Assignment - 1 New Edited

Arjun Jayachandran
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***Abstract*—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

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svn co https://github.com/arjunjc93/Assignment−1
    _new.git
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1 Vectors

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Which can be written as: $\begin{bmatrix} 3 & -4 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{1.1.1}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix}$$

$$\xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

Thus,

$$x = 2, y = 3 \tag{1.1.2}$$

is the solution for the two equations. Let this point be **P**

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.3}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of

$$3x - 4y + 6 = 0$$

with the x axis:
equation of x axis is

$$y = 0$$

we have 2 equations:

$$3x - 4y + 6 = 0$$

$$y = 0$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.4}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{1}{3} R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{4}{3} R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = -2, y = 0 \tag{1.1.5}$$

is the solution for the two equations. Let this point be $\mathbf{Q}$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.6}$$

is the point of intersection of the line

$$3x - 4y + 6 = 0$$

with the x axis

c) To find out intersection of

$$3x + y - 9 = 0$$

with the x axis:
equation of x axis is

$$y = 0$$

we have 2 equations:

$$3x + y - 9 = 0$$

$$y = 0$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.7}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = 3, y = 0 \tag{1.1.8}$$

is the solution for the two equations. Let this point be $\mathbf{R}$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

is the point of intersection of the line

$$3x + y - 9 = 0$$

with the x axis.

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix},$$

$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} and$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

represent the vertices of the triangle formed by the lines

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

with the X-axis.
P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis.
R is the point at which $3x + y - 9 = 0$ meets the X-axis.

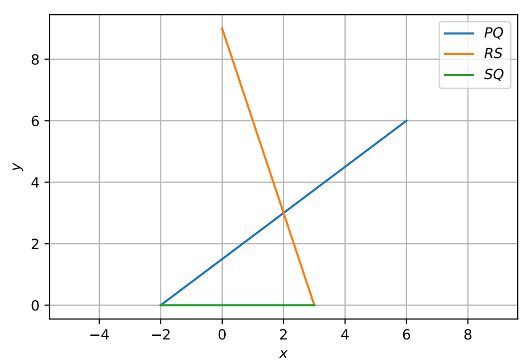

Fig. 1. Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$
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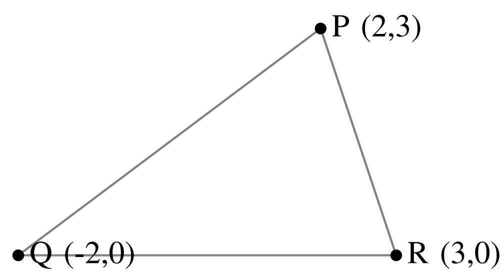

Tikz-Diagram

The vector equation of a line passing through two points with position vector A & B is:

$$\mathbf{r} = \mathbf{A} + t(\mathbf{B} - \mathbf{A})$$

where, t is a scalar.
Let $\mathbf{T}$ represent the line $3x + 4y - 6 = 0$,

$\therefore \mathbf{T} = \mathbf{P} + t(\mathbf{Q} - \mathbf{P})$

$$\Rightarrow \mathbf{T} = \left[\left[\begin{pmatrix}2\\3\end{pmatrix} + t\left[\left[\begin{pmatrix}-2\\0\end{pmatrix}\right] - \left[\begin{pmatrix}2\\3\end{pmatrix}\right]\right]\right]\right]$$

$$\therefore \mathbf{T} = \left[\left[\begin{pmatrix}2\\3\end{pmatrix} + t\left[\begin{pmatrix}4\\3\end{pmatrix}\right]\right]\right]$$

where,

$$x = 2 + 4t$$

$$y = 3 + 3t$$

Let $\mathbf{U}$ represent the line $3x - y + 9 = 0$,

$\therefore \mathbf{U} = \mathbf{P} + t(\mathbf{R} - \mathbf{P})$

$$\Rightarrow \mathbf{U} = \left[\left[\begin{pmatrix}2\\3\end{pmatrix} + t\left[\left[\begin{pmatrix}3\\0\end{pmatrix}\right] - \left[\begin{pmatrix}2\\3\end{pmatrix}\right]\right]\right]\right]$$

$$\therefore \mathbf{U} = \left[\left[\begin{pmatrix}2\\3\end{pmatrix} + t\left[\begin{pmatrix}1\\-3\end{pmatrix}\right]\right]\right]$$

where,

$$x = 2 + t$$

$$y = 3 - 3t$$

Let $\mathbf{V}$ represent the x axis

$$y = 0$$

,

$\therefore \mathbf{V} = \mathbf{Q} + t(\mathbf{R} - \mathbf{Q})$

$$\Rightarrow \mathbf{V} = \left[\left[\begin{pmatrix}-2\\0\end{pmatrix} + t\left[\left[\begin{pmatrix}3\\0\end{pmatrix}\right] - \left[\begin{pmatrix}-2\\0\end{pmatrix}\right]\right]\right]\right]$$

$$\therefore \mathbf{V} = \left[\left[\begin{pmatrix}-2\\0\end{pmatrix} + t\left[\begin{pmatrix}5\\0\end{pmatrix}\right]\right]\right]$$

where,

$$x = -2 + 5t$$

$$y = 0$$