NuCypher KMS: Mining

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This paper describes mining mechanisms and economics in NuCypher KMS. It includes inflation rates, mechanisms to incentivize long-term stakers and estimates of the number of tokens generated by nodes running in typical modes. Also, optimal strategies for stakers who may be affected by market volatility are proposed.

I. MOTIVATION

In the future, NuCypher KMS nodes will be exclusively paid via network fees. But initially, when adoption of the network is relatively low, miners running the nodes necessary for network operation will need to be subsidized via an inflation schedule.

The distribution of miner compensation should have the following properties:

- All the inflation is distributed to stakers who run the nodes, proportional to their stake;
- The amount of work (and, hence, fees earned) is proportional to stake;
- Stakers are incentivized (by a higher compensation rate) to run long-term nodes;
- High inflation doesn't depreciate the price in order to keep liquidity for new stakers;
- Stakers are incentivized to stay online all the time.

In the paper we address all these points, calculate expected earnings of miners who run nodes and devise optimal mining strategies.

II. HISTORICAL EXAMPLES OF INFLATION

Let's review inflation schedules of different cryptocurrency projects: DASH [1] and ZCash [2].

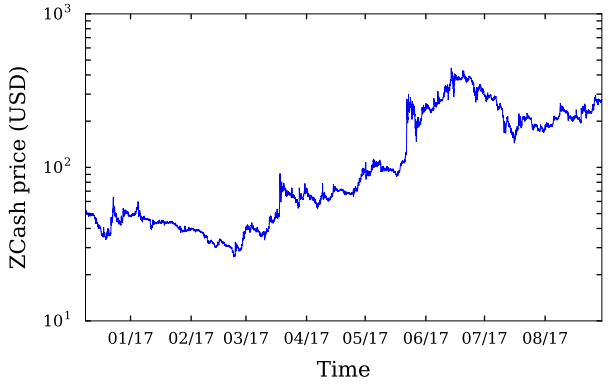

FIG. 1: Historical price of ZCash in logarithmic scale. Note the minimum at 23 Feb 2017

DASH has a hybrid Proof-of-Work (POW) and Proof-of-Stake (POS) model, with 45% of inflation going to POW miners, 45% to staking master nodes, and 10% reserved for budget proposals [3]. After the first year, its emission was 18.42% APR, decreasing by 1/14 every 383 days. With this setting, 60% of DASH coins are locked in masternodes for staking. It's unclear how inflation rate affects the price (and if it does here), but the useful data point is that there are 60% of coins locked for staking. Perhaps, that is a reasonalbe starting point to expect in a network where staking is an option.

ZCash is very interesting because it started with an extremely high inflation rate. This caused a short-term price drop (even though the market capitalization was growing) (Fig. 1). But on 23 Feb 2017, the price started going up. ZCash block rewards yield 50 ZEC every 10 min, and ZEC supply at Feb 23 was 727k ZEC. This corresponds to 360% APR. It is even more remarkable given the fact that miners who mined ZEC are likely

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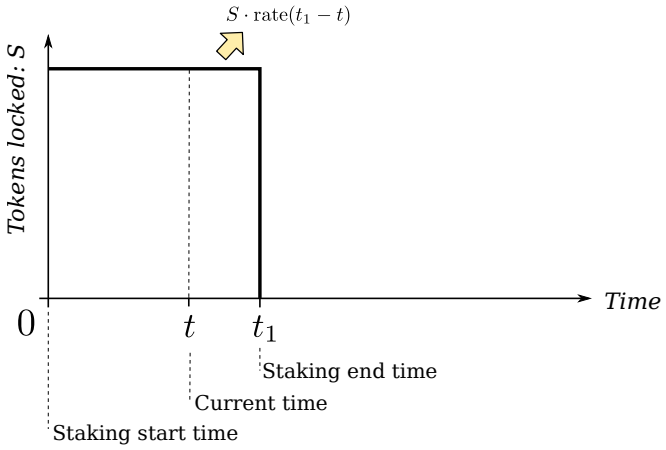
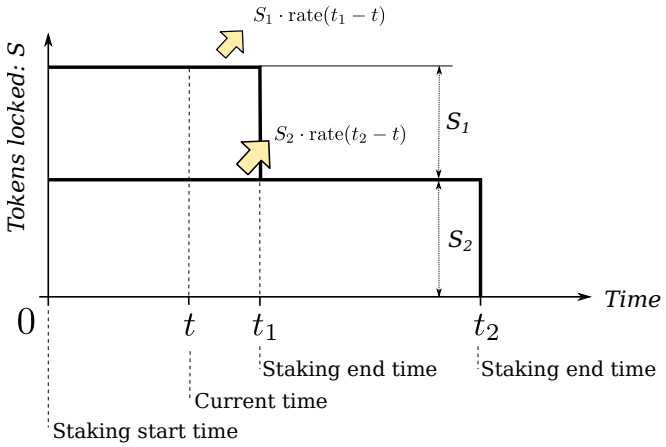


(a) Unlocking in a single step



(b) Splitting into two steps

FIG. 2: When staked, an unlock time t_1 is specified. At any time the unlock time can be increased (but not decreased). The stake can optionally be split into two parts where only one part is extended to t_2.

selling and exchanging the proceeds into other currencies to cover expenses. This gives us information about the maximum allowable inflation which doesn't create too much downward pressure on the price.

