# ANALYSIS OF FLOOR-SHOE STATIC FRICTION NEEDED TO CONTROL A WHEELCHAIR, GURNEY, OR PALLET JACK ON A RAMP MANUALLY

**George Sotter[1], Steven Stone[2], and Michael J. McCarthy[3]**

[1]Sotter Engineering Corporation, dba Safety Direct America
26705 Loma Verde
Mission Viejo, CA 92691, USA
Corresponding author's e-mail: sottergeo@aol.com

[2]Scientific Testing Technologies, LLC
23052 Alcalde Drive, Suite A
Laguna Hills, CA 92653

[3]University of California at Irvine
Department of Mechanical Engineering
Irvine, CA 92697

**Abstract:** By the definition of coefficient of friction, a pedestrian weighing more automatically gets additional frictional force to help control his own mass: that is, a 220-pound person gets twice as much friction as a 110-pound person. However, when the person is controlling a separate load, that load does not necessarily add to the weight on the feet. A theoretical analysis of static conditions indicates the minimum friction coefficient needed to prevent pedestrian slipping on ramps. The controlled load might be a person on a wheelchair or a gurney; freight on a trolley; steel roofing dragged across a roof in construction, etc. The minimum safe coefficient increases with ramp angle and with a weight ratio: the weight of the separate load the walking person is controlling divided by the weight of the walking person. The minimum coefficient decreases with an increase in the angle (with respect to horizontal) through which force is exerted on the load.

## 1. INTRODUCTION

A high coefficient of friction between shoe (or bare foot) and floor is necessary for safe pedestrian traction. In the past, various workers have used one of two different assumptions for the minimum safe coefficient of friction needed for a ramp: (1) 0.80, constant for all ramps regardless of slope or any load controlled by the pedestrian; and (2) a minimum value for level floors, plus an increment equal to the tangent of the ramp angle. Neither assumption is adequate in general.

By the very definition of coefficient of friction, a pedestrian weighing more automatically gets more frictional force to control his or her own larger mass: that is, a 220-pound person gets twice as much friction as a 110-pound person. If either person is carrying a shopping bag, the weight of the bag adds more frictional force on his or her shoes, fortunately providing more friction to control the extra mass. However, when the person is controlling a separate load such as a shopping cart, that load does not add to the weight on the feet. The result may be that the additional frictional force needed to control a heavy separate load is not available.

In this paper, a theoretical analysis of static conditions indicates the minimum coefficient of friction needed to prevent pedestrian slipping on ramps while manually controlling a load. The load might be a person on a wheelchair or a gurney; freight on a trolley or pallet jack; merchandise in a shopping cart; or construction material being dragged across a sloped roof.

The Access Board of the United States Department of Justice administers the Americans with Disabilities Act (ADA). The Access Board has suggested a minimum coefficient of friction of 0.80 for ramps at angles from 2.86 degrees (slope of 1 in 20) up to 7.12 degrees (slope of 1 in 8). The Building Department of the City of Los Angeles has mandated this requirement for a number of years.

The 0.80 minimum was based on research into movement of unaccompanied disabled persons on ramps. It doesn't consider the manual movement of separate loads, such as a wheelchair pushed by another person, or freight moved manually by means of a trolley or pallet jack.

Coefficient of friction is, by definition, the ratio of frictional force to normal force, the latter typically being the component of weight that is applied at a right angle to the surface. Elementary physics specifies that the coefficient of friction is a constant for a given shoe-floor combination.

Naturally, large loads require large tractive forces to move them manually. When a person *carries* a load, the constant coefficient of friction means that she automatically gets a larger tractive force because of the added weight on her feet. For instance, if she carries 15 pounds of groceries, the added 15 pounds is borne by her shoe bottoms, and because of the constant ratio of tractive force to weight on the shoes, her tractive force increases accordingly.

This fortunate situation doesn't occur when someone *pushes or pulls* a load rather than carrying it. Again, a larger load requires a larger tractive force. However, in this case the tractive force of the shoes doesn't necessarily increase — the added weight is not all being borne by the shoes, but mostly by the wheels of the wheelchair, trolley, etc. Therefore, slipping of the walking person is more likely when the load is separate rather than carried.

