Industrial Organization II: Problem Set 2

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Exercise 2

(a) SPNE equilibrium n_t^* for generic market t

The SPNE n_t^* for a generic market t is given by

$$n_t^* = \sum_i^K \mathbb{1}\left[\pi_{i,t}(n_t^* > 0 | x_t, z_{i,t}; \beta, \gamma, \delta, \rho)\right]$$

Our sequential entry in order of heterogeneous profitability (ϕ_{it}) assumption ensures that the SPNE is unique.

(b) Simulating the data

See attached code.

(c) Moments

Using the conditional mean restrictions $\mathbb{E}\left[u_t | X_t, Z_{it}\right] = 0$ and $\mathbb{E}\left[\xi_t | X_t, Z_{it}\right] = 0$, we can construct the following moments:

$$\begin{aligned}
\mathbb{E}\left[\xi_{it} X_t\right] &= 0 \\
\mathbb{E}\left[u_t X_t\right] &= 0 \\
\mathbb{E}\left[\xi_{it} Z_{it}\right] &= 0 \\
\mathbb{E}\left[u_t Z_{it}\right] &= 0 \\
\mathbb{E}\left[\xi_{it}\right] &= 0 \\
\mathbb{E}\left[u_t\right] &= 0
\end{aligned}$$

The expectation is taken across potential entrants and markets, so that the dimension of each of the above expectations is 1. Note that we have 6 moments to estimate 5 parameters, so we are overidentified.

*Discussed with Joe Battles, Sean Kong, Feng Lin, Hugo Lopez, Haruka Uchida.

(d) Coding up the GMM objective

This is all in the attached code. However, we would like to make a few remarks:

1. To approximate the probability of entry of each firm and equilibrium number of firms in each market, we take $S = 500$ draws from the distribution of shocks.

2. We include all potential entrants in the construction of the moments. Berry (1992) uses a subset of potential entrants in each market (he chooses the 4 firms with the highest city output share), but this is to ensure that each market had the same number of I_{it} equations, since markets generally do not have the same number of potential entrants. In the case of this problem set, however, each market has the same number of potential entrants, so we do not need to choose a subset of firms for each market. Note that we are assuming that we observe Z_{it} even for firms that do not enter market t.

3. We set the weighting matrix equal to identity throughout the whole estimation. This is justified because we do not care about standard errors in this question.

4. When we optimize the GMM objective, we set the maximum number of iterations to 300. Every time we have tried the optimization, it seems like the maximum number of iterations is reached before convergence. We have tried increasing the number of iterations but to no avail. So, we decided to stick with 300 iterations for the sake of time.

(e) GMM objective plots around true parameter values

See figures 1-5 below. All the plots look as they should, expect for the plot for ρ: the objective is *maximized* rather than minimized at the true value of ρ[1]. There are also many local minima in this plot, which would lead to problems in the optimization (e.g., the estimate of ρ may get stuck at the wrong minimum).

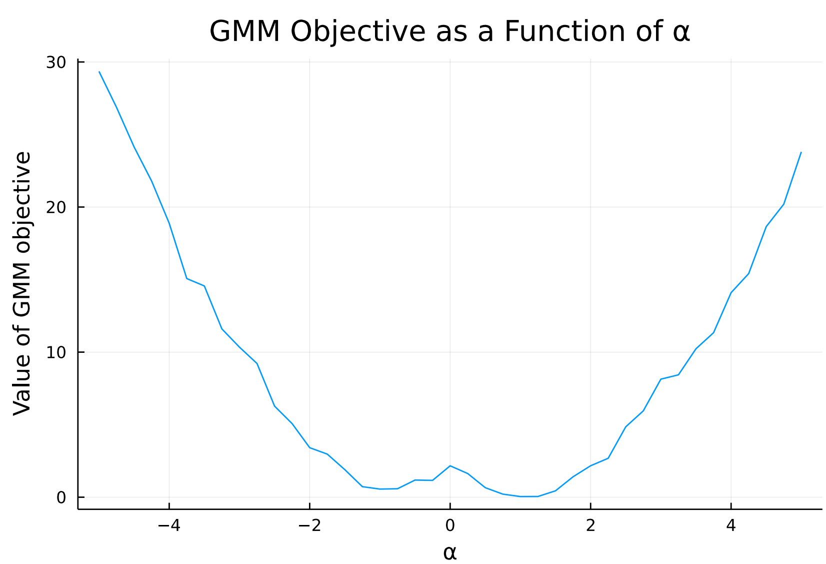

Figure 1: GMM Objective as a function of α (true $\alpha = 1$)

[1]We have spoken with other people about this and it seems that they have similar issues.

