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EE18B104

EE6320 - RF Integrated Circuits

Assignment 1

$\boxed{\text{ANALYSIS}}$

1. (a) $P_{out} = 1W$ (desired), $R_{ant} = 50\Omega$ $(= R_s)$

$f_{op} = 1GHz$

W/o Matching Network

$$P_{max} = \frac{V_{rms}^2}{R_s} = \left(\frac{5}{2}\right)^2 \times \frac{1}{2\times 50} W = \frac{6.25}{100} W$$

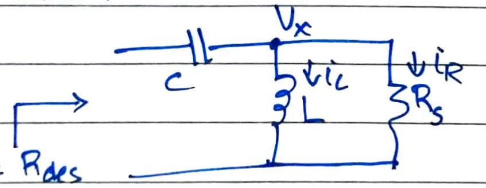
$$P_{max} = 62.5 mW$$

(b) Desired impedance $= R_{des}$

$$P = 1W = \frac{V_{rms}^2}{R_{des}} = \frac{1}{2} \frac{\times 2.5^2}{R_{des}}$$

$$R_{des} = \frac{2.5^2}{2} = 3.125\Omega$$

(c) $\because R_{des} < R_s$, the following L-match network must be used:



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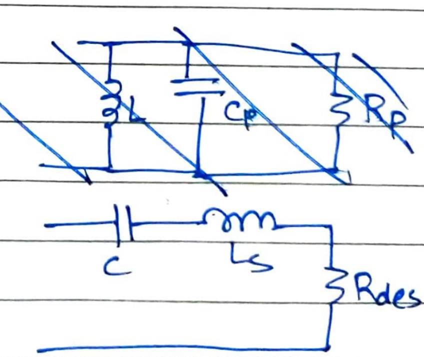


$R_{in} = R_{des}$

@ f_{op}

$$R_{des} = \frac{R_s}{1+Q^2}$$

$$\omega_0 = \frac{1}{\sqrt{C L_s}}$$

$$Q = \omega_0 \times \frac{\frac{1}{2} L i_L^2}{\frac{1}{2} R_s i_R^2}$$

$$= \frac{\omega_0 L}{R_s}\left(\frac{V_x}{\omega_0 L}\right)^2\left(\frac{R_s}{V_x}\right)^2 = \frac{R_s}{\omega_0 L} = Q_{series} = \frac{\omega_0 L_s}{R_{des}} = \frac{1}{\omega_0 C R_{des}}$$

$$\frac{\omega_0 L_s}{R_{des}} = \frac{R_s}{\omega_0 L} \Rightarrow L_s = \frac{R_s R_{des}}{\omega_0^2 L} = \frac{R_s^2}{\omega_0^2 L(1+Q^2)} = \frac{L Q^2}{1+Q^2}$$

Further,

$$\frac{R_s}{\omega_0 L} = Q = \frac{1}{\omega_0 C R_{des}}$$

$$C = \frac{\omega_0 L}{\omega_0 R_{des} R_s}$$

$$= \frac{L}{R_s R_{des}}$$

$$\omega_0 = \frac{1}{\sqrt{L_s C}}$$ → so that Z_{in} @ $\omega_0 = R_{des}$

$R_{des} = 3.125\Omega$

$R_s = 50\Omega$

$$1+Q^2 = \frac{R_s}{R_{des}} = \frac{50}{3.125} = 16$$

$$Q = \sqrt{15} = 3.87298$$

$$L_s C = L\left(\frac{Q^2}{1+Q^2}\right) \cdot \frac{L}{50 \times 3.125}$$

$$= L^2\left(\frac{15}{16}\right) \times \frac{1}{156.25} = L^2 \times 6 \times 10^{-3}$$

$$L_s C = \frac{1}{\omega_0^2} = \frac{1}{4\pi^2 f_0^2} = \frac{1}{4 \times \pi^2 \times 10^{18}}$$

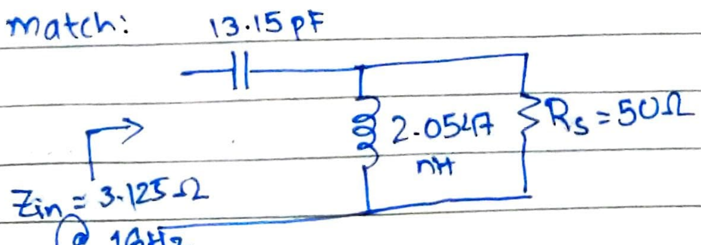
$$L^2 \times 6 \times 10^{-3} = \frac{1}{3.94784 \times 10^{19}}$$

$$L^2 = 4.2217 \times 10^{-18}\ H^2$$

$$\boxed{L = 2.0547\ nH}$$

$$C = \frac{2.0547 \times 10^{-9}}{50 \times 3.125}\ F = 13.15\ pF$$

L-match:



d) Theoretical 3-dB BW:

$$Q = \frac{\omega_0}{3dB\ BW}$$

~~$3dB\ BW = \frac{\omega_0}{\sqrt{15}} = \frac{2\pi \times 10^9}{3.87298}$~~

~~$BW = 1.622\ GHz$~~

$$BW \text{ in Hz} = \frac{f_0}{Q}\ Hz$$

$BW = 2.58199 \times 10^8\ Hz$

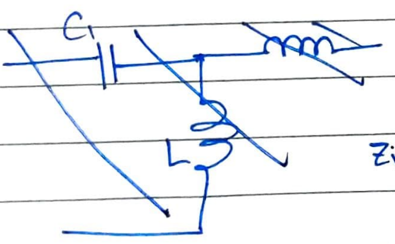
$BW = 258.199\ MHz$ → theoretical

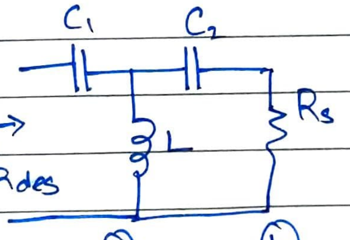
BW from simulation: ~~244.3597 MHz~~ 255.956 MHz