*Static* coefficient refers to friction force that exists while the shoe and floor are at rest relative to each other. High static coefficients of friction are associated with safe conditions with regard to slipping. Adequate static coefficients are necessary, though not sufficient, for safety. *Dynamic* coefficient is especially important for wet or lubricated floors.

American Society for Testing and Materials test method ASTM C 1028-96 provides one method of assessing static coefficients of friction, using a laboratory grade of Neolite to represent shoe bottom material. A standard ceramic tile provides a calibration point and a resulting correction factor. A static coefficient of friction of 0.60 or higher (to Neolite) is often specified for safety on a level floor. This is suggested by the U.S. Department of Justice's Access Board, the Ceramic Tile Institute of America, and the City of Los Angeles. The 0.60 minimum includes little or no safety factor. For some flooring, the static coefficient of friction to Neolite is lower than 0.60 only when the walking surface is wet or otherwise lubricated. However, it is lower than 0.80 for many types of flooring, even when the floor is clean and dry.

This investigation considers what increment should be added to the minimum static coefficient of friction to provide safety on a ramp. Results can aid in selection or chemical treatment of flooring for slip-resistant ramps and in selection of footwear for work on ramps. In a pedestrian situation, the actual coefficient of friction is a result of the combination of the flooring and the footwear in the ambient environment and other conditions of use.

## 2. ANALYSIS

Figure 1 shows the situation of interest. A walking person (line CA), with weight W, center of gravity at point C, and feet at point A, is pulling on an object ("load"), DB. The load has weight P, center of gravity at D, and floor contact at B. Floor friction resists the motion of both the walking person and the load. The coefficient of friction of the walking person is $\mu_A$, and that of the load is $\mu_B$.

The walking person and load won't topple over if their force moments are in equilibrium. In order to satisfy moment equilibrium at A, the walking person must lean through an angle θ with respect to vertical. Similarly, at B the load must lean through an angle β. In the case of a wheelchair, the contact point at B is offset from the center of gravity by the angle β.

The force, T, exerted by the walking person to control the load, acts through the angle γ with respect to horizontal. For the analysis we want to specify α, γ, W, P, and $\mu_B$ and then calculate T, θ, and $\mu_A$.

Let the distance between A and C be denoted as L. The requirement that the moments at AC be in equilibrium requires that

$$T L \sin (\pi/2 - \theta - \gamma) = W L \sin \theta$$

or,

$$T = W \sin \theta / \cos (\theta + \gamma)$$

The distance from B to D is defined as λ. For the moments at BD to be in equilibrium,

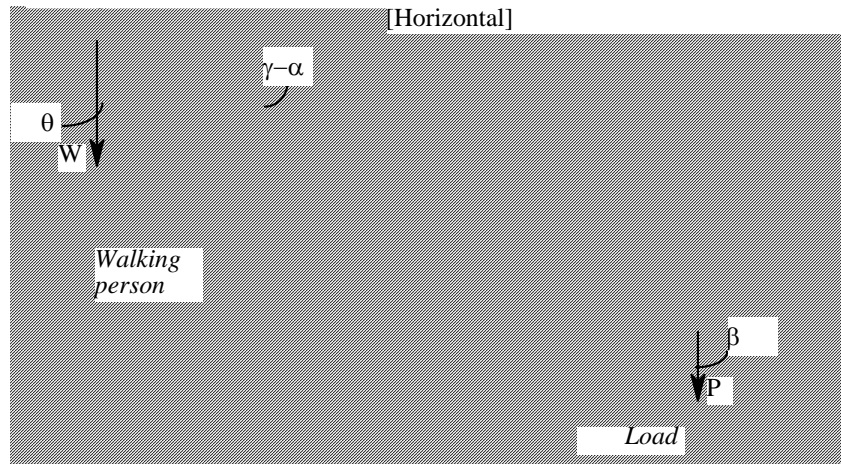$$T \lambda \sin (\pi/2 - \gamma + \beta) = P \lambda \sin \beta$$

