Exam 4:

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Question 1

a)

```
clear;
syms s w t A real;
G(s) = (100*s+100)/(s*(s^2+16*s+100))
```

G(s) =

$$\frac{100\,s + 100}{s\,\left(s^2 + 16\,s + 100\right)}$$

```
U = .5 * sin(20*t)
```

U = $0.5000 \sin(20\,t)$

```
A_input = .5;
w_input = 20;

G_jw=subs(G(s),s,1i*w)
```

G_jw =

$$-\frac{1\,(100 + 100\,w\,\mathrm{i})\,\mathrm{i}}{w\,\left(-w^2 + 16\,w\,\mathrm{i} + 100\right)}$$

```
[num,den]=numden(G_jw);
num = num/1i; % bad matlab dont move i
den = den*1i; % bad matlab dont move i
num = expand(num)
```

num = $-100 - 100\,w\,\mathrm{i}$

```
den = expand(den)
```

den = $-1\,w^3\,\mathrm{i} - 16\,w^2 + 100\,w\,\mathrm{i}$

```
a = real(num)
```

a = -100

```
b = imag(num)
```

b = $-100\,w$

```
c = real(den)
```

c = $-16\,w^2$

```
d = imag(den)
```

d = $100\,w - w^3$

```
A_G_jw = sqrt(a^2+b^2)/sqrt(c^2+d^2)
```

A_G_jw =

$$\frac{100\,\sqrt{w^2+1}}{\left((100\,w-w^3)^2+256\,w^4\right)^{0.5000}}$$

```
Phase_G_jw = atan2(b,a)-atan2(d,c)
```

Phase_G_jw = $-\mathrm{atan2}(6.2500\,w - 0.0625\,w^3, -w^2) + \mathrm{atan2}(-100\,w, -100)$

```
y_ss = A*A_G_jw*sin(w*t + Phase_G_jw)
```

y_ss =

$$\frac{100\,A\,\sin(t\,w + \mathrm{atan2}(-100\,w,-100) - \mathrm{atan2}(6.2500\,w - 0.0625\,w^3, -w^2))\,\sqrt{w^2+1}}{\left((100\,w-w^3)^2+256\,w^4\right)^{0.5000}}$$

```
%subsituting in input values
y_ss = subs(y_ss,[A, w], [A_input,w_input])
```

y_ss = $0.1141\,\sin(20\,t + 0.7677)$

b)

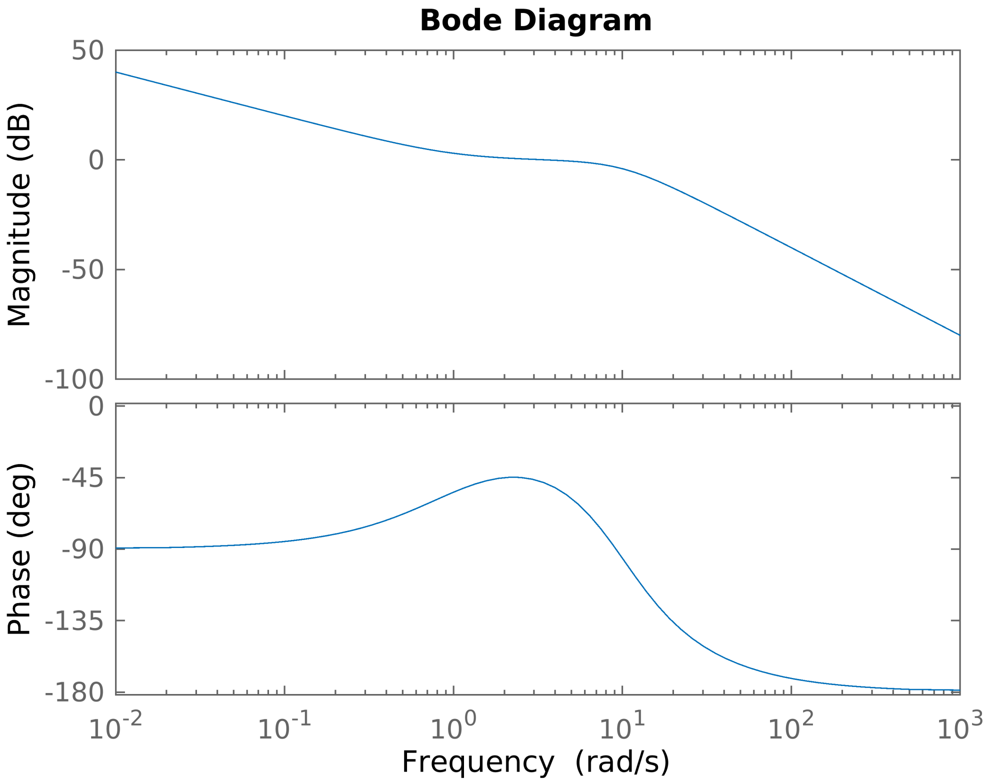
```
clear;
close all;
s= tf('s');
G = (100*s+100)/(s*(s^2+16*s+100))
```

```
G =

      100 s + 100
  --------------------
  s^3 + 16 s^2 + 100 s

Continuous-time transfer function.
```

```
bode(G)
```




```
[mag,phase,wout] = bode(G,20);
mag =  mag * .5
```

```
mag = 0.1141
```

```
phase = deg2rad(phase) + pi %converting from cos to sin
```

```
phase = 0.7677
```