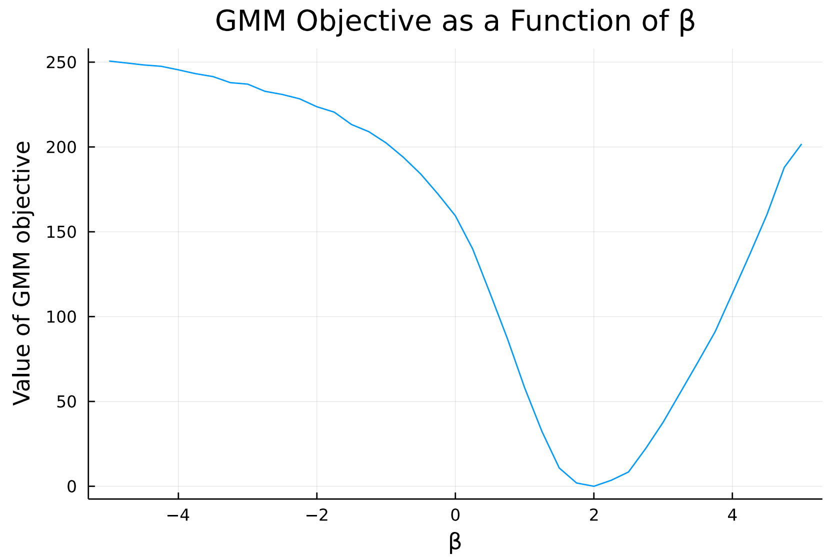

Figure 2: GMM Objective as a function of β (true $\beta = 2$)

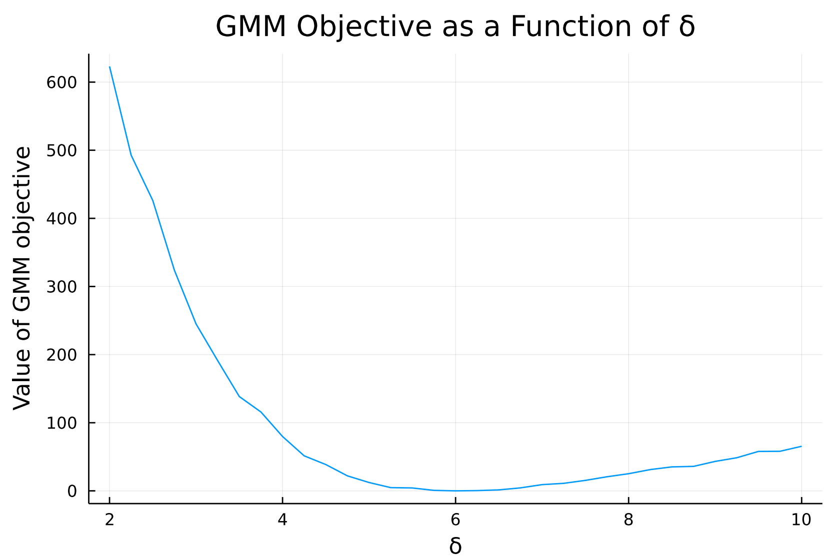

Figure 3: GMM Objective as a function of δ (true $\delta = 6$)

