

Module-2

Finite Automata



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Language Recognizers : An example of Finite Automata

An automaton is an abstract model of a digital computer.

Finite Automata(FA) is the simplest machine to recognize patterns.



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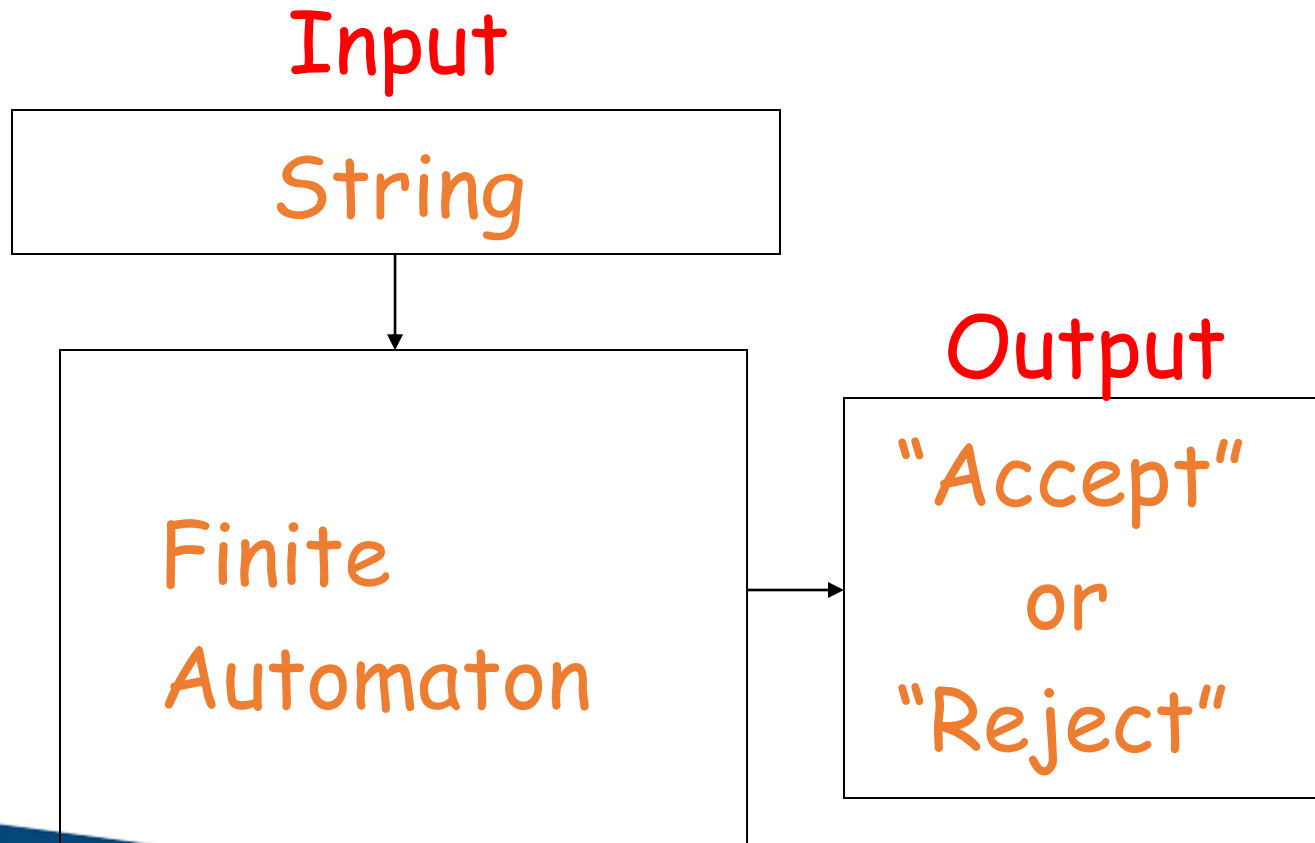
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Finite Automaton



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Representation of Finite Automata

Finite Automata is represented by -

1. Transition Graph
2. Transition Table
3. Regular Expression



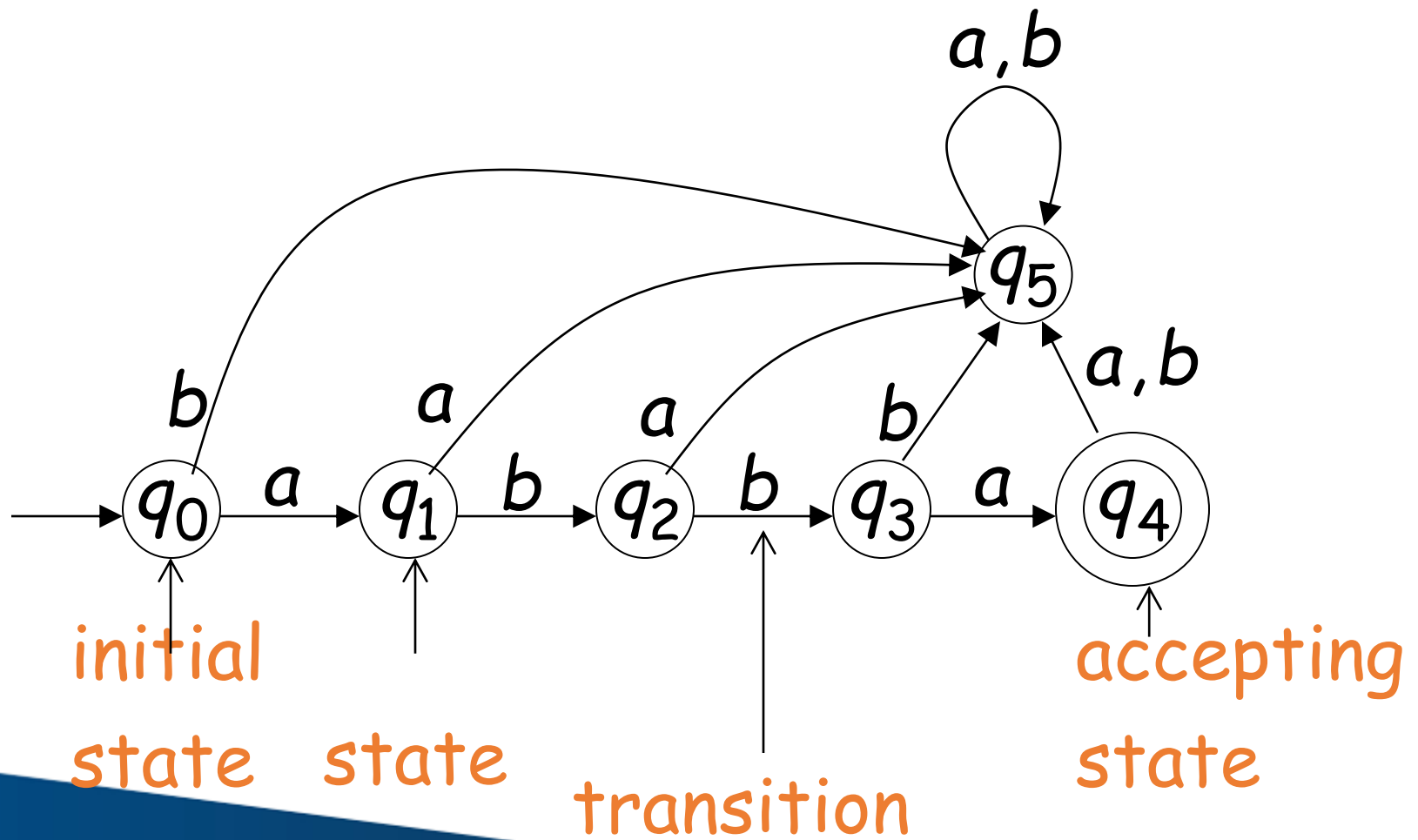
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Transition Graph



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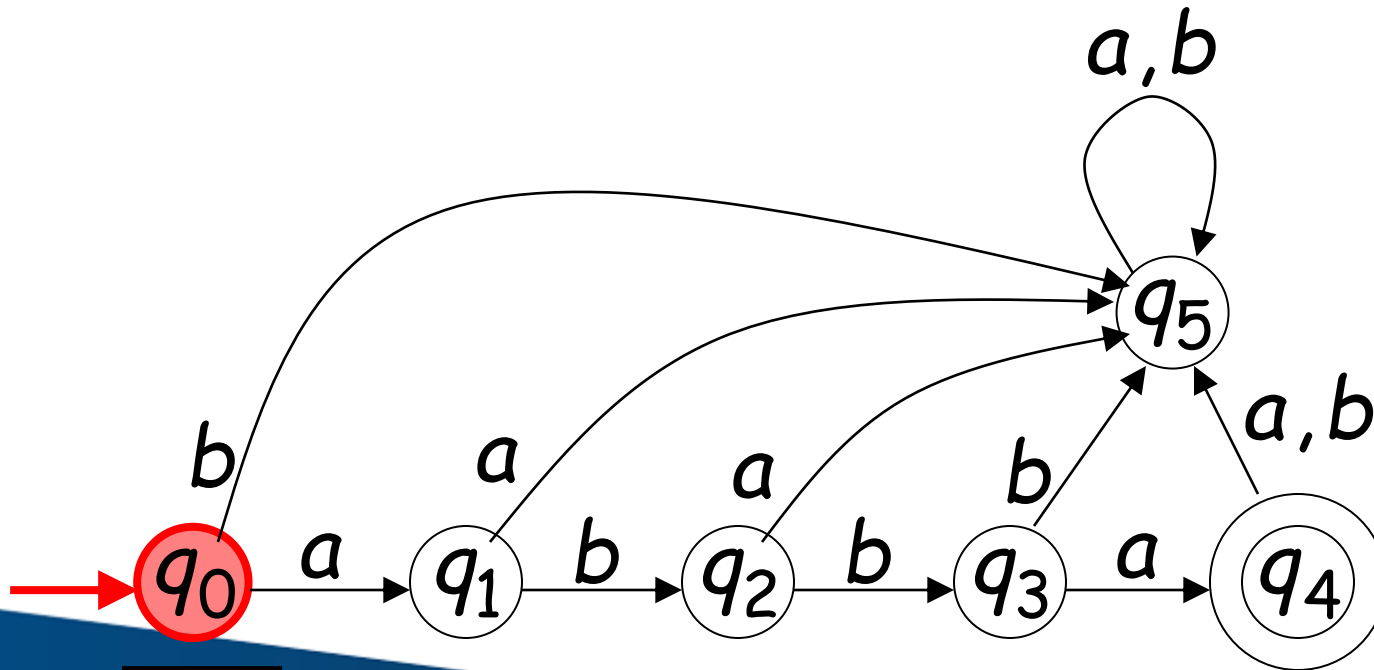
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Initial Configuration



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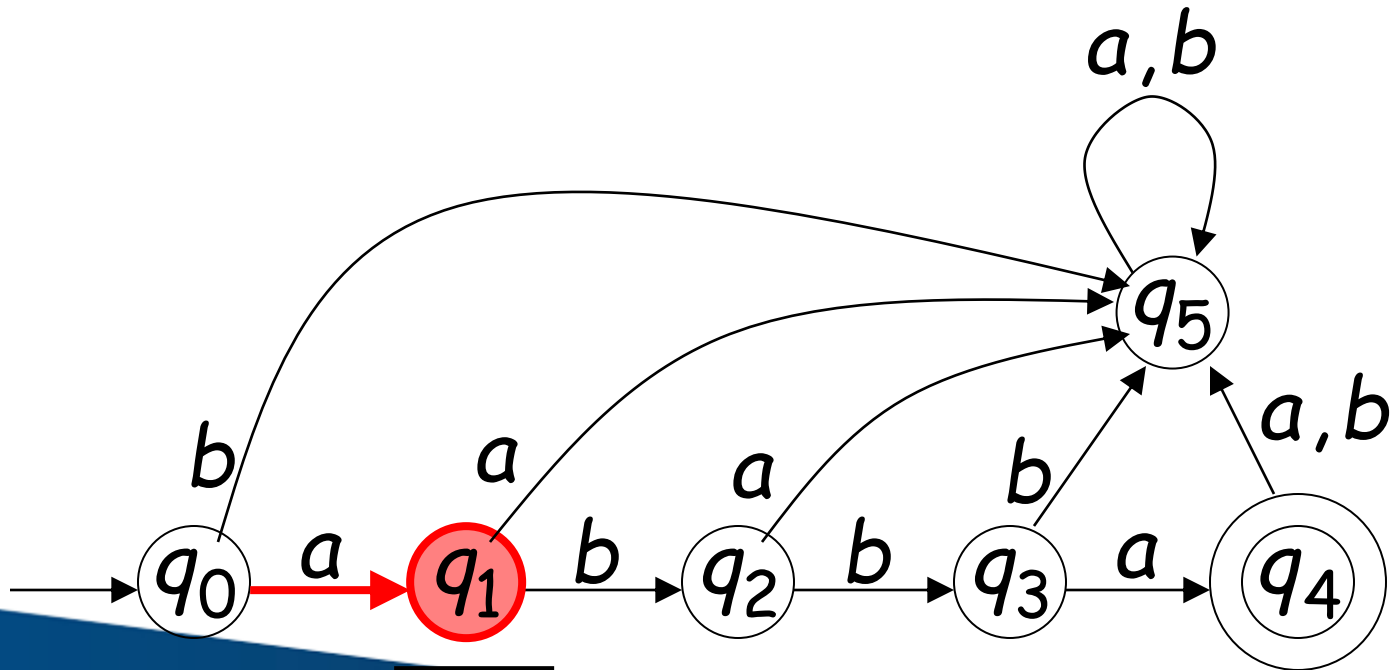
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Reading the Input



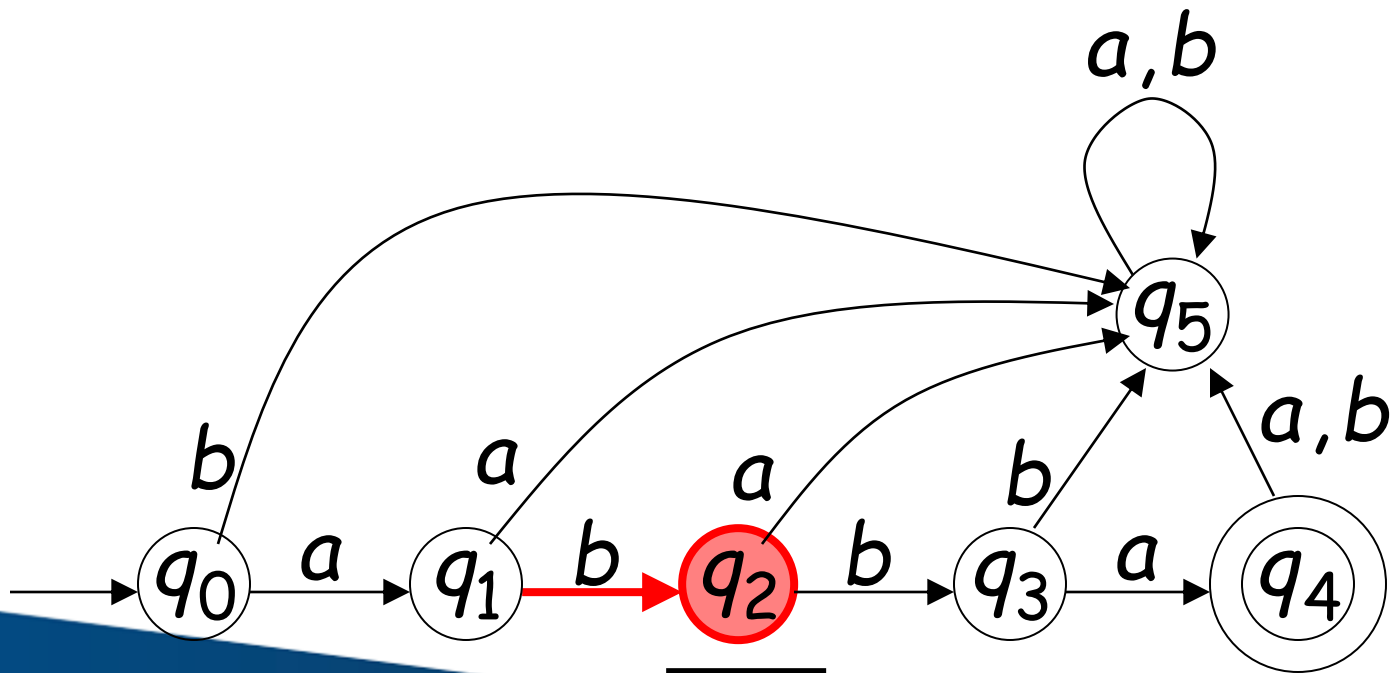
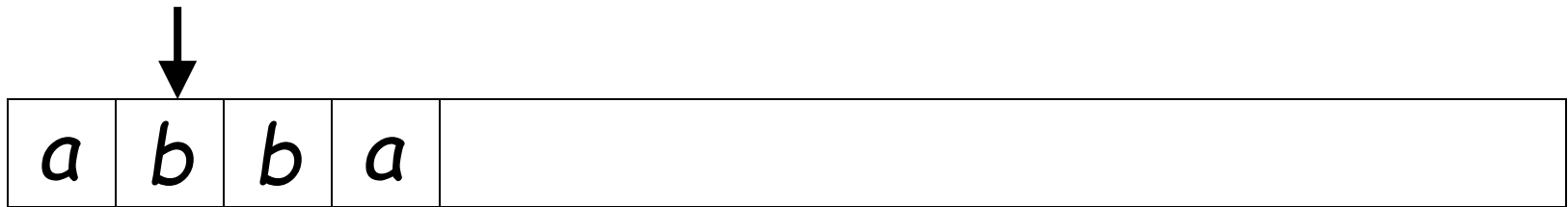
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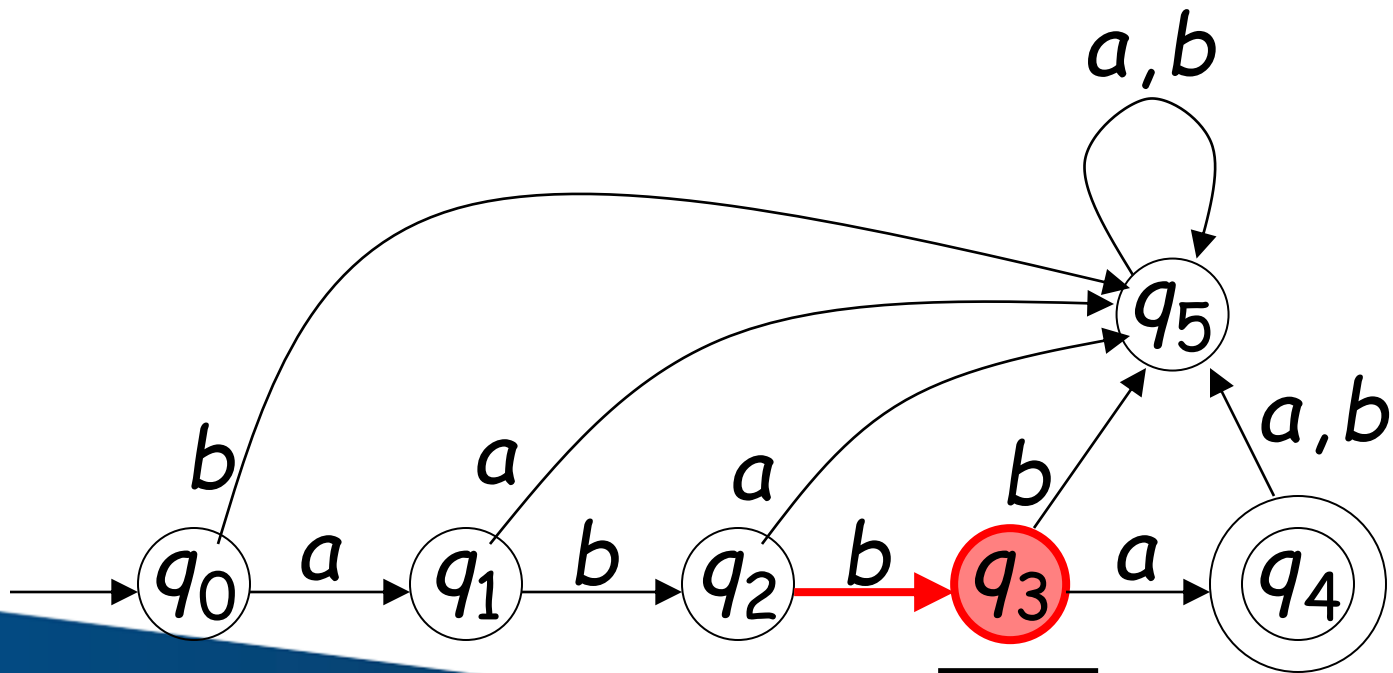
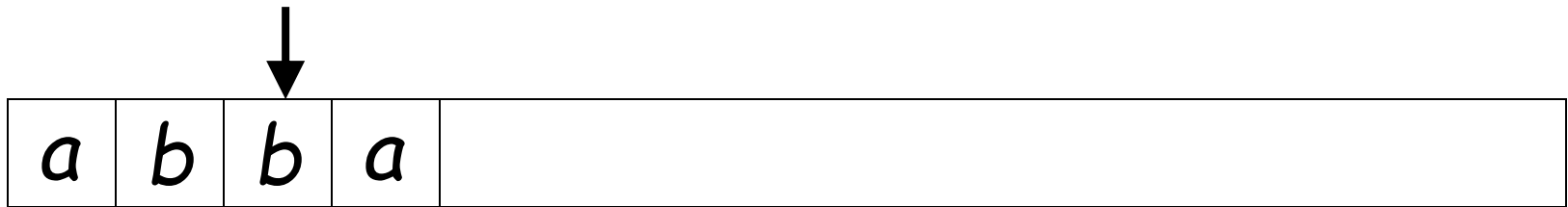
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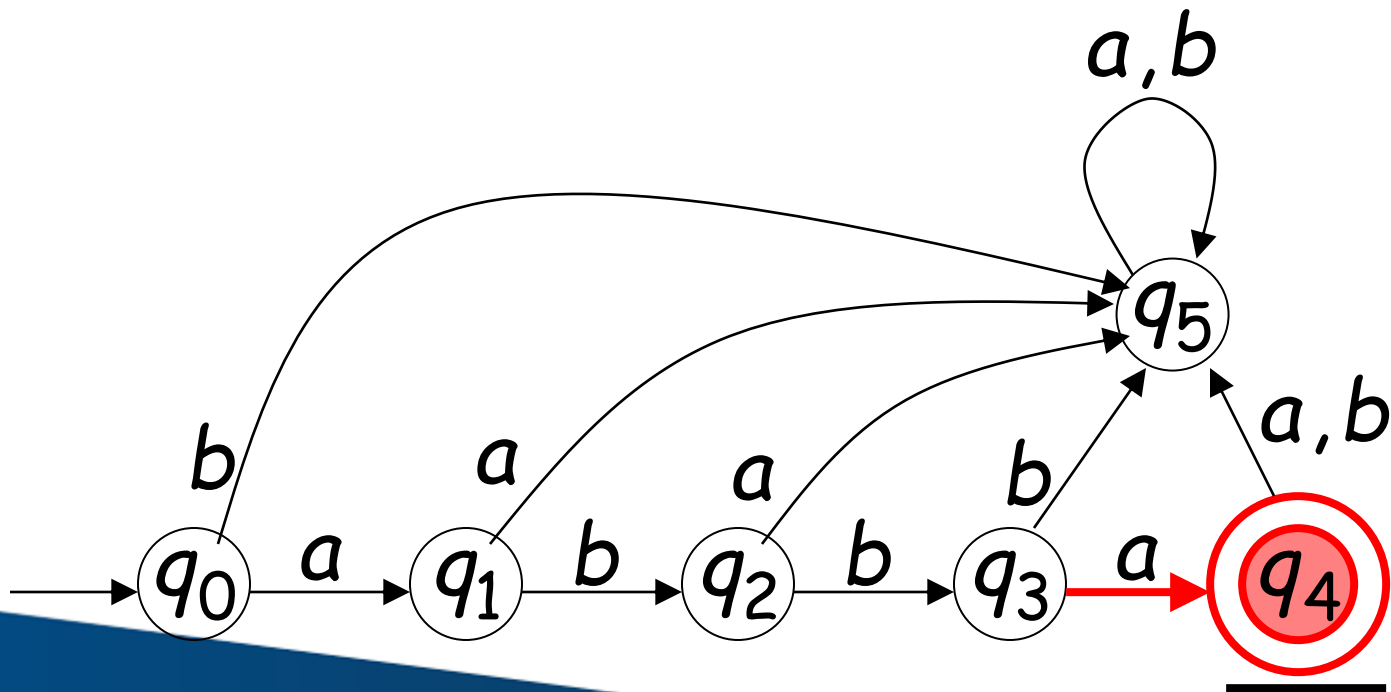
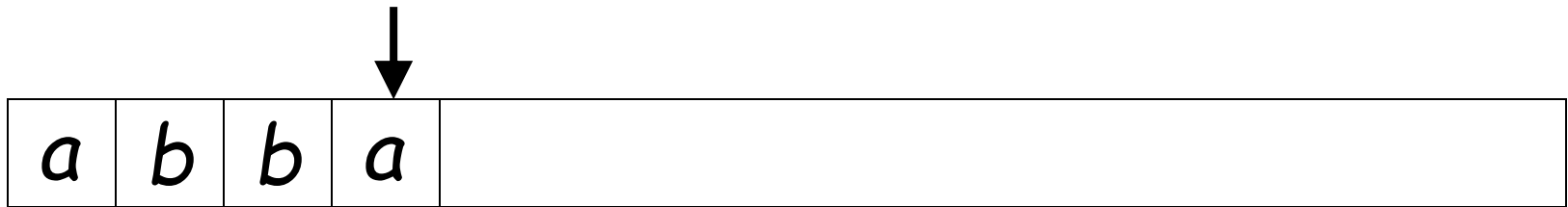
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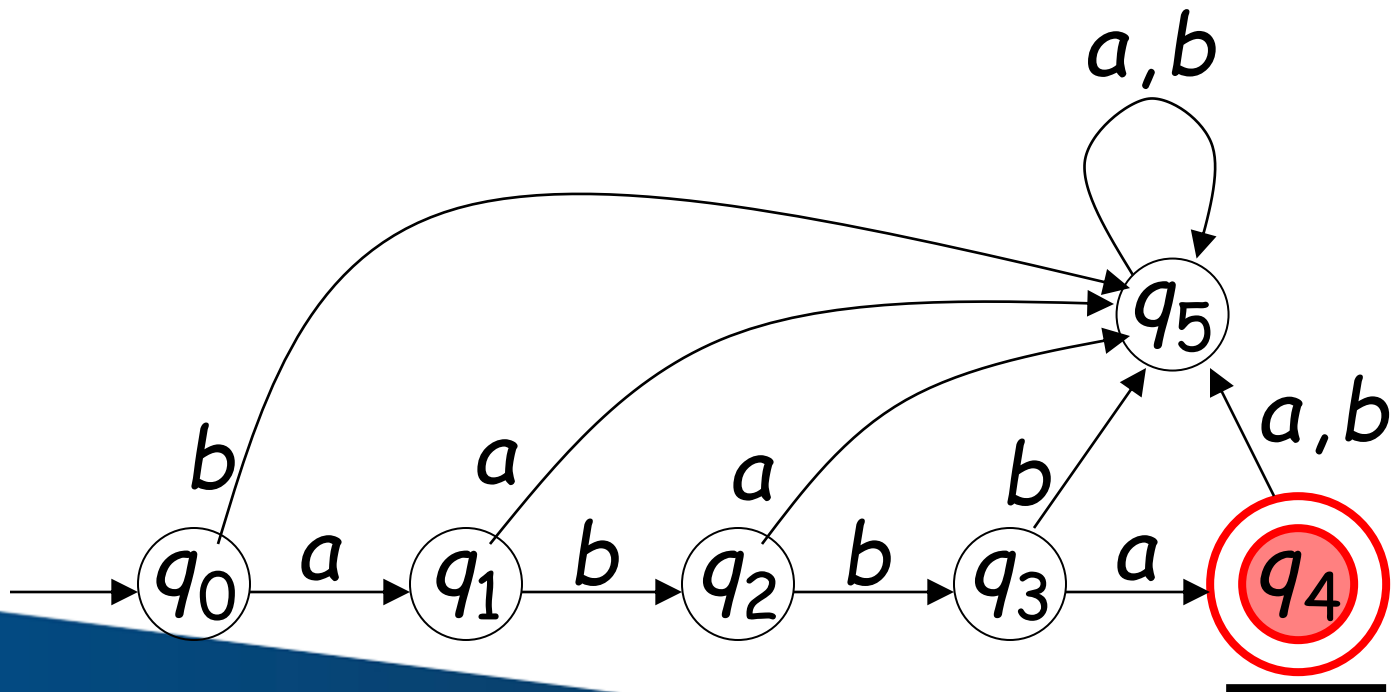
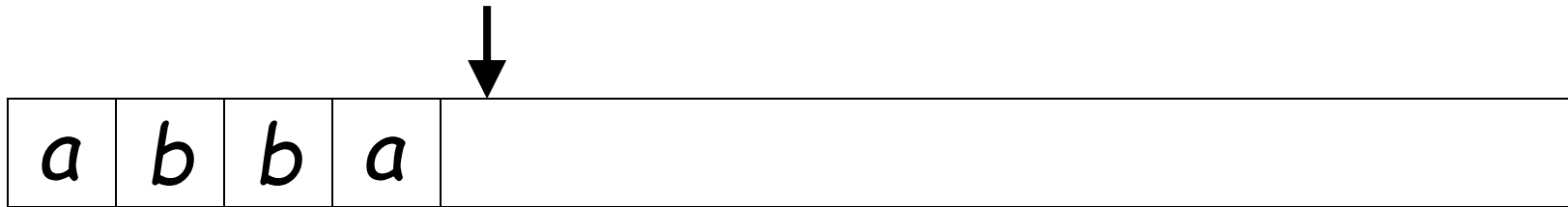
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accept



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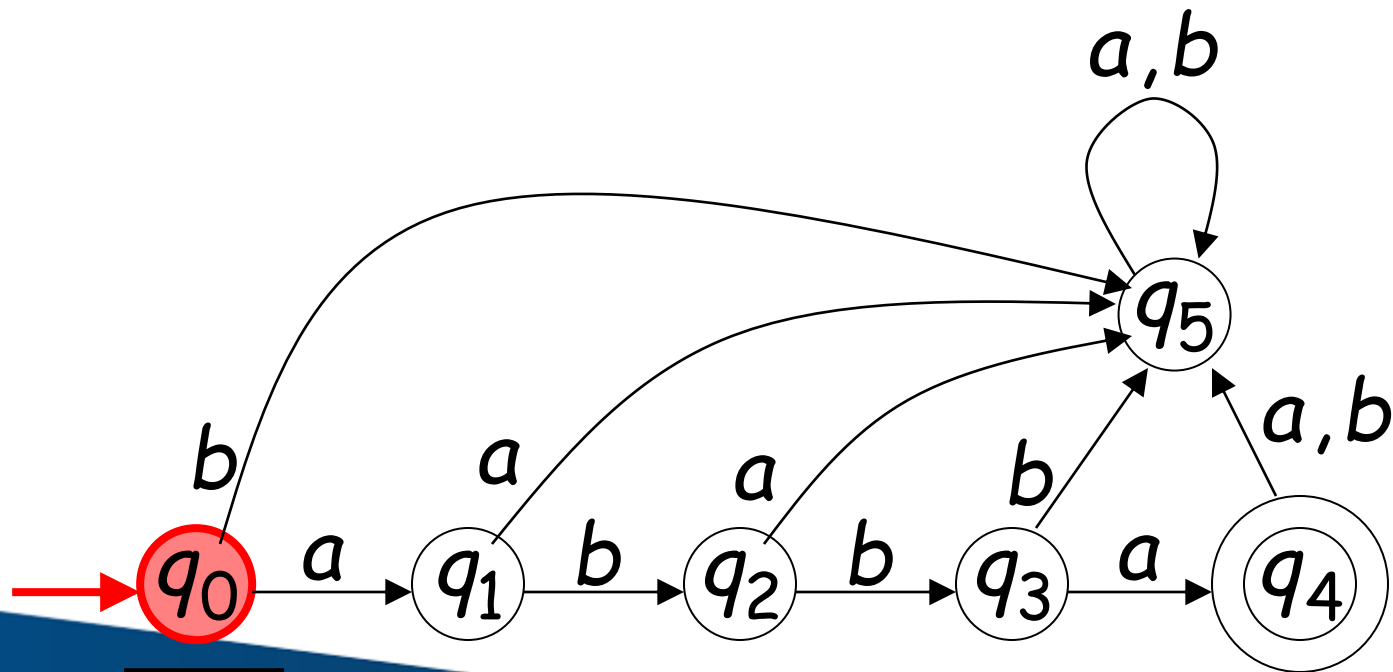
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Rejection

