Estimating the sleep need of adolescents using nonlinear mixed effects modelling

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Introduction

How much sleep is enough sleep? Using changes in lapses during the psychomotor vigilance task (PVT), Van Dongen et al. (2003) estimated that it is close to 8.16h. In their study, participants were split into different sleep restriction conditions and lapses were analysed for each of these conditions across several days. Lapses in a PVT task are when participants miss reacting to a stimulus that they are supposed to. In general, the number of lapses increases as the amount of sleep restriction increases. Van Dongen et al. modelled this alternatively as lapses increasing as the amount of wakefulness during the previous day increase beyond a certain critical value. This was formulated mathematically as follows:

$$lapses = b(excess)^{\theta}$$

Here, b represents rate of change of lapses per unit change in the nonlinear part of the equation. Excess denotes sleep in excess of the critical waking duration. Theta accommodates nonlineaerity in the relationship. Excess can then be formulated as the difference between Cumulative Wake Time (CWT) across a number of days and the critical waking duration multiplied across the number of days.

$$lapses = b(CWT - critical * day)^{\theta}$$

Van Dongen's study was based on a population of adults between the ages 21 and 38. In this analysis, I use the same approach but for adolescents between the ages of 15 and 19. The data here is from the Need for sleep studies (Lo et al, 2016; 2017; 2019; 2020). There are four different sleep conditions - 5h, 6.5 h, 8h and 9h across the studies. The baseline sleep for all sleep conditions was 9h. Participants took the neurobehavioral tests three times every day across the sleep restriction days. For the analysis, I need the average lapses across days. These studies also had periods of recovery sleep, either inbetween or at the end. However, only the baseline sleep plus the days of sleep restriction until the first sleep recovery period is sufficient for this analysis.

Part 1 - with aggregated test conditions

The Dataset

The dataset for this analysis came from NFS1, NFS2, NFS4 and NFS5. Here is a section of the original dataset. Most of the columns have been omitted for the sake of presentation here.

##		subj	gender	day_num	${\tt test_num}$	kss	${\tt pvt_median_rt}$	pvt_sd_rt	<pre>pvt_lapses</pre>
##	1	NFS001	${\tt Female}$	1	1	6	229	78	1
##	2	NFS001	Female	1	2	7	250	90	2
##	3	NFS001	Female	1	3	4	244	68	1
##	4	NFS001	Female	2	1	4	245	59	1
##	5	NFS001	Female	2	2	4	253	44	0
##	6	NFS001	Female	2	3	5	253	52	0

Data cleaning

First, I selected only the columns that were required for the analysis.

```
##
       subj day_num
                          group lapses
## 1 NFS001
                   1 nonap_5hx7
## 2 NFS001
                   1 nonap_5hx7
                                      2
                   1 nonap_5hx7
## 3 NFS001
                                      1
## 4 NFS001
                   2 nonap_5hx7
                                      1
## 5 NFS001
                   2 nonap 5hx7
                                      0
## 6 NFS001
                   2 nonap_5hx7
                                      0
```

Since participants did the PVT task three times a day, I summarised the results to get the average PVT lapses for each day.

```
## # A tibble: 6 x 4
               subj, group [1]
## # Groups:
     subj
            group
                       day_num lapses
     <chr>
           <chr>
                         <int>
                                <dbl>
## 1 NFS001 nonap 5hx7
                             1 1.33
## 2 NFS001 nonap_5hx7
                             2 0.333
## 3 NFS001 nonap 5hx7
                             3
                                5
## 4 NFS001 nonap_5hx7
                             4
                                6.33
## 5 NFS001 nonap_5hx7
                             5 7
## 6 NFS001 nonap 5hx7
                             6 17.3
```

I then imported the TST data.

```
##
       subj TST day_num study
## 1 NFS001 7.17
                        1
                           NFS1
## 2 NFS001 3.47
                        2
                           NFS1
## 3 NFS001 3.97
                           NFS1
                        4
## 4 NFS001 4.20
                           NFS1
## 5 NFS001 4.30
                        5
                           NFS1
## 6 NFS001 4.23
                        6
                           NFS1
```

Then I merged the TST dataset and the NFS dataset.

```
##
       subj day_num
                          group
                                     lapses TST study
## 1 NFS001
                   1 nonap_5hx7
                                  1.3333333 7.17
                                                   NFS1
## 2 NFS001
                   2 nonap_5hx7
                                  0.3333333 3.47
                                                   NFS1
## 3 NFS001
                   3 nonap_5hx7
                                  5.0000000 3.97
                                                   NFS1
## 4 NFS001
                   4 nonap_5hx7
                                  6.3333333 4.20
                                                   NFS1
## 5 NFS001
                   5 nonap_5hx7
                                 7.0000000 4.30
                                                   NFS1
## 6 NFS001
                   6 nonap_5hx7 17.3333333 4.23
                                                   NFS1
```

I created a new column that denotes their total bed time during the previous night. On baseline days, they had 9h of sleep and their bed time varies based on their sleep condition on the other days. The day numbers were also made to align across the different studies and start with day 1 being the baseline day.

