# Random phenomenon

### Random experiment (phenomenon)

An experiment or phenomenon which produces different outcomes, even though it is repeated in same manner every time, is called a random experiment or a random phenomenon

Examples: throw a die, sensor reading, defects in a product

## Prediction perspective:

### Random phenomenon

Any phenomenon that cannot be predicted accurately given infinite past is said to be random. Alternatively, there exists no known mathematical function that can accurately describe the process

# Sample Space / Population

## Sample Space / Population

It is the set of all possible outcomes of a random phenomenon. It is usually denoted by “S”

* If the outcomes are discrete-valued, we have a discrete sample space (eg: throw of a die, scores in a game)
* When the outcomes are continuous-valued, we have a continuous sample space (eg: ambient temperature, gas pressure)

Sample space for rainfall is {“yes”,“no”}, here the event is just one either “yes” or “no”. Sample space is all possible options which could be the outcome. Similarly for a single die, sample space = {1,2,3,4,5,6}, however the event is just one of the possible options, or if we are only interested in outcomes which are less than 4, then the event would {1,2,3}

An event is a subset of a sample space.

* In a two coin toss experiment, an event is , while , here I am only interested in scenarios where only one head shows up
* A collection of sets (events) is said to be exhaustive if

# Probability Basics and Axioms

Whenever outcomes that are equally likely, the probability of each outcome is 1/N

For a discrete sample space, the probability of an event , denoted as , is the sum of the probabilities of the outcome in .

## Axioms

If is the sample space and is an event in any random experiment,

1. (one of the events has to occur!)
2. (probabilities are always non-negative values less than unity)
3. For two mutually exclusive events and ,
4. If is the complement of an event ,

Probabilities on sets:

## Conditional probability:

The conditional probability of an event given an , , denoted as is

Example:

A = event that it rains,

B = event that Rahul is carrying an umbrella

= what is the probability that Rahul is carrying an umbrella, given it is raining

Example:

Table below lists the classification of 940 wafers in a semiconductor manufacturing process.

|  |  |  |  |
| --- | --- | --- | --- |
| Contamination | Center | Edge | Total |
| Low | 514 | 68 | 582 |
| High | 112 | 246 | 358 |
| Total | 626 | 314 | 940 |

If is the event corresponding to high contamination and is the event that the wafer is in the center of the sputtering tool, determine

Answer:

# Independent events

## Independence

Two events and are independent if and only if

If Rahul will always take the Umbrella, and it has nothing to do with rain, then both are totally independent events

Example:

A day’s production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Suppose two parts are selected from the batch, but the first part is replaced before the second part is selected. What is the part that the second part is defective (denoted as ) given that the first part is defective (denoted as )?

Answer:

# Random Variable

A random variable is a mapping ( or point function) from a sample space onto the real line such that to each element in set of there corresponds a unique real number.

* Effectively we replace our original (abstract) sample space by a new (concrete) sample space.
  + Eg: head and tail of a toss are mapped to [1,0]
* If the experiment itself yields some physical quantity that is real valued, then no further mapping is required

Randomness is not a characteristic of a process, but it is rather a reflection of our (lack of) knowledge and understanding of that process

# Probability Distribution

The natural recourse to dealing with uncertainties is to list all possible outcomes and assign a chance to each of those outcomes