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Assignment - 1

Arjun Jayachandran MD/2020/702

Abstract—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.

Download all and latex-tikz codes from

svn co https://github.com/arjunjc93/Assignment-1 new.git

1 Vectors (CBSE-Math-X-2006-Set 2-Q.11)

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines: Which is written in vector form:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \end{pmatrix} \tag{1.1.1}$$

and

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 9 \end{pmatrix} \tag{1.1.2}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1.1.3}$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \tag{1.1.4}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{1.1.5}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix}$$

$$\stackrel{R_2 \leftarrow R_1}{\longleftarrow} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\stackrel{R_1 \leftarrow \frac{R_1}{3}}{\longleftarrow} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix}$$

$$\stackrel{R_2 \leftarrow R_2 - 3R_1}{\longleftarrow} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix}$$

$$\stackrel{R_2 \leftarrow \frac{1}{5}R_2}{\longleftarrow} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\stackrel{R_1 \leftarrow R_1 - \frac{1}{3}R_2}{\longleftarrow} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.6}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of 1.1.1 with the x axis:

equation of x axis is

$$(0 \ 1)(x) = (0)$$
 (1.1.7)

we have 2 equations:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \end{pmatrix} \tag{1.1.8}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \tag{1.1.9}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.10}$$

This can be reduced as follows: $\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix}$

$$\stackrel{R_1 \leftarrow \frac{1}{3}R_1}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\stackrel{R_1 \leftarrow R_1 + \frac{4}{3}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.11}$$

is the point of intersection of the line 1.1.1 with the x axis.

c) To find out intersection of 1.1.2 with the x axis:

equation of x axis is

$$(0 1)(x) = (0) (1.1.12)$$

we have 2 equations:

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 9 \end{pmatrix} \tag{1.1.13}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \end{pmatrix} \tag{1.1.14}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.15}$$

This can be reduced as follows:

$$\begin{pmatrix}
0 & 1 & 0 \\
R_1 \leftarrow \frac{1}{3}R_1 & \begin{pmatrix} 1 & \frac{1}{3} & 3 \\
0 & 1 & 0 \end{pmatrix}$$

$$\stackrel{R_1 \leftarrow R_1 - \frac{1}{3}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 3 \\
0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.16}$$

is the point of intersection of the line 1.1.2 with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2\\3 \end{pmatrix} \tag{1.1.17}$$

$$\mathbf{Q} = \begin{pmatrix} -2\\0 \end{pmatrix} \tag{1.1.18}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.19}$$

(1.1.20)

represent the vertices of the triangle formed by the lines 1.1.1 & 1.1.2 with the X-axis.

P is the vertex of the triangle. Q is the point at which 3x - 4y + 6 = 0 meets the X-axis. R is the point at which 3x + y - 9 = 0 meets the X-axis.

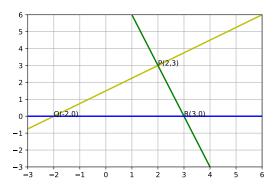


Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$

