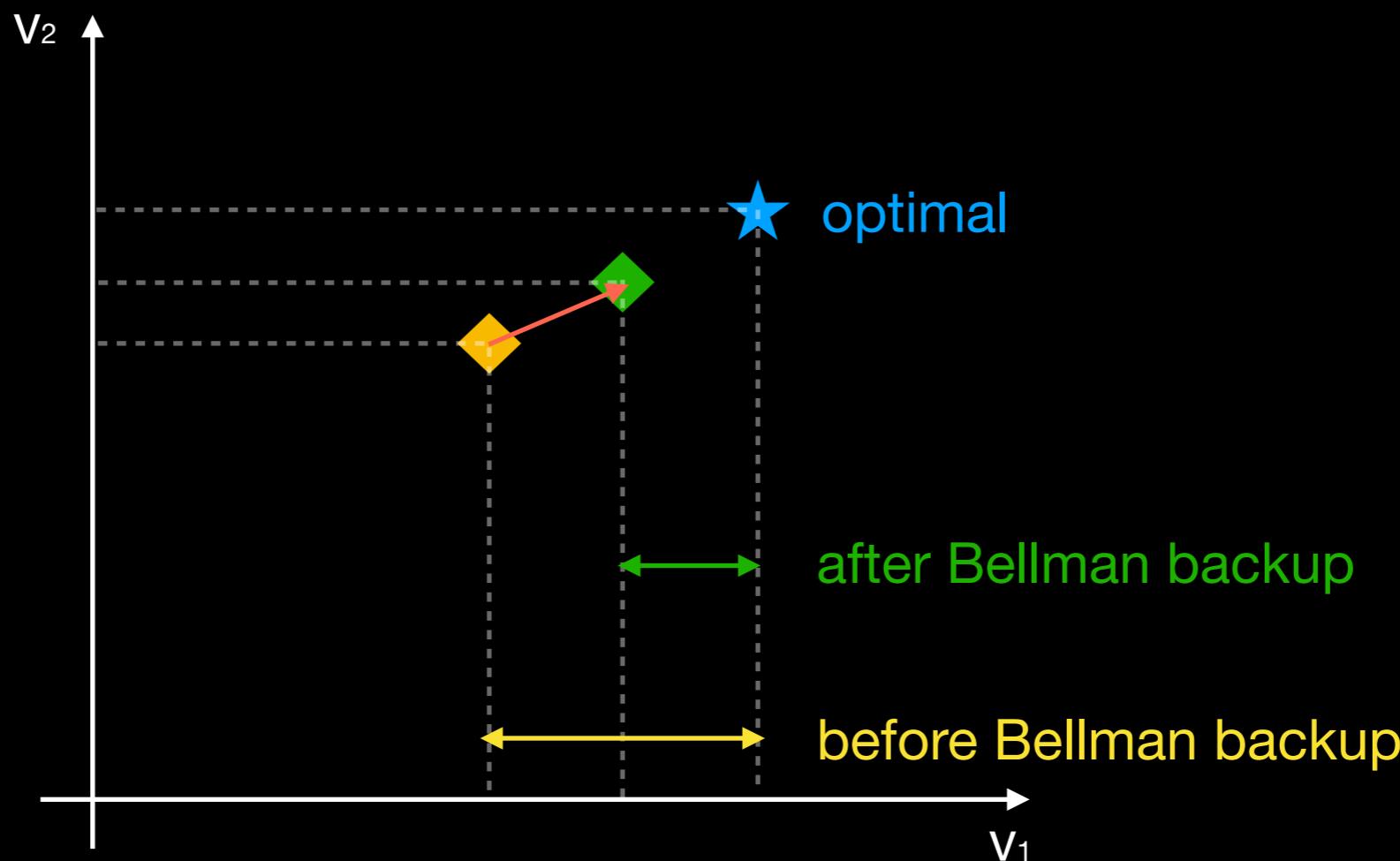


Value of $\bullet = P(\text{path}) * \text{Value}(\text{path})$



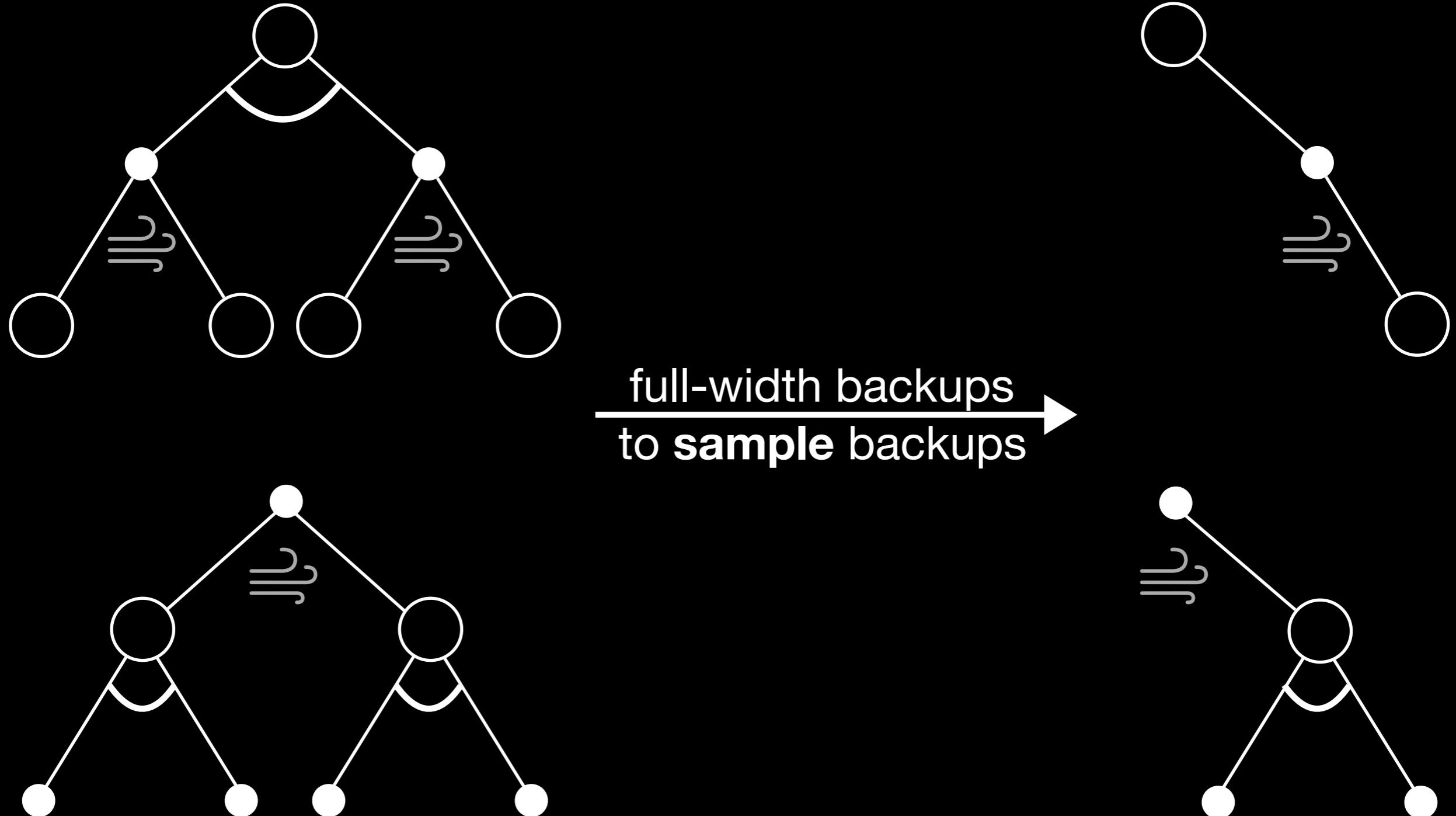
$$q_*(s, a) = r_s^a + \gamma \sum_{s'} P_{ss'}^a \max_{a'} q_*(s', a')$$

