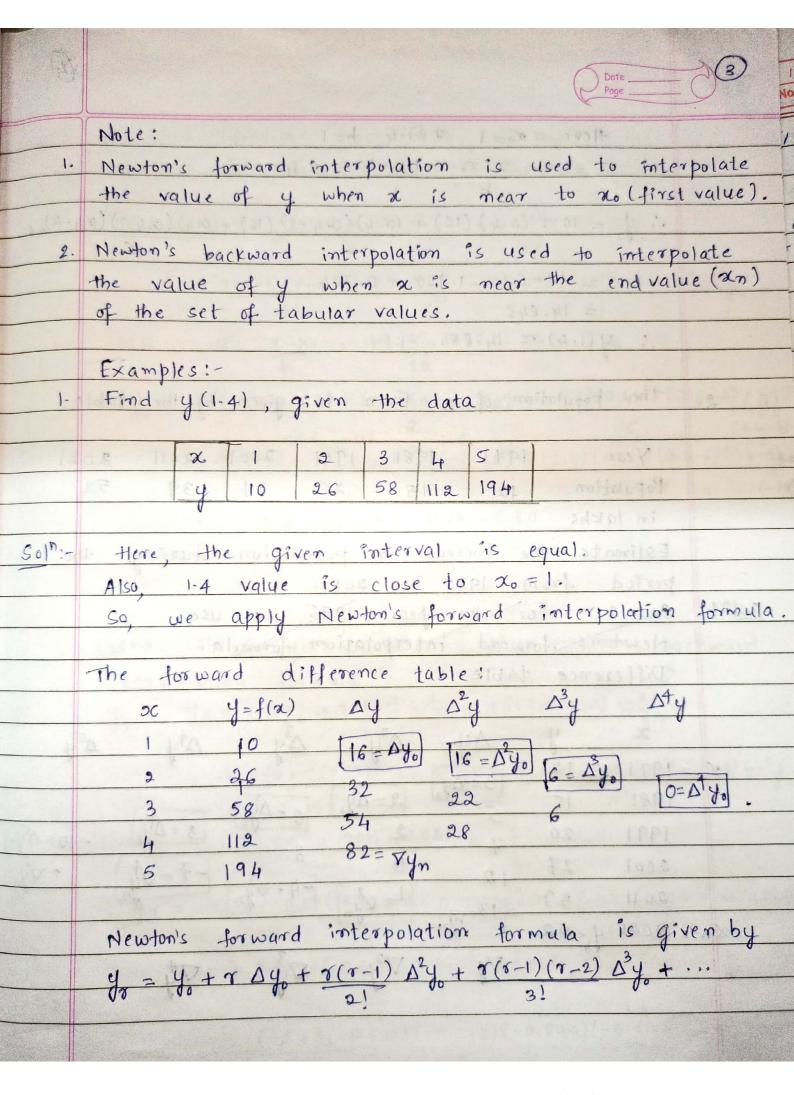
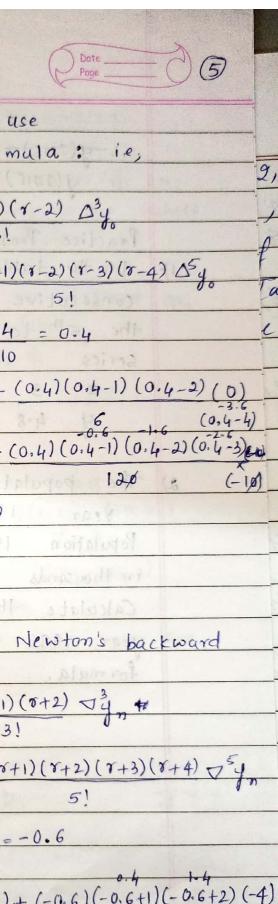
| | Module 2: Numerical Interpolation, Differentiation & |
|------|--|
| | Integration Date Page 0 |
| | Numerical Interpolation: |
| 1 | Interpolation with equal intervals (XOR) |
| | rewions forward interpolation formula |
| | (b) Newton's backward interpolation formula |
| | |
| | Interpolation: |
| | Suppose we have the values of y=f(a) for a |
| | set of values of x as follows: |
| | n: x. x. x2 · · · · · · · · · |
| | y: y. y. y2 yn. |
| | a) Newton's detwind intropolation formula: |
| 100 | Then, the process of finding the value of y |
| | Then, the process of finding the value of y Corresponding to the value of a between no fan |
| | is called interpolation. |
| | - I the test (-b) the test (-b) the test of the |
| | Also the process of computing the value of the function |
| | y outside the given range of x is called |
| | extrapolation. |
| | remail doite and aid the Miles the Action of |
| e.g: | χ : 20 25 30 35 40 45 $y = f(\alpha)$: 354 332 291 260 231 204 |
| | $q = f(\alpha) : 354 \qquad 352 \qquad 211 \qquad 260 \qquad 251 \qquad 209$ |
| | (1) Then of 22 - Interpolation. |
| 430 | (1) Finding y when x=23 -> Interpolation. (2) f(28) -> Interpolation. |
| | 1 |
| | (3) Finding y when n=18 -> Extrapolation: (4) \$(46) -> Extrapolation. |
| | 14) J(46) Literature Line grant in 18 18 18 18 18 18 18 18 18 18 18 18 18 |
| | How to create a difference table: |
| e. | The state of the s |
| | carried brown does not be the things of the contraction |
| | |

