Fuzzy Propositions

OBJECTIVES

1. Introduce fuzzy propositions.
2. Learn to evaluate truth value of a fuzzy proposition.
3. Understand fuzzy quantifiers
4. Understand linguistic hedges

- The truth value of a fuzzy proposition is a matter of degree expressed by a fraction in the unit interval [0, 1]. For example,

  "Tina is young"

  is a fuzzy proposition. Depending upon the age of Tina the truth value of the proposition assume any value, say 0.63.

- Simple fuzzy propositions may be classified into four types:

  1. unconditional and unqualified proposition
  2. unconditional and qualified proposition
  3. conditional and unqualified proposition
  4. conditional and qualified proposition

1. Unconditional and unqualified proposition: The canonical form of this type of proposition, *p* is shown below:

p: $\mathcal{V}$ is F.

where, $\mathcal{V}$ is a variable on which fuzzy set F is defined.

Example of such a proposition is:

p: *temperature* is *high*.

Here, *temperature* is the variable $\mathcal{V}$ and *high* is the fuzzy set *F*.

Truth value, $T(p)$ of such a proposition is given by membership grade of the particular value, *v* of temparature in the fuzzy set *F*. That is,

$T(p) = F(v)$.

This is illustrated by the example in Fig. 1.

Fuzzy Propositions

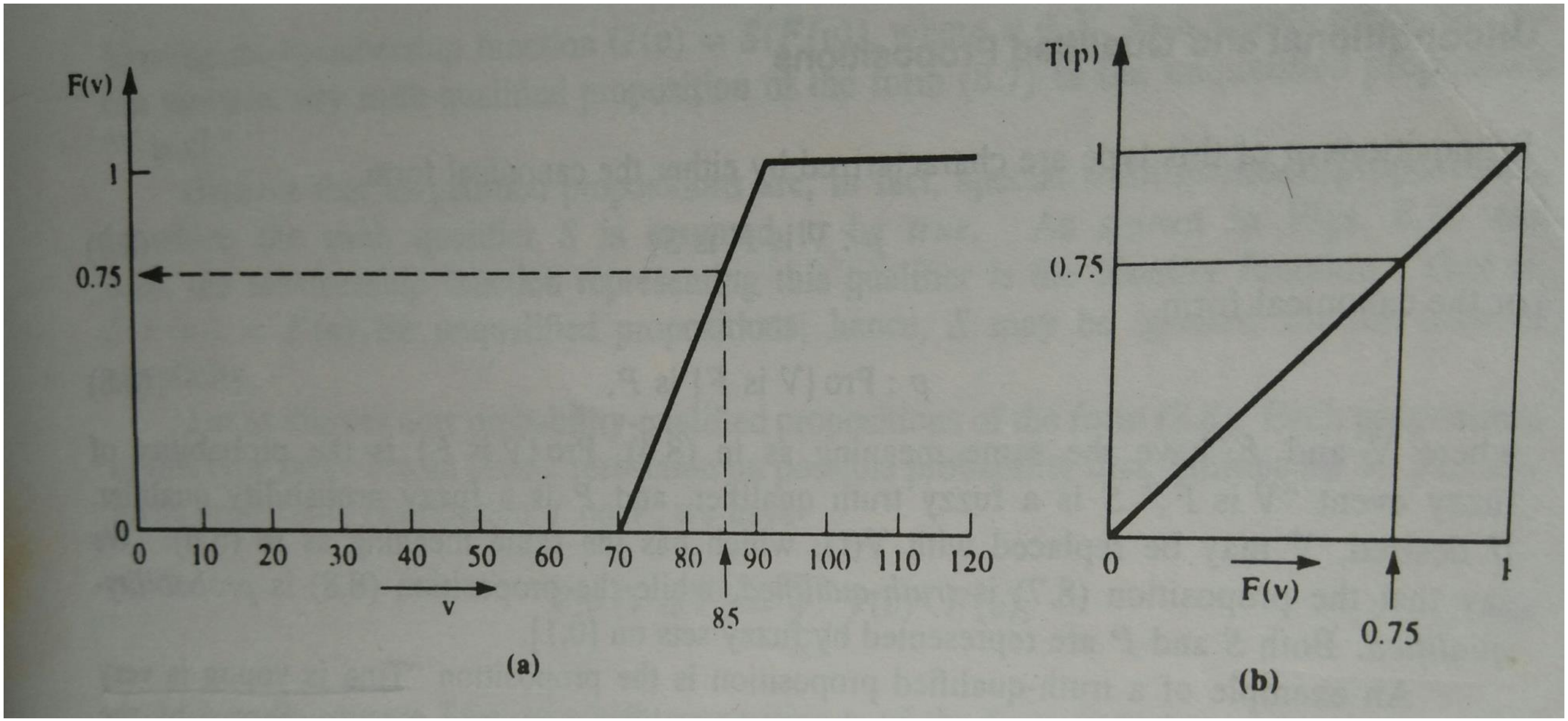

Fig. 1. Components of fuzzy proposition: *p*: Temperature(*v*) is high (*F*).

2. Unconditional and qualified proposition: Canonical form of this type of proposition is,

p: $\mathcal{V}$ is F is S.

Where S is a fuzzy truth qualifier.

An example of such a truth-qualified proposition is:

p: *Tina* is *young* is *very true*.

Here, *very* is the truth qualifier represented by S.

The degree of truth, $T(p)$, of any truth-qualified proposition p is given for each value v of the undelying variable by the equation

$$T(p) = S(F(v))$$

The process can be understood with Fig. 2.