III. MINING PROTOCOL

A miner commits to stay available for at least time T. For that, they specify an unlocking time t_1, where minimal lock time $t_1 - t$ should be not less than $T_{\min} =$ 1 month. The number of coins locked for staking l should be no less than $S_{\min}$.

At any point, a miner can split their stake or any piece of their stake into two pieces (Fig. 2). The size of each piece should be not less than $S_{\min}$. The reason miners may want to split their stake is because their compensation rate depends on the lock time T, which will be discussed in more details later. At any point the miner can increase (but not decrease) T and add more tokens to their stake.

IV. GENERAL INFLATION PROPERTIES

A. Initial inflation

Let's assume that NuCypher KMS will have the same number of tokens locked as DASH: $\lambda = 60\%$. Thus, we'll have $1 - \lambda = 40\%$ in circulation. If the inflation rate is I, then the adjusted inflation rate (i.e. inflation as if the locked tokens didn't exist) of tokens in circulation will be:

$$I^* = \frac{I}{1-\lambda}, \tag{1}$$

and we should be comparing I^* with historical examples of inflation. If we take $I^* = 350\%$ (turnover point of ZCash price in an overall bullish market), the corresponding inflation I will be 140% APR.

To err on the side of caution, we set the starting inflation to be $I_0 = 100\%$ APR (or, in other words, 1/365 per day).

B. Inflation decay

Initially, inflation subsidizes mining, but payments for re-encryption services will generate the majority or all of the revenues of miners in the long run. If all miners have the same, maximum compensation rate, we choose the inflation rate to decay by factor of 2 in $T_{1/2} = 2$ years. The inflation, depending on time passed from the Genesis t, looks like:

$$I(t) = I_0 \cdot 2^{-\frac{t}{T_{1/2}}} = I_0 \exp\left[-\ln 2 \frac{t}{T_{1/2}}\right]. \tag{2}$$

In this case, the dependence of the token supply on the time t is:

$$S(t) = S_0 + \int_0^t I(t)\,dt = S_0 + \frac{I_0 T_{1/2}}{\ln 2}\left[1 - 2^{-\frac{t}{T_{1/2}}}\right], \quad (3)$$

Let's call relative initial annual inflation i_0, and then $I_0 = i_0 S_0$. For 100% APR, $i_0 = 1$ and $I_0 = S_0$ per year, and the maximum number of tokens which will ever be created is:

$$S_{\max} = S(\infty) = S_0\left(1 + \frac{i_0 T_{1/2}}{\ln 2}\right) \approx 3.89\,S_0, \quad (4)$$

where S_0 is initial number of tokens.

C. Implementation of the exponential decay in a smart contract

Complex functions like exponentials, if implemented in smart contracts, would be quite costly. Fortunately, the exponential is a solution of a differential equation where inflation is proportional to the amount of not yet mined tokens:

$$I(t) = \frac{\ln 2}{T_{1/2}}\left(S_{\max} - S(t)\right) \quad (5)$$

$$dS = I(t)\,dt, \quad (6)$$

where $S(t)$ is the current token supply with $S(0) = S_0$ and the time step dt can actually be equal to the mining period (1 day). Each mining node can trivially calculate its dS in a smart contract using very few operations and the coin supply S from the last period. So, the amount of tokens mined for the node i and the time period t will be:

$$ds_{i,t} = \frac{l_i}{L}\frac{\ln 2}{T_{1/2}}\left(S_{\max} - S_{t-1}\right), \quad (7)$$

$$dS_t = \sum_i ds_{i,t}, \quad (8)$$

where l_i is the number of tokens locked by the miner i, L is the total number of tokens locked. Instead of calculating all the sum over i, each miner i can add her portion $ds_{i,t}$.

