

```
In [5]: import numpy as np
import pandas as pd
import matplotlib.patches as mpatches
import matplotlib.pyplot as plt
import matplotlib
import statsmodels
from statsmodels.distributions.empirical_distribution import ECDF
import statsmodels.formula.api as smf
from stargazer.stargazer import Stargazer
import cycler
import warnings

matplotlib.rcParams['text.usetex'] = True
matplotlib.rcParams['text.latex.preamble'] = \
    r'\usepackage{amssymb} \usepackage{amsmath} \usepackage{xcolor}' + \
    r'\renewcommand*\familydefault{\sfdefault}'
matplotlib.rcParams['pgf.texsystem'] = 'pdflatex'
matplotlib.rcParams['pgf.preamble'] = \
    r'\usepackage[utf8x]{inputenc} \usepackage{amssymb}' + \
    r'\usepackage[T1]{fontenc} \usepackage{amsmath} \usepackage{sansmath}'

ax = np.newaxis
norm = np.linalg.norm
warnings.filterwarnings("ignore")
kernel_est = statsmodels.nonparametric.kde.KDEUnivariate
```

```
In [6]: data = pd.read_csv('ps3_auction.csv')
data.head()
```

```
Out[6]:   Experiment Period Subject Value BidC3_o Unnamed: 5 BidNC Market BidC3 BidC6 Ncont Size6 Vh BidC6.1
0            3      5       1   9.21     8.00      .     .      1    8.00    8.50    0      0   30    8.50
1            3      5       2   4.38     4.00      .     .      0    4.00    4.25    0      0   30    4.25
2            3      5       3  28.05    21.05      .     .      0   21.05   22.05    0      0   30   22.05
3            3      5       4  26.44    20.44      .     .      1   20.44   21.44    0      0   30   21.44
4            3      5       5  21.49    18.49      .     .      0   18.49   19.49    0      0   30   19.49
```

Part 1 - Question 1

```
In [7]: data['bid_RNBN3'] = data['Value'] * 2/3
data['bid_RNBN6'] = data['Value'] * 5/6

data['diff_BN3'] = data['BidC3'] - data['bid_RNBN3']
data['diff_BN6'] = data['BidC6'] - data['bid_RNBN6']
```

```
In [13]: fig3, ax3 = plt.subplots(1, 2, figsize=(10, 5), tight_layout=True, dpi=400, sharey='row')
color = plt.cm.inferno(np.linspace(0.1, 0.9, 3))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax3[0].plot(data['Value'], data['bid_RNBN3'], label=r'$N = 3$ Equilibrium Bid')
ax3[0].plot(data['Value'], data['BidC3'], label=r'$N = 3$ Actual Bid', marker='x', linestyle='')

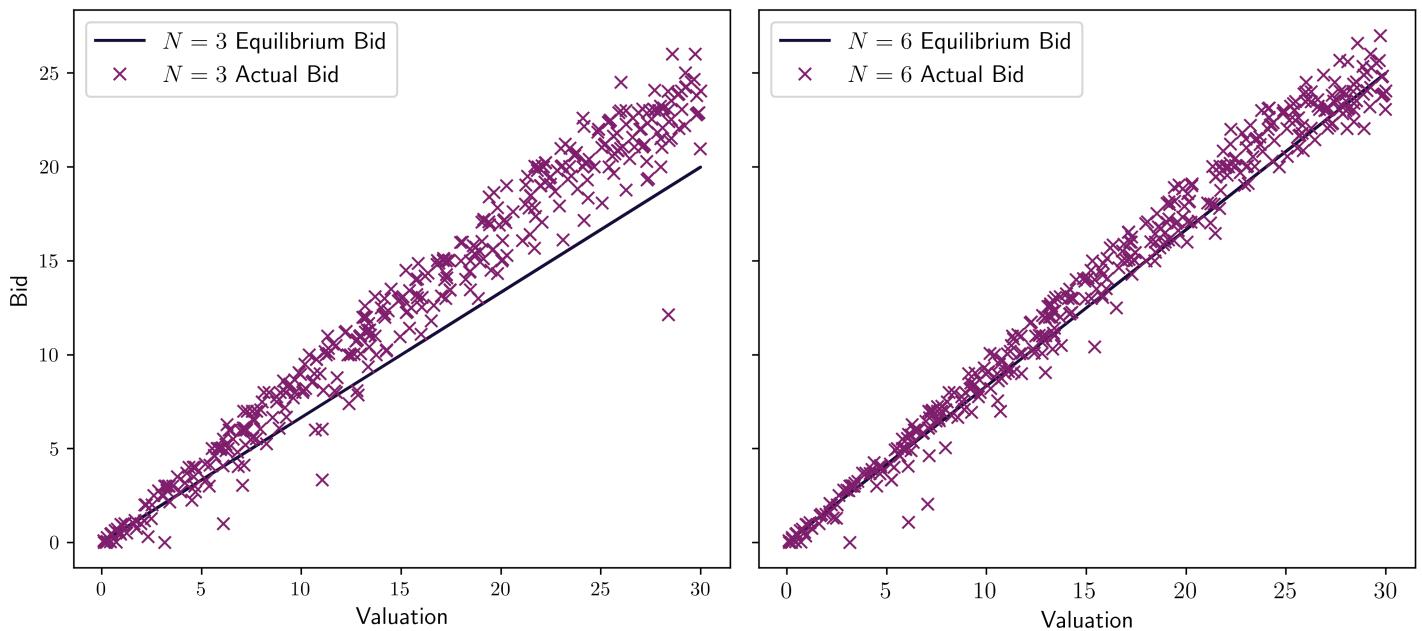
ax3[1].plot(data['Value'], data['bid_RNBN6'], label=r'$N = 6$ Equilibrium Bid')
ax3[1].plot(data['Value'], data['BidC6'], label=r'$N = 6$ Actual Bid', marker='x', linestyle='')

ax3[0].legend(fontsize=12, ncol=1);
ax3[0].set_ylabel('Bid', fontsize=12);
ax3[0].set_xlabel('Valuation', fontsize=12);

ax3[1].legend(fontsize=12, ncol=1);
ax3[1].set_xlabel('Valuation', fontsize=12);

fig3.suptitle('Comparison of Equilibrium and Actual bids', fontsize=20, y=0.99);
plt.xticks(fontsize=12);
plt.yticks(fontsize=12);
```

Comparison of Equilibrium and Actual bids



```
In [14]: def return_empirical_cdf(grid, exp, num_bid):
```

```
    """
    Computes the empirical CDF over a specified 1D grid for the bids within an experiment
    and given number of players.
    :param grid: Support of empirical CDF.
    :param exp: Experiment number.
    :param num_bid: Number of bidders.
    :return: CDF over the support.
    """

    if 3 <= exp <= 5:

        count_below = lambda x : data.query('Experiment == {} and BidC{} <= {}'.format(exp, num_bid, x)).Experiment.count()
        count_total = data.query('Experiment == {}'.format(exp)).Experiment.count()

    else:

        count_below = lambda x : data.query('BidC{} <= {}'.format(num_bid, x)).Experiment.count()
        count_total = data.Experiment.count()

    count_data = np.vectorize(count_below)(grid)

    return count_data/count_total
```

```
In [15]: def compute_util_max_bid(grid, cdf, num_bid):
```

```
    v_grid = grid
    b_grid = grid
    util = (v_grid[:, ax] - b_grid[ax, :]) * (cdf[np.array(b_grid * 100, dtype=int)]) ** (num_bid - 1)

    b_max = util.max(axis=1)

    return b_max
```

Pretty sure that Equation 4 is wrong as printed. I match the optimization errors given in Table 1 when I use the following equation:

$$\omega(b, v; N, e) = \left[\max_{b'} (v - b') \cdot \hat{Q}(b'; N, e)^{N-1} \right] - \hat{\pi}(b, v; N, e)$$