Q from simulation: 3.9069

Q theoretical: 3.873

e) High-pass T-match





$$Q_L = \frac{R_I}{\omega L_1}\ ;\ Q_R = \frac{R_I}{\omega L_2}$$

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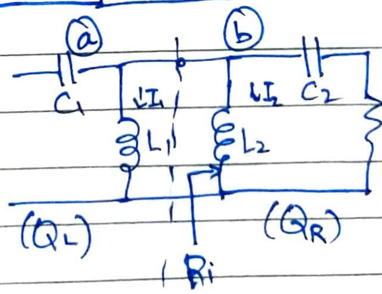


$$Q_{net} = \frac{\omega \times \frac{1}{2}(L_1 I_1^2 + L_2 I_2^2)}{\frac{1}{2}(I_I^2 R_I)}$$

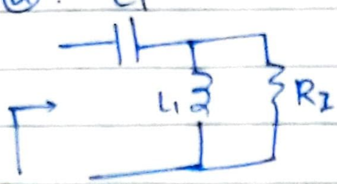
where $I_1 \omega L_1 = I_2 \omega L_2 = V_I = I_I R_I$

$$Q_{net} = \frac{\omega \left(\frac{I_I^2 R_I^2}{\omega^2}\right)\left(\frac{1}{L_1} + \frac{1}{L_2}\right)}{I_I^2 R_I} = \frac{R_I}{\omega L_1} + \frac{R_I}{\omega L_2}$$

$$Q_{net} = Q_L + Q_R$$

Now, ~~the~~ subcircuits ⓐ and ⓑ resemble L-matches.

(a):

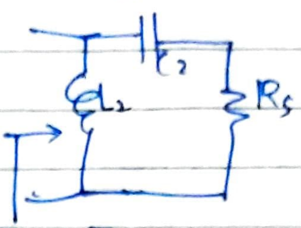


$Z_{in} = R_{des}$

$$R_{des} = \frac{R_I}{1+Q_L^2}$$

$$Q_L = \sqrt{\frac{R_I}{R_{des}} - 1}$$

(b):



$Z_{in} = R_I$

$$R_I = R_s(1+Q_R^2)$$

$$Q_R = \sqrt{\frac{R_I}{R_s} - 1}$$

$$Q = Q_L + Q_R = 5$$

$$\therefore \sqrt{\frac{R_I}{3.125} - 1} + \sqrt{\frac{R_I}{50} - 1} = 5$$

$$\Rightarrow \frac{R_I}{3.125}\left(1 - \frac{1}{16}\right) = 25 - 10\sqrt{\frac{R_I}{50} - 1}$$

$$0.3R_I = 25 - 10\sqrt{\frac{R_I}{50} - 1}$$

$$100\left(\frac{R_I}{50} - 1\right) = 625 - 15R_I + 0.09R_I^2$$

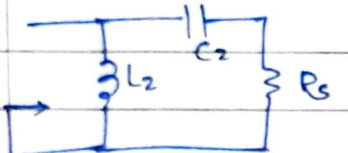
$$0.09R_I^2 - 17R_I + 725 = 0$$

$R_I = 123.842\Omega$ or $R_I = 65.04$

Results in $\sqrt{\frac{R_I}{R_{des}} - 1} > 5$

$$\therefore \underline{R_I = 65.0472\Omega}$$

L-match b:



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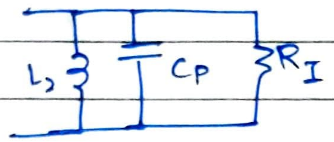


R_I

$$Q_R = \sqrt{\frac{R_I}{R_{des}} - 1} = \sqrt{\frac{65.0472}{50} - 1}$$

$$= 0.5486$$

$$Q_R = \frac{1}{\omega C_2 R_s} = \omega C_p R_I = \frac{R_I}{\omega L_2}$$

$$L_2 = \frac{R_I}{\omega \times Q_R} = \frac{65.0472}{2\pi \times 10^9 \times 0.5486} \text{ H}$$

$$L_2 = 1.88709 \times 10^{-8} \text{ H}$$

$$= 18.8709 \text{ nH}$$

$$\omega = \frac{1}{\sqrt{L_2 C_p}} \Rightarrow C_p = \frac{1}{\omega^2 L_2} = \frac{10^8}{4\pi^2 \times 10^{18} \times 1.88709}$$

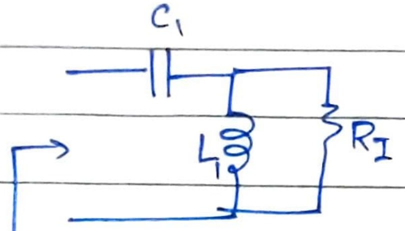
$$= 1.3423 \text{ pF}$$

$$C_2 = \cancel{C_R Q^2} \; \frac{C_p(1+Q_R^2)}{Q_R^2} = 1.3423 \times 4.32268 \text{ pF}$$

