Assignment - 3

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***Abstract*—This is a simple document to learn about vectors and matrices and present it using latex, draw figures using Python, Latex.**

Download all and latex-tikz codes from

```
svn co https://github.com/arjunjc93/Assignment−2.
    git
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1 Vectors
(Points and Vectors by G V V Sharma Exercises-Q.2.26)

1.1. The two adjacent sides of a parallelogram are $\begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$. Find out the unit vector parallel to its diagonal. Also, find its area.

Solution:

a) Let

$$\mathbf{A} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \tag{1.1.1}$$

be the adjacent sides of the parallelogram. Let $\mathbf{D}$ be the diagonal of the parallelogram. Then,

$$\mathbf{D} = \mathbf{A} + \mathbf{B} \tag{1.1.2}$$

$$\mathbf{D} = \begin{pmatrix} 3 \\ -6 \\ -8 \end{pmatrix} \tag{1.1.3}$$

$$\|\mathbf{D}\| = \sqrt{(3)^2 + (-6)^2 + (-8)^2} = \sqrt{109} \tag{1.1.4}$$

Let $\mathbf{U}$ be the unit vector of $\mathbf{D}$ which can be found as follows:

$$\mathbf{U} = \frac{\mathbf{D}}{\|\mathbf{D}\|} \tag{1.1.5}$$

Solving the above equation gives the unit vector $\mathbf{U}$ which is parallel to the diagonal $\mathbf{D}$.

$$\therefore \boxed{\mathbf{U} = \frac{1}{\sqrt{109}} \begin{pmatrix} 3 \\ -6 \\ -8 \end{pmatrix}} \tag{1.1.6}$$

The area of the parallelogram is defined as

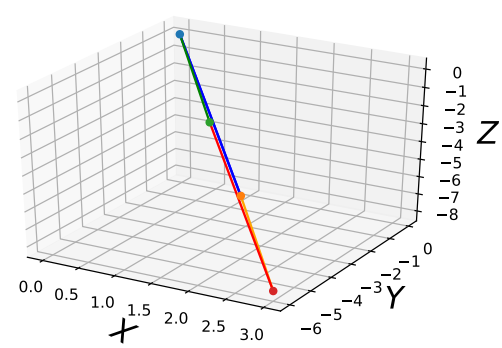
$$\|\mathbf{A} \times \mathbf{B}\| \tag{1.1.7}$$

where

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B3 \end{pmatrix} \tag{1.1.8}$$

$$= \begin{pmatrix} 0 & 5 & -4 \\ -5 & 0 & -2 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \tag{1.1.9}$$

Thus the desired area is

$$\|\mathbf{A} \times \mathbf{B}\| = \sqrt{(-2)^2 + (1)^2 + (0)^2} \tag{1.1.10}$$

$$= \sqrt{5} \tag{1.1.11}$$