Bellman backups

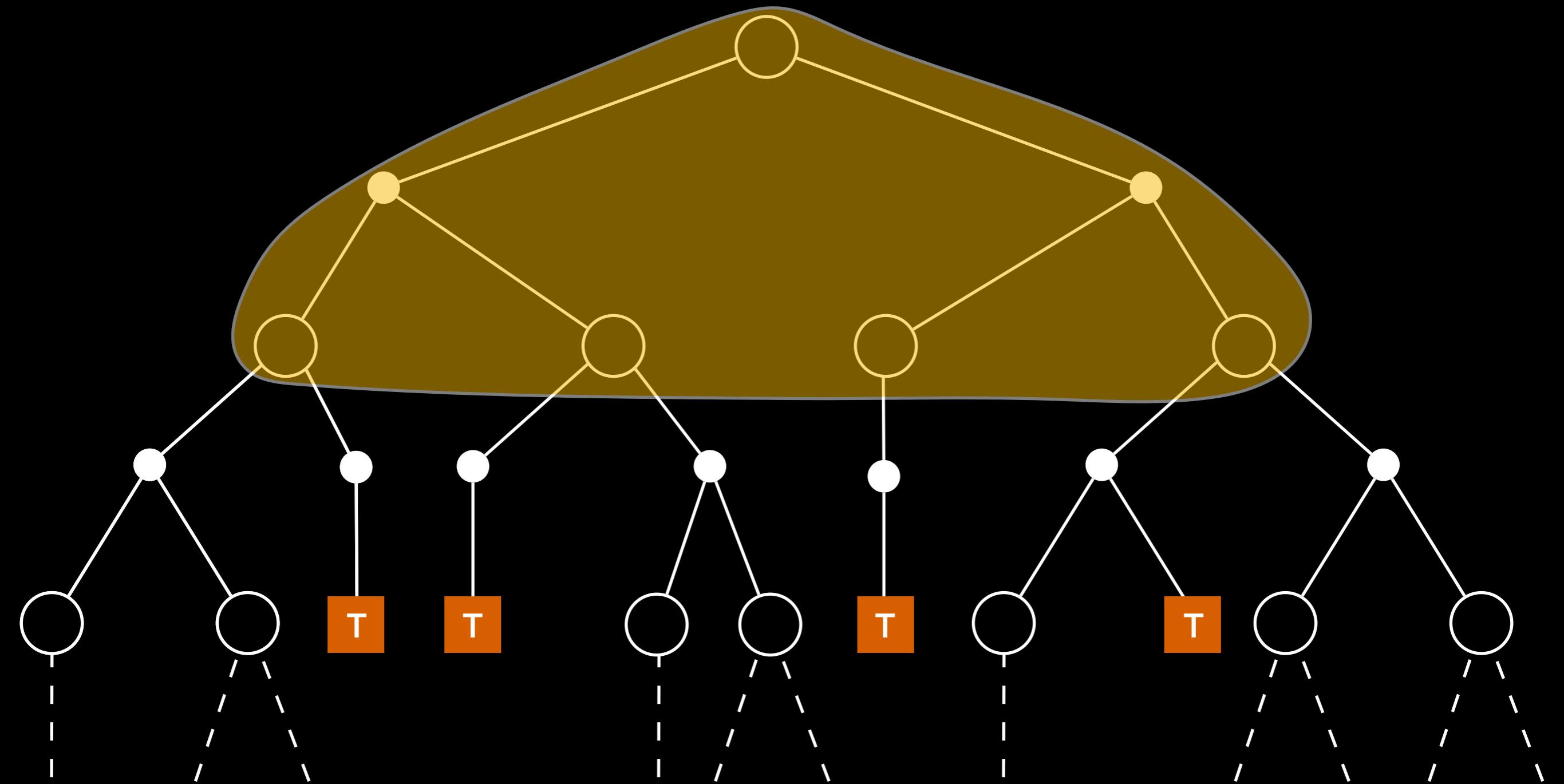


largest distance between values
decreases after Bellman backups

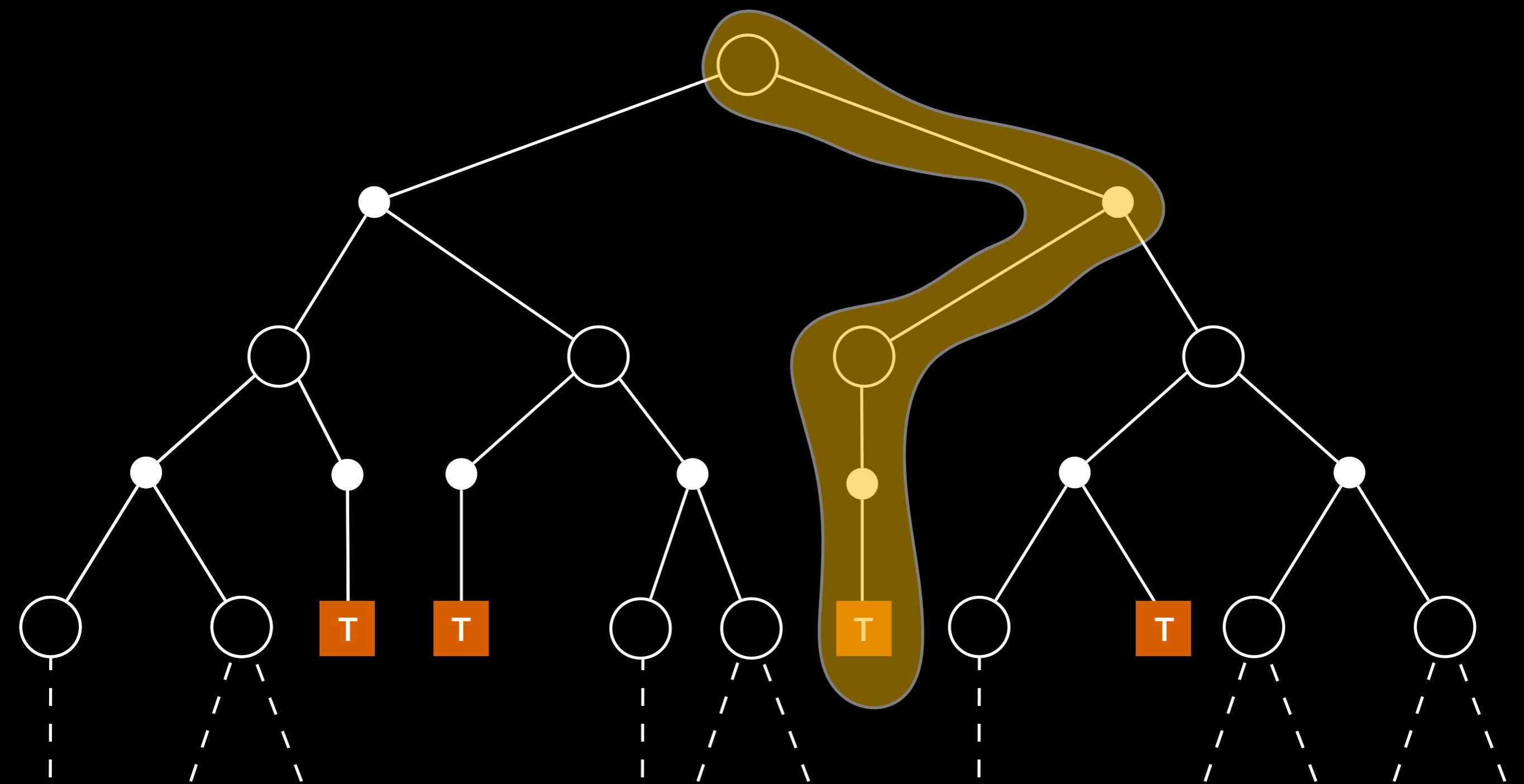
From DP to Learning



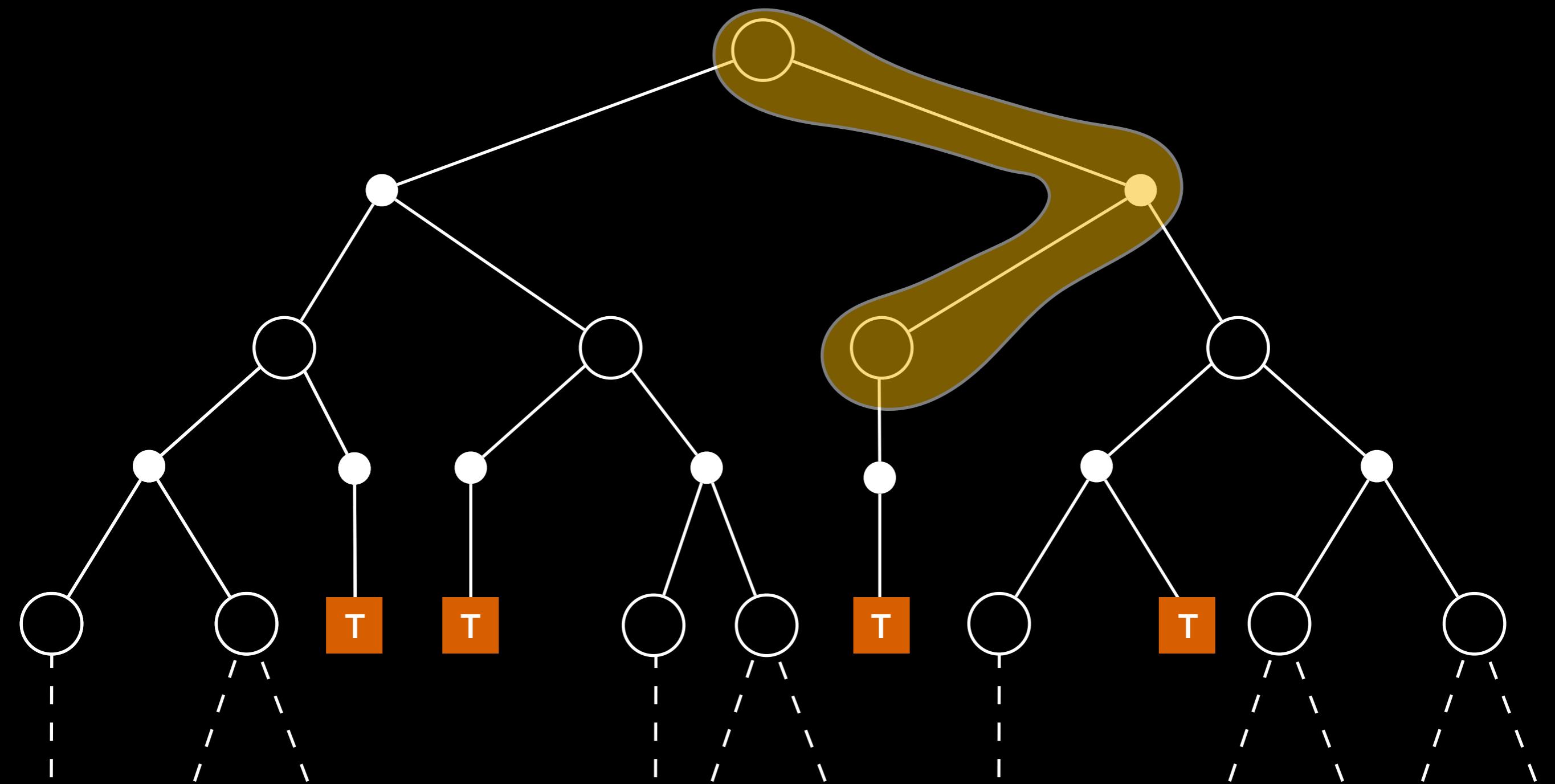
Full-width Backup



Backup with Sample Return



Backup with Guess



Incremental Updates

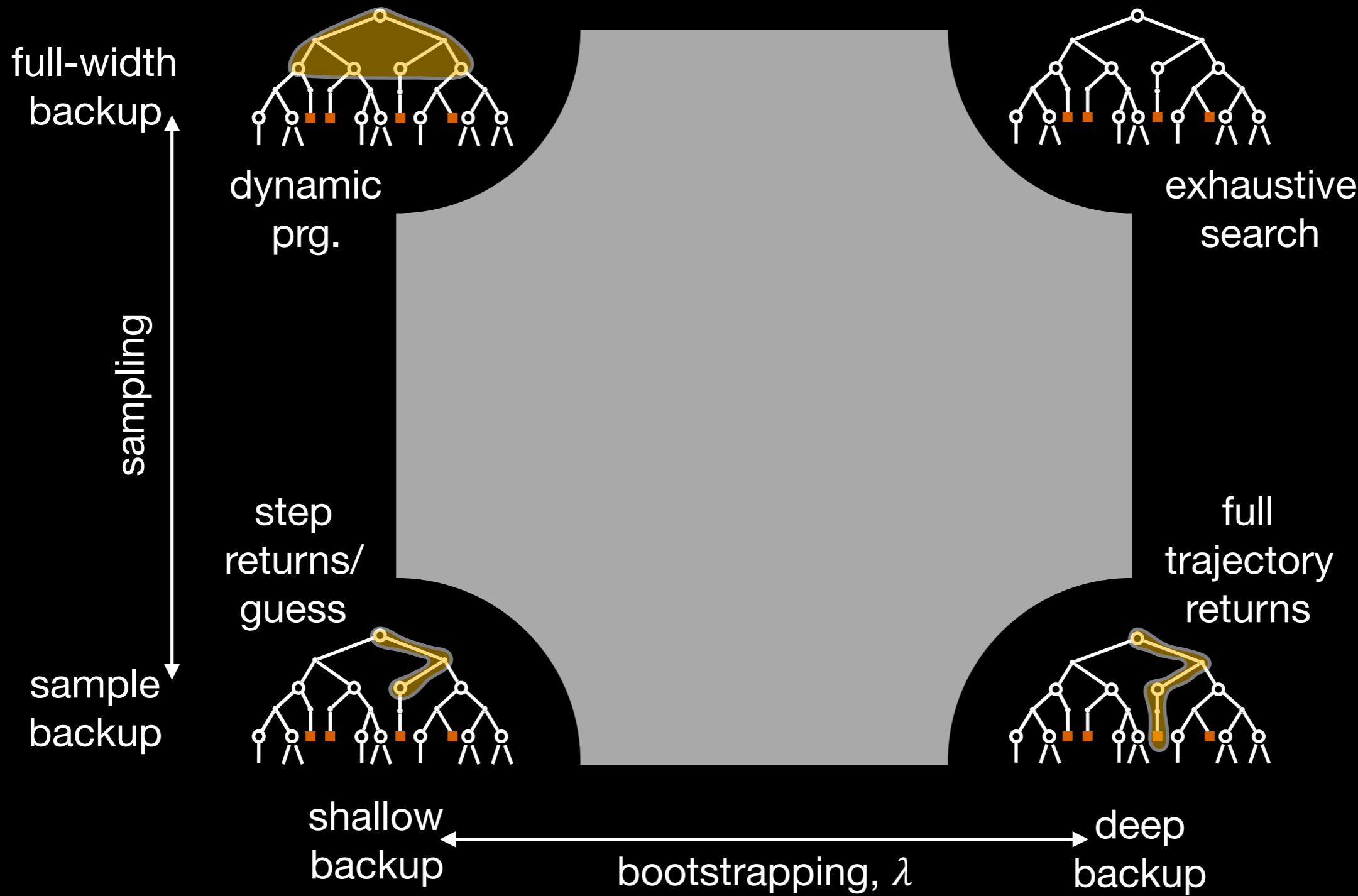
$$E\{R\} \approx \mu_k = \frac{1}{k} \sum_{\tau=1}^k R_\tau \quad \text{batched}$$

$$\mu_k = \mu_{k-1} + \frac{1}{k} (R_k - \mu_{k-1}) \quad \text{incremental}$$

$$\mu_k = \mu_{k-1} + \alpha (R_k - \mu_{k-1})$$

running
(saw this in
Q-learning!)