##		subj	day_num	group	lapses	TST	study	TBT	condition
##	1	NFS001	1	nonap_5hx7	1.3333333	7.17	NFS1	9	5
##	2	NFS001	2	nonap_5hx7	0.3333333	3.47	NFS1	5	5
##	3	NFS001	3	nonap_5hx7	5.0000000	3.97	NFS1	5	5
##	4	NFS001	4	nonap_5hx7	6.3333333	4.20	NFS1	5	5
##	5	NFS001	5	nonap_5hx7	7.0000000	4.30	NFS1	5	5
##	6	NFS001	6	nonap_5hx7	17.3333333	4.23	NFS1	5	5

Finally I calculated the cumulative wake duration for each participant based on the TBT and TST estimates.

```
## # A tibble: 6 x 10
   # Groups:
                subj [1]
##
     subj
             day_num group
                                  lapses
                                            TST study
                                                         TBT condition TWT_tbt TWT_tst
     <chr>>
                                   <dbl>
                                                                  <dbl>
                                                                           <dbl>
               <dbl> <chr>
                                          <dbl> <chr>
                                                       <dbl>
                                                                                    <dbl>
## 1 NFS001
                    1 nonap_5hx7
                                   1.33
                                           7.17 NFS1
                                                           9
                                                                      5
                                                                              15
                                                                                     16.8
## 2 NFS001
                   2 nonap_5hx7
                                   0.333
                                           3.47 NFS1
                                                           5
                                                                      5
                                                                              34
                                                                                     37.4
                                                           5
                                                                      5
                                                                              53
## 3 NFS001
                   3 nonap_5hx7
                                   5
                                           3.97 NFS1
                                                                                     57.4
                                   6.33
                                                           5
                                                                      5
                                                                              72
## 4 NFS001
                    4 nonap_5hx7
                                           4.2
                                                NFS1
                                                                                     77.2
                    5 nonap 5hx7
                                                                      5
## 5 NFS001
                                   7
                                           4.3
                                                           5
                                                                              91
                                                                                     96.9
                                                NFS1
                                                                      5
## 6 NFS001
                    6 nonap 5hx7 17.3
                                           4.23 NFS1
                                                           5
                                                                             110
                                                                                    117.
```

Clean data

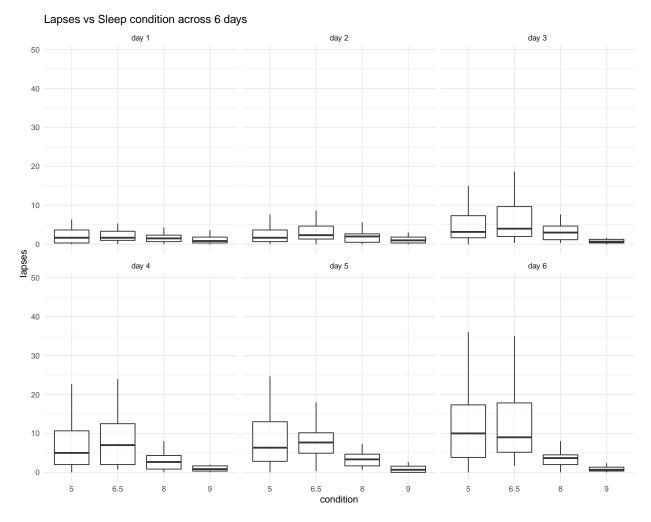
Summing up, the last baseline day plus the first five sleep manipulation days were used for the analysis from the NFS studies. 'Lapses' represents mean lapses on a particular day (day_num). TST represents the total sleep time on the previous night. TBT represents the total bed time on the previous night. Total Wake Time was calculated as the cumulative total wakeful duration across days based on both the TST estimate [(TWT_tst = 24 - TST_tbt) x day_num] and TBT estimate [(TWT_tbt = 24 - TST_tbt) x day_num]. After cleaning, there were 834 observations in total (6 days x 139 subjects).

```
## # A tibble: 6 x 10
  # Groups:
               subj [1]
            day_num group
##
     subj
                                 lapses
                                          TST study
                                                       TBT condition TWT_tbt TWT_tst
##
     <chr>
              <dbl> <chr>
                                  <dbl> <dbl> <chr>
                                                                <dbl>
                                                                        <dbl>
                                                                                 <dbl>
                                                     <dbl>
## 1 NFS001
                   1 nonap_5hx7
                                 1.33
                                         7.17 NFS1
                                                         9
                                                                    5
                                                                           15
                                                                                  16.8
                                                                           34
## 2 NFS001
                   2 nonap_5hx7
                                 0.333
                                         3.47 NFS1
                                                         5
                                                                    5
                                                                                 37.4
## 3 NFS001
                   3 nonap_5hx7
                                 5
                                         3.97 NFS1
                                                         5
                                                                    5
                                                                           53
                                                                                 57.4
                   4 nonap_5hx7
                                 6.33
                                         4.2
                                                                    5
                                                                           72
                                                                                 77.2
## 4 NFS001
                                              NFS1
                                                         5
## 5 NFS001
                   5 nonap_5hx7
                                 7
                                         4.3
                                              NFS1
                                                         5
                                                                    5
                                                                           91
                                                                                 96.9
## 6 NFS001
                   6 nonap_5hx7 17.3
                                         4.23 NFS1
                                                         5
                                                                    5
                                                                          110
                                                                                 117.
```