a	b	a	
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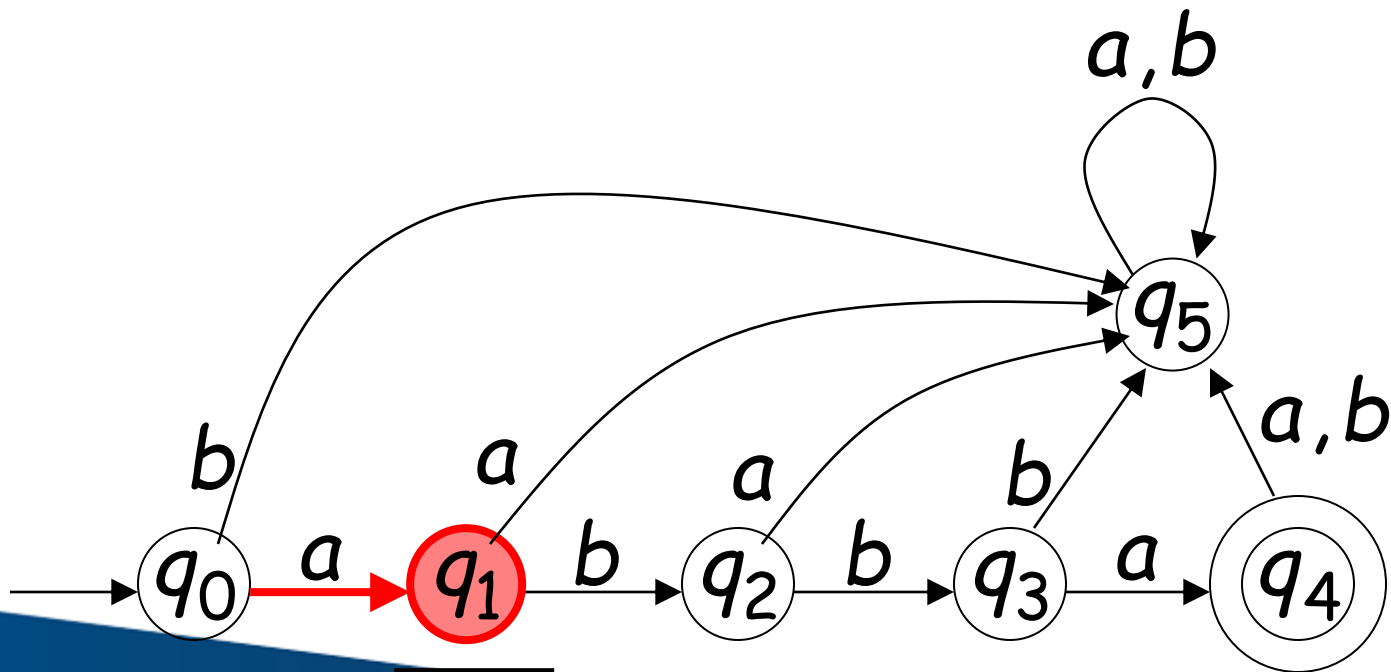
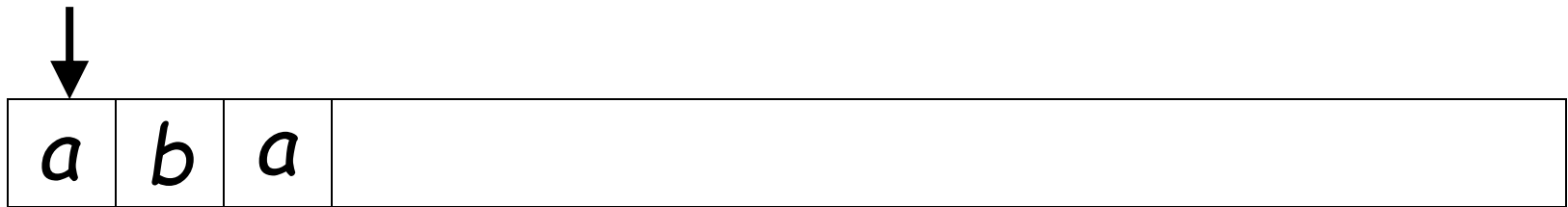
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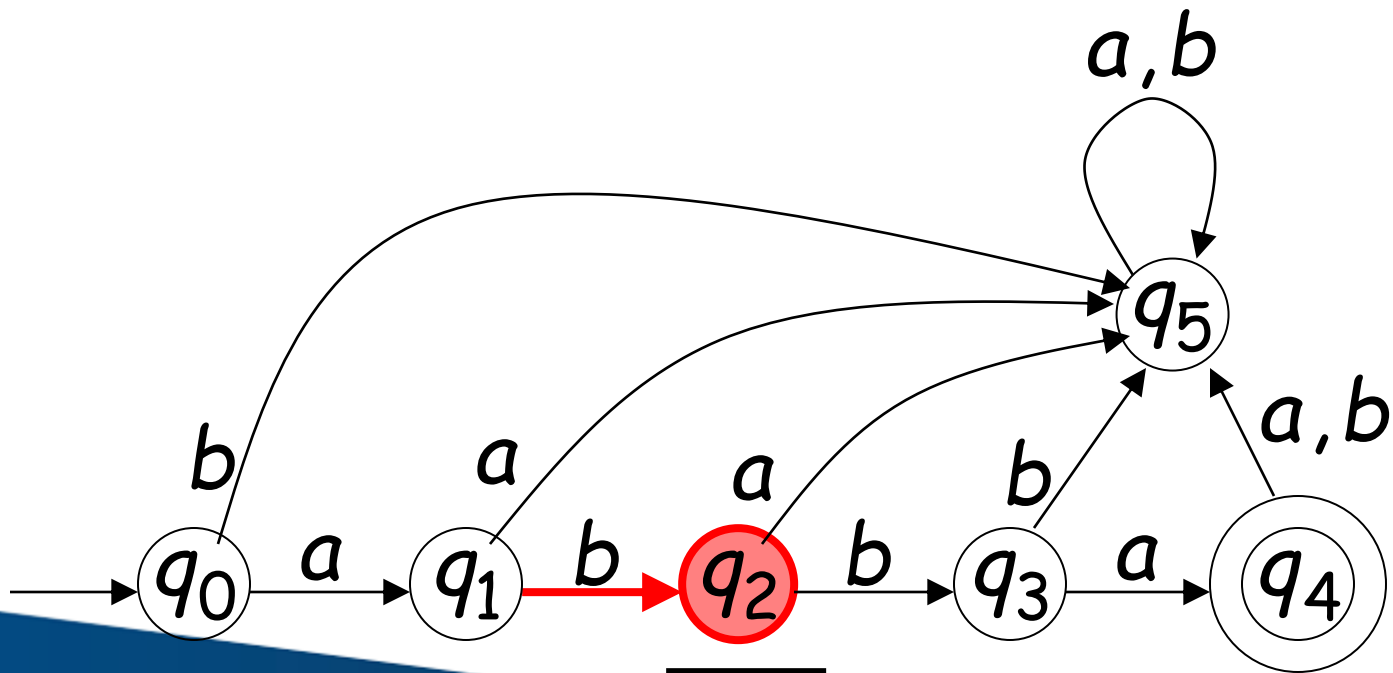
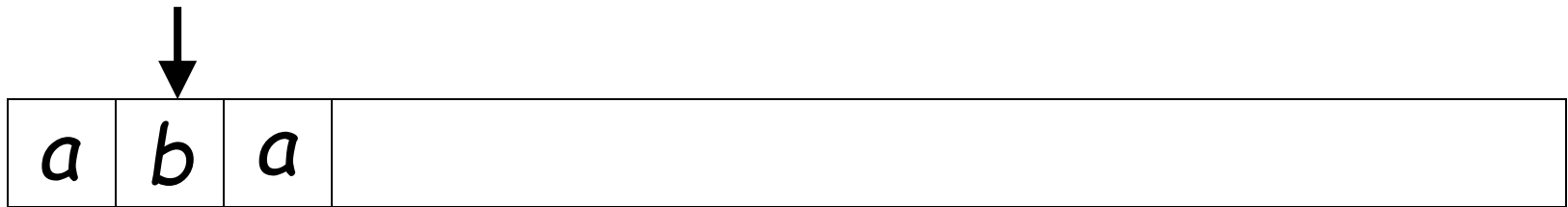
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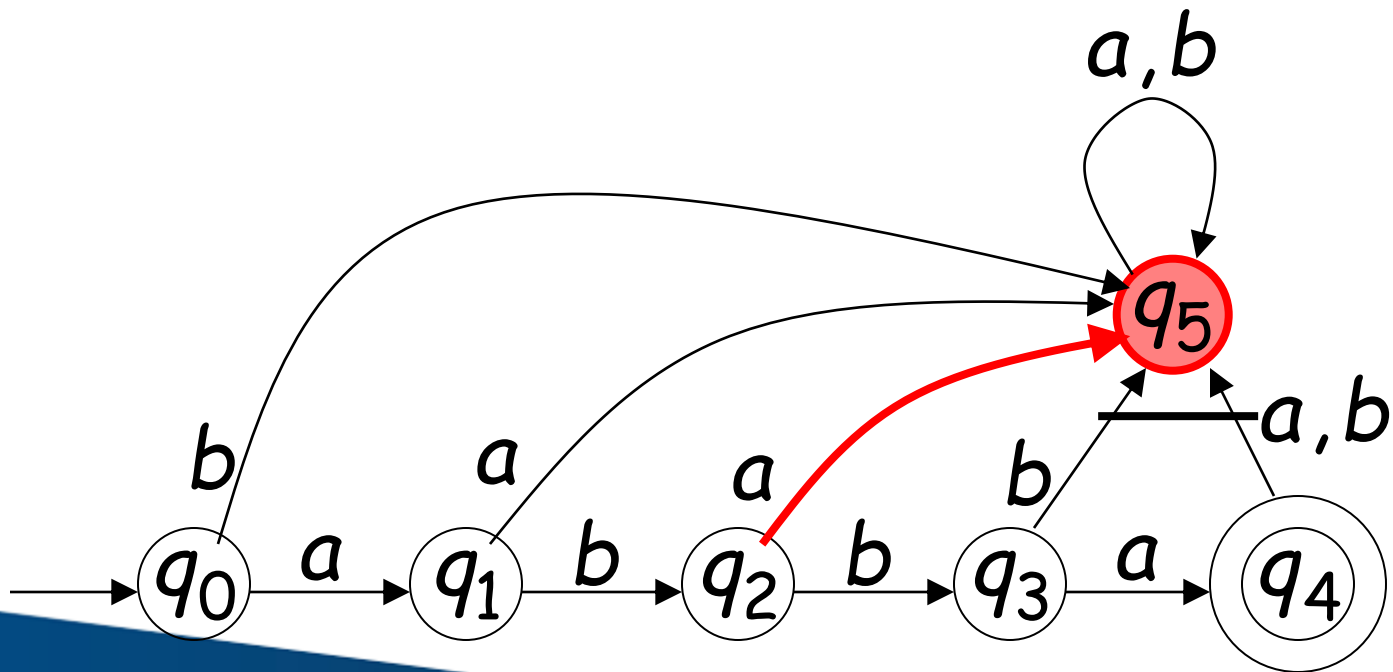
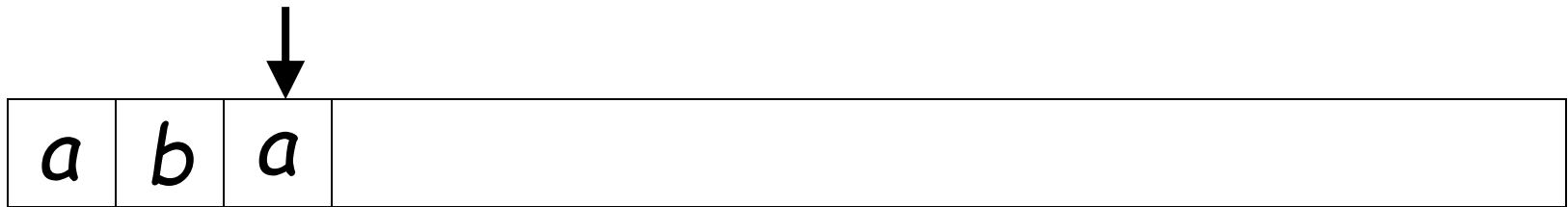
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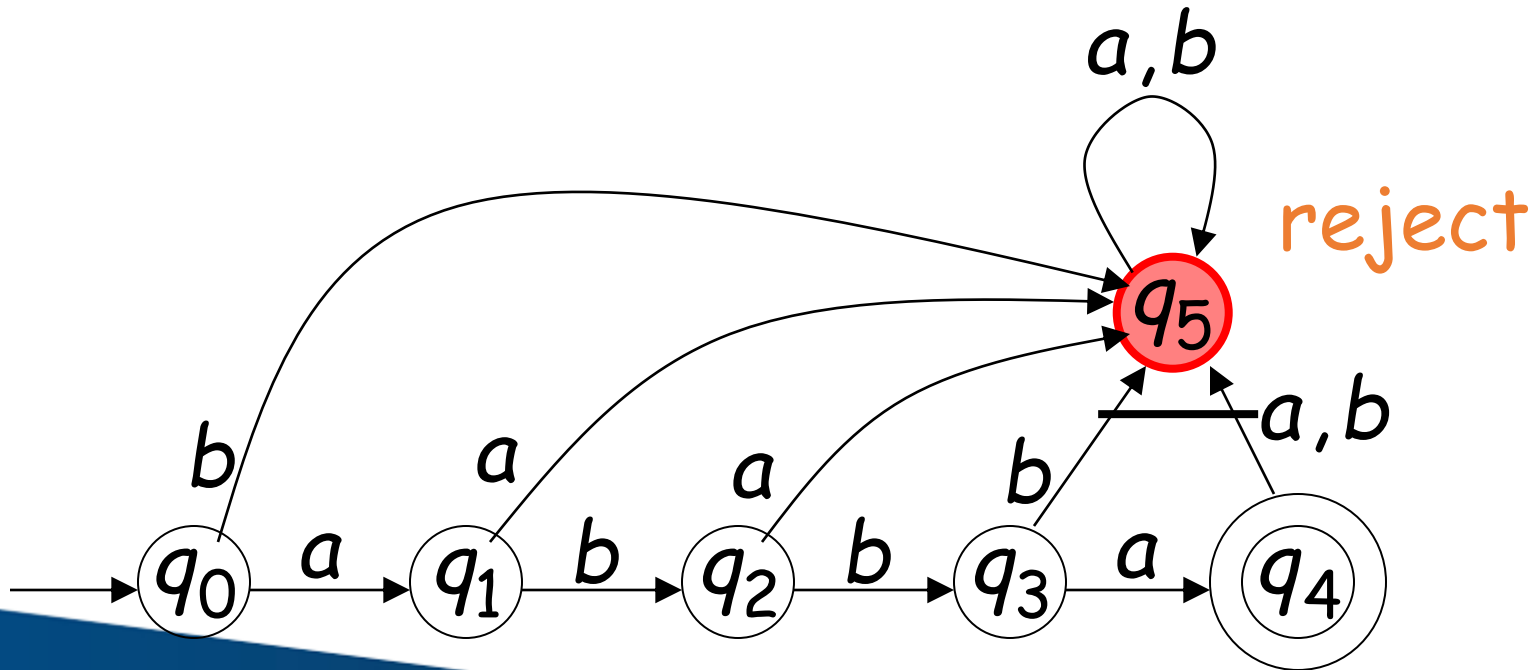
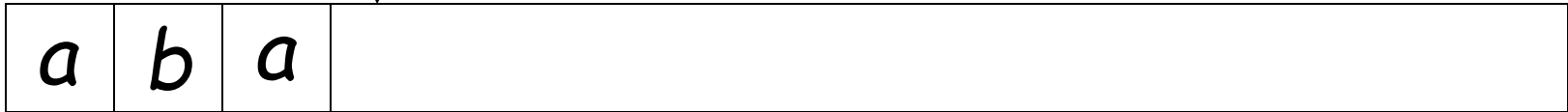


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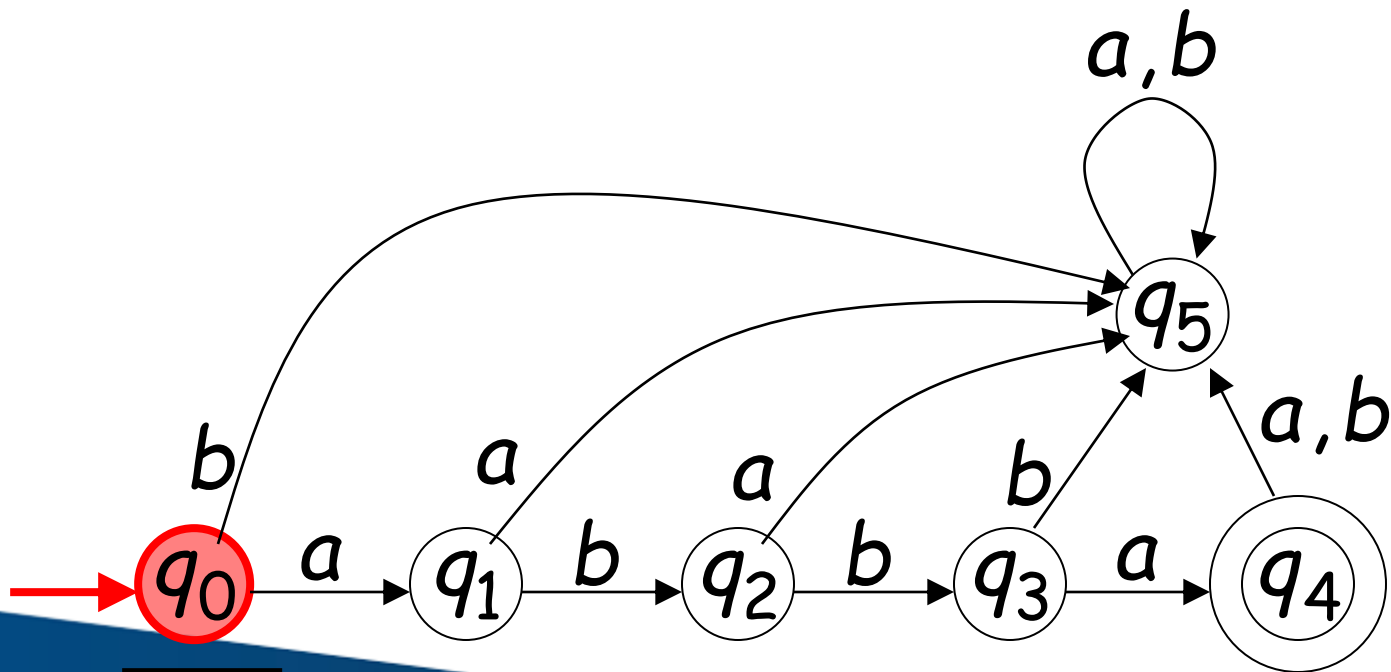
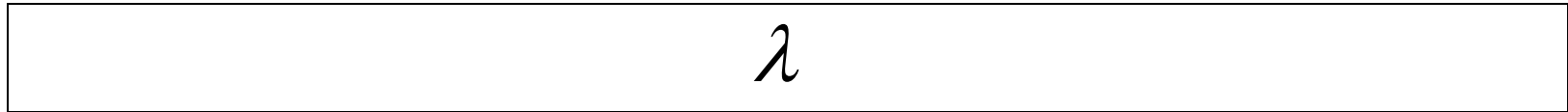
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Another Rejection



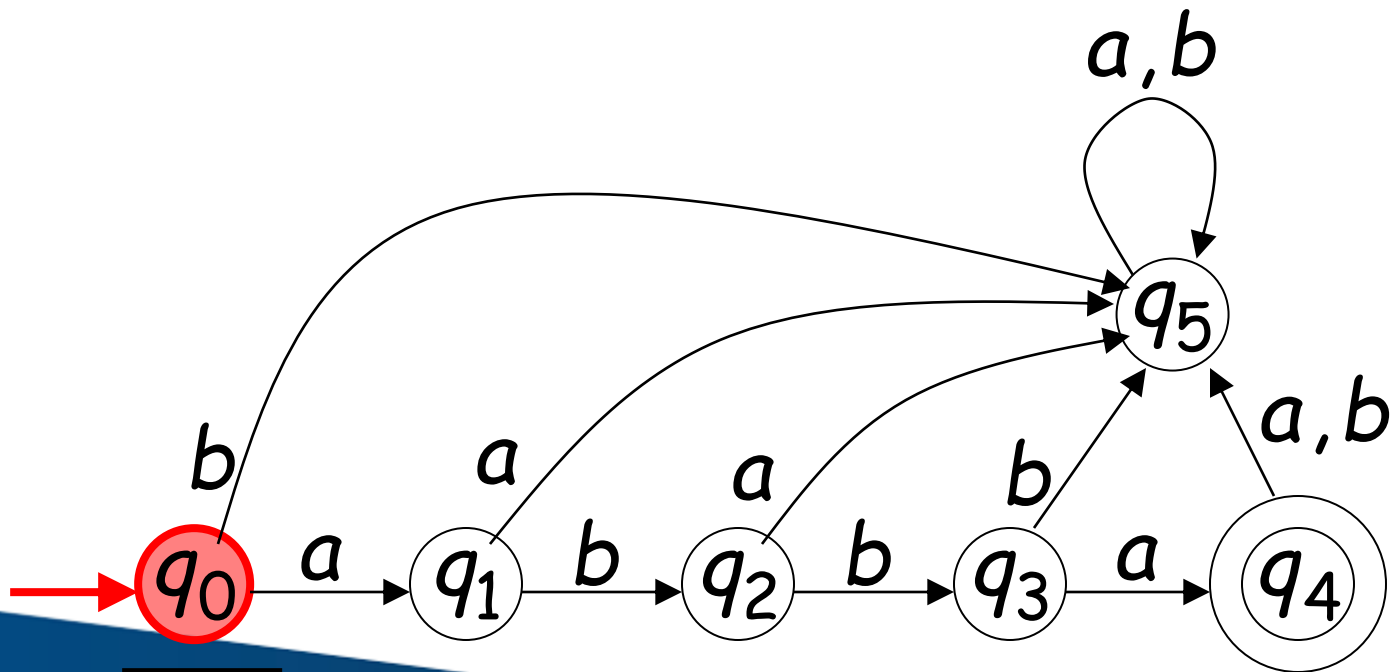
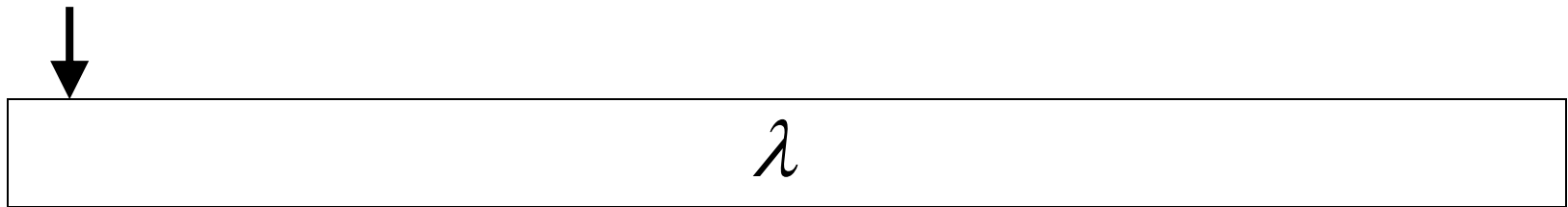
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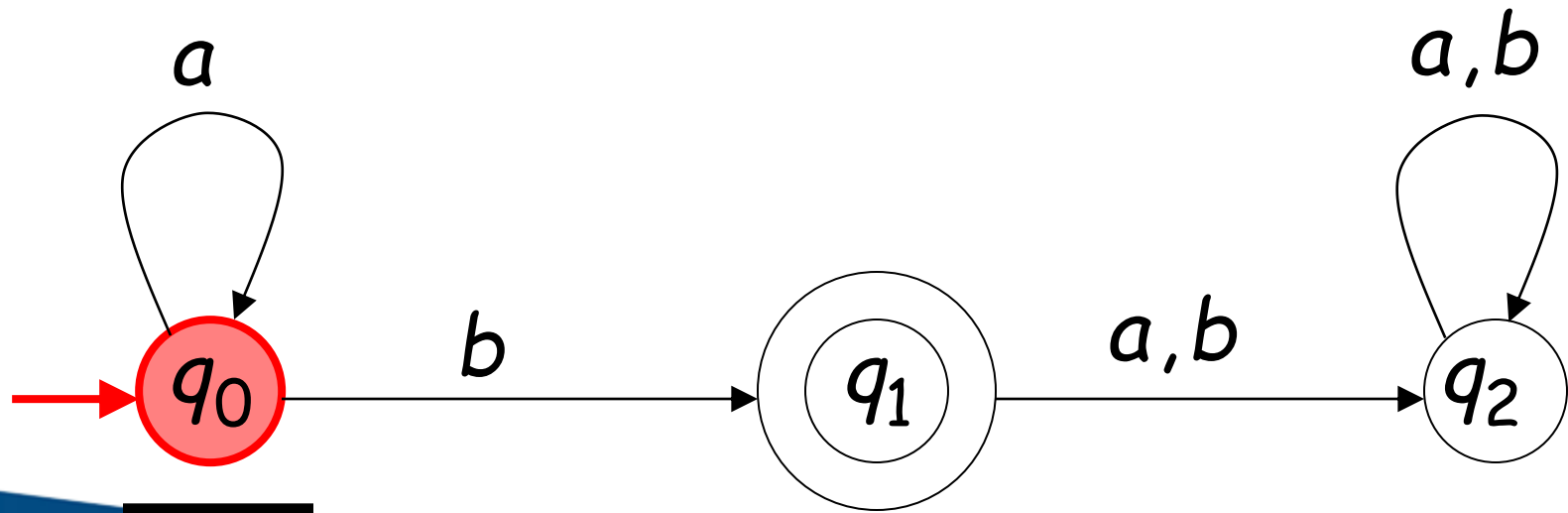
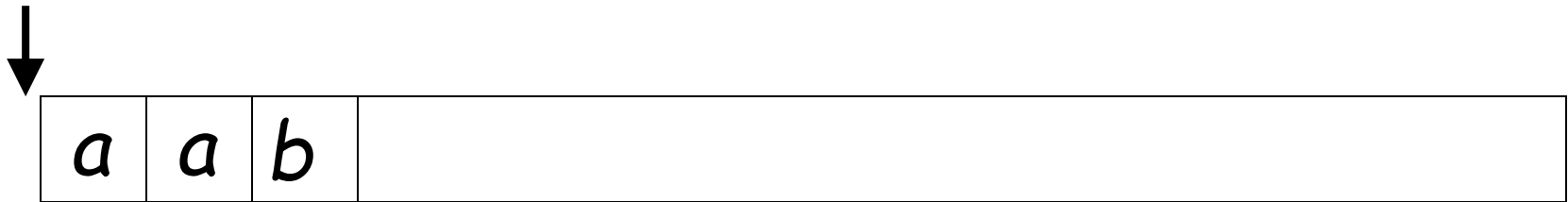


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Another Example

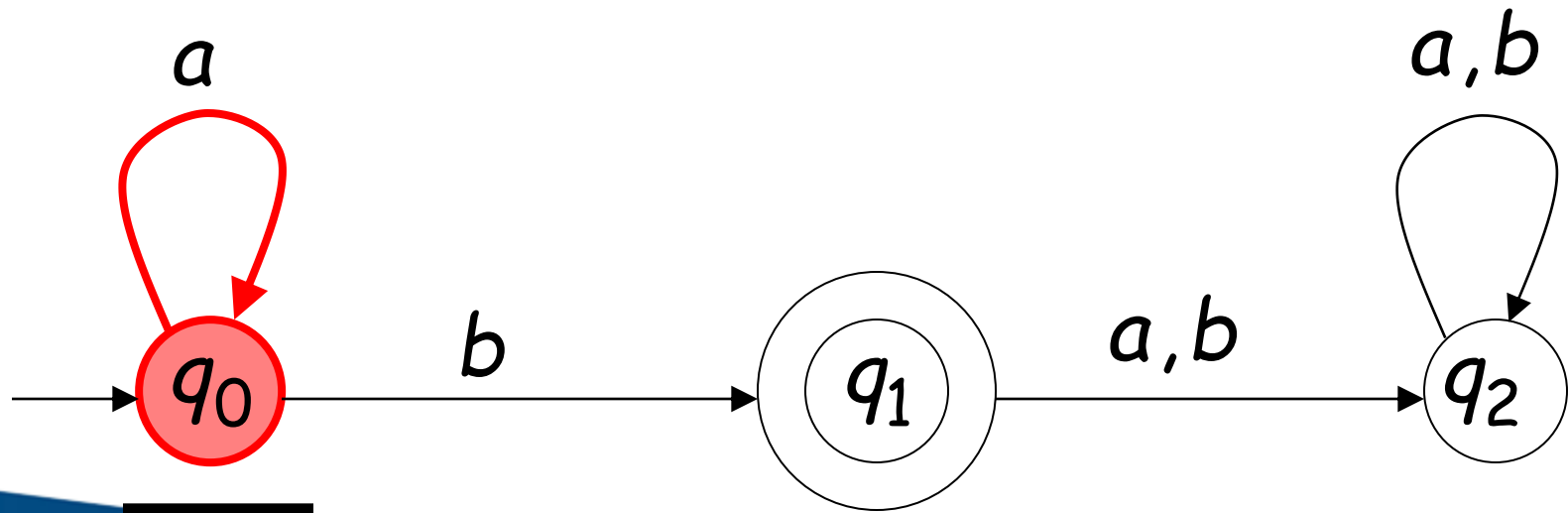
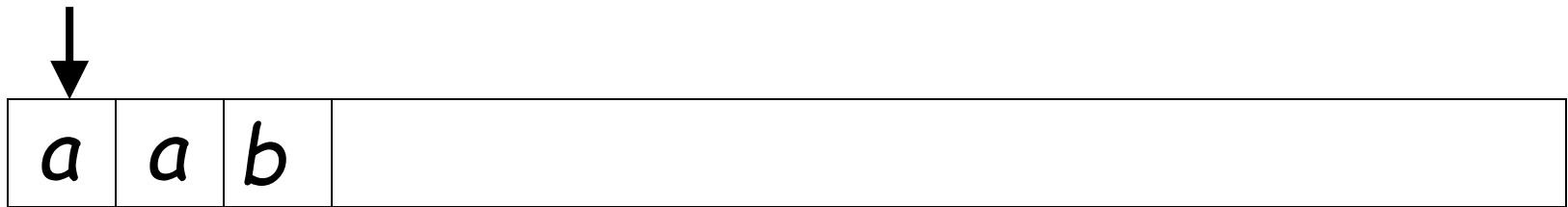


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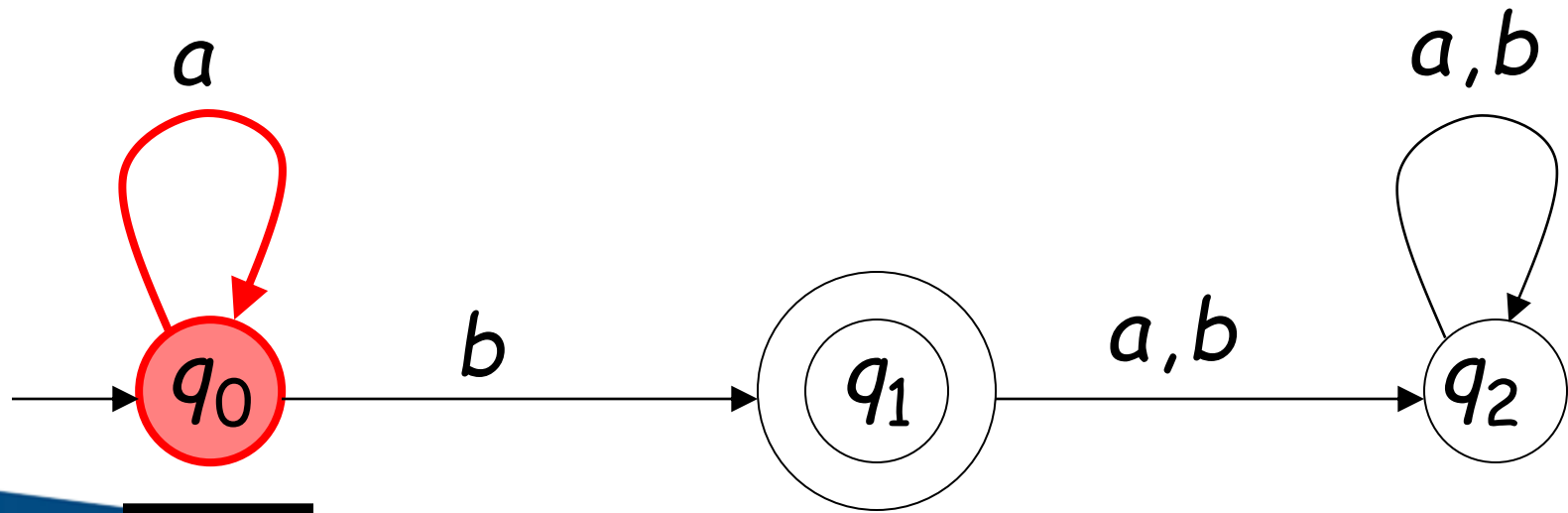
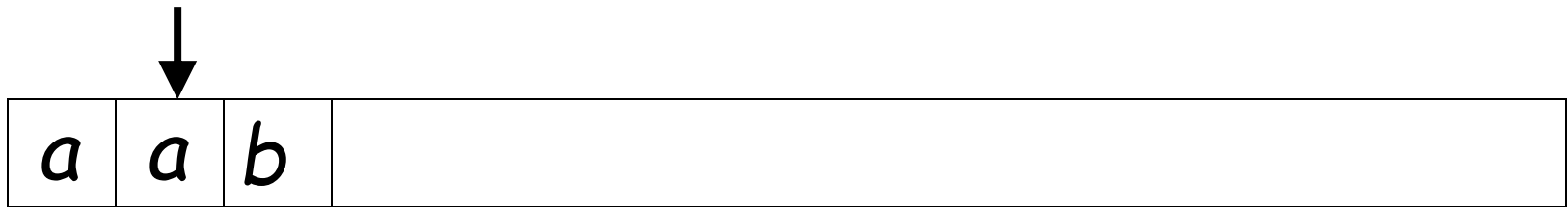


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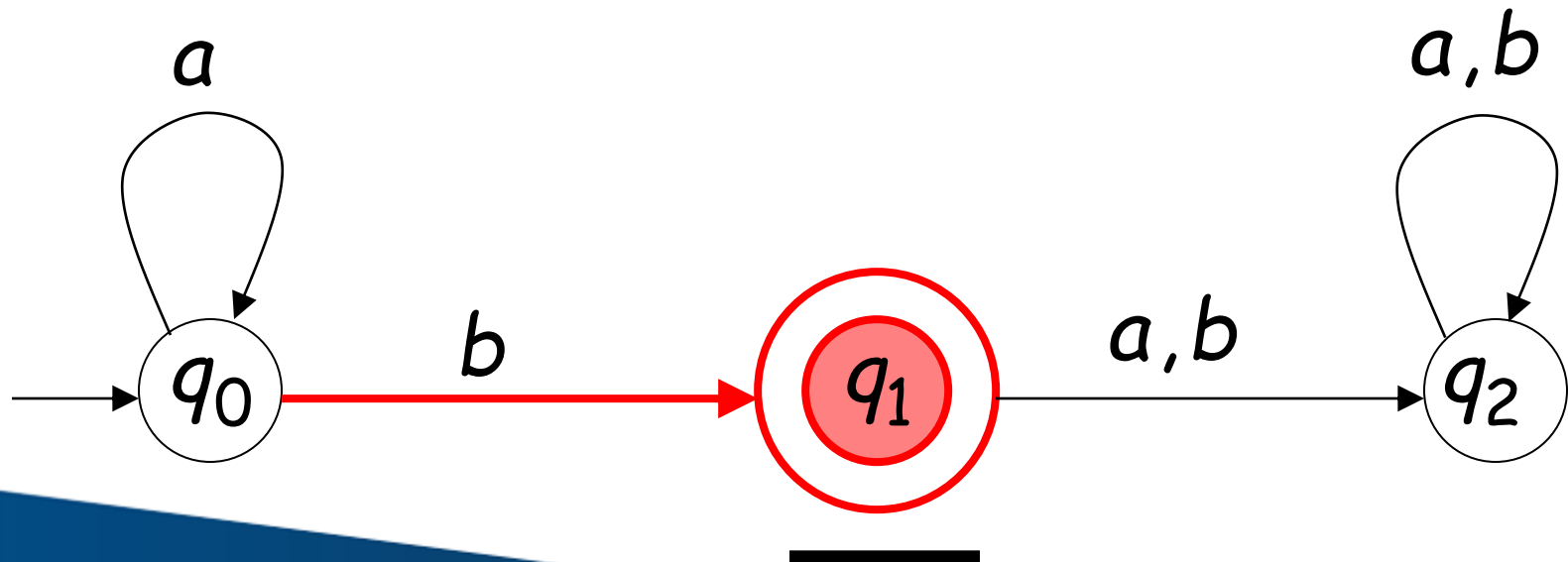
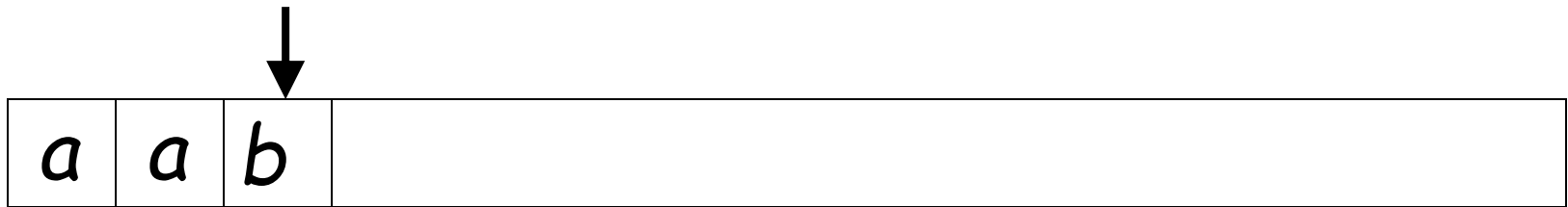


