Advanced Industrial Organization II
Problem Set 2

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February 12, 2021

Due on February 26th. Please upload a zip file containing your code and a pdf with your answers to canvas. You may have one partner in completing this problem set.

1 Entry Game

There are two firms, $i = 1, 2$ considering whether to enter markets $t = 1, ..., T$. The firm would like to enter if and only if net utility of entry is non-negative:

$$y_{it} = 1\{-\delta y_{-i,t} + \alpha x_t + \epsilon_{it} \geq 0\} \tag{1}$$

The following subquestions (1.1, 1.2, 1.3) are independent and separate. The only thing they all share is equation (1).

1.1 Complete Information

The firms observe $(\epsilon_{1t}, \epsilon_{2t})$, but the econometrician does not observe these. $(\epsilon_{1t}, \epsilon_{2t})$ is distributed iid across t as f_ϵ. Find the lower bound and upper bound of $Pr(y_{1t} = 1, y_{2t} = 0 \,|\, x_t)$ in the spirit of Ciliberto and Tamer (2008).

1.2 Demand Estimation with Endogenous Entry

Suppose once firm i enters market t, the firm's market share S_{it} will be the following:

$$S_{it} = z_{it}\beta + \gamma p_{it} + \xi_{it} \tag{2}$$

The structural errors $(\epsilon_{1t}, \epsilon_{2t}, \xi_{1t}, \xi_{2t})$ are distributed iid across t as $f_{\epsilon\xi}$. z_{it} is exogenous product characteristics. p_{it} is the optimal price set by firm i if it enters the market. The econometrician observes the share and price of the firms that enter the market.

1. Can we get consistent estimates of β and γ from an OLS regression on equation (2)? Why or why not?

2. Suppose we have valid price instruments, can we get consistent estimates from an IV regression on equation (2)? Why or why not?

3. Can you find a lower bound and an upper bound of $Pr(\xi_{1t} \leq e \,|\, y_{1t} = 1, y_{2t} = 0, x_t, z_{1t}, z_{2t})$ for a given scalar e? (Hint: the realized market structure is determined by ϵ, given x and parameter values.)

1.3 Incomplete Information

Now let ϵ_{it} be the private information of the firm i, and all the other variables are common knowledge to both firms. In this setting they play a static Bayesian Nash equilibrium. The econometrician observes $(y_{1t}, y_{2t}, x_t)_t$. Assume $\epsilon \sim Logistic(0,1)$ is iid across firms and markets. $x_t \in \{1,2\}$ with $\Pr(x_t = 1) = .5$ is also independent.

1. What is firm i's strategy? What is the behavior of firm i from the other firm's point of view?

2. Compute all Bayesian Nash equilibria for $(\alpha, \delta) = (1,1), (3,6)$, respectively. Numerical solutions are fine.

3. From now on, let $(\alpha, \delta) = (3,6)$. Suppose both firms play the same symmetric strategy, Estimate (α, δ) in the same spirit of Seim (2006) and run a Monte Carlo Simulation to see whether it is consistent. Consider $T = 1000$ and $S = 50$ for the simulation. Fix a random seed for replicability. (Hint: In constructing the likelihood, one's probability to enter the market depends on the other's decision. It can be obtained by solving a fixed point problem. Note that, as observed in part 2, their strategies also depend on (α, δ) and x. The number of unknowns is less with the symmetry assumption.)

4. Suppose the firms do not necessarily use the symmetric strategy. The probability that equilibrium $k = 1, ..., K$ is selected, if the model exhibits multiple equilibria, is $\lambda_{it}^k(x_t) \propto \exp(k/2)$, where the equilibria are ordered in a way the firm 1's choice probability in equilibrium k, $p_{1t}^k(x_t)$, satisfies $p_{1t}^k(x) \geq p_{1t}^{k+1}(x)$ for all $k = 1, ..., K-1$ and given x. Propose an estimator of (α, δ) and run a Monte Carlo Simulation to see whether it is consistent. (Hint: Consider the equilibrium selection probabilities as mixture weights. You need to introduce auxiliary parameters to consistently estimate the parameters of interest. Note that players are aware of which equilibrium is selected. Although not required, it would be a good exercise to estimate parameters using the estimator in part 3.)

5. Let u_t be an iid binary variable with $\Pr(u_t = 1|x_t) = \frac{1}{1+x_t}$, which is unobserved to the econometrician. Let the equilibrium selection probabilities be $\lambda_{it}^k(x_t, u_t) \propto \exp((k + u_t)/2)$. Does the estimator you propose above give consistent estimates? Discuss.

