UMA1477 Prob theory & Stochastic Processes
UMA1478 Prob & Statistics.

Part A. **key**

1. $P(B_i/A) = \dfrac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}$ ——— (2)

2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} ce^{x}dx + \int_{0}^{\infty} ce^{-x}dx = 1$ } ——— (1)

$c = 1/2$ — (1)

3. $np = 0.8 \times 8 = 6.4$ ——— (2)

4. $V\left(\frac{x}{2}\right) = 1$ ——— (2)

5. $P(4<x<5) = F(5) - F(4)$ —— (1)

$= \frac{9}{100}$ — (1)

6. mgf $M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r'$ — (1)

$= (1-2t)^{-2}$ — (1)

Part B.

7. $a = \frac{1}{81}$ — (1)

$P(x<3) = \frac{1}{9}$

$P(x \geq 3) = \frac{8}{9}$ } — (1)

$P(0<x<5) = \frac{8}{27}$ — (1)

Distn.

x = x	F(x)
0	1/81
1	4/81
2	
3	1/9
4	16/81
5	25/81
6	4/9
7	49/81
8	64/81
	1

— (3)

8 $M_{x+y}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}$ — (2)

$M_{x-y}(t) = e^{\lambda_1(e^t-1)} \, e^{\lambda_2(e^t-1)}$ — (2)

9 $P(x = k+t \mid x \geq k) = P(x=t)$ — (2)

proof — (4)

10 $P(F) = \frac{1}{4}$

$P(T) = \frac{3}{4}$

$P(H/F) = 1$

$P(H/T) = 1/16$

$P(F/H) = ?$ — (2)

Using BT

$$P(F/H) = \frac{P(F) \cdot P\left(\frac{H}{F}\right)}{P(T) \cdot P\left(\frac{H}{T}\right) + P(F) \cdot P\left(\frac{H}{F}\right)} \quad \text{— (4)}$$

$$= \frac{\frac{1}{4}(1)}{\frac{3}{4} \cdot \frac{1}{16} + \frac{1}{4} \cdot 1} + \frac{1}{\frac{3}{16}+1} \quad \text{— 4}$$

$$= \frac{1}{19/16} = \frac{16}{19} = 0.842 = 84.2\%$$

12. $P(X = x) = {}^nC_x p^x q^{n-x}$

$M_x(t) = E(e^{tx}) = (pe^t + q)^n$ — (3)

Mean $= \mu_1' = [M_x'(t)]_{t=0} = np$ — (2)

$\mu_2' = [M_x''(t)]_{t=0} = n(n-1)p^2 + np$ — (3)

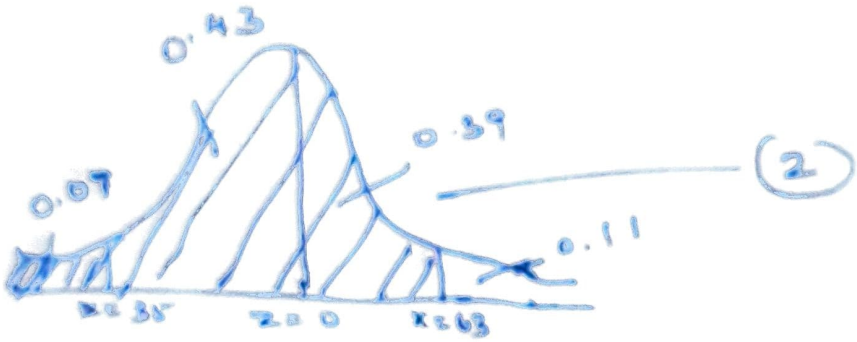
Var $= \mu_2 = \mu_2' - \mu_1'^2 = npq$ — (2)

11. $a = \frac{1}{2}$ ——— (2)

12.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x < 1 \\ \frac{1}{2}\left(x - \frac{1}{2}\right) & 1 \le x < 2 \\ \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$ (5)

13.



(2)

$\mu = 50.3$ ——— (2)

$\sigma = 10.33$. ——— (4)

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