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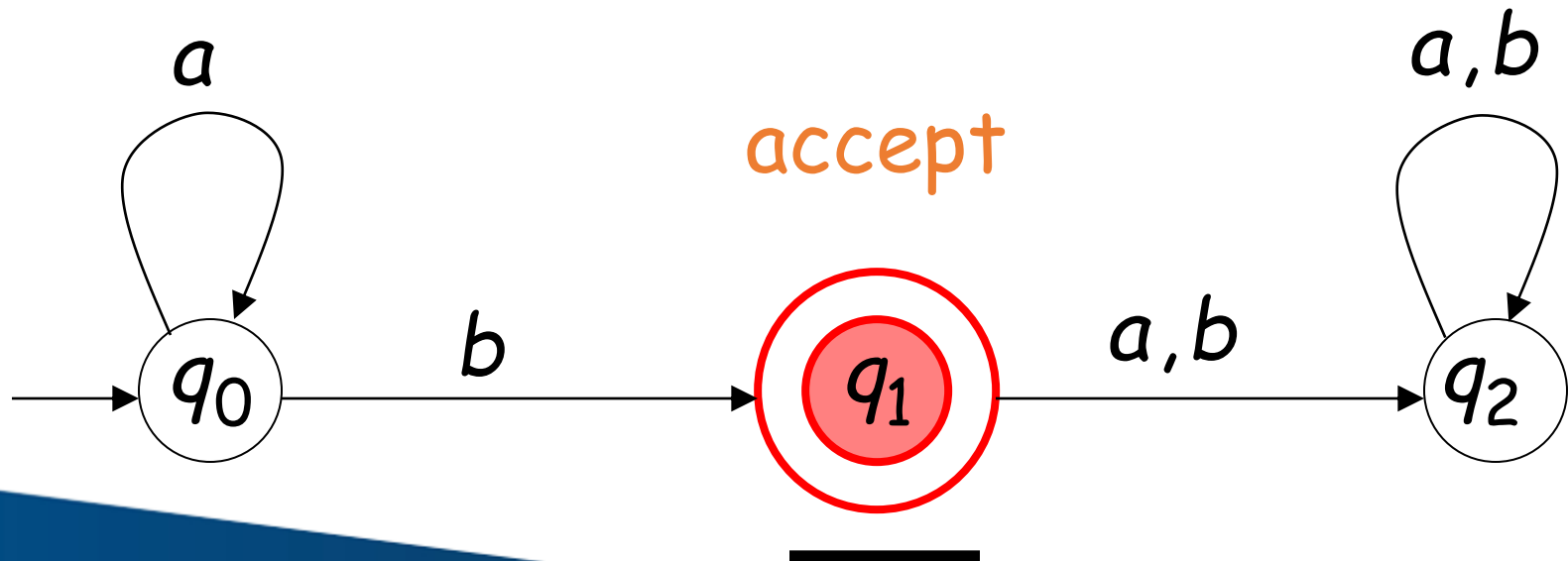
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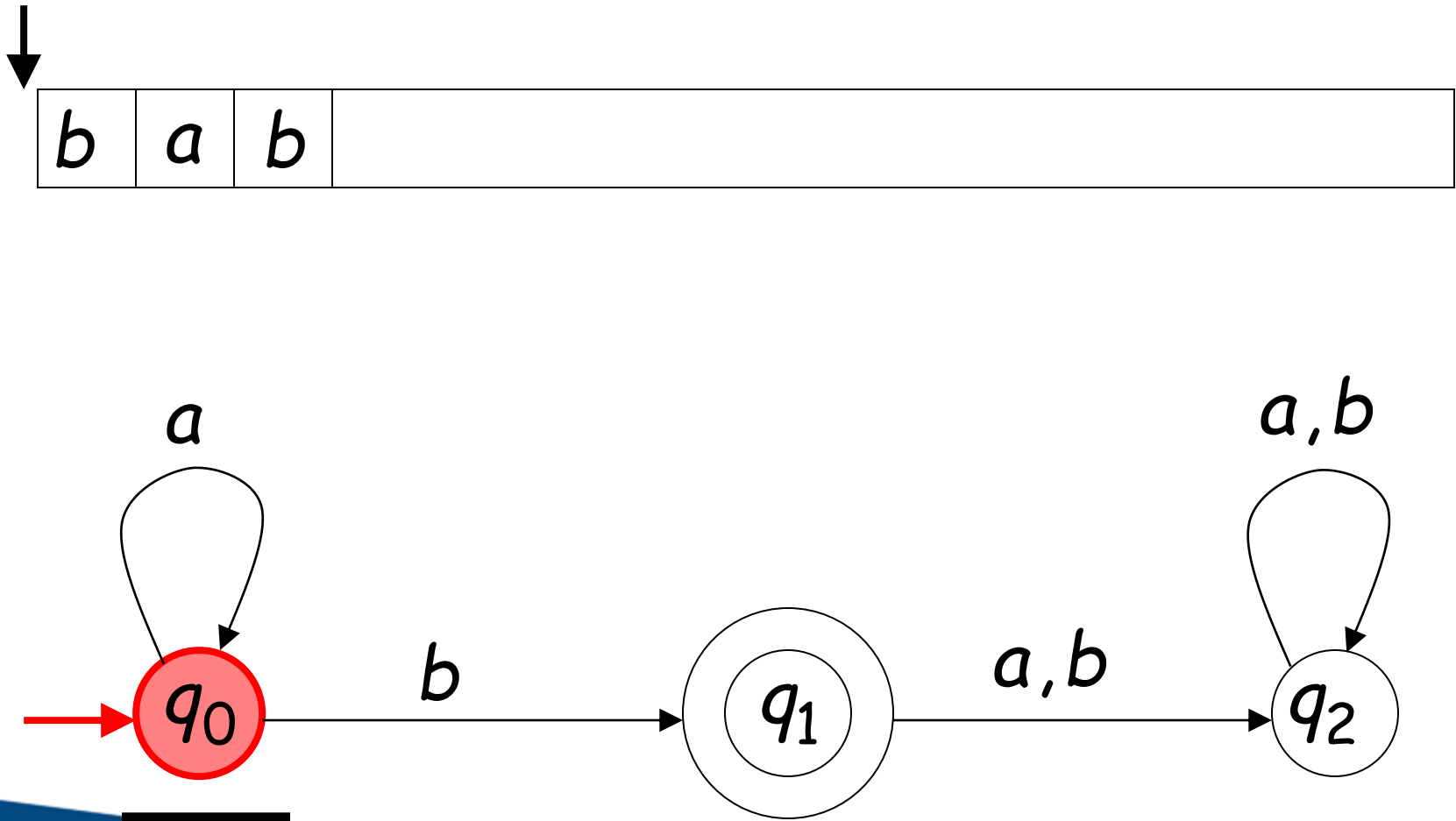
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Rejection Example



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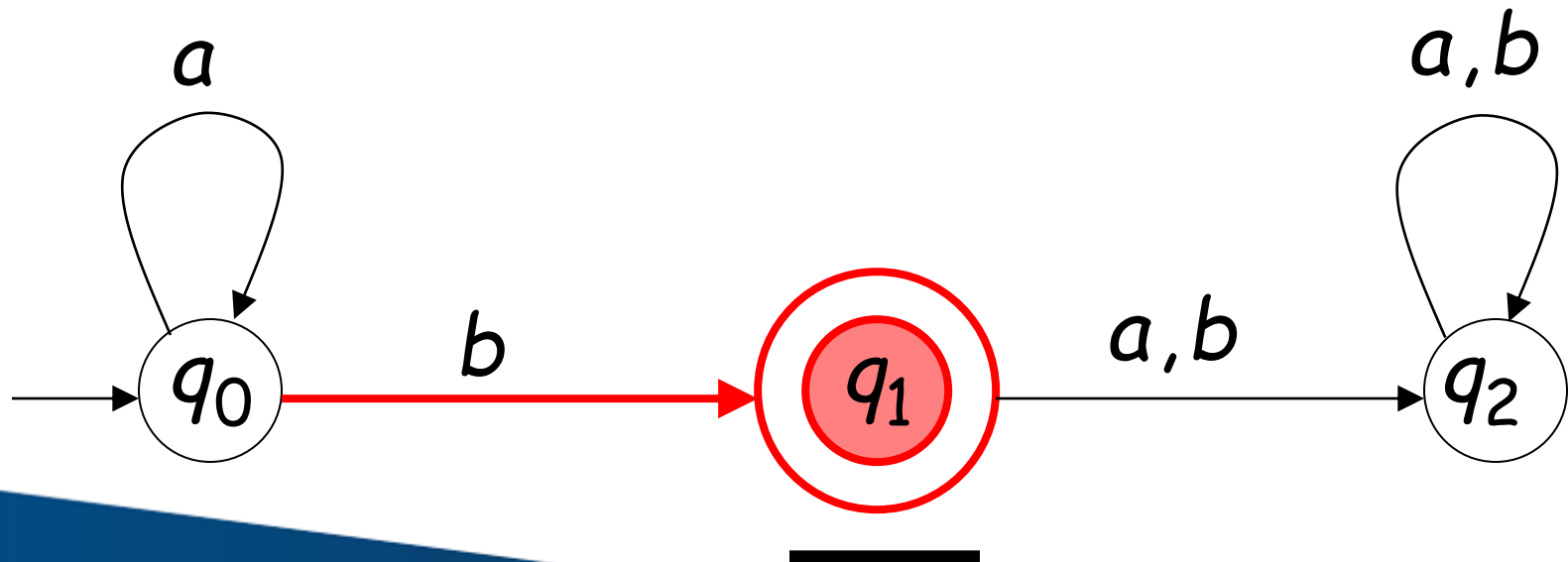
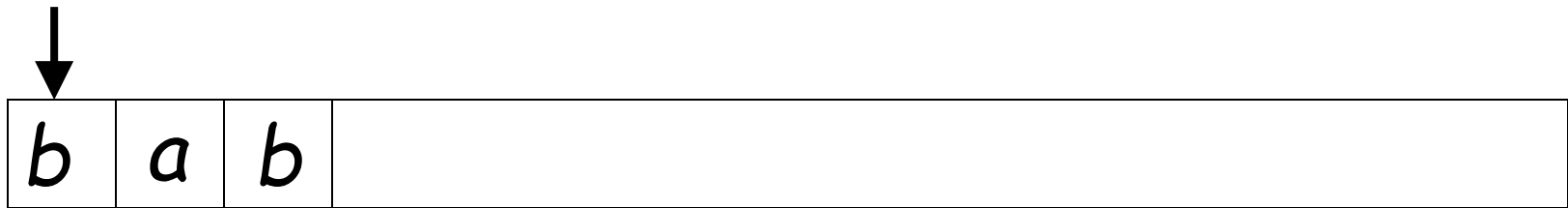
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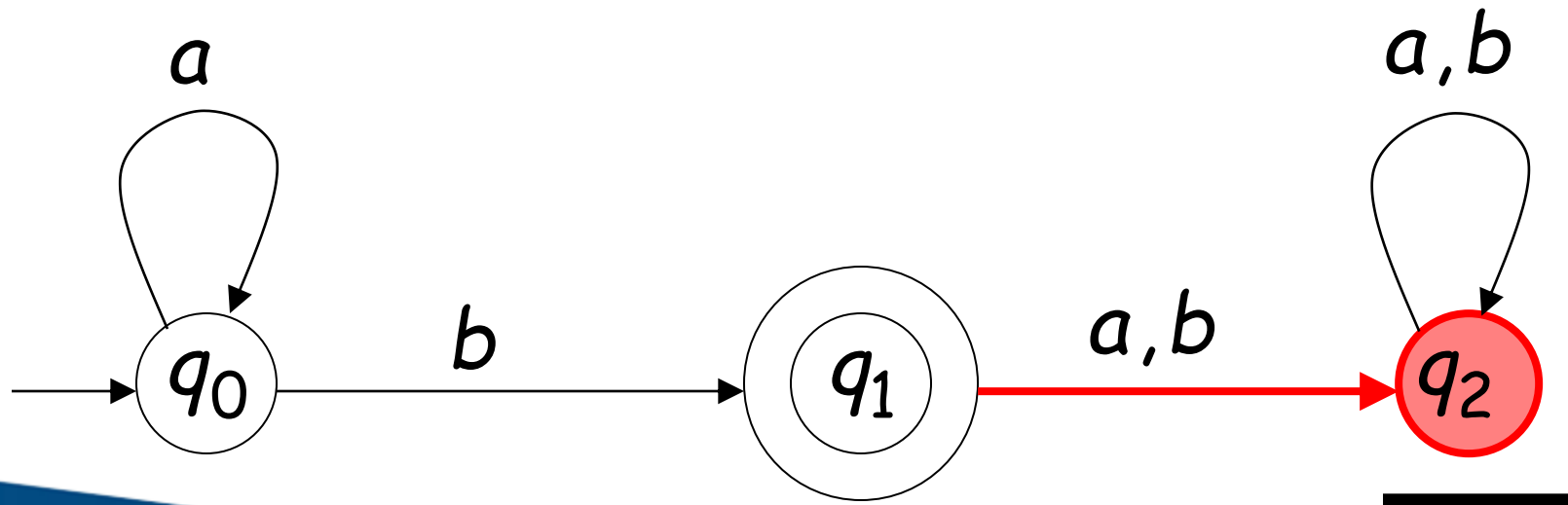
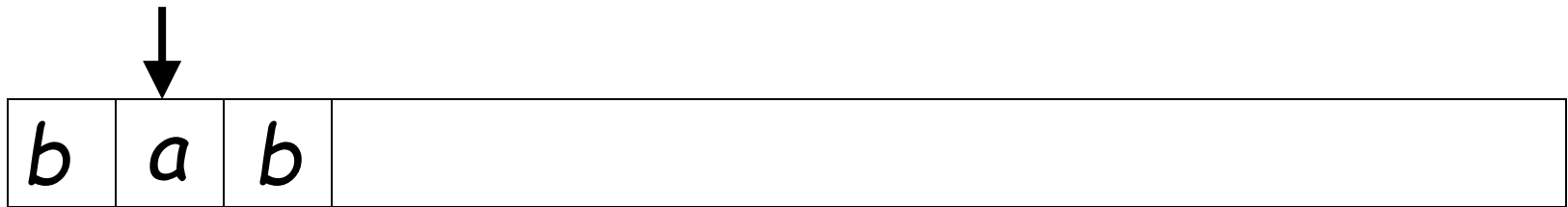


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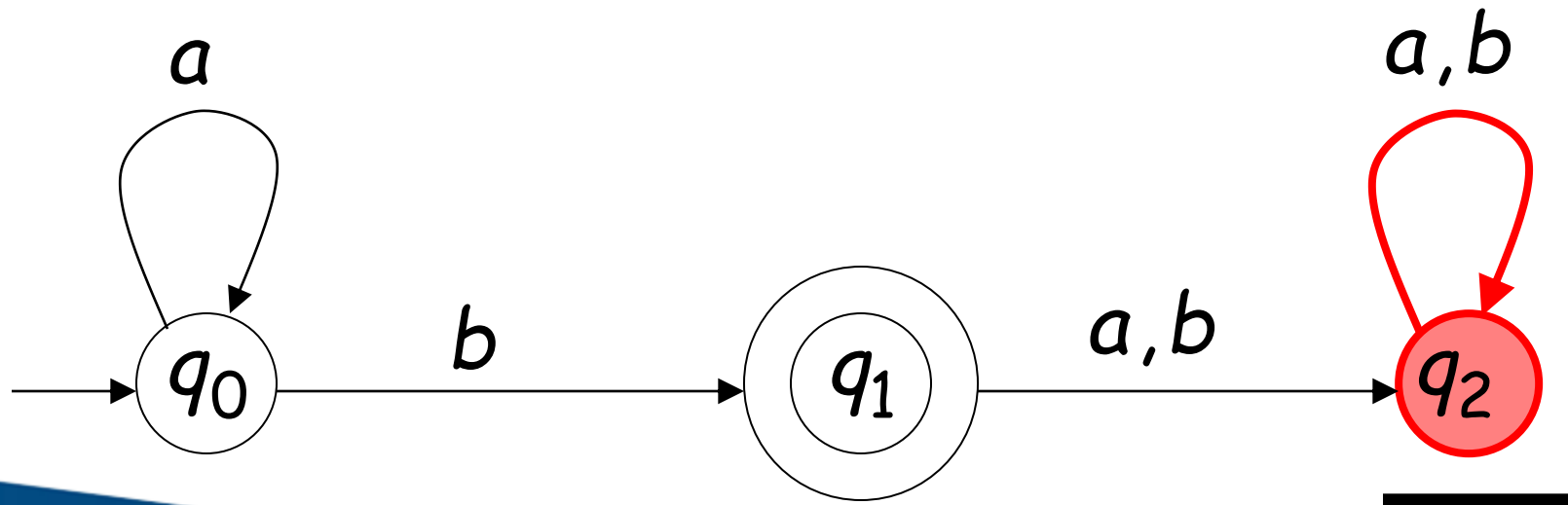
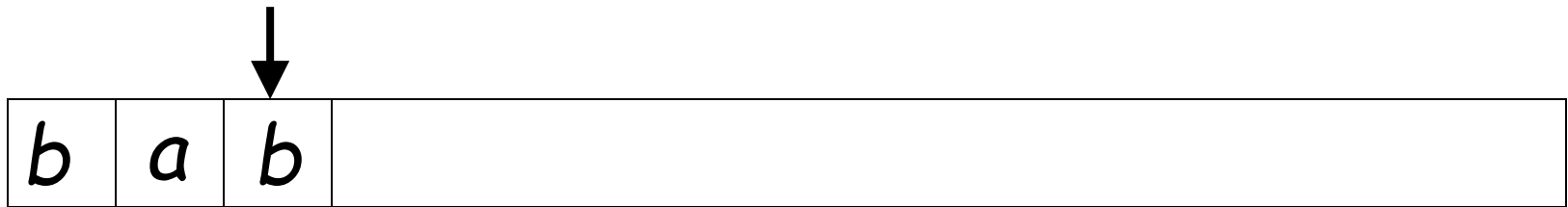


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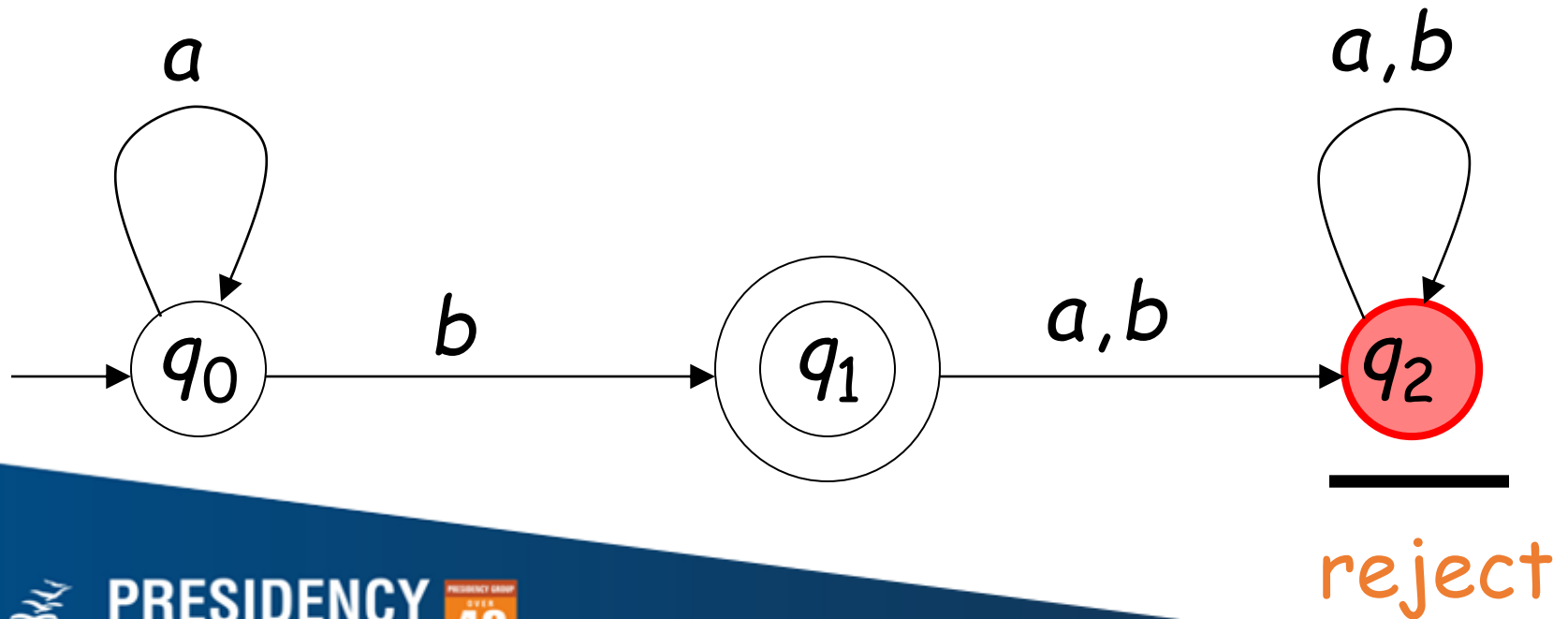
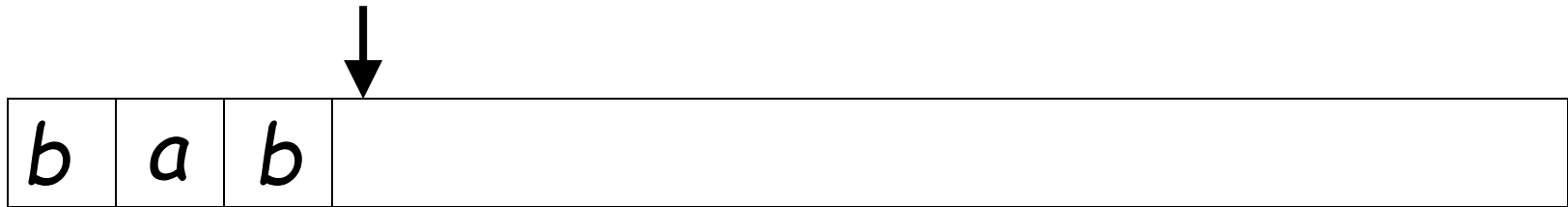
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Languages Accepted by FAs

Definition:

The language $L(M)$ contains all input strings accepted by M

$$L(M) = \{ \text{strings that bring } M \text{ to an accepting state} \}$$



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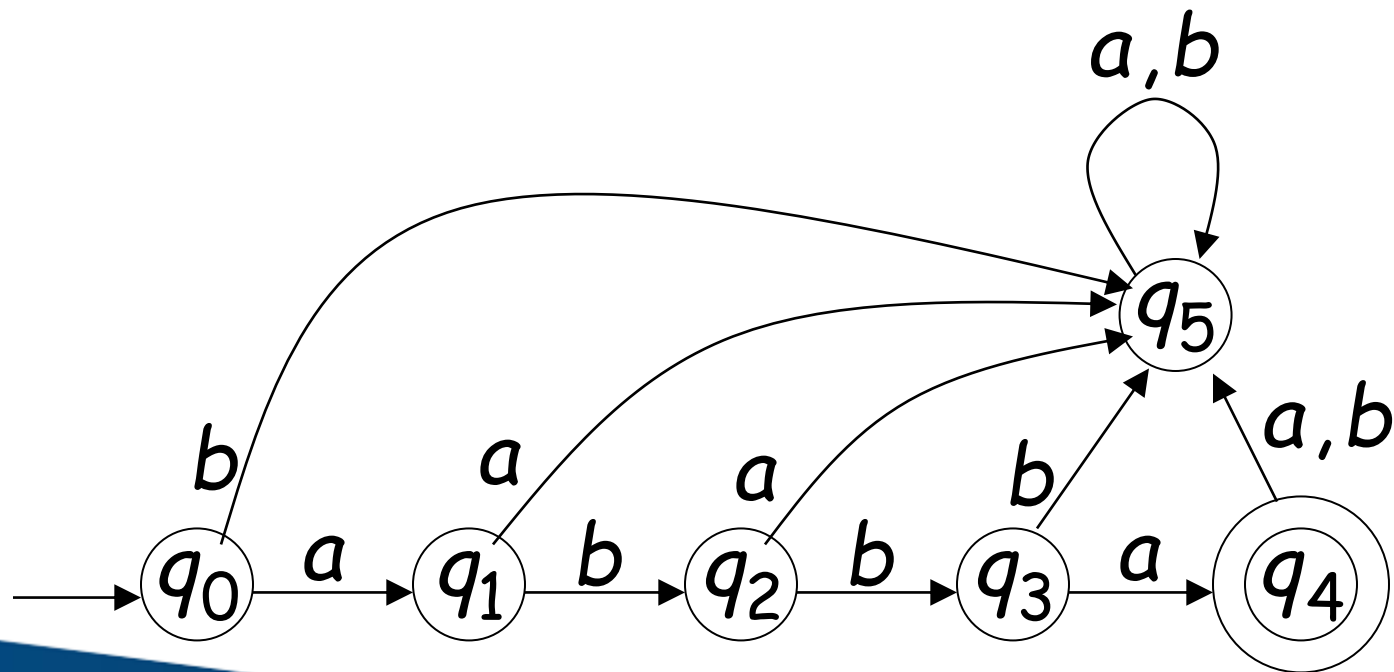


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Example

$$L(M) = \{abba\}$$

M



accept



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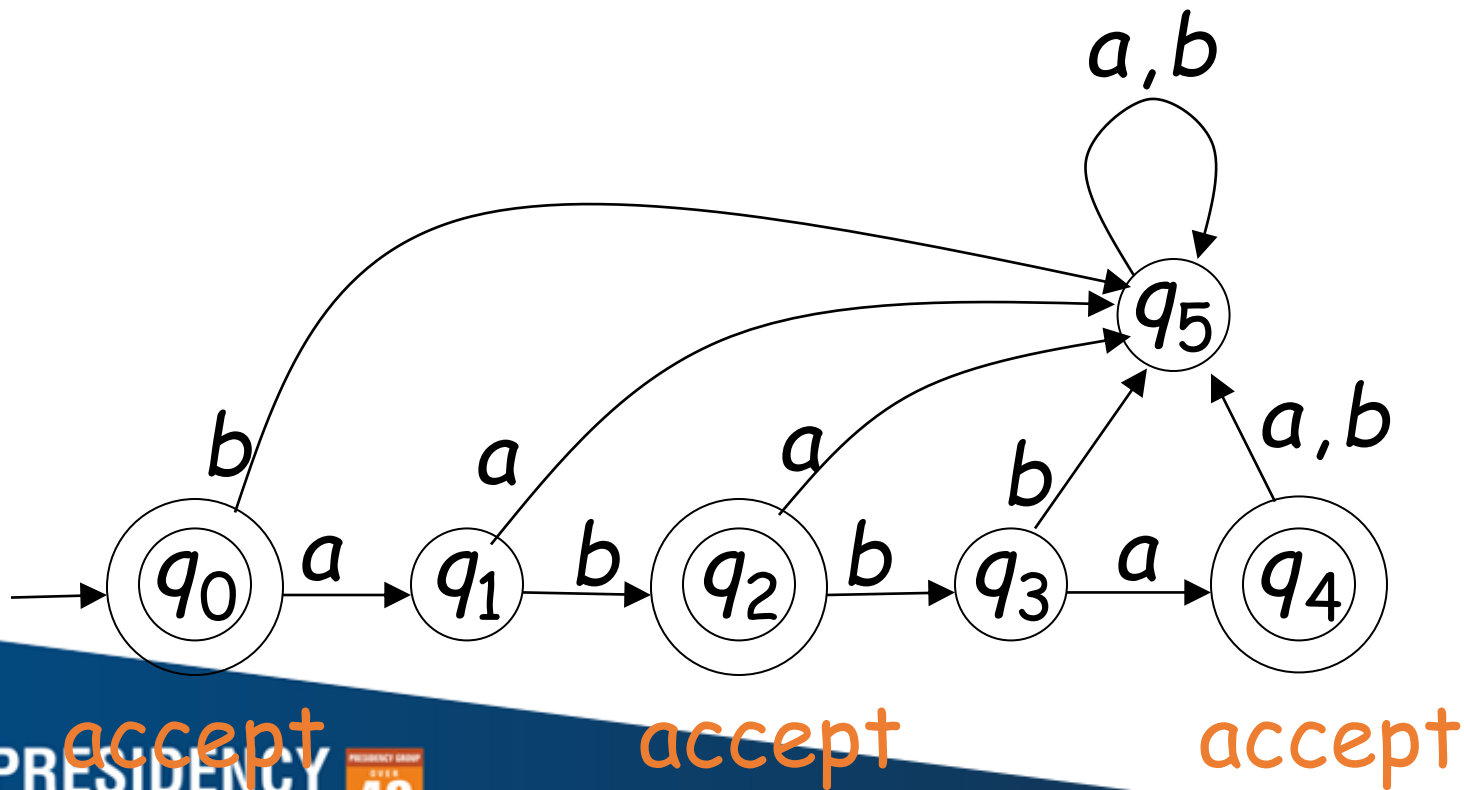
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Example

$$L(M) = \{\lambda, ab, abba\}$$

M



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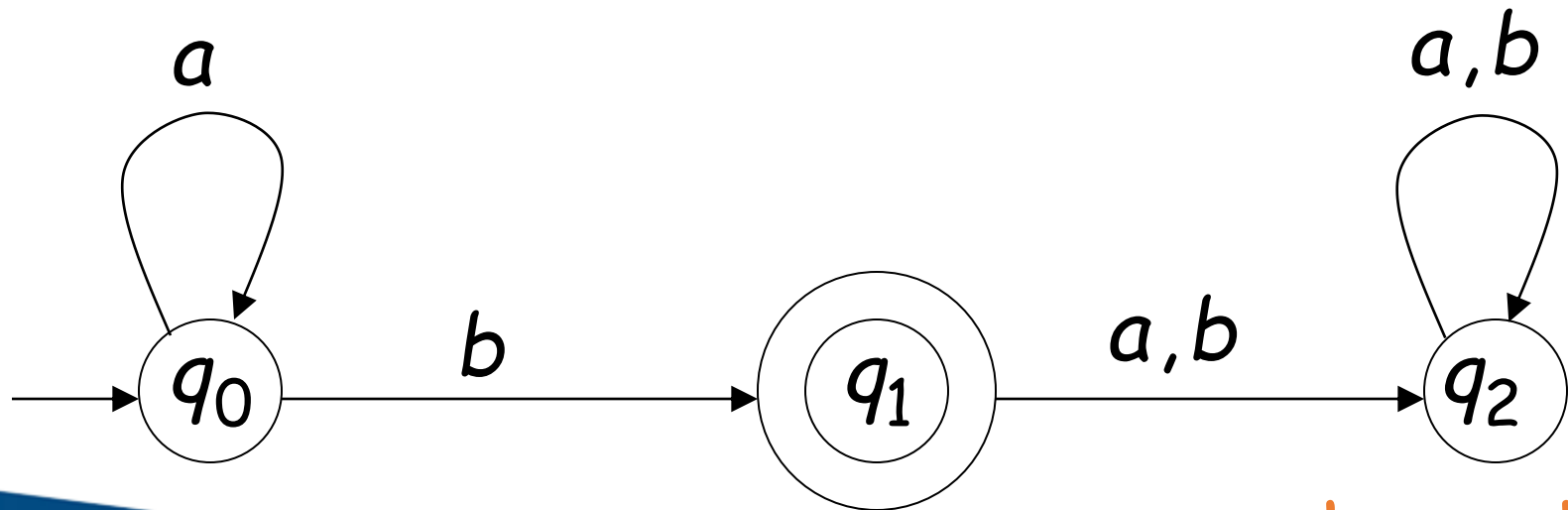
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Example

$$L(M) = \{a^n b : n \geq 0\}$$



accept

trap state/
dead state



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Formal Definition

- Finite Automaton (FA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

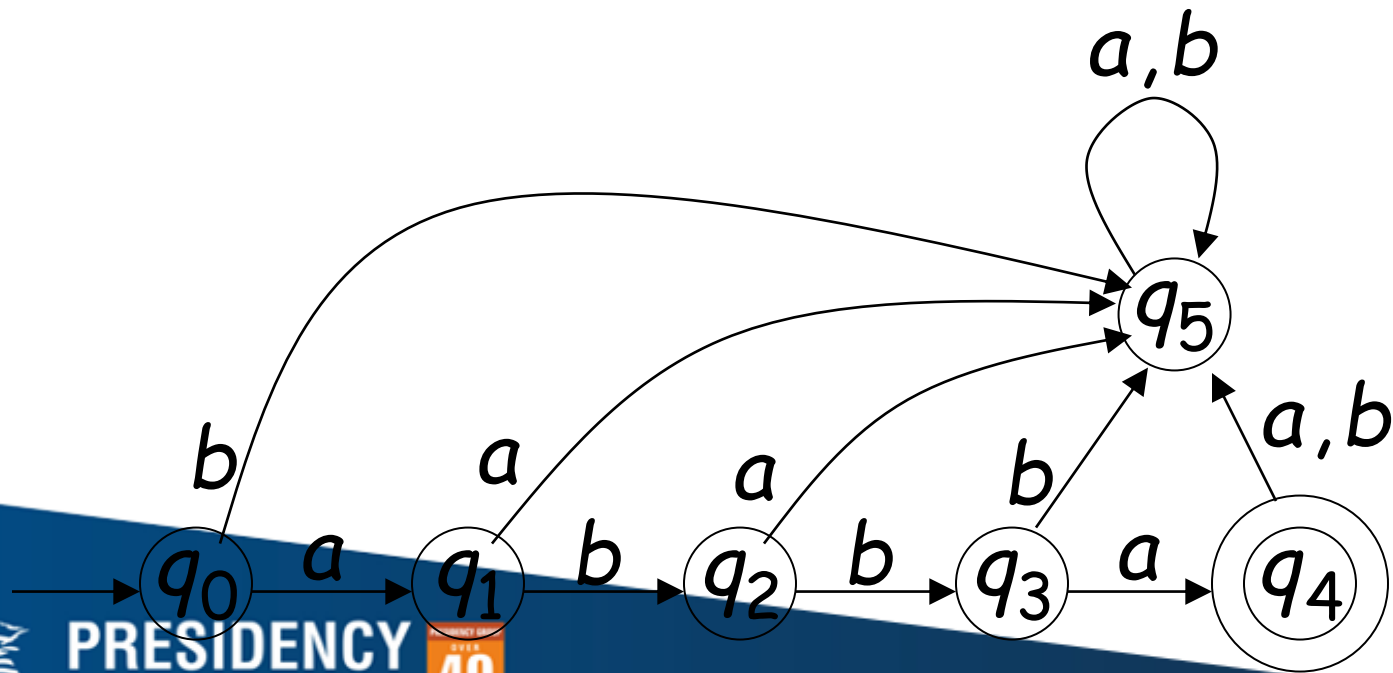
Σ : input alphabet

δ : transition function

q_0 : initial state

Input Alphabet Σ

- $\Sigma = \{a, b\}$



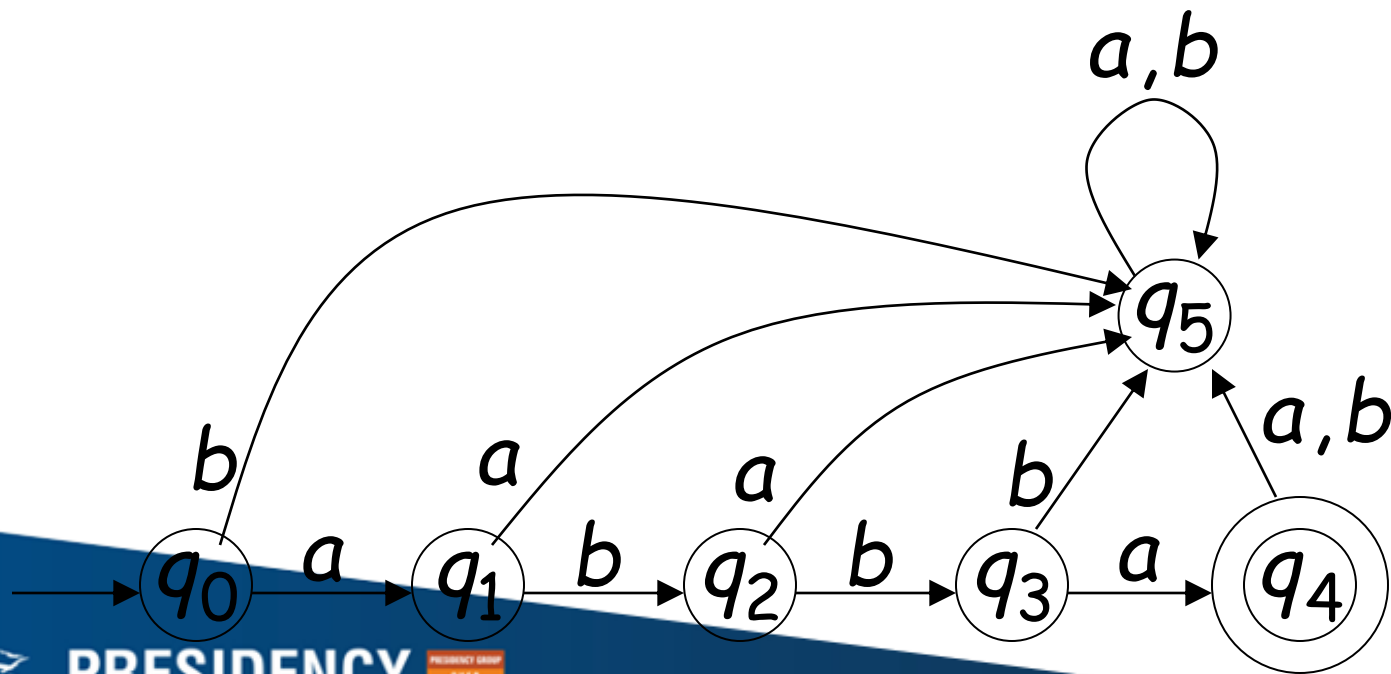
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Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$



