Assignment - 1

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***Abstract*—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

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svn co https://github.com/arjunjc93/Assignment−1
    _new.git
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1 Vectors
(CBSE-Math-X-2006-Set 2-Q.11)

1.1. Draw the graphs of the following equations:

$$3x - 4y + 6 = 0 \tag{1.1.1}$$
$$or \begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = -6 \tag{1.1.2}$$
$$3x + y - 9 = 0 \tag{1.1.3}$$
$$or \begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{1.1.4}$$

Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

Solution:

a) We have equations of two lines: Which is written in vector form:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = -6 \tag{1.1.5}$$

and

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{1.1.6}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1.1.7}$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 3 & -4 \\ 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -6 \\ 9 \end{pmatrix} \tag{1.1.8}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \tag{1.1.9}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 3 & 1 & 9 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_2]{R_2 \leftarrow R_1} \begin{pmatrix} 3 & 1 & 9 \\ 3 & -4 & -6 \end{pmatrix} \tag{1.1.10}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 3 & -4 & -6 \end{pmatrix} \tag{1.1.11}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & -5 & -15 \end{pmatrix} \tag{1.1.12}$$
$$\xleftrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 3 \end{pmatrix} \tag{1.1.13}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3} R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix} \tag{1.1.14}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.15}$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of (1.1.5) with the x axis:
equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \end{pmatrix} = 0 \tag{1.1.16}$$

we have 2 equations:

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = -6 \tag{1.1.17}$$
$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{1.1.18}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.19}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & -4 & -6 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.20}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{4}{3}R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.21}$$

$$\tag{1.1.22}$$

$$\therefore \mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.23}$$

is the point of intersection of the line (1.1.5) with the x axis.

c) To find out intersection of (1.1.6) with the x axis:
equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{1.1.24}$$

we have 2 equations:

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{1.1.25}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{1.1.26}$$

Augmented matrix for above is:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.27}$$

This can be reduced as follows:

$$\begin{pmatrix} 3 & 1 & 9 \\ 0 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 1 & \frac{1}{3} & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.28}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{1}{3}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix} \tag{1.1.29}$$

$$\tag{1.1.30}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.31}$$

is the point of intersection of the line (1.1.6) with the x axis.

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.32}$$

$$\mathbf{Q} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.1.33}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{1.1.34}$$

$$\tag{1.1.35}$$

represent the vertices of the triangle formed by the lines (1.1.5) & (1.1.6) with the X-axis.

P is the vertex of the triangle. Q is the point at which $3x - 4y + 6 = 0$ meets the X-axis. R is the point at which $3x + y - 9 = 0$ meets the X-axis.

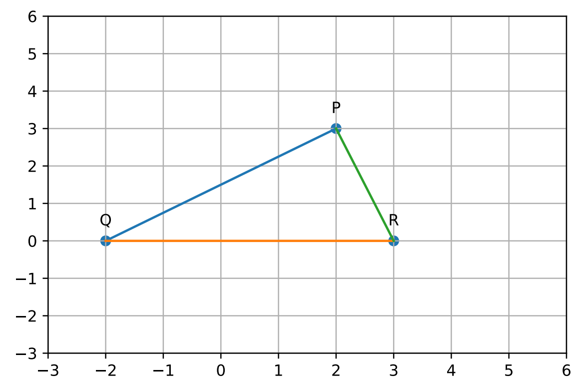

Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 2 & 3 \end{pmatrix}$
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