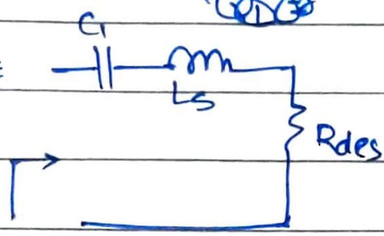
$$C_2 = 5.80233 \text{ pF}$$

$$L_2 = 18.8709 \text{ nH}$$

L-match a:



$Z_{in} = R_{des}$ $\equiv$



R_{des}

$$Q_L = \sqrt{\frac{R_I}{R_{des}} - 1} = 4.45142$$

$$L_s = L\left(\frac{Q^2}{1+Q^2}\right)$$

$$Q_L = \frac{R_I}{\omega L_1} = \frac{\omega L_s}{R_{des}} = \frac{1}{\omega C_1 R_{des}}$$

$$\omega = \frac{1}{\sqrt{L_s C_1}}$$

$$C_1 = \frac{1}{\omega R_{des} Q_L} = \frac{1}{2\pi\times10^9\times3.125\times4.45142}\ \text{F}$$

$$C_1 = 1.14412\times10^{-11}\ \text{F} = 11.4412\ \text{pF}$$

$$L_s = \frac{1}{\omega^2 C_1} = \frac{10^{12}}{4\pi^2\times10^{18}\times11.4412}\ \text{H} = 2.21395\ \text{nH}$$

$$L_1 = L_s\left(\frac{1+Q_L^2}{Q_L^2}\right) = \cancel{2.23}\ 2.32568\ \text{nH}$$

$$L = L_1 \| L_2 = \cancel{1.99887\times10^{-9}\ \text{H}}\ \ 2.07\ \text{nH}$$

$$L = \cancel{1.9989\ \text{nH}}\ 2.07\ \text{nH}$$

Simulation Results:

$BW = \cancel{209.563}\ 204.125$ MHz

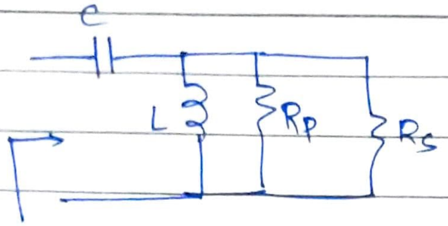
$Q = \frac{1\text{GHz}}{BW} = \cancel{4.772}\ 4.899$

S_{dB} @ 1GHz = ~~-20dB~~ -60dB

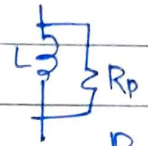
T-match parameters:

$L = 1.999$ nH, $C_1 = 11.4412$ pF, $C_2 = 5.8023$ pF

(h)



$\cancel{R_P} \equiv$

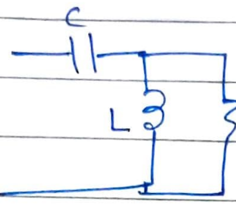


$Q = \dfrac{R_p}{\omega L}$

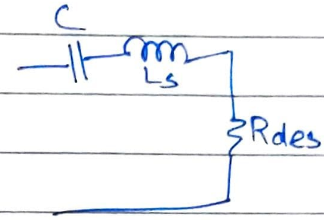
$R_p = 15\omega L$

$Z_{in} = R_{des}$

$\equiv$



$R_s' = R_p \| R_s \equiv$



$R_p \| R_s = R_s'$

For $L = 2.0547\,nH$,

$R_p = 193.651\,\Omega$

$$R_{des} = \frac{R_s'}{1+Q^2}$$

$$Q = \frac{R_s'}{\omega L} = \frac{\omega L_s}{R_{des}} = \frac{1}{\omega C R_{des}}$$

$$R_s' = \frac{50 \times 15\omega L}{50 + 15\omega L} = \frac{4.71239 \times 10^{12} L}{50 + 9.4248 \times 10^{10} L}$$

$$Q = \sqrt{\frac{R_s'}{R_{des}} - 1} = \sqrt{\frac{1.50796 \times 10^{12} L - 50 - 9.4248 \times 10^{10} L}{50 + 9.4248 \times 10^{10} L}}$$

$$Q = \sqrt{\frac{1.4137 \times 10^{12} L - 50}{9.4248 \times 10^{10} L + 50}} = \sqrt{\frac{2.8274 \times 10^{10} L - 1}{1.88496 \times 10^{9} L + 1}}$$

$$C = \frac{1}{\omega R_{des} Q} = \frac{1}{2\pi \times 10^9 \times 3.125\, Q}$$

$$C = \sqrt{\frac{1.885 \times 10^9 L + 1}{2.827 \times 10^{10} L - 1}} \times \frac{1}{1.964 \times 10^{10}}$$

$$L_s = \frac{1}{\omega^2 C} = \sqrt{\frac{2.827 \times 10^{10} L - 1}{1.885 \times 10^9 L + 1}} \times \frac{1.964 \times 10^{10}}{4\pi^2 \times 10^{18}}$$

$$L_s = \sqrt{\frac{2.827 \times 10^{10} L - 1}{1.885 \times 10^9 L + 1}} \times 4.975 \times 10^{-10}\ H$$

$$L = \frac{R_s' R_{des}}{\omega^2 L_s} = L_s \left(\frac{1+Q^2}{Q^2}\right)$$

$$L = 4.975 \times 10^{-10} \left(\frac{3.0155 \times 10^{10} L}{\sqrt{1.885 \times 10^9 L + 1}} \times \frac{1}{\sqrt{2.827 \times 10^{10} L - 1}} \right)$$

$$\Rightarrow (1.885 \times 10^9 L + 1)(2.827 \times 10^{10} L - 1) = \cancel{2.47506 \times 10^{+9}}\ 225.063$$

$$5.3289 \times 10^{19} L^2 + 2.6385 \times 10^{10} L - 226.063 = 0$$

$L = 1.8269\,nH$ or $L = -2.322\,nH$ (✗ capacitive)