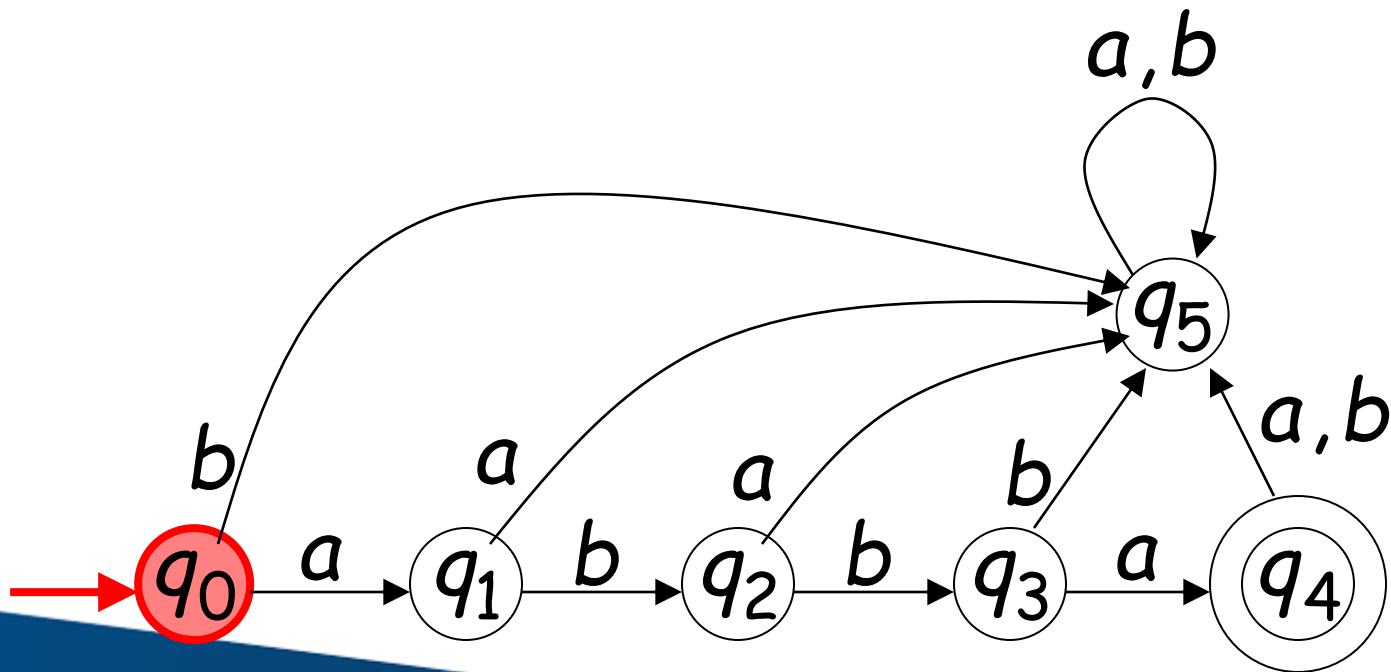
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Initial State q_0

•



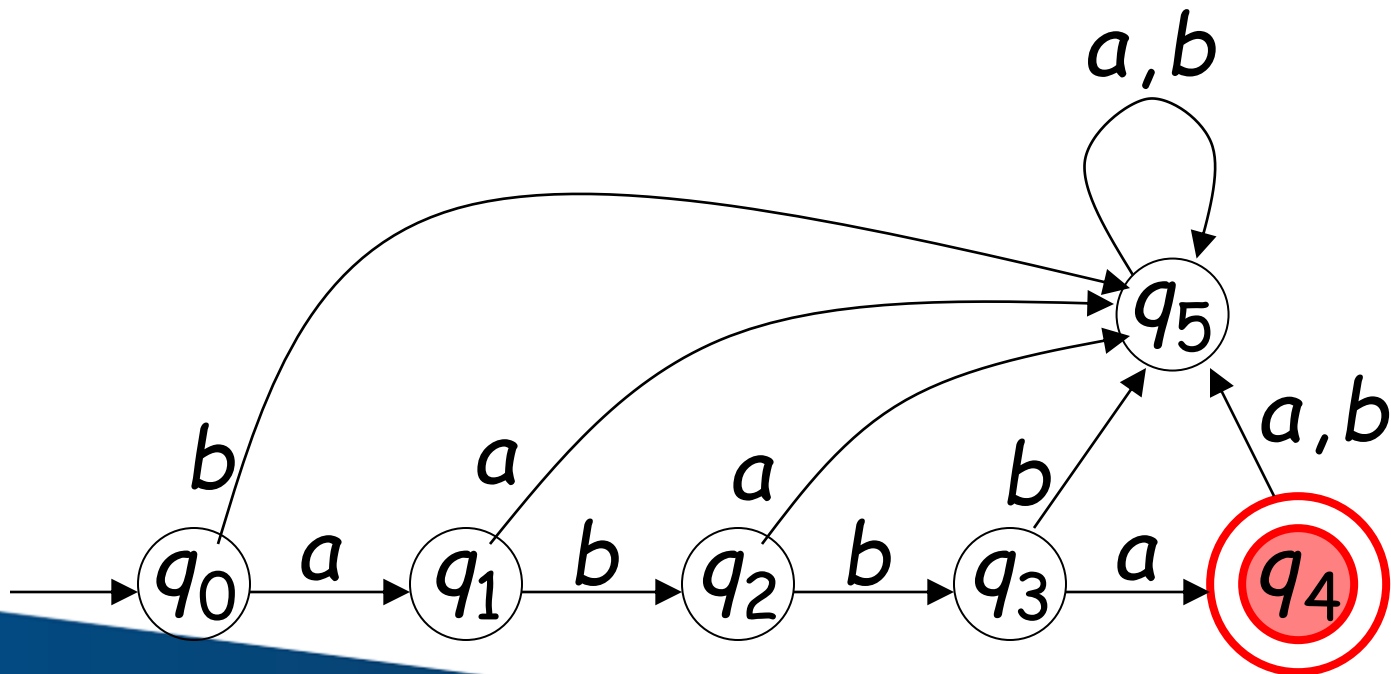
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Set of Accepting States F

- $F = \{q_4\}$



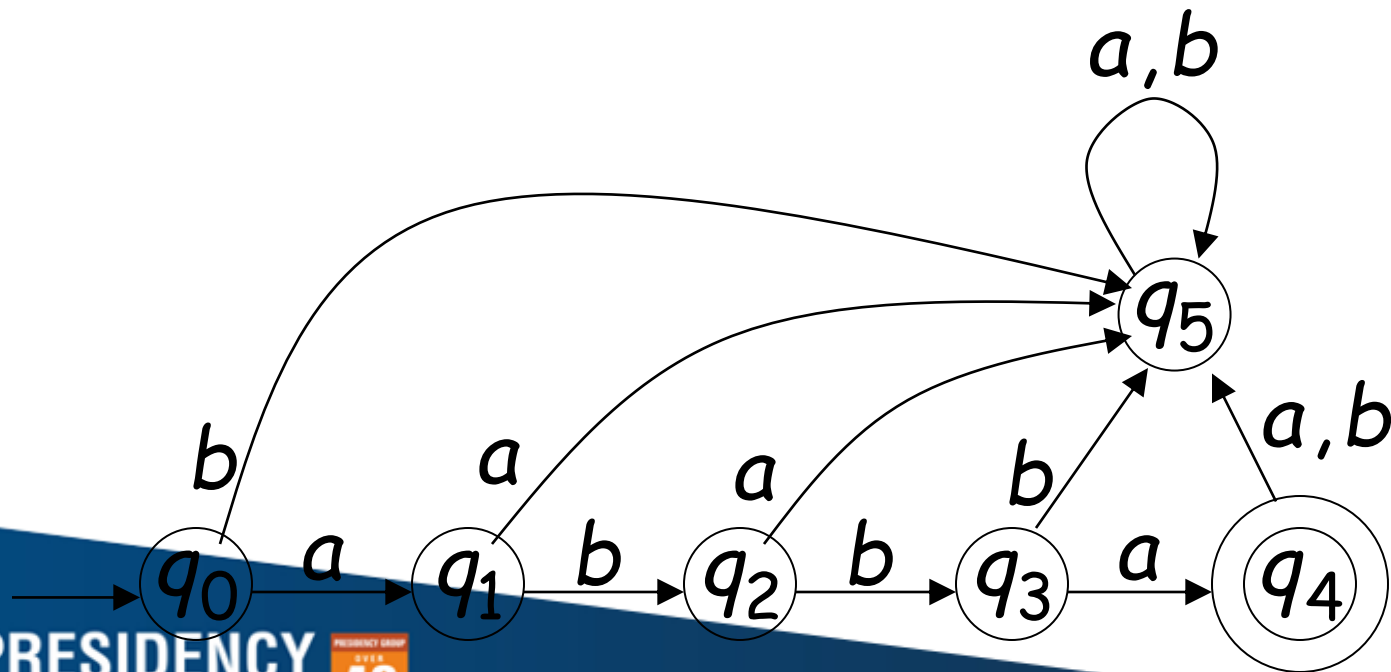
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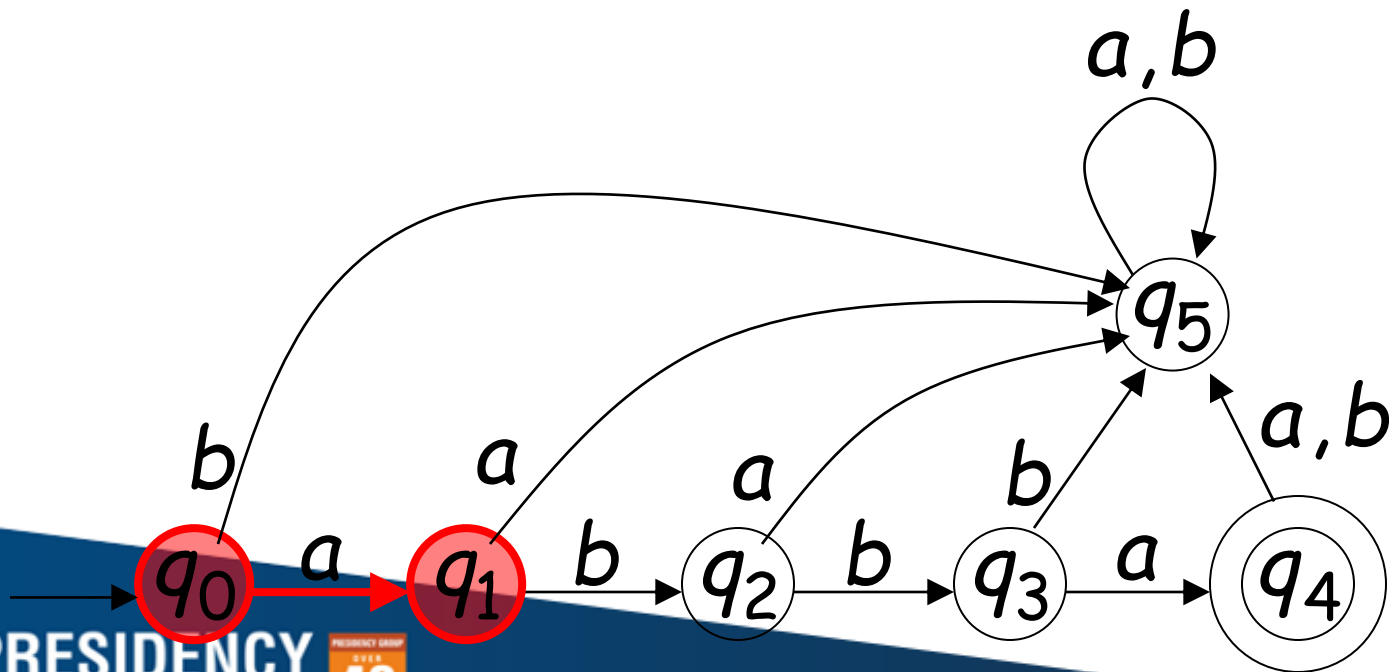


Transition Function δ

- $$\delta : Q \times \Sigma \rightarrow Q$$



$$\delta(q_0, a) = q_1$$

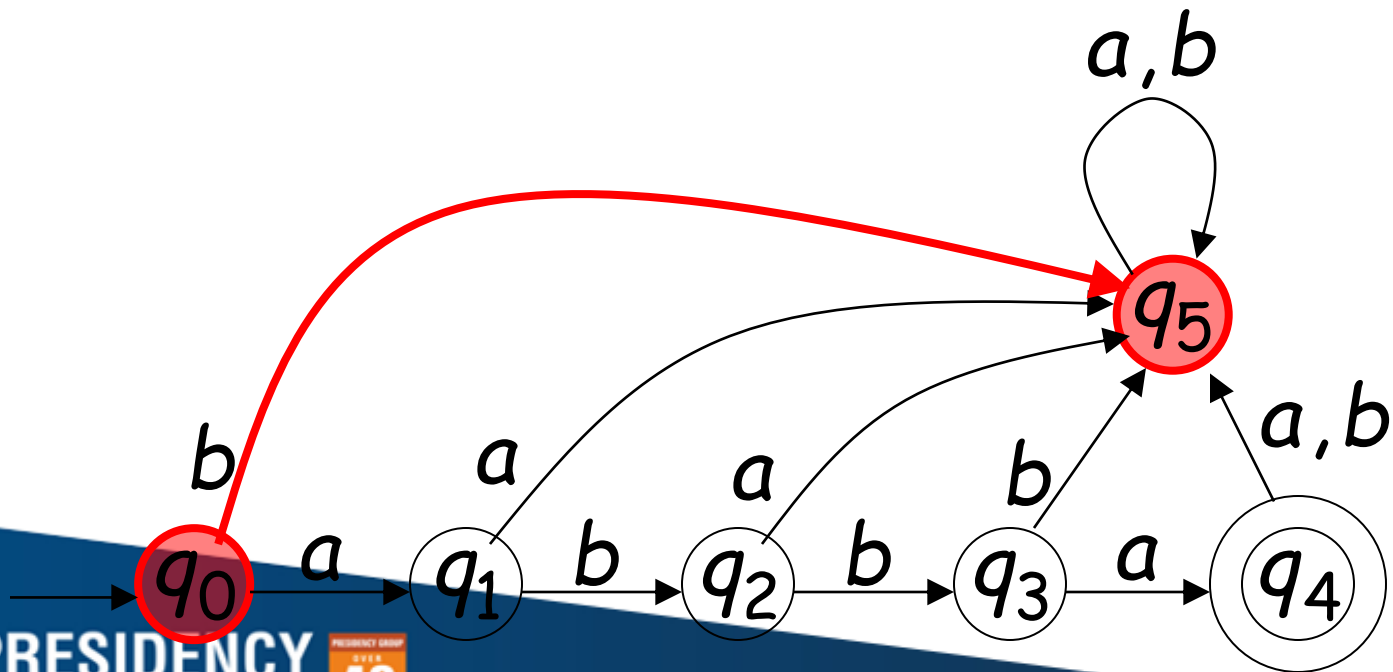


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$$\delta(q_0, b) = q_5$$

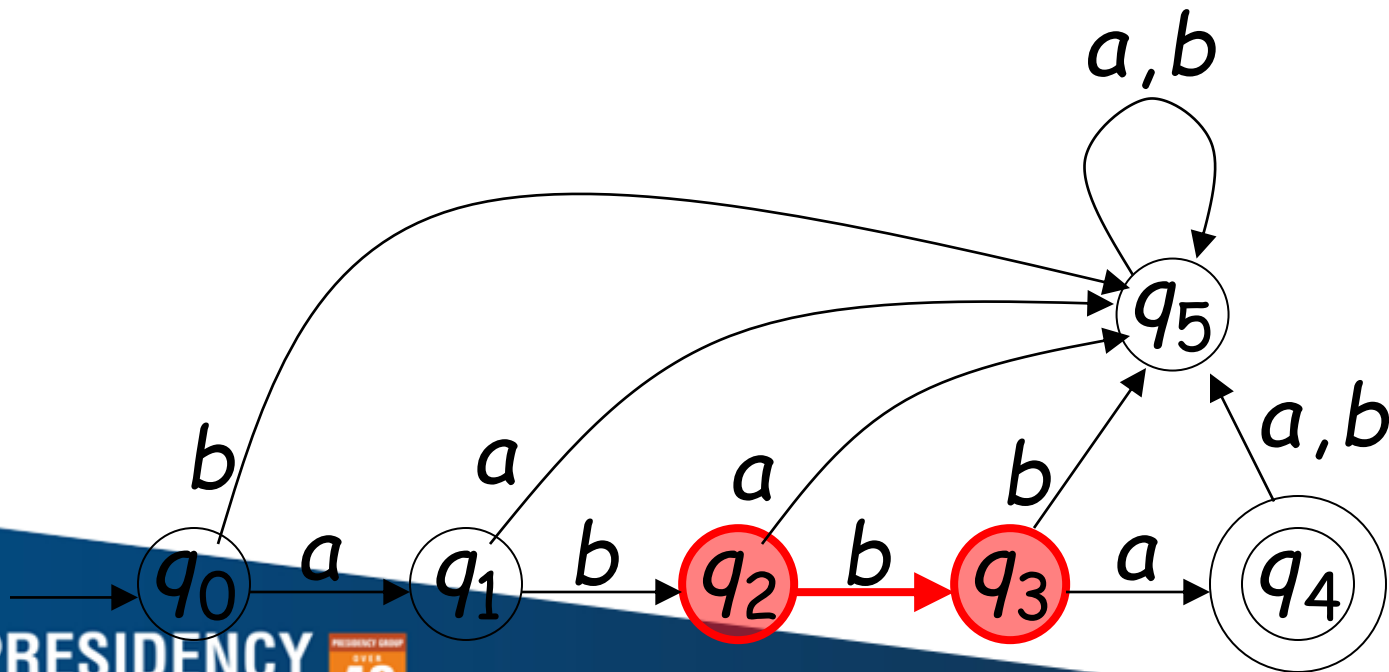


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$$\delta(q_2, b) = q_3$$



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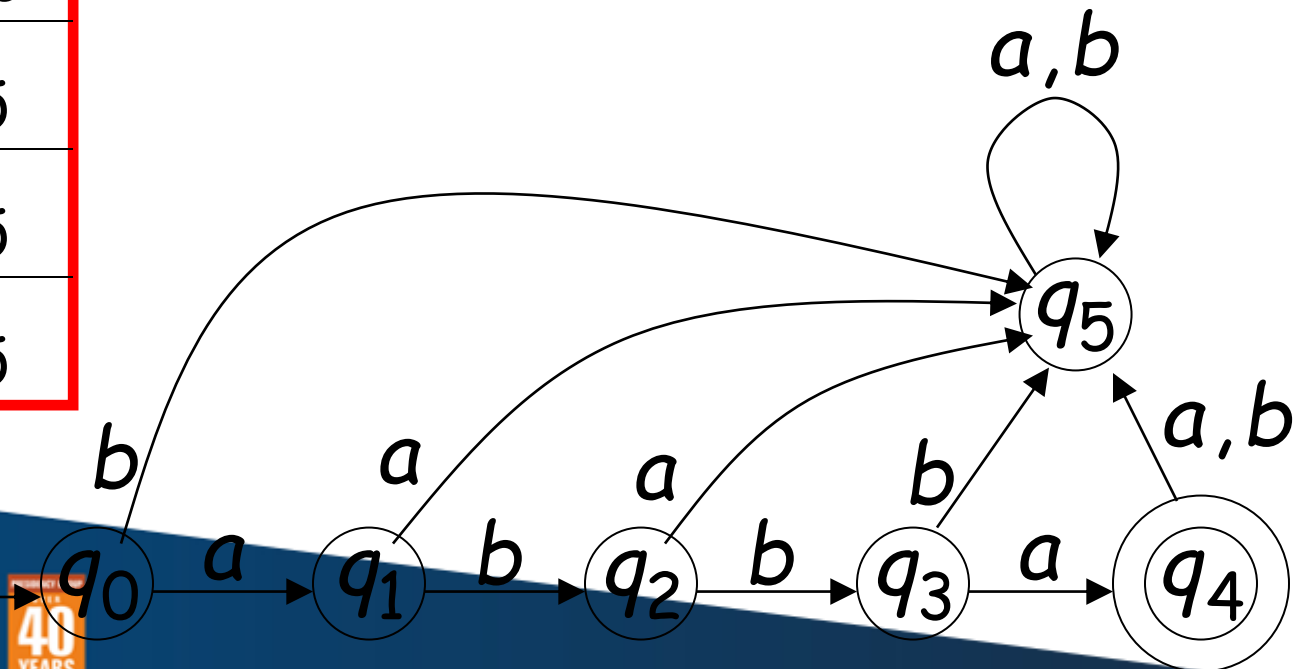
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Transition Table

δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5^*	q_5	q_5



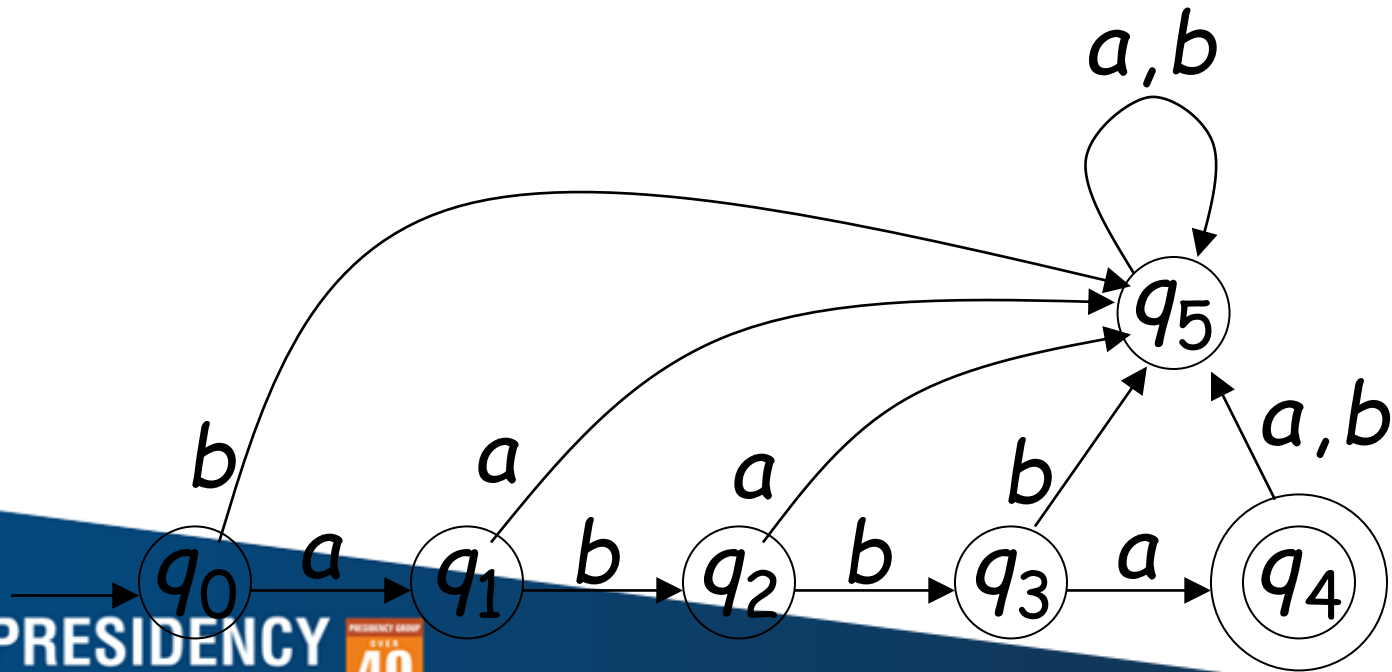
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Extended Transition Function δ^*

- $$\delta^*: Q \times \Sigma^* \rightarrow Q$$

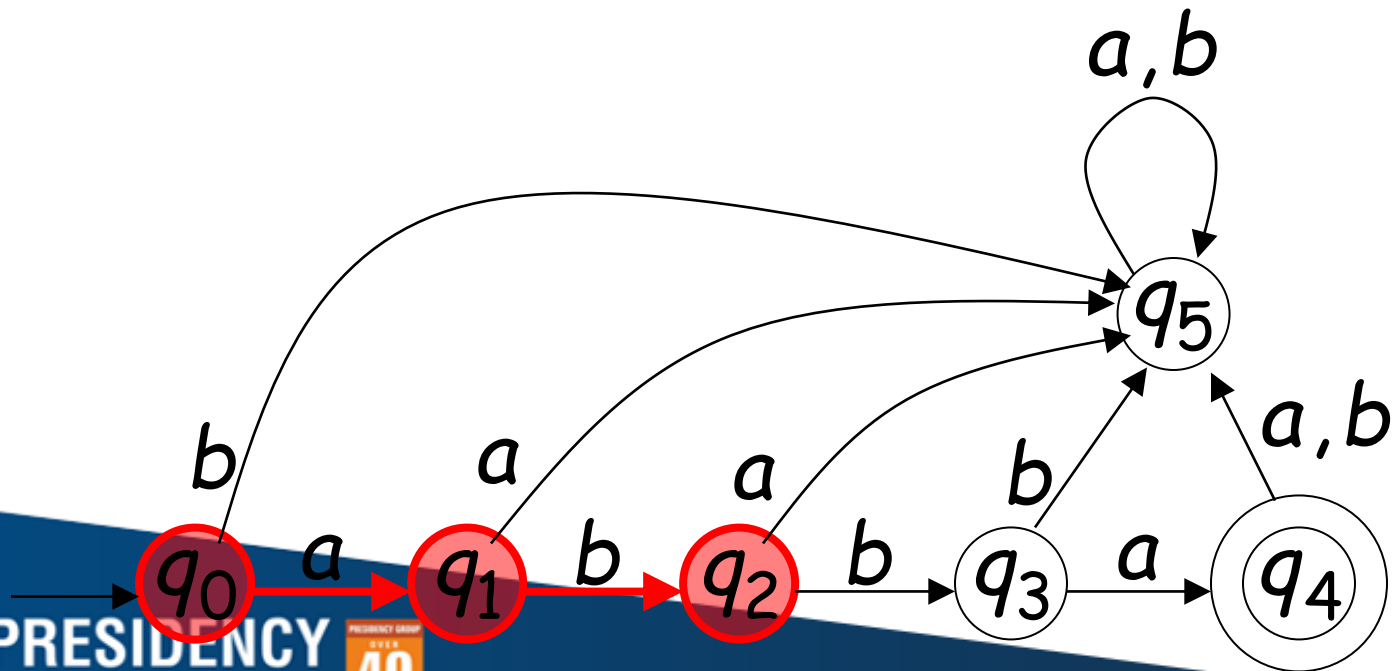


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$$\delta^*(q_0, ab) = q_2$$

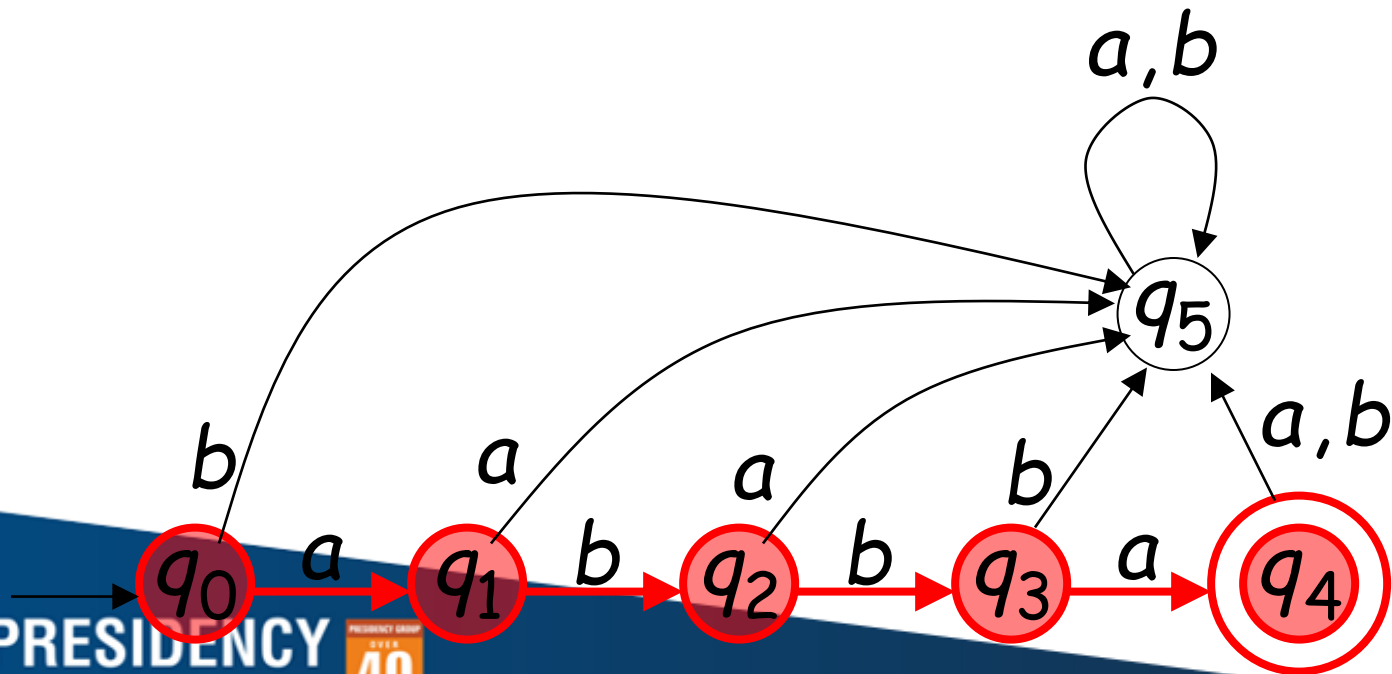


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$$\delta^*(q_0, abba) = q_4$$

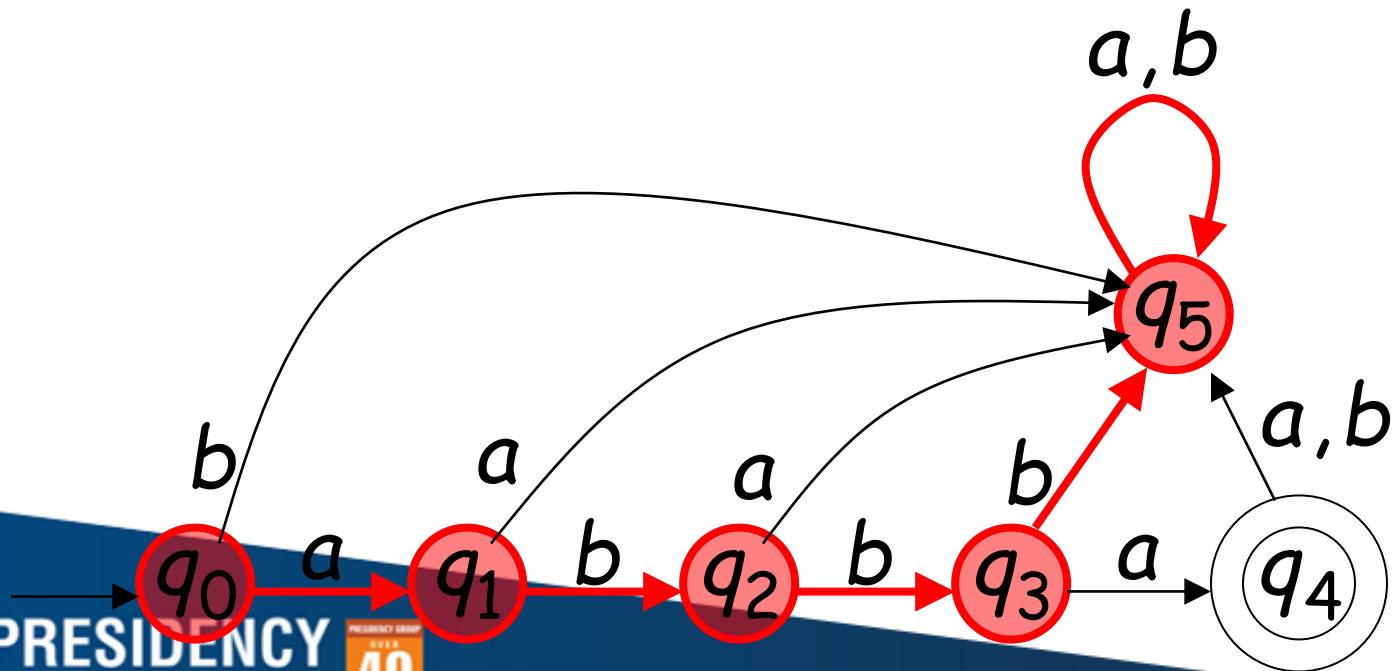


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$$\delta^*(q_0, abbbaa) = q_5$$

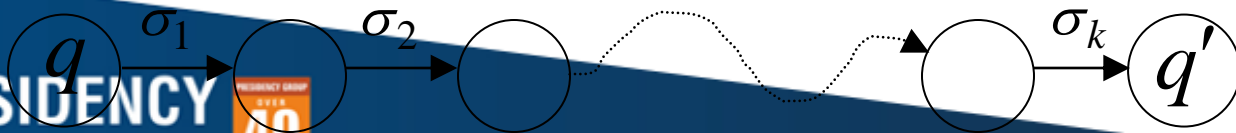


Observation: if there is a walk from q to q'
with label w then

$$\delta^*(q, w) = q'$$



$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



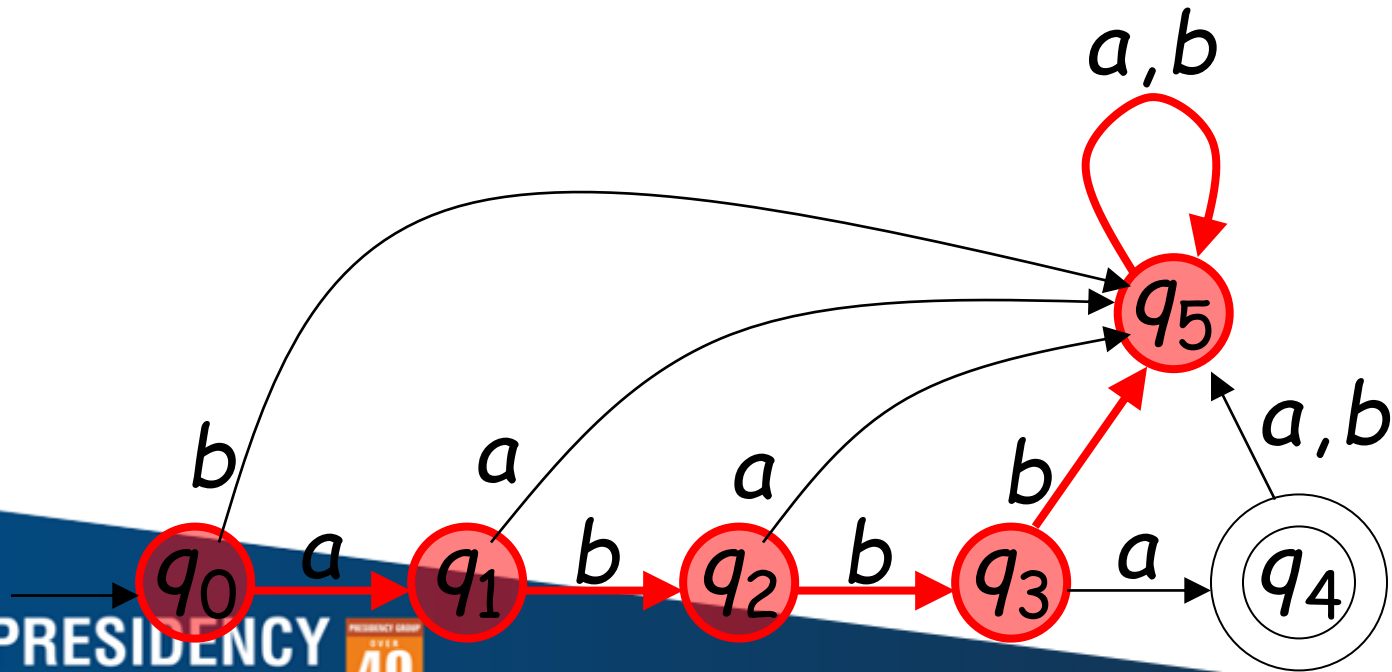
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Example: There is a walk from q_0 to q_5
with label $abbbaa$

$$\delta^*(q_0, abbbaa) = q_5$$



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Recursive Definition

$$\delta^*(q, \lambda) = q$$

$$\delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$$



$$\left. \begin{array}{l} \delta^*(q, w\sigma) = q' \\ \delta(q_1, \sigma) = q' \end{array} \right\} \Rightarrow \delta^*(q, w\sigma) = \delta(q_1, \sigma)$$

$$\left. \begin{array}{l} \delta^*(q, w\sigma) = \delta(q_1, \sigma) \\ \delta^*(q, w) = q_1 \end{array} \right\} \Rightarrow \delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$$



