Assignment 1 Machine Learning 10-701

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1 Probability Review [Ahmed]

1.1 Why just 2 variables? Let's go for 3

1.1.1

From the law of conditional probability,

$$\frac{\Pr(A, B \mid C)}{\Pr(B \mid C)} = \frac{\Pr(A, B, C)}{\Pr(C)} \frac{\Pr(C)}{\Pr(B, C)}$$

$$= \frac{\Pr(A, B, C)}{\Pr(B, C)}$$

$$= \frac{\Pr(A, B, C)}{\Pr(B, C)}$$

$$= \frac{\Pr(A, D)}{\Pr(D)}$$

$$= \Pr(A \mid D)$$

$$= \Pr(A \mid B, C)$$

1.1.2

$$\sum_{B} \Pr(A, B \mid C) = \sum_{B} \frac{\Pr(A, B, C)}{\Pr(C)}$$

$$= \frac{\Pr(A, C)}{\Pr(C)}$$

$$= \Pr(A \mid C)$$

1.1.3

Using result from Problem 1.1.1 and Problem 1.1.2,

$$\sum_{B} \Pr(A \mid B, C) \Pr(B \mid C) = \sum_{B} \Pr(A, B \mid C)$$

$$= \Pr(A \mid C)$$

1.2 Evaluating Test Results

1.2.1

The probability that a transation succeeds given that it was handled by A2 is

$$\Pr(Success \mid A = 2) = \frac{\Pr(Success, A = 2)}{\Pr(A = 2)}$$

$$= \frac{|Success, A = 2|}{|A = 2|}$$

$$= \frac{2150}{2150 + 500}$$

$$= 0.811$$

1.2.2

If we recommend A2 then we need to see that

$$\Pr\left(Success \mid A=2\right) \geq \Pr\left(Success \mid A=1\right)$$

$$0.811 \geq \frac{6000}{6000+1700}$$

$$0.811 \geq 0.779$$

1.2.3

The statement about the probability of *A*2 handling a transaction successfully given *A*1 handled it successfully is given by,

$$\Pr(A2_{success} = 1 \mid A1_{success} = 1) = \frac{\Pr(A2_{success} = 1, A1_{success} = 1)}{\Pr(A1_{success} = 1)}$$

$$\geq \frac{\Pr(A2_{success} = 1) + \Pr(A1_{success} = 1) - 1}{\Pr(A1_{success} = 1)}$$

$$= \frac{\frac{2150}{2150 + 500} + \frac{6000}{6000 + 1700} - 1}{\frac{6000}{6000 + 1700}}$$

$$= 0.757$$

$$\leq 0.70$$

1.3 Monty Hall Problem

Solving for Pr (car3 | open2, choose1) we get,

$$\begin{split} \Pr\left(\textit{car3} \mid \textit{open2}, \textit{choose1}\right) &= \frac{\Pr\left(\textit{car3}, \textit{open2} \mid \textit{choose1}\right)}{\Pr\left(\textit{open2} \mid \textit{choose1}\right)} \\ &= \frac{\Pr\left(\textit{open2} \mid \textit{choose1}, \textit{car3}\right) \Pr\left(\textit{car3} \mid \textit{choose1}\right)}{\Pr\left(\textit{open2} \mid \textit{choose1}\right)} \\ &= \frac{1 \times \frac{1}{3}}{\frac{1}{2}} \\ &= \frac{2}{3} \end{split}$$

and solving for Pr (car1 | open2, choose1) we get,

$$\begin{split} \Pr\left(\textit{car1} \mid \textit{open2}, \textit{choose1}\right) &= \frac{\Pr\left(\textit{car1}, \textit{open2} \mid \textit{choose1}\right)}{\Pr\left(\textit{open2} \mid \textit{choose1}\right)} \\ &= \frac{\Pr\left(\textit{open2} \mid \textit{choose1}, \textit{car1}\right) \Pr\left(\textit{car1} \mid \textit{choose1}\right)}{\Pr\left(\textit{open2} \mid \textit{choose1}\right)} \\ &= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} \\ &= \frac{1}{3} \end{split}$$

which gives us,

$$\frac{\Pr\left(car3\mid open2, choose1\right)}{\Pr\left(car1\mid open2, choose1\right)} = \frac{\frac{2}{3}}{\frac{1}{3}}$$

$$= 2$$

- 2 Regression [Leila]
- 2.1 Linear Regression
- 2.2 Ridge Regression

- 3 Classification [Dougal]
- 3.1 Drawing decision boundaries
- 3.2 Defeating classifiers

4 Coding Competition [Carlton]