Part 1 - Question 2 and 3

```
In [17]: fig4, ax4 = plt.subplots(1, 2, figsize=(10, 5), tight_layout=True, dpi=400, sharey='row')
color = plt.cm.inferno(np.linspace(0, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

support_cdf = np.arange(start=0.0, stop=30.01, step=0.01)
```

```

data['Profit_B3'], data['Profit_B6'] = 0., 0.
data['OptError_B3'], data['OptError_B6'] = 0., 0.
cdf_3, cdf_6 = dict(), dict()

# Span over experiments.
for ix, val in enumerate(data.Experiment.unique()):

    # Compute empirical CDFs for N=3 and N=6.
    cdf_3['Exp{}'.format(val)] = return_empirical_cdf(grid=support_cdf, exp=val, num_bid=3)
    cdf_6['Exp{}'.format(val)] = return_empirical_cdf(grid=support_cdf, exp=val, num_bid=6)

    # Compute estimated expected profits.
    exp_index = data['Experiment'] == val
    data.loc[exp_index, 'Profit_B3'] = (data.loc[exp_index, 'Value'] - data.loc[exp_index, 'BidC3']) * (cdf_3['Exp{}'.format(val)])
    data.loc[exp_index, 'Profit_B6'] = (data.loc[exp_index, 'Value'] - data.loc[exp_index, 'BidC6']) * (cdf_6['Exp{}'.format(val)])

    # Compute utility maximizing bids
    util_max_3 = compute_util_max_bid(support_cdf, cdf_3['Exp{}'.format(val)], 3)
    util_max_6 = compute_util_max_bid(support_cdf, cdf_6['Exp{}'.format(val)], 6)

    # Compute optimization errors.
    data.loc[exp_index, 'OptError_B3'] = util_max_3[np.array(data.loc[exp_index, 'Value'] * 100, dtype=int)] - data.loc[exp_index, 'Value']
    data.loc[exp_index, 'OptError_B6'] = util_max_6[np.array(data.loc[exp_index, 'Value'] * 100, dtype=int)] - data.loc[exp_index, 'Value']

    # Plot CDFs.
    ax4[0].plot(support_cdf, cdf_3['Exp{}'.format(val)], label=r'$N = 3$', Experiment '{}'.format(val))
    ax4[1].plot(support_cdf, cdf_6['Exp{}'.format(val)], label=r'$N = 6$', Experiment '{}'.format(val))

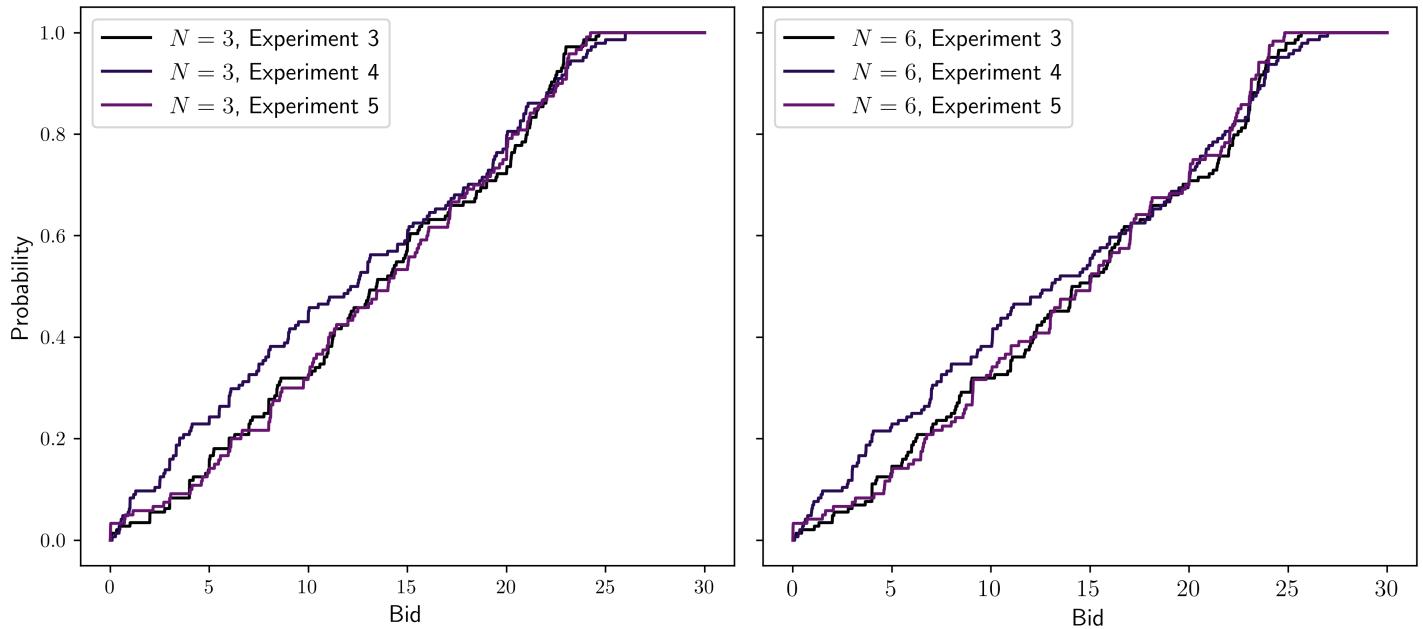
ax4[0].legend(fontsize=12, ncol=1);
ax4[0].set_ylabel('Probability', fontsize=12);
ax4[0].set_xlabel('Bid', fontsize=12);

ax4[1].legend(fontsize=12, ncol=1);
ax4[1].set_xlabel('Bid', fontsize=12);

fig4.suptitle('Comparison of CDFs Across Experiments', fontsize=20, y=0.99);
plt.xticks(fontsize=12);
plt.yticks(fontsize=12);

```

Comparison of CDFs Across Experiments



We see that the distribution of bids varies slightly between the three experiments.

Part 1 - Question 4

I have Table 1 replicated below in a less refined format. `diff_` refers to the difference in observed and equilibrium Nash bids, `Profit_` refers to the estimated expected profits, and `OptError_` refers to the optimization error of the bidder.

```
In [ ]: pd.set_option('display.float_format', lambda x: '%.3f' % x)
```

```
data[['diff_BN3', 'diff_BN6', 'Profit_B3',
      'Profit_B6', 'OptError_B3', 'OptError_B6']].describe().to_markdown()
```

	diff_BN3	diff_BN6	Profit_B3	Profit_B6	OptError_B3	OptError_B6
mean	2.4441	0.6479	1.3326	0.5791	0.3961	0.1170
std	1.8633	1.0547	1.6095	1.0559	0.4928	0.2521
25%	1.0575	0.0350	0.0804	0.0008	0.0271	0.0002
50%	2.4633	0.5583	0.6237	0.0461	0.1914	0.0099
75%	3.8333	1.4138	2.0501	0.5682	0.6172	0.1181