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$$\delta^*(q_0, ab) =$$

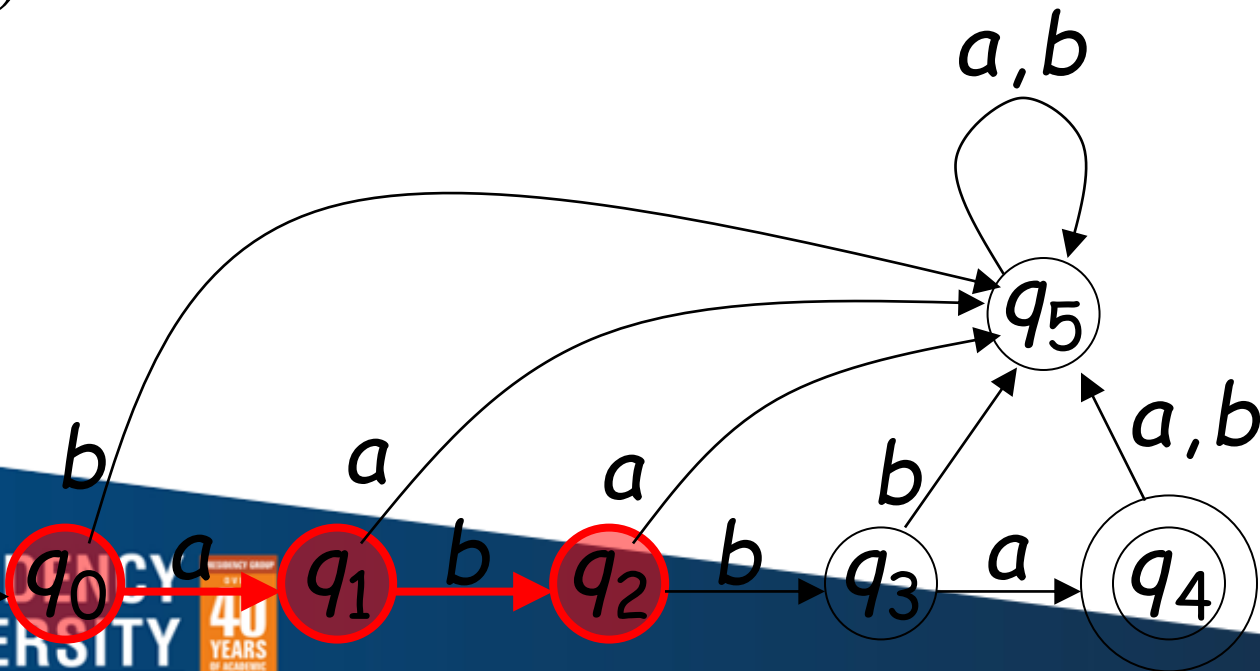
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

q_2



Check the string acceptance of abba

- $\delta^*(q_0, abba) = \delta(\delta^*(q_0, abb), a)$
- $= \delta(\delta(\delta^*(q_0, ab), b), a)$
- $= \delta(\delta(\delta(\delta^*(q_0, a), b), b), a)$
- $= \delta(\delta(\delta(\delta(\delta^*(q_0, \lambda), a), b), b), b), a)$
- $= \delta(\delta(\delta(\delta(q_0, a), b), b), a)$
- $= \delta(\delta(\delta(q_1, b), b), a)$
- $= \delta(\delta(q_2, b), a)$
- $= \delta(q_3, a)$
- $= q_4 \in F$
- String abba is accepted as q_4 is a final state



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Language Accepted by FAs

- For a FA $M = (Q, \Sigma, \delta, q_0, F)$
- Language accepted by M
- $L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$



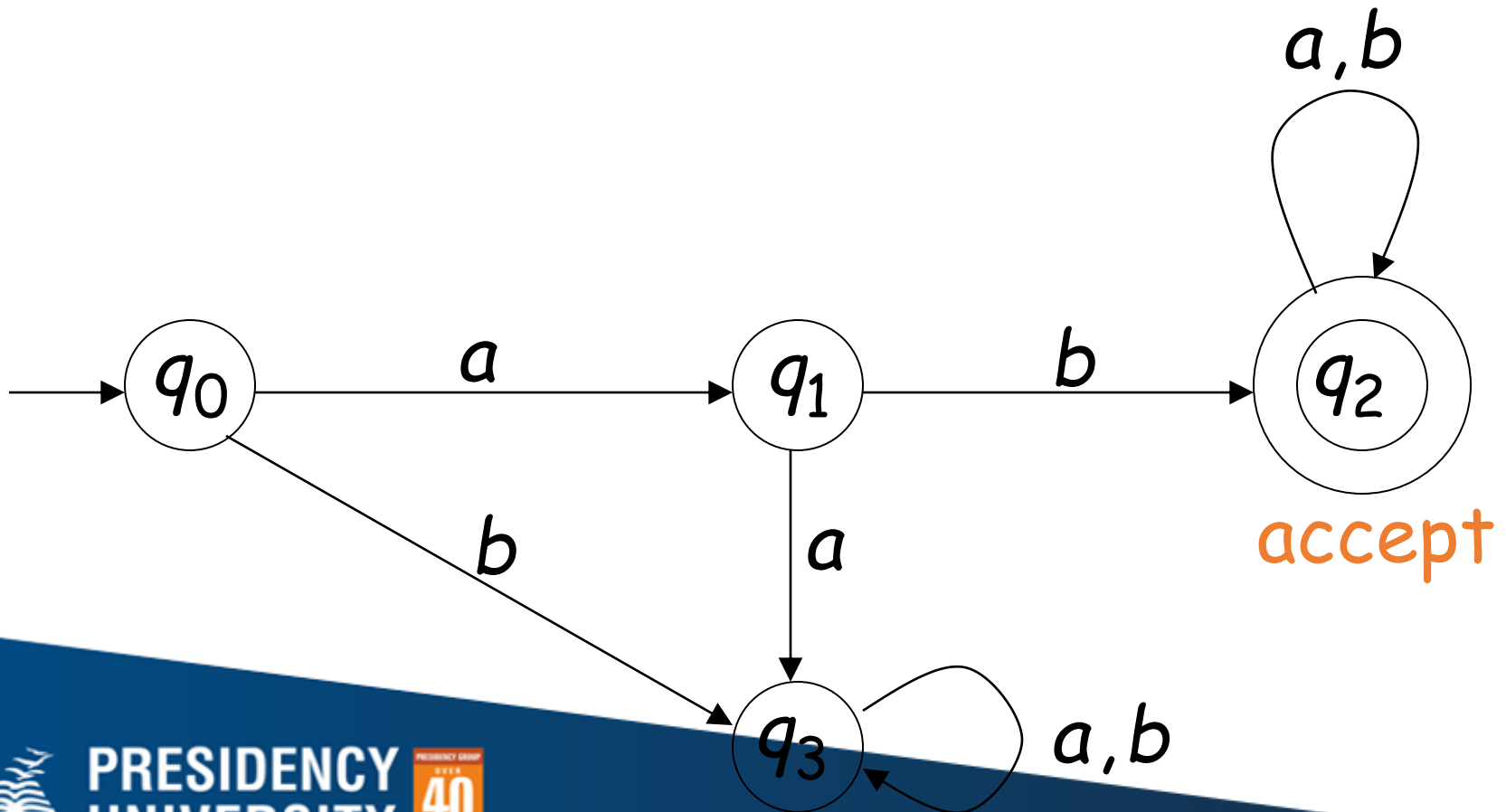
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Example

$L(M) = \{ \text{all strings with prefix } ab \}$



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L=strings with substring '101' over $\{0, 1\}$



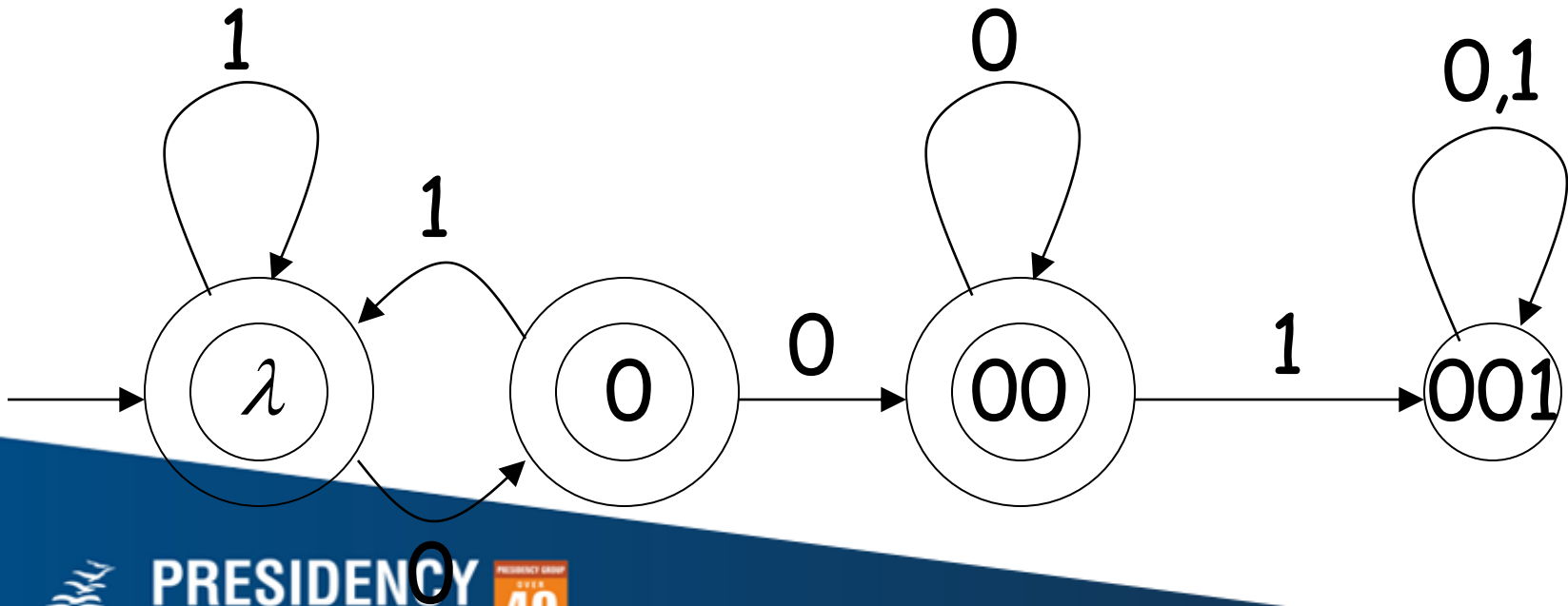
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Example

• $L(M) = \{ \text{all strings without substring } 001 \}$



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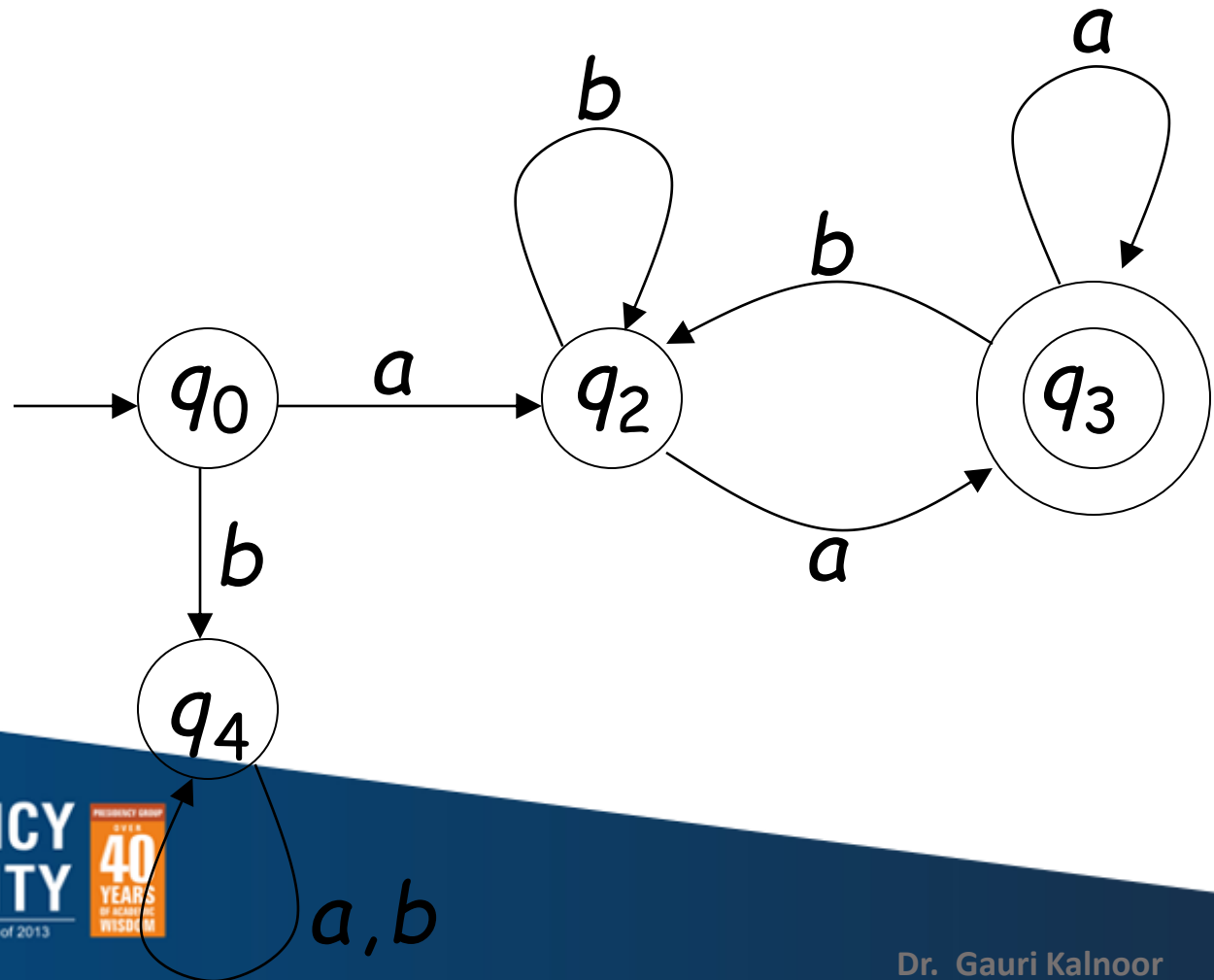
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Example

$$L(M) = \{awa : w \in \{a,b\}^*\}$$

•



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Regular Languages

- **Definition:**

- A language L is regular if there is
- FA M such that $L = L(M)$

- **Observation:**

- All languages accepted by FAs
- form the family of regular languages
-



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Examples of regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$

$\{awa : w \in \{a,b\}^*\}$ $\{a^n b : n \geq 0\}$

$\{\text{all strings with prefix } ab\}$

$\{\text{all strings without substring } 001\}$

There exist automata that accept these
Languages



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There exist languages which are not Regular:

Example: $L = \{a^n b^n : n \geq 0\}$

There is no FA that accepts such a language

(we will prove this later in the class)



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Deterministic Finite Automata

- Every State should have transition over every input symbol
- There should be only One next state for each transition
- **DFA is defined as**

$$M = (Q, \Sigma, \delta, q_0, F)$$



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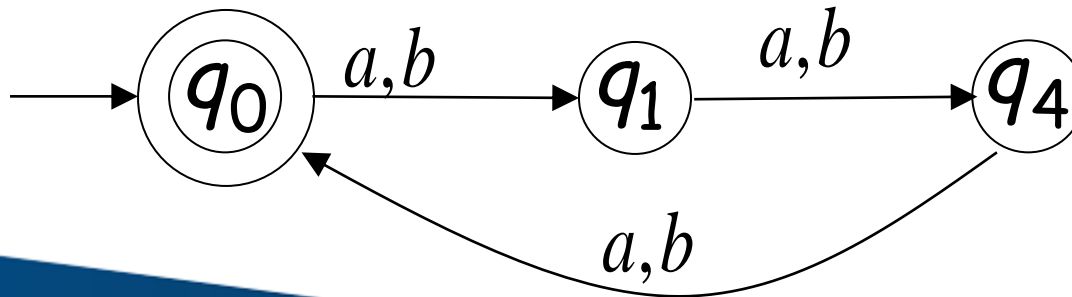


DFA Example

Design DFA that accepts language $L = \{ w : |w| \bmod 3 = 0 \}$
over $\Sigma = \{a, b\}$

Solution

- Strings accepted = $\{\epsilon, aaa, bbb, aba, aab, bab, aaabbb, ababab, \dots\}$
- Strings rejected = $\{a, b, ab, ba, abab, baba, bbaa, aaabb, \dots\}$
- Transition Diagram



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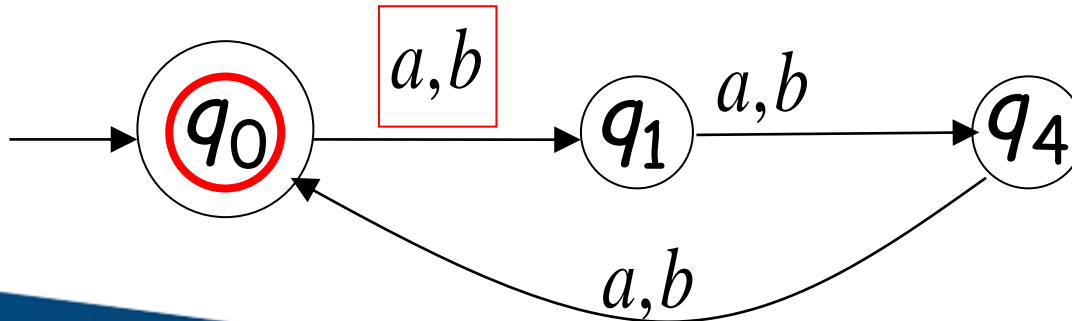
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DFA Example

➤ Consider Sample String : **a a b**

↑



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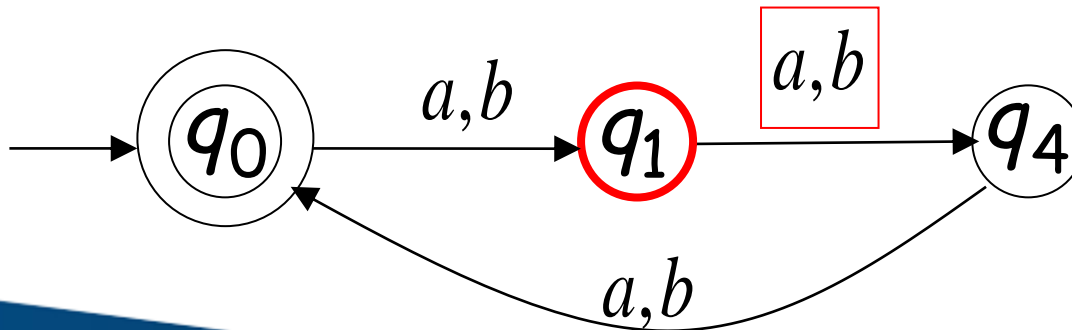
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DFA Example

➤ Consider Sample String : **a a b**

↑



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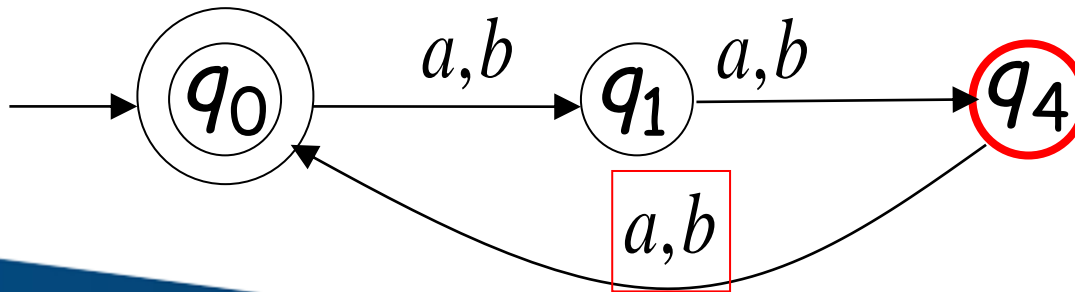
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DFA Example

➤ Consider Sample String : **a a b**

↑



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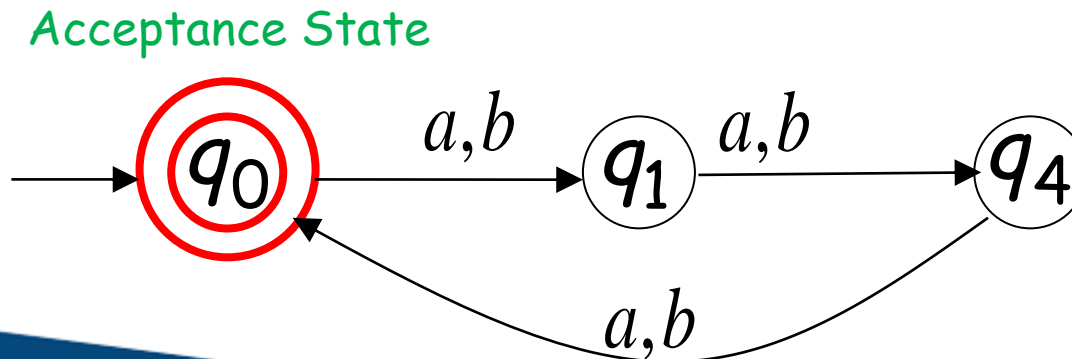
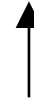
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DFA Example

- Consider Sample String : **a a b**
- String is accepted



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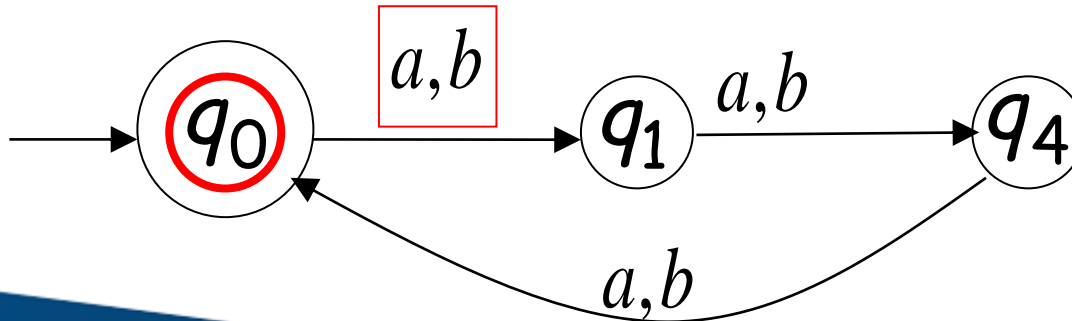
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DFA Example

➤ Consider Another Sample String : **b a b a**

↑



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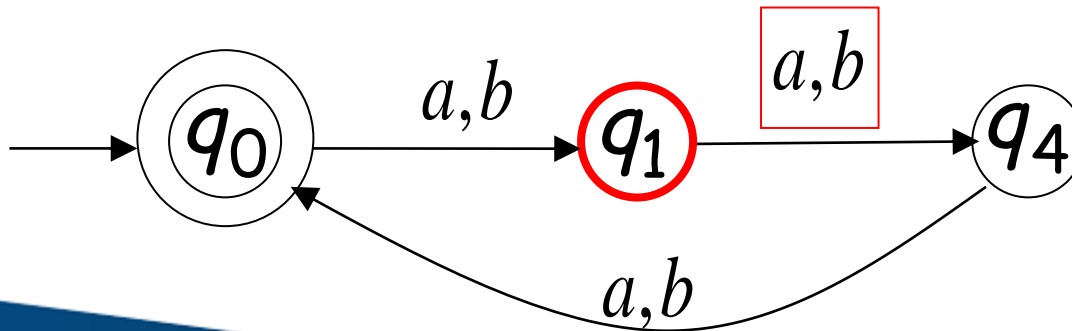
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DFA Example

➤ Consider Another Sample String : **b a b a**

↑



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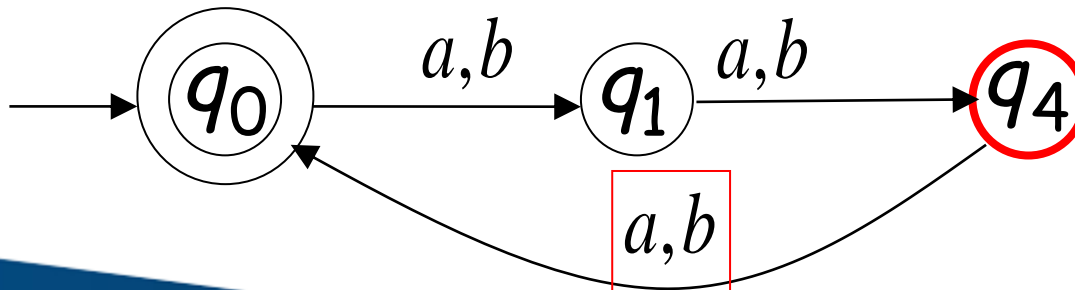
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DFA Example

➤ Consider Another Sample String : **b a b a**

↑



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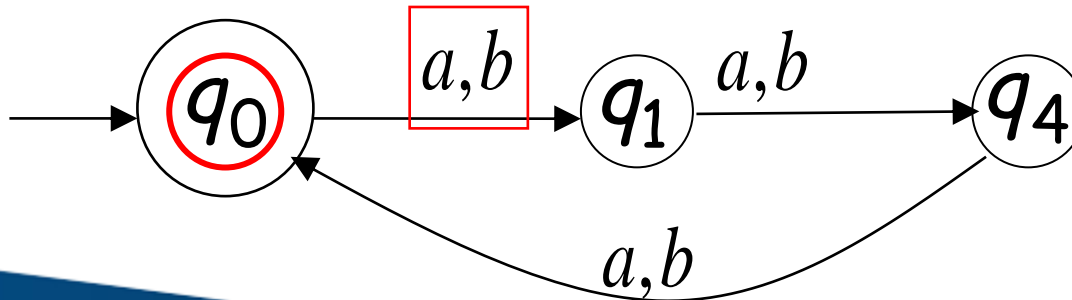
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DFA Example

➤ Consider Another Sample String : **b a b a**



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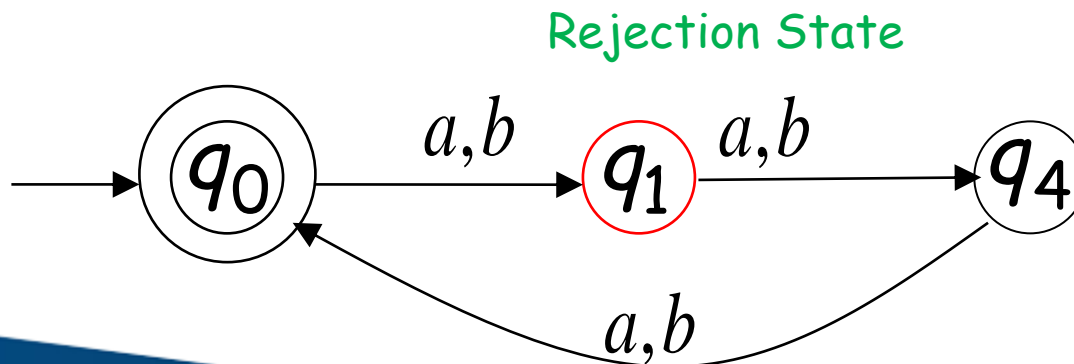
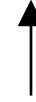
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DFA Example

➤ Consider Another Sample String : **b a b a**

➤ String is rejected



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DFA Example

- DFA Tuples $M=(Q, \Sigma, \delta, q_0, F)$
- $Q= \{q_0, q_1, q_4\}$
- $\Sigma= \{a, b\}$
- δ = Transition function represented by Transition table
- q_0 = Initial State
- $F= \{q_0\} \rightarrow$ Acceptance State



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DFA Example

➤ Transition Table

Q	Σ	a	b
→*q0		q1	q1
q1		q4	q4
q4		q0*	q0*



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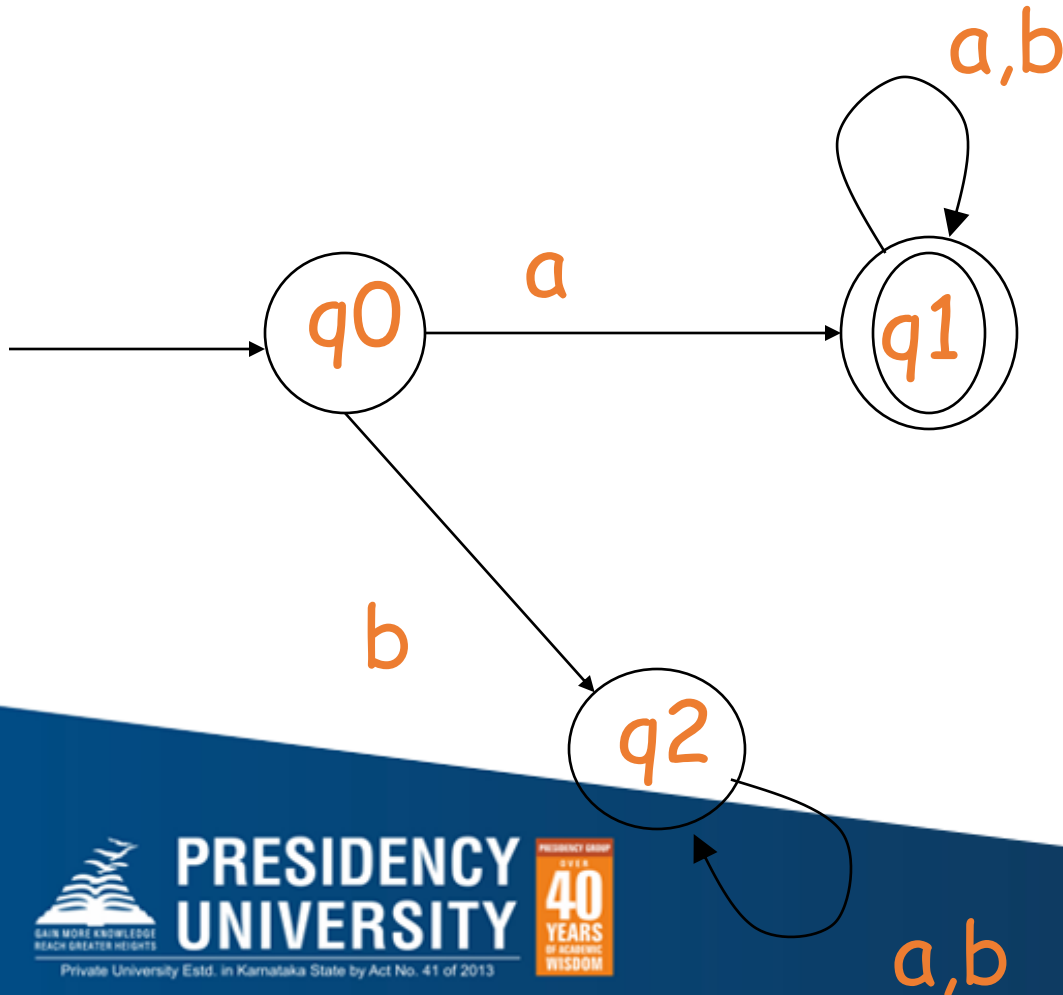
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DFA - Starts with a , $\Sigma=\{a,b\}$

- $L = \{ a, aa, ab, aaa, aab, aba, \dots \}$



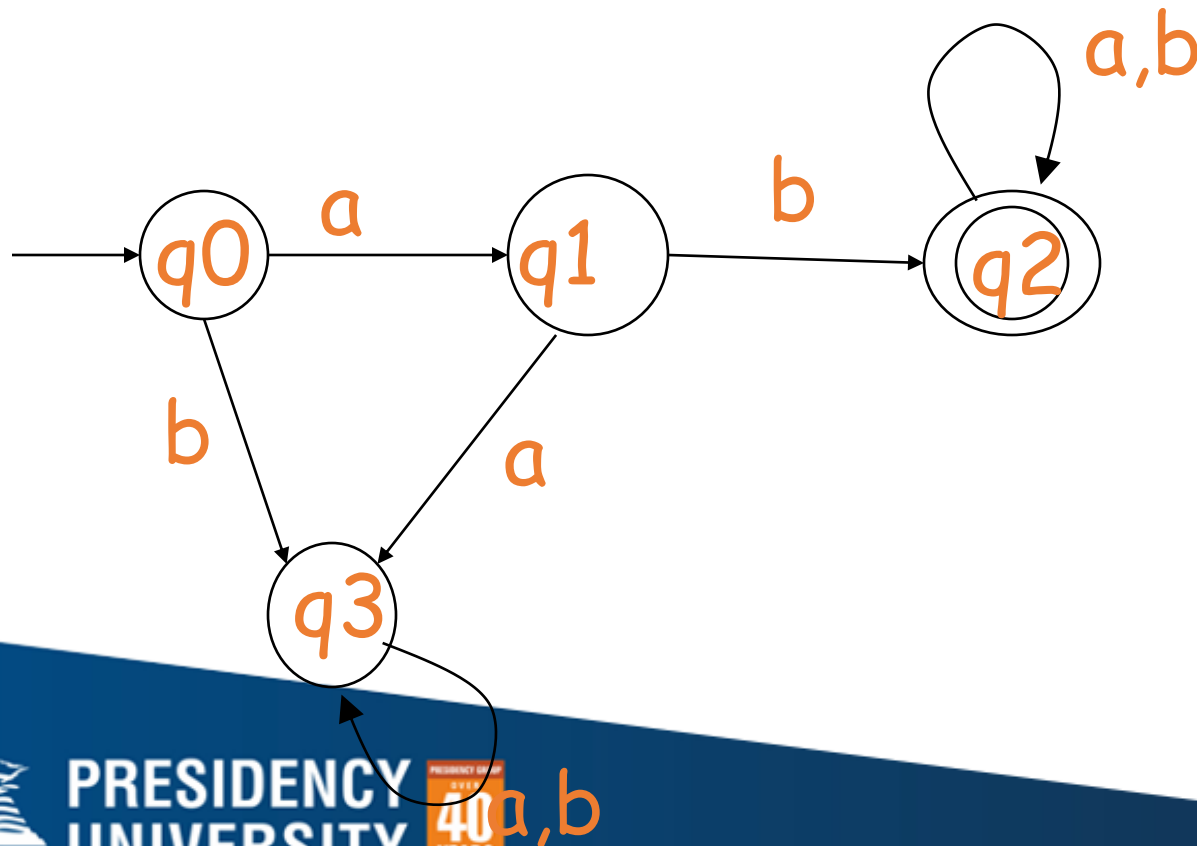
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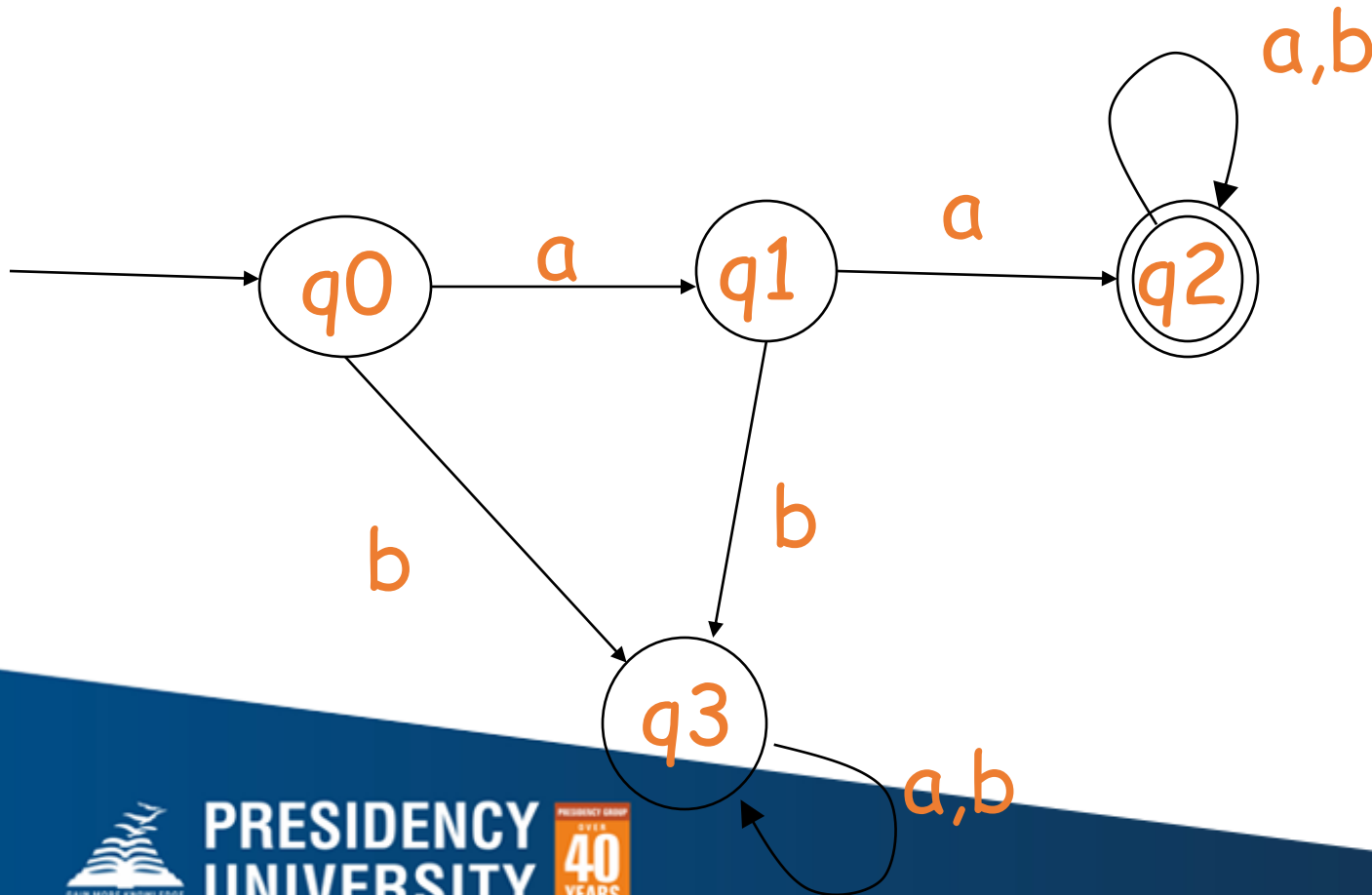
DFA - Starts with ab , $\Sigma=\{a,b\}$

