Sri Sivasubramaniya Nadar college of Engineering.

Department of Mathematics

UMA1478 – Probability and Statistics

CAT-3 Answer key

Part-A

(1) Residual $Q_3 = Q - Q_1 - Q_2$

$$= \sum \sum (x_{ij} - \bar{x}_{i*} - \bar{x}_{*j} + \bar{x})^2$$

(2)

(1) The number of replications of each treatment is equal to the number of treatments in LSD, whereas there is no such restrictions on treatments and replication in RBD.

(2) LSD can be performed on a square field, while RBD can be performed either on a square or a rectangular field.

(3) LSD is known to be suitable for the case when the number of treatments is between 5 and 12, whereas RBD can be used for any number of treatments.

(3) (1) Randomisation (2) Replication (3) Local control

(4) $n = 150$, $\bar{p} = 0.075$

$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 11.25 + 3\sqrt{11.25(1-0.075)}$

$= 11.25 + 9.68 = 20.93$

$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 1.57$

(5)

$n = 6$, $\bar{\bar{x}} = 2.1126$, $\bar{R} = 0.0084$

$$\bar{\bar{x}} \mp \frac{3\bar{R}}{d_2} = 2.1126 + \frac{3 \times 0.0084}{2.534} = 2.123$$

(6) The size of the sample varies from sample to sample. Hence we cannot use np-chart. Construct p-chart, $0.75\bar{n} \le n_i \le 1.25\bar{n}$ $\forall i$

Part - B

(7) H_0: The treatments are not significantly different.

Treatment	Yield (x_{ij})				T_i	T_i^2	n_i	T_i^2/n_i
A	5	7	3	1	16	256	4	64
B	4	4	7	—	15	225	3	75
C	3	5	1	—	9	81	3	27
Total					40 = T		N = 10	166

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 200 - \frac{40^2}{10} = 40$$

$$Q_1 = \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = 166 - 160 = 6$$

$$Q_2 = Q - Q_1 = 40 - 6 = 34$$

S.V	S.S	d.f	M.S	F_0
Bet classes	$Q_1 = 6$	$h-1 = 2$	3.0	$\frac{4.86}{3}$
Within classes	$Q_2 = 34$	$N-h = 7$	4.86	$= 1.62$
Total	$Q = 40$	$N-1 = 9$	—	—

From F-table, $F_{5\%}(7, 2) = 19.35$

Since $F_0 < F_\alpha$, H_0 is accepted.

(8) ~~n = 150~~ $\sum np = 73$

$$n\bar{p} = \frac{73}{N} = \frac{73}{\cancel{15}} = \cancel{7.3}\ \frac{73}{15} = 4.87$$

$$\bar{p} = \frac{4.87}{10} = .487$$

$CL = n\bar{p} = 4.87$

$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = \cancel{\sqrt{4.87 - 3}}$

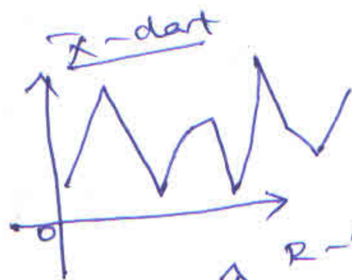
$= 4.87 - 3\sqrt{4.87(1-.487)} = 4.87 - 4.74 = .13$

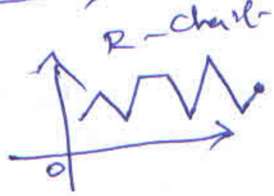
$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 4.87 + 4.74$
$= 9.61$

Since sample no's 3 & 4 lie outside the control limits, the process is out of control.

(9) $\bar{\bar{X}} = \frac{1}{N}\sum_i \bar{X}_i = 44.2$

$\bar{R} = \frac{1}{N}\sum_i R_i = 5.8$





from Table, n=5
$A_2 = 0.577$, $D_4 = 2.115$, $D_3 = 0$

$\bar{X}$-chart

$LCL = \bar{\bar{X}} - A_2\bar{R} = 40.85$
$UCL = \bar{\bar{X}} + A_2\bar{R} = 47.55$
$CL = \bar{\bar{X}} = 44.2$

R-chart

$CL = \bar{R} = 5.8$
$LCL = \bar{R}D_3 = 0$
$UCL = \bar{R}D_4 = 12.27$

The process is under control.

Part-C

(10) Rewriting the data,

Blocks	Crops A	B	C
1	47	49	48
2	57	49	53
3	49	52	52
4	49	40	58

(11) Since the number of defective in 10 samples, each of size 50, are given, we may construct either no. of defective (np) chart or proportion of defectives (p) chart.

$$\sum np = 25$$

$$\therefore n\bar{p} = \frac{1}{N}\sum np = 2.5$$

$$\& \ \bar{p} = \frac{1}{n} \times n\bar{p} = \frac{2.5}{50} = 0.05$$

<u>np-chart</u>

$$CL = n\bar{p} = 2.5$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = -2.12$$

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 7.12$$

<u>p-chart</u>

$$CL = \bar{p} = 0.05$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = -0.042$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.142$$

The proportion of defective (p) for the given samples are 0.04, 0.02, 0.02, 0.04, 0.06, 0.10, 0.10, 0.02, 0.04, 0.06.

The process is under control.