Risk-Neutral Bayes-Nash model

Part 2 - Questions 1 and 2

```
In [19]: # Compute pooled empirical CDF estimates
cdf_3['Pool'] = ECDF(data.BidC3)
cdf_6['Pool'] = ECDF(data.BidC6)
```

```
In [20]: pdf_3, pdf_6 = dict(), dict()

# Compute experiment specific PDF estimates using Gaussian kernel.

for ix, val in enumerate(data.Experiment.unique()):

    kernel3 = kernel_est(endog=data.query('Experiment == {}'.format(val))['BidC3']).fit(kernel='gau', bw='silverman')
    kernel6 = kernel_est(endog=data.query('Experiment == {}'.format(val))['BidC6']).fit(kernel='gau', bw='silverman')

    # Compute empirical CDFs for N=3 and N=6.
    pdf_3['Exp{}'.format(val)] = kernel3
    pdf_6['Exp{}'.format(val)] = kernel6

kernel3 = kernel_est(endog=data['BidC3']).fit(kernel='gau', bw='silverman')
kernel6 = kernel_est(endog=data['BidC6']).fit(kernel='gau', bw='silverman')

pdf_3['Pool'] = kernel3
pdf_6['Pool'] = kernel6
```

```
In [21]: fig5, ax5 = plt.subplots(1, 2, figsize=(10, 5), tight_layout=True, dpi=400, sharey='row')
color = plt.cm.inferno(np.linspace(0, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

# Span over experiments.
for ix, val in enumerate(data.Experiment.unique()):

    # Plot CDFs.
    ax5[0].plot(support_cdf, pdf_3['Exp{}'.format(val)].evaluate(support_cdf),
                 label=r'$N = 3$', Experiment={}.format(val))
    ax5[1].plot(support_cdf, pdf_6['Exp{}'.format(val)].evaluate(support_cdf),
                 label=r'$N = 6$', Experiment={}.format(val))

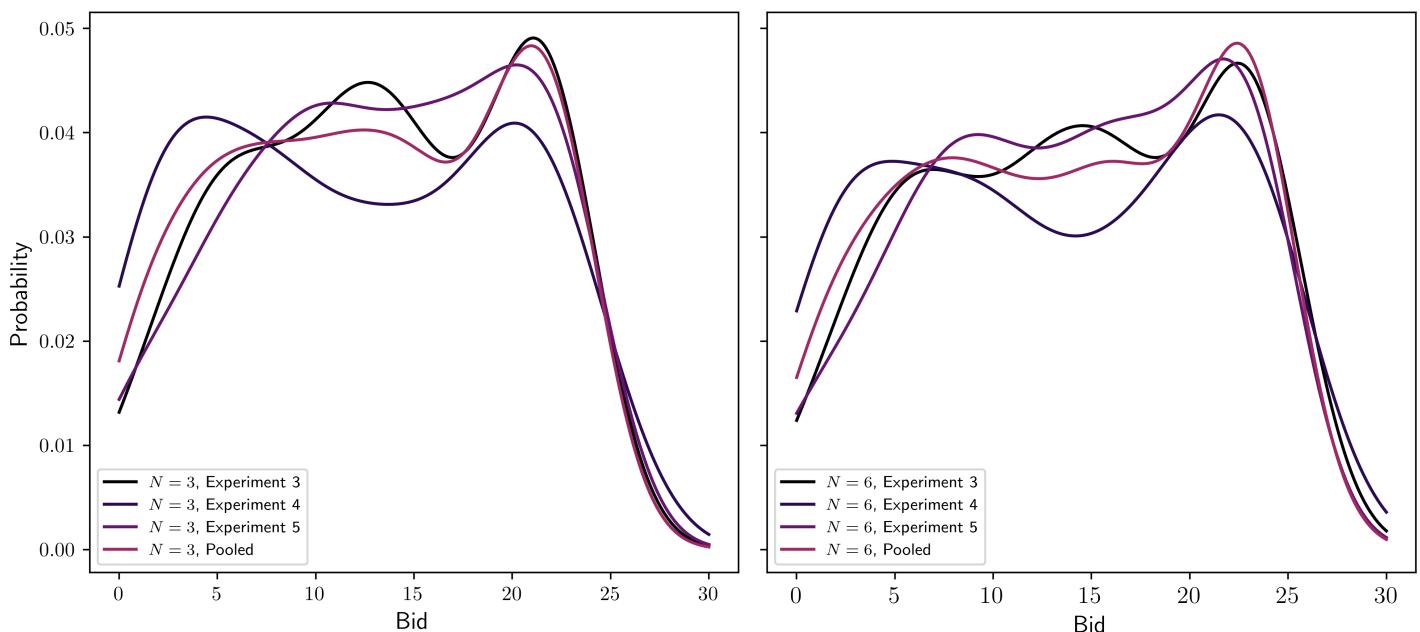
    ax5[0].plot(support_cdf, pdf_3['Pool'].evaluate(support_cdf),
                 label=r'$N = 3$', Pooled='')
    ax5[1].plot(support_cdf, pdf_6['Pool'].evaluate(support_cdf),
                 label=r'$N = 6$', Pooled='')

    ax5[0].legend(fontsize=8, ncol=1, loc=3);
    ax5[0].set_ylabel('Probability', fontsize=12);
    ax5[0].set_xlabel('Bid', fontsize=12);

    ax5[1].legend(fontsize=8, ncol=1, loc=3);
    ax5[1].set_xlabel('Bid', fontsize=12);

fig5.suptitle('Comparison of PDFs Across Experiments', fontsize=20, y=0.99);
plt.xticks(fontsize=12);
plt.yticks(fontsize=12);
```

Comparison of PDFs Across Experiments



```
In [22]: data['EstVal_3_RN'] = data['BidC3'] + \
    cdf_3['Pool'](data['BidC3']) / \
    (2 * pdf_3['Pool'].evaluate(np.array(data['BidC3'])))

data['EstVal_6_RN'] = data['BidC6'] + \
    cdf_6['Pool'](data['BidC6']) / \
    (5 * pdf_6['Pool'].evaluate(np.array(data['BidC6'])))

data['EstVal_3_E3_RN'] = data['BidC3'] + \
    cdf_3['Exp3'][np.array(data['BidC3'] * 100, dtype=int)] / \
    (2 * pdf_3['Exp3'].evaluate(np.array(data['BidC3'])))

data['EstVal_6_E3_RN'] = data['BidC6'] + \
    cdf_6['Exp3'][np.array(data['BidC6'] * 100, dtype=int)] / \
    (5 * pdf_6['Exp3'].evaluate(np.array(data['BidC6'])))

data['EstVal_3_E4_RN'] = data['BidC3'] + \
    cdf_3['Exp4'][np.array(data['BidC3'] * 100, dtype=int)] / \
    (2 * pdf_3['Exp4'].evaluate(np.array(data['BidC3'])))

data['EstVal_6_E4_RN'] = data['BidC6'] + \
    cdf_6['Exp4'][np.array(data['BidC6'] * 100, dtype=int)] / \
    (5 * pdf_6['Exp4'].evaluate(np.array(data['BidC6'])))

data['EstVal_3_E5_RN'] = data['BidC3'] + \
    cdf_3['Exp5'][np.array(data['BidC3'] * 100, dtype=int)] / \
    (2 * pdf_3['Exp5'].evaluate(np.array(data['BidC3'])))

data['EstVal_6_E5_RN'] = data['BidC6'] + \
    cdf_6['Exp5'][np.array(data['BidC6'] * 100, dtype=int)] / \
    (5 * pdf_6['Exp5'].evaluate(np.array(data['BidC6'])))
```

Part 2 - Question 3

```
In [23]: fig6, ax6 = plt.subplots(1, 2, figsize=(10, 5), tight_layout=True, dpi=400, sharey='row')
color = plt.cm.inferno(np.linspace(0.1, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax6[0].plot(data.Value, data.EstVal_3_RN, label=r'Pooled', marker='x', linestyle='', markersize=4, alpha=0.5)
ax6[0].plot(data.Value, data.EstVal_3_E3_RN, label=r'Experiment 3', marker='p', linestyle='', markersize=4, alpha=0.5)
ax6[0].plot(data.Value, data.EstVal_3_E4_RN, label=r'Experiment 4', marker='o', linestyle='', markersize=4, alpha=0.5)
ax6[0].plot(data.Value, data.EstVal_3_E5_RN, label=r'Experiment 5', marker='s', linestyle='', markersize=4, alpha=0.5)

ax6[0].axline([0, 0], [1, 1], label=r'$45^\circ$ line', color='k')

ax6[1].plot(data.Value, data.EstVal_6_RN, label=r'Pooled', marker='x', linestyle='', markersize=4, alpha=0.5)
ax6[1].plot(data.Value, data.EstVal_6_E3_RN, label=r'Experiment 3', marker='p', linestyle='', markersize=4, alpha=0.5)
ax6[1].plot(data.Value, data.EstVal_6_E4_RN, label=r'Experiment 4', marker='o', linestyle='', markersize=4, alpha=0.5)
ax6[1].plot(data.Value, data.EstVal_6_E5_RN, label=r'Experiment 5', marker='s', linestyle='', markersize=4, alpha=0.5)
```

```

ax6[1].axline([0, 0], [1, 1], label= r'$\circlearrowleft$ line', color='k')

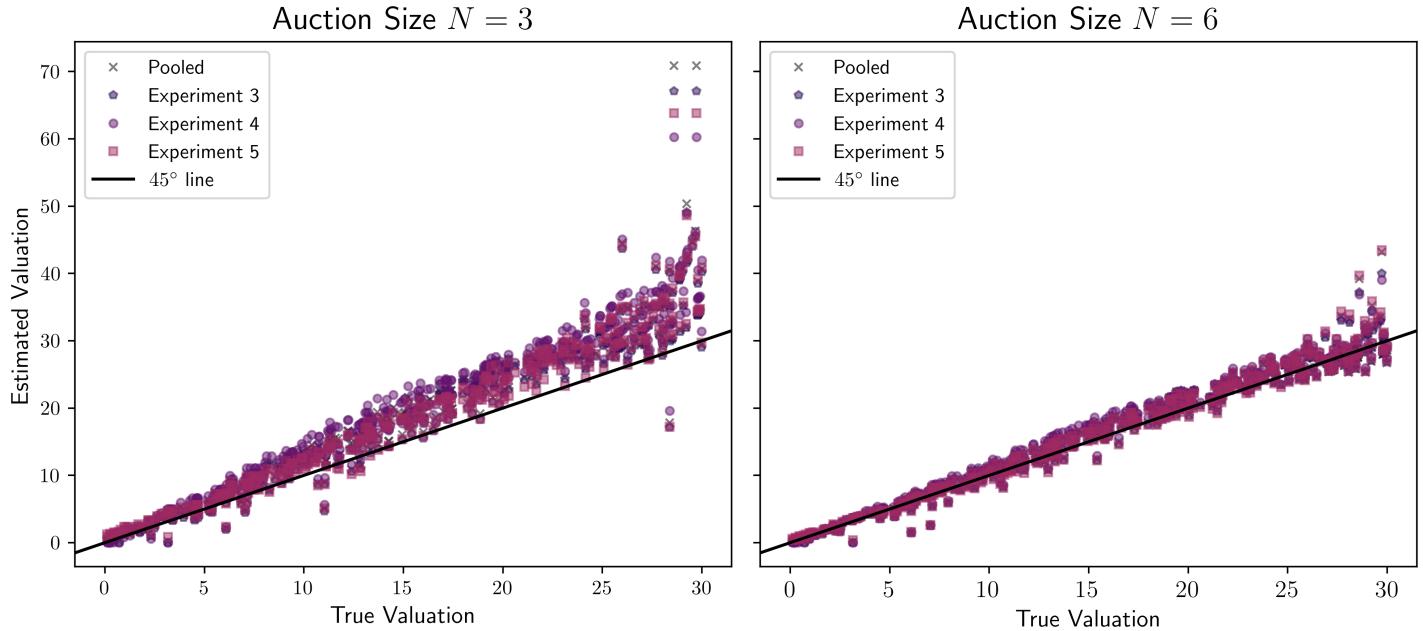
ax6[0].legend(fontsize=10, ncol=1, loc=2);
ax6[0].set_ylabel('Estimated Valuation', fontsize=12);
ax6[0].set_xlabel('True Valuation', fontsize=12);
ax6[0].set_title(r'Auction Size $N = 3$', fontsize=16);

ax6[1].legend(fontsize=10, ncol=1, loc=2);
ax6[1].set_xlabel('True Valuation', fontsize=12);
ax6[1].set_title(r'Auction Size $N = 6$', fontsize=16);

fig6.suptitle('Risk Neutral Value Estimation', fontsize=20, y=0.99);
plt.xticks(fontsize=12);
plt.yticks(fontsize=12);

