Assignment - 1 New Edited

Arjun Jayachandran
MD/2020/702

***Abstract*—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

```
svn co https://github.com/arjunjc93/Assignment−1
    _new.git
```

1 Vectors

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines:

$$3x - 4y + 6 = 0 \tag{1.1.1}$$

$$3x + y - 9 = 0 \tag{1.1.2}$$

Which can be written as: $\begin{bmatrix} 3 & -4 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{1.1.3}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix}$$

$$\xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

Thus,

$$x = 2, y = 3 \tag{1.1.4}$$

is the solution for the two equations. Let this point be **P**

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.5}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of 1.1.1 with the x axis:
equation of x axis is

$$y = 0 \tag{1.1.6}$$

we have 2 equations:

$$3x - 4y + 6 = 0 \tag{1.1.7}$$

$$y = 0 \tag{1.1.8}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.9}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{1}{3} R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{4}{3} R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = -2, y = 0 \tag{1.1.10}$$

is the solution for the two equations. Let this point be $\mathbf{Q}$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.11}$$

is the point of intersection of the line 1.1.1 with the x axis.

c) To find out intersection of 1.1.2 with the x axis:
equation of x axis is

$$y = 0 \tag{1.1.12}$$

we have 2 equations:

$$3x + y - 9 = 0 \tag{1.1.13}$$
$$y = 0 \tag{1.1.14}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.15}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = 3, y = 0 \tag{1.1.16}$$

is the solution for the two equations. Let this point be $\mathbf{R}$
.

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.17}$$

is the point of intersection of the line 1.1.2 with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.18}$$
$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.19}$$
$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.20}$$
$$\tag{1.1.21}$$

represent the vertices of the triangle formed by the lines 1.1.1 & 1.1.2 with the X-axis.

P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis. R is the point at which $3x + y - 9 = 0$ meets the X-axis.

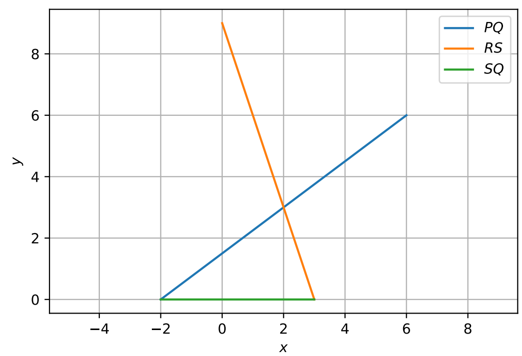
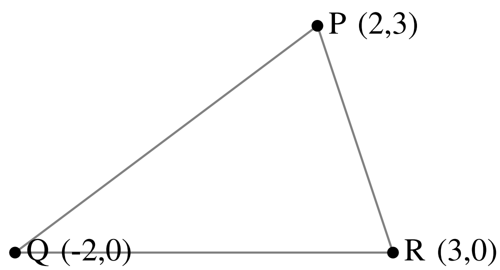


Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$
.



Tikz-Diagram

The vector equation of a line passing through two points with position vector $\mathbf{A}$ & $\mathbf{B}$ is:

$$\mathbf{r} = \mathbf{A} + t(\mathbf{B} - \mathbf{A}) \tag{1.1.22}$$

where, t is a scalar.
Let $\mathbf{T}$ represent the line 1.1.1,

$$\mathbf{T} = \mathbf{P} + t(\mathbf{Q} - \mathbf{P}) \tag{1.1.23}$$
$$\Rightarrow \mathbf{T} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.24}$$
$$\mathbf{T} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right]\right]\right] \tag{1.1.25}$$

where,

$$x = 2 + 4t \tag{1.1.26}$$
$$y = 3 + 3t \tag{1.1.27}$$

Let **U** represent the line 1.1.2,

$$\mathbf{U} = \mathbf{P} + t(\mathbf{R} - \mathbf{P}) \tag{1.1.28}$$
$$\Rightarrow \mathbf{U} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\left[\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.29}$$
$$\mathbf{U} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\begin{pmatrix} 1 \\ -3 \end{pmatrix}\right]\right]\right] \tag{1.1.30}$$

where,

$$x = 2 + t \tag{1.1.31}$$
$$y = 3 - 3t \tag{1.1.32}$$

Let **V** represent the x axis

$$y = 0 \tag{1.1.33}$$

$$\mathbf{V} = \mathbf{Q} + t(\mathbf{R} - \mathbf{Q}) \tag{1.1.34}$$
$$\Rightarrow \mathbf{V} = \left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} + t\left[\left[\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.35}$$
$$\mathbf{V} = \left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} + t\left[\begin{pmatrix} 5 \\ 0 \end{pmatrix}\right]\right]\right] \tag{1.1.36}$$

where,

$$x = -2 + 5t \tag{1.1.37}$$
$$y = 0 \tag{1.1.38}$$