$L = 1.8269\,nH$

$R_p = 15 \times 2\pi \times 10^9 \times L = 172.181\ \Omega$

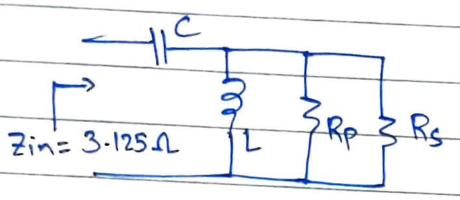
$Q = 3.3763$

$C = 15.0844\,pF$

Simulation Results:

$BW = 274.632\,MHz$

$Q = 3.64$



(g) With noise:

$R_p = 193.651\,\Omega$

$L = 2.0547\,nH$

$C = 13.15\,pF$

$BW = 307.6932\,MHz$

$Q = 3.249$

Result Waveforms

L-Match Network

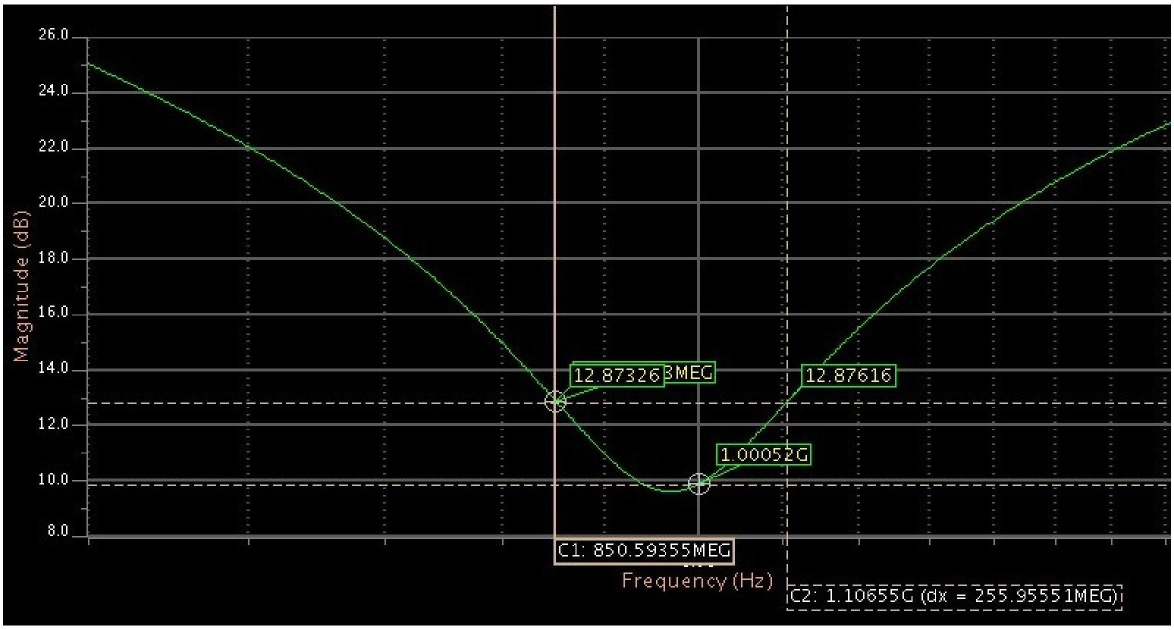


Figure 1: Z_{dB} of L-match network. BW = 255.956 MHz, Q = 3.9069. $Q_{theoretical} = 3.873$

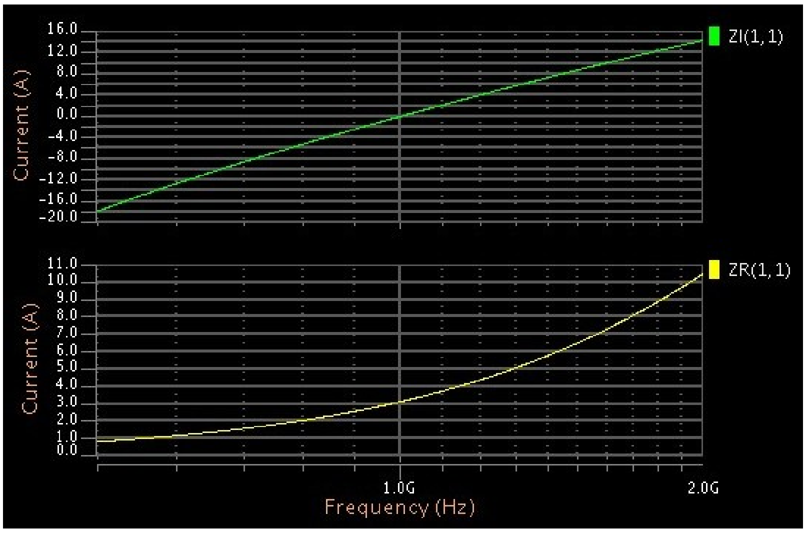


Figure 2: Z_{in} of L-match network. $Z_r \approx 3.125\Omega$, $Z_i = 0\Omega$ at 1GHz

