Runge-Kutta 4th Order Methods :-

→ It provides the approximate value of y for a given point x.

→ It is used for finding the increment K of y corresponding to an increment h of x from the initial value problem

$$\frac{dy}{dx} = f(x,y); \; y(x_0) = y_0$$

formula given by:
$$K_1 = hf(x_0, y_0)$$
$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$
$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$
$$K_4 = h(f(x_0 + h, y_0 + K_3))$$

Required approximate value. $y_1 = y_0 + K$.

is given by.

$$\boxed{K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4).}$$

(3) Using Runge-Kutta Method of order 4, find at $x = 0.2$, given that $10\frac{dy}{dx} = x^2+y^2$, with $y(0) = 1$ and $h = 0.1$.

Soln →
$$10\frac{dy}{dx} = x^2+y^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{10} = f(x,y).$$

$x_0 = 0$, $y_0 = 1$, $h = 0.1$.
$x_1 = 0.1$, $x_2 = 0.2$

By fourth order Runge Kutta Method,

$$K_1 = hf(x_0, y_0) = 0.1 f(0,1) = (0.1)(0.1)$$
$$= 0.01.$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$
$$= (0.1) f(0.05, 1.005)$$
$$= (0.1)(0.1013) = 0.0101.$$

$$K_3 = hf\left(x_0 + h/2, y_0 + \frac{K_2}{2}\right)$$
$$= (0.1) f(0.05, 1.0051)$$
$$= (0.1)(0.1013)$$
$$= 0.0101.$$

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$K_4 = h(f(x_0+h, y_0+K_3))$
$= (0.1) f(0.1, 1.0101)$
$= (0.1)(0.103)$
$= 0.0103.$

$y_1 = y_0 + K.$

where $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$
$= \frac{1}{6}\left(0.01 + 2(0.0101) + 2(0.0101) + 0.0103\right)$
$= 0.0101.$

and Hence $y_1 = y_0 + K$
$= 1 + 0.0101$
$y_1 = 1.0101$ at $x_1 = 0.1$

Again taking (x_1, y_1) in place of (x_0, y_0) and repeat the process.

$K_1 = hf(x_1, y_1) = (0.1) f(0.1, 1.0101)$
$= (0.1)(0.103)$
$= 0.0103$

$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$
$= (0.1) f(0.15, 1.0153)$
$= (0.1)(0.1053) = 0.0105$

$K_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$
$= (0.1) f(0.15, 1.0154)$
$= (0.1)(0.1054) = 0.0105$

$K_4 = hf(x_1+h,\ y_1+K_3)$
$= (0.1) f(0.2,\ 1.0207)$
$= (0.1)(0.1082)$
$= 0.0108$

Hence $y_2 = y_1 + K$

where $K = \frac{1}{6}\{K_1 + 2K_2 + 2K_3 + K_4\}$
$= \frac{1}{6}\{0.0103 + 2(0.0105) + 2(0.0105) + 0.0108\}$
$= \frac{1}{6}(0.0631) = 0.01051$

and Hence $y_2 = y_1 + K$

or $y_2 = 1.0207$ at $x_2 = 0.2$.

(4) Using Runge-Kutta Method, of order 4, find $y(0.2)$ given that

$$\frac{dy}{dx} = \frac{y-x}{y+x} \quad \text{with } y(0)=1, \text{ and}$$

$h = 0.1$.

Soln $\rightarrow$ Given that $\frac{dy}{dx} = \frac{y-x}{y+x} = f(x,y)$

$x_0 = 0,\ y_0 = 1,\ h = 0.1$
$x_1 = 0.1$
$x_2 = 0.2$.

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fourth order Runge kutta Method;

$K_1 = hf(x_0, y_0) = (0.1) f(0,1) = (0.1)(1) = 0.1$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$

$= (0.1) f(0.05, 1.05)$

$= (0.1)(0.9091) = 0.0909$

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$

$= (0.1) f(0.05, 1.0455)$

$= (0.1)(0.9087)$

$= 0.0909$

$K_4 = h f(x_0 + h, y_0 + K_3)$

$= (0.1) f(0.1, 1.0909)$

$= (0.1)(0.8321) = 0.0832$

$y_1 = y_0 + K$

where $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= \frac{1}{6}(0.1 + 2(0.0909) + 2(0.0909) + 0.0832)$

$= 0.09113$

$y_1 = y_0 + K = 1.0911$ at $x_2 = 0.2$

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Again taking $(x_1 y_1)$ in place of (x_0, y_0) and repeat the process.

$K_1 = h(f(x_1, y_1) = (0.1) f(0.1, 1.0911)$
$= (0.1)(0.8321)$
$= 0.0832$

$K_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2})$

$= (0.1) f(0.15, 1.1327)$
$= (0.1)(0.7661)$
$= 0.0766.$

$K_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2})$

$= (0.1) f(0.15, 1.1294)$
$= (0.1)(0.7655)$
$= 0.0766.$

$K_4 = hf(x_1 + h, y_1 + K_3)$
$= (0.1) f(0.2, 1.1677)$
$= (0.1)(0.7075)$
$= 0.0708$

$y_2 = y_1 + K$

where $K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.0832 + 2(0.0766) + 2(0.0766) + 0.0708]$

$= 0.07673.$

Hence $y_2 = y_1 + K = 1.0911 + 0.07673$

$y_2 = 1.1678$ at $x_2 = 0.2$