Lab6

Fitting and Plotting of Binomial distribution & Poisson distribution

BASICS IN PROBABILITY:-

1. If you want to pick five numbers at random from the set 1:50, then you can

```
> sample(1:40,5)
[1] 15 36 10 40 24

> sample(1:40,5)
[1]  3 27  5  8 38

> sample(1:40,5)
[1] 19 17 37 20  6
```

2. Sampling with replacement is suitable for modelling coin tosses or throws of a die.

Roll a die(it gives different results)

```
> sample(1:6,10,replace=TRUE)

[1] 4 4 4 1 4 6 3 5 4 2

> sample(1:6,10,replace=TRUE)

[1] 3 3 4 3 2 1 4 2 6 3
```

3. ## roll 2 die. Even fancier # replace when rolling dice

```
> dice = as.vector(outer(1:6,1:6,paste))
> sample(dice,5,replace=TRUE)
[1] "1 2" "5 2" "4 4" "6 2" "6 4"

> sample(dice,5,replace=TRUE)
[1] "4 3" "3 6" "1 6" "4 2" "4 6"
```

4. #Toss a coin

```
> sample(c("H","T"),10,replace=TRUE)

[1] "H" "H" "H" "H" "T" "T" "T" "H" "H" "H"

> sample(c("H","T"),10,replace=TRUE)

[1] "T" "T" "H" "H" "T" "H" "H" "H" "T" "T"
```

5. # Combination

$\binom{10}{3}$ *OR* $10c_3$

```
> choose(10,3)
[1] 120
```

$\binom{20}{6}$

```
> choose(20,6)
[1] 38760
```

$\binom{30}{5}$

```
> choose(30,5)
[1] 142506
```

6. #permutation (there is no separate permutation function in R)

```
> #P.nk <- factorial(n) / factorial(n-k)
> n=10
> k=5
> P <- factorial(n) / factorial(n-k)
> P
[1] 30240
```

7. Give all binomial coefficients for $\binom{10}{x}$

```
> choose(10,0:10)

 [1]   1  10  45 120 210 252 210 120  45  10   1
```

8. Use a loop to print the first several rows of pasacal's triangle.

```
> for(n in 0:10)print(choose(n,0:n))
[1] 1
[1] 1 1
[1] 1 2 1
[1] 1 3 3 1
[1] 1 4 6 4 1
[1]  1  5 10 10  5  1
[1]  1  6 15 20 15  6  1
[1]  1  7 21 35 35 21  7  1
[1]  1  8 28 56 70 56 28  8  1
 [1]   1   9  36  84 126 126  84  36   9   1
 [1]   1  10  45 120 210 252 210 120  45  10   1
```

Binomial Distribution

The **binomial distribution** is a discrete probability distribution. It describes the outcome of *n* independent trials in an experiment. Each trial is assumed to have only two outcomes, either success or failure. If the probability of a successful trial is *p*, then the probability of having *x* successful outcomes in an experiment of *n* independent trials is as follows.

$$P[X = x] = \binom{n}{x} p^x q^{n-x}, x = 0,1,\ldots\ldots n$$

Mean $\mu_1' = np$ **Variance:** $\mu_2 = npq$

Syntax:-

For a binomial(n,p) random variable X, the R functions involve the abbreviation "binom":

```
dbinom(k,n,p)  # binomial(n,p) density at k: Pr(X = k)

pbinom(k,n,p)  # binomial(n,p) CDF at k: Pr(X <= k)

qbinom(P,n,p)  # binomial(n,p) P-th quantile

rbinom(N,n,p)  # N binomial(n,p) random variables

help(Binomial) # documentation on the functions related
               # to the Binomial distribution
```

Problem1. Find the Probability of getting two '4' among ten dice

>dbinom(2,size=10,prob=1/6)
[1] 0.29071

Problem 2: Find the P(2) by using binomial probability formula

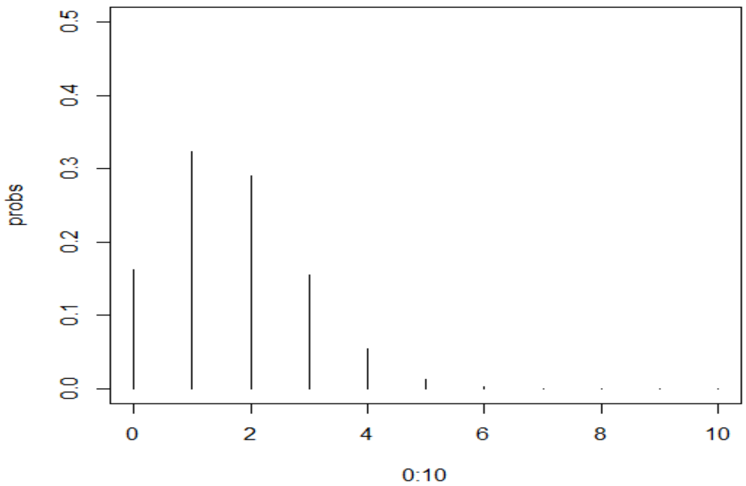
> choose(10,2)*(1/6)^2*(5/6)^8
[1] 0.29071