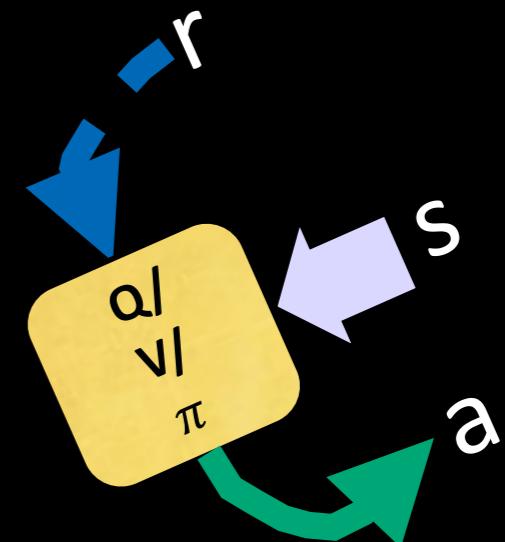
Sample and Bootstrap



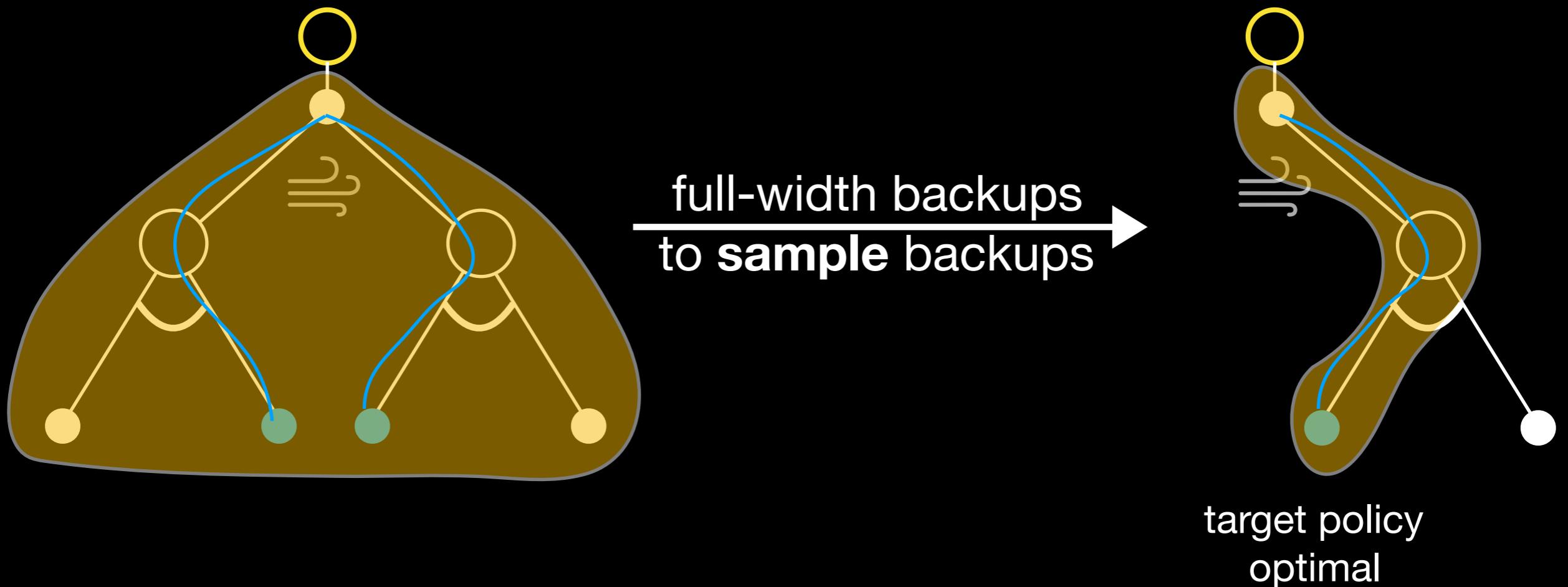
It all comes down to:

estimating returns

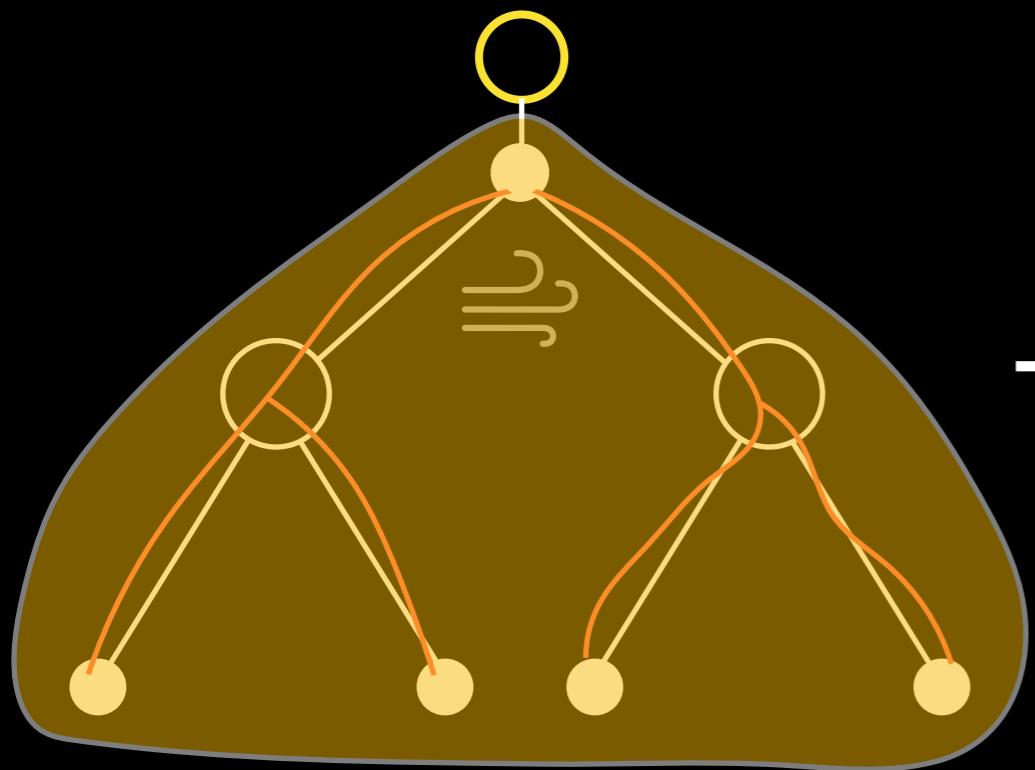
optimising
towards achieving
returns



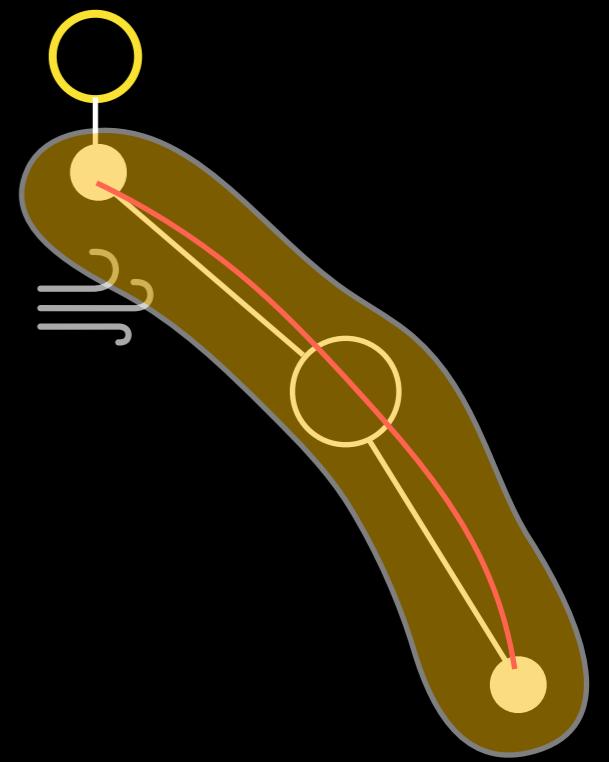
Q-learning



SARSA



full-width backups
to sample backups



target policy
same as
behaviour policy

scaling up RL with
function approximation

Approximate Q-learning

e.g. linear approximation

$$Q_\theta(s,a) = \theta_0 f_0(s,a) + \theta_1 f_1(s,a) + \dots + \theta_n f_n(s,a)$$

$$Q_{\text{target}} = (r_s^a + \gamma \max_{a'} Q(s', a'))$$

$$\theta \leftarrow \theta - \alpha \nabla_\theta \frac{1}{2} \left(Q_{\text{target}} - Q_\theta(s, a) \right)^2$$

gradient updates equivalent to tabular Q updates

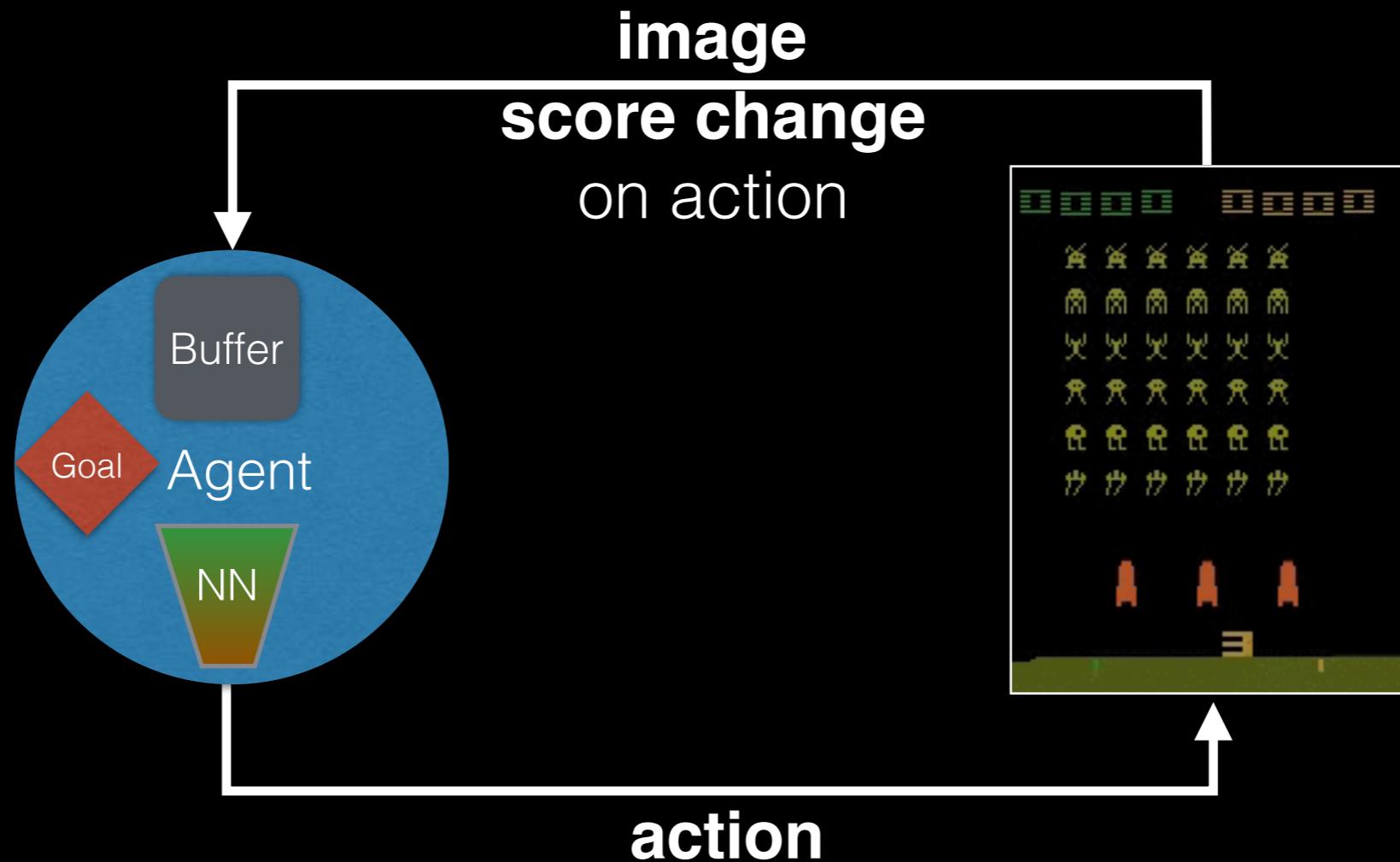
Say $\theta \in \mathbb{R}^{\|S\|\|x\|\|A\|}$, so $Q_\theta(s,a) = \theta_{sa}$