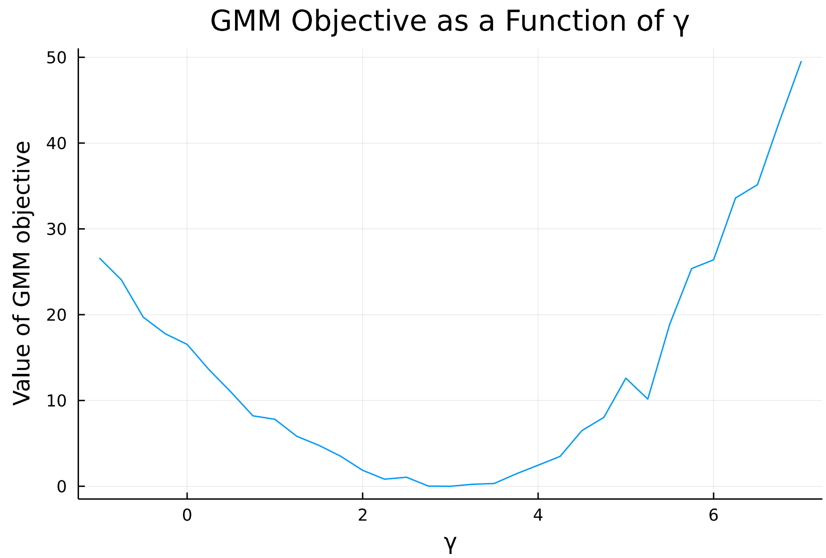

Figure 4: GMM Objective as a function of γ (true $\gamma = 3$)

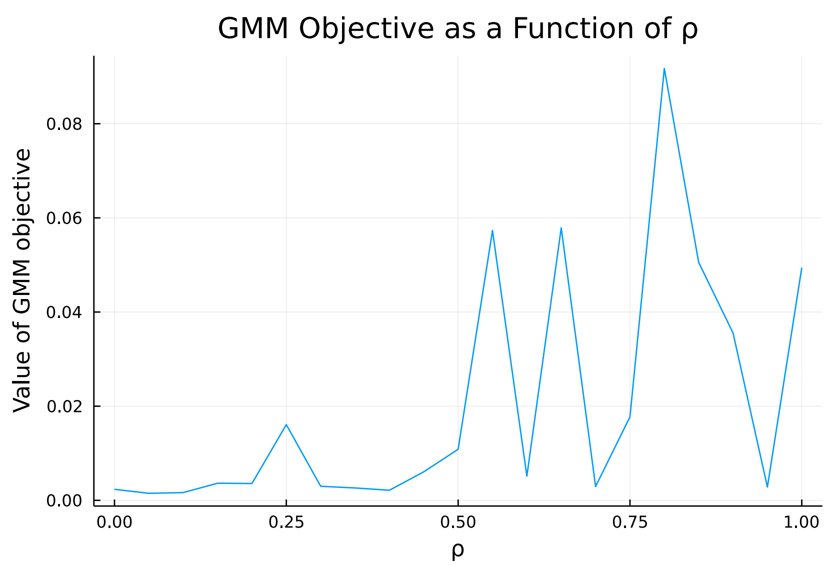

Figure 5: GMM Objective as a function of ρ (true $\rho = 0.8$)

(f),(g) GMM parameter estimates for different initial conditions

We tried 3 different sets of initial conditions:

1. $(\alpha, \beta, \delta, \gamma, \rho) = (1.1, 2.1, 6.2, 3.1, 0.85)$
2. $(\alpha, \beta, \delta, \gamma, \rho) = (1.5, 2.5, 7.0, 4.0, 0.95)$
3. $(\alpha, \beta, \delta, \gamma, \rho) = (0.9, 1.9, 5.8, 2.9, 0.75)$

For each of the initial conditions, we estimate the parameters 20 times. Figures 6-8 below show histograms (normalized) with a fitted density for these estimates. Generally, it looks like the initial conditions matter for the parameter estimates. When the initial conditions are further from the true parameters (figure 8), the resulting estimates are also not close to the true parameters. Note also that there were some issues with estimating ρ. ρ must be between 0 and 1, but some of the estimates were out of this range[2].