| | Différence table. | | | | | | |
|------|---|--|--|--|--|--|--|
| | $\Delta^2 \cup \Delta^3 \cup \Delta^4 $ | | | | | | |
| | x y Dy Dy | | | | | | |
| Xo | $= 0 y_0 = 1 2 = \Delta y_0 G = \Delta^2 y_0 C \Delta^3 y$ | | | | | | |
| | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | | | | | |
| | 2 11 20 10 | | | | | | |
| | 3 | | | | | | |
| | 4 69 62 30-721 | | | | | | |
| | 92 = 7/1 | | | | | | |
| Nn | = 6 Ja = 12 - A The Ty | | | | | | |
| | 7 you 7 you | | | | | | |
| | Newton's forward interpolation formula: | | | | | | |
| a) | - W - X - X - X - X - X - X - X - X - X | | | | | | |
| | | | | | | | |
| | to by said of the sollow soll of projected the | | | | | | |
| Ball | $y_r = y_0 + r \Delta y_0 + \frac{r(r-1)}{2!} \Delta^2 y_0 + \frac{r(r-1)(r-2)}{2!} \Delta^3 y_0 + \cdots,$ | | | | | | |
| 2000 | 21 31 31 | | | | | | |
| | where r is any real no., r= x-xo, who | | | | | | |
| | h | | | | | | |
| | no -> first value of x of h is the steplength. | | | | | | |
| | also, Dy, D2y, D3y, are forward differences. | | | | | | |
| | the section of the feet was the section of the sect | | | | | | |
| b) | Newton's backward interpolation formula: | | | | | | |
| | The value of y=f(x) at x=2n+rh is given | | | | | | |
| | .03176fry 631mile 40 11 (32) 9 11 (2) 31 11 | | | | | | |
| | y= yn+ 7 Vyn+ x(x+1) Vyn+ x(x+1)(x+2) Vyn+ | | | | | | |
| | 21 31 | | | | | | |
| | where 'r' is any real number, $r = x - x + x + x + x + x + x + x + x + x +$ | | | | | | |
| | an - last value of | | | | | | |
| | an \Rightarrow last value of $x \neq h \Rightarrow$ step length also, ∇y , $\nabla^2 y$, $\nabla^3 y$ are back | | | | | | |
| | also, ∇y , $\nabla^2 y$, $\nabla^3 y$, are backward differences. | | | | | | |