$$Q_{\text{target}} = r_s^a + \gamma \max_{a'} Q(s', a')$$

$$\begin{aligned}\theta_{sa} &\leftarrow \theta_{sa} - \alpha \nabla_{\theta_{sa}} \frac{1}{2} \left(Q_{\text{target}} - \theta_{sa} \right)^2 \\ \theta_{sa} &\leftarrow \theta_{sa} - \alpha \left(-Q_{\text{target}} + \theta_{sa} \right) \\ \theta_{sa} &\leftarrow \theta_{sa} + \alpha \left(Q_{\text{target}} - \theta_{sa} \right) \\ \theta_{sa} &\leftarrow (1 - \alpha) \theta_{sa} + \alpha Q_{\text{target}}\end{aligned}$$

tabular
equivalent

DQN



Human-level control through deep reinforcement learning,
Mnih et. al., Nature 518, Feb 2015

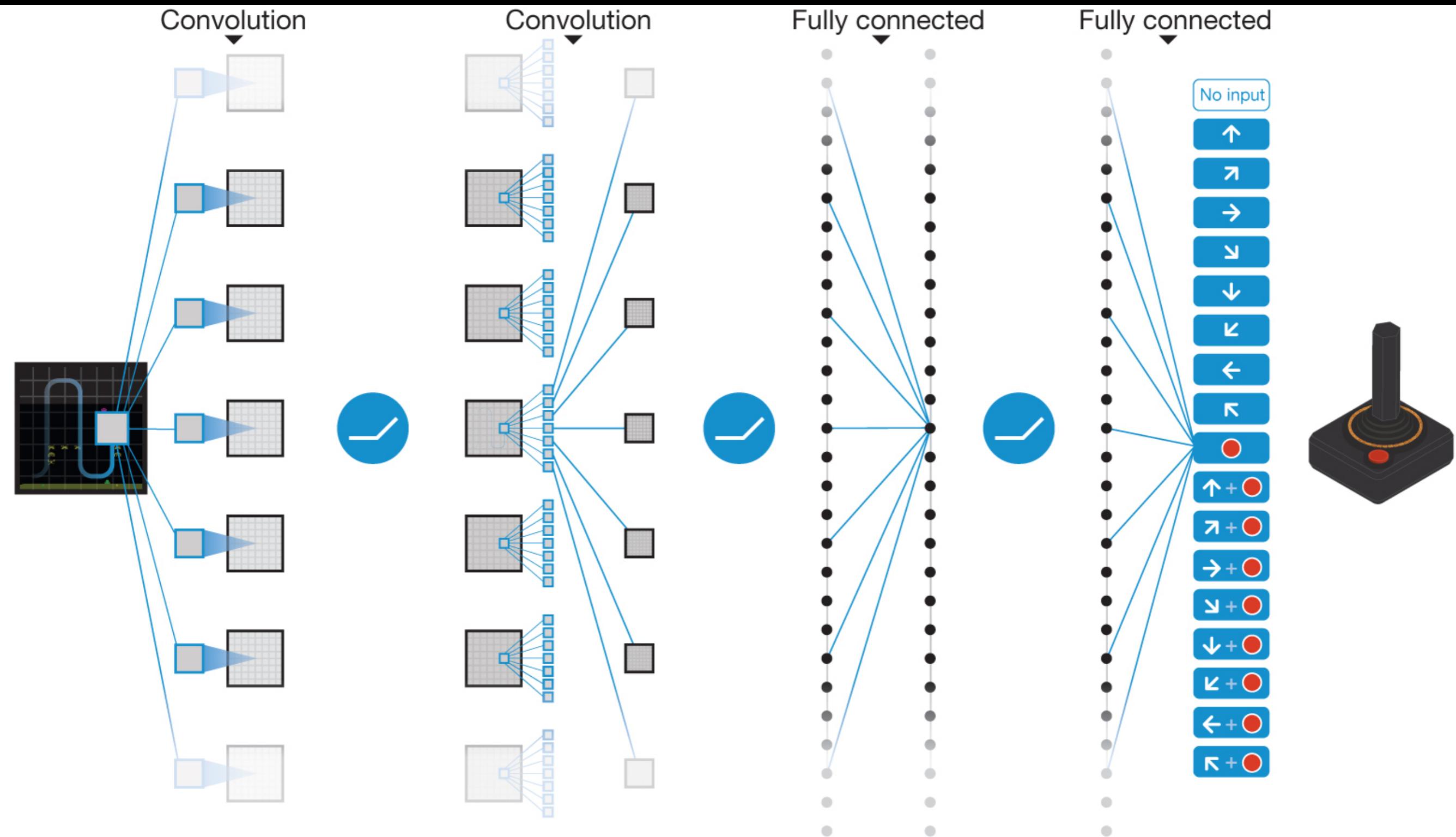
human level game control

- pixel input
- 18 joystick/button positions output
- change in game score as feedback
- convolutional net representing Q
- backpropagation for training!

Human-level control through deep reinforcement learning,
Mnih et. al., Nature 518, Feb 2015

<http://www.nature.com/nature/journal/v518/n7540/full/nature14236.html>

neural network



backpropagation

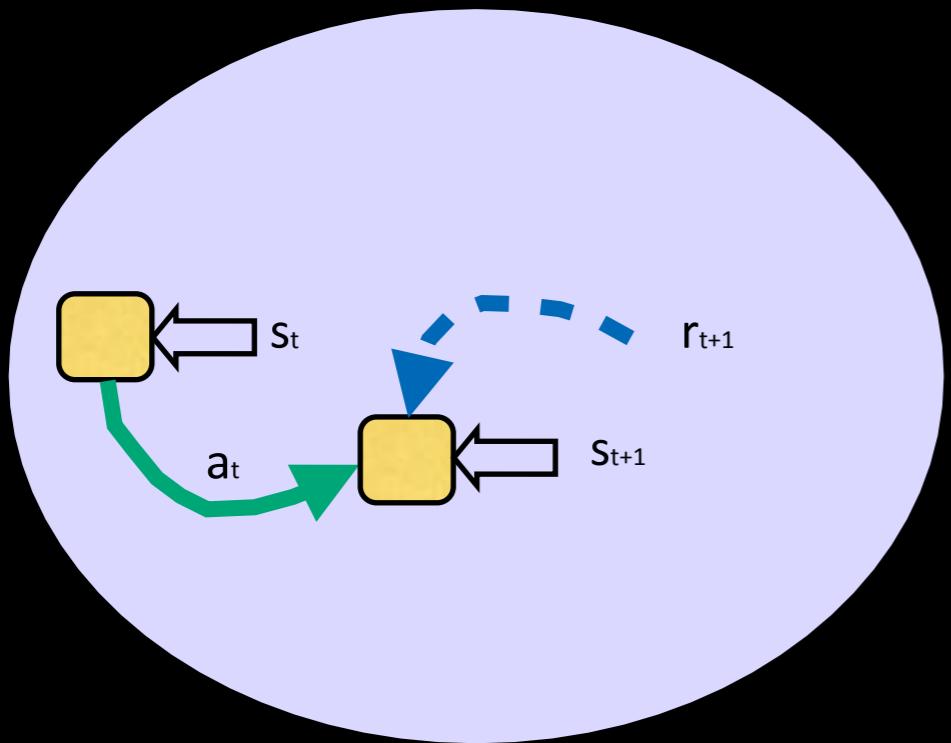
What is the **target** against which to minimise error?

$$\mathcal{L}(w) = \mathbb{E} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

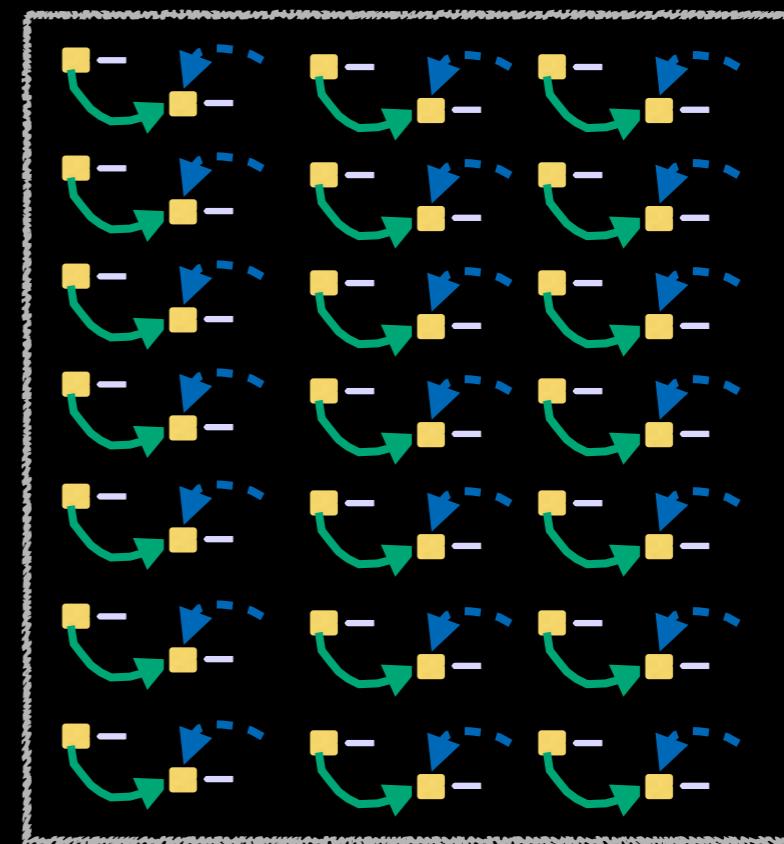
target

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E} \left[\left(r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right) \frac{\partial Q(s, a, w)}{\partial w} \right]$$