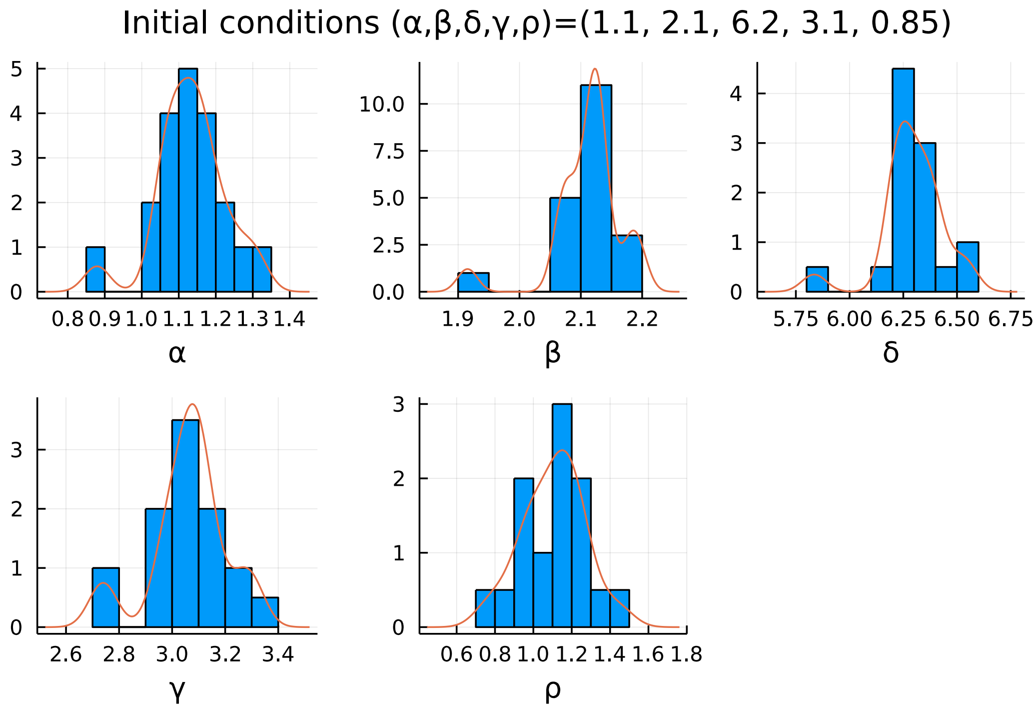

Figure 6: GMM estimates for initial conditions $(\alpha, \beta, \delta, \gamma, \rho) = (1.1, 2.1, 6.2, 3.1, 0.85)$

[2]In the code for the GMM objective, we replace ρ with 1 (0) if the previous iteration's estimate is greater than 1 (less than 0) so that when simulating the data there is never an issue with taking the square root of a negative number. However, this does not mean that the minimizing value of ρ estimated by the optimizer is between 0 and 1. Also, once again, other people seem to have had similar issues.