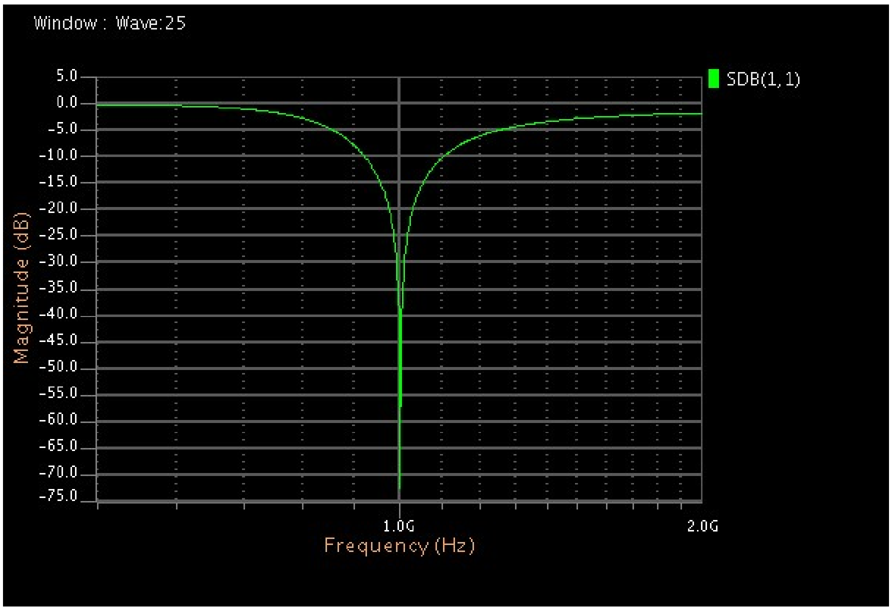


Figure 3: S_{dB} of L-match network.

T-Match Network

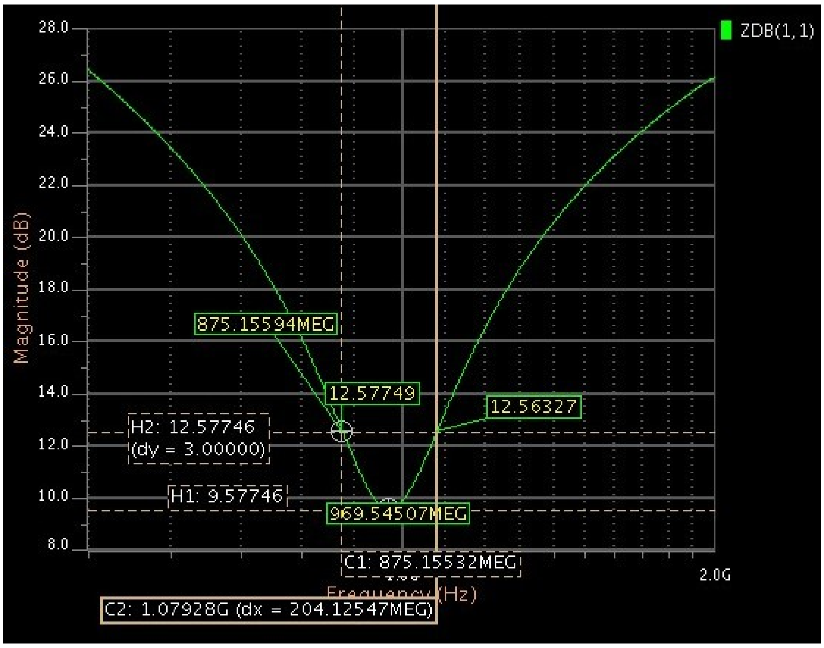


Figure 4: Z_{dB} of L-match network. BW = 204.125 MHz, Q = 4.899. $Q_{theoretical} = 5$

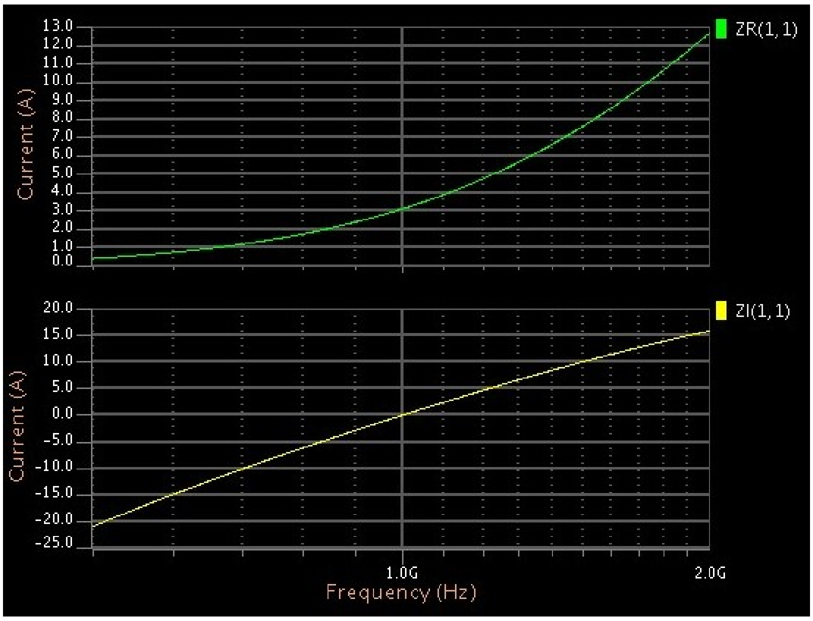


Figure 5: Z_{in} of T-match network. $Z_r \approx 3.125\Omega$, $Z_i = 0\Omega$ at 1GHz