experience replay buffer



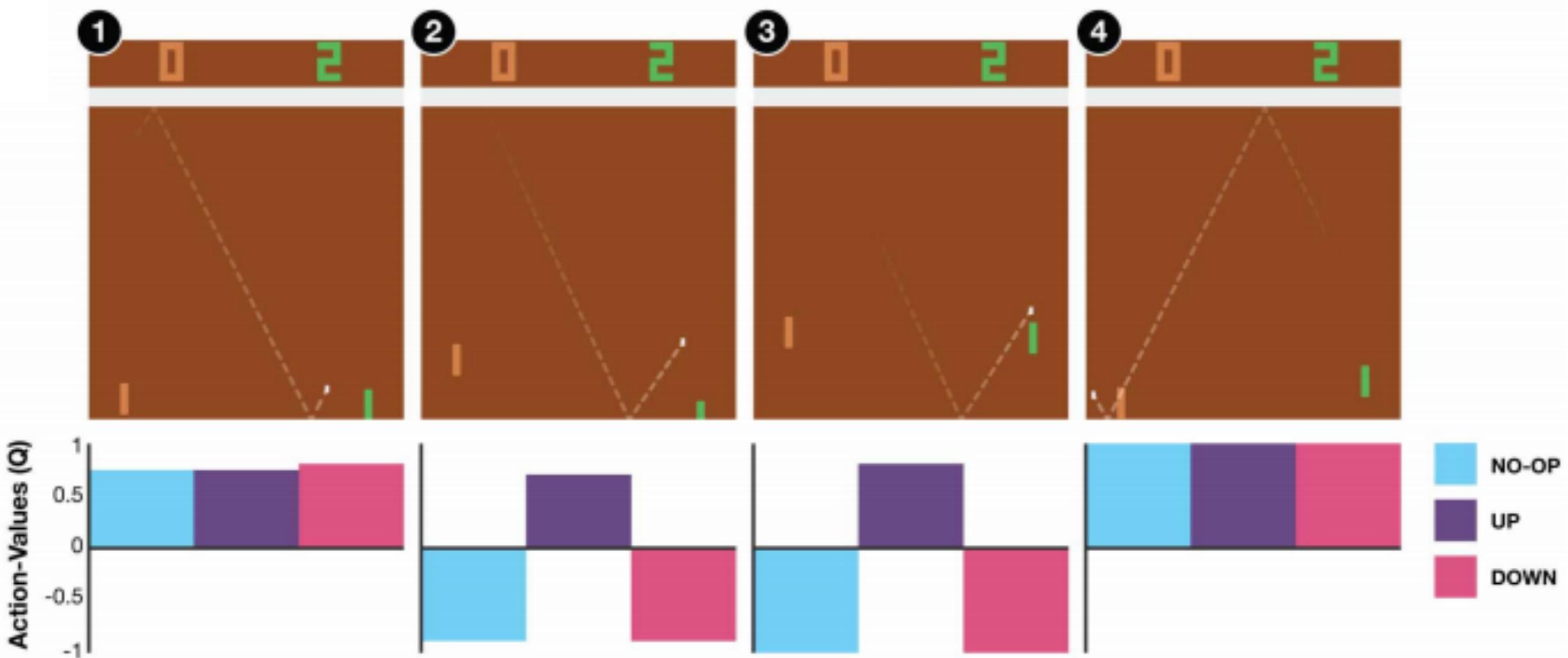
save transition in
memory



randomly **sample**
from memory
for training
= i.i.d

freeze
target

$$\left(r + \gamma \max_{a'} Q(s', a', \mathbf{w}^-) - Q(s, a, \mathbf{w}) \right)^2$$



[https://storage.googleapis.com/deepmind-media/dqn/
DQNNaturePaper.pdf](https://storage.googleapis.com/deepmind-media/dqn/DQNNaturePaper.pdf)

however
training is

SLOOOOO...W

parallelise...

Parallel Asynchronous Training

value and **policy** based methods

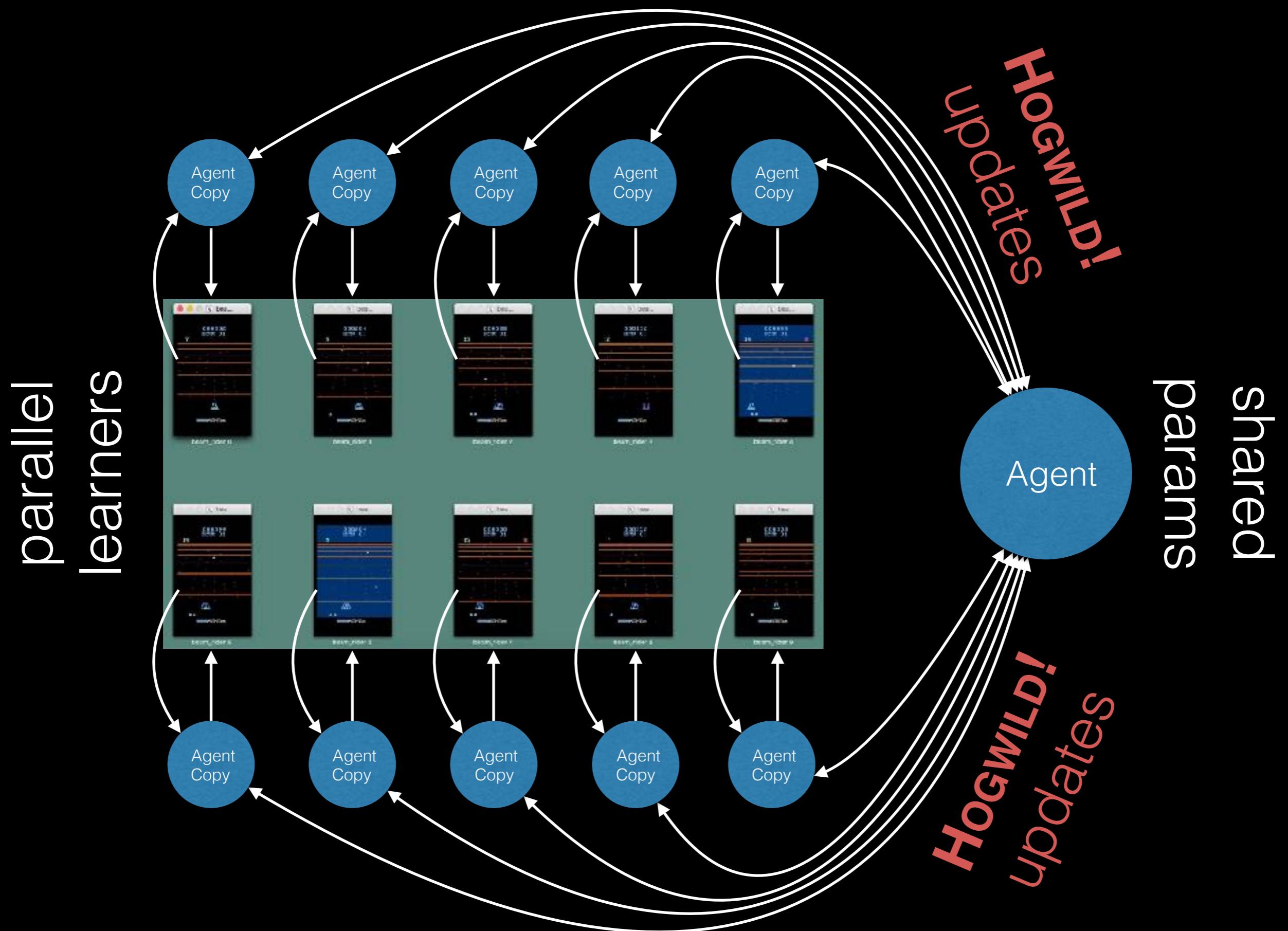


<https://youtu.be/0xo1Ldx3L5Q>

parallel
agents

shared
parameters

lock-free
updates



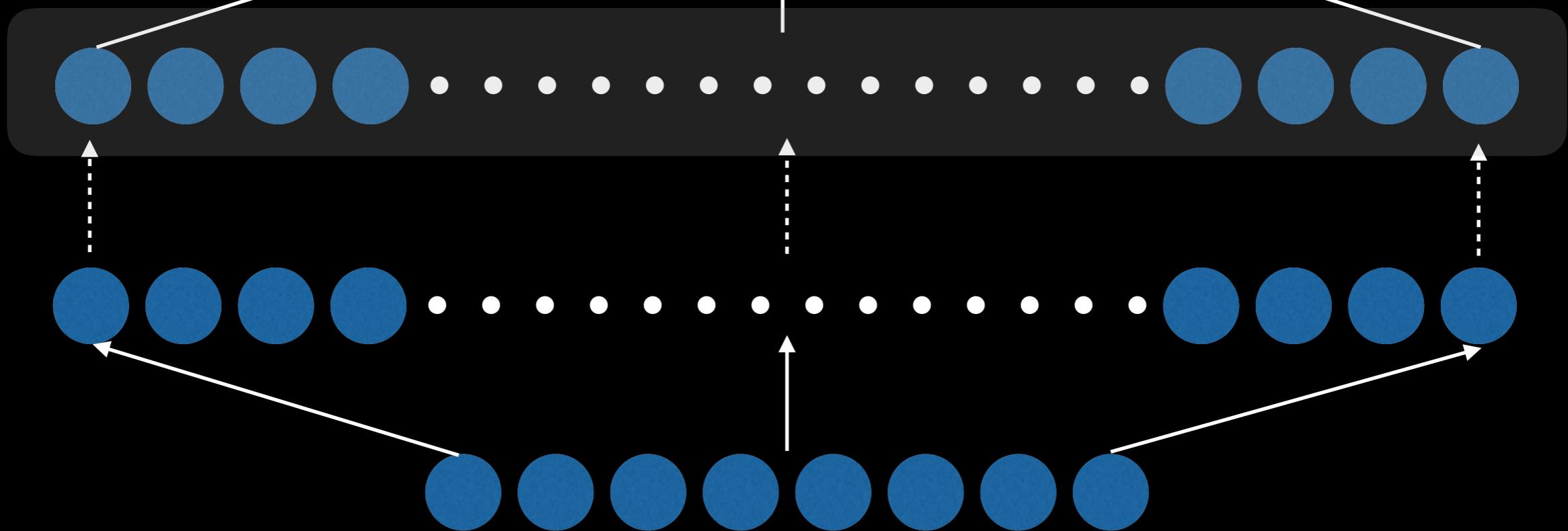
Policy Based

π

policy $\pi(a|s)$

$\pi(\text{north}|s)$ $\pi(\text{south}|s)$ $\pi(\text{east}|s)$ $\pi(\text{west}|s)$

