Advanced Industrial Organization II
Problem Set 1

DUE: February 2nd before class.

Please typeset your assignment in Latex. For the coding part feel free to use your preferred programming language. Please comment your code clearly, but not excessively, and use low-levels commands. The clarity of your code and the overall assignment will be evaluated. Submit your assignment through Canvas in a single zip file. You can work in a group of two and submit a single file per group.

Exercise 1

1) Let X_i with $i = 1, ..., N$ be a sequence of independent type 1 extreme value random variables with location parameter μ_i and scale parameter $\sigma > 0$ (T1EV(μ_i,σ)). The c.d.f is given by:

$$\Pr\{X_i \leq x|\mu_i, \sigma\} = \exp\left(-\exp\left(-\frac{x-\mu_i}{\sigma}\right)\right)$$

Derive the distribution of $Y = \max_i\{X_i\}$.

2) Let X and Y two independent T1EV random variables with location parameters μ_x and μ_y respectively and common scale parameter $\sigma > 0$. Derive the distribution of $X - Y$.

3) Consider an individual who has to choose one product among N possible alternatives. The utility derived from alternative j is given by:

$$u_j = \mu_j + \epsilon_j$$

where μ_j is non-random and ϵ_j are independent and identically distributed T1EV(0,1). Derive the probability that alternative j is chosen.

4) Consider a market with J products indexed by $j = 1, ..., J$, an outside good denoted by $j = 0$ and a large number of consumers indexed by $i \in \mathcal{I}$ each of whom only buys one of the products. Consumer i's indirect utility from consuming product j is given by:

$$
\begin{aligned}
u_{ij} &= \alpha(y_i - p_j) + \epsilon_{ij} \quad \text{for } j = 1, ..., J \\
u_{i0} &= \epsilon_{i0} \quad \text{for } j = 0
\end{aligned}
$$

where p_j is the price of product j, y_i is consumer i's income and ϵ_{ij} is an idiosyncratic taste shock that makes products horizontally differentiated.

i) Assume ϵ_{ij} are i.i.d T1EV(0,1). Denote consumer i's individual choice probability of selecting product j as $s_j(i)$. Derive $s_j(i)$ and compute $\frac{\partial s_j(i)}{\partial y_i}$. Interpret your results.

ii) Assume ϵ_{ij} are i.i.d T1EV(0,1). Derive the market share of product j, s_j and compute own and cross-price elasticities. Are the latter reasonable? Explain.

iii) Assume that $\epsilon_{ij} = \beta_i x_j$ where x_j, represents a non-random product characteristic that consumers value, and β_i represents an idiosyncratic taste shock for that same characteristic. Moreover, assume that $x_j > 0$, $x_0 = 0$.

   - Assume that $\beta_i \equiv \beta$ for all i. Derive product j market share, s_j. Interpret your results.
   - Assume that β_i are i.i.d Uniform $\left[0, \overline{\beta}\right]$ with $\overline{\beta}$ sufficiently large. Derive product j market share, s_j and compute own and cross-price elasticities. Are the latter reasonable? Explain and compare with your findings in points (ii) above. (For simplicity assume that $\frac{p_i - p_j}{x_i - x_j} \geq \frac{p_j - p_k}{x_j - x_k}$ whenever $x_i \geq x_j \geq x_k$)

iv) Assume that $\epsilon_{ij} = \beta_i x_j + \nu_{ij}$ where x_j represents a non-random product characteristic, β_i represents an idiosyncratic taste shock for that same characteristic and v_{ij} are i.i.d T1EV(0,1). Moreover, assume that β_i are i.i.d with generic c.d.f $F(\cdot)$. Derive product j market share and compute own and cross-price elasticities. Explain and compare with your findings in points (ii) above.

v) Assume, as in point (i) above, that ϵ_{ij} are i.i.d T1EV(0,1). Moreover, suppose we want to measure welfare at given prices $(p_1, ..., p_J)$ as

$$
W \equiv \mathbb{E}\left[\max_{j=0,\dots,J} u_{ij}\right].
$$

- Rewrite W as a function of the market share of the outside option s_0.[1]
- Suppose that a new product $J+1$ is introduced in the market. What happens to W? Interpret your results.

Exercise 2

The file ps1_ex2.csv contains data on $I = 4000$ individual choices among $J = 30$ products and an outside good denoted by $j = 31$. Assume individual i's indirect utility of consuming product j is given by:[2]

$$
\begin{aligned}
u_{ij} &= x_j'\beta + \xi_j + d_i'\Gamma x_j + \epsilon_{ij} \quad \text{for } j = 1, ..., 30 \\
u_{ij} &= \epsilon_{ij} \quad \text{for } j = 31,
\end{aligned}
$$

where x_j is a K dimensional vector of product characteristics, ξ_j is an unobserved product characteristic, d_i is an L dimensional vector of individual observable demographics and ϵ_{ij} is an i.i.d $T1EV(0,1)$ taste shock. Our goal is to estimate the coefficients on product characteristics enclosed in the K dimensional vector β and all interaction coefficients between product characteristics and individual demographics enclosed in the $(L \times K)$ matrix Γ.