D. Mining rate and staking time

We want to incentivize miners to serve re-encryption policies for at least 1 year. However, short-term stakers are still useful and should be rewarded. We will give the full compensation ($\kappa = 1$) to the stakers who are committed to stake at least $T_1 = 1$ year, however those who stake for $T_{\min} = 1$ month will get close to half the compensation ($\kappa \approx 0.54$) (Fig. 3). The individual daily compensation rate for a miner looks as:

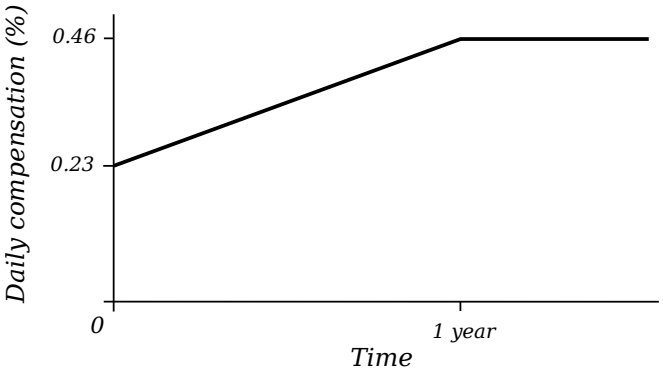


FIG. 3: Dependence of the compensation rate on staking duration. We assume 60% of all tokens locked for staking

$$\kappa = \left(0.5 + 0.5\frac{\min(T_i, T_1)}{T_1}\right) \quad (9)$$

$$T_{i,\text{initial}} \geq T_{\min}, \quad (10)$$

$$\delta s_{i,t} = \kappa\,\frac{l_i}{L}\frac{\ln 2}{T_{1/2}}\left(S_{\max} - S_{t-1}\right). \quad (11)$$

$$(12)$$

The unlocking time T_i means the time left to unlock the tokens $t_1 - t$. The initial T_i cannot be set smaller than 1 month, but it eventually becomes smaller than that as the time passes, and the miner gets close to unlocking the stake.

This has implications on the global token economy. Firstly, if stakers, despite smaller compensation, prefer to stake for shorter time periods, that results in a smaller daily token emmission. Since miners will likely prefer shorter stake times during bear markets, reducing the issuance rate during that time will provide better price support and stability as a side benefit.

Interestingly, $\kappa < 1$ prolongs the compensation half-decay time $T^*_{1/2} = T_{1/2}/\kappa^*$, where κ^* is the mean staking parameter. If all the stakers have $\kappa^* = \kappa = 0.5$, this prolongs $T_{1/2}$ to be 4 years instead of 2.

The total supply over time (Eq. 3) at $\kappa^* \neq 1$ will then

look like:

$$S(t) = S_0 \left[1 + \frac{i_0 \kappa^* T^*_{1/2}}{\ln 2} \left(1 - 2^{-\frac{t}{T^*_{1/2}}}\right)\right]. \qquad (13)$$

V. MINING STRATEGIES AND EXPECTED COMPENSATION

In this section, we look at three possibilities: a miner liquidating all the compensation while extending the lock time (Stake and Earn), a miner adding all the compensation to their current stake (Restake), and a miner waiting for their stake to unlock after time T (Spindown). Each of these possibilities could have different distributions of κ. Let's consider $\kappa = 1$ and $\kappa = 0.5$ as two marginal values. Let's take the amount of tokens locked to be $\lambda = 60\%$, as in DASH. We'll plot graphs of daily compensation, as well as calculate the compensation during the first year in each of these scenarios.

A. Stake and Earn: Liquidate mining compensation

This is the simplest case. The total amount of tokens staked in the network can be expressed as $L = \lambda S$. The amount of stake stays constant in this case and equal to $s_i = l$, and the mining rate (i.e. the cumulative compensation) is:

$$\frac{dr}{dt} = \kappa \frac{l}{\lambda S(t)} \frac{\ln 2}{T_{1/2}} \left(S_{\max} - S(t)\right). \qquad (14)$$

Daily compensation dr/dt plotted for $\lambda = 0.6$ is shown on Fig. 4.

When we substitute $S(t)$ from Eq. 13 and integrate over time, we find total compensation:

$$r(t) = l \frac{\kappa}{\kappa^* \lambda} \ln \frac{S(t)}{S_0}. \qquad (15)$$

If $\kappa = 1$ (staking for 1 year+) and $\lambda = 60\%$ (60% of all nodes in the network are staking), miner's compensation starts from 0.46% per day in NU tokens, or 100.2% during the first year of staking.

We should note that if other miners stake for less than a year ($\kappa^* < 1$), the inflation rate decays slower, and the compensation over a given period will be higher.