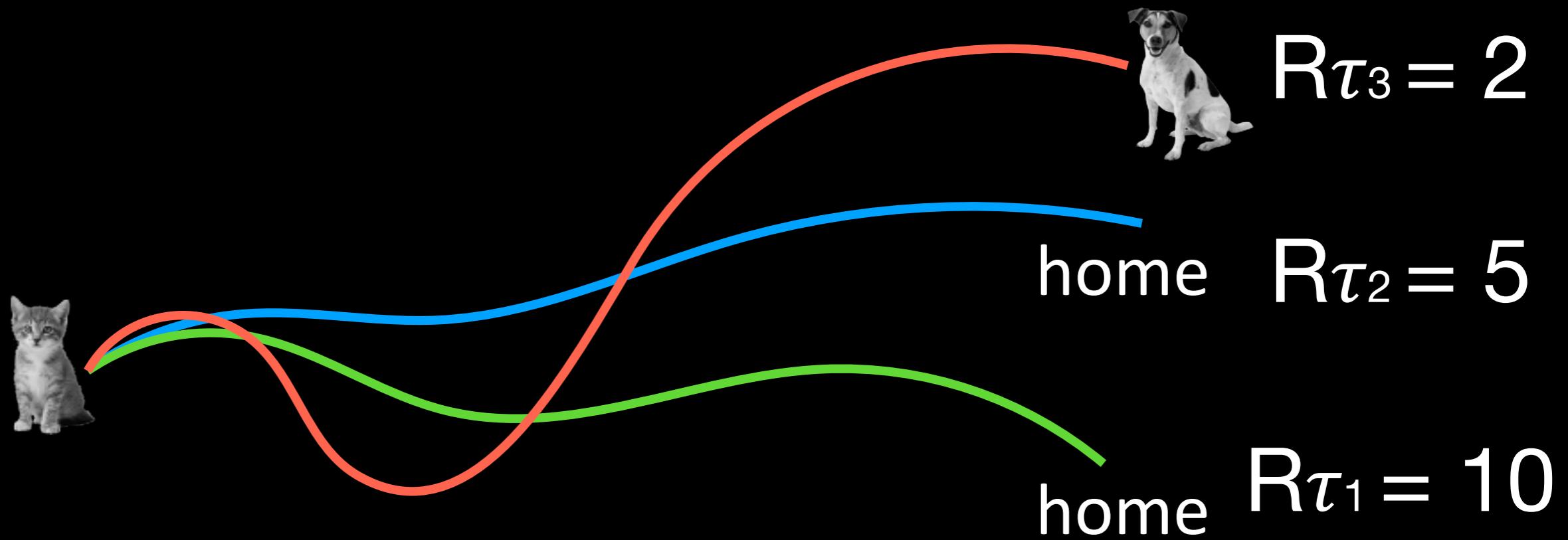
features



state s

Intuition

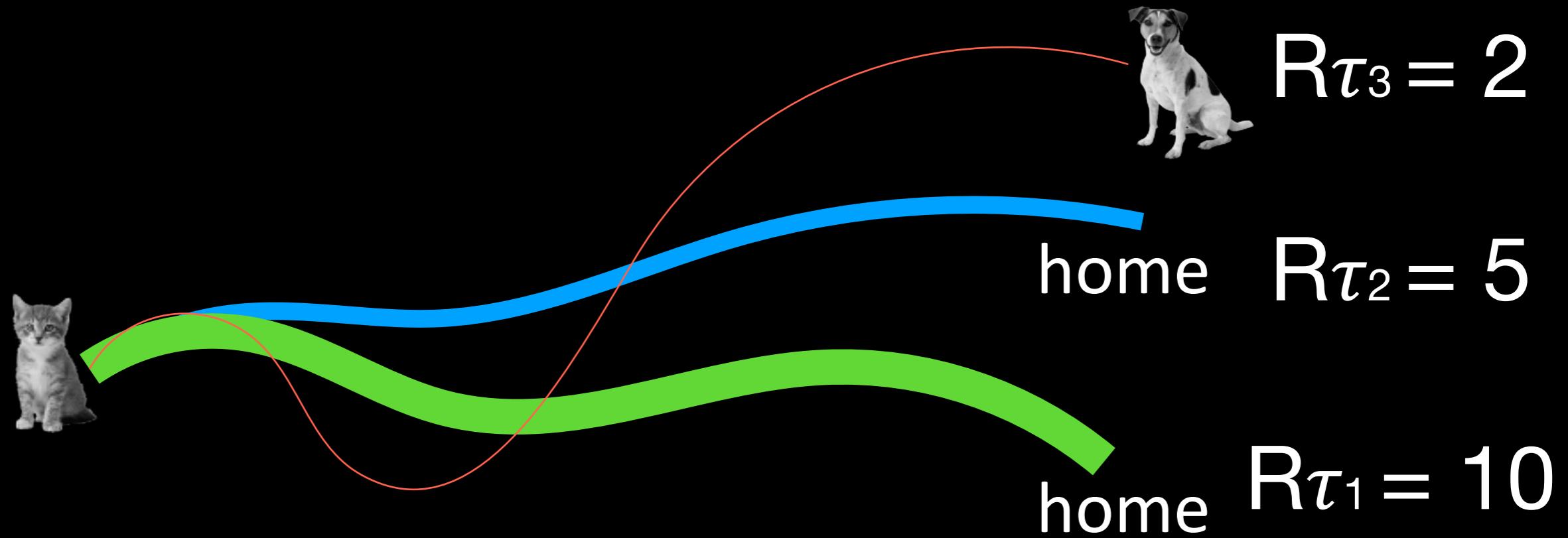
$$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$$



Intuition

$\pi(a|s)$ along path with high return higher

$$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$$



probabilities are relative

Revisiting the Objective

$$\tau : s_1, a_1, r_1^1, s_2, a_2, r_2^2, \dots, s_{H-1}, a_{H-1}, r_{H-1}^{H-1}$$

$$\max_{\theta} \mathbb{E}_{\tau} \left\{ \sum_{t=0}^{H-1} r_{s_t}^{a_t} \mid \pi_{\theta} \right\}$$



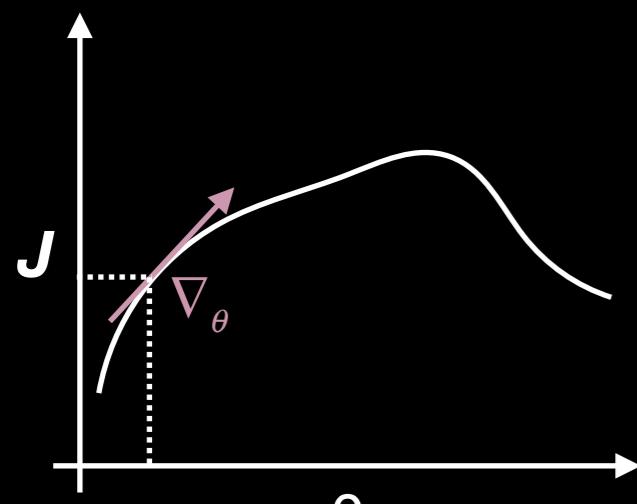
$$\max_{\theta} J(\theta) = \max_{\theta} \sum_{\tau} P(\tau \mid \theta) R(\tau)$$

Samples → Gradient

$$J(\theta) = \sum_{\tau} P(\tau | \theta) R(\tau)$$

$$\max_{\theta} J(\theta)$$

$$\theta \leftarrow \theta + \nabla_{\theta} J(\theta)$$

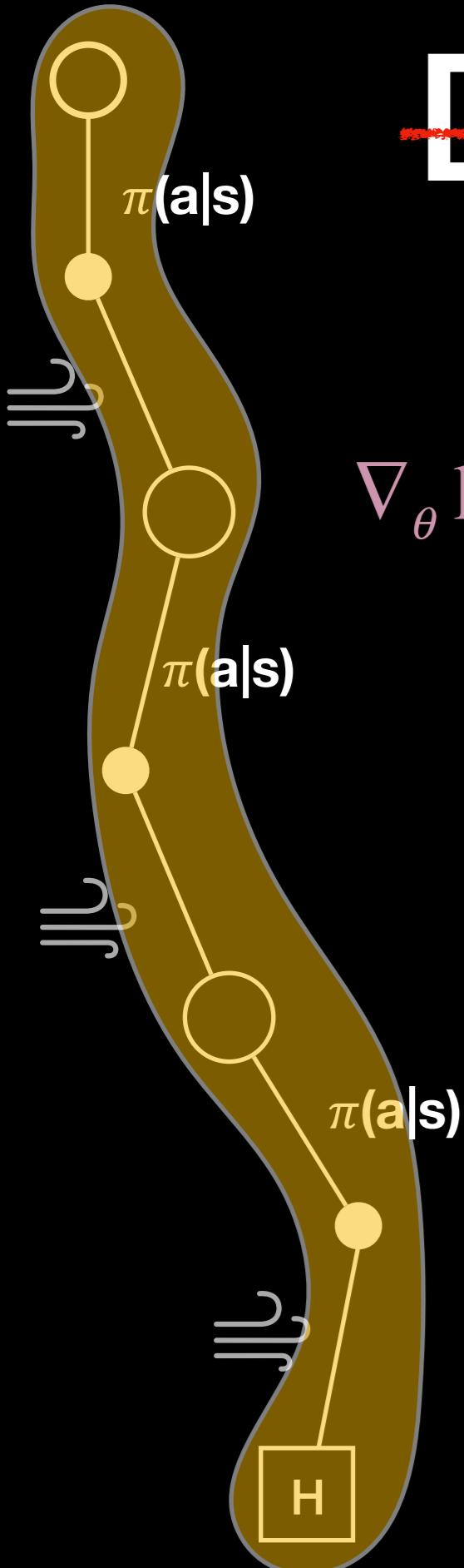


gradient
via
sampling

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \nabla_{\theta} \sum_{\tau} P(\tau | \theta) R(\tau) \\ &= \sum_{\tau} \nabla_{\theta} P(\tau | \theta) R(\tau) \\ &= \sum_{\tau} \frac{P(\tau | \theta)}{P(\tau | \theta)} \nabla_{\theta} P(\tau | \theta) R(\tau) \\ &= \sum_{\tau} P(\tau | \theta) \frac{\nabla_{\theta} P(\tau | \theta)}{P(\tau | \theta)} R(\tau) \\ &= \sum_{\tau} P(\tau | \theta) \nabla_{\theta} \log P(\tau | \theta) R(\tau)\end{aligned}$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} \log P(\tau^{(i)} | \theta) R(\tau^{(i)})$$

Dynamics Model



$$\begin{aligned}
 \nabla_{\theta} \log P(\tau | \theta) &= \nabla_{\theta} \log \left[\prod_{t=0}^{H-1} P(s_{t+1} | s_t, a_t) \cdot \pi_{\theta}(a_t | s_t) \right] \\
 &\stackrel{\text{dynamics model}}{\Rightarrow} \sum_{t=0}^{H-1} \log P(s_{t+1} | s_t, a_t) + \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t | s_t) \\
 &= \nabla_{\theta} \sum_{t=0}^{H-1} \log \pi_{\theta}(a_t | s_t) = \sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)
 \end{aligned}$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \left(\sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right) R(\tau^{(i)})$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{m} \sum_{i=1}^m \left(\sum_{t=0}^{H-1} \nabla_{\theta} \log \pi_{\theta}(a_t^{(i)} | s_t^{(i)}) \right) R(\tau^{(i)})$$

For each action a_t in state s_t
during each trajectory m

$$\underline{\nabla_{\theta} \log \pi_{\theta}(a_t | s_t)} \underline{R(\tau)}$$

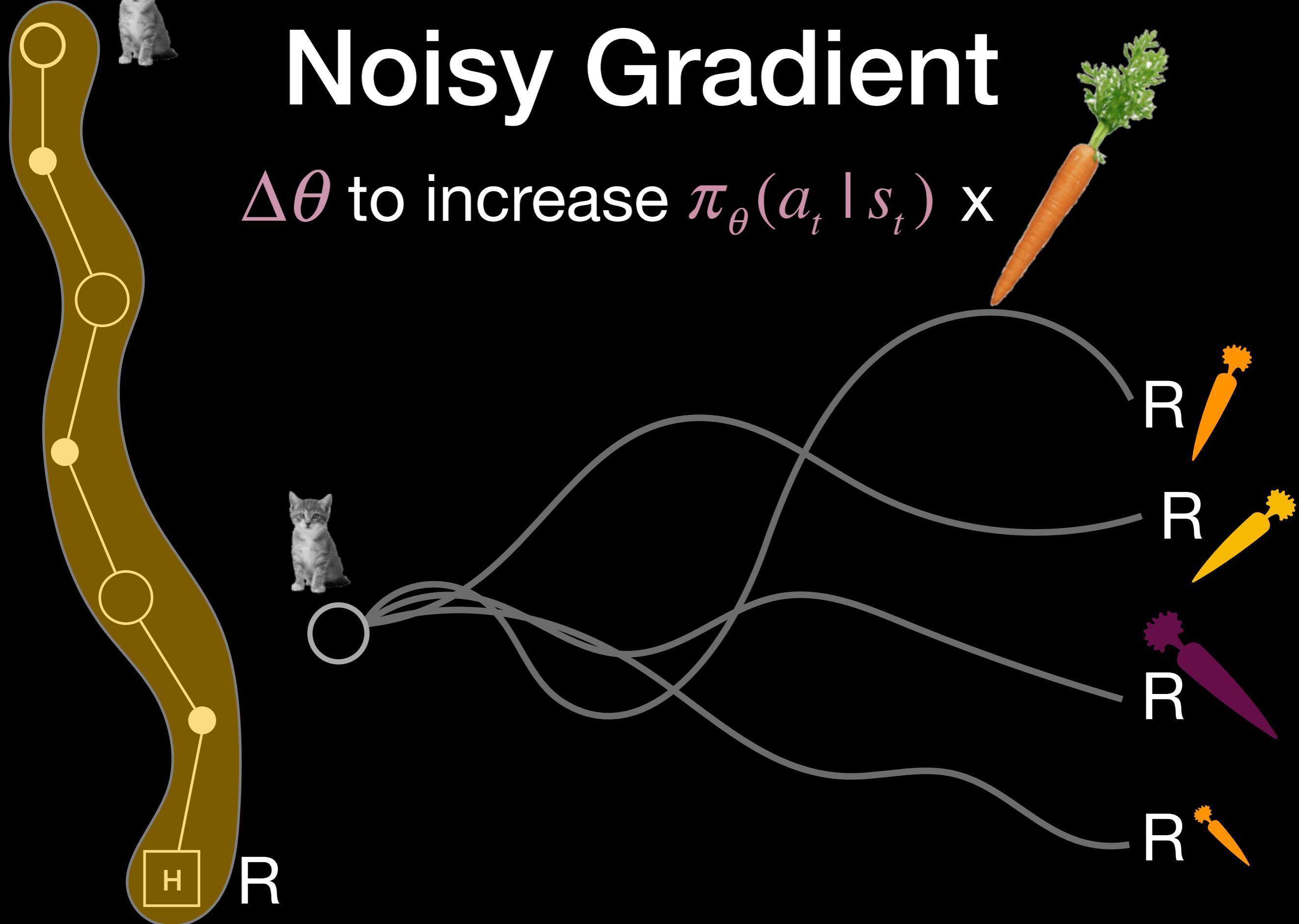
$\Delta \theta$ to increase $\pi_{\theta}(a_t | s_t)$ x

