Advanced Industrial Organization II
Problem Set 1

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Due on February 3rd before class starts. Please upload a zip file containing your code and a pdf with your answers to canvas. You can have one partner in completing this problem set.

1 The Data

In the file labeled `psetOne.csv` you will find a .csv containing cross-sectional data for markets of motorcycles. In each market there are potentially three brands, the characteristics for the bike are price, engine size, bike type, and brand FEs. You will use this data to estimate demand parameters for the motorcycle market.

The columns labeled $z[1-4]$ are valid, exogenous instruments.

2 The model

Consider a unit mass of consumers choosing over products $j \in \{0, 1, 2, ..., J\}$ in markets $t = 1, 2, ..., T$. Each consumer i's indirect utility of purchasing product j is:

$$u_{ijt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \tag{1}$$

where x_{jt} is observable product characteristics and p_{jt} is price. ξ_{jt} is unobserved (by researchers) product characteristics. ϵ_{ijt} is unobserved (by researchers) product/consumer match value that is distributed iid as T1EV. Product 0 is the outside option. We normalize the utility of choosing the outside option as $u_{i0t} = \epsilon_{i0t}$. Let θ denote (α, β).

1. Derive the aggregate market share s_{jt} for product j in market t.
2. Derive the elasticity matrix for a market with J goods. Describe the IIA property and the associated concerns. What are some possible remedies?
3. Suppose the researchers observe market shares s_{jt} in the data. Are observed zero market shares consistent with the model we have just described? If not, why and discuss some potential solutions.
4. Define mean utilities $\delta_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}$. Show that δ_{jt} can be written as a function of market shares by inversion. Discuss to what extent does the inversion depend on the functional form assumptions.

5. Are the OLS estimates based on the computed mean utilities above consistent for demand parameters α and β? If not, propose solutions on how to get consistent estimates.

6. What is ξ in this setting? Discuss the validity of both Haussman and BLP instruments in this setting. Regardless of your answers, do not include these as additional instruments for computation.

7. Consider a firm that potentially sells multiple products in a market where price is set simultaneously by each firm. Derive the FONCs for the firm. Combine these FONC's for all firms for a fixed-point equation describing the equilibrium.

8. Open the data set. Consider market $t = 17$. Ignore cost data, setting $MC = 0$, simulate values for ξ such that $\xi \sim \mathcal{N}(0, 1)$. Use $\theta = [-3, 1, 1, 2, -1, 1]$ to obtain the share equation for each good and solve the fixed-point equation for the optimal prices. Interpret these prices as markups, and describe any patterns at the firm level. Is this related to IIA?

Now we allow for preference heterogeneity, so the preference parameters are β_i and α_i. Assume the following distribution

$$\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \bar{\alpha} \\ \bar{\beta} \end{pmatrix}, \mathrm{diag}(\sigma_\alpha^2, \sigma_\beta^2) \right) \qquad \boldsymbol{\theta} = [\alpha, \boldsymbol{\beta}, \boldsymbol{\sigma}]$$

For identification purposes, only σ_α and $\sigma_{\beta_{cc}}$ are non-zero. (cc denotes the characteristic labeled engine size).

9. Write a share prediction function sHat (Do not be afraid to use multiple functions). This function should take $\boldsymbol{\delta}, X, \boldsymbol{\sigma}, \zeta, I$ as inputs and return a vector of predicted shares $\hat{s}$. Verify this function for these test cases.
   - $\boldsymbol{\sigma} = 0$, $\boldsymbol{\delta} = 0$, $J = 3$
   - $\boldsymbol{\sigma} = 0$, $J = 3$, $\delta_1 = 40$, $\boldsymbol{\delta}_{-1} = 20$. (Shares should not be one)
   - $\boldsymbol{\sigma} = .1$, $\boldsymbol{\delta} = 0$, $J = 3$, $I = 20$, $X = 0$. How much did this change your shares? Try for other values of X.

10. Write a share inversion function (Do not be afraid to use multiple functions).

    This function should take $s, X, I, \boldsymbol{\sigma}, \zeta, J$ as inputs and return a δ such that $\hat{s}(\delta, X, \boldsymbol{\sigma}, \zeta, I) = s$.

    Verify this function using the shares that were output from your test cases previously.

11. Write the objective function for the estimation problem in terms of an implicit function $\xi(X, \theta, s)$. Write the function so that it can take any arbitrary weighting matrix W. What is the objective value for $\boldsymbol{\theta} = 0$, using the weighting matrix $W = (ZZ')^{-1}$?

12. Write a gradient function. (Do not be afraid of making this multiple functions). Verify your gradient using Forward-Stepwise Automatic differention. If your programming language does not support AD, use finite differences, but expect error. For Julia, use `ForwardDiff`; for Python use `Jax`.

13. Estimate your results using Two-stage GMM, beginning with the TSLS weighting matrix. Report point estimates as well as the standard errors.

14. Compare your results to pyBLP (This does not need to be done in python, you can call pyBLP from R, matlab, and Julia.)

15. For market $t = 17$, print the elasticity matrix. Is IIA still present? How close are we to IIA? Was this worth the effort of adding hetereogeneity? Feel free to use your supply-side results from the homogenous section.