i) What are the coefficients in β capturing? What about the coefficients in Γ?

ii) Denote by $\delta_j \equiv x_j'\beta + \xi_j$ the mean utility of product j (where clearly $\delta_{31} = 0$). Write down the likelihood of the data as a function of the parameters (δ, Γ) where $\delta = (\delta_j)_{j=1}^J$.

iii) Derive the FOC of the likelihood with respect to δ_j. Interpret your result.

iii) Derive the FOC of the likelihood with respect to Γ. Interpret your results.

iv) Obtain MLE estimates for (δ, Γ).

v) Exploiting your MLE estimates for the product specific intercepts $\hat{\delta}_{MLE}$ you now want to estimate β. Propose a moment condition that (if true) would allow you to consistently estimate β.

vi) Use the proposed moment condition to obtain an estimate of β.

[1] Recall that if $X_i \sim T1EV(\mu, \sigma)$, then $\mathbb{E}[X_i] = \mu + \sigma k$ where k is the Euler-Mascheroni constant.

[2] Note that in the data $x_{31} = 0$.

Exercise 3

The file ps1_ex3.csv contains aggregate data on a large number $T = 1000$ of markets in which $J = 6$ products compete between each other together with an outside good $j = 0$. The utility of consumer i is given by:

$$\begin{aligned} u_{ijt} &= -\alpha p_{jt} + \beta x_{jt} + \xi_{jt} + \epsilon_{ijt} \quad j = 1, ..., 6 \\ u_{i0t} &= \epsilon_{i0t} \end{aligned}$$

where p_{jt} is the price of product j in market t, x_{jt} is an observed product characteristic, ξ_{jt} is an unobserved product characteristic and ϵ_{ijt} is i.i.d T1EV(0,1). Our goal is to to estimate demand parameters (α, β) and perform some counterfactual exercise.

i) Assuming that the variable z in the dataset is a valid instrument for prices, write down the moment condition that allows you to consistently estimate (α, β) and obtain an estimate for both parameters.

ii) For each market, compute own and cross-product elasticities. Average your results across markets and present them in a $J \times J$ table whose (i, j) element contains the (average) elasticity of product i with respect to an increase in the price of product j. What do you notice?

iii) Using your demand estimates, for each product in each market recover the marginal cost c_{jt} implied by Nash-Bertrand competition. For simplicity, you can assume that in each market each product is produced by a different firm (i.e., there is no multi-products firms). Report the average (across markets) marginal cost for each product. Could differences in marginal costs explain the differences in the average (across markets) market shares and prices that you observe in the data?

iv) Suppose that product $j = 1$ exits the market. Assuming that marginal costs and product characteristics for the other products remain unchanged, use your estimated marginal costs and demand parameters to simulate counterfactual prices and market shares in each market. Report the resulting average prices and shares.

v) Finally, for each market compute the change in firms profits and in consumer welfare following the exit of firm $j = 1$. Report the average changes across markets. Who wins and who loses?

Exercise 4

The file ps1_ex4.csv contains aggregate data on $T = 600$ markets in which $J = 6$ products compete between each other together with an outside good $j = 0$. The utility of consumer i is given by:

$$
\begin{aligned}
u_{ijt} &= \tilde{x}'_{jt}\beta + \xi_{jt} + \tilde{x}'_{jt}\Gamma v_i + \epsilon_{ijt} \quad j = 1, ..., 6 \\
u_{i0t} &= \epsilon_{i0t}
\end{aligned}
$$

where x_{jt} is a vector of observed product characteristics including the price, ξ_{jt} is an unobserved product characteristic, v_i is a vector of unobserved taste shocks for the product characteristics and ϵ_{ijt} is i.i.d T1EV(0,1). Our goal is to to estimate demand parameters (β, Γ) using the BLP algorithm. As you can see from the data there are only two characteristics $\tilde{x}_{jt} = (p_{jt}, x_{jt})$, namely prices and an observed measure of product quality. Moreover, there are several valid instruments z_{jt} that you will use to construct moments to estimate (α, Γ). Finally, you can assume that Γ is lower triangular e.g.,

$$
\Gamma = \begin{bmatrix} \gamma_{11} & 0 \\ \gamma_{21} & \gamma_{22} \end{bmatrix}
$$

such that $\Gamma\Gamma' = \Omega$ is a positive definite matrix and that v_i is a 2 dimensional vector of i.i.d random taste shocks.

i) Assume that $v_i \sim N(0, I)$ so that $\Gamma v_i \sim N(0, \Omega)$. Implement the BLP routine to estimate (β, Γ)

ii) For each market, compute cross and own product elasticities. Average your results across markets and present them in a $J \times J$ table whose (i, j) element contains the (average) elasticity of product i with respect to an increase in the price of product j. What's the main difference when compared with the table of elasticities you found in Exercise 3 point (ii)?

iii) Look at the average (across markets) prices, shares and observed quality of the products you observe in the data. Based on your estimated Γ, what do you think could be driving differences in prices and market shares?

iv) (Optional) compare your results with PyBLP (This does not need to be done in python, you can call pyBLP from R, matlab, and Julia.)