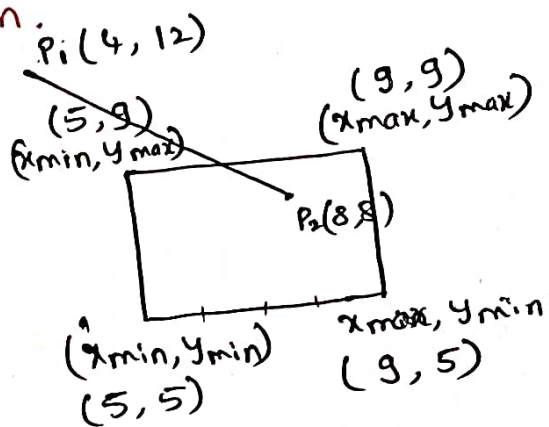


Liang-Barzky Numerical Problems ::

①. Clip a line $P_1(4, 12)$ and $P_2(8, 8)$ with clipping window $(x_{\min}, y_{\min}) = (5, 5)$ and $(x_{\max}, y_{\max}) = (9, 9)$ using Liang-Barzky line clipping algorithm.



Soln:- Given $P_1(4, 12)$ and $P_2(8, 8)$

$$(x_{\min}, y_{\min}) = (5, 5)$$

$$(x_{\max}, y_{\max}) = (9, 9)$$

$$\Delta x = x_2 - x_1 = 8 - 4 = 4$$

$$\Delta y = y_2 - y_1 = 8 - 12 = -4$$

using Liang-Barzky line clipping algorithm

$$P_1 = -\Delta x = -4$$

$$P_2 = \Delta x = 4$$

$$P_3 = -\Delta y = -(-4) = 4$$

$$P_4 = \Delta y = -4$$

$$a_1 = x_1 - x_{\min} = 4 - 5 = -1$$

$$a_2 = x_{\max} - x_1 = 9 - 4 = 5$$

$$a_3 = y_1 - y_{\min} = 12 - 5 = 7$$

$$a_4 = y_{\max} - y_1 = 9 - 12 = -3$$

Case 1: $P_k = 0$ (Not Required $\because P_k > 0$ & $P_k < 0$)

Case 2: $P_k < 0$

$$P_1 = -4$$

$$P_4 = -4$$

$$a_1 = -1$$

$$a_4 = -3$$

$$t = a_1 / P_1 = -1 / -4 = 1/4$$

$$t = a_4 / P_4 = -3 / -4 = 3/4$$

$$t_1 = \max(0, a/P) \Rightarrow \max(0, 1/4, 3/4)$$

$$t_1 \Rightarrow 3/4$$

Case 3: $P_k > 0$

$$P_2 = 4$$

$$P_3 = 4$$

$$a_2 = 5$$

$$a_3 = 7$$

$$t = a_2 / P_2 = 5/4$$

$$t = a_3 / P_3 = 7/4$$

$$t_2 = \min(1, a/P) \Rightarrow \min(1, 5/4, 7/4)$$

$$t_2 \Rightarrow 1$$

Finally,

(i)

$$t_1 \leq t_2$$

$$\frac{3}{4} \leq t_2$$

$$\frac{3}{4} \leq 1 \quad \checkmark$$

(ii) $t_1 \geq 0$

$$\frac{3}{4} \geq 0 \quad \checkmark$$

$$\text{So } x'_1 = x_1 + t_1 \Delta x \\ = 4 + \frac{3}{4}(4)$$

$$\boxed{x'_1 = 7}$$

$$y'_1 = y_1 + t_1 \Delta y \\ = 12 + \frac{3}{4}(-4)$$

$$= 12 - 3 \\ \boxed{y'_1 = 9}$$

$$\therefore \boxed{P'_1(x, y) \Rightarrow P'_1(7, 9)}$$

(iii) $t_2 \leq 1$

$$1 \leq 1 \quad \checkmark$$

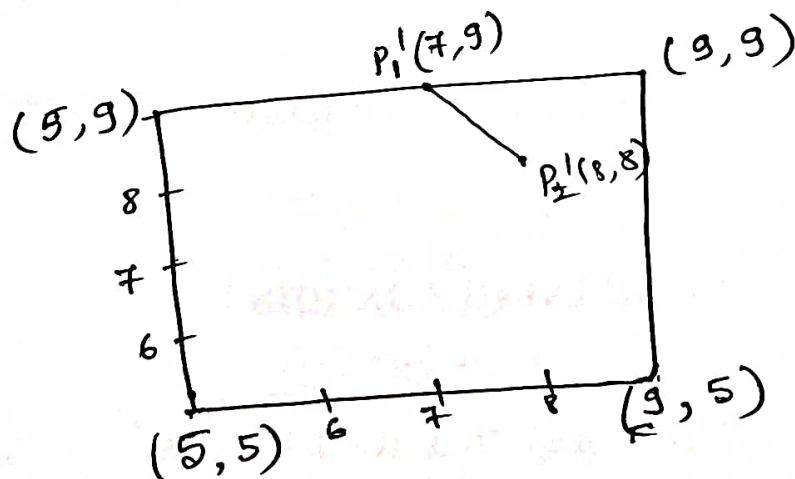
$$\text{So } x'_2 = x_1 + t_2 \Delta x \\ = 4 + 1(4)$$