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Case 1: for the year 1975, we use Newton's forward interpolation formula: ie, y= y+ x Dy+ x(x-1) Δ2y+ x(x-1)(x-2) Δ3y. $+ r(r-1)(r-2)(r-3) \Delta^{4}y + r(r-1)(r-2)(r-3)(r-4) \Delta^{5}y$ 4! Here, $r = x - x_0 = 1975 - 1971 = 4 = 0.4$ $\frac{1}{3} = 12 + (0.4)(3) + (0.4)(0.4-1)(2) + (0.4)(0.4-1)(0.4-2)(0)$ + (0.4) (0.4-1) (0.4-2) (0.4-3) (8) + (0.4) (0.4-1) (0.4-2) (0.4-3) (0.4-3) (0.4-1) (0.4-2) (0.4-3)The soft and any 248 and a land the for 120 = 12 + 1.2 + 0.24 + 0 - 0.12 - 0.30 = 13.2 - 0.66 ,: y (1975) = 12.54 Casez: For the year 2015, we use Newton's backward interpolation formula, y= yn+ r ryn+ r (r+1) ryn+ r (r+1) (r+2) ryn+ + x (x+1)(x+2)(x+3) \tag{4} + x(x+1)(x+2)(x+3)(x+4) \sqrt{5} Here, $x = \frac{x - x_n}{h} = \frac{2015 - 2021}{10} = \frac{-6}{10} = -0.6$ $y_{1} = 52 + (-0.6)(13) + (-0.6)(-0.6+1)(1) + (-0.6)(-0.6+1)(-0.6+2)(-4)$ + (-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-7)+ (-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)(-10)126

| 0= | 52 - | 7.8 | Ŧ 0. | 12+ | 0.2 | 22+ | 0.24 | +0.23 |
|----|------|-----|------|-----|-----|-----|------|-------|
|----|------|-----|------|-----|-----|-----|------|-------|

y(2015) = 44.77

Practice Problems:

In the table given below, the values of y are consciutive terms of a series of which 14.5 9s the 5th term. Find the first of tenth term of the series

4 4.8 8.4 14.5 23.6 36.2 52.8 73.9

+ (+0(2)6,0641)(0.6+2)(-0.5)(+3)(-1)+

-)(4+3,0-)(8+3,0-)(5+3,0-)(113,0-)(3,0-)

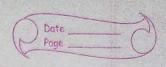
2) The population of a town is given by the table

Year 1951 1961 1971 1981 1991

Population 19.96 39.65 58.81 77.21 94.61

in thousands

Calculate the increase in population from the year 1995 to 1985 using appropriate interpolation formula.



Interpolation formulae for unequal intervals: Divided différences: Let f(x), f(x1) ... f(xn) be the values of an of x: xo, x1,..., xin at unequal intervals. The first order divided differences are defined as, $f(x_0 x_1) = f(x_1) - f(x_0), \quad f(x_1 x_2) = f(x_2) - f(x_1),$ $x_1 - x_0, \quad x_1 - x_1,$... $f(x_{n-1}, x_n) = f(x_n) - f(x_{n-1})$. 2n - 2(n-1 The second order divided differences are $f(x_0x_1x_2) = f(x_1x_2) - f(x_0x_1), f(x_1x_2x_3) = f(x_2x_3) - f(x_1x_2)$ $x_2 - x_0$ $x_3 - x_1$... $f(x_{n-2}, x_{n-1}, x_n) = f(x_{n-1}x_n) - f(x_{n-2}x_{n-1})$ 2(n-2(n-2 Similarly, the higher order divided differences are defined. Divided difference table:

| 1. | We have the following two methods: Newton's divided difference formula Lagrange's Interpolation formula. |
|----|--|
| 1. | Newton's divided différence formula: is given by |
| en | $y = f(x) = f(x_0) + (x_0) f(x_0x_1) + (x_0) (x_0) (x_0x_1) \cdot f(x_0x_1x_2)$ + + (x_0) (x_1) (x_x_0), f(x_0x_1) (x_n_1), f(x_0x_1 x_n) |
| 1- | Examples:- (1-mx)+- (mx)+- (mx)+ |
| | The state of the s |
| | FH = (exex) + = (exex) + (exex |
| | $\frac{dx_1 - dx_2 + \dots + dx_n}{dx_n - dx_n} = \frac{dx_n - dx_n}{dx_n - dx_n} = \frac{dx_n - dx_n}{dx_n} = \frac{dx_n - dx_n}{dx_n} = \frac{dx_n}{dx_n} = \frac{dx_n}$ |
| | Classical abe bigber coder divided |
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