Winter 2021 BUS 37904 Professor Jean-Pierre Dubé Professor Günter J. Hitsch

## Advanced Quantitative Marketing: Switching Cost Assignment

In this assignment you will solve a model of purchasing in the presence of a loyalty or reward program. The model follows Hartmann and Viard (2008). In each period t = 0, 1, ... a consumer decides whether to buy a product or service. Purchase is denoted by  $a_t = 1$ , and  $a_t = 0$  indicates the choice of the outside option. We assume that purchase coincides with consumption and that the payoff from purchase is  $\delta + \epsilon_{1t}$ , where  $\epsilon_{1t}$  is a latent utility draw. The purchase price is constant, and hence one can think of  $\delta$  as the product valuation net of price,  $\delta = \tilde{\delta} - p$ . The payoff when not making a purchase is  $\epsilon_{0t}$ . All  $\epsilon_{jt}$  draws are independent and Type I Extreme Value distributed.

The consumer accumulates reward points (credits) when making a purchase.  $x_t$  denotes the number of reward points that a consumer holds at the beginning of period t. If the consumer has accumulated  $x_t = N$  rewards in the past and she makes another purchase then she receives a reward with value r. Hence, if she decides to purchase when  $x_t = N$  her payoff is  $\delta + r + \epsilon_{1t}$ . Upon making a purchase the consumer receives one credit, and  $x_{t+1} = x_t + 1$  if  $x_t < N$ . However, if the consumer receives a reward the balance of credits resets to zero  $(x_{t+1} = 0)$  if  $x_t = N$  and  $a_t = 1$ .

Consumers are forward looking and maximize the expected present discounted value of payoffs.  $\beta < 1$  is the discount factor.

Your task is to numerically solve and analyze this dynamic purchase decision problem.

- 1. Verify that this problem is a special case of a dynamic discrete choice problem as defined in class. Correspondingly state the specific form of the utility function  $u_j(x)$  and the transition probability f(x'|x,a).
- 2. Numerically solve for the choices-specific value functions on the state space  $\mathbb{X}$ . Your code should allow for a solution based on arbitrary parameter values  $\beta$ ,  $\delta$ , r, N.
- 3. Following Hartmann and Viard (2008) define the switching cost

$$c(x) \equiv (v_1(x) - v_0(x)) - (v_1(0) - v_0(0)).$$

Explain this definition, and in particular why  $v_1(x) - v_0(x)$  would not be a suitable measure of switching costs.

4. Define the type of a consumer as  $\delta$ . Correspondingly, high types are consumers with large product valuations, and low types are consumers who value the product less. A central claim in Hartmann and Viard (2008) is that high types generally face lower switching costs than low types. Re-examine this claim based on the numerical model solution and an examination of the switching cost function c(x) (plotting c(x) helps!). Can you find

- violations of the claim for specific parameter values? How is the answer to this question affected by the value of the discount factor  $\beta$ ?
- 5. Let  $\sigma_j(x)$  be the conditional choice probability (CCP) of action j given state x. Exploiting the form of the implied CCP's by the Type I Extreme Value assumption on  $\epsilon_{jt}$  show that

$$\log(\sigma_1(x)) - \log(\sigma_0(x)) = v_1(x) - v_0(x).$$

Correspondingly, re-express the switching cost as  $\exp(c(x))$  and discuss the interpretation of this measure.

- 6. Instead of analyzing type-specific switching costs based on the whole schedule of switching costs c(x) try to summarize c(x) across states for different types. Think of the best way to quantify the average switching cost faced by a specific consumer, and again re-examine if higher types generally face lower switching costs than lower types based on this average. Instead of characterizing consumers by their type  $\delta$  one can also characterize them by their average (in a well-defined sense) CCP  $\sigma_1(x)$  or by the ratio  $\sigma_1(x)/\sigma_0(x)$ . Repeat your analysis of average switching costs for different consumers types, where consumer type is now measured using the CCPs.
- 7. Summarize your conclusions from questions 1-6 in a write-up. Show supporting figures and/or tables. Be brief and concise.

## References

HARTMANN, W. R. AND V. B. VIARD (2008): "Do frequency reward programs create switching costs? A dynamic structural analysis of demand in a reward program," *Quantitative Marketing and Economics*, 6, 109–137.