Problem 3: Find the table for BIN(n=10,P=1/6)

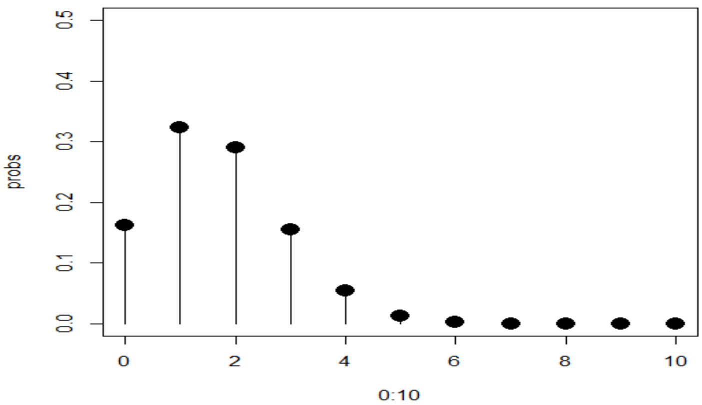
```
> probs=dbinom(0:10,size=10,prob=1/6)
> data.frame(0:10,probs)
   X0.10        probs
1      0 1.615056e-01
2      1 3.230112e-01
3      2 2.907100e-01
4      3 1.550454e-01
5      4 5.426588e-02
6      5 1.302381e-02
7      6 2.170635e-03
8      7 2.480726e-04
9      8 1.860544e-05
10     9 8.269086e-07
11    10 1.653817e-08
> 
```

Problem4: BINOMIAL PROBABILITY PLOTS :Draw a Plot for the Binomial distribution Bin(n=10,p=1/6)

>plot(0:10,probs,type="h",xlim=c(0,10),ylim=c(0,0.5))

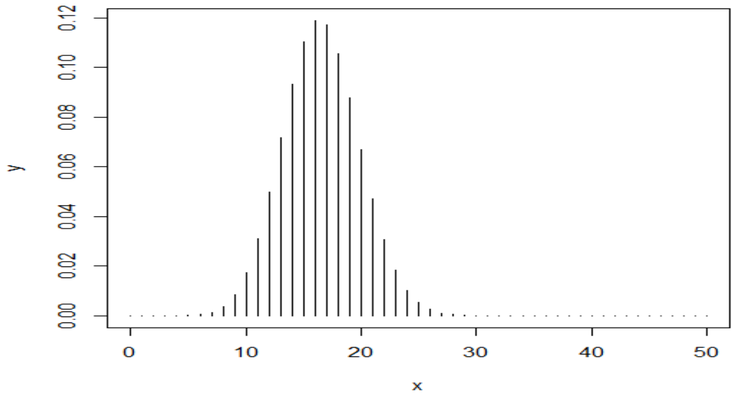


>points(0:10,probs,pch=16,cex=2)



Problem 5: Plot Binomial distribution with n=50 and P=0.33

```
> x=0:50
>(x,size=50,prob=0.33)
> plot(x,y,type="h")
```



Problem 6 : For a Binomial(7,1/4) random variable named X,

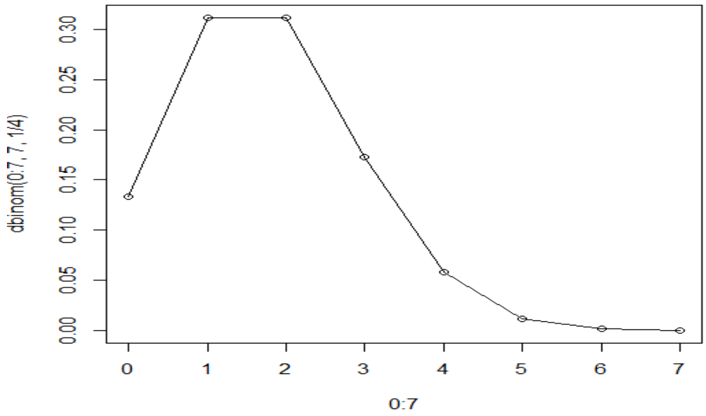
i. Compute the probability of two success
ii. Compute the Probablities for whole space
iii. Display those probabilities in a table
iv. Show the shape of this binomial Distribution

Solution:

```
> dbinom(2,7,1/4)                    # probability of two success
[1] 0.3114624

> dbinom(0:7,7,1/4)                  # probabilities for whole space

[1] 1.334839e-01 3.114624e-01 3.114624e-01 1.730347e-01 5.767822e-02

[6] 1.153564e-02 1.281738e-03 6.103516e-05
```

```
> P=data.frame(0:7,dbinom(0:7,7,1/4))          #probabilities in a Table
> round(P,4)
  X0.7 dbinom.0.7..7..1.4.
1    0              0.1335
2    1              0.3115
3    2              0.3115
4    3              0.1730
5    4              0.0577
6    5              0.0115
7    6              0.0013
8    7              0.0001
```

```
>plot(0:7,dbinom(0:7,7,1/4),type="o")          #shape of the Distribution
```



Problem 7: Suppose there are twelve multiple choice questions in an English class quiz. Each question has five possible answers, and only one of them is correct. Find the probability of having four or less correct answers if a student attempts to answer every question at random.

