

Assignment - 6

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Abstract—This is a simple document to learn about writing vectors, matrices and quadratic forms using latex, draw figures using Python, Latex.

Download all python and latex-tikz codes from
svn co <https://github.com/arjunjc93/Assignment-6>.
git

1 QUADRATIC FORMS G V V SHARMA EXERCISE 2.4

1.1. Find the points on the curve

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0 \quad (1.1.1)$$

at which the tangents are parallel to the x-axis.

Solution: Given curve,

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 3 = 0 \quad (1.1.2)$$

where,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{V}^{-1}, \mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, f = 0 \quad (1.1.3)$$

$$\because |\mathbf{V}| > 0 \quad (1.1.4)$$

given curve is ellipse. For an ellipse the point of contact for the tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (1.1.5)$$

where,

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (1.1.6)$$

For the tangents parallel to x-axis, the direction and normal vectors are

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.1.7)$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (1.1.8)$$

$$= \pm 2 \quad (1.1.9)$$

By substituting $\kappa, \mathbf{n}, \mathbf{V}^{-1}$ in (1.1.5)

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (1.1.10)$$

$$= \begin{pmatrix} -1 \\ \pm 2 \end{pmatrix} \quad (1.1.11)$$

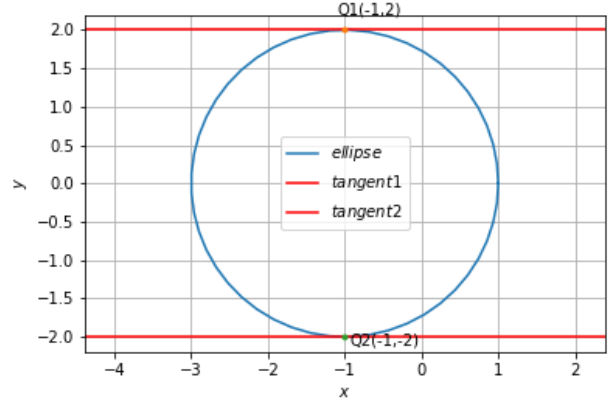


Fig. 1.1. Ellipse with tangents parallel to x axis at points $\mathbf{q} = \begin{pmatrix} -1 \\ \pm 2 \end{pmatrix}$