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Part -1

PART – I (setting up)

① Discretize the systems dynamics :

Given :

$\dot{x} = v_x$, $m\dot{v}_x = -(u_1+u_2)\sin\theta$, $\dot{\theta} = \omega$

$\dot{y} = v_y$, $m\dot{v}_y = (u_1+u_2)\cos\theta$, $I\dot{\omega} = r(u_1-u_2)$

$z = [x, v_x, y, v_y, \theta, \omega]^T$

$x_{n+1} = x_n + \Delta t\, v_{x_n}$

$y_{n+1} = y_n + \Delta t\, v_{y_n}$

$v_{x_{n+1}} = v_{x_n} + \Delta t\left(-\frac{(u_1+u_2)\sin\theta}{m}\right)$

$v_{y_{n+1}} = v_{y_n} + \Delta t\left(\frac{(u_1+u_2)\cos\theta - mg}{m}\right)$

$\theta_{n+1} = \theta_n + \Delta t\,\omega_n$

$\omega_{n+1} = \omega_n + \Delta t\left(\frac{r(u_1+u_2)}{I}\right)$

$\therefore z_{n+1} = z_n + \Delta t\,\frac{dy}{dt}$

② Given : $v_x = 0$, $v_y = 0$, $\omega = 0$

substituting the values we get :-

$0 = -(u_1+u_2)\sin\theta$

$\boxed{u_1 = -u_2}$

& $\frac{mg}{\cos\theta} = u_1+u_2$

$\theta = 0 \therefore \cos\theta = 1$

$\boxed{u_1+u_2 = mg}$

③ If $\theta = 0$:

$m\dot{v}_x = -(u_1+u_2)\sin\theta$

$F_x = 0$

$\dot{\theta} = 0$

$m\dot{v}_x = (u_1+u_2) - mg$

$f_y = u_1+u_2 - mg$

$\therefore$ we see that F_x is 0 meaning when $\theta = 0$ there is no force in the x direction and hence it is not possible for it to move.