(12) We subtract 16 from the given values.

$$Q = \sum\sum x_{ij}^2 - T^2/N = 69 - 1/9 = 68.89$$

$$Q_1 = \frac{1}{n}\sum T_i^2 - T^2/N = 35 - 1/9 = 34.89$$

	E_1	E_2	E_3	T_i	T_i^2/n	$\sum_j x_{ij}^2$
D_1	0 (B_1)	1 (B_2)	4 (B_3)	5	8.33	17
D_2	0 (B_2)	5 (B_1)	-1 (B_1)	4	5.33	26
D_3	-1 (B_3)	-4 (B_1)	-3 (B_2)	-8	21.33	26
T_j	-1	2	0	$T=1$	$\sum \frac{T_i^2}{n} = 35$	69
T_j^2/n	0.33	1.33	0	$\sum \frac{T_j^2}{n} = 1.66$		
$\sum_i x_{ij}^2$	1	42	26	69		

Rearranging the data values according to the burners

Burner	x_k			T_k	T_k^2/n
B_1	0	-1	-4	-5	8.33
B_2	1	0	-3	-2	1.33
B_3	4	5	1	8	21.33
Total				$T=1$	$\sum \frac{T_k^2}{n} = 31$

$$Q_2 = \frac{1}{n}\sum T_j^2 - \frac{T^2}{n} = 1.67 - \frac{1}{9} = 1.56$$

$$Q_3 = \frac{1}{n}\sum T_k^2 - \frac{T^2}{n} = 31 - \frac{1}{9} = 30.89$$

$$Q_4 = Q - Q_1 - Q_2 - Q_3 = 1.55$$

S.V	S.S	d.f	M.S	F_0
Bet' rows	Q_1	$n-1=2$	17.445	$\frac{17.445}{0.775} = 22.51$
Bet' cols	Q_2	$n-1=2$	.780	$\frac{.780}{0.775} = 1.01$
Bet' letters	Q_3	$n-1=2$	15.445	$\frac{15.445}{0.775} = 19.93$
Residual	Q_4	$(n-1)(n-2) = 2$	0.775	
Total	Q	$n^2-1=8$		

From F-table $F_{5\%}(2,2) = 19.00$

Since $F_0 (19.93) > F_{5\%}$, there is significant diff bet' the burners.

Incidentally, since $F_0 > F_{5\%}$, the diff. bet' the days is significant and since $F_0 < F_{5\%}$ for the cols, the diff. bet' the engine is not significant.

(13)

$$\bar{\bar{X}} = \frac{1}{N} \sum \bar{X}_i = 63.19$$

$$\bar{R} = \frac{1}{N} \sum R_i = 22.6$$

i	1	2	3	4	5	6	7	8	9	10
$\sum x_i$	375	346	361	397	381	406	382	388	349	405
$\overline{x}_i$	62.5	57.7	60.2	66.2	63.5	67.7	63.7	64.7	58.2	67.5
R_i	25	31	19	24	20	49	17	14	16	11

For $\overline{X}$-chart

$CL = \overline{\overline{X}} = 63.19$

$LCL = \overline{\overline{X}} - A_2 \overline{R}$

$= 52.27$

$UCL = \overline{\overline{X}} + A_2 \overline{R}$

$= 74.11$

For R-chart

$CL = \overline{R} = 22.6$

$LCL = D_3 \overline{R} = 0$

$UCL = D_4 \overline{R} = 45.29$

note $LCL < UCL$, but R (sample 6) ≥ 45.29

Hence, the process is under control.

Now, we remove sample no. 6,

$\overline{\overline{X}} = \frac{1}{9} \times 564.2 = 62.69$

$\overline{R} = \frac{1}{9} \times 177 = 19.67$

$\overline{X}$-chart

$CL = \overline{\overline{X}} = 62.69$

$LCL = \overline{\overline{X}} - A_2 \overline{R}$

$= 53.19$

$UCL = \overline{\overline{X}} + A_2 \overline{R}$

$= 72.19$

R-chart

$CL = \overline{R} = 19.67$

$LCL = D_3 \overline{R} = 0$

$UCL = D_4 \overline{R} = 39.42$

The process is under control

The tolerance lts are $\overline{\overline{X}} \mp \frac{3\overline{R}}{d_2}$

$= 62.69 \mp 3 \times \frac{19.67}{2.97}$

we shift the origin to 90.

Blocks	A	B	C	T_i	$\frac{T_i^2}{k}$	$\sum_j x_{ij}^2$
1	-3	-1	-2	-6	$\frac{36}{3}=12$	14
2	1	-1	3	3	$9/3=3$	11
3	-1	2	2	3	$9/3=3$	9
4	-1	-10	1	-10	$\frac{100}{3}=33.33$	102
T_j	-4	-10	4	-10	51.33	
T_j^2/h	$\frac{16}{4}=4$	$\frac{100}{4}=25$	$\frac{16}{4}=4$	33		136
$\sum_i x_{ij}^2$	12	106	18			

$$Q = \sum\sum x_{ij}^2 - T^2/N = 136 - \frac{(-10)^2}{12} = 127.67$$

$$Q_1 = \frac{1}{k}\sum T_i^2 - T^2/N = 51.33 - \frac{(-10)^2}{12} = 42.99$$

$$Q_2 = \frac{1}{h}\sum T_j^2 - T^2/N = 33 - \frac{(-10)^2}{12} = 24.67$$

$$Q_3 = Q - Q_1 - Q_2 = 60.01$$

S.V	S.S	d.f	M.S	F_0
Bet^n rows	Q_1	$h-1=3$	6	$6/1.67=3.6$
Bet^n cols	Q_2	$k-1=2$	4	$4/1.67=2.4$
Residual	Q_3	$(h-1)(k-1)=6$	1.67	—
Total	Q	$hk-1=11$	—	

From F tables,

$F_{5\%}(3, 6) = 4.76$

$F_{5\%}(2, 6) = 5.14$

Since $F_0 < F_{5\%}$, bet' rows, the diff. bet' the rows is not significant.

$F_0 < F_{5\%}$, bet' cols, the diff bet' the cols is not significant.

$\therefore$ The varieties of crops do not differ significantly w.r.t the yield.

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