```

Risk Neutral Value Estimation



```

In [26]: fig7, ax7 = plt.subplots(3, 1, figsize=(7, 7), tight_layout=True, dpi=400)
color = plt.cm.inferno(np.linspace(0.5, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax7[0].hist(data.Value, 40, alpha=0.5, histtype='bar', ec='black');
ax7[0].set_title('Actual Valuation', fontsize=12)
ax7[0].set_ylabel(r'Frequency', fontsize=10);

ax7[1].hist(data.EstVal_3_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[1].set_title('$N = 3$', fontsize=12);
ax7[1].set_ylabel(r'Frequency', fontsize=10);

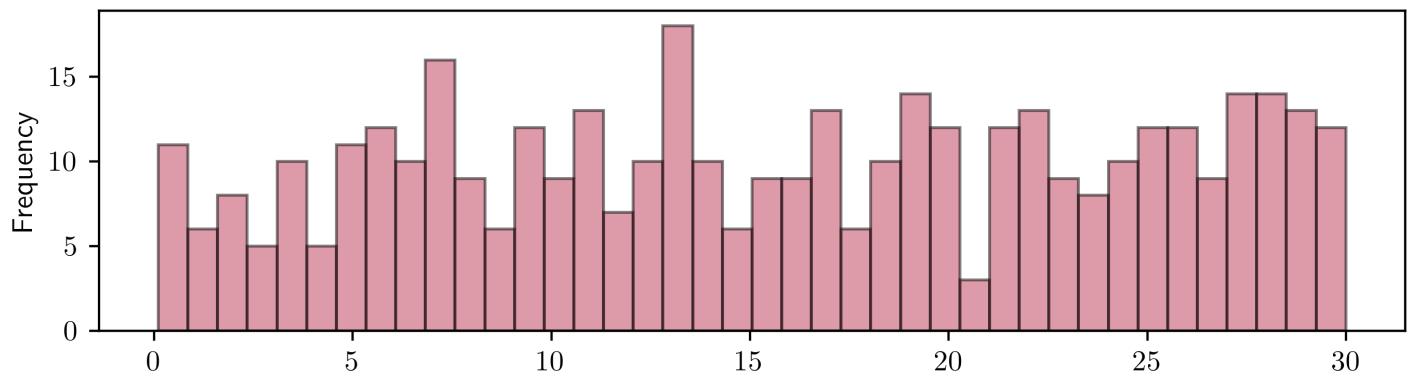
ax7[2].hist(data.EstVal_6_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[2].set_title('$N = 6$', fontsize=12)
ax7[2].set_xlabel(r'Valuation', fontsize=10);
ax7[2].set_ylabel(r'Frequency', fontsize=10);

fig7.suptitle('Comparing Risk Neutral Estimated Values, Pooled', fontsize=16, y=0.99);

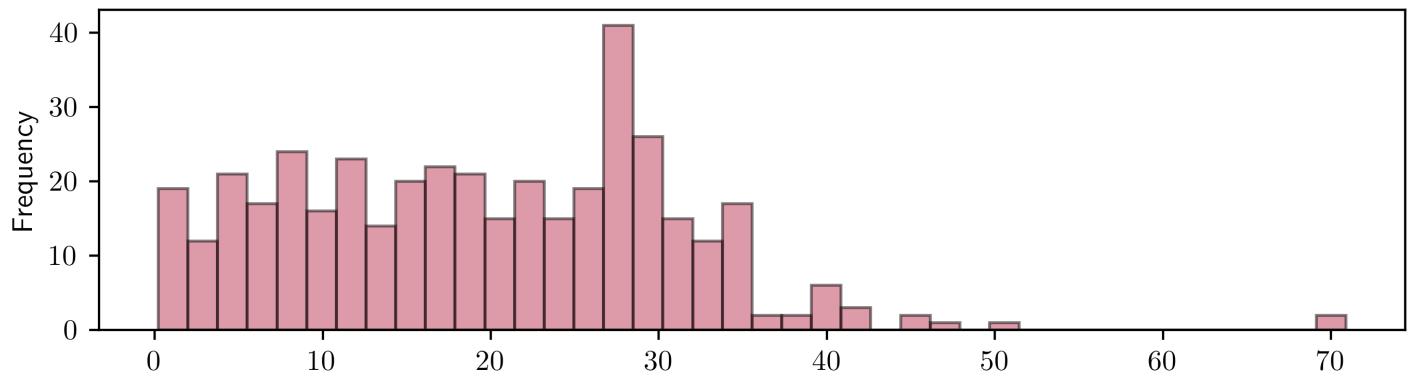
```