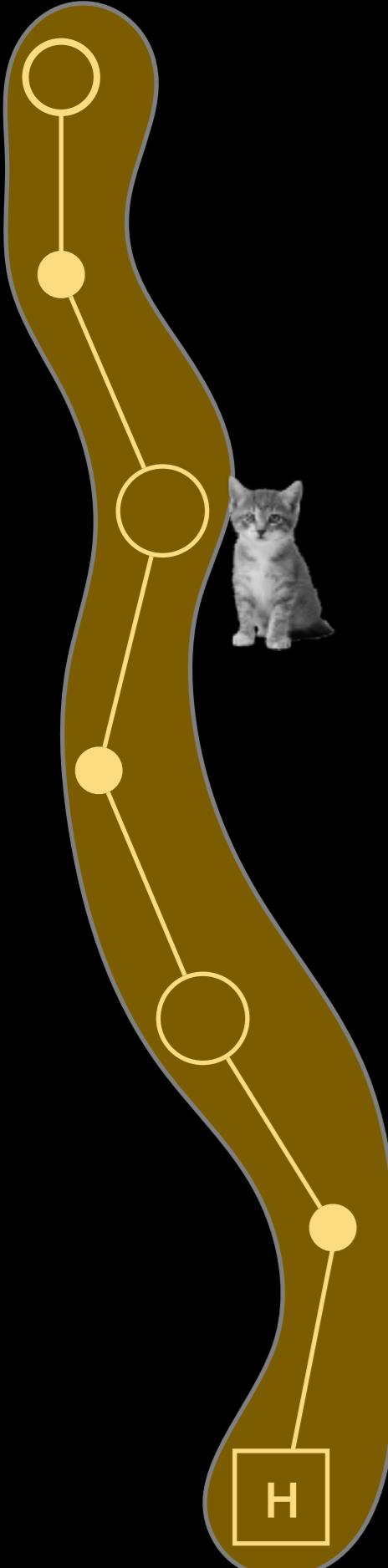




Noisy Gradient

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto$





$R(\tau_t \text{ onwards})$

Reduce Noise

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto$

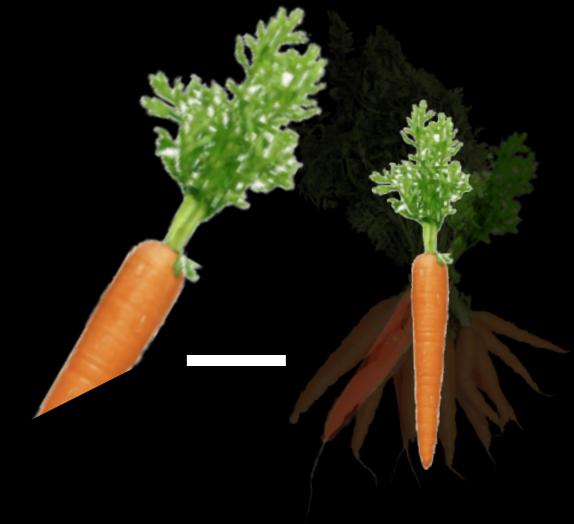


R time t onwards

$R(\tau_t \text{ onwards}) - b$

Reduce Noise

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto$



baseline b
(how much is action
better than average)



$V = E\{R | s\}$



$$R(\tau_t \text{ onwards}) - V(s_t)$$

Reduce Noise

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto$



baseline b
(how much is action
better than average)



$$V = E\{R | s\}$$



Actor-Critic

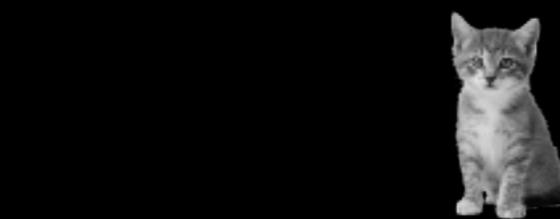
Reduce Noise

$$Q(s_t, a_t) - V(s_t)$$

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto$

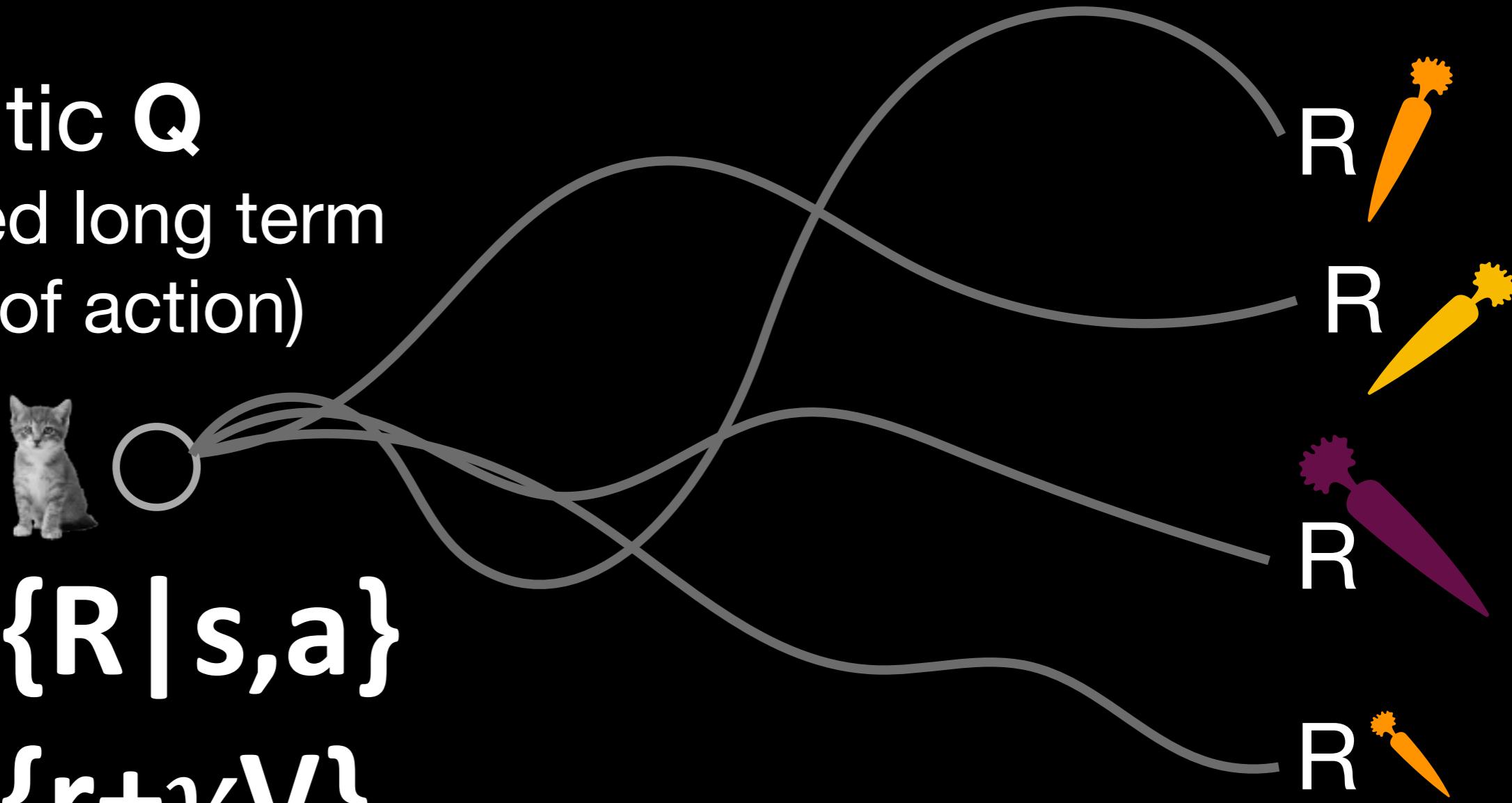


critic **Q**
(expected long term
value of action)



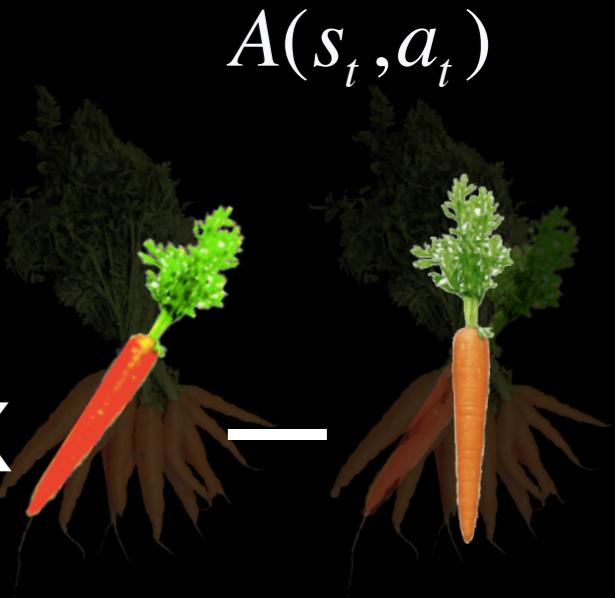
$$Q = E\{R | s, a\}$$

$$= E\{r + \gamma V\}$$



Reduce Noise

$\Delta\theta$ to increase $\pi_\theta(a_t | s_t) \propto -$



$$A = Q - V$$

(advantage of
an action)



(how much is action
better than average)



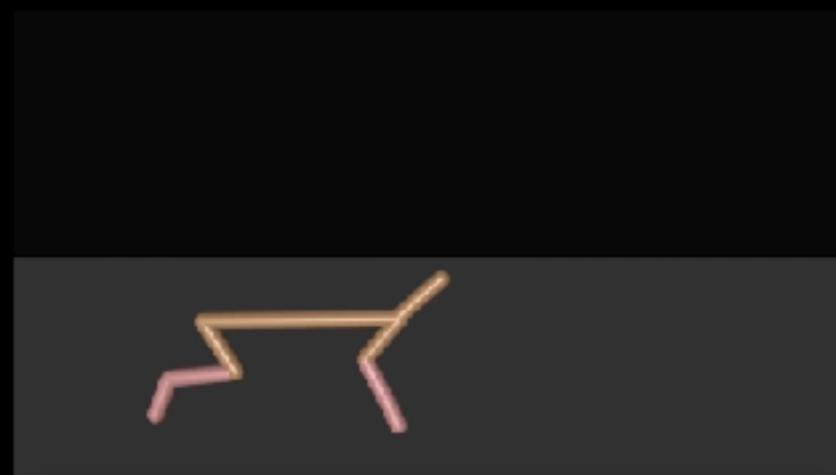
parallelise...

