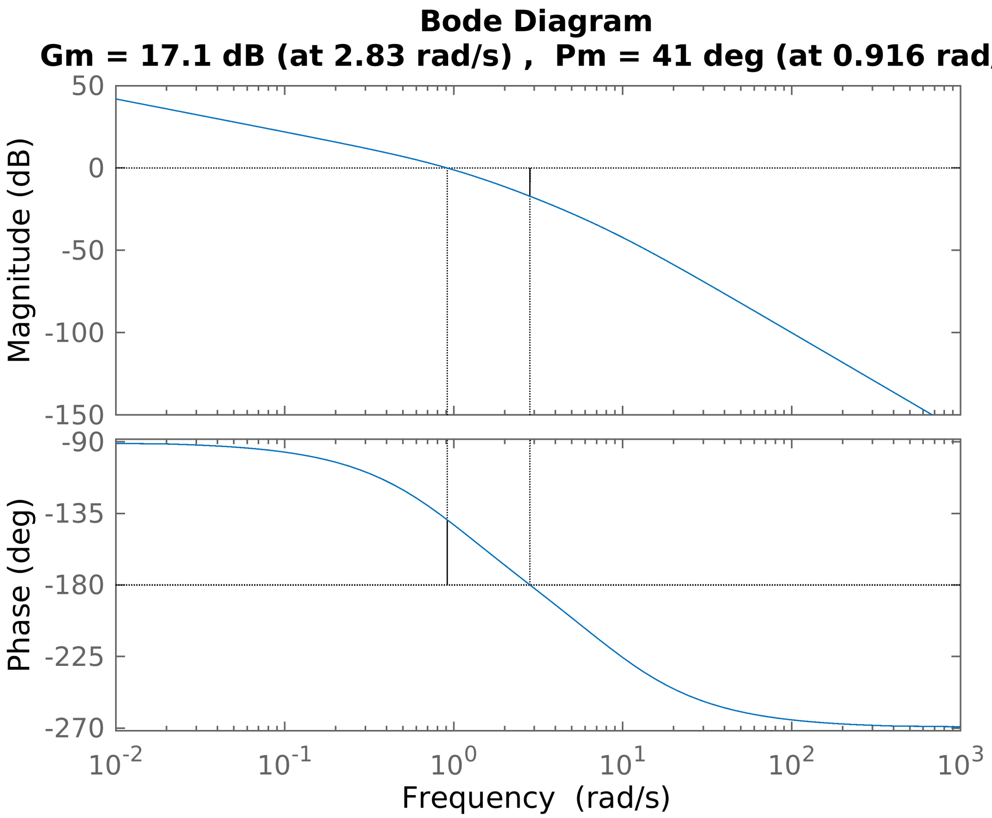
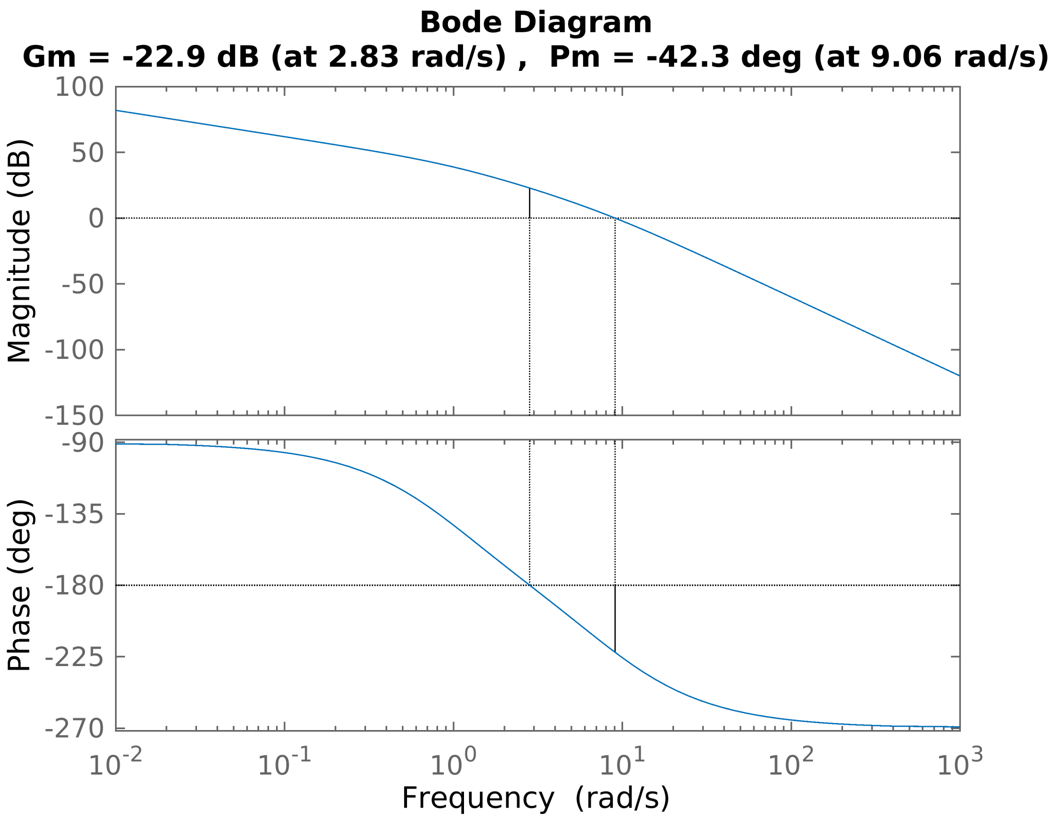
Question 1

```
clear;
close all;
s = tf('s');
gain_list = [10 1000];
G = 1/(s*(s+1)*(s+8));
hold off;
for k = gain_list
    margin(k*G)
    [~,~,phase_crossover,gain_crossover] = margin(k*G)
end
```




```
phase_crossover = 2.8284
gain_crossover = 0.9158
```




```
Warning: The closed-loop system is unstable.
phase_crossover = 2.8284
gain_crossover = 9.0643
```