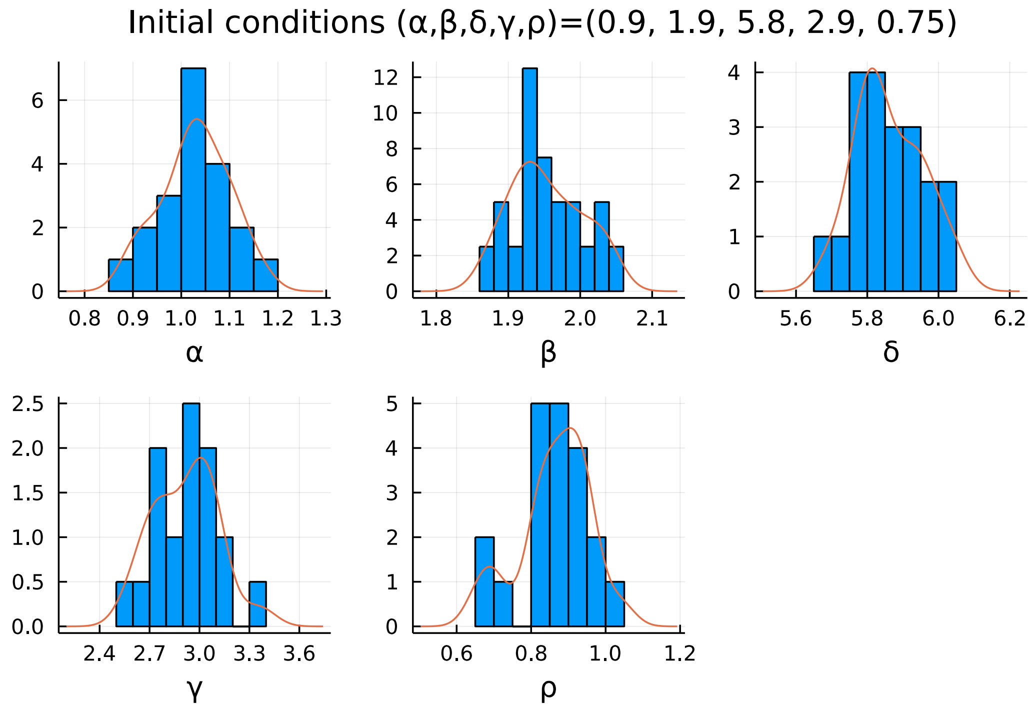

Figure 7: GMM estimates for initial conditions $(\alpha, \beta, \delta, \gamma, \rho) = (1.5, 2.5, 7.0, 4.0, 0.95)$

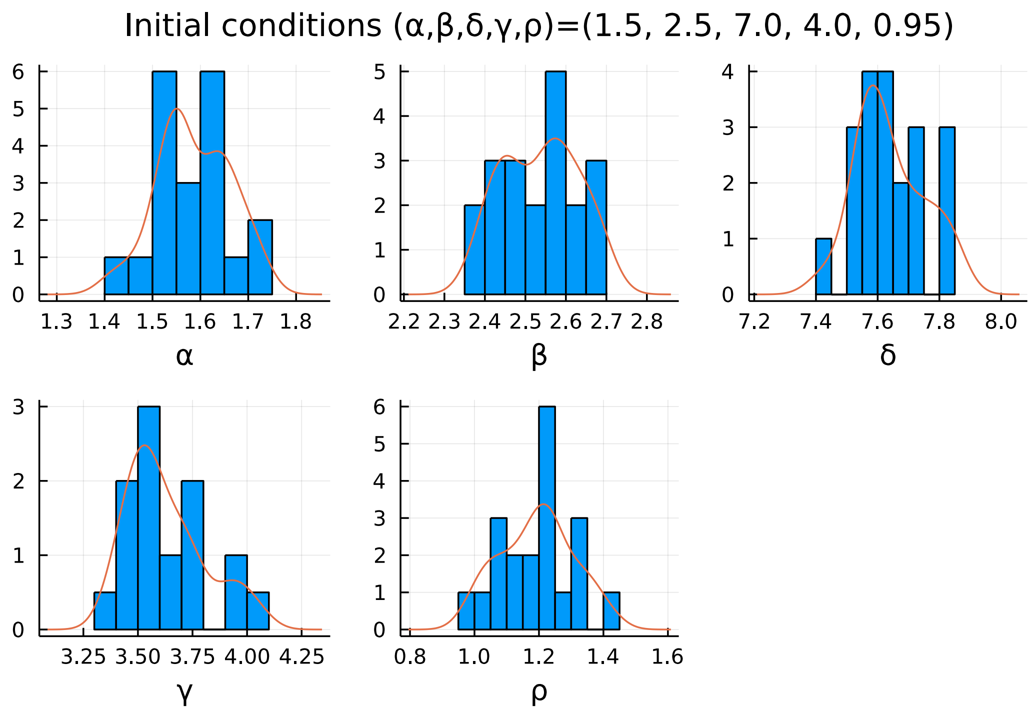

Figure 8: GMM estimates for initial conditions $(\alpha, \beta, \delta, \gamma, \rho) = (0.9, 1.9, 5.8, 2.9, 0.75)$

Additionally, we report our point estimates in table 1. Each estimate is an average of the 20 parameter estimates that we obtained.

Initial condition set	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\gamma}$	$\hat{\rho}$
1	1.133	2.110	6.293	3.062	1.105
2	1.029	1.953	5.862	2.918	0.863
3	1.587	2.536	7.639	3.627	1.194

Table 1: GMM parameter estimates for each set of initial conditions

References

Berry, Steven T., "Estimation of a Model of Entry in the Airline Industry," *Econometrica*, 1992, *60* (4), 889–917.