Fuzzy Propositions

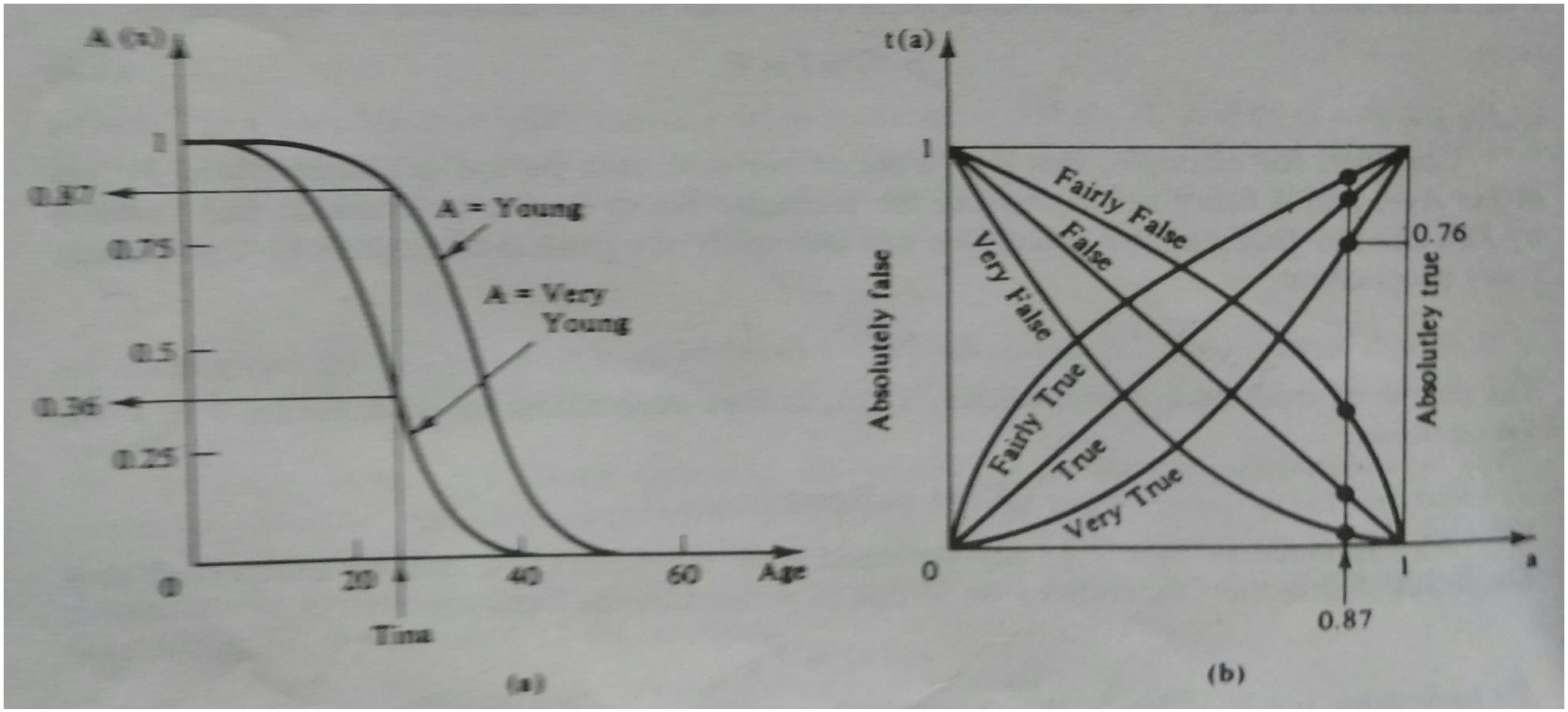

Fig. 2 Truth values of a fuzzy proposition

3. Conditional and unqualified proposition: Proposition *p* of this type are expressed by the canonical form

p: If $\mathscr{X}$ is *A*, then $\mathscr{Y}$ is *B*,

whrer $\mathscr{X}$ and $\mathscr{Y}$ are variables whose values are in sets *X* and *Y*, respectively, and *A and B* are fuzzy sets on *X and Y. These propositions may also be viewed as propositions of the form*

$(\mathscr{X}, \mathscr{Y})$ is *R*,

where *R* is a fuzzy relation that is determined for each $x \in \mathscr{X}$ and $y \in \mathscr{Y}$ by the formula

$$R(x, y) = \partial(A(x), B(y))$$

here, ∂ denotes a suitable fuzzy implication like Lukasiewicz implication

$$\partial(a, b) = \min(1, 1 - a + b)$$

4. Conditional and qualified proposition: One canonical form of this type of proposition is

p: If $\mathcal{X}$ is A, then $\mathcal{Y}$ is B is S

Truth value of such a proposition can be evaluated by combining the methods used for the previous types of propositions.

Fuzzy quantifiers: Fuzzy quantifiers are fuzzy numbers that take part in fuzzy propositions. There are two kinds.

1. **First kind:** Fuzzy quantifiers of this kind are defined on the set of real numbers and characterize linguistic terms such as *about 10*, *much more than 100*, *at least about 5*, and so on.
2. **Second kind:** Fuzzy quantifiers of this kind are defined on [0, 1] and characterize linguistic terms such as *almost all*, *about half*, *most* etc.

Examples of fuzzy quantifiers of the second kind are shown in Fig. 3.