Question 2

```
clear;
sympref('FloatingPointOutput',true);
syms K s w real;
G(s) = K/(s*(s^2+s+4))
```

G(s) =

$$\frac{K}{s\ (s^2+s+4)}$$

```
H = 1
```

H = 1

```
GH=G(w*1i)*H
```

GH =

$$-\frac{1\,K\,\mathrm{i}}{w\ (-w^2+1\,w\,\mathrm{i}+4)}$$

```
[num,den]=numden(GH);

den = expand(den);
a = real(num)
```

a = $-K$

```
b = imag(num)
```

b = 0

```
c = real(den)
```

c = w^2

```
d = imag(den)
```

d = $w^3 - 4\,w$

```
amp = simplify(sqrt(a^2+b^2)/sqrt(c^2+d^2))
```

amp =

$$\frac{|K|}{|w|\ \left(w^4 - 7\,w^2 + 16\right)^{0.5000}}$$

```
phase = simplify(atan(b/a)-atan(d/c))
```

phase =

$$-\mathrm{atan}\left(\frac{w^2-4}{w}\right)$$

```
phase_eq = phase == deg2rad(-180 + 50) + pi % the + pi here is so the eq solves between
```

phase_eq =

$$-\mathrm{atan}\left(\frac{w^2-4}{w}\right) = 0.8727$$

```
S = eval(solve(phase_eq));

wn = S(2)
```

wn = 1.4910

```
gain_eq = subs(amp,w,wn) == 1
```

gain_eq = $0.2891\,|K| = 1$

```
S = eval(solve(gain_eq));
Kn = S(2)
```

Kn = 3.4585

```
phase_crossover_eq = phase == deg2rad(-180) + pi
```

phase_crossover_eq =

$$-\operatorname{atan}\left(\frac{w^2-4}{w}\right) = 0$$

```
S = eval(solve(phase_crossover_eq));
phase_crossover = S(2)
```

phase_crossover = 2

```
amp
```

amp =

$$\frac{|K|}{|w|\ (w^4-7\,w^2+16)^{0.5000}}$$

```
gain_margin = eval(subs(amp,[K, w], [Kn, phase_crossover]))
```

gain_margin = 0.8646

```
gain_margin_decibels = 20*log(gain_margin)
```

gain_margin_decibels = -2.9091

Question 3

```
clear;
sympref('FloatingPointOutput',true);
syms K s w real;
G1(s)= K*(s+.1)/(s+.5)
```

G1(s) =

$$\frac{K\ (s+0.1000)}{s+0.5000}$$

```
G2(s)=10/(s*(s+1))
```

G2(s) =

$$\frac{10}{s\ (s+1)}$$

```
H = 1
```

H = 1

```
GH = G1(w*1i)* G2(w*1i)*H
```

GH =

$$-\frac{10\,K\,(0.1000+1\,w\,\mathrm{i})\,\mathrm{i}}{w\,(1+1\,w\,\mathrm{i})\,(0.5000+1\,w\,\mathrm{i})}$$

```
[num,den]=numden(GH);

den = expand(den);
a = real(num)
```

a = $20\,K\,w$

```
b = imag(num)
```

b = $-2\,K$

```
c = real(den)
```

c = $w-2\,w^3$

```
d = imag(den)
```

d = $3\,w^2$

```
amp = simplify(sqrt(a^2+b^2)/sqrt(c^2+d^2))
```

amp =

$$\frac{2\,|K|\,\sqrt{100\,w^2+1}}{|w|\,(4\,w^4+5\,w^2+1)^{0.5000}}$$

```
phase = simplify(atan(b/a)-atan(d/c))
```

phase =

$$\mathrm{atan}\left(\frac{3\,w}{2\,w^2-1}\right)-\mathrm{atan}\left(\frac{0.1000}{w}\right)$$

```
phase_eq = phase == deg2rad(-180 + 50) + pi % the + pi here is so the eq solves between
```

phase_eq =

$$\mathrm{atan}\left(\frac{3\,w}{2\,w^2-1}\right)-\mathrm{atan}\left(\frac{0.1000}{w}\right)=0.8727$$

```
S = eval(solve(phase_eq));

wn = S
```

wn = 1.4384

```
gain_eq = subs(amp,w,wn) == 1
```

gain_eq = $3.7577\,|K| = 1$

```
S = eval(solve(gain_eq));
Kn = S(2)
```

```
Kn = 0.2661
```

Phase margin lies above phase = -180 so gain margin is infinity