Comparing Risk Neutral Estimated Values, Pooled

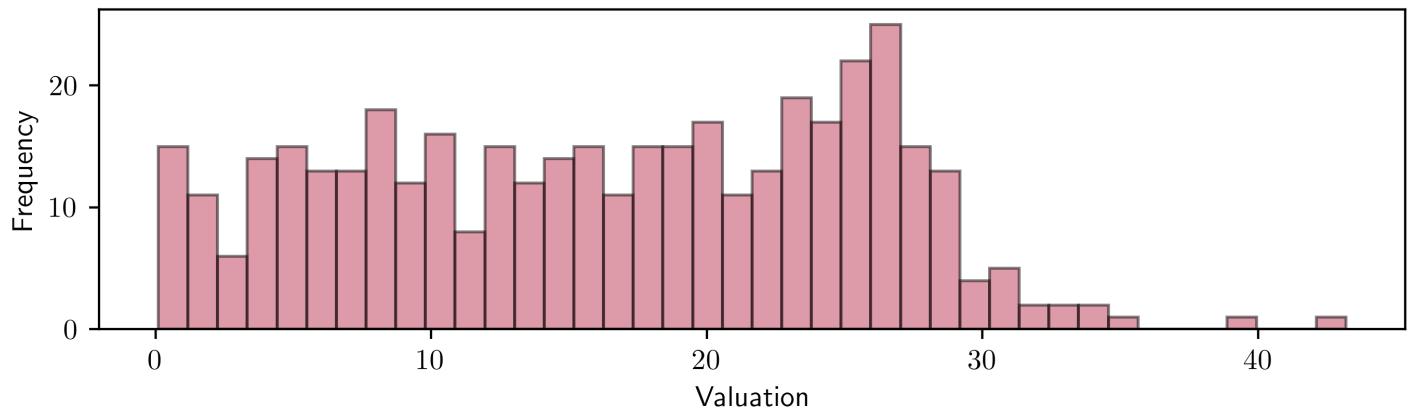
Actual Valuation



$N = 3$



$N = 6$



```
In [25]: fig7, ax7 = plt.subplots(3, 1, figsize=(7, 7), tight_layout=True, dpi=400)
color = plt.cm.inferno(np.linspace(0.5, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

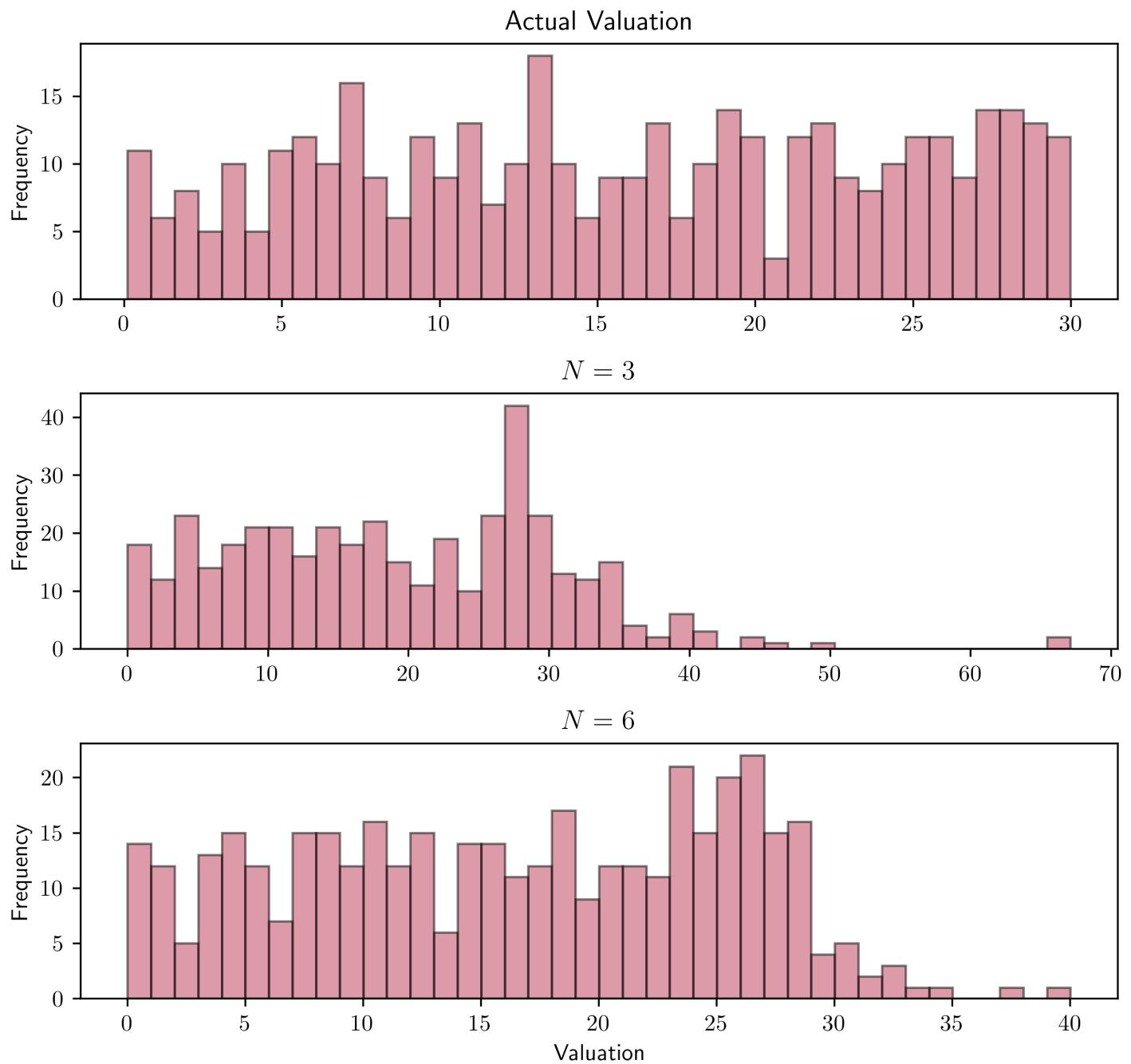
ax7[0].hist(data.Value, 40, alpha=0.5, histtype='bar', ec='black');
ax7[0].set_title('Actual Valuation', fontsize=12)
ax7[0].set_ylabel(r'Frequency', fontsize=10);

ax7[1].hist(data.EstVal_3_E3_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[1].set_title('$N = 3$', fontsize=12);
ax7[1].set_ylabel(r'Frequency', fontsize=10);

ax7[2].hist(data.EstVal_6_E3_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[2].set_title('$N = 6$', fontsize=12);
ax7[2].set_xlabel(r'Valuation', fontsize=10);
ax7[2].set_ylabel(r'Frequency', fontsize=10);

fig7.suptitle('Comparing Risk Neutral Estimated Values, Experiment 3', fontsize=16, y=0.99);
```

Comparing Risk Neutral Estimated Values, Experiment 3



```
In [27]: fig7, ax7 = plt.subplots(3, 1, figsize=(7, 7), tight_layout=True, dpi=400)
color = plt.cm.inferno(np.linspace(0.5, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax7[0].hist(data.Value, 40, alpha=0.5, histtype='bar', ec='black');
ax7[0].set_title('Actual Valuation', fontsize=12)
ax7[0].set_ylabel(r'Frequency', fontsize=10);

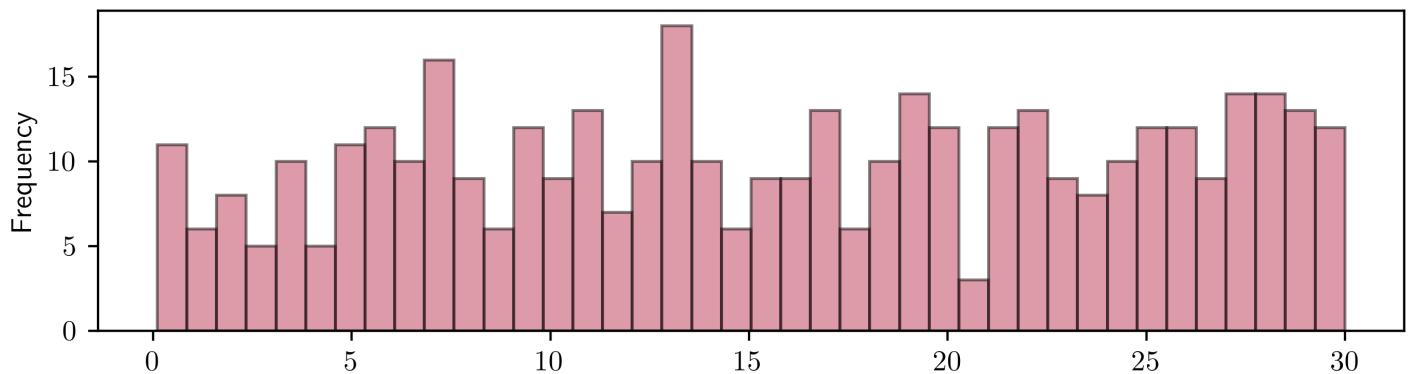
ax7[1].hist(data.EstVal_3_E4_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[1].set_title('$N = 3$', fontsize=12);
ax7[1].set_ylabel(r'Frequency', fontsize=10);

ax7[2].hist(data.EstVal_6_E4_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[2].set_title('$N = 6$', fontsize=12);
ax7[2].set_xlabel(r'Valuation', fontsize=10);
ax7[2].set_ylabel(r'Frequency', fontsize=10);

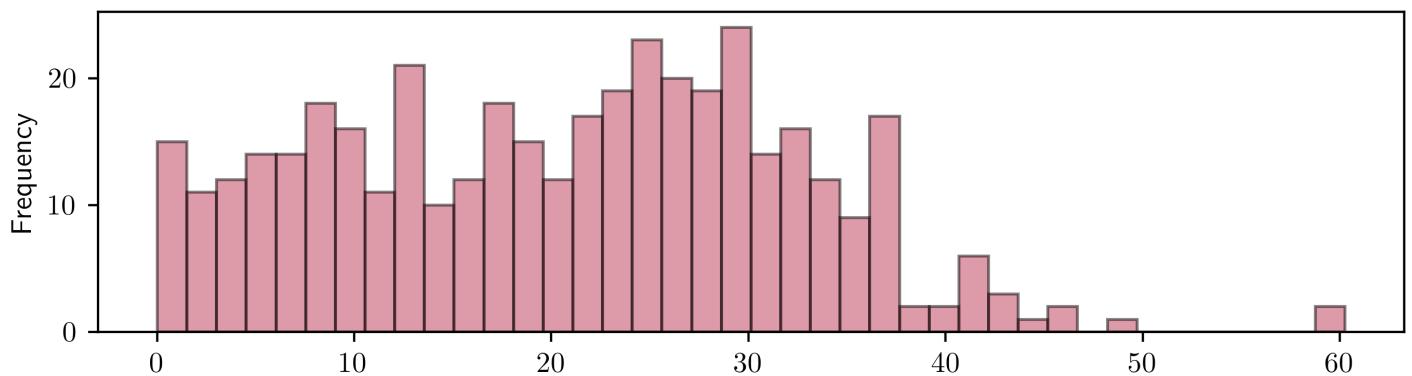
fig7.suptitle('Comparing Risk Neutral Estimated Values, Experiment 4', fontsize=16, y=0.99);
```