**Figure 1. Situation of interest**

or,

$$T = (P \sin \beta)/\cos (\gamma - \beta)$$

Parallel to the plane at A,

$$F_A - W \sin \alpha = T \cos (\gamma - \alpha)$$

Perpendicular to the plane,

$$N_A - W \cos \alpha - T \sin (\gamma - \alpha) = 0$$

$$F_A = W \{\sin \alpha + [\sin \theta \cos (\gamma - \alpha)/\cos (\theta + \gamma)]\}$$

$$N_A = W \{\cos \alpha + [\sin \theta \sin (\gamma - \alpha)/\cos (\theta + \gamma)]\}$$

Parallel to the plane at D,

$$F_B - (P \sin \alpha) + T \cos (\gamma - \alpha) = 0$$

Perpendicular to the plane,

$$N_B - P \cos \alpha + T \sin (\gamma - \alpha) = 0$$

$$F_B = P \{\sin \alpha - [\sin B \cos (\gamma - \alpha)/\cos (\gamma - B)]\}$$

$$N_B = P \{\cos \alpha - [\sin B \sin (\gamma - \alpha)/\cos (\gamma - B)]\}$$

The limiting friction coefficients F/N, where F is frictional force and N is force normal to the surface, are

$$\mu_A = F_A /N_A = [\sin \alpha \cos (\theta + \gamma) + \sin \theta \cos (\gamma - \alpha)]/[\cos \alpha \cos (\theta + \gamma) + \sin \theta \sin (\gamma - \alpha)]$$

$$\mu_B = F_B /N_B = [\sin \alpha \cos (\gamma - \beta) - \sin \beta \cos (\gamma - \alpha)]/[\cos \alpha \cos (\gamma - \beta) - \sin \beta \sin (\gamma - \alpha)]$$

Now we solve for $\beta$ and $\theta$

$$\cot \beta = p \cot \theta - [(p + 1) \tan \gamma]$$

where $p = P/W$. That is, the weight of the load (e.g. wheelchair plus its passenger, or pallet truck plus its freight) is normalized by dividing by the weight of the person walking.

$$\cot \theta = [(\cot \beta) + (p+1) \tan \gamma]/p$$
$$\cot \theta = \{[\mu_B (\tan \alpha) + 1]/[(\tan \alpha) - \mu_B]\}/p + [(p+1)(\tan \gamma )]/p$$

$$\mu_A = [\cot \theta \sin \alpha + \cos \alpha]/[\cot \theta \cos \alpha - \sin \alpha]$$

or,

$$\tan \theta = p[\tan \alpha - \mu_b]/\{[\mu\beta + (p+1) \tan \gamma] \tan \alpha + [1 - \mu_\beta (p+1) \tan \gamma]\}$$

and

$$\mu_A = [(\tan \theta \cos \alpha) + \sin \alpha]/[\cos \alpha - (\tan \theta \sin \alpha)]  = \tan (\alpha + \theta)$$

The pulling force, T, is

$$T = P [\sin \alpha - \mu_B \cos \alpha]/[\cos (\gamma - \alpha) - \mu_B \sin (\gamma - \alpha)]$$

As mentioned previously, the value $\mu_A$ is only what's required to maintain static conditions. To obtain a result for practical application, we shall assume that this is an incremental value that can be added to the value of 0.60 that is sometimes considered a requirement for safe walking on a level floor when using laboratory-grade Neolite heel material.

## 3. RESULTS AND DISCUSSION

Figure 2 summarizes the results for the case in which a person pulls or controls a weight by exerting a force at the same angle as the surface of the ramp. Use of Figure 2 requires knowing the ramp angle and the ratio, P/W, of the load to the weight of the walking person or persons. The definitions, again, are:

P        Weight of the load; e.g., the combined weight of the passenger in a wheelchair plus the wheelchair, or the combined weight of freight and trolley or pallet jack

W        Weight of the walking person(s) controlling the movement of the load

Given these parameters, Figure 2 yields an estimated minimum coefficient of friction necessary to avoid slipping. For example, consider a ramp of 7.12 degrees (slope 1:8), and a 150-pound person controlling a 300-pound load including pallet jack. The ratio P/W = 300/150 = 2.0, and the top curve of the graph (angle 7.12 degrees) indicates a minimum static coefficient of friction of 0.97. Since the actual coefficient of friction of most shoe-floor combinations is 1.0 or less, the 300-pound load is probably the maximum safe one for preventing slips in this example. The ramp angles shown in Fig. 2 are referenced in the Americans with Disabilities Act as slopes of 1:20 (2.86°), 1:12, 1:10, and 1:8 (7.12°).