Fuzzy Quantifiers

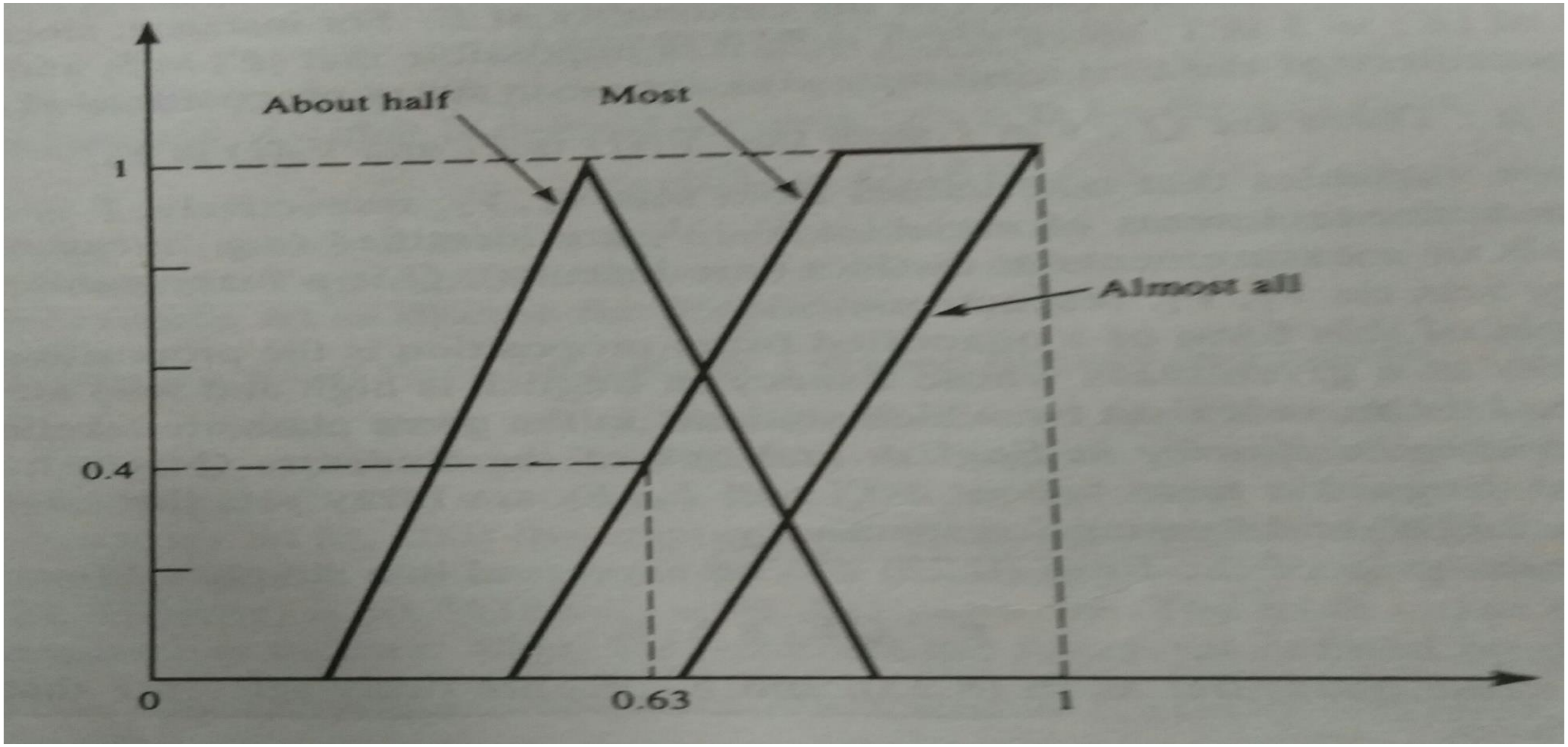

Fig. 3 Fuzzy quantifiers of the second kind.

- **Linguistic Hedges**: Linguistic hedges (or simply hedges) are special linguistic terms by which other linguistic terms are modified.
- Example: Linguistic terms such as *very*, *more or less*, *fairly*, *extremely* etc.
- Hedges can be used to modify fuzzy predicates, fuzzy truth values, and fuzzy probabilities. For example,
- "x is very young is true"
  "x is young is very true"
  "x is very young is very true"

- Any linguistic hedge, H, may be interpreted as a unary operation, h, called a modifier, on the interval [0, 1].
- For example, the hedge *very* is often interpreted as $h(a)=a^2$, the hedge *fairly* is interpreted as $h(a)=\sqrt{a}$ ($a\in[0, 1]$).
- The modifier h is called strong if $h(a)<a$ for all $a\in[0, 1]$, the modifier called weak if $h(a)>a$ for all $a\in[0, 1]$. The special modifier $h(a)=a$ is called an identity modifier.
- A strong modifier strengthens a fuzzy predicate to which it is applied, and consequently it reduces the truth value of the associated proposition. Opposite is the case with weak modifiers.