Visualising the dataset

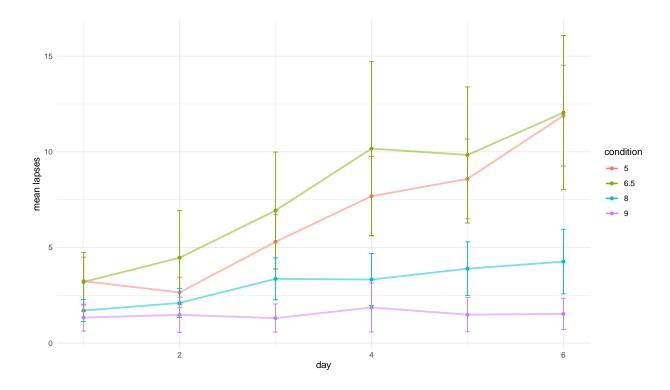
For this analysis, I have mainly focused on no nap conditions. Hence the visualizations below do not include the conditions that had nap time.

Boxplot - Lapses vs Sleep Condition



As it can be seen, the number of lapses increases across the days as the level of sleep restriction increases. This can be noted more clearly in the following charts

Line plot - Lapses vs Day



Estimating Sleep Need

TBT based estimate

Summary I'm using the nlme() function from the nlme library to fit a nonlinear mixed effects model to my data.

```
## Nonlinear mixed-effects model fit by maximum likelihood
## Model: lapses ~ b * (TWT_tbt - crit * day_num)^theta
## Data: data.nonap
## AIC BIC logLik
## 4360.464 4384.035 -2175.232
##
```

```
## Random effects:
## Formula: crit ~ 1 | subj
              crit Residual
## StdDev: 1.696446 2.472632
## Fixed effects: b + theta + crit ~ 1
           Value Std.Error DF t-value p-value
         3.276916 0.30539792 683 10.72999
## b
## theta 0.667553 0.02602681 683 25.64869
                                                0
## crit 16.213213 0.17853896 683 90.81051
                                                0
## Correlation:
        b
               theta
## theta -0.904
## crit 0.342 -0.222
##
## Standardized Within-Group Residuals:
##
          Min
                       Q1
                                  Med
                                               QЗ
## -4.08657132 -0.66240176 -0.01365964 0.60578292 5.07993341
## Number of Observations: 824
## Number of Groups: 139
RMSE
## [1] 3.158632
```

intervals(TBT.nonap.lapses)

Estimates and their 95% confidence intervals

```
## Approximate 95% confidence intervals
##
## Fixed effects:
##
             lower
                         est.
                                   upper
         2.6783772 3.2769161 3.8754551
## b
## theta 0.6165443 0.6675534 0.7185624
## crit 15.8633011 16.2132135 16.5631259
## attr(,"label")
## [1] "Fixed effects:"
##
## Random Effects:
   Level: subj
##
             lower
                       est.
                               upper
## sd(crit) 1.42516 1.696446 2.019372
## Within-group standard error:
     lower est.
                       upper
## 2.341592 2.472632 2.611006
```

TST based estimate

Summary

```
## Nonlinear mixed-effects model fit by maximum likelihood
    Model: lapses ~ b * (TWT_tst - crit * day_num)^theta
##
    Data: data.nonap
##
         AIC
                  BIC
                         logLik
##
     4406.687 4430.258 -2198.344
##
## Random effects:
## Formula: crit ~ 1 | subj
##
              crit Residual
## StdDev: 1.402444 2.50208
## Fixed effects: b + theta + crit ~ 1
            Value Std.Error DF t-value p-value
##
         3.622969 0.31543308 683 11.48570
## theta 0.674452 0.02582812 683 26.11308
                                                 0
## crit 17.131346 0.14233562 683 120.35881
                                                 0
## Correlation:
##
               theta
        b
## theta -0.889
## crit 0.334 -0.208
## Standardized Within-Group Residuals:
                       Q1
                                  Med
                                               QЗ
## -4.29474310 -0.62959079 -0.02426273 0.57142563 4.97521379
## Number of Observations: 824
## Number of Groups: 139
RMSE
```

[1] 3.084051

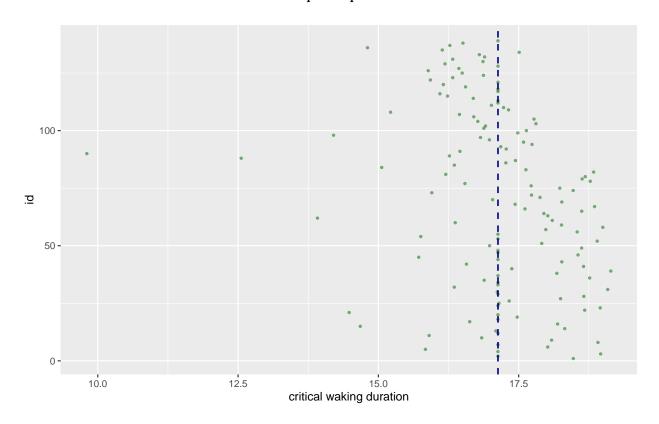
intervals(TST.nonap.lapses)