The four points on the extreme left-hand side of Fig. 2 are for a weight ratio of zero. They indicate that when there is no load except one being carried (not pushed or pulled), the weight of the walking person doesn't affect the required minimum coefficient of friction — and that the minimum required is always below 0.80. This is consistent with the Access Board recommendation, mentioned earlier, for ramps — as long as there is no separate load, such as a wheelchair or pallet jack.

Figure 2 illustrates the important principle that the minimum coefficient of friction for safety depends on the angle of the ramp and on the ratio of weights, P/W. This means, for instance, that for a given load and ramp angle a lighter person is more likely to slip than a heavier person. (This isn't true when the person is *carrying* the load.) The issue regarding

weight here isn't *strength,* but rather shoe traction. Also, and rather obviously, all ramps are not the same (as assumed in the Access Board's simplified recommendation); the minimum coefficient of friction required for safety depends on how steep the ramp is.

Next, consider the case in which the pallet jack's handle is attached at a point that, on a level floor, is lower than the walking person's hands. The force from the walking person's arms is then effectively tending to lift the pallet jack *up relative to the ramp's surface,* rather than just pulling it *parallel* to the surface. This results in an opposing force of extra weight (and therefore extra traction) at the walking person's feet.
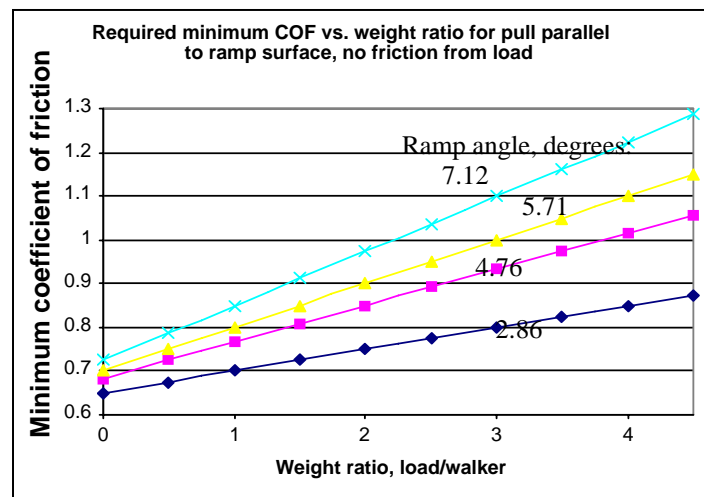


Figure 2. Calculated minimum safe static coefficient of friction
for a ramp when pull is parallel to the ramp surface

Figure 3 shows the case in which the pull is at 45 degrees relative to horizontal. In this case the required minimum coefficient of friction is lower because of the added weight applied to the feet by the load. For a weight ratio of 2.0 and a ramp angle of 7.12 degrees the minimum coefficient of friction for safety is 0.91, compared to 0.97 in the previous example.

Other factors not considered in the model of Figures 1–3 are acceleration and deceleration, which require more traction than controlling at a constant speed; and dynamic effects associated with each individual person's gait, which can also lead to increased tendency toward slipping. These additional effects, if included, could further increase the calculated increment in coefficient of friction needed for safety.

Coefficient of friction is affected not only by the ramp's surface, but by the footwear bottoms. Here, the situation is different for employees than it is for visitors to a workplace: the employee's footwear is potentially subject to control, or at least to good advice.