- $L = \{ ab, aba, abb, \dots \}$



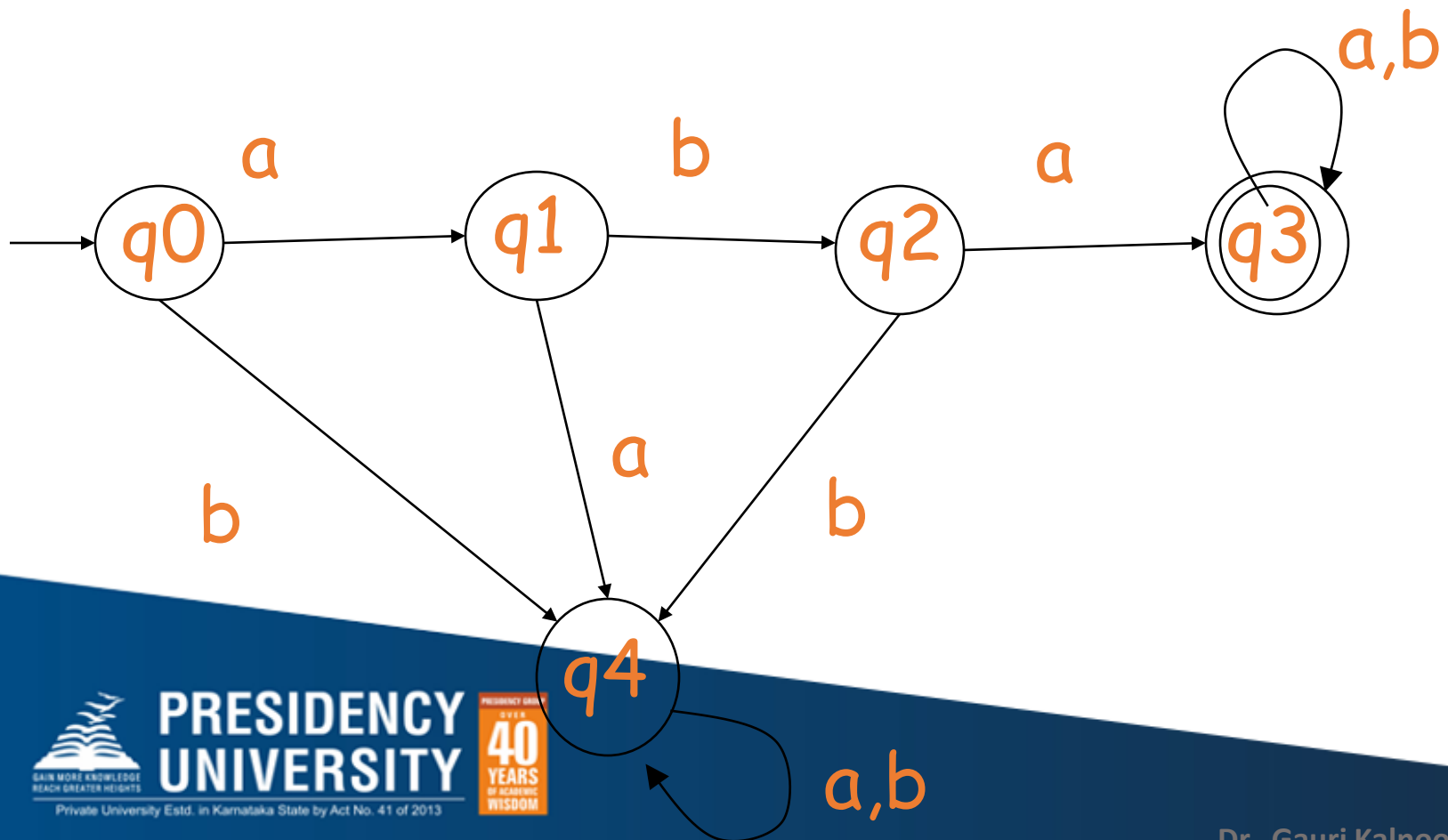
DFA - Starts with aa , $\Sigma=\{a,b\}$

- $L = \{aa, aab, aaba, \dots\}$



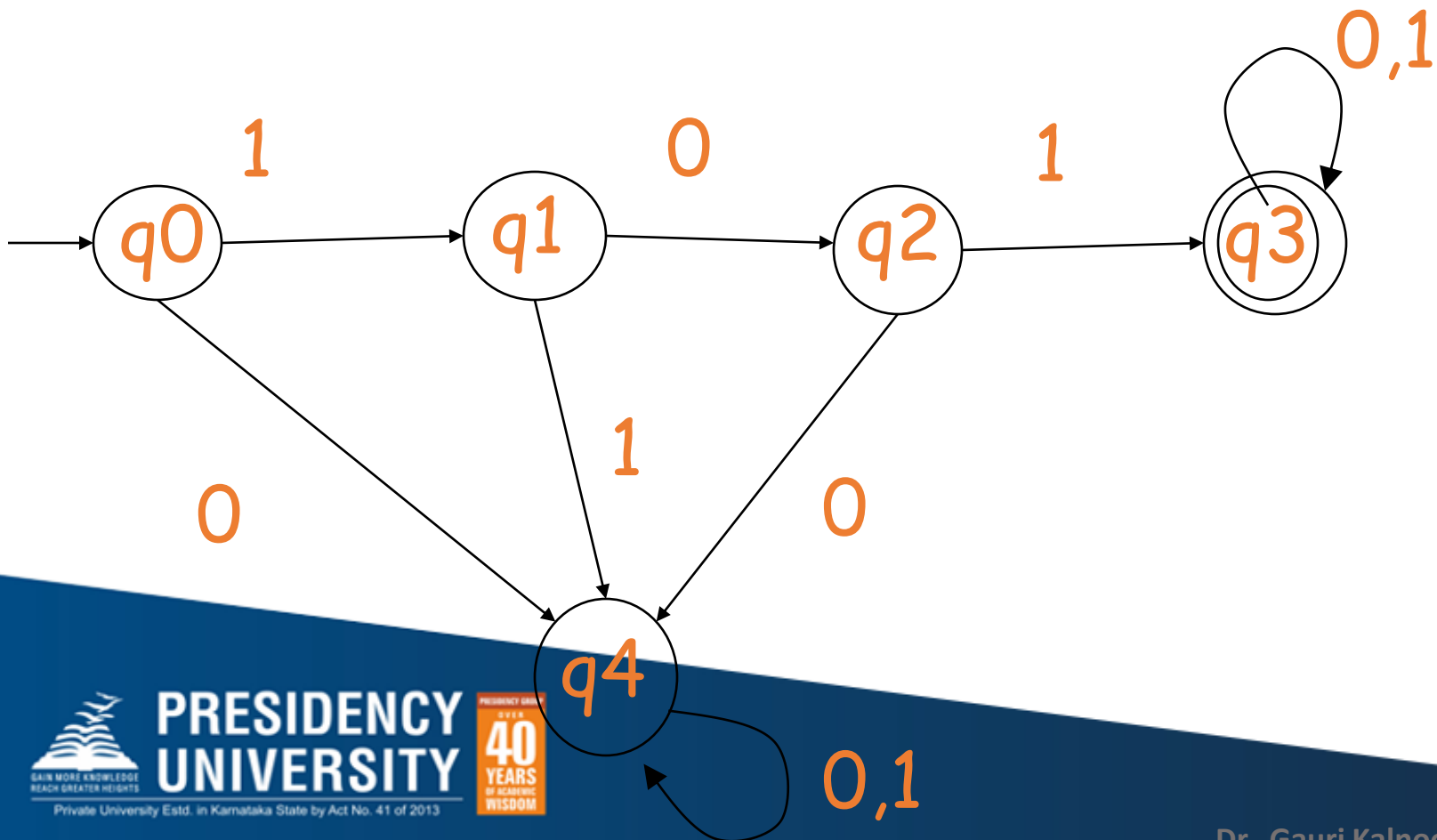
DFA - Starts with aba , $\Sigma=\{a,b\}$

- $L=\{ aba, abaa, abab, abaaa, \dots \}$



DFA - Starts with 101 , $\Sigma=\{0,1\}$

- $L=\{ 101,1010,1011,101101,\dots\}$



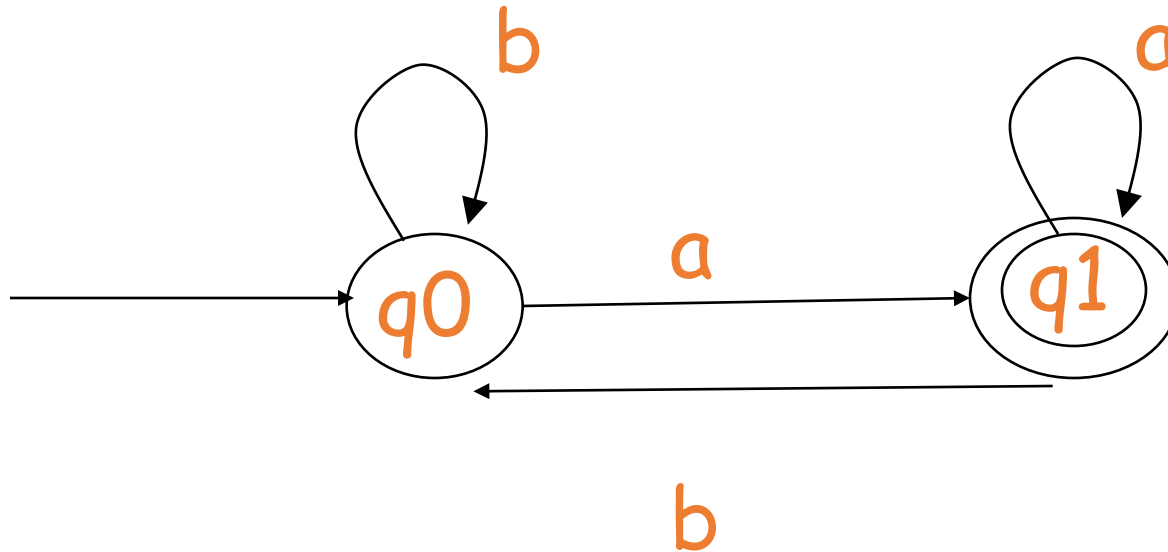
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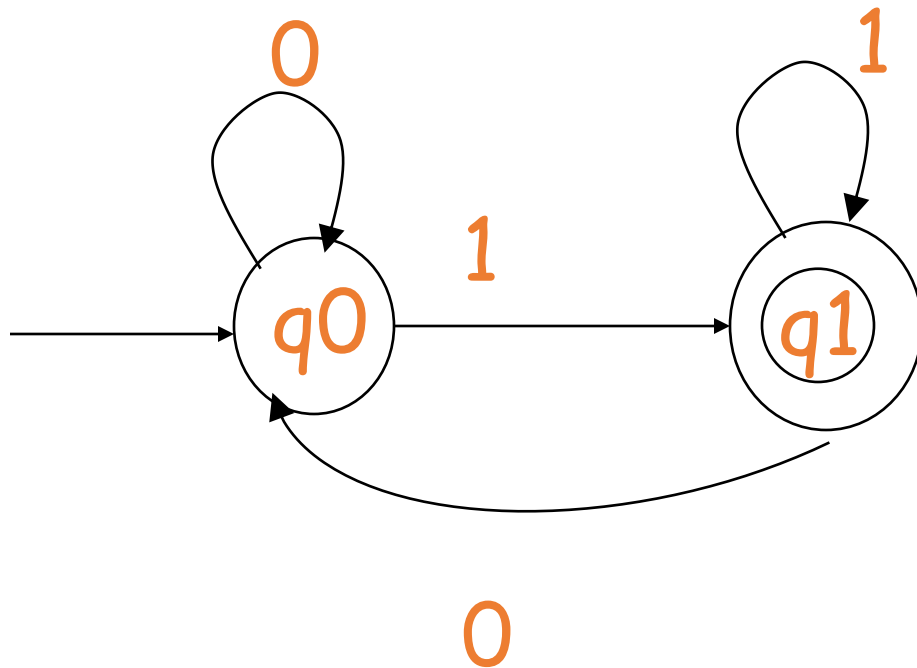
Ends with a

- $L = \{a, aa, ba, aaa, aba, \dots\}$



Ends with 1 , $\Sigma = \{0,1\}$

- $L = \{1, 01, 11, \dots\}$



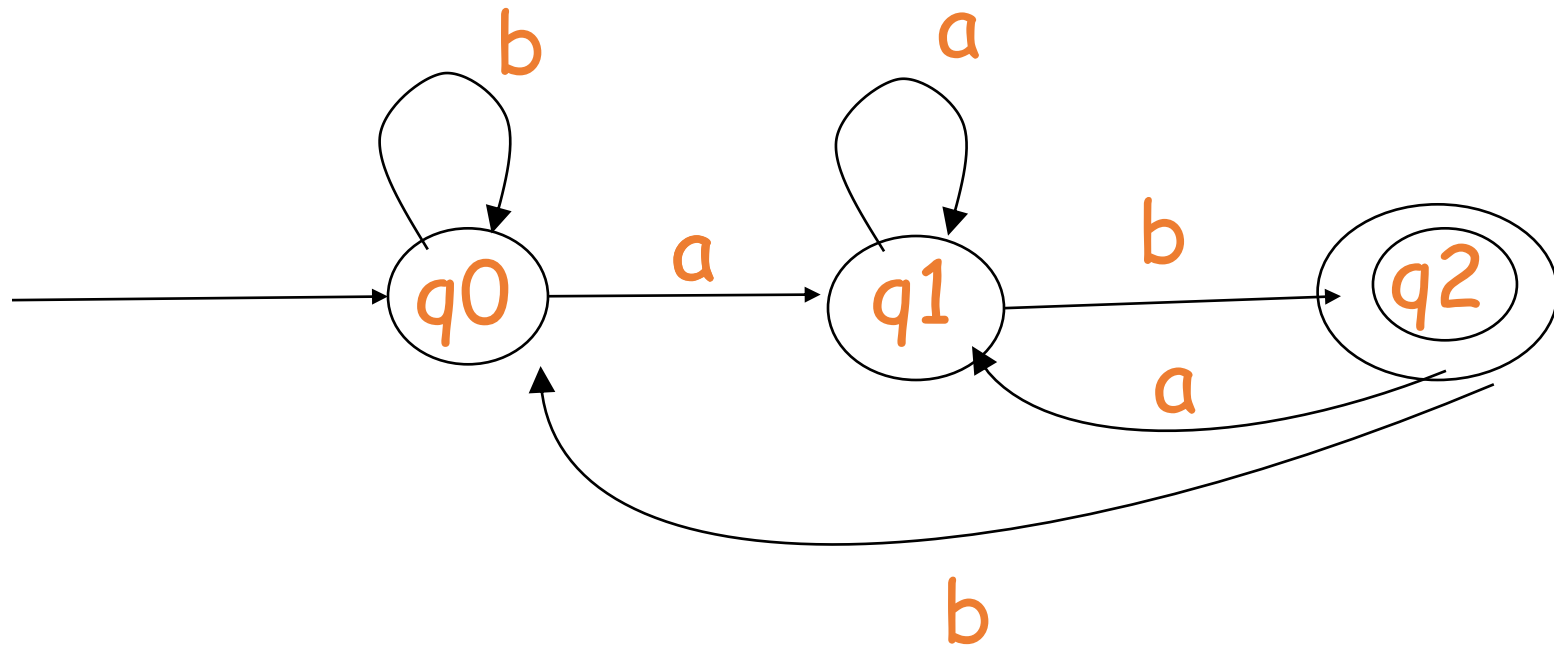
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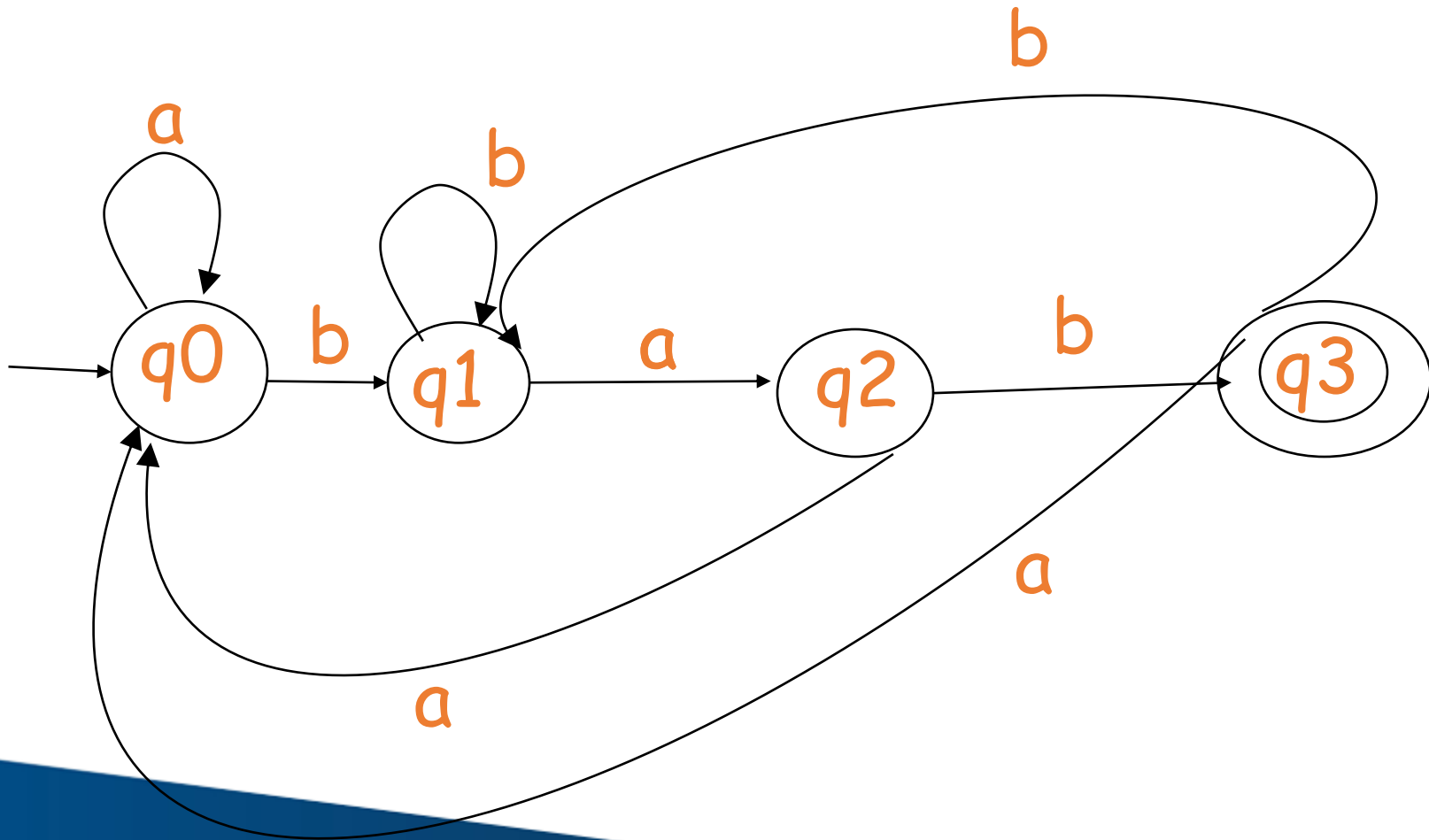


Ends with ab

- $L = \{ab, aab, bab, aaab, bbbbab, abab, \dots\}$



Ending with bab

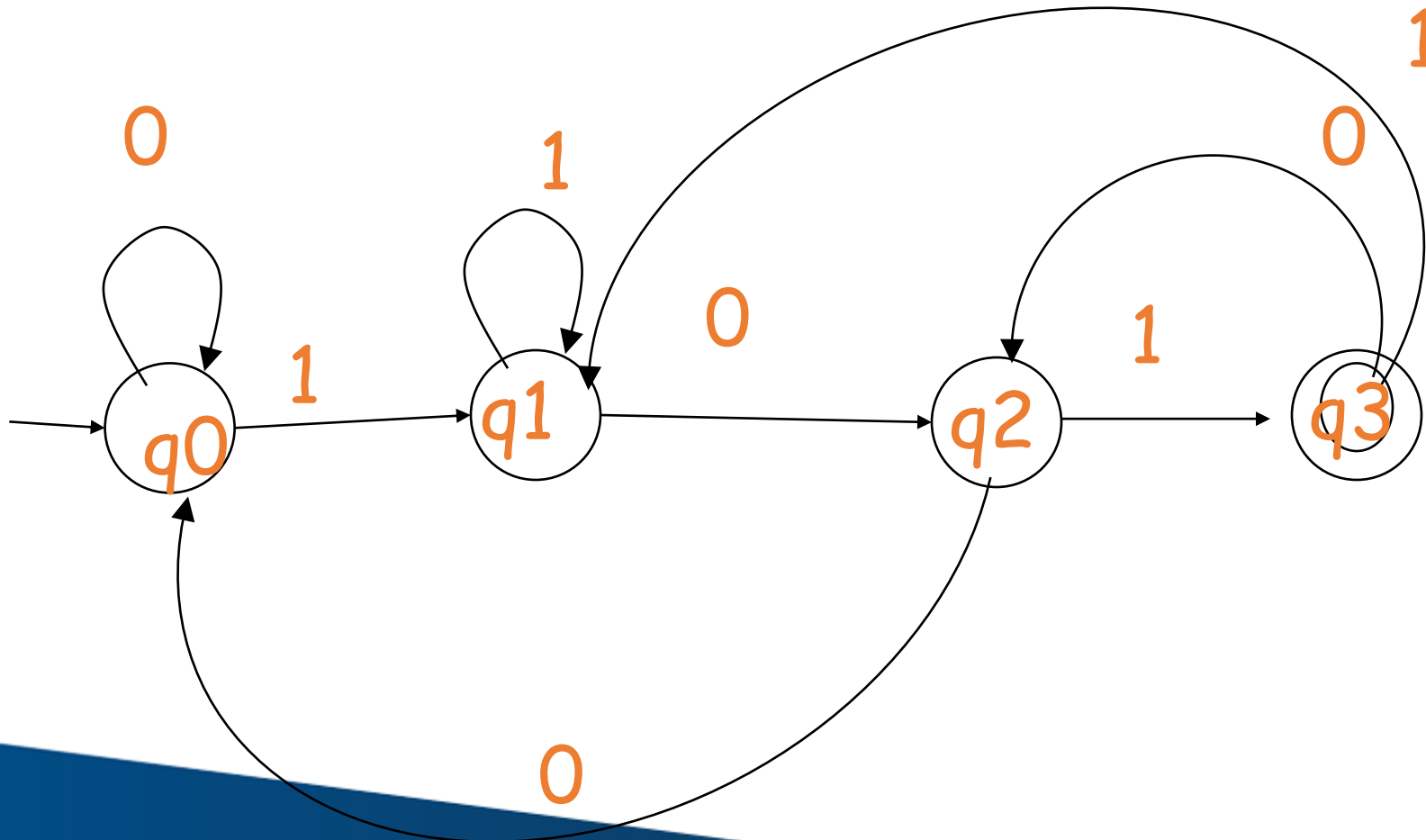


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- DFA – Ends with 101, $\Sigma = \{0,1\}$
 $L = \{101, 0101, 1101, 000101, 111101, \dots\}$



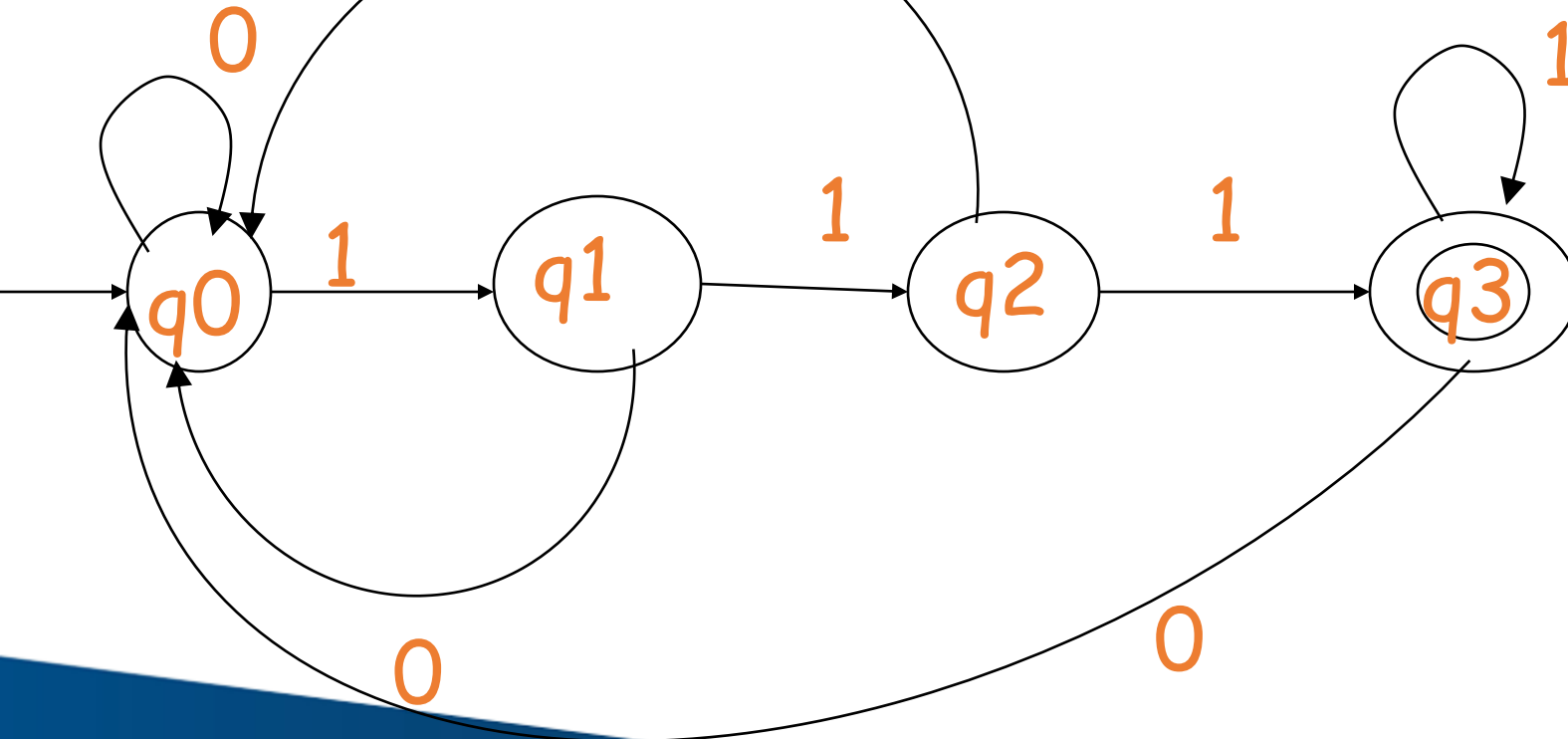
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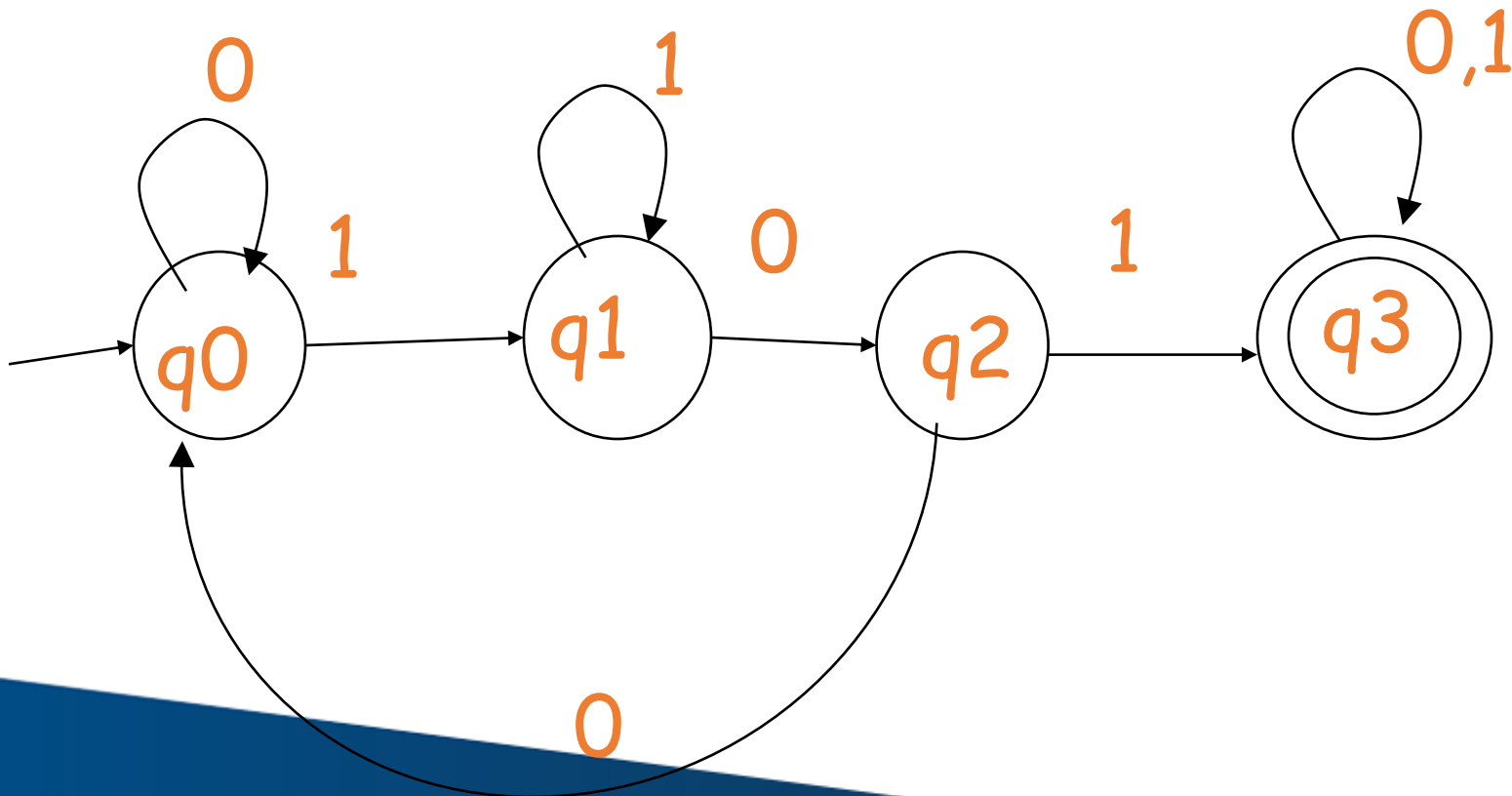
DFA – Ends with 111 , $\Sigma=\{0,1\}$

- $L = \{111, 0111, 11111, \dots\}$



DfA – Substring 101

- $L = \{ 101, 0101, 1101, 1010, 1011, 00010111, \dots \}$



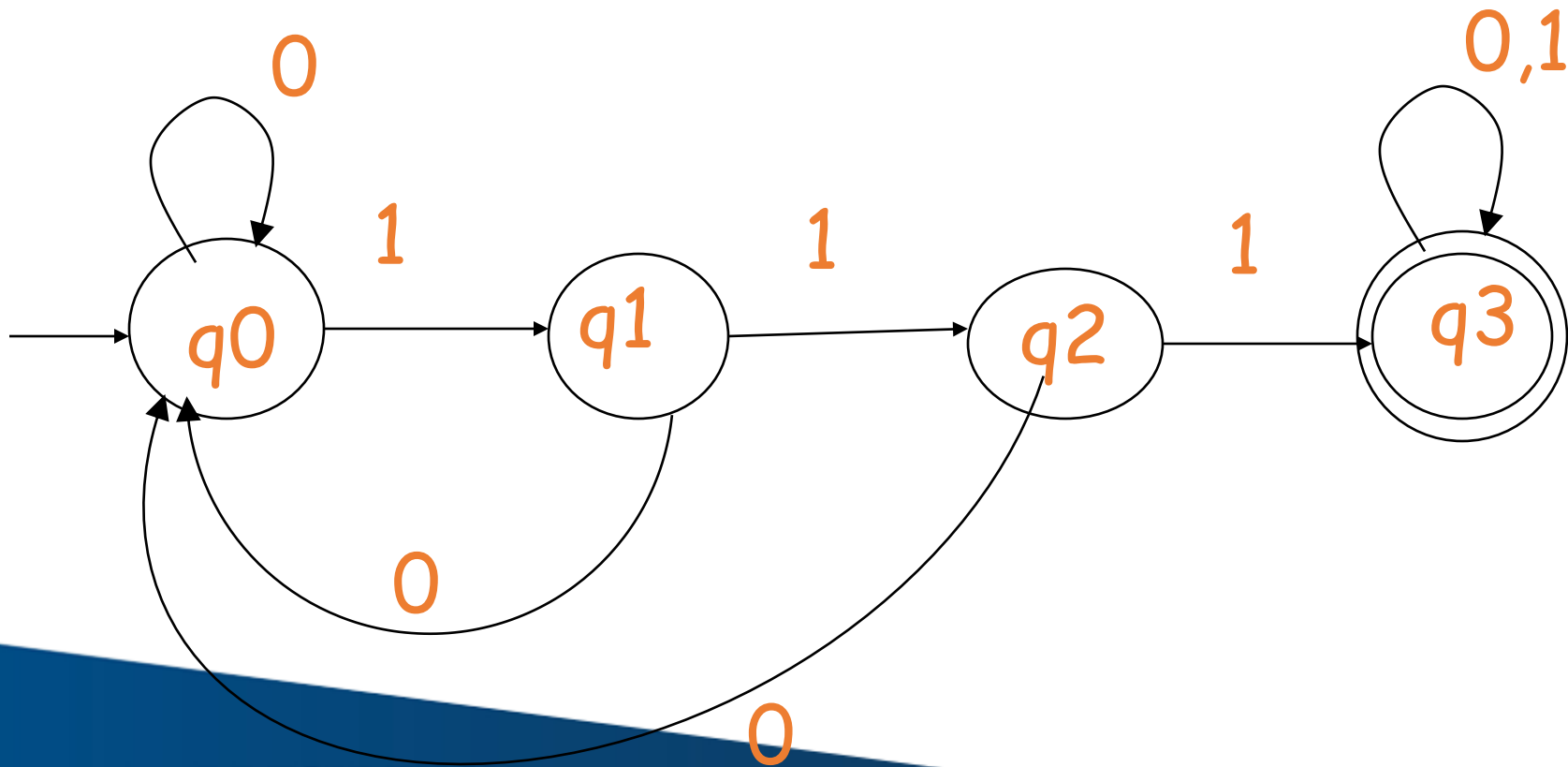
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DFA- Contains 111