Question 2

$$G(s) = \frac{200s + 10}{s(s+1)(s^2 + 16s + 100)}$$

$$G(s) = \frac{10\left(\frac{s}{.05} + 1\right)}{s(s+1)(s^2 + 16s + 100)}$$

$$G(s) = \frac{1}{10}\frac{\left(\frac{s}{.5} + 1\right)}{s(s+1)\left(\frac{s^2}{10^2} + \frac{1.6}{10}s + 1\right)}$$

Poles:

Constant Gain: $K = \frac{1}{10}$ BLUE

20*log10(K) = -20 db

Pole at Origin: $\frac{1}{(s)}$ PURPLE

First Order Pole: $\frac{1}{(s+1)}$ YELLOW

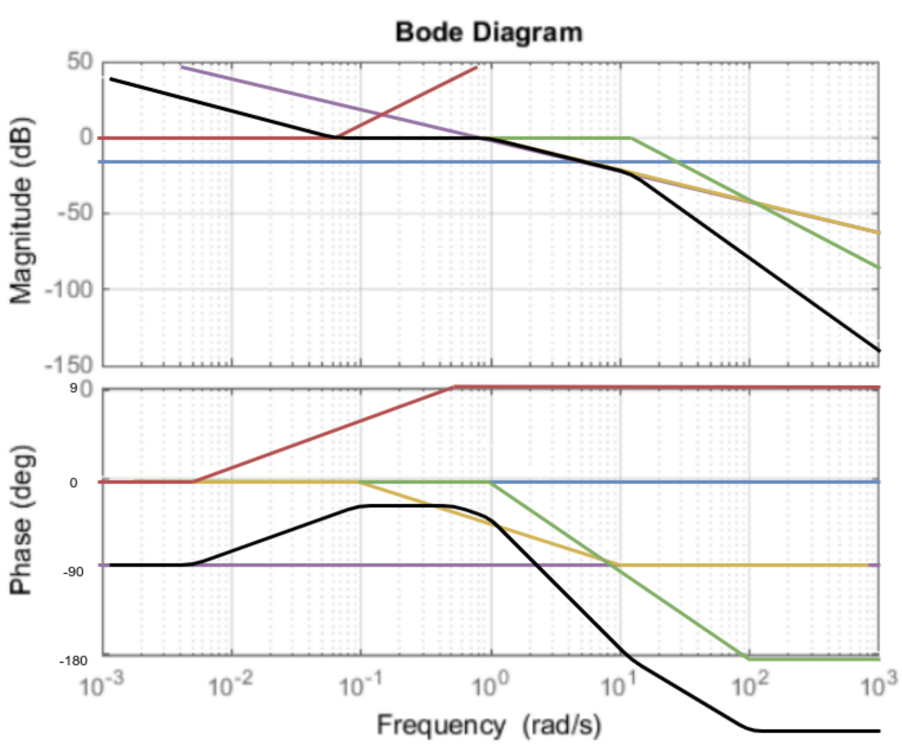
$\omega_n = 1$

First Order Pole: $\left(\frac{s}{.05}+1\right)$ RED

$\omega_n = .5$

Second Order: $\frac{1}{\left(\frac{s^2}{10^2}+\frac{1.6}{10}s+1\right)}$ GREEN

$\omega_n = 10$



Question 3

a)

cg = 2 rad/s

cp = $\sqrt{3*4}$ =3.46 rad/s

Gm = 10 db

Pm = 40 deg

b)

System is stable as both Gm and Pm are positive

c)

```
clear;
p=pi;
zeta = 40/100
```

```
zeta = 0.4000
```

```
z=zeta;
syms pi zeta real;
Mp = exp(-zeta*pi/sqrt(1-zeta^2))
```

```
Mp =
```

$$e^{-\frac{\pi\,\zeta}{(1-\zeta^2)^{0.5000}}}$$

```
Mp = subs(Mp,[pi,zeta],[p,z])
```

Mp = 0.2538

d)

the inital slope of of the curve is -20dB/dec and indicates the precense of one pole at the origin

Question 4

```
clear;
Pm = 30 %deg
```

```
Pm = 30
```

```
Wg = 15 %rad/s
```

```
Wg = 15
```

```
Kg = 1
```

```
Kg = 1
```

```

p = pi;
zeta = 30/100
```

```
zeta = 0.3000
```

```
z=zeta;
syms pi zeta real;
```

```
Mp = exp(-zeta*pi/sqrt(1-zeta^2))
```

Mp =

$$e^{-\frac{\pi \zeta}{\left(1-\zeta^2\right)^{0.5000}}}$$

```
Mp = subs(Mp,[pi,zeta],[p,z])
```

Mp = 0.3723

```
P_desired = 45%
```

```
P_desired = 45
```

$$C = K\ \frac{\mathrm{Ts}+1}{\alpha \mathrm{Ts}+1}$$

```
P_max = P_desired-Pm
```

```
P_max = 15
```

```
alpha = (1-sind(P_max))/(1+sind(P_max))
```

```
alpha = 0.5888
```

```
T = 1/(sqrt(alpha)*Wg)
```

```
T = 0.0869
```

```
K = sqrt(alpha)/Kg
```

```
K = 0.7673
```

```
s = tf('s')
```

```
s =

  s

Continuous-time transfer function.
```

```
C = K*(T * s +1)/(alpha*T*s+1)
```

```
C =

  0.06667 s + 0.7673
  ------------------
    0.05116 s + 1

Continuous-time transfer function.
```