In the American Society for Testing and Materials (ASTM) C 1028 static friction test of flooring, Neolite is a surrogate for what is considered a minimum reasonably safe soling. Actual solings in use can be better or worse than this. Improved traction from footwear is one way of dealing with a situation in which traction is inadequate. Controlling footwear is feasible in some work situations, but is not feasible where the public has access. There are no safety standards for footwear traction in the United States, and most footwear manufacturers give no information or warnings regarding traction for their products. A jury verdict may be necessary to establish whether the shoes' heels or soles were safe.

When coefficient of friction of the walking person's shoes with the floor isn't high enough for safety when controlling a load, the situation can be improved by the walking person getting help from others (more weight on more pairs of shoes to supply additional tractive force), or by using a mechanically powered wheelchair or pallet jack.

When the floor is wet or otherwise lubricated, *hydroplaning* of a moving shoe can occur and lead to a slip. For this reason, a high static coefficient of friction is not sufficient for safety. Surface micro-roughness of an adequate magnitude and type (e.g., sharp peaks) can help prevent hydroplaning (Sotter, 2000). Roughness of soles is helpful, but roughness of

the floor is more effective — or at least it is when the flooring is harder than the sole material. The analysis in this paper doesn't consider hydroplaning.

The equation for T allows calculation of the pulling force. This is the force necessary to maintain static conditions, and it is independent of the weight of the walking person. More strength (larger pulling force) is required when pulling up as a price for the increased traction. Part of the pulling force goes into a lifting force on the load, which increases the weight on the shoes to yield the friction benefit, providing safety with a slightly lower static coefficient of friction.

Friction from the load is negligible in many cases in which wheels support the weight, provided that the wheel bearings are lubricated and in good condition. High friction from the load occurs, for instance, when workers drag construction materials across a working surface.
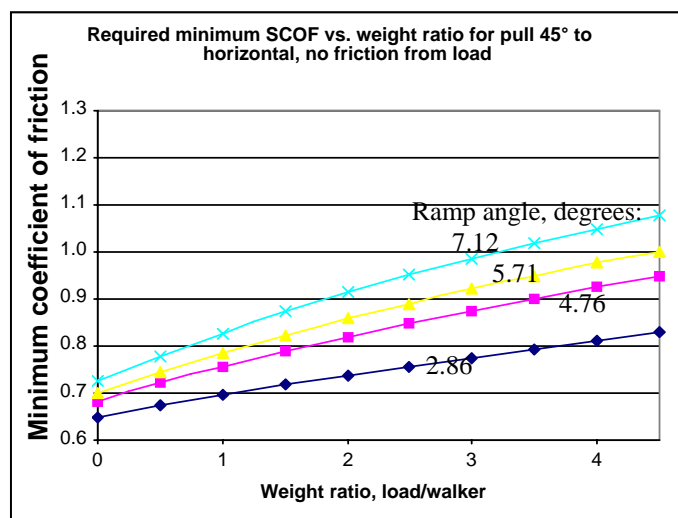


**Figure 3. Calculated minimum safe static coefficient of friction for pull 45° upward**

Apart from frictional forces, ergonomic considerations affect safety on a ramp. Wilkinson (1997) points out that the force that a person is *capable* of exerting on the wheelchair or pallet jack might be well in excess of what is *safe* for that person in the long run for repetitive tasks involving pushing or pulling. Thus, injury could result even though a slip does not occur. (When a slip does occur, injury may result even if there's no fall.)

Figure 2 indicates that the ADA-related minimum of 0.80 for static coefficient of friction of a ramp may not always be adequate for safety. Consider the case of a 110-pound nurse wheeling a 240-pound patient down a 5.5-degree portion of a ramp in a wheelchair. The Americans with Disabilities Act allows such a ramp for as much as a 6-inch rise, which in this case could be a length along the plane of the ramp of some five feet. Interpolating on Figure 2 for 5.5 degrees and weight ratio of 240/110 = 2.18 indicates that a minimum static coefficient of friction of approximately 0.91 is required — well above the 0.80 ADA suggested minimum.

In the healthcare industry, gurneys are used even more often than wheelchairs, and manpower shortages in U.S. healthcare mean that there are not always enough personnel to push gurneys without overtaxing those who do the pushing. This can lead to slips and ergonomic injuries to healthcare personnel, as well as a hazard to patients.

## 4. REFERENCES

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