- $L = \{ 111, 0111, 1111, 000111000, 11111000, \dots \}$



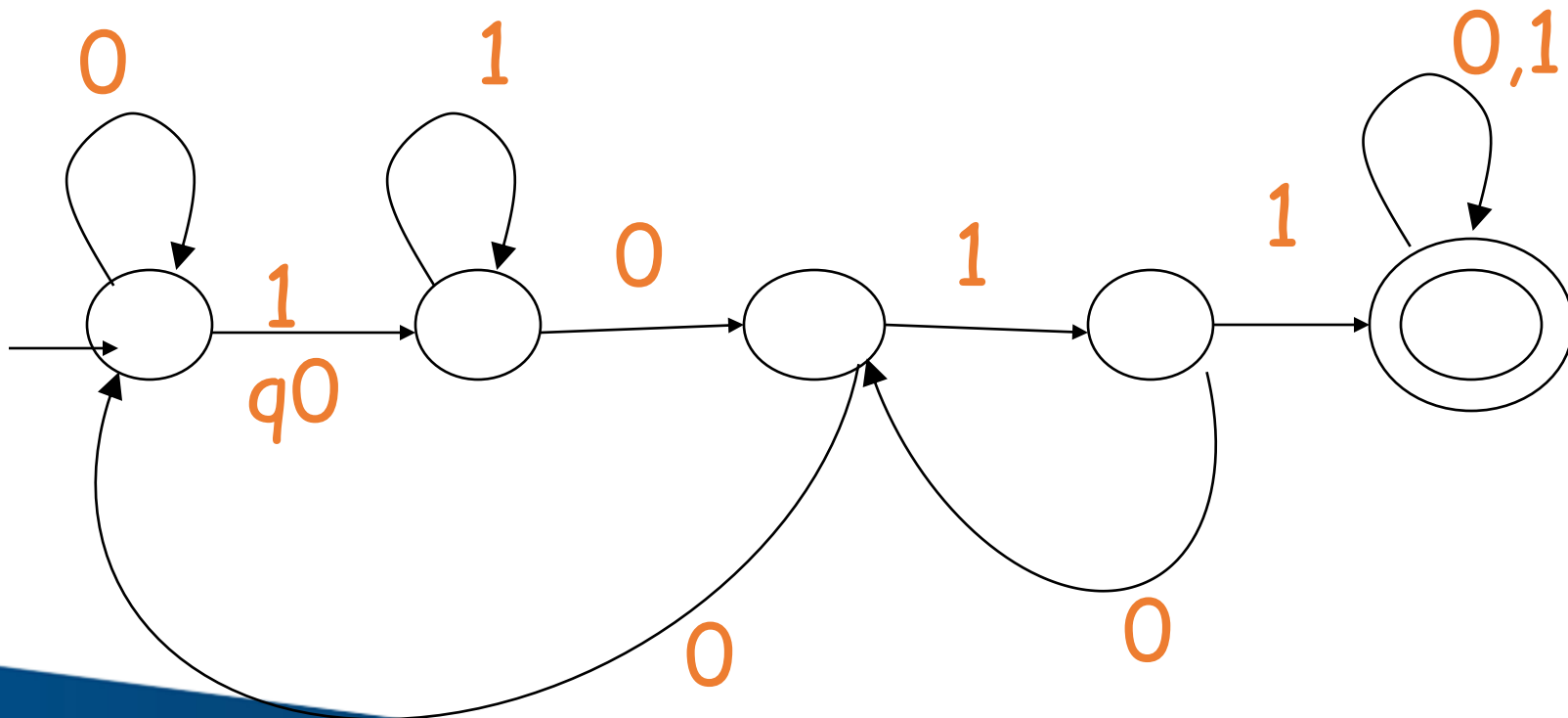
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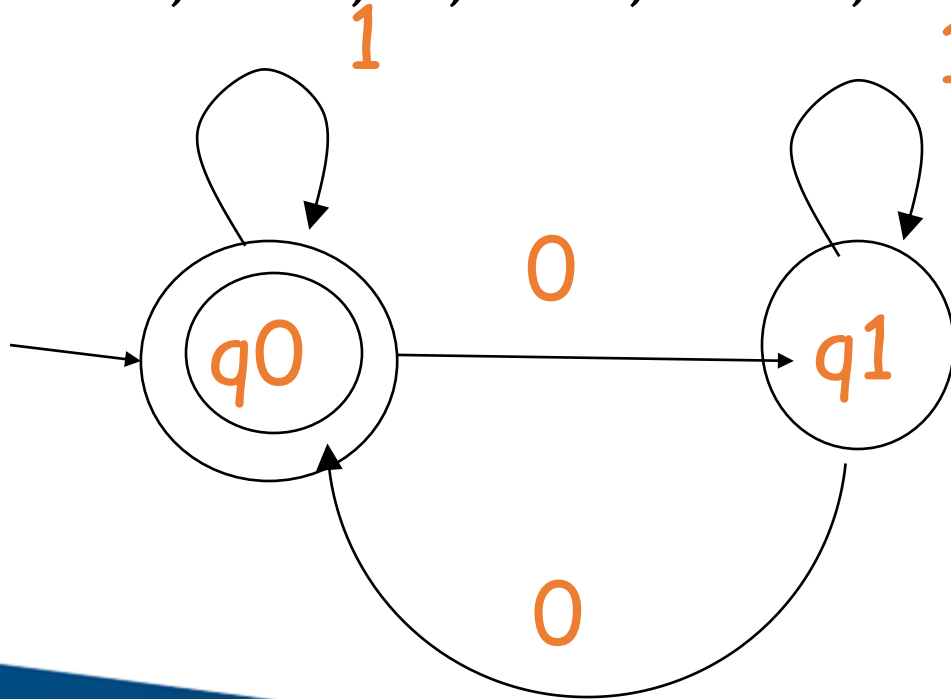
DFA – Contains 1011

- $L = \{ 1011, 01011, 11011, 00010111, \dots \}$



DFA- Even no of 0s, $\Sigma=\{0,1\}$ or $n_0(w) \bmod 2 = 0$

- $L = \{ \lambda, 1, 1111, 00, 0000, 11100, 1110000, 0000111, \dots \}$

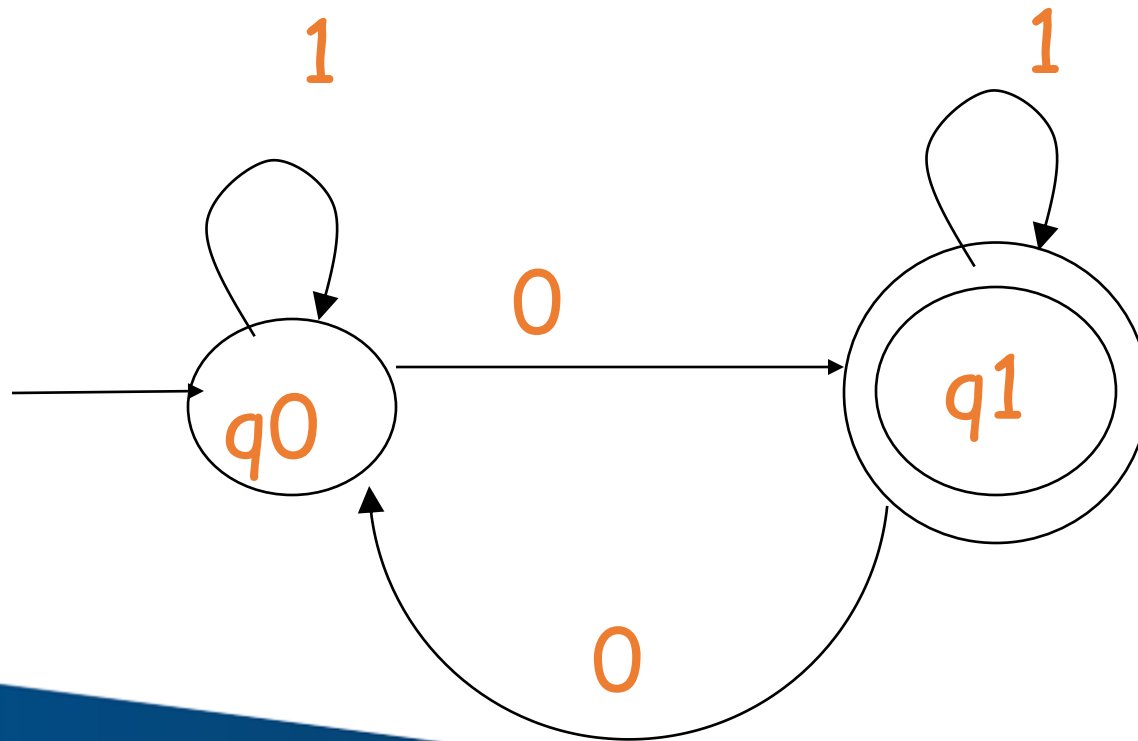


Transition	0	1
*-> q_0	q_1	q_0
q_1	q_0	q_1

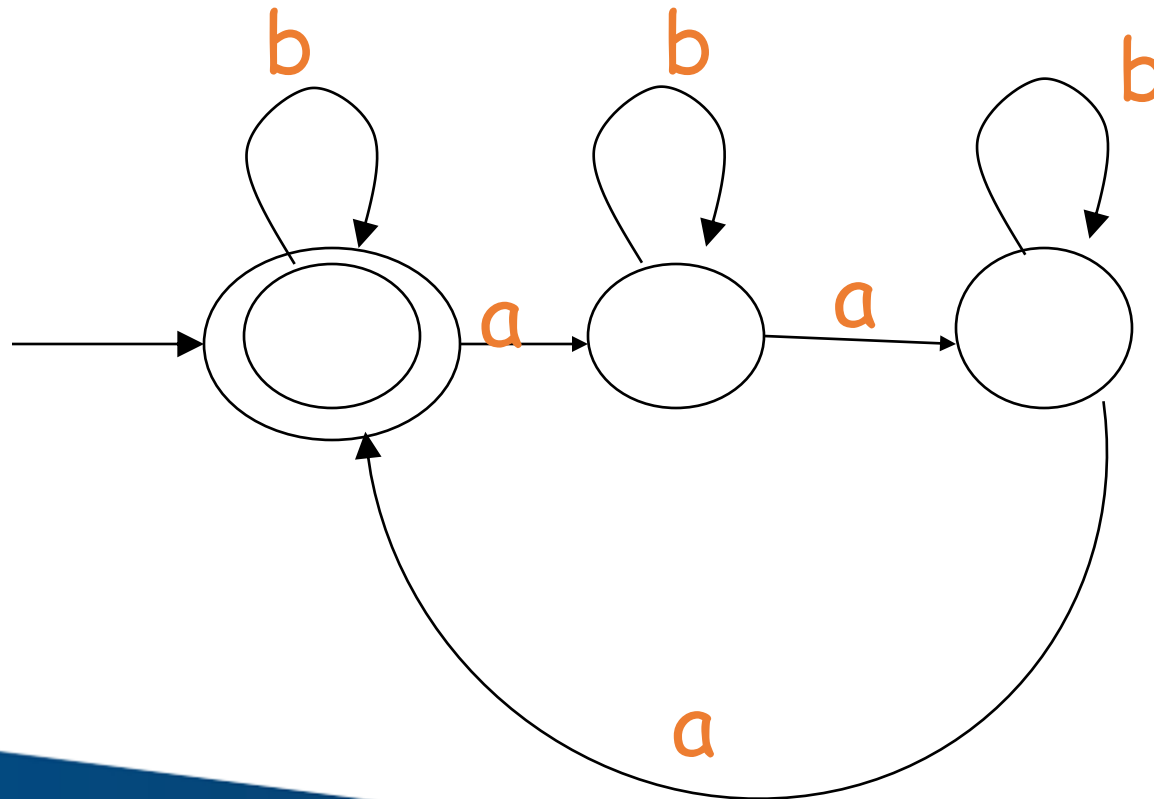


DFA – Odd no of 0's , $\Sigma = \{0,1\}$ or $n0(w) \bmod 2 = 1$

- $L = \{0, 01, 1110, 000, 000001111, \dots\}$



DFA - $na(w) \bmod 3 = 0, \Sigma = \{a, b\}$
 $L = \{ \lambda, bbbb, aaabbb, aaaaaabbb, \dots \}$

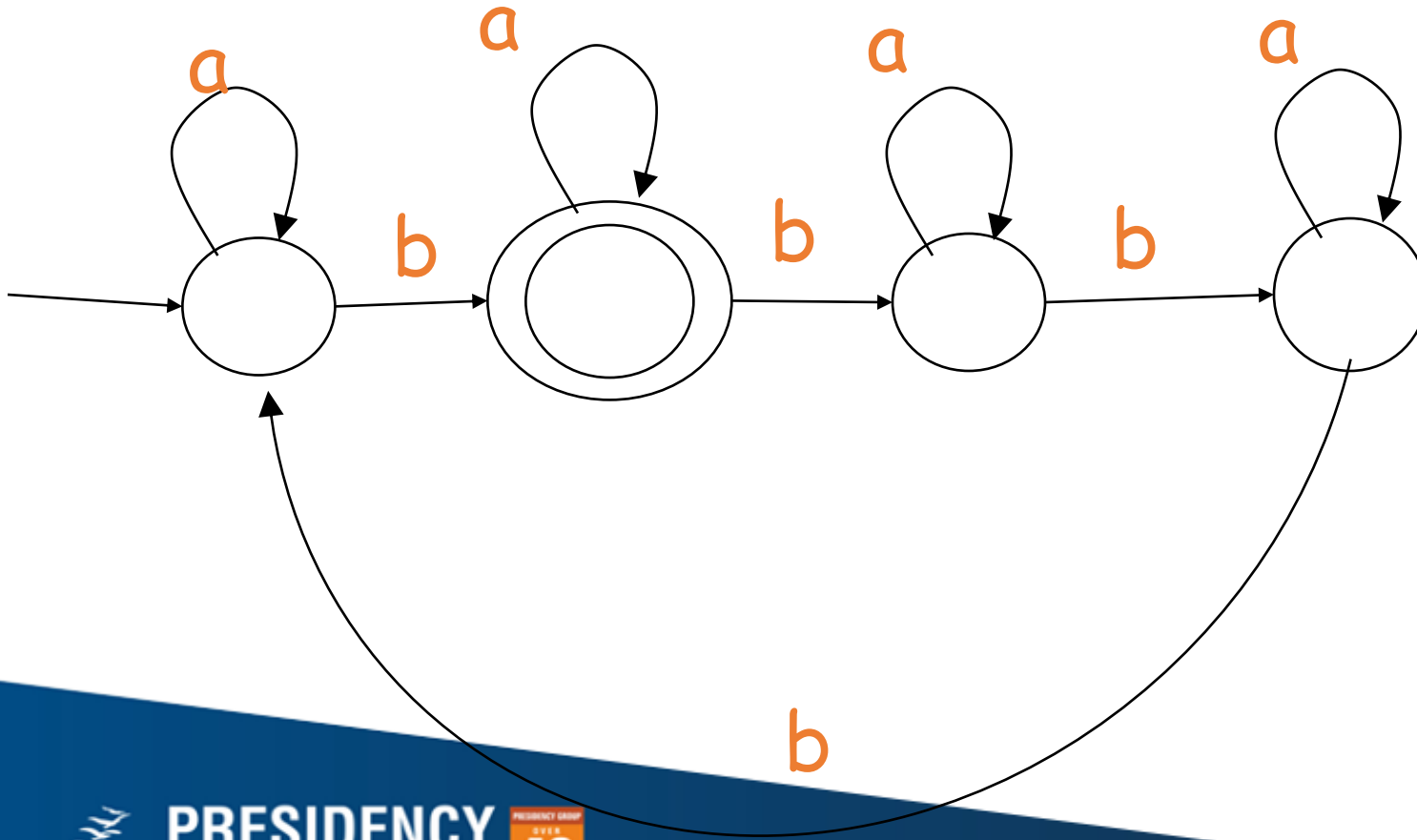


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DFA- $nb(w) \bmod 4 = 1$, $\Sigma = \{a, b\}$

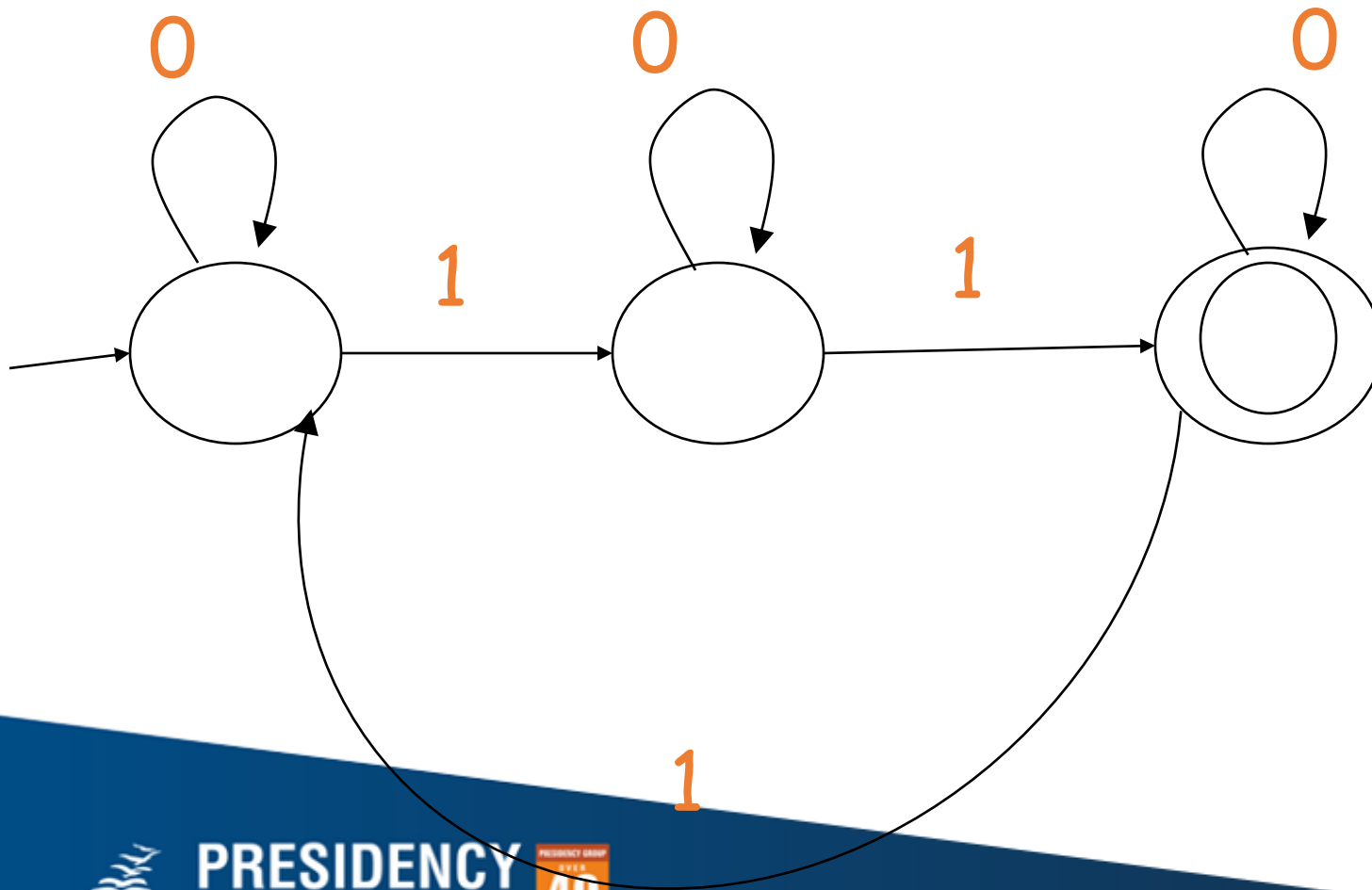


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DfA – $n1(w) \bmod 3 = 2$, $\Sigma = \{0, 1\}$

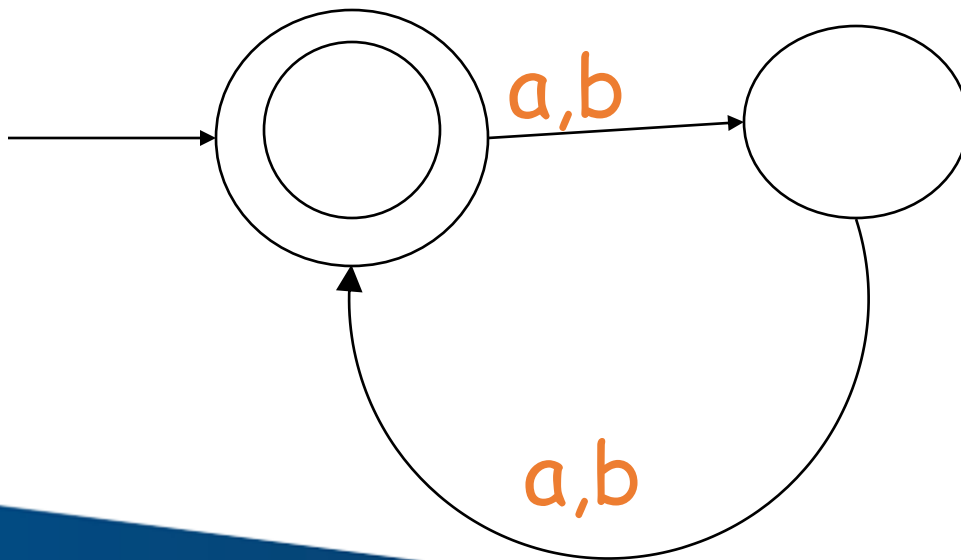


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Dfa $-lwlmod2=0$,



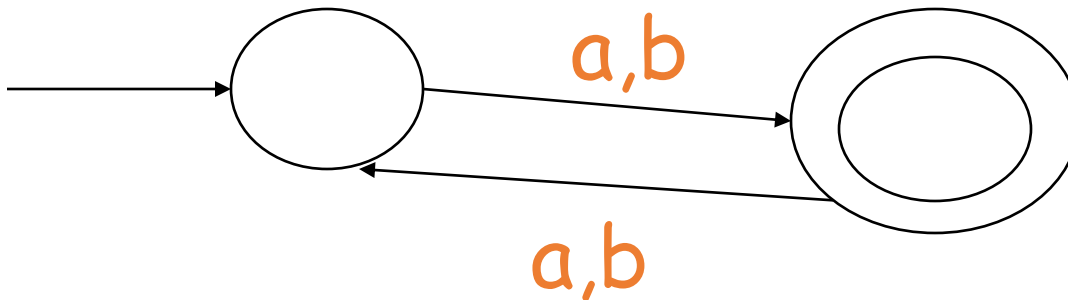
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DFA – $l \bmod 2 = 1$

$L = \{a, b, aaa, aba, aab, baa, bbb, bab, a, bba, \dots\}$

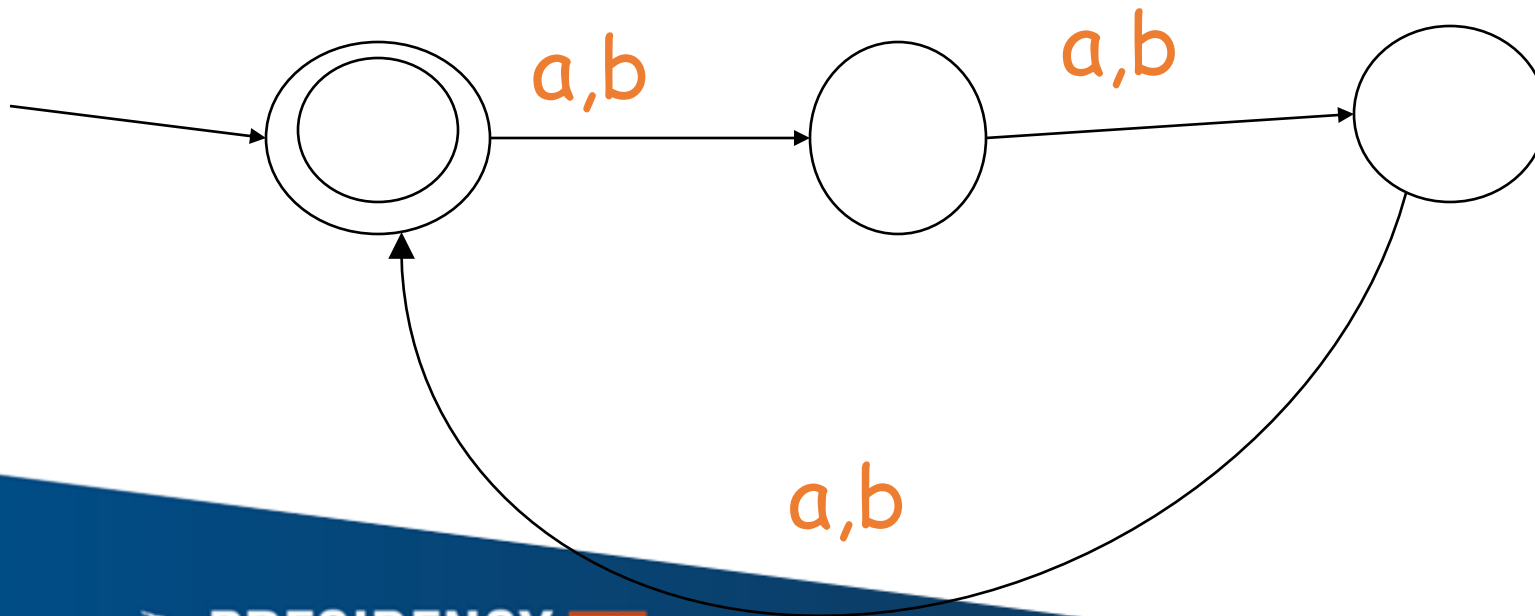


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DFA - $l \bmod 3 = 0$, $\Sigma = \{0,1\}$

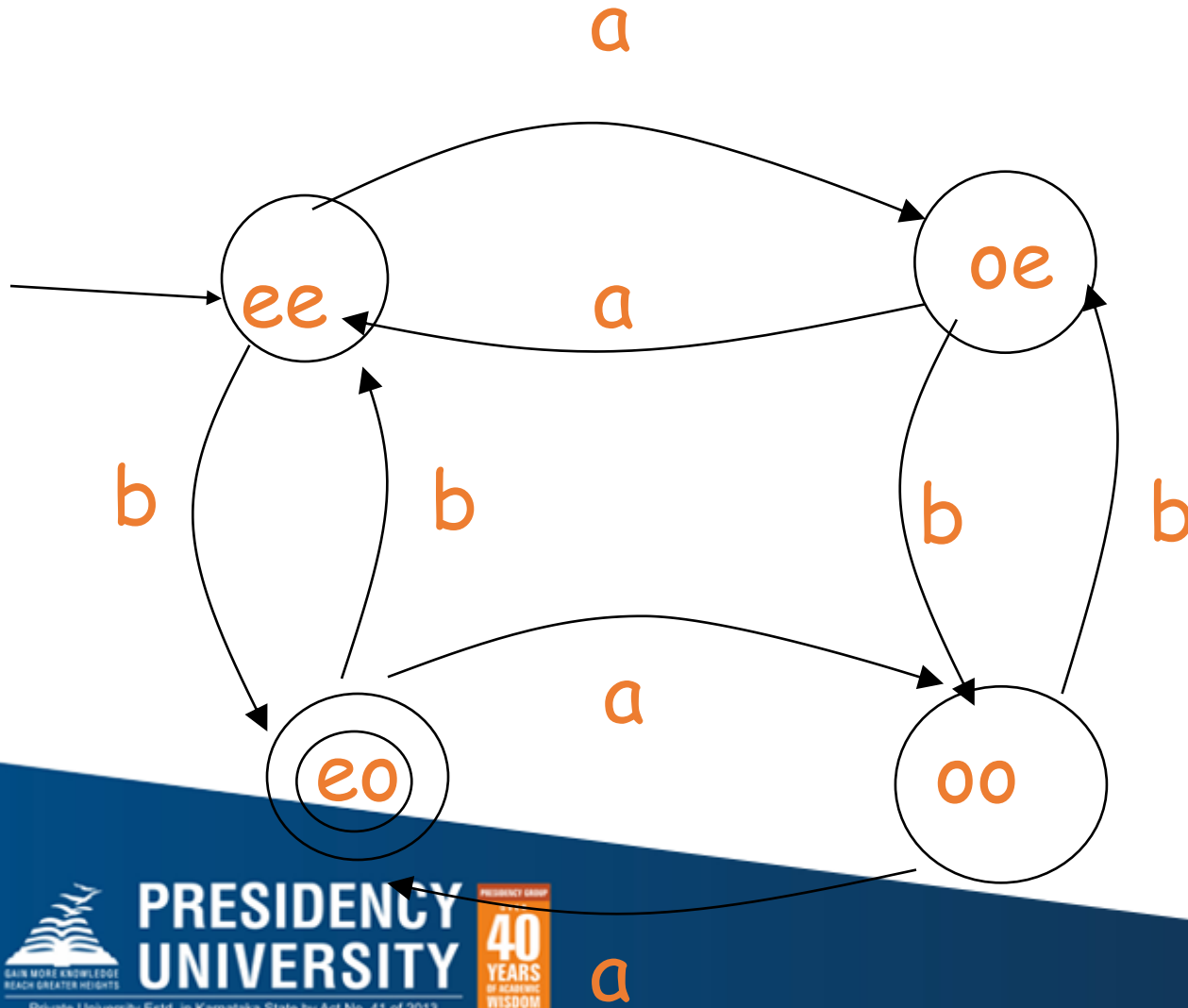


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No of a's is even and no of b's odd- eo



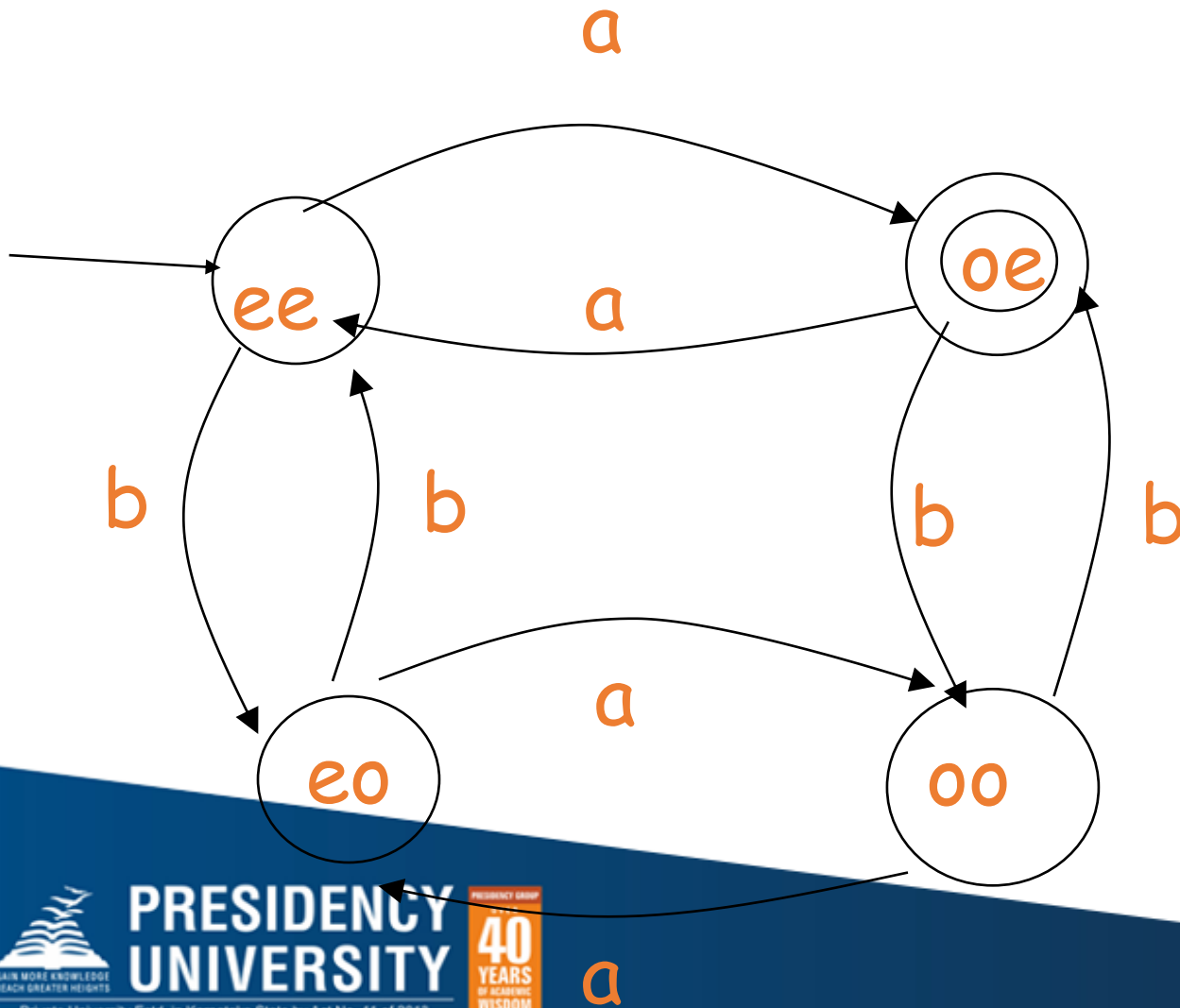
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a

No of a's is odd and no of b's even=oe



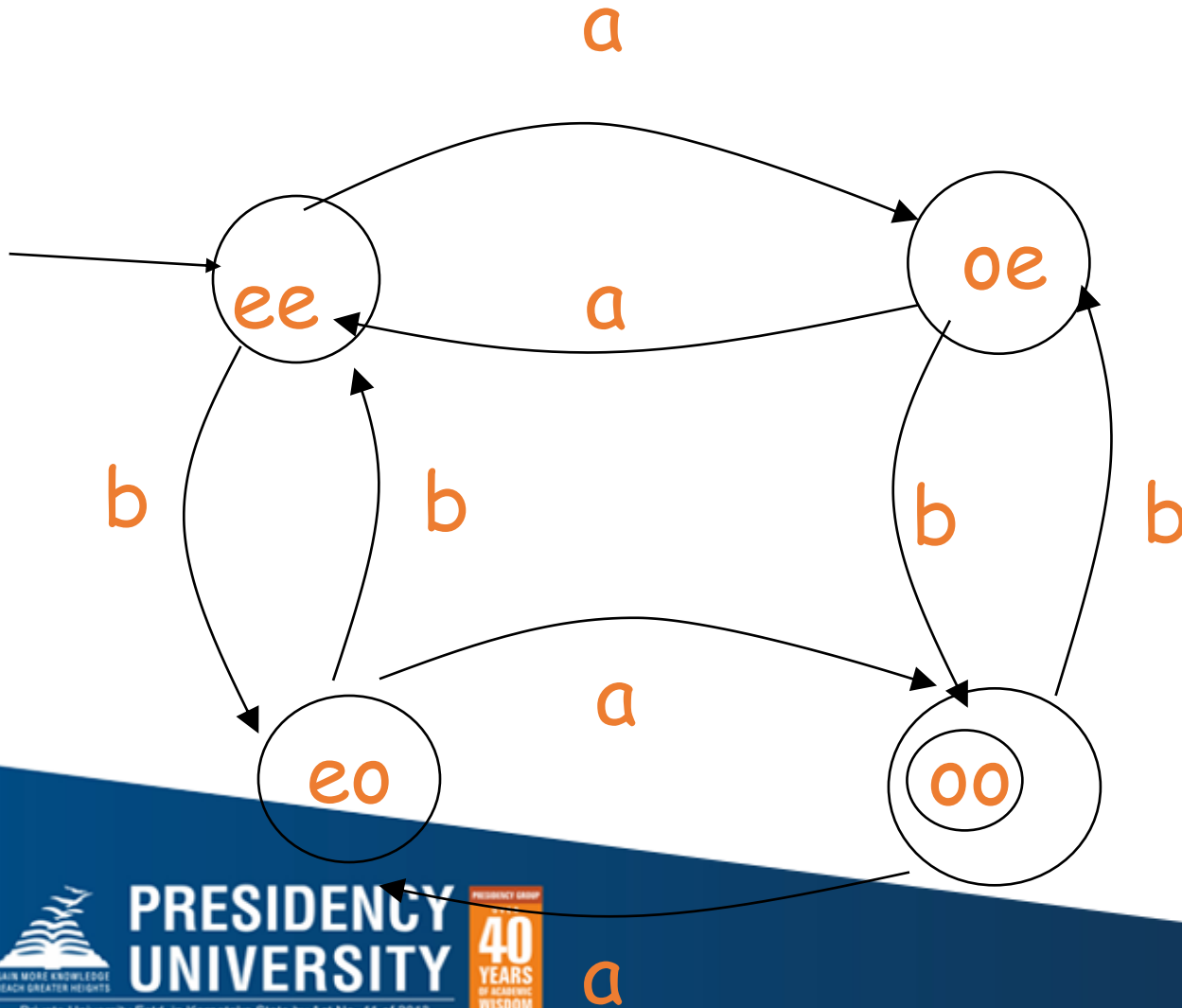
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a

