Notes for Probability by Varadhan, published by AMS

Arjun Krishnan

Behzad Mehrdad

CHAPTER 1

Weak Convergence

1. Page 19

We'll use the shorthand CF to represent characteristic function because of its frequent use in this chapter. The symbol $\phi(t)$ is most often used to represent the CF. The first exercise is straightforward calculation.

EXERCISE 1. (1) The CF of degenerate distribution at a is

$$\int e^{itx} d\delta_a(x) = e^{ita}$$

(2) The CF of the binomial distribution is

$$\sum_k \binom{n}{k} p^k (1-p)^{n-k} e^i tk = (pe^{it} + 1 - p)^n$$

Theorem 2.1 and it's proof are clearly explained.

EXERCISE 2. Show that if $\int |x| d\alpha(x) < \infty$, then $\phi(t)$ is continuously differentiable and $\phi'(0) = i \int x d\alpha$.

We first have to show that $\phi(t)$ is differentiable and so we consider the quotient,

$$\frac{\phi(t + \Delta t) - \phi(t)}{\Delta t} =$$