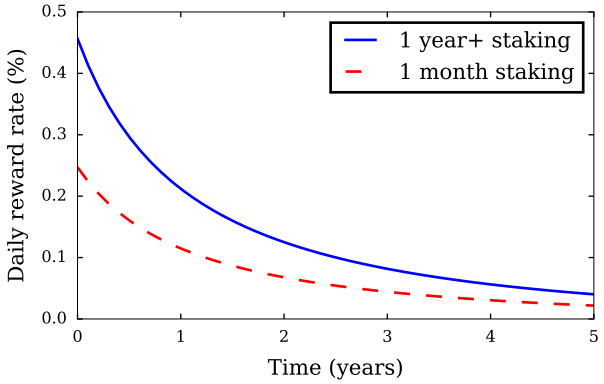


FIG. 4: Daily compensation over time assuming 60% tokens locked for lock times of 1 year and 1 month

B. Restake mining compensation

Instead of liquidating mining compensation, it could be restaked into the node in order to increase the miner's stake and, provided the node's hardware is powerful enough to support the additional workload, get higher compensation. In this case, the actual stake l is constantly increasing with time:

$$\frac{dl}{dt} = \kappa \frac{l}{\lambda S(t)} \frac{\ln 2}{T_{1/2}} \left(S_{\max} - S(t)\right). \qquad (16)$$

If we substitute $S(t)$ from Eq. 13 and solve this differential equation against l, we get:

$$l(t) = l(0) \left[\frac{S(t)}{S_0}\right]^{\frac{\kappa}{\kappa^* \lambda}}. \qquad (17)$$

If $\kappa = 1$ (staking for 1 year+) and $\lambda = 60\%$ (60% of all nodes in the network are staking), miner's compensation starts from 0.46% per day in NU tokens, or $l(1) - l(0) = 177.5\%$ during the first year of staking. The difference between restaking and taking the reward is shown at Fig. 5.

C. Take mining compensation and spindown

When the node spins down, the miner doesn't extend the time for end of staking t_1, and the compensation is constantly decreasing as the time left to unlock becomes smaller and smaller, effectively decreasing κ gradually towards 0.5. That's the default behavior: to avoid that,

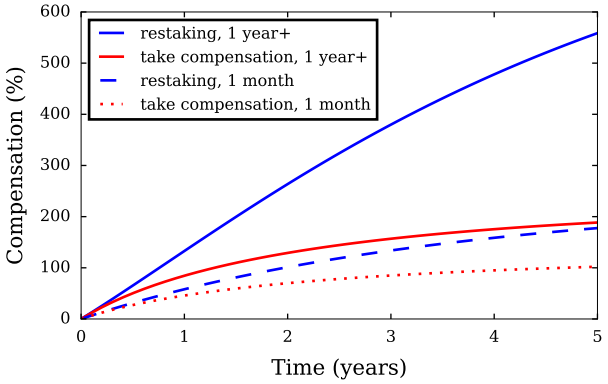


FIG. 5: Total amount of compensation produced by staking with relocking (blue) and without (red), when staking for 1 year or more (solid lines) or 1 month.

the miner should set t_1 large enough, or increase t_1 periodically.

D. Edge case: connection problems

If a miner is found to be non-operational and/or cannot confirm its activity, the tokens aren't unlocked (and compensation isn't earned). It's not fatal for the miner if that happens (i.e. their entire stake won't be slashed), however their tokens will remain locked without earning compensation, until they satisfy their commitment. So if the miner commits to stake for at least a year, it implies a year of operation.

Connection problems may also be downtime when the miner upgrades their node's software, which is an absolutely legitimate reason to be offline for a short time.

VI. TLDR

A. How much will I be earnining if I run a node?

It depends on how early you start (the earlier, the better), and for how long you commit to provide re-encryption services. If you commit to work for 1 month, you'll be getting approximately 54% of what you'll be getting if you commit for 1 year or more. Also the compensation is inversely proportional to the total amount of tokens staked by all the participants. Finally, if you choose to automatically restake the tokens, this will increase your total compensation because your stake will be increasing as well as the amount of work done for the network.

For example, if 60% of tokens in the system are always staked, you'd be earning 0.46% on day 1. With this amount of stakers, if you withdraw all the earned tokens instead of restaking them, you'd earn 100.2% of your stake in the first year. But if you restake all your tokens, in the same first year you'd earn 177.5% of the tokens you staked.

B. How many tokens will ever be in existence?

We'll start with 1 billion tokens, and the maximum amount of tokens ever mined will be 3.89 billion.

C. What's the inflation rate?

The inflation rate will depend on how many short-term miners and long-term miners are in the system. Depending on this, the initial inflation will be between 50% APR (when all miners are very short term) and 100% APR (when everyone commits for a long term). The inflation will decay exponentially every day, halving sometime between 2 years (if all the miners are long term) and 4 years (if all the miners are short term).

[1] Evan Duffield and Daniel Diaz, "Dash: A privacy-centric crypto-currency," (2015).

[2] "Zcash documentation," .

[3] "Official dash documentation: Emission rate," .