Estimates and their 95% confidence intervals

```
## Approximate 95% confidence intervals
##
##
    Fixed effects:
##
              lower
                          est.
                                    upper
## b
          3.0047630 3.6229695 4.2411760
## theta 0.6238322 0.6744519 0.7250715
   crit 16.8523870 17.1313457 17.4103045
   attr(,"label")
   [1] "Fixed effects:"
##
##
    Random Effects:
##
    Level: subj
##
               lower
                         est.
## sd(crit) 1.193168 1.402444 1.648426
##
##
    Within-group standard error:
##
      lower
                est.
                        upper
## 2.369793 2.502080 2.641752
```

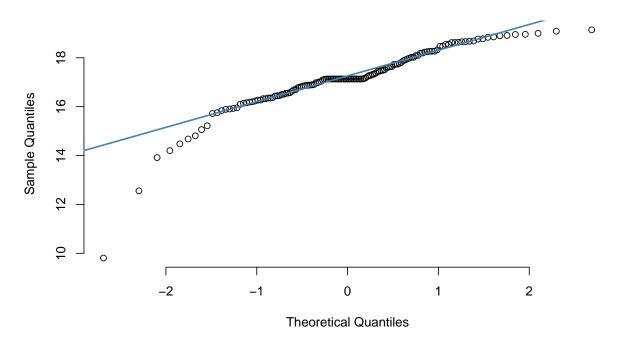
Visualising the results of TST based estimate

Dot Plot - Critical Wake Durations across participants



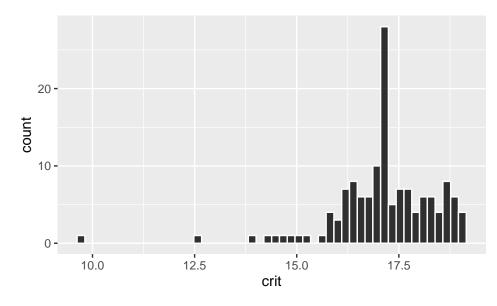
QQ Plot - Normality of distribution



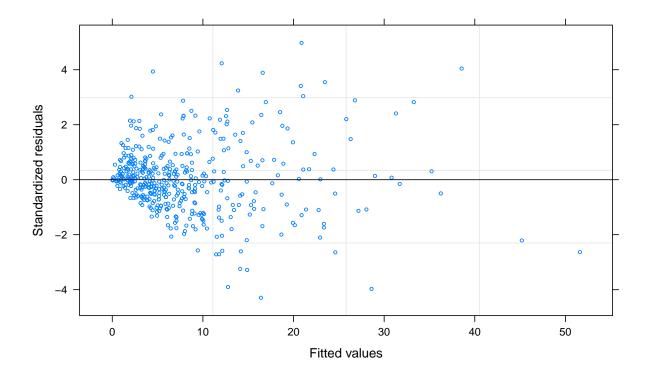


Histogram - Normality of distribution

The plot indicates a left skew. I checked the histogram

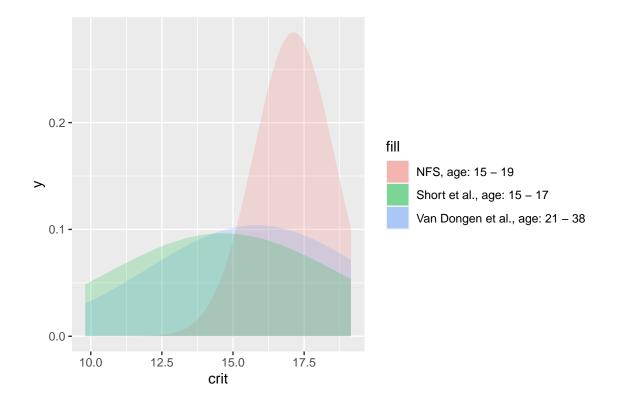


Residuals Plot



Comparison against estimates based on the literature

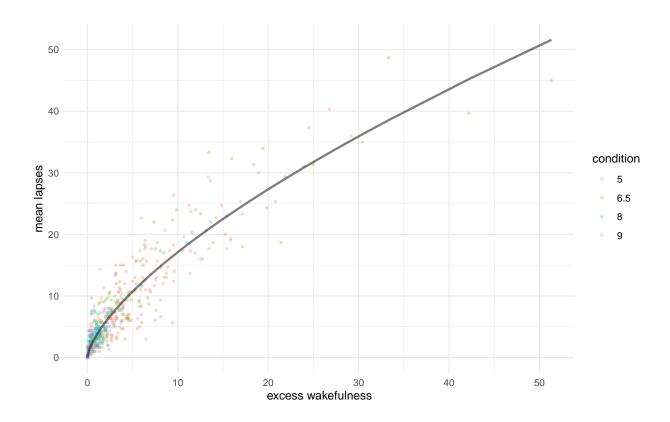
I compared the distribution obtained from this estimate with Van Dongen et al. (2003) and Short et al. (2018) estimates for the critical waking duration