For example:

p_1: John is young,

p_2: John is very young,

p_3: John is fairly young.

Let, the linguistic hedges *very* and *fairly* be represented by strong modifier a^2 and weak modifier $\sqrt{a}$. Assume that John is 26, and Y represents the fuzzy set *young*, $Y(26)=0.8$. Then,

very young(26) $=0.8^2=0.64$,

fairly young(26)$=\sqrt{0.8}=0.89$.

A convenient class of function that can be used as modifier is $h_\alpha(a) = a^\alpha$, where α is a parameter whose value is a +ve real number. When $\alpha<1$, h_α is a weak modifier, when $\alpha >1$, h_α is a strong modifier.

Another *example:*

Suppose we define fuzzy sets *Small* and *Large* on Y={1,2,3,4,5} as:

$$Small = \frac{1}{1} + \frac{0.8}{2} + \frac{0.6}{3} + \frac{0.4}{4} + \frac{0.2}{5} \quad Large = \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}$$

Obtain the fuzzy sets representing:

i) *Very small*

ii) *Very very large*

Solution:

$$very\ small = \frac{1^2}{1} + \frac{0.8^2}{2} + \frac{0.6^2}{3} + \frac{0.4^2}{4} + \frac{0.2^2}{5}$$
$$= \frac{1}{1} + \frac{0.64}{2} + \frac{0.36}{3} + \frac{0.16}{4} + \frac{0.04}{5}$$

$$very\ very\ large = \frac{(.2^2)^2}{1} + \frac{(.4^2)^2}{2} + \frac{(.6^2)^2}{3} + \frac{(.8^2)^2}{4} + \frac{(1^2)^2}{5}$$
$$= \frac{.0016}{1} + \frac{.0256}{2} + \frac{.1296}{3} + \frac{.4096}{4} + \frac{1}{5}$$

Fuzzy Inference

Introduction

- Fuzzy inference is the process of obtaining new knowledge through existing knowledge.
- Knowledge is most commonly represented in the form of rules or proposition for example "if x is A then y is B" (Where A and B are linguistic values defined by fuzzy sets on universes of discourse X and Y). A rule is also called a fuzzy implication.
- "x is A" is called the antecedent or premise and "y is B" is called the consequence or conclusion.

Important Inferring Processes

- Generalized modus Ponens (GMP) - Latin for "The way that affirms by affirming"
- Generalized modus Tollens (GMT) - Latin for "The way that denies by denying"
- Hypothetical Syllogism

$$\begin{array}{c} p \to q \\ \underline{p \qquad} \\ q \end{array} \qquad \begin{array}{c} p \to q \\ \underline{\neg q \qquad} \\ \neg p \end{array} \qquad \begin{array}{c} p \to q \\ \underline{q \to r} \\ p \to r \end{array}$$

GMP

p: If X is A than Y is B (Analytically known)

q: If X is A' (Analytically known)

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than Y is B' (Analytically unknown)

Where, A, B, A', B' are fuzzy terms.
A' and B' are some predicate with different linguistic hedges. To compute the membership function of B' the min – max composition of fuzzy set A' with R(x,y) which is known as implication rule is used.

$$B' = A' o R(x,y)$$

In terms of membership function

$$\mu_{B'}(y) = \max(\min(\mu_{A'}(x), \mu_R(x,y)))$$

where $\mu_{B'}(y)$, $\mu_{A'}(A')$, $\mu_R(x,y)$ are membership function of B', A' and implication relation respectively.

GMT

p: If X is A than Y is B (Analytically known)

q: If Y is B'   (Analytically known)

than X is A'   (Analytically unknown)

Where, A, B, A', B' are fuzzy terms.

$$A' = B' o\, R(x, y)$$

In terms of membership function

$$\mu_{A'}(y) = \max(\min(\mu_{B'}(x), \mu_R(x, y)))$$

where $\mu_{B'}(y)$, $\mu_{A'}(A')$, $\mu_R(x,y)$ are membership function of B', A' and implication relation respectively.

Thank You!