

Assignment - 1 New Edited

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Abstract—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.

Download all and latex-tikz codes from

svn co https://github.com/arjunjc93/Assignment-1_new.git

1 VECTORS

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0$$

$$3x + y - 9 = 0$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines:

$$3x - 4y + 6 = 0 \quad (1.1.1)$$

Which is written in vector form:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{X} = \begin{pmatrix} -6 \end{pmatrix} \quad (1.1.2)$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.1.3)$$

and

$$3x + y - 9 = 0 \quad (1.1.4)$$

Which is written in vector form:

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 9 \end{pmatrix} \quad (1.1.5)$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.1.6)$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{X} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \quad (1.1.7)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 9 & 9 \end{pmatrix} \quad (1.1.8)$$

This can be reduced as follows:

$$\begin{aligned} & \begin{pmatrix} 3 & -4 & -6 \\ 3 & 9 & 9 \end{pmatrix} \\ & \xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 9 & 9 \\ 3 & -4 & -6 \end{pmatrix} \\ & \xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & 3 & 3 \\ 3 & -4 & -6 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 3 & 3 \\ 0 & -13 & -15 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow -\frac{1}{13}R_2} \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & \frac{15}{13} \end{pmatrix} \\ & \xleftrightarrow{R_1 \leftarrow R_1 - 3R_2} \begin{pmatrix} 1 & 0 & \frac{3}{13} \\ 0 & 1 & \frac{15}{13} \end{pmatrix} \end{aligned}$$

Thus,

$$x = 2, y = 3 \quad (1.1.9)$$

is the solution for the two equations. Let this point be \mathbf{P}

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1.1.10)$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of 1.1.1 with the x axis:

equation of x axis is

$$y = 0 \quad (1.1.11)$$

we have 2 equations:

$$(3 \ -4) \mathbf{X} = (-6) \quad (1.1.12)$$

$$(0 \ 1) \mathbf{X} = (0) \quad (1.1.13)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.14)$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & -\frac{4}{3} & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{4}{3}R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = -2, y = 0 \quad (1.1.15)$$

is the solution for the two equations. Let this point be **Q**

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (1.1.16)$$

is the point of intersection of the line 1.1.1 with the x axis.

- c) To find out intersection of 1.1.4 with the x axis:

equation of x axis is

$$y = 0 \quad (1.1.17)$$

we have 2 equations:

$$(3 \ 1) \mathbf{X} = (9) \quad (1.1.18)$$

$$(0 \ 1) \mathbf{X} = (0) \quad (1.1.19)$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.20)$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

Thus,

$$x = 3, y = 0 \quad (1.1.21)$$

is the solution for the two equations. Let this point be **R**

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.1.22)$$

is the point of intersection of the line 1.1.4 with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1.1.23)$$

$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (1.1.24)$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (1.1.25)$$

$$(1.1.26)$$

represent the vertices of the triangle formed by the lines 1.1.1 & 1.1.4 with the X-axis.

P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis. R is the point at which $3x + y - 9 = 0$ meets the X-axis.

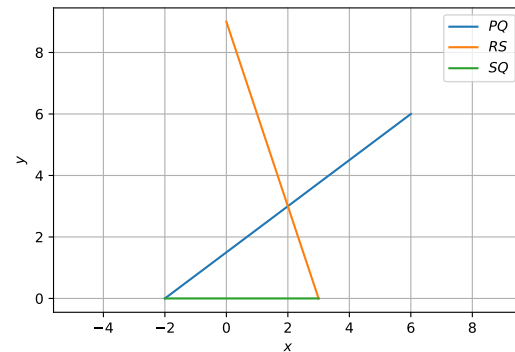
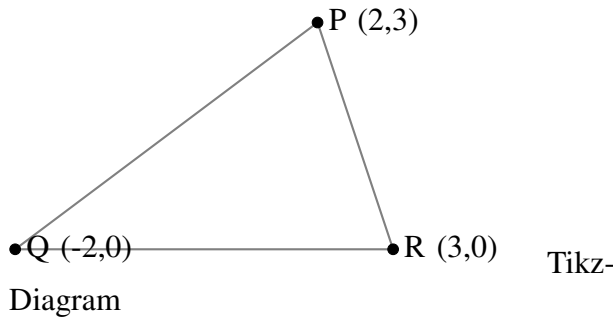


Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$



The vector equation of a line passing through two points with position vector **A** & **B** is:

$$\mathbf{r} = \mathbf{A} + t(\mathbf{B} - \mathbf{A}) \quad (1.1.27)$$

where, t is a scalar.

Let **T** represent the line 1.1.1,

$$\mathbf{T} = \mathbf{P} + t(\mathbf{Q} - \mathbf{P}) \quad (1.1.28)$$

$$\Rightarrow \mathbf{T} = \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix} + t \left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right] \right] \quad (1.1.29)$$

$$\mathbf{T} = \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} -4 \\ -3 \end{bmatrix} \right] \quad (1.1.30)$$

where,

$$x = 2 - 4t \quad (1.1.31)$$

$$y = 3 - 3t \quad (1.1.32)$$

Let **U** represent the line 1.1.4,

$$\mathbf{U} = \mathbf{P} + t(\mathbf{R} - \mathbf{P}) \quad (1.1.33)$$

$$\Rightarrow \mathbf{U} = \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix} + t \left[\begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right] \right] \quad (1.1.34)$$

$$\mathbf{U} = \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right] \quad (1.1.35)$$

where,

$$x = 2 + t \quad (1.1.36)$$

$$y = 3 - 3t \quad (1.1.37)$$

Let **V** represent the x axis

$$y = 0 \quad (1.1.38)$$

$$\mathbf{V} = \mathbf{Q} + t(\mathbf{R} - \mathbf{Q}) \quad (1.1.39)$$

$$\Rightarrow \mathbf{V} = \left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} + t \left[\begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right] \right] \quad (1.1.40)$$

$$\mathbf{V} = \left[\begin{bmatrix} -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right] \quad (1.1.41)$$

where,

$$x = -2 + 5t \quad (1.1.42)$$

$$y = 0 \quad (1.1.43)$$