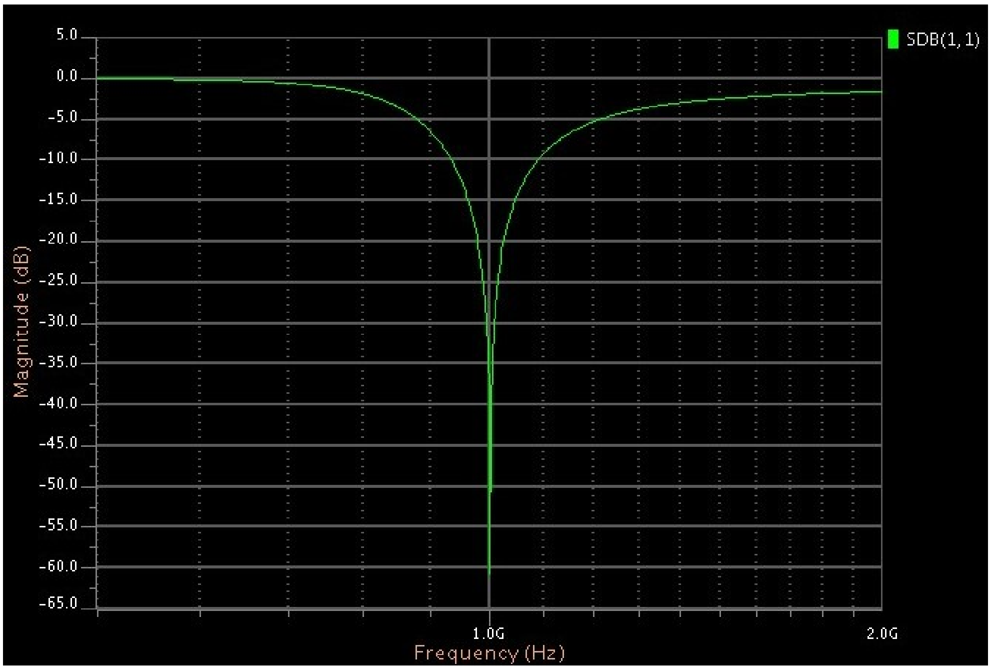


Figure 6: S_{dB} of T-match network.

L-Match with Noisy Inductors

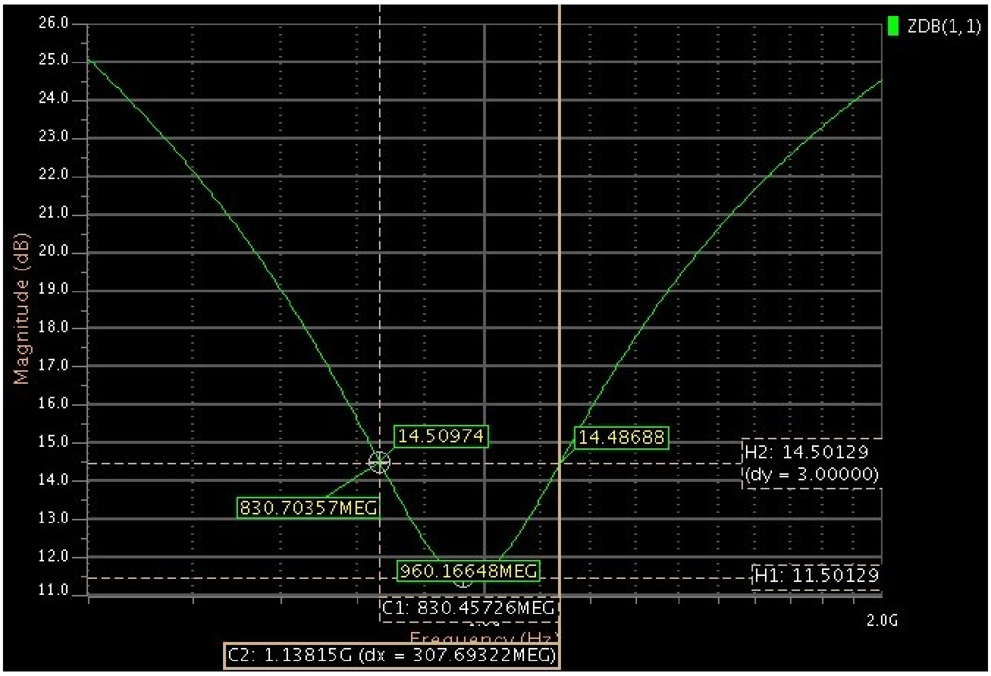


Figure 7: Z_{dB} of L-match network. R_p = 193.651Ω, BW = 307.693 MHz, Q = 3.249. $Q_{theoretical}$ = 3.873

