

Assignment - 1

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Abstract—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.

Download all and latex-tikz codes from

svn co https://github.com/arjunjc93/Assignment-1_new.git

1 VECTORS

(CBSE-MATH-X-2006-SET 2-Q.11)

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines: Which is written in vector form:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \end{pmatrix} \quad (1.1.1)$$

and

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 9 \end{pmatrix} \quad (1.1.2)$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.1.3)$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \quad (1.1.4)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 9 & 9 \end{pmatrix} \quad (1.1.5)$$

This can be reduced as follows:

$$\begin{aligned} & \begin{pmatrix} 3 & -4 & -6 \\ 3 & 9 & 9 \end{pmatrix} \\ & \xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 9 & 9 \\ 3 & -4 & -6 \end{pmatrix} \\ & \xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & 3 & 3 \\ 3 & -4 & -6 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 3 & 3 \\ 0 & -13 & -15 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow -\frac{1}{13}R_2} \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & \frac{15}{13} \end{pmatrix} \\ & \xleftrightarrow{R_1 \leftarrow R_1 - 3R_2} \begin{pmatrix} 1 & 0 & \frac{3}{13} \\ 0 & 1 & \frac{15}{13} \end{pmatrix} \end{aligned}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1.1.6)$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of 1.1.1 with the x axis:

equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \quad (1.1.7)$$

we have 2 equations:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \end{pmatrix} \quad (1.1.8)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \quad (1.1.9)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.10)$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} &\xleftarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \\ &\xleftarrow{R_1 \leftarrow R_1 + \frac{4}{3}R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (1.1.11)$$

is the point of intersection of the line 1.1.1 with the x axis.

- c) To find out intersection of 1.1.2 with the x axis:

equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \quad (1.1.12)$$

we have 2 equations:

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 9 \end{pmatrix} \quad (1.1.13)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \quad (1.1.14)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.15)$$

This can be reduced as follows:

$$\begin{aligned} &\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \\ &\xleftarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \\ &\xleftarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.1.16)$$

is the point of intersection of the line 1.1.2 with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1.1.17)$$

$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (1.1.18)$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.1.19)$$

$$(1.1.20)$$

represent the vertices of the triangle formed by the lines 1.1.1 & 1.1.2 with the X-axis.

P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis. R is the point at which $3x + y - 9 = 0$ meets the X-axis.

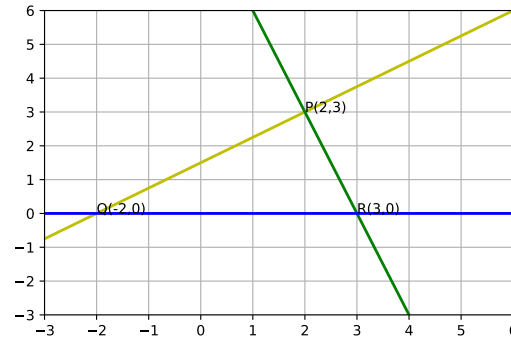


Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$