Assignment - 1 New Edited

Arjun Jayachandran
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***Abstract*—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

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svn co https://github.com/arjunjc93/Assignment−1_new.git
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1 Vectors

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines:

$$3x - 4y + 6 = 0 \tag{1.1.1}$$

Which is written in vector form:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{X} = \begin{pmatrix} -6 \end{pmatrix} \tag{1.1.2}$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1.1.3}$$

and

$$3x + y - 9 = 0 \tag{1.1.4}$$

Which is written in vector form:

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 9 \end{pmatrix} \tag{1.1.5}$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1.1.6}$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{X} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \tag{1.1.7}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{1.1.8}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix}$$

$$\xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

Thus,

$$x = 2, y = 3 \tag{1.1.9}$$

is the solution for the two equations. Let this point be $\mathbf{P}$

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.10}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of 1.1.1 with the x axis:

equation of x axis is

$$y = 0 \tag{1.1.11}$$

we have 2 equations:

$$3x - 4y + 6 = 0 \tag{1.1.12}$$
$$y = 0 \tag{1.1.13}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.14}$$

This can be reduced as follows:
$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{4}{3}R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$
Thus,

$$x = -2, y = 0 \tag{1.1.15}$$

is the solution for the two equations. Let this point be **Q**

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.16}$$

is the point of intersection of the line 1.1.1 with the x axis.

c) To find out intersection of 1.1.4 with the x axis:
equation of x axis is

$$y = 0 \tag{1.1.17}$$

we have 2 equations:

$$3x + y - 9 = 0 \tag{1.1.18}$$
$$y = 0 \tag{1.1.19}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.20}$$

This can be reduced as follows:
$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$
Thus,

$$x = 3, y = 0 \tag{1.1.21}$$

is the solution for the two equations. Let this point be **R**
.

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.22}$$

is the point of intersection of the line 1.1.4 with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.23}$$
$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.24}$$
$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.25}$$
$$\tag{1.1.26}$$

represent the vertices of the triangle formed by the lines 1.1.1 & 1.1.4 with the X-axis.

P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis. R is the point at which $3x + y - 9 = 0$ meets the X-axis.

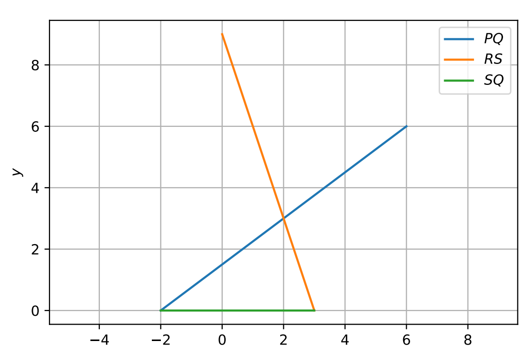

Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$
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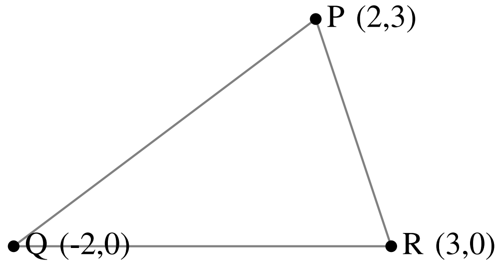

Tikz-Diagram

The vector equation of a line passing through two points with position vector **A** & **B** is:

$$\mathbf{r} = \mathbf{A} + t(\mathbf{B} - \mathbf{A}) \tag{1.1.27}$$

where, t is a scalar.
Let **T** represent the line 1.1.1,

$$\mathbf{T} = \mathbf{P} + t(\mathbf{Q} - \mathbf{P}) \tag{1.1.28}$$

$$\Rightarrow \mathbf{T} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.29}$$

$$\mathbf{T} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right]\right]\right] \tag{1.1.30}$$

where,

$$x = 2 + 4t \tag{1.1.31}$$

$$y = 3 + 3t \tag{1.1.32}$$

Let **U** represent the line 1.1.4,

$$\mathbf{U} = \mathbf{P} + t(\mathbf{R} - \mathbf{P}) \tag{1.1.33}$$

$$\Rightarrow \mathbf{U} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\left[\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.34}$$

$$\mathbf{U} = \left[\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\left[\begin{pmatrix} 1 \\ -3 \end{pmatrix}\right]\right]\right] \tag{1.1.35}$$

where,

$$x = 2 + t \tag{1.1.36}$$

$$y = 3 - 3t \tag{1.1.37}$$

Let **V** represent the x axis

$$y = 0 \tag{1.1.38}$$

$$\mathbf{V} = \mathbf{Q} + t(\mathbf{R} - \mathbf{Q}) \tag{1.1.39}$$

$$\Rightarrow \mathbf{V} = \left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} + t\left[\left[\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right] - \left[\begin{pmatrix} -2 \\ 0 \end{pmatrix}\right]\right]\right]\right] \tag{1.1.40}$$

$$\mathbf{V} = \left[\left[\begin{pmatrix} -2 \\ 0 \end{pmatrix} + t\left[\begin{pmatrix} 5 \\ 0 \end{pmatrix}\right]\right]\right] \tag{1.1.41}$$

where,

$$x = -2 + 5t \tag{1.1.42}$$

$$y = 0 \tag{1.1.43}$$