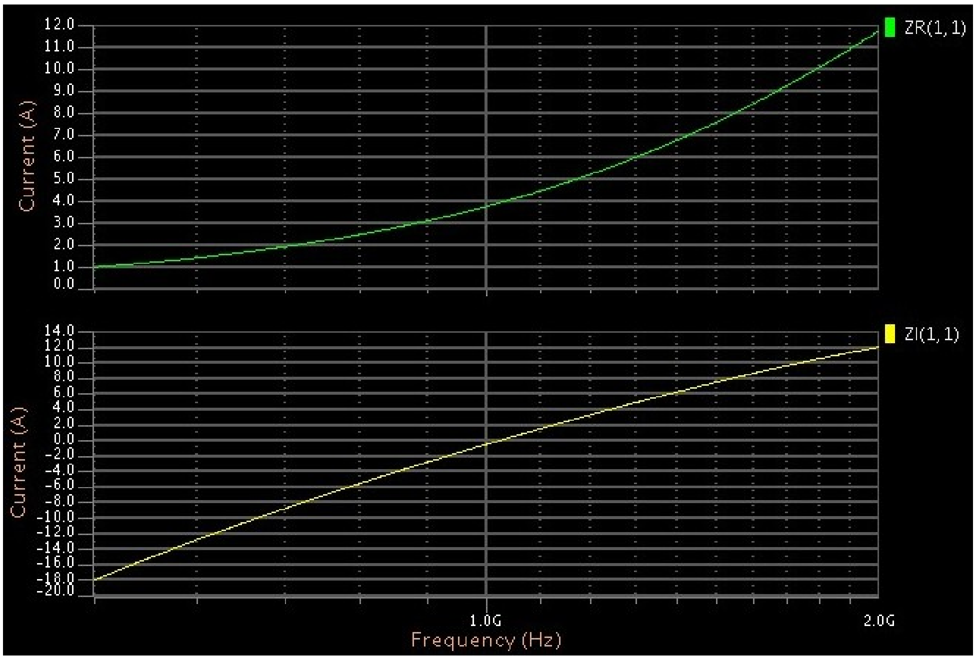


Figure 8: Z_{in} of L-match network. Notice deviation from desired values

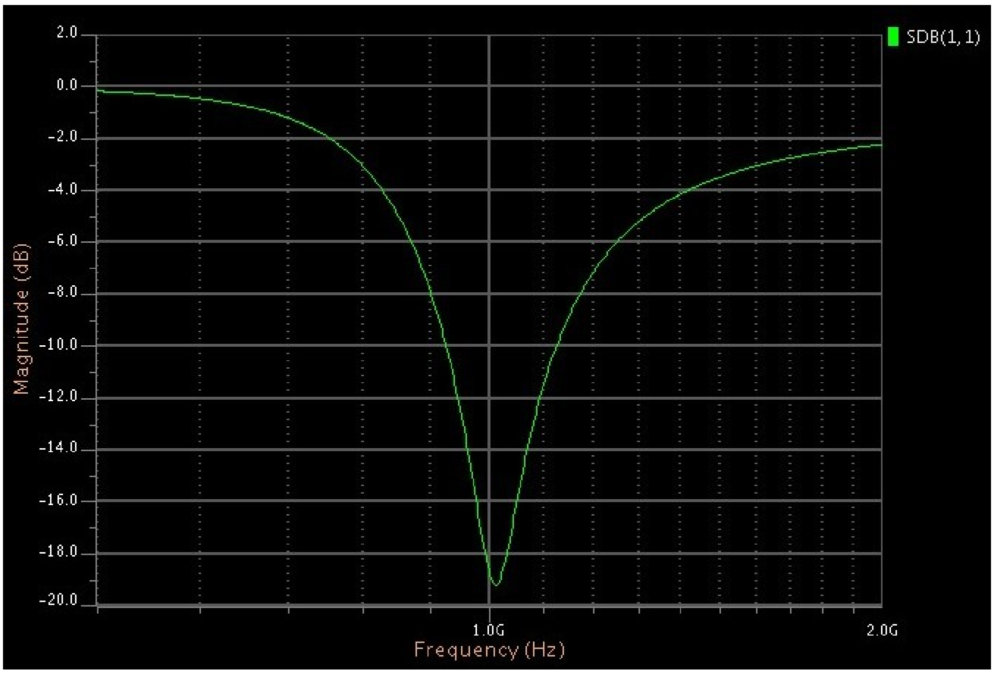


Figure 9: S_{dB} of L-match network. The minima in S_{dB} is now shifted away from the operating frequency, and is only $\approx -19dB$