Comparing Risk Neutral Estimated Values, Experiment 4

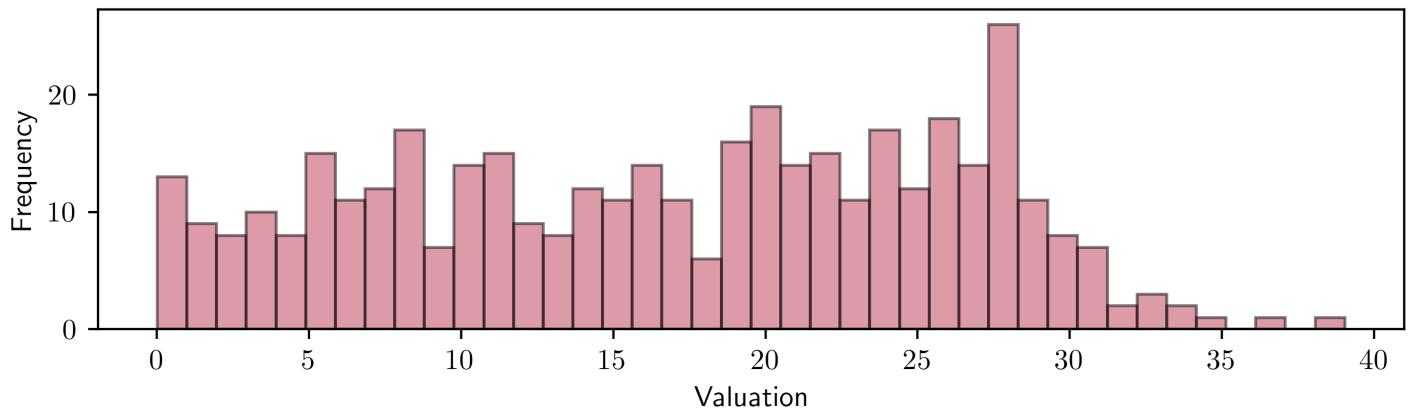
Actual Valuation



$N = 3$



$N = 6$



```
In [28]: fig7, ax7 = plt.subplots(3, 1, figsize=(7, 7), tight_layout=True, dpi=400)
color = plt.cm.inferno(np.linspace(0.5, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax7[0].hist(data.Value, 40, alpha=0.5, histtype='bar', ec='black');
ax7[0].set_title('Actual Valuation', fontsize=12)
ax7[0].set_ylabel(r'Frequency', fontsize=10);

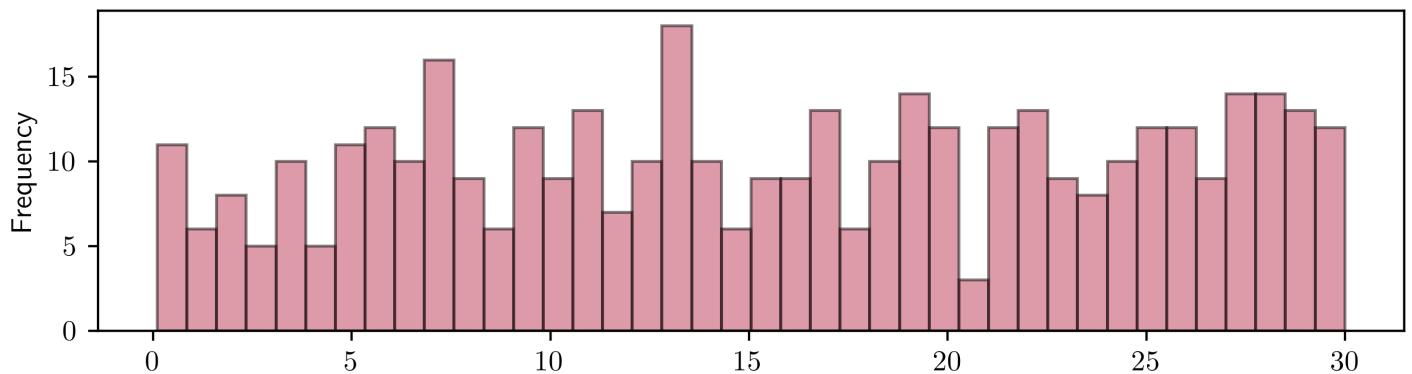
ax7[1].hist(data.EstVal_3_E5_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[1].set_title('$N = 3$', fontsize=12);
ax7[1].set_ylabel(r'Frequency', fontsize=10);

ax7[2].hist(data.EstVal_6_E5_RN, 40, alpha=0.5, label=r'Pooled',
            histtype='bar', ec='black');
ax7[2].set_title('$N = 6$', fontsize=12);
ax7[2].set_xlabel(r'Valuation', fontsize=10);
ax7[2].set_ylabel(r'Frequency', fontsize=10);

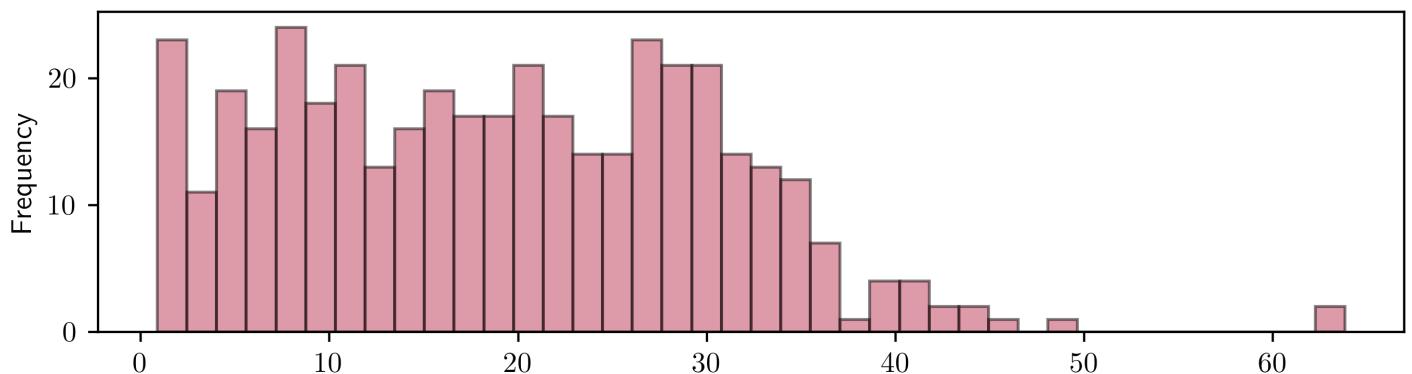
fig7.suptitle('Comparing Risk Neutral Estimated Values, Experiment 5', fontsize=16, y=0.99);
```