Mean lapses vs excess wakefulness

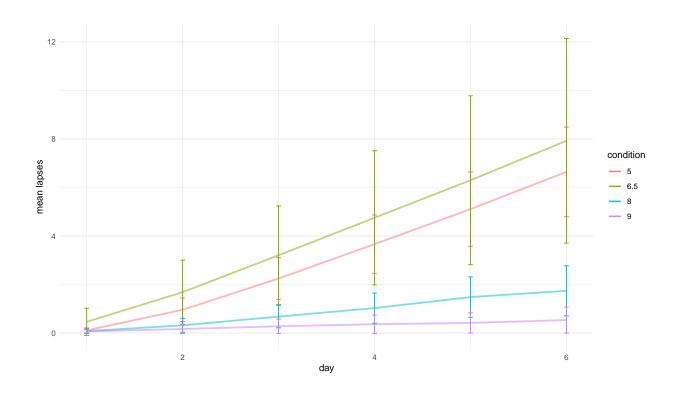
$$lapses = 3.62 * (excess)^{0.67}$$

The line represents predicted number of lapses, dots represent the actual number of lapses observed.



Predicted Lapses vs Day

$$lapses = 3.62 * (CWT - 17.13 * day)^{0.67}$$



To conclude, the estimate of critical waking duration (17.13) was greater than that of Van Dongen's estimate (15.84). Our results seem to suggest that for the average adolescent (based on the sample used for this study), 6.87h of sleep would be sufficient to prevent the build of neurobehavioral deficits at least in the context of the PVT task.

Part 2 - without aggregating test conditions

In this part of the analysis, I won't be aggregating the test results.

The Dataset

I reloaded the nfs dataset again for the analysis.

##		subj	gender	day_num	${\tt test_num}$	kss	${\tt pvt_median_rt}$	pvt_sd_rt	<pre>pvt_lapses</pre>
##	1	NFS001	${\tt Female}$	1	1	6	229	78	1
##	2	NFS001	Female	1	2	7	250	90	2
##	3	NFS001	Female	1	3	4	244	68	1
##	4	NFS001	Female	2	1	4	245	59	1
##	5	NFS001	Female	2	2	4	253	44	0
##	6	NFS001	Female	2	3	5	253	52	0

Data cleaning

Again, similar to part 1, I selected only the columns that were required for the analysis.

##		subj	day num	group	lapses
##	1	NFS001	<i>3</i> –	nonap_5hx7	1
##	2	NFS001	1	nonap_5hx7	2
##	3	NFS001	1	nonap_5hx7	1
##	4	NFS001	2	$nonap_5hx7$	1
##	5	NFS001	2	${\tt nonap_5hx7}$	0
##	6	NFS001	2	${\tt nonap_5hx7}$	0
##	7	NFS001	3	$nonap_5hx7$	6
##	8	NFS001	3	nonap_5hx7	6

I created a new column called ST that indicated the TBT from the past night

##		subj	day_num	group	lapses	TBT	condition
##	1	NFS001	1	nonap_5hx7	1	9	5
##	2	NFS001	1	nonap_5hx7	2	9	5
##	3	NFS001	1	nonap_5hx7	1	9	5
##	4	NFS001	2	nonap_5hx7	1	5	5
##	5	NFS001	2	nonap_5hx7	0	5	5
##	6	NFS001	2	nonap_5hx7	0	5	5
##	7	NFS001	3	nonap_5hx7	6	5	5
##	8	NFS001	3	nonap_5hx7	6	5	5

Next, I used the TBT data from each night to calculate the cumulative total wake time based on the TBT measures until each test time. For example, if test 1 is at 10 AM and this is day 3, the cumulative wake duration would be the (Total hours since start of first wake time after the end of baseline sleep - cumulative TBT since start of protocol). Additionally, for conditions with nap, I subtracted the nap duration from the cumulative wake duration for test in which participants had a nap preceding the test and during the day.

[1] 3996

```
group lapses TBT condition test TWT wake_time test_time WD
##
       subj day_num
## 1 NFS001
                  1 nonap_5h
                                   1
                                                  5
                                                       1
                                                          19
## 2 NFS001
                  1 nonap_5h
                                   2
                                       9
                                                  5
                                                       2
                                                          24
                                                                      6
                                                                               15 9
## 3 NFS001
                  1 nonap_5h
                                   1
                                       9
                                                  5
                                                       3
                                                          29
                                                                      6
                                                                               20 14
## 4 NFS001
                                       5
                                                  5
                                                          38
                                                                      6
                                                                               10
                                                                                   4
                  2 nonap_5h
                                   1
                                                       1
## 5 NFS001
                  2 nonap 5h
                                   0
                                       5
                                                  5
                                                       2 43
                                                                      6
                                                                               15
                                                                                   9
                  2 nonap_5h
## 6 NFS001
                                   0
                                                  5
                                                       3 48
                                                                      6
                                                                               20 14
                                       5
## 7 NFS001
                  3 nonap 5h
                                   6
                                       5
                                                  5
                                                       1
                                                          57
                                                                      6
                                                                               10
                                                                                   4
## 8 NFS001
                  3 nonap_5h
                                   6
                                       5
                                                  5
                                                       2
                                                          62
                                                                      6
                                                                               15
                                                                                  9
##
       type
## 1 no nap
## 2 no nap
## 3 no nap
## 4 no nap
## 5 no nap
## 6 no nap
## 7 no nap
## 8 no nap
```