2 Dynamics

2.1 The Data

In the file labeled `psetTwo.csv` you will find mileage data for the bus managed by one Harold Zurcher. For this exercise, there are no covariates.

2.2 The model

Each time period, Harold Zurcher chooses to perform maintenance on the bus, or to replace the engine. Let his flow utilities be given by the following function

$$u(x_t, d_t) + \epsilon_{a,t} = \begin{cases} -\theta_1 x_t - \theta_2 \left(\frac{x_t}{100}\right)^2 + \epsilon_{0,t} & \text{if } d_t = 0 \\ -\theta_3 + \epsilon_{1,t} \text{ if } d_t = 1 \end{cases} \tag{3}$$

Where x_t is the current mileage of the bus, d_t is the choice of Harold Zurcher, and θ is a vector of parameters.

Each choice also contains unobserved utility $\epsilon_{a,t}$ that are distributed independent T1EV.

Harold Zurcher maximizes his lifetime discounted utility, discounted by β, where the state x_t evolves according to

$$p(x_{t+1}|x_t, d_t) = \begin{cases} g(x_{t+1} - 0) & \text{if } d_t = 1 \\ g(x_{t+1} - x_t) & \text{if } d_t = 0 \end{cases} \tag{4}$$

That is, replacing the engine regenerates the mileage to 0.

2.3 Questions

1. How can you recover engine replacement from the mileage data? Store these decisions in a separate variable.
2. Discuss the conditional independence assumption.
3. Discretize the domain of x_t into K chunks. Estimate the Markov Transition probability $p(x_{t+1}|x_t, d_t)$. This should be two $K \times K$ stochastic matrices, depending on action d_t. Make your own choice of K.
4. Define the expected value function $EV(x, d) = \int V_\theta(y, \epsilon)p(d\epsilon)p(dy|x, d)$. Show the following fixed point equation holds:

$$EV(x, d) = \int \log \left(\sum_j \exp(u(y, j) + \beta EV(y, j)) \right) p(dy|x, d) \tag{5}$$

5. Derive the conditional choice probabilities using $EV(x, d)$ and θ.

2.3.1 Nested Fixed Point

1. Reduce the state space of EV using the regenerative property.
2. Rewrite the fixed point equation as a matrix equation.
3. Write a function that solves the fixed-point equation using Rust's poly-algorithm.
4. Write a function that computes the likelihood of the sample for any parameter θ.
5. Write a function that computes the the gradient of the likelihood for any parameter θ. Compare this against Automatic Differentiation.
6. Estimate the model parameters θ. Use $\beta = .999$

2.3.2 Nested Pseudo Likelihood (Optional)

1. Derive the inversion from CCPs to conditional value functions v_k.
2. Derive an equation for the smoothed value function as a function of θ, CCPs p_k, and Conditional Transition matrices F_d. (Solve the fixed point equation in smoothed value function for V_θ.)
3. Estimate the conditional choice probabilities non-parametrically.
4. Derive the sequential policy estimators using the likelihood of the choice probabilities given by the smoothed value function.
5. Solve this estimator once, using the CCPs as your initial P_0. Is this close to your NXFP estimates?
6. Iterate the sequential policy estimators 10 times. How close to the NXFP estimates are they?

2.3.3 BBL (Optional)

1. Write a function that implements Forward Simulation to approximate the value function $v(k, d|\theta)$ for any θ.
2. Use the Hotz-Miller inversion to derive the optimal choice rule $\delta(p_k, \epsilon)$ as a function of only CCPs and epsilon.
3. Write a function that computes the BBL loss function $g(x, \epsilon, \theta)$.

$$g(x, \epsilon, \theta) = v(x, \delta^c(x, \epsilon, \theta^*), \theta) + \epsilon(\delta^c(x, \epsilon, \theta^*)) - v(x, \delta(x, \epsilon, \theta^*), \theta) - \epsilon(\delta(x, \epsilon, \theta^*))$$

4. Use a frequency estimator to estimate the unconditional distribution of the states x_t.
5. Estimate θ using the BBL loss function, using Monte Carlo Integration for ϵ and x.

$$\theta \in \arg\min_\theta \int_x \int_\epsilon (\min\left\{g(x, \epsilon, \theta), 0\right\})^2 dH(x, \epsilon)$$