$$\boxed{x'_2 = 8}$$

$$y'_2 = y_1 + t_2 \Delta y \\ = 12 + 1(-4) \\ = 12 - 4$$

$$\boxed{y'_2 = 8}$$

$$\therefore \boxed{P'_2(x, y) = P'_2(8, 8)}$$



②. clip a line $P_1(10, 30)$ and $P_2(80, 90)$ with respective clipping Window $(x_{min}, y_{min}) = (20, 20)$ and $(x_{max}, y_{max}) = (90, 70)$ using Liang - Barsky line clipping algorithm.

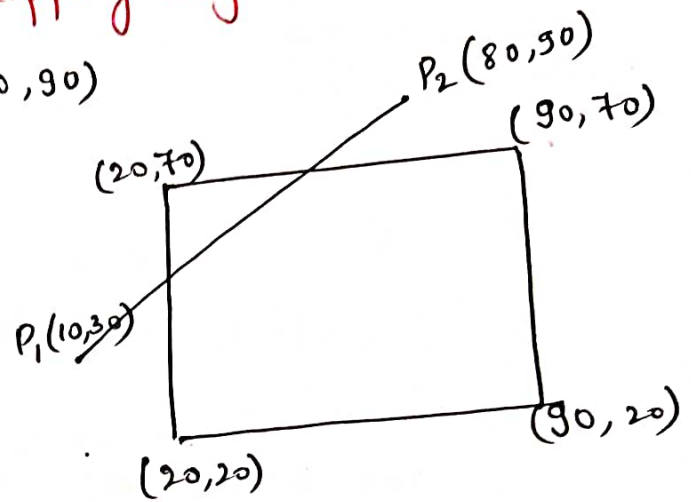
Soln:- Given $P_1(10, 30)$ and $P_2(80, 90)$

$$(x_{min}, y_{min}) = (20, 20)$$

$$(x_{max}, y_{max}) = (90, 70)$$

$$\Delta x = x_2 - x_1 = 80 - 10 = 70$$

$$\Delta y = y_2 - y_1 = 90 - 30 = 60$$



Using Liang - Barsky line clipping algorithm

$$P_1 = -\Delta x = -70 \quad \alpha_1 = x_1 - x_{min} = 10 - 20 = -10$$

$$P_2 = \Delta x = 70 \quad \alpha_2 = x_{max} - x_1 = 90 - 10 = 80$$

$$P_3 = -\Delta y = -60 \quad \alpha_3 = y_1 - y_{min} = 30 - 20 = 10$$

$$P_4 = \Delta y = 60 \quad \alpha_4 = y_{max} - y_1 = 70 - 30 = 40$$

Case 1: $P_k \neq 0$ (Not Required)

Case 2: $P_k < 0$

$$P_1 = -70 \quad \alpha_1 = -10$$

$$P_3 = -60 \quad \alpha_3 = 10$$

$$t_1 = \max(0, \alpha/P) \Rightarrow \max(0, 10/-70, -10/-60)$$

$$t_1 \Rightarrow 10/70$$

Case 3: $P_k > 0$

$$P_2 = 70 \quad \alpha_2 = 80 \quad t = 80/70$$

$$P_4 = 60 \quad \alpha_4 = 40 \quad t = 40/60$$

$$t_2 = \min(1, \alpha/P) \Rightarrow \min(1, 80/70, 40/60)$$

$$t_2 \Rightarrow (40/60)$$

Finally,

(i) $t_1 \leq t_2$

$$\frac{10}{70} \leq \frac{40}{60} \checkmark$$

(ii) $t_1 \geq 0$

$$\frac{10}{70} \geq 0 \checkmark$$

$$\begin{aligned} \text{So } x'_1 &= x_1 + t_1 \Delta x \\ &= 10 + \left(\frac{10}{70}\right)(70) \\ &= 10 + 10 \end{aligned}$$

$$\boxed{x'_1 = 20}$$

$$\begin{aligned} y'_1 &= y_1 + t_1 \Delta y \\ &= 30 + \left(\frac{10}{70}\right)(60) \end{aligned}$$

$$= 38.57$$

$$\boxed{y'_1 \approx 39}$$

$$\therefore \boxed{P'_1(x, y) = (20, 39)}$$

(iii) $t_2 \leq 1$

$$\frac{40}{60} \leq 1 \checkmark$$

$$\begin{aligned} \text{So } x'_2 &= x_1 + t_2 \Delta x \\ &= 10 + \left(\frac{40}{60}\right)(70) \\ &= 56.66 \end{aligned}$$

$$\boxed{x'_2 \approx 57}$$

$$\begin{aligned} y'_2 &= y_1 + t_2 \Delta y \\ &= 30 + \left(\frac{40}{60}\right)(60) \end{aligned}$$

$$= 30 + 40$$

$$\boxed{y'_2 = 70}$$

$$\therefore \boxed{P'_2(x, y) = (57, 70)}$$