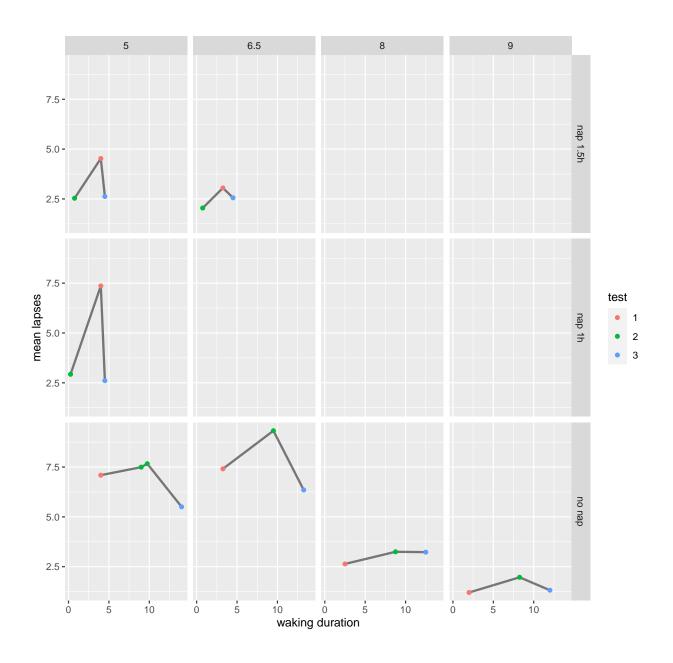
Clean dataset

Final dataset after cleaning

```
subj day_num
                        group lapses TBT condition test TWT wake_time test_time WD
## 1 NFS001
                                                  5
                  1 nonap_5h
                                   1
                                        9
                                                       1
                                                           19
                                                                      6
                                                                                10
                                                  5
                                                       2
## 2 NFS001
                  1 nonap_5h
                                   2
                                        9
                                                           24
                                                                      6
                                                                                15
                                                                                   9
## 3 NFS001
                                        9
                                                  5
                                                       3
                                                          29
                                                                      6
                                                                                20 14
                  1 nonap_5h
## 4 NFS001
                  2 nonap_5h
                                   1
                                       5
                                                  5
                                                       1
                                                          38
                                                                      6
                                                                                10
                                                                                   4
## 5 NFS001
                  2 nonap_5h
                                   0
                                       5
                                                  5
                                                       2
                                                          43
                                                                      6
                                                                                15
                                                                                   9
## 6 NFS001
                  2 nonap_5h
                                   0
                                                  5
                                                       3
                                       5
                                                          48
                                                                      6
                                                                                20 14
## 7 NFS001
                  3 nonap_5h
                                   6
                                        5
                                                  5
                                                       1
                                                          57
                                                                      6
                                                                                10
                                                                                   4
## 8 NFS001
                  3 nonap_5h
                                   6
                                                  5
                                                       2
                                                                      6
                                        5
                                                          62
                                                                                15 9
##
       type
## 1 no nap
## 2 no nap
## 3 no nap
## 4 no nap
## 5 no nap
## 6 no nap
## 7 no nap
## 8 no nap
```

Visualising the Dataset

mean lapses against time awake (from sleep at night or nap)

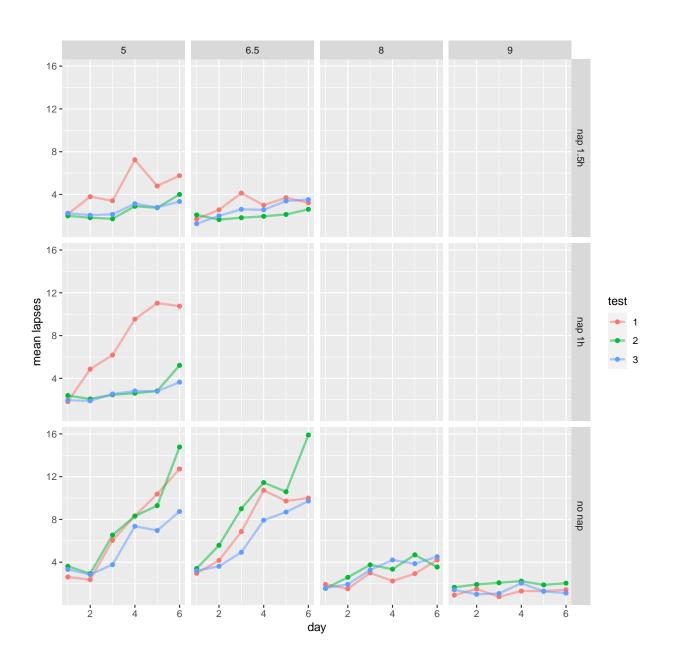


Mean lapses vs Day

```
#mean lapses vs Day

data.nfs %>% mutate(across(group, factor, levels=c("control_9h","nonap_8h","nap90_6pt5h",
    ggplot(aes(x = day_num, y = lapses, color = as.factor(test))) +
    stat_summary(fun = mean, geom = "line", size = 1, alpha = 0.5) +
    stat_summary(fun = mean, geom = "point") +
```

```
#stat_summary(fun.data = mean_cl_normal, geom = "errorbar", aes(group = as.factor(condition)), width
labs(x = "day", y = "mean lapses", title = " ", color = "test") +
facet_grid(~ type ~condition)
```