Parallel Asynchronous Training

value and **policy** based methods



<https://youtu.be/0xo1Ldx3L5Q>



<https://youtu.be/Ajjc08-iPx8>

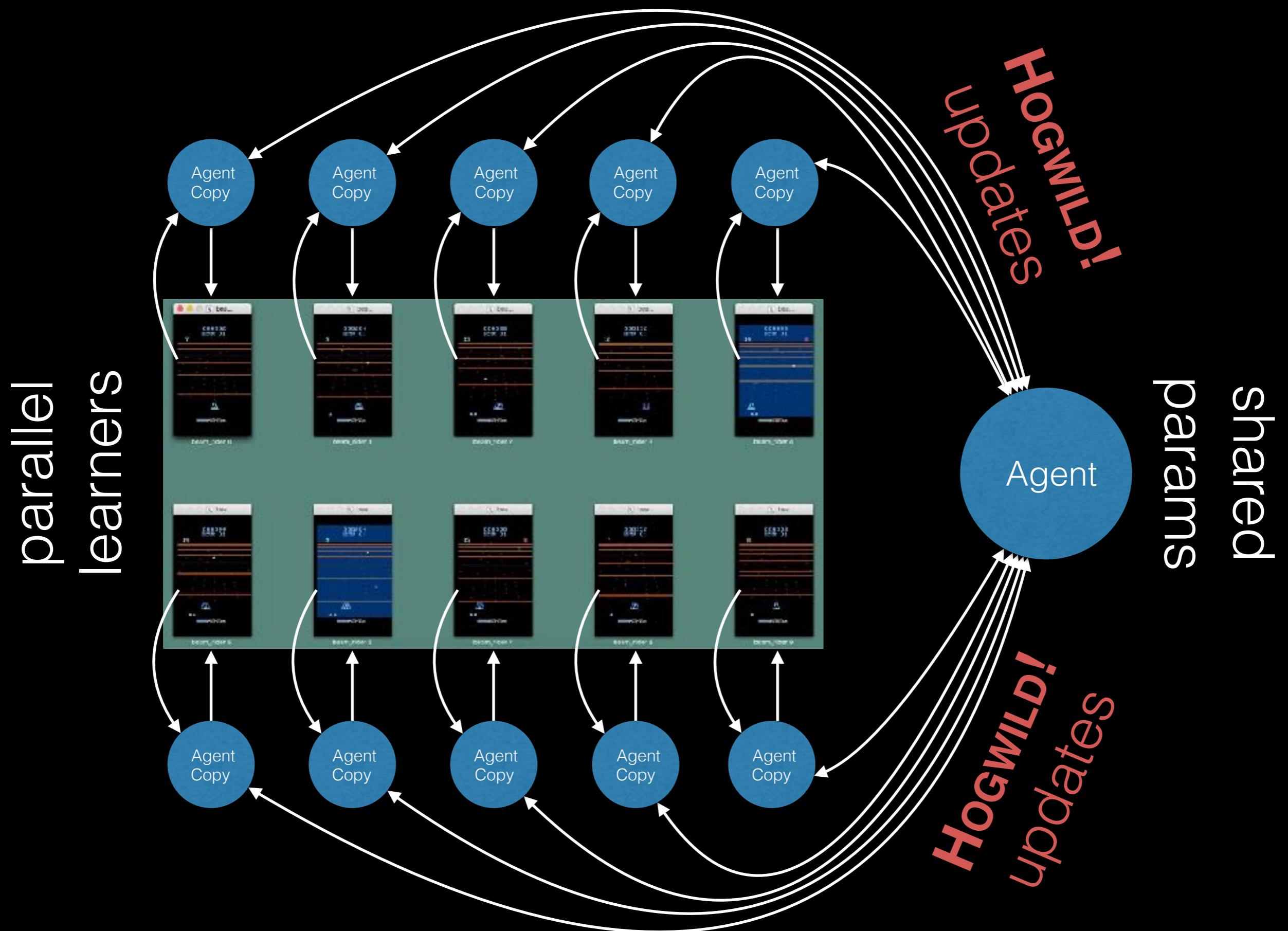


<https://youtu.be/nMR5mjCFZCw>

parallel
agents

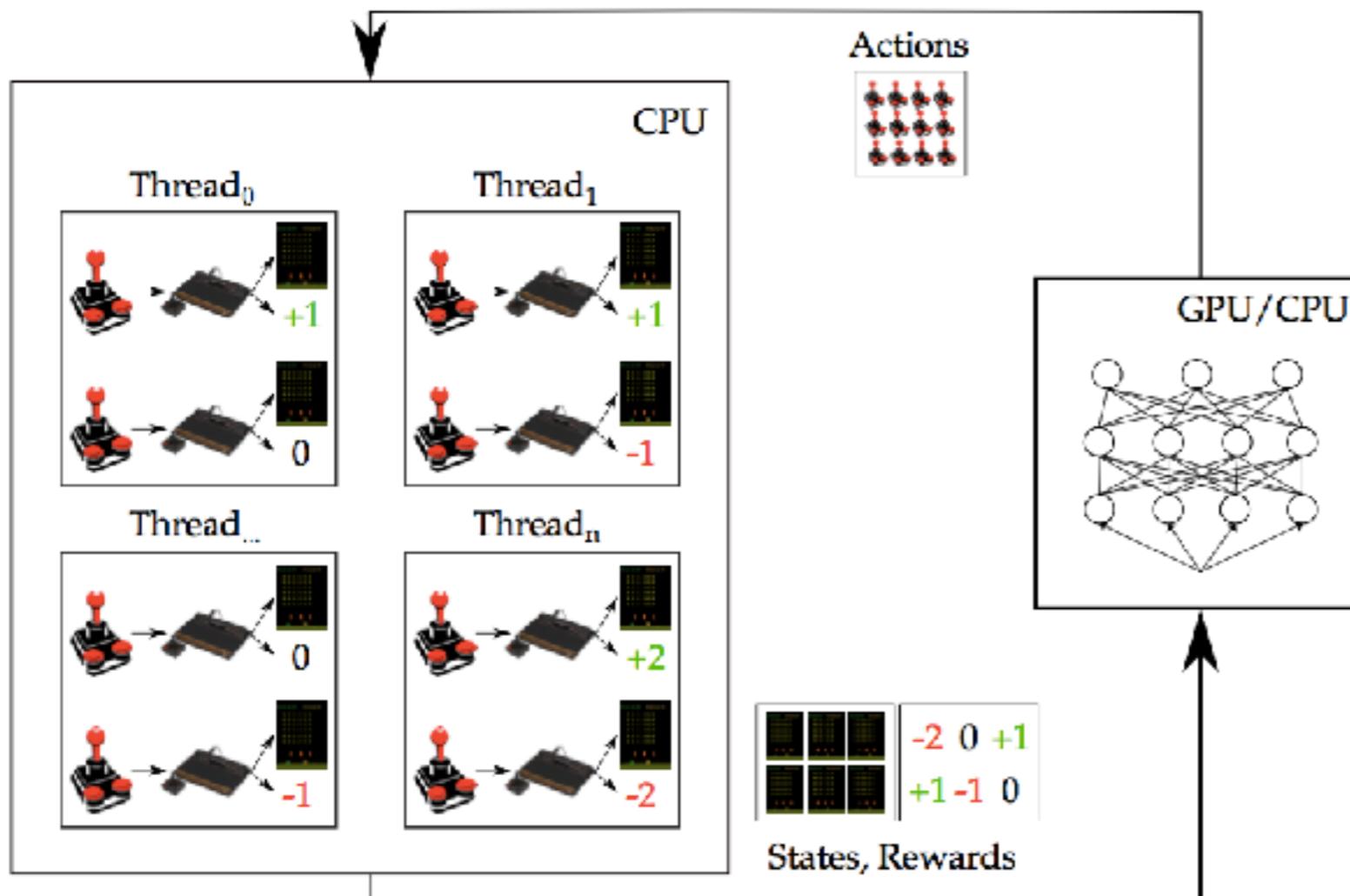
shared
parameters

lock-free
updates



PAAC

(Parallel Advantage Actor-Critic)



1 GPU/CPU
Reduced
 training time
SOTA
 performance

<https://github.com/alfredvc/paac>

Efficient Parallel Methods for Deep Reinforcement Learning,
 A. V. Clemente, H. N. Castejón, and A. Chandra, **RLDM 2017**



Alfredo
Clemente

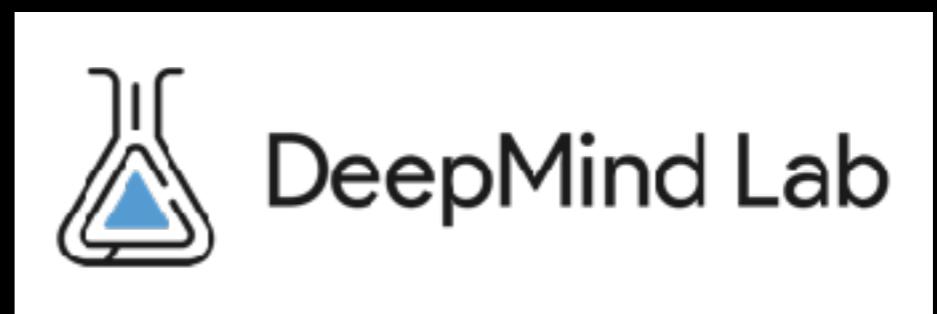
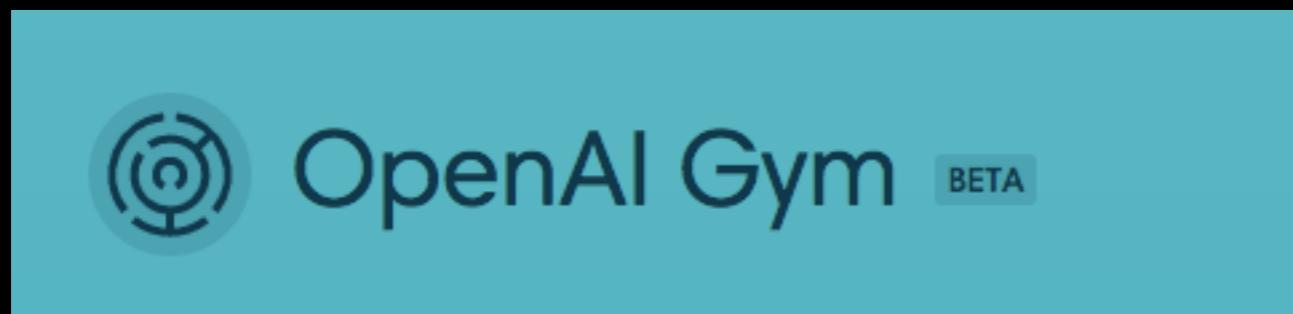
code for you to play with...

Rich Sutton's book examples (**exhaustive, must try!**):
<https://github.com/ShangtongZhang/reinforcement-learning-an-introduction>

Telenor's implementation of **asynchronous parallel methods**:
<https://github.com/traai/async-deep-rl>

Alfredo's **faster parallel methods**:
<https://github.com/alfredvc/paac>

++...





Inspired to
code/apply RL?

Next lecture:
Applications (and some **hacking**)
November 21, 2017

<https://join.slack.com/t/deep-rl-tutorial/signup>