④ If $\theta = \pi/2$

$m\dot{v}_x = -(u_1+u_2)\sin\pi/2$

$m\dot{v}_x = -(u_1+u_2)$

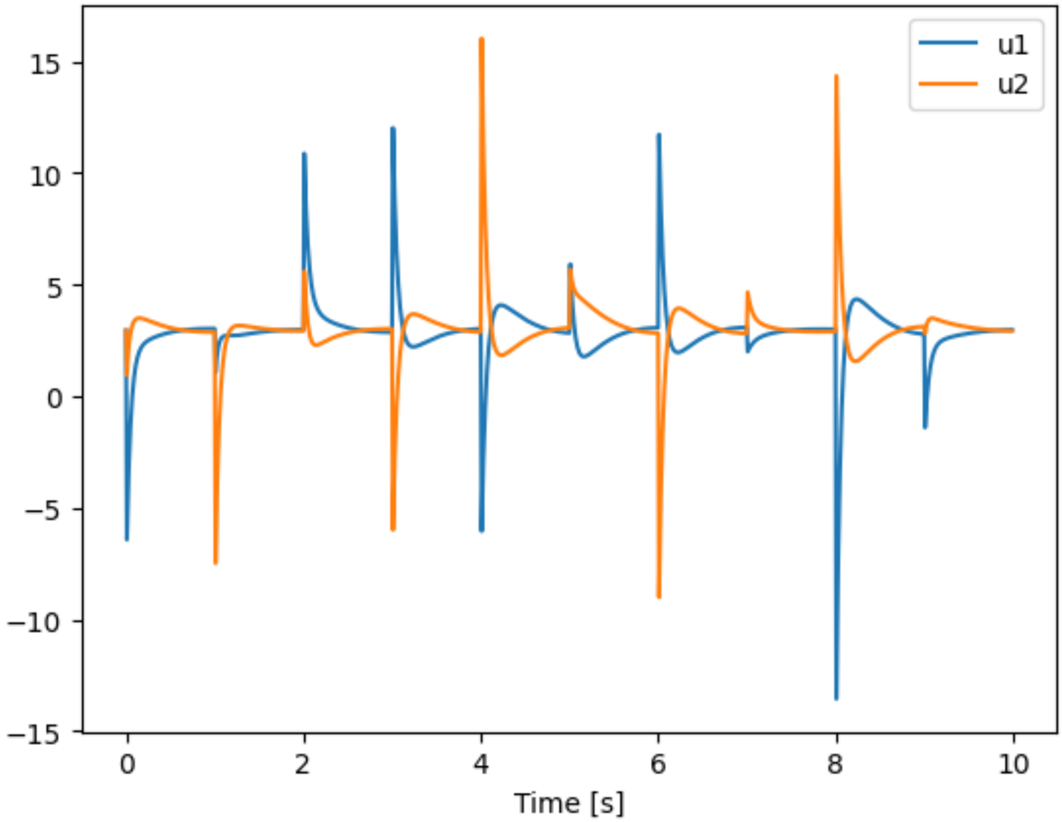
$m\dot{v}_y = (u_1+u_2)\cos\pi/2 - mg$

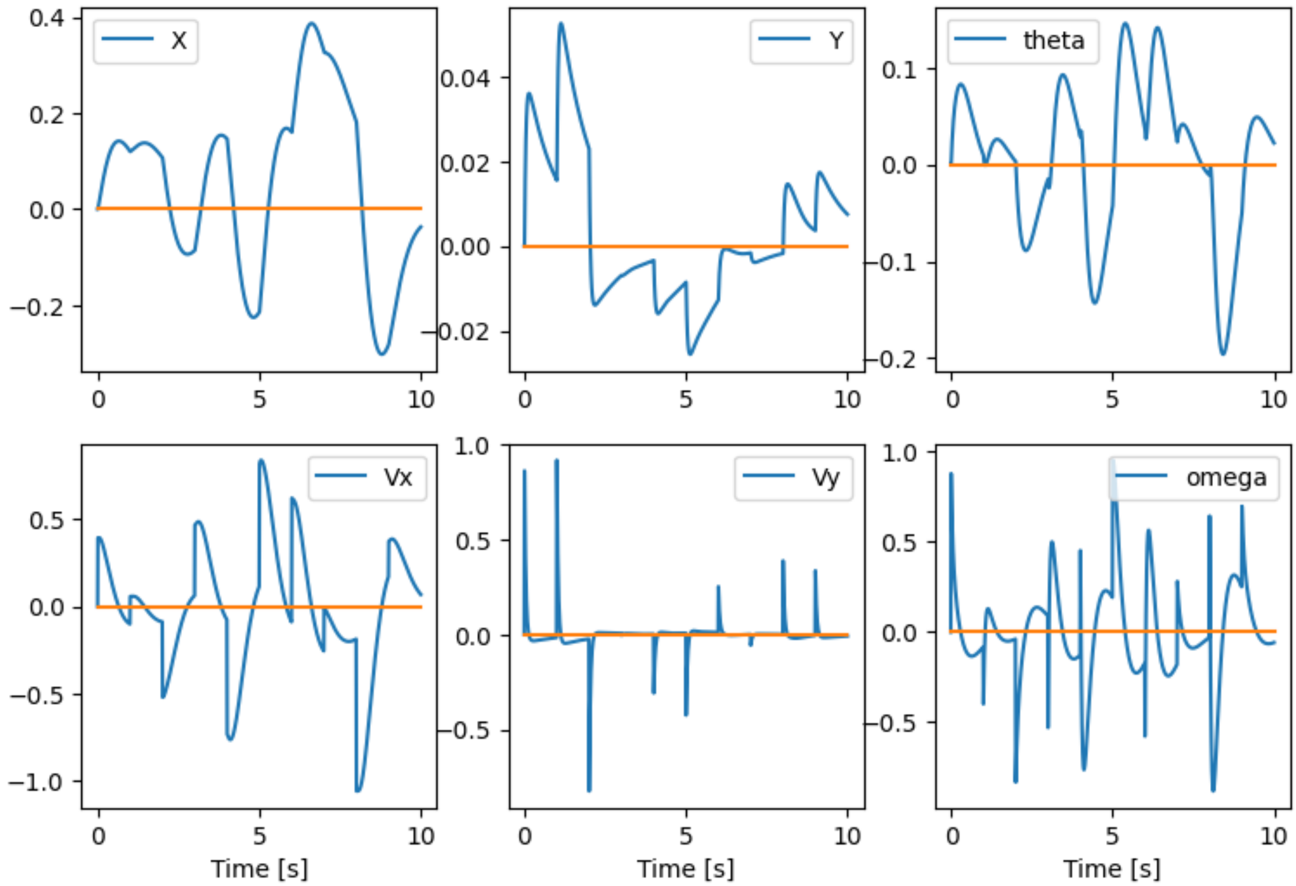
$m\dot{v}_y = -mg$

$\therefore$ We clearly see that the robot cannot be at rest when $\theta = \pi/2$.

Part -2

Infinite-horizon LQR





Part -3

4. One of the main benefit of this design is that it tracks the position of the robot.

An issue with this design is that it is not suitable for much more complex tasks for example like in a real robot.

5. When theta = pi/4 we see that the y position is tracked whereas the x position isn't tracked.

Part -4

7. Benefit of this approach is that it allows us to calculate the position of the robot at every state.

An issue is that it can not be used for very complex tasks

9. Application of this controller can be run on a real controller in the real world. However we will have to keep in mind the computation cost associated with this naive approach and therefore there are more efficient approaches with ILQR.