No of a's is odd and no of b's odd-oo



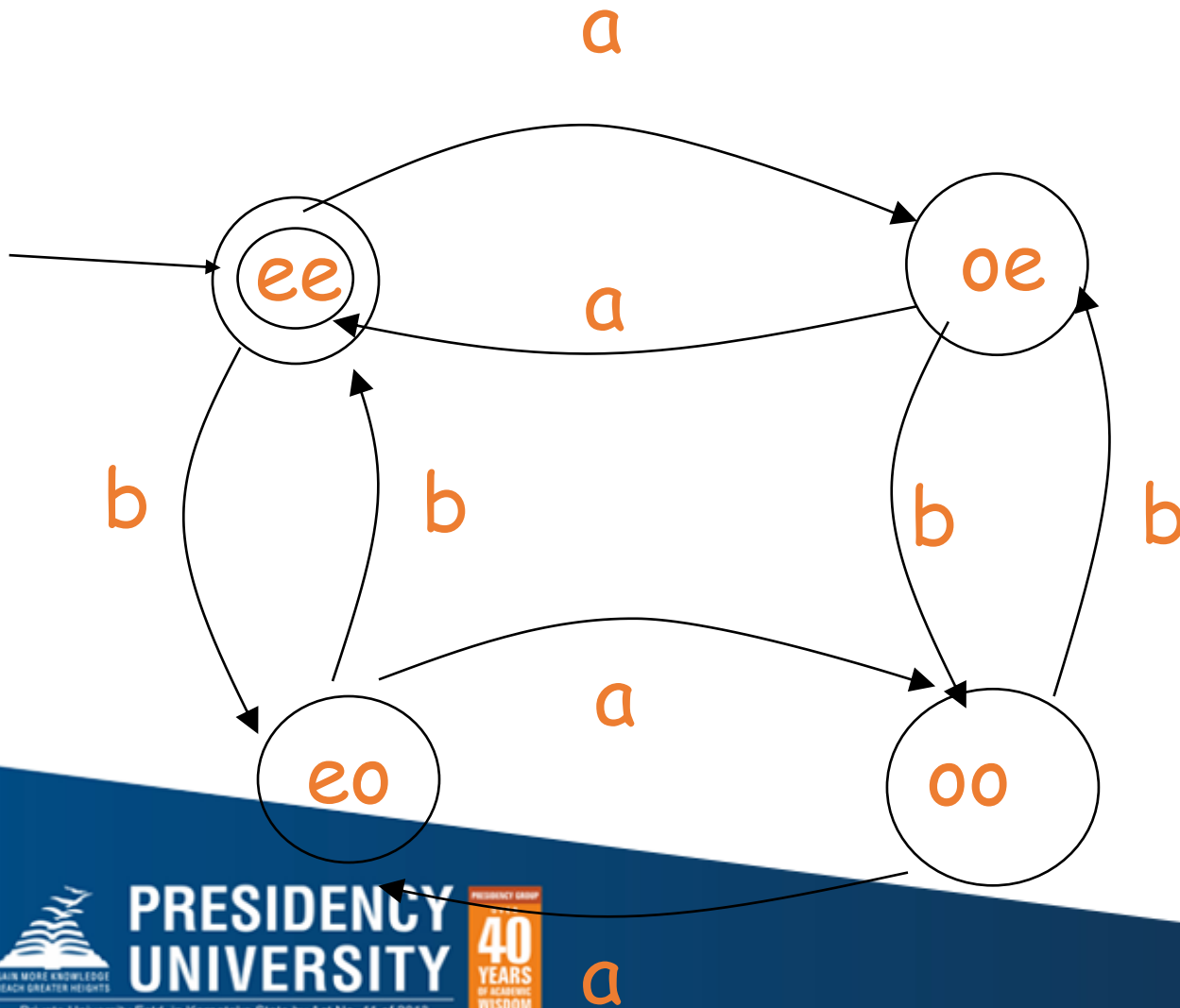
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a

No of a's is even and no of b's even-
ee



NFA Definition

NFA is defined as $M = (Q, \Sigma, \delta, q_0, F)$

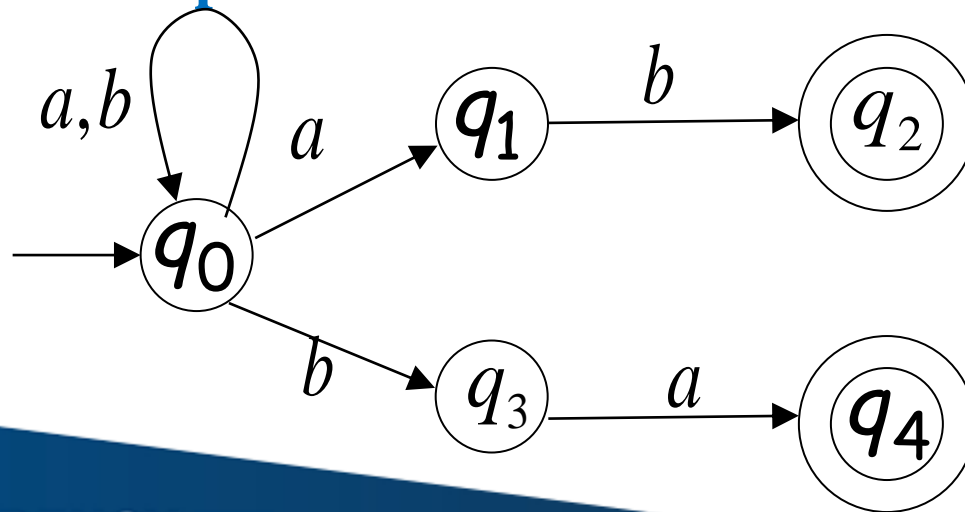
Where, Q = set of states

Σ = input alphabet

δ = transition function $\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$

$q_0 \rightarrow$ Start / Initial State

$F \rightarrow$ set of acceptance state



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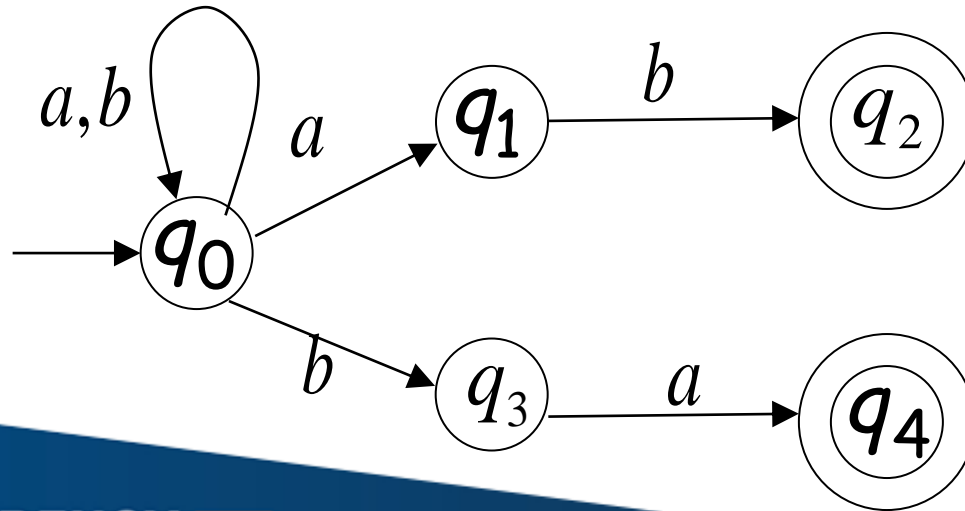


NFA Example

Design NFA that accepts language of strings ending with ab/ba over $\Sigma = \{a, b\}$

Solution

- Strings accepted= $\{ab, ba, abab, baba, bbab, aaaba, \dots\}$
- Strings rejected= $\{\epsilon, a, b, aa, bb, aaa, abb, aaabbb, \dots\}$
- Transition Diagram

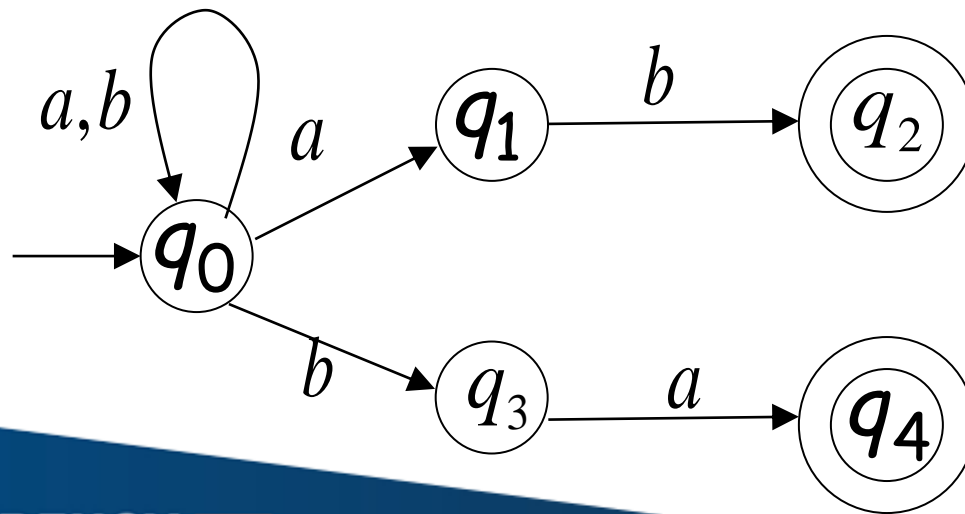


NFA to DFA Conversion Example

Convert NFA to DFA that accepts language of strings ending with ab/ba over $\Sigma = \{a, b\}$

Solution

- Draw NFA for given language
- Convert that NFA into DFA using subset construction method



NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider $\{q_0\}$ as Start State
- $\delta(q_0, a) = \{q_0, q_1\}$ newly formed state
- $\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(\{q_1, a\})$
 $= \{q_0, q_1\} \cup \{\emptyset\}$
 $= \{q_0, q_1\}$
- $\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(\{q_1, b\})$
 $= \{q_0, q_3\} \cup \{q_2\}$
 $= \{q_0, q_2, q_3\}$

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0, q1}	{q0, q3}
{q0, q1}	{q0, q1}	{q0, q2, q3}



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NFA to DFA Conversion Example

- Using Subset Construction Method
- Consider $\{q_0\}$ as Start State
- $\delta(q_0, a) = \{q_0, q_1\}$ newly formed state
- $\delta(\{q_0, q_1\}, a) = \delta(q_0, a) \cup \delta(\{q_1, a\})$

$$= \{q_0, q_1\} \cup \{\emptyset\}$$

$$= \{q_0, q_1\}$$

- $\delta(\{q_0, q_1\}, b) = \delta(q_0, b) \cup \delta(\{q_1, b\})$
- $= \{q_0, q_3\} \cup \{q_2\}$
- $= \{q_0, q_2, q_3\}$

- $\{q_0, q_2, q_3\}$ is newly formed state

- $\delta(\{q_0, q_2, q_3\}, a) = \delta(q_0, a) \cup \delta(\{q_2, a\}) \cup \delta(\{q_3, a\})$
- $= \{q_0, q_1\} \cup \{\emptyset\} \cup \{q_4\}$
- $= \{q_0, q_1, q_4\}$

- $\delta(\{q_0, q_2, q_3\}, b) = \delta(q_0, b) \cup \delta(\{q_2, b\}) \cup \delta(\{q_3, b\})$
- $= \{q_0, q_3\} \cup \{\emptyset\} \cup \{\emptyset\} = \{q_0, q_3\}$

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0, q1}	{q0, q3}
{q0, q1}	{q0, q1}	{q0, q2, q3}
{q0, q2, q3}	{q0, q1, q4}	{q0, q3}



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NFA to DFA Conversion Example

- $\{q_0 q_3\}$ is newly formed state
- $\delta(\{q_0 q_3\}, a) = \delta(q_0, a) \cup \delta(\{q_3, a\})$
 $= \{q_0, q_1\} \cup \{q_4\}$
 $= \{q_0 q_1 q_4\}$
- $\delta(\{q_0 q_3\}, b) = \delta(q_0, b) \cup \delta(\{q_3, b\})$
 $= \{q_0, q_3\} \cup \{\emptyset\}$
 $= \{q_0 q_3\}$

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3}	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}



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NFA to DFA Conversion Example

- $\{q_0 q_3\}$ is newly formed state
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 $= \{q_0, q_3\} \cup \{\emptyset\}$
 $= \{q_0 q_3\}$
- $\{q_0 q_1 q_4\}$ is newly formed state
- $\delta(\{q_0 q_1 q_4\}, a) = \delta(q_0, a) \cup \delta(\{q_1, a\}) \cup \delta(\{q_4, a\})$
 $= \{q_0, q_1\} \cup \{\emptyset\} \cup \{\emptyset\}$
 $= \{q_0 q_1\}$
- $\delta(\{q_0 q_1 q_4\}, b) = \delta(q_0, b) \cup \delta(\{q_1, b\}) \cup \delta(\{q_4, b\})$
 $= \{q_0, q_3\} \cup \{q_2\} \cup \{\emptyset\}$
 $= \{q_0 q_2 q_3\}$

NFA	a	b
q0	{q0, q1}	{q0, q3}
q1	-	{q2}
q2	-	-
q3	{q4}	-
q4	-	-

DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3}	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}
{q0 q1 q4}	{q0 q1}	{q0 q2 q3}



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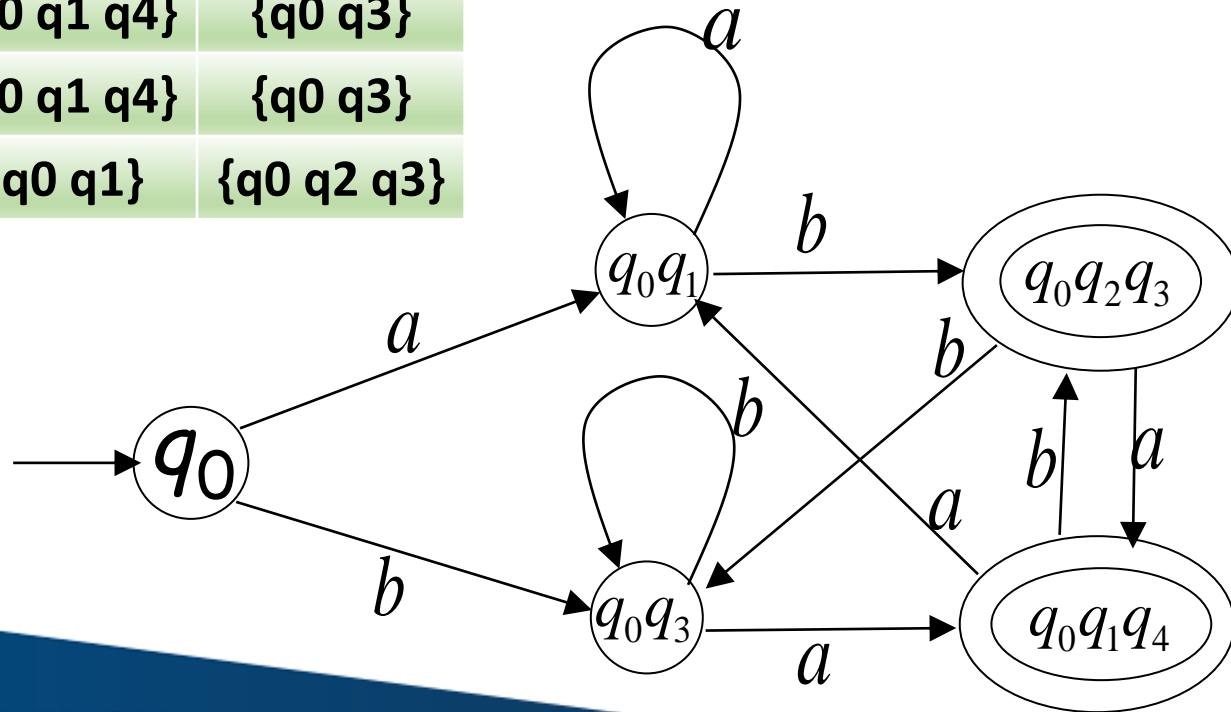
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NFA to DFA Conversion Example

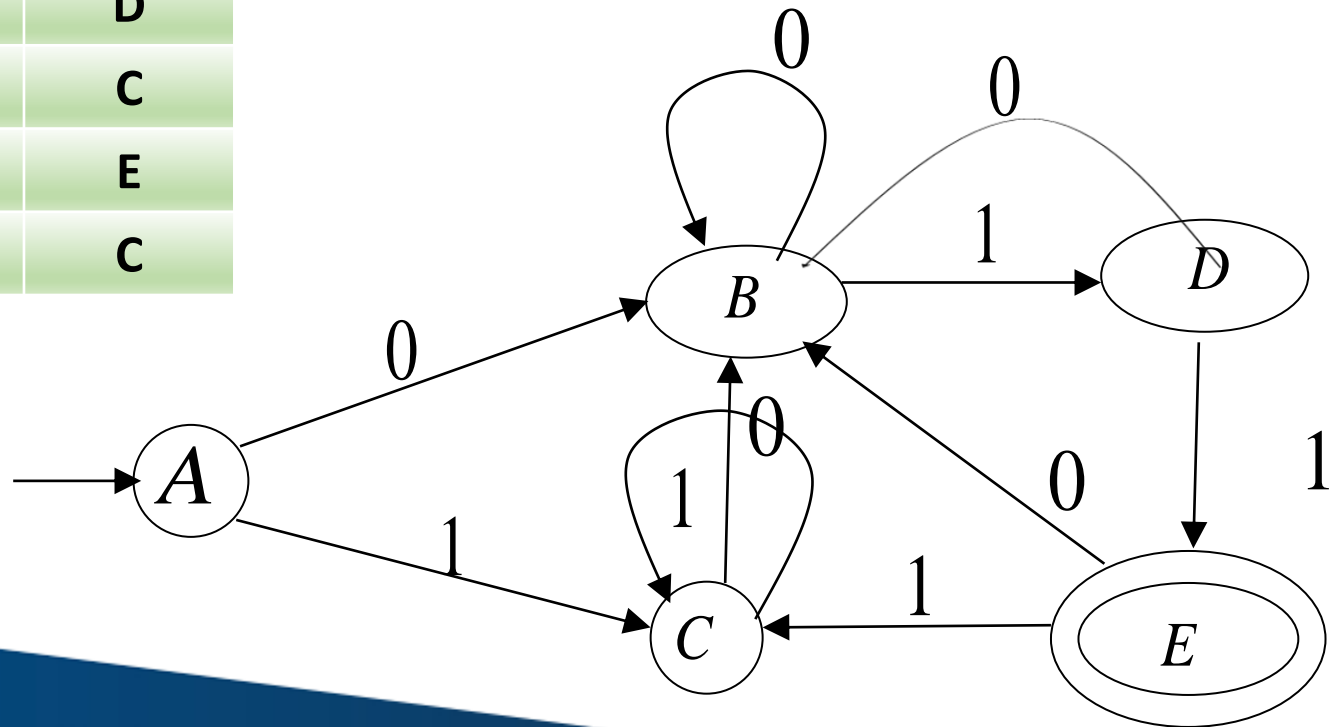
DFA	a	b
q0	{q0 q1}	{q0 q3}
{q0 q1}	{q0 q1}	{q0 q2 q3}
{q0 q2 q3}	{q0 q1 q4}	{q0 q3}
{q0 q3}	{q0 q1 q4}	{q0 q3}
{q0 q1 q4}	{q0 q1}	{q0 q2 q3}



Minimization of DFA Example

Minimize the following DFA using state equivalence method

DFA	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C



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Minimization of DFA Example

Minimize the following DFA

➤ Use State Equivalence Method

➤ Write 0' Equivalence as $\{A, B, C, D\} \{E\} \rightarrow 0$'s equivalence

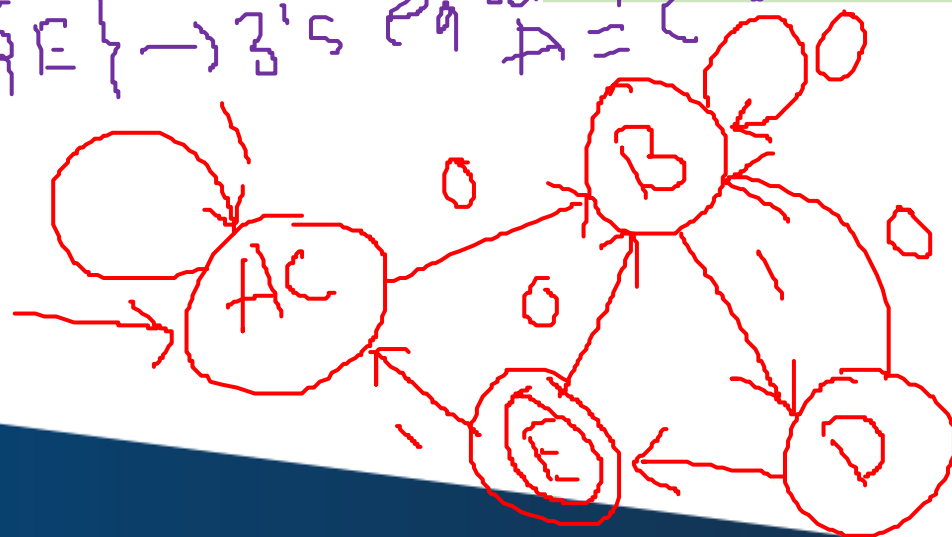
➤ Write 1' Equivalence $\{A, B, C, D\} \{E\} \rightarrow 1$'s equivalence

➤ Write 2' Equivalence $\{A, B, C\} \{D\} \{E\} \rightarrow 2$'s equivalence

➤ Write 3' Equivalence $\{A, C\} \{B\} \{D\} \{E\} \rightarrow 3$'s equivalence

➤ Write 4' Equivalence $\{A, C\} \{B\} \{D\} \{E\}$

DFA	0	1
A	B	C
B	B	D
C	B	C
D	B	E
E*	B	C



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Minimization of DFA Example

Minimize the following DFA

➤ Use State Equivalence Method

➤ Write 0'Equivalence as

{A, B, C, D} {E}

➤ Write 1'Equivalence

{A, B, C} {D} {E}

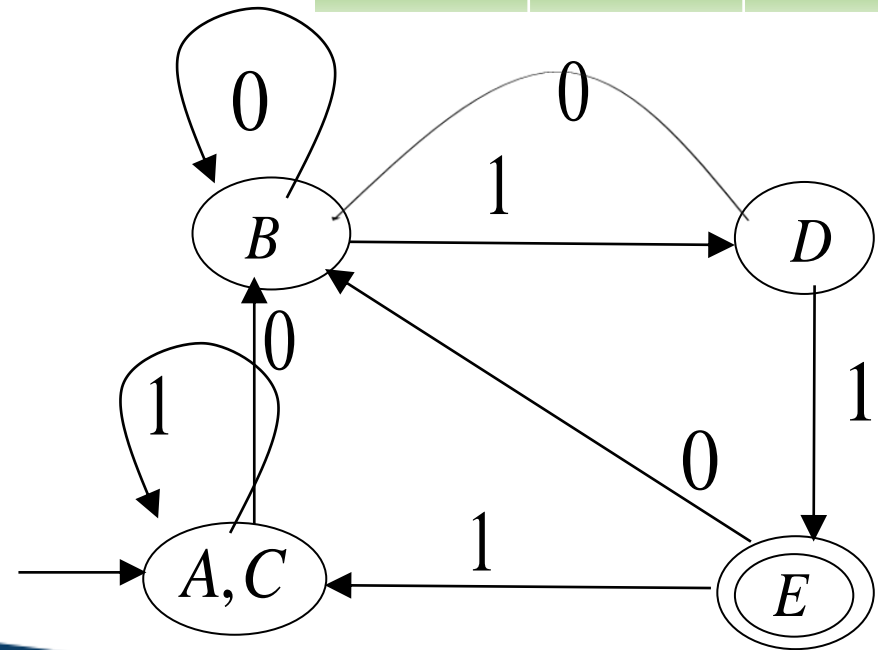
➤ Write 2'Equivalence

{A, C} {B} {D} {E}

➤ Write 3'Equivalence

{A, C} {B} {D} {E}

DFA	0	1
→ A, C	B	C
B	B	D
D	B	E
E*	B	C

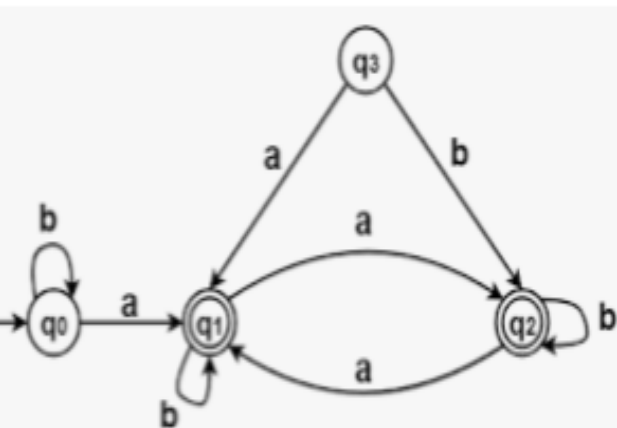


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$\{q_0, q_3\} \subseteq \{q_1, q_2\} \rightarrow 0's \text{ equivalent}$

$$\delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_0$$

$$\delta(q_3, a) = q_1 \quad \delta(q_3, b) = q_2$$

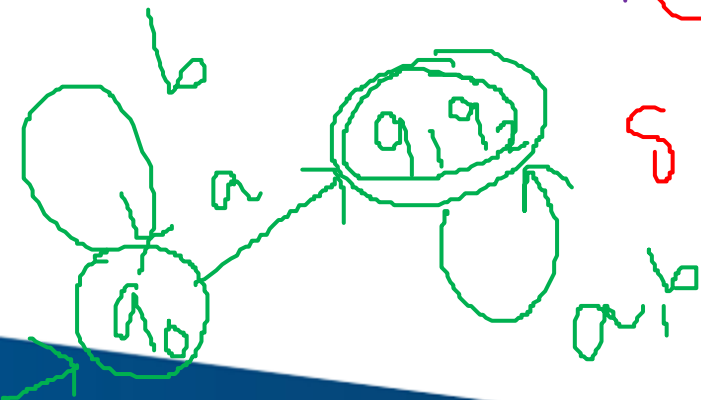
$\{q_0\} \subseteq \{q_3\} \subseteq \{q_1, q_2\} \rightarrow 1's \quad q_0 \neq q_3$

$\{q_0\} \subseteq \{q_3\} \subseteq \{q_1, q_2\} \rightarrow 2's$

$$\delta(q_1, a) = q_2 \quad \delta(q_1, b) = q_1$$

$$\delta(q_2, a) = q_1 \quad \delta(q_2, b) = q_2$$

$$q_1 = q_2$$



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ϵ -NFA (Finite Automaton with Epsilon Transitions)

- A transition with an empty string is called an ϵ -transition
- The formal definition of ϵ -NFA is represented through 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where,
- Q is a finite set of all states $(q_0, q_1, q_2, \dots, q_n)$ where n is finite number
- Σ is a finite set of symbols called the alphabet. i.e. $\{0, 1\}$,
- $\delta : Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$ is a **total function called as transition function**
- q_0 is the initial state from where any input is processed ($q_0 \in Q$).
- F is a set of final state/states where F will be subset (\subseteq) of Q .



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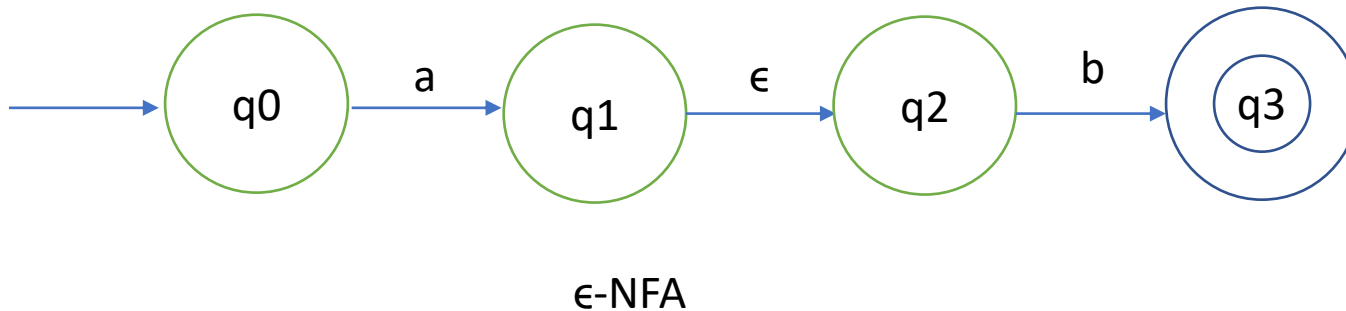
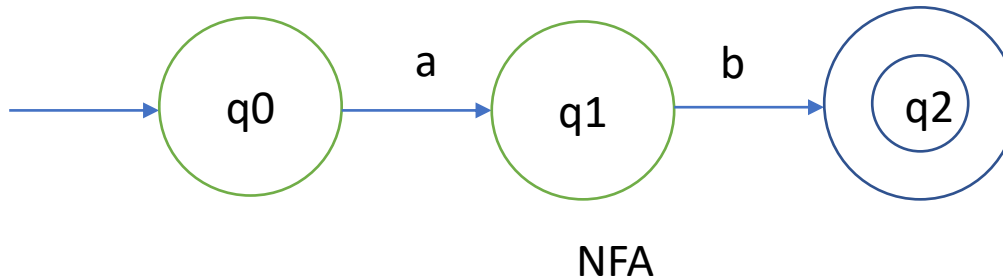
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Examples of Epsilon-NFA

- Draw a Epsilon NFA which accept the string “ab”.



Example 2

Draw an Epsilon Finite Automata which can accept the string “a or b or c”



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Conversion of ϵ -NFA to DFA

- The method for converting the NFA with ϵ to DFA is explained below –

Step 1 – Consider $M = \{Q, \Sigma, \delta, q_0, F\}$ is NFA with ϵ . We have to convert this NFA with ϵ to equivalent DFA denoted by

$$M_0 = (Q_0, \Sigma, \delta_0, q_0, F_0)$$

Then obtain,

$$\epsilon\text{-closure}(q_0) = \{p_1, p_2, p_3, \dots, p_n\}$$

then $[p_1, p_2, p_3, \dots, p_n]$ becomes a start state of DFA

now $[p_1, p_2, p_3, \dots, p_n] \in Q_0$

- **Step 2** – We will obtain δ transition on $[p_1, p_2, p_3, \dots, p_n]$ for each input.

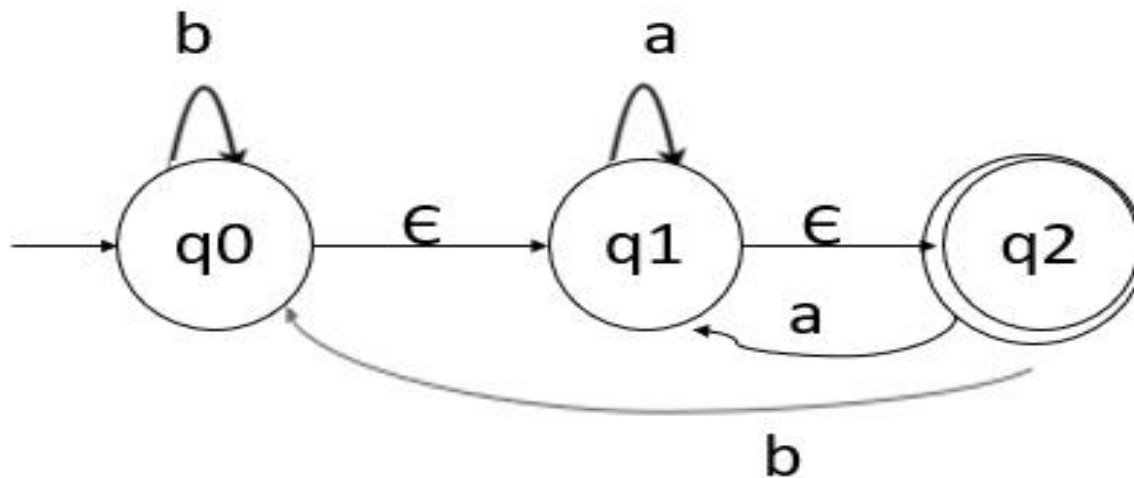
$$\delta([p_1, p_2, p_3, \dots, p_n], a) = \epsilon\text{-closure}(\delta(p_1, a) \cup \delta(p_2, a) \cup \dots \cup \delta(p_n, a)) = \bigcup_{i=1 \text{ to } n} \epsilon\text{-closure}(\delta(p_i, a))$$

Where a is input $\in \Sigma$

- Step 3 – The state obtained $[p_1, p_2, p_3, \dots, p_n] \in Q_0$.
- The states containing final state in p_i is a final state in DFA.

Example of Conversion

- Convert the following NFA with epsilon to equivalent DFA



- To convert this NFA with epsilon, we will first find the ϵ -closures, as given below –

ϵ -closure(q_0) = { q_0, q_1, q_2 }

ϵ -closure(q_1) = { q_1, q_2 }

ϵ -closure(q_2) = { q_2 }

- Let us start from **ϵ -closure of start state**, as mentioned below –
- When, ϵ -closure(q_0) = { q_0, q_1, q_2 }, we will call this state as A.
- Now, let us find transition on A with every input symbol, as shown below –



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- $\delta'(A, a) = \epsilon\text{-closure}(\delta(A, a))$
 $= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), a)$
 $= \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$
 $= \epsilon\text{-closure}(\Phi \cup q_1 \cup q_1)$
 $= \epsilon\text{-closure}(q_1)$
 $= \{q_1, q_2\}$ let us call it as state B



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- $\delta'(A, b) = \varepsilon\text{-closure}(\delta(A, b))$
 $= \varepsilon\text{-closure}(\delta(q_0, q_1, q_2), b)$
 $= \varepsilon\text{-closure}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b))$
 $= \varepsilon\text{-closure}(q_0 \cup \Phi \cup q_0)$
 $= \varepsilon\text{-closure}(q_0)$
 $= \{q_0, q_1, q_2\}$ its nothing but state A



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- $\delta'(B, a) = \varepsilon\text{-closure}(\delta(B, a))$
 $= \varepsilon\text{-closure}(\delta(q_1, q_2), a)$
 $= \varepsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a))$
 $= \varepsilon\text{-closure}(q_1 \cup q_2)$
 $= \varepsilon\text{-closure}(q_1)$
 $= \{q_1, q_2\}$ its nothing but state B



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- $\delta'(B, b) = \epsilon\text{-closure}(\delta(B, b))$
 $= \epsilon\text{-closure}(\delta(q_1, q_2), b)$
 $= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$
 $= \epsilon\text{-closure}(\Phi \cup q_0)$
 $= \epsilon\text{-closure}(q_0)$
 $= \{q_0, q_1, q_2\}$ its nothing but state A



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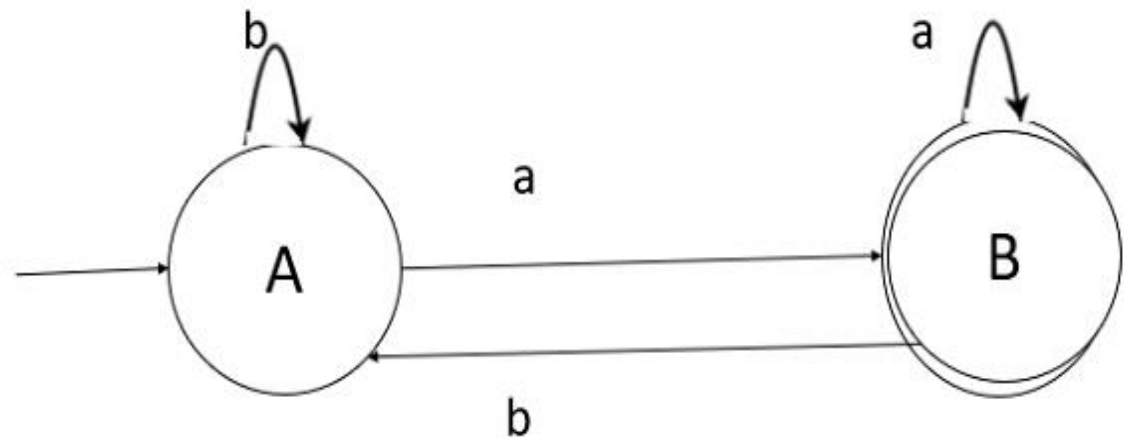
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- Hence, the transition table for the generated DFA is as follows –

States\inputs	a	b
A	B	A
B	B	A



$$F' = \{A, B\}$$



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- As, $A = \{q_0, q_1, q_2\}$ in which the final state q_2 lies. Hence, A is the final state.
- In $B = \{q_1, q_2\}$ the state q_2 lies. Hence, B is also the final state.



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END of MODULE 2



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Exercises



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