Comparing Risk Neutral Estimated Values, Experiment 5

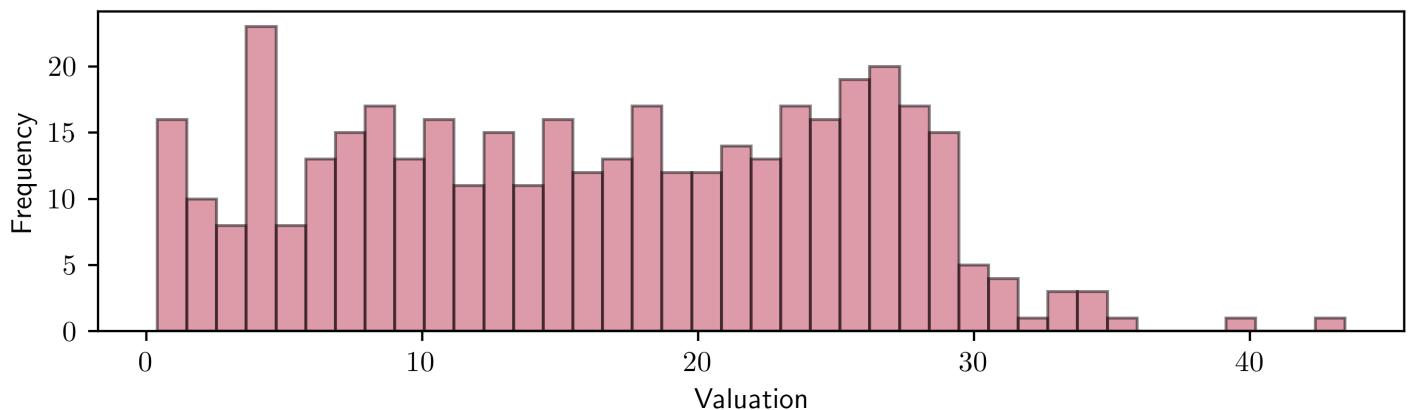
Actual Valuation



$N = 3$



$N = 6$



```
In [29]: n_obs = len(data.Experiment)
```

```
risk_neutral_norms = {
    'L1 Norm': [norm(data.Value - data.EstVal_3_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_3_E3_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_3_E4_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_3_E5_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_6_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_6_E3_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_6_E4_RN, ord=1)/n_obs,
                norm(data.Value - data.EstVal_6_E5_RN, ord=1)/n_obs],
    'L2 Norm': [norm(data.Value - data.EstVal_3_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_3_E3_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_3_E4_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_3_E5_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_6_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_6_E3_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_6_E4_RN, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_6_E5_RN, ord=2)/(n_obs ** 0.5)]
}
```

```
risk_neutral_norms=pd.DataFrame(
    data=risk_neutral_norms,
```

```

        index=['$N = 3$', 'Pooled', '$N = 3$', 'Experiment 3', '$N = 3$', 'Experiment 4',
        '$N = 3$', 'Experiment 5', '$N = 6$', 'Pooled', '$N = 6$', 'Experiment 3',
        '$N = 6$', 'Experiment 4', '$N = 6$', 'Experiment 5']
    )

```

Part 2 - Question 4

	L1 Norm	L2 Norm
$N = 3$, Pooled	3.86074	5.49069
$N = 3$, Experiment 3	3.53455	5.12834
$N = 3$, Experiment 4	4.95483	6.15492
$N = 3$, Experiment 5	3.51949	5.04657
$N = 6$, Pooled	1.1498	1.67484
$N = 6$, Experiment 3	1.07434	1.52228
$N = 6$, Experiment 4	1.36673	1.77798
$N = 6$, Experiment 5	1.09736	1.68132

Risk-Averse Bayes-Nash Model

Part 3 - Question 1

```
In [31]: bid_percentile = {
    'Number': np.arange(0, 101, 1),
    'Bid3': data.BidC3.quantile(np.arange(0., 1.01, 0.01)),
    'Bid6': data.BidC6.quantile(np.arange(0., 1.01, 0.01))
}

bid_percentile = pd.DataFrame(bid_percentile)
```

```
In [32]: bid_percentile['CDF3'] = cdf_3['Pool'](bid_percentile['Bid3'])
bid_percentile['CDF6'] = cdf_6['Pool'](bid_percentile['Bid6'])
bid_percentile['PDF3'] = pdf_3['Pool'].evaluate(np.array(bid_percentile['Bid3']))
bid_percentile['PDF6'] = pdf_6['Pool'].evaluate(np.array(bid_percentile['Bid6']))
bid_percentile['d_bid'] = bid_percentile['Bid3'] - bid_percentile['Bid6']
bid_percentile['rhs'] = bid_percentile['CDF6']/(5 * bid_percentile['PDF6']) - \
    bid_percentile['CDF3']/(2 * bid_percentile['PDF3'])
```

Part 3 - Question 2

```
In [33]: m1 = smf.ols('d_bid ~ rhs - 1', data=bid_percentile).fit()
m2 = smf.ols('d_bid ~ rhs - 1', data=bid_percentile.query('5 <= Number <= 95')).fit()
m3 = smf.ols('d_bid ~ rhs - 1', data=bid_percentile.query('25 <= Number <= 75')).fit()
res1 = Stargazer([m1, m2, m3])
res1.custom_columns(['All Data', '5-95', '25-75'], [1, 1, 1])
res1.show_confidence_intervals(True)
res1.show_degrees_of_freedom(False)
res1.rename_covariates({'rhs': 'θ'})
res1.significant_digits(4)
res1.title('Risk Aversion Parameter Estimates')
```

```
In [34]: res1
```

Out [34]: Risk Aversion Parameter Estimates

Dependent variable: d_bid			
	All Data	5-95	25-75
	(1)	(2)	(3)
θ	0.1500*** (0.1293 , 0.1707)	0.2284*** (0.2123 , 0.2445)	0.2453*** (0.2280 , 0.2626)
Observations	101	91	51
R ²	0.6737	0.8982	0.9421
Adjusted R ²	0.6705	0.8971	0.9409
Residual Std. Error	0.5447	0.3105	0.2330
F Statistic	206.5022***	794.4993***	813.4665***

Note: * p<0.1; ** p<0.05; *** p<0.01

Part 3 - Question 3

In [35]: theta = m3.params.squeeze()

```
In [36]: data['EstVal_3_RA'] = data['BidC3'] + theta * \
cdf_3['Pool'](data['BidC3']) / \
(2 * pdf_3['Pool'].evaluate(np.array(data['BidC3'])))

data['EstVal_6_RA'] = data['BidC6'] + theta * \
cdf_6['Pool'](data['BidC6']) / \
(5 * pdf_6['Pool'].evaluate(np.array(data['BidC6'])))
```

```
In [37]: fig8, ax8 = plt.subplots(1, 2, figsize=(10, 5), tight_layout=True, dpi=400, sharey='row')
color = plt.cm.inferno(np.linspace(0, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax8[0].plot(data.Value, data.EstVal_3_RA, label=r'Pooled', marker='x', linestyle='', markersize=4, alpha=0.5)
ax8[0].axline([0, 0], [1, 1], label=r'$45^\circ$ line', color='k')

ax8[1].plot(data.Value, data.EstVal_6_RA, label=r'Pooled', marker='x', linestyle='', markersize=4, alpha=0.5)
ax8[1].axline([0, 0], [1, 1], label=r'$45^\circ$ line', color='k')

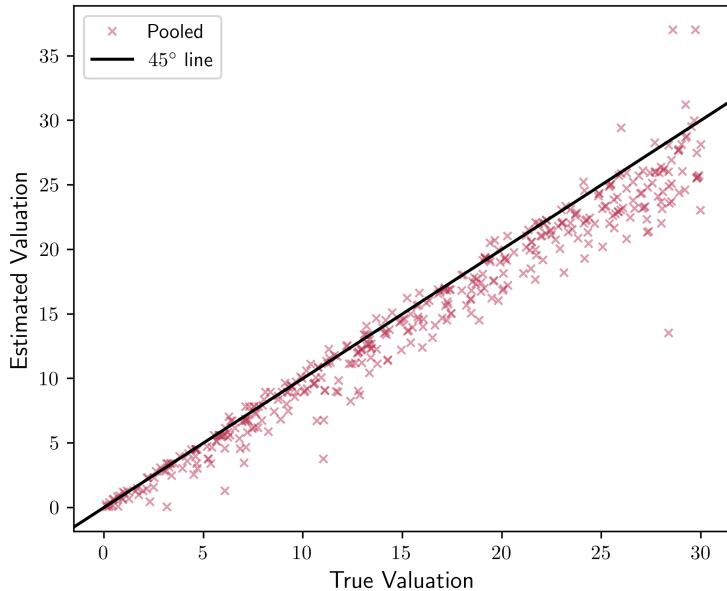
ax8[0].legend(fontsize=10, ncol=1, loc=2);
ax8[0].set_ylabel('Estimated Valuation', fontsize=12);
ax8[0].set_xlabel('True Valuation', fontsize=12);
ax8[0].set_title(r'Auction Size $N = 3$', fontsize=16)

ax8[1].legend(fontsize=10, ncol=1, loc=2);
ax8[1].set_xlabel('True Valuation', fontsize=12);
ax8[1].set_title(r'Auction Size $N = 6$', fontsize=16)

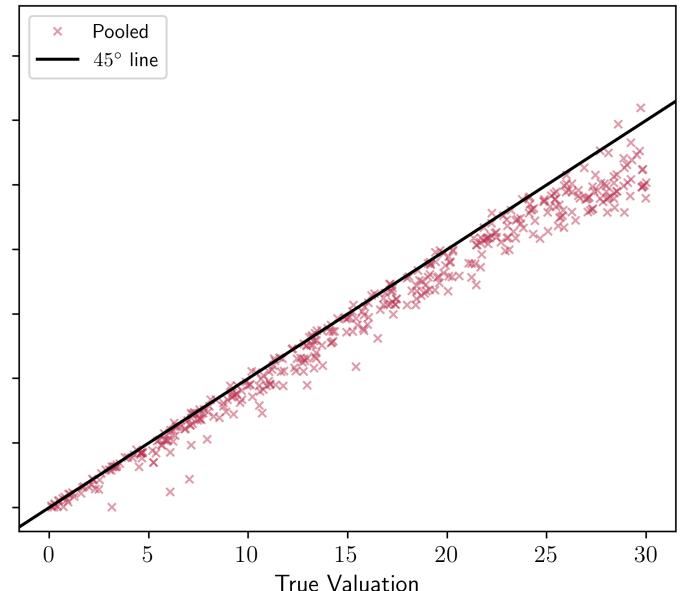
fig8.suptitle(r'Risk Averse Value Estimation, $\theta = {:.4f}$.format(theta),
              fontsize=20, y=0.99);
plt.xticks(fontsize=12);
plt.yticks(fontsize=12);
```

