Assignment - 2

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***Abstract*—This is a simple document to learn about vectors and matrices and present it using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

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svn co https://github.com/arjunjc93/Assignment-2.git
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1 Vectors
(Points and Vectors by G V V Sharma Exercises-Q.2.10)

1.1. In each of the following, find the value of k for which the points are collinear

a) $\begin{pmatrix}7\\-2\end{pmatrix}, \begin{pmatrix}5\\1\end{pmatrix}, \begin{pmatrix}3\\k\end{pmatrix}$

b) $\begin{pmatrix}8\\1\end{pmatrix}, \begin{pmatrix}k\\-4\end{pmatrix}, \begin{pmatrix}2\\-5\end{pmatrix}$

Solution:

a) Let

$$\mathbf{A} = \begin{pmatrix}7\\-2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}5\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\k\end{pmatrix} \tag{1.1.1}$$

Then,

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}-2\\3\end{pmatrix}, \mathbf{C}-\mathbf{A} = \begin{pmatrix}-4\\k+2\end{pmatrix} \tag{1.1.2}$$

and

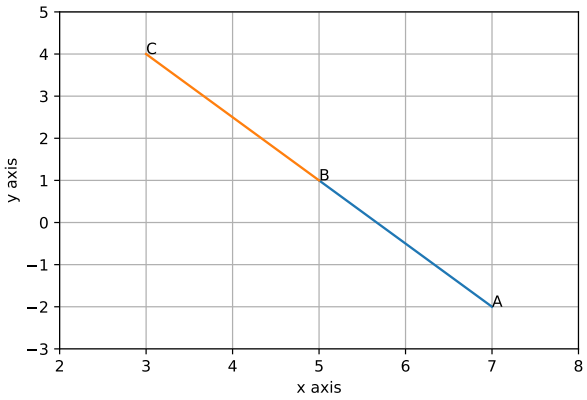
$$\mathbf{M} = \begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^T \tag{1.1.3}$$

$$= \begin{pmatrix}-2 & 3\\-4 & k+2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix}-2 & 3\\0 & 4-k\end{pmatrix} \tag{1.1.4}$$

$$\implies rank(\mathbf{M}) = 1 \iff R_2 = \mathbf{0}$$

or

$$4-k = 0 \implies k = 4$$



b) Let

$$\mathbf{A} = \begin{pmatrix}8\\1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}k\\-4\end{pmatrix}, \mathbf{C} = \begin{pmatrix}2\\-5\end{pmatrix} \tag{1.1.5}$$

Then,

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}k-8\\-5\end{pmatrix}, \mathbf{C}-\mathbf{A} = \begin{pmatrix}-6\\-6\end{pmatrix} \tag{1.1.6}$$

and

$$\mathbf{M} = \begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^T \tag{1.1.7}$$

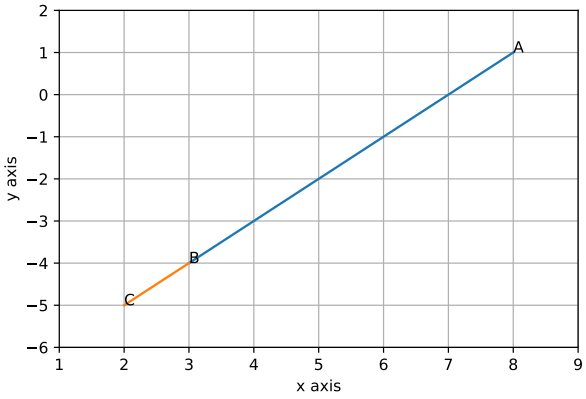
$$= \begin{pmatrix}k-8 & -5\\-6 & -6\end{pmatrix} \xleftrightarrow{R_1 \leftarrow -\frac{1}{5}R_1} \begin{pmatrix}\frac{8-k}{5} & 1\\-6 & -6\end{pmatrix} \tag{1.1.8}$$

$$\xleftrightarrow{R_2 \leftarrow 6R_1 + R_2} \begin{pmatrix}\frac{8-k}{5} & 1\\\frac{18-6k}{5} & 0\end{pmatrix} \tag{1.1.9}$$

$$\implies rank(\mathbf{M}) = 1 \iff R_2 = \mathbf{0}$$

or

$$\frac{18-6k}{5} = 0 \implies k = \frac{18}{6} \implies k = 3$$


A
B
C
x axis
y axis