Estimating Sleep Need

All conditions

```
library(nlme)
```

```
#inclusive of nap conditions
sleep.allconditions.lapses <- nlme(lapses ~ b*(TWT - crit*day_num)^theta,</pre>
                   data = data.nfs,
                   fixed = b + crit + theta ~ 1,
                   random = crit ~ 1,
                   groups = ~ subj,
                   start = c(b = 2, crit = 14, theta = 0.5),
                   na.action = na.omit
                   )
summary(sleep.allconditions.lapses)
```

Summary

```
## Nonlinear mixed-effects model fit by maximum likelihood
    Model: lapses ~ b * (TWT - crit * day_num)^theta
##
    Data: data.nfs
##
         AIC
                  BIC
                         logLik
##
     26137.06 26168.53 -13063.53
## Random effects:
## Formula: crit ~ 1 | subj
##
                 crit Residual
## StdDev: 0.000572914 6.366384
##
## Fixed effects: b + crit + theta ~ 1
            Value Std.Error DF t-value p-value
##
         0.205547 0.0461620 3771 4.45272
## crit 13.822926 0.3283707 3771 42.09549
                                                0
## theta 1.077504 0.0588213 3771 18.31828
## Correlation:
##
        b
               crit
## crit
        0.641
## theta -0.959 -0.412
## Standardized Within-Group Residuals:
                     Q1
                               Med
## -1.7681189 -0.5314576 -0.1973687 0.2172085 7.9065137
## Number of Observations: 3995
## Number of Groups: 222
RMSE
## [1] 6.366383
Estimates and their 95% confidence intervals
## Approximate 95% confidence intervals
##
## Fixed effects:
##
             lower
```

est.

upper

```
## b     0.1150758     0.2055468     0.2960177
## crit     13.1793665     13.8229262     14.4664858
## theta     0.9622231     1.0775043     1.1927856
## attr(,"label")
## [1] "Fixed effects:"
```

Only no nap conditions

Summary

```
## Nonlinear mixed-effects model fit by maximum likelihood
    Model: lapses ~ b * (TWT - crit * day_num)^theta
##
##
    Data: data.nonap
##
        AIC
                 BIC
                        logLik
##
    17379.7 17408.93 -8684.849
##
## Random effects:
## Formula: crit ~ 1 | subj
##
                 crit Residual
## StdDev: 0.001138025 7.244334
##
## Fixed effects: b + crit + theta ~ 1
            Value Std.Error DF t-value p-value
## b
         0.115809 0.0348742 2411 3.320769 9e-04
## crit 12.440110 0.5334087 2411 23.321908
                                             0e+00
## theta 1.199507 0.0748659 2411 16.022063
                                             0e+00
## Correlation:
##
        b
               crit
## crit 0.563
## theta -0.953 -0.299
## Standardized Within-Group Residuals:
         Min
                     Q1
                               Med
                                           QЗ
                                                     Max
## -1.7177180 -0.5190705 -0.2433830 0.1732791 6.8882463
## Number of Observations: 2555
## Number of Groups: 142
```

RMSE

```
## [1] 7.244333
```

Estimates and their 95% confidence intervals

```
## Approximate 95% confidence intervals
##
## Fixed effects:
## lower est. upper
## b 0.04746279 0.1158091 0.1841554
## crit 11.39473708 12.4401098 13.4854826
## theta 1.05278470 1.1995067 1.3462287
## attr(,"label")
## [1] "Fixed effects:"
```

Only nap conditions

Summary

```
## Nonlinear mixed-effects model fit by maximum likelihood
##
    Model: lapses ~ b * (TWT - crit * day_num)^theta
##
    Data: data.nap
##
        AIC
                 BIC
                       logLik
##
     7669.34 7695.702 -3829.67
##
## Random effects:
  Formula: crit ~ 1 | subj
               crit Residual
## StdDev: 0.8947836 3.182258
##
## Fixed effects: b + crit + theta ~ 1
            Value Std.Error
                              DF
                                    t-value p-value
         1.737617 0.18002255 1358
                                    9.65222
## b
## crit 17.568085 0.11425575 1358 153.76105
                                                   0
## theta 0.734852 0.04226477 1358 17.38686
## Correlation:
##
        b
               crit
        0.341
## crit
## theta -0.904 -0.195
## Standardized Within-Group Residuals:
         Min
                     Q1
                               Med
                                            QЗ
                                                      Max
## -3.2553146 -0.5700844 -0.1210714 0.4116992 6.6657784
## Number of Observations: 1440
## Number of Groups: 80
```

