

## Advanced Quantitative Marketing: Switching Cost Assignment

In this assignment you will solve a model of purchasing in the presence of a loyalty or reward program. The model follows Hartmann and Viard (2008). In each period  $t = 0, 1, \dots$  a consumer decides whether to buy a product or service. Purchase is denoted by  $a_t = 1$ , and  $a_t = 0$  indicates the choice of the outside option. We assume that purchase coincides with consumption and that the payoff from purchase is  $\delta + \epsilon_{1t}$ , where  $\epsilon_{1t}$  is a latent utility draw. The purchase price is constant, and hence one can think of  $\delta$  as the product valuation net of price,  $\delta = \tilde{\delta} - p$ . The payoff when not making a purchase is  $\epsilon_{0t}$ . All  $\epsilon_{jt}$  draws are independent and Type I Extreme Value distributed.

The consumer accumulates reward points (credits) when making a purchase.  $x_t$  denotes the number of reward points that a consumer holds at the beginning of period  $t$ . If the consumer has accumulated  $x_t = N$  rewards in the past and she makes another purchase then she receives a reward with value  $r$ . Hence, if she decides to purchase when  $x_t = N$  her payoff is  $\delta + r + \epsilon_{1t}$ . Upon making a purchase the consumer receives one credit, and  $x_{t+1} = x_t + 1$  if  $x_t < N$ . However, if the consumer receives a reward the balance of credits resets to zero ( $x_{t+1} = 0$  if  $x_t = N$  and  $a_t = 1$ ).

Consumers are forward looking and maximize the expected present discounted value of payoffs.  $\beta < 1$  is the discount factor.

Your task is to numerically solve and analyze this dynamic purchase decision problem.

1. Verify that this problem is a special case of a dynamic discrete choice problem as defined in class. Correspondingly state the specific form of the utility function  $u_j(x)$  and the transition probability  $f(x'|x, a)$ .
2. Numerically solve for the choices-specific value functions on the state space  $\mathbb{X}$ . Your code should allow for a solution based on arbitrary parameter values  $\beta, \delta, r, N$ .
3. Following Hartmann and Viard (2008) define the switching cost

$$c(x) \equiv (v_1(x) - v_0(x)) - (v_1(0) - v_0(0)).$$

Explain this definition, and in particular why  $v_1(x) - v_0(x)$  would not be a suitable measure of switching costs.

4. Define the *type* of a consumer as  $\delta$ . Correspondingly, high types are consumers with large product valuations, and low types are consumers who value the product less. A central claim in Hartmann and Viard (2008) is that high types generally face lower switching costs than low types. Re-examine this claim based on the numerical model solution and an examination of the switching cost function  $c(x)$  (plotting  $c(x)$  helps!). Can you find

violations of the claim for specific parameter values? How is the answer to this question affected by the value of the discount factor  $\beta$ ?

5. Let  $\sigma_j(x)$  be the conditional choice probability (CCP) of action  $j$  given state  $x$ . Exploiting the form of the implied CCP's by the Type I Extreme Value assumption on  $\epsilon_{jt}$  show that

$$\log(\sigma_1(x)) - \log(\sigma_0(x)) = v_1(x) - v_0(x).$$

Correspondingly, re-express the switching cost as  $\exp(c(x))$  and discuss the interpretation of this measure.

6. Instead of analyzing type-specific switching costs based on the whole schedule of switching costs  $c(x)$  try to summarize  $c(x)$  across states for different types. Think of the best way to quantify the average switching cost faced by a specific consumer, and again re-examine if higher types generally face lower switching costs than lower types based on this average. Instead of characterizing consumers by their type  $\delta$  one can also characterize them by their average (in a well-defined sense) CCP  $\sigma_1(x)$  or by the ratio  $\sigma_1(x)/\sigma_0(x)$ . Repeat your analysis of average switching costs for different consumers types, where consumer type is now measured using the CCPs.
7. Summarize your conclusions from questions 1-6 in a write-up. Show supporting figures and/or tables. Be brief and concise.

## References

- HARTMANN, W. R., AND V. B. VIARD (2008): "Do frequency reward programs create switching costs? A dynamic structural analysis of demand in a reward program," *Quantitative Marketing and Economics*, 6(2), 109–137.