Optimised L-Match Network accounting for Noisy Inductors

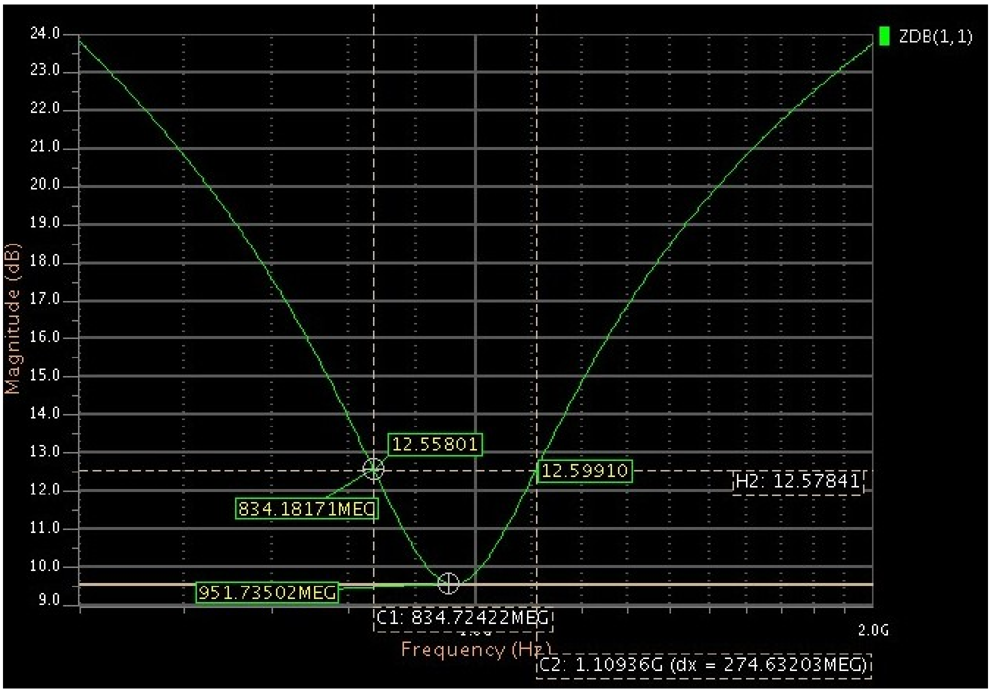


Figure 10: Z_{dB} of L-match network. $R_p = 172.181\Omega$, BW = 274.632 MHz, Q = 3.64

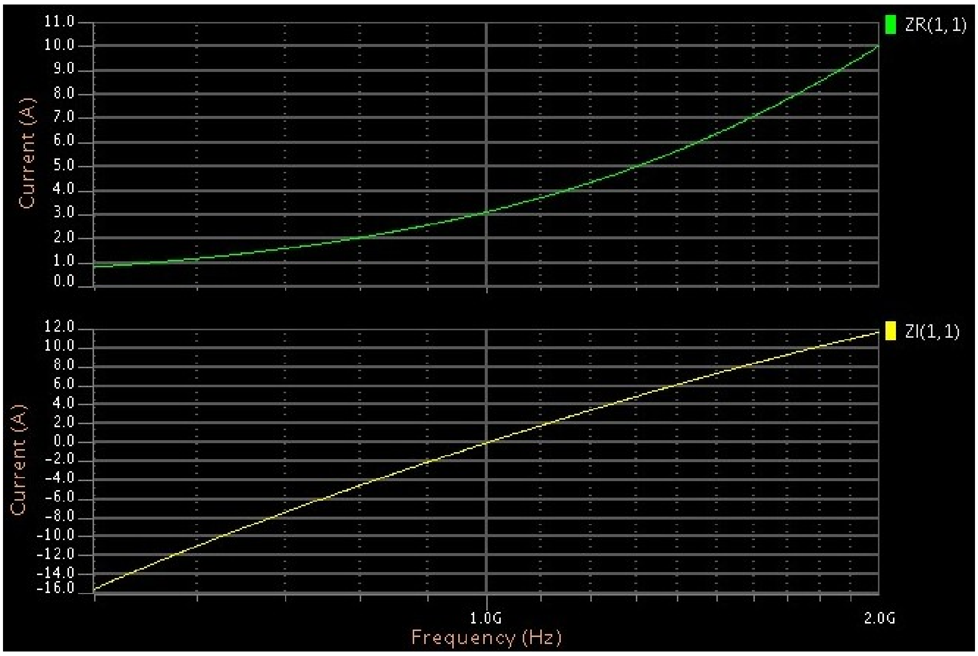


Figure 11: Z_{in} of L-match network.

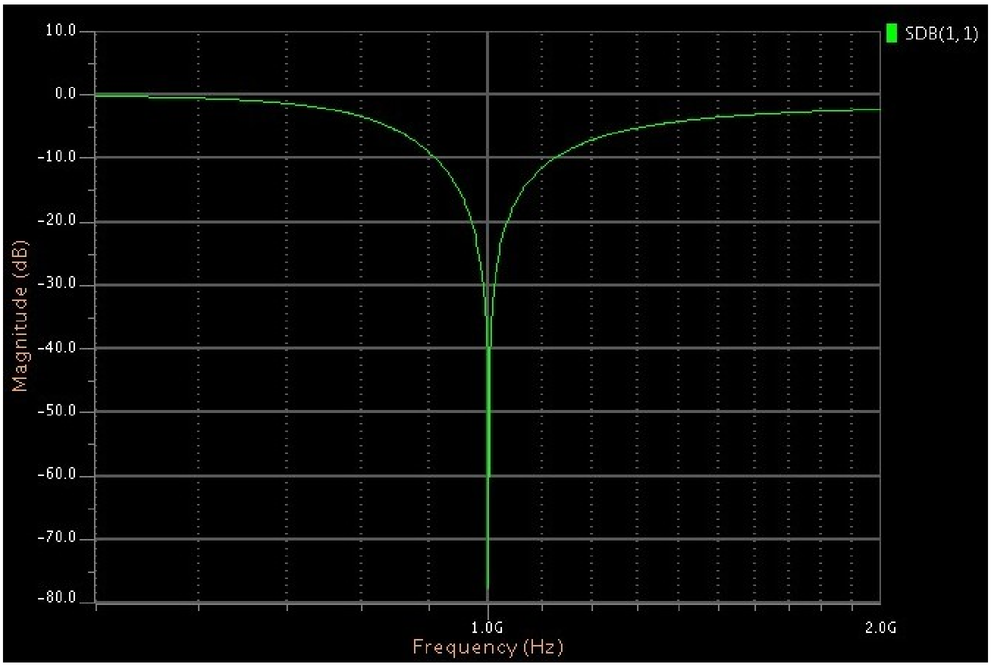


Figure 12: S_{dB} of L-match network.

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