

Assignment - 2

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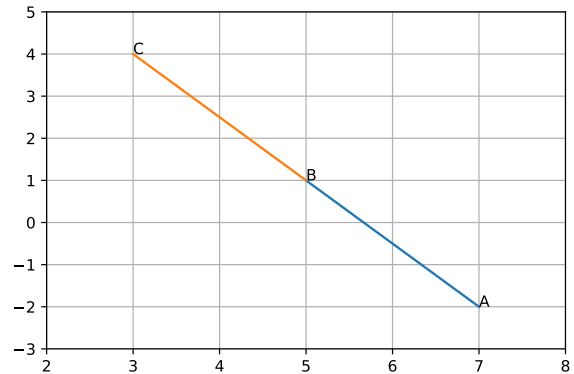
Abstract—This is a document concerning Assignment 2 of AI course for Medical Devices, NIPER Hyderabad.

Download all and latex-tikz codes from

svn co https://github.com/arjunjc93/Assignment-1_new.git

1 VECTORS

(POINTS AND VECTORS BY G V V SHARMA
EXERCISES-Q.2.10)



1.1. In each of the following, find the value of k for which the points are collinear

- a) $\begin{pmatrix} 7 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ k \end{pmatrix}$
 b) $\begin{pmatrix} 8 \\ 1 \end{pmatrix}, \begin{pmatrix} k \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

Solution:

a) Let

$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix} \quad (1.1.1)$$

Then,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ k+2 \end{pmatrix} \quad (1.1.2)$$

and

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (1.1.3)$$

$$= \begin{pmatrix} -2 & 3 \\ -4 & k+2 \end{pmatrix} \xrightarrow{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix} -2 & 3 \\ 0 & 4-k \end{pmatrix} \quad (1.1.4)$$

$$\Rightarrow \text{rank}(\mathbf{M}) = 1 \iff R_2 = \mathbf{0}$$

or

$$4 - k = 0 \implies k = 4$$

b) Let

$$\mathbf{A} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k \\ -4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad (1.1.5)$$

Then,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k-8 \\ -5 \end{pmatrix}, \mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -6 \end{pmatrix} \quad (1.1.6)$$

and

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T \quad (1.1.7)$$

$$= \begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{5}R_1} \begin{pmatrix} \frac{8-k}{5} & 1 \\ -6 & -6 \end{pmatrix} \quad (1.1.8)$$

$$\xrightarrow{R_2 \leftarrow 6R_1 + R_2} \begin{pmatrix} \frac{8-k}{5} & 1 \\ \frac{18-6k}{5} & 0 \end{pmatrix} \quad (1.1.9)$$

$$\implies \text{rank}(\mathbf{M}) = 1 \iff R_2 = \mathbf{0}$$

or

$$\frac{18-6k}{5} = 0 \implies k = \frac{18}{6} \implies k = 3$$