\mathbf{RMSE}

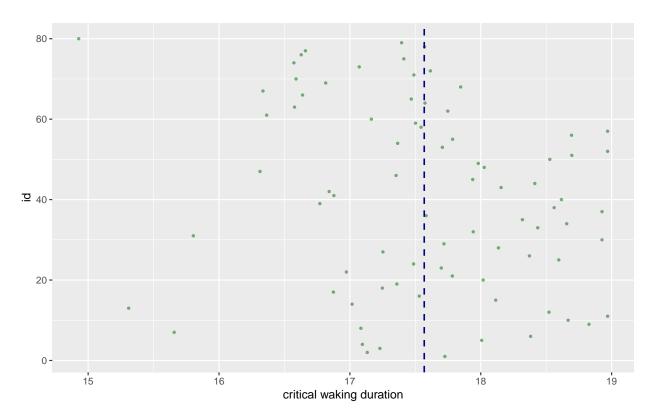
[1] 3.639682

Estimates and their 95% confidence intervals

```
## Approximate 95% confidence intervals
##
##
    Fixed effects:
##
              lower
                          est.
                                    upper
## b
          1.3848326 1.7376171
                                2.0904015
  crit 17.3441815 17.5680848 17.7919881
##
   theta 0.6520269 0.7348518 0.8176767
  attr(,"label")
   [1] "Fixed effects:"
##
##
##
    Random Effects:
##
    Level: subj
##
                lower
                           est.
                                   upper
## sd(crit) 0.7469947 0.8947836 1.071812
##
##
    Within-group standard error:
##
      lower
                est.
                        upper
## 3.064016 3.182258 3.305062
```

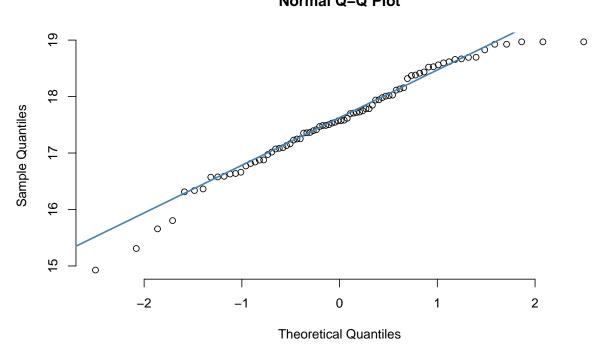
Visualising the results of TBT based estimate - only nap

Dot Plot - Critical Wake Durations across participants



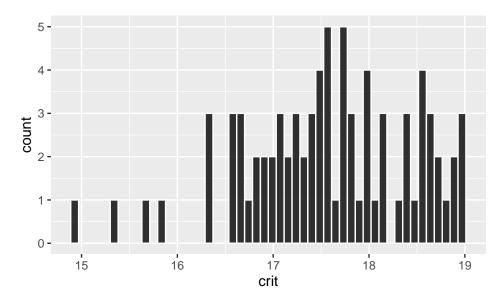
QQ Plot - Normality of distribution



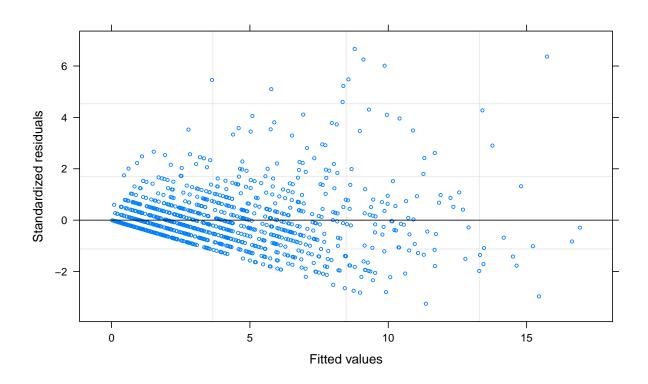


Histogram - Normality of distribution

The plot indicates a left skew. I checked the histogram



Residuals Plot



Summary of all results

```
##
                                          measure
                                                       RMSE sleep_need.estimate
## 1
                 aggregated tests (TST) - no nap 3.084051
                                                                       6.868654
## 2
                 aggregated tests (TBT) - no nap 3.158632
                                                                       7.786787
             tests not aggregated (TBT) - no nap 7.244333
                                                                      11.559890
\#\# 4 tests not aggregated (TBT) - all conditions 6.366383
                                                                      10.177074
           tests not aggregated (TBT) - nap only 3.639682
## 5
                                                                       6.431915
## 6
                Van Dongen et al. (TST) - no nap
                                                         NA
                                                                       8.160000
## 7
                     Short et al. (TST) - no nap
                                                         NA
                                                                       9.350000
##
     theta.estimate
## 1
          0.6744519
## 2
          0.6675534
## 3
          1.1995067
## 4
          1.0775043
## 5
          0.7348518
## 6
          0.6700000
## 7
          0.5900000
```