Risk Averse Value Estimation, $\theta = 0.2453$

Auction Size $N = 3$



Auction Size $N = 6$



```
In [39]: fig9, ax9 = plt.subplots(3, 1, figsize=(7, 7), tight_layout=True, dpi=400)
color = plt.cm.inferno(np.linspace(0.5, 1, 8))
plt.rcParams['axes.prop_cycle'] = cycler.cycler('color', color)

ax9[0].hist(data.Value, 40, alpha=0.5, histtype='bar', ec='black');
ax9[0].set_title('Actual Valuation', fontsize=12);
ax9[0].set_ylabel(r'Frequency', fontsize=10);

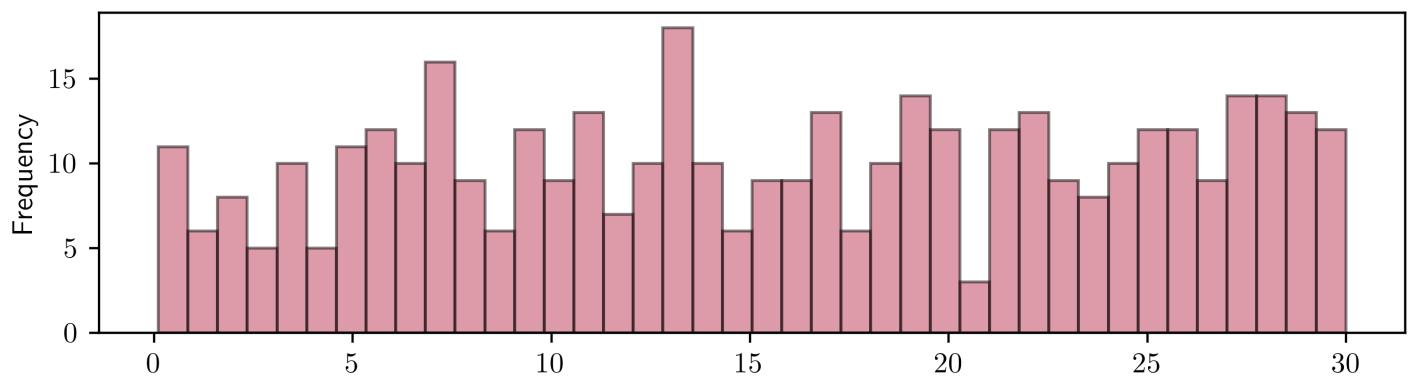
ax9[1].hist(data.EstVal_3_RA, 40, alpha=0.5, histtype='bar', ec='black');
ax9[1].set_title('$N = 3$', fontsize=12);
ax9[1].set_ylabel(r'Frequency', fontsize=10);

ax9[2].hist(data.EstVal_6_RA, 40, alpha=0.5, histtype='bar', ec='black');
ax9[2].set_title('$N = 6$', fontsize=12);
ax9[2].set_xlabel(r'Valuation', fontsize=10);
ax9[2].set_ylabel(r'Frequency', fontsize=10);

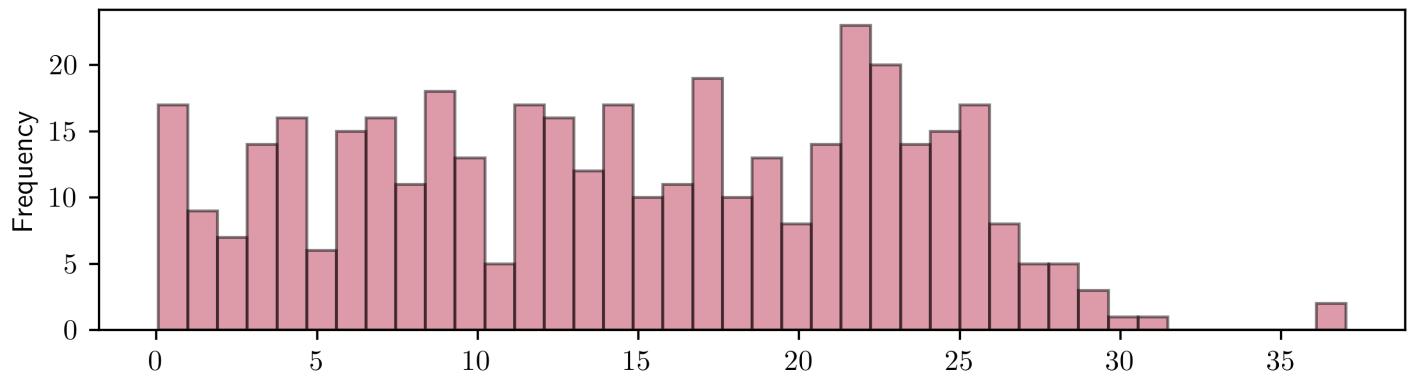
fig9.suptitle('Comparing Risk Averse Estimated Values', fontsize=16, y=0.99);
```

Comparing Risk Averse Estimated Values

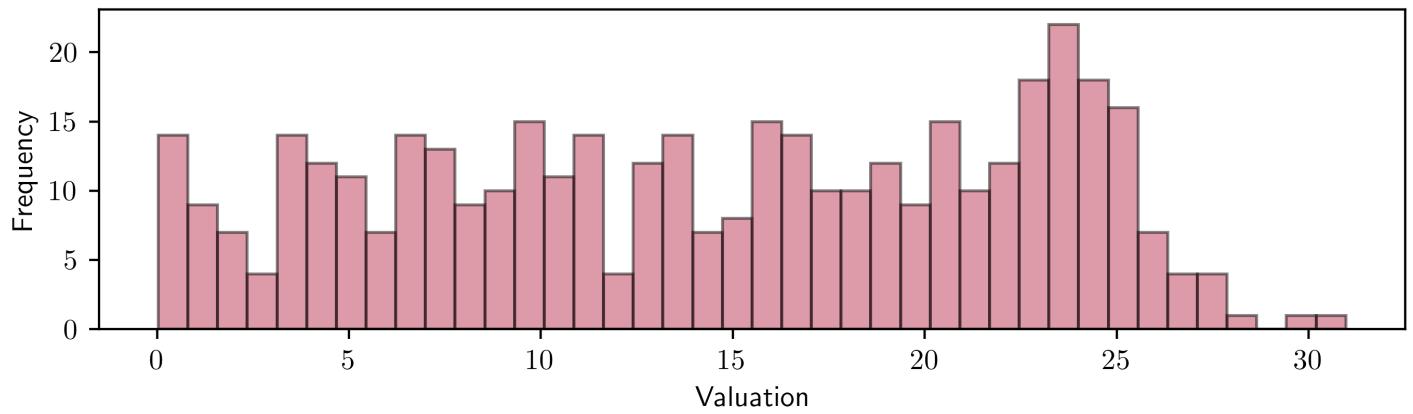
Actual Valuation



$N = 3$



$N = 6$



```
In [40]: n_obs = len(data.Experiment)
```

```
risk_averse_norms = {
    'L1 Norm': [norm(data.Value - data.EstVal_3_RA, ord=1)/n_obs,
                norm(data.Value - data.EstVal_6_RA, ord=1)/n_obs],
    'L2 Norm': [norm(data.Value - data.EstVal_3_RA, ord=2)/(n_obs ** 0.5),
                norm(data.Value - data.EstVal_6_RA, ord=2)/(n_obs ** 0.5)]
}

risk_averse_norms = pd.DataFrame(
    data=risk_averse_norms,
    index=['$N = 3$', '$N = 6$']
)
```

	L1 Norm	L2 Norm
$N = 3$	1.45746	2.16691
$N = 6$	1.38659	1.90917

```
In [ ]:
```