Assignment - 6

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***Abstract*—This is a simple document to learn about writing vectors, matrices and quadratic forms using latex, draw figures using Python, Latex.**

Download all python and latex-tikz codes from

```
svn co https://github.com/arjunjc93/Assignment-6.git
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1 Quadratic Forms
G V V Sharma
Exercise 2.4

1.1. Find the points on the curve

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} - 3 = 0 \tag{1.1.1}$$

at which the tangents are parallel to the x-axis.
Solution: Given curve,

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} - 3 = 0 \tag{1.1.2}$$

where,

$$\mathbf{V} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \mathbf{V}^{-1}, \mathbf{u} = \begin{pmatrix}1\\0\end{pmatrix}, f = 0 \tag{1.1.3}$$

$$\because |\mathbf{V}| > 0 \tag{1.1.4}$$

given curve is ellipse. For an ellipse the point of contact for the tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{1.1.5}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{1.1.6}$$

For the tangents parallel to x-axis, the direction and normal vectors are

$$\mathbf{m} = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{n} = \begin{pmatrix}0\\1\end{pmatrix} \tag{1.1.7}$$

$$\kappa = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{1.1.8}$$

$$= \pm 2 \tag{1.1.9}$$

By substituting $\kappa, \mathbf{n}, \mathbf{V}^{-1}$ in (1.1.5)

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{1.1.10}$$

$$= \begin{pmatrix}-1\\ \pm 2\end{pmatrix} \tag{1.1.11}$$

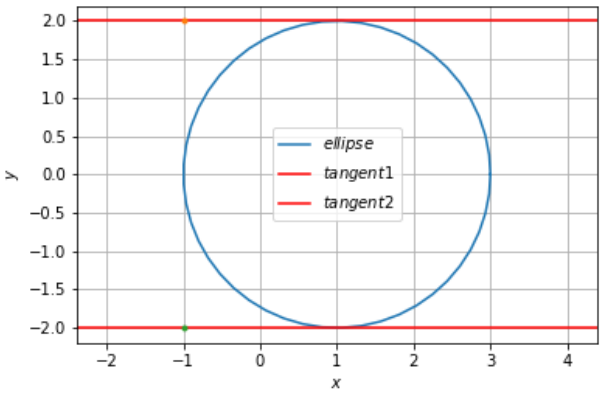

Fig. 1.1. Ellipse with tangents parallel to x axis at points $\mathbf{q} = \begin{pmatrix}-1\\ \pm 2\end{pmatrix}$