Solution

Since only one out of five possible answers is correct, the probability of answering a question correctly by random is 1/5=0.2. We can find the probability of having exactly 4 correct answers by random attempts as follows.

R CODE:-

```
> dbinom(4, size=12, prob=0.2)
[1] 0.1329
```

To find the probability of having four or less correct answers by random attempts, we apply the function dbinom with $x = 0,\ldots,4$.

```
> dbinom(0, size=12, prob=0.2) +dbinom(1, size=12, prob=0.2) + dbinom(2, size=12, prob=0
.2) + dbinom(3, size=12, prob=0.2) + dbinom(4, size=12, prob=0.2)
[1] 0.9274
```

Or

```
> sum(dbinom(x=0:4,size=12,prob=0.2))

[1] 0.9274445
```

or

Alternatively, we can use the cumulative probability function for binomial distribution pbinom.

```
> pbinom(4, size=12, prob=0.2)
[1] 0.92744
```

The probability of four or less questions answered correctly by random in a twelve question multiple choice quiz is 92.7%.

Problem 8: If 10% of the Screws produced by an automatic machine are defective, find the probability that out of 20 screws selected at random, there are

(i) Exactly 2 defective
(ii) At least 2 defectives
(iii) Between 1 and 3 defectives (inclusive)

Code:-

(i)
```
> # Exactly 2 defective P=0.10 n=20
> dbinom(2,20,0.10)
[1] 0.2851798
```

(ii)
```
> 1-dbinom(1,20,0.10)
[1] 0.7298297
```

(iii)
```
> #Between 1 and 3 defectives (inclusive)
> x=sum(dbinom(1:3,20,0.10))
> x
[1] 0.74547
```

Relationship between mean and variance :-

Problem 9: Show that Binomial distribution variance is less than mean with Binomial variable follows (7,1/4)

```
> n=7
> p=1/4
> x=dbinom(0:7,n,p)
> x
[1] 1.334839e-01 3.114624e-01 3.114624e-01 1.730347e-01 5.767822e-02
[6] 1.153564e-02 1.281738e-03 6.103516e-05
> Ex=sum(x*p)
> Ex
[1] 0.25
> var=sum((x-Ex)^2*x)
> var
[1] 0.008062817
```

THE POISON DISTRIBUTION:

If the number of Bernoulli trials of a random experiment is fairly large and the probability of success is small it becomes increasingly difficult to compute the binomial probabilities. For values of n and p such that $n \geq 150$ and $p \leq 0.05$, the poisson distribution serves as an excellent approximation to the binominal distribution.

The random variable X is said to follow the Poisson distribution if and only if

$$p\left[X = x\right] = \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}, x = 0,1,2,.....$$

Assumptions:-

1. Number of Bernoulli trials (n) is indefinitely large, $(n \rightarrow \infty)$
2. The trials are independent.
3. Probability of success (p) is very small, $(p \rightarrow 0)$

   λ = np is constant, $\lambda = np \Rightarrow p = \dfrac{\lambda}{n}$
4. Mean and variance in poison distribution are equal

Syntax:-

```
dpois(x, lambda, log = FALSE)
ppois(q, lambda, lower.tail = TRUE, log.p = FALSE)
qpois(p, lambda, lower.tail = TRUE, log.p = FALSE)
rpois(n, lambda)
```

Problem 1:

a. #P(x=5) with parameter 7
```
> dpois(x=5,lambda=7)
[1] 0.1277167
```

b. #P(x=0)+P(x=1)+……….+P(x=5)
```
> dpois(x=0:5,lambda=7)

[1] 0.000911882 0.006383174 0.022341108 0.052129252 0.091226192 0.127716668
```

c. > #P(x<=5)
```
> sum(dpois(0:5,lambda=7))
[1] 0.3007083
```
Or

```
> ppois(q=4,lambda=7,lower.tail=T)
[1] 0.1729916
```

d. > ppois(q=12,lambda=7,lower.tail=F)
```
[1] 0.02699977
```

Problem 2 : Check the relationship between mean and variance in Poisson distribution(4) with n=100

```
> X.val=0:100
> P.val=dpois(X.val,4)
> EX=sum(X.val*P.val)              #mean
> EX
[1] 4
> sum((X.val-EX)^2*P.val)          #variance
[1] 4
```

Problem 3 : Compute Probabilities and cumulative probabilities of the values between 0 and 10 for the parameter 2 in poisson distribution.

```
> dpois(0:10,2)                              # probabilities

 [1] 1.353353e-01 2.706706e-01 2.706706e-01 1.804470e-01 9.022352e-02
 [6] 3.608941e-02 1.202980e-02 3.437087e-03 8.592716e-04 1.909493e-04
[11] 3.818985e-05
```

Or

```
> P=data.frame (0:10,dpois(0:10,2))
> round (P,4)
```

```
   X0.10 dpois.0.10..2.
1      0         0.1353
2      1         0.2707
3      2         0.2707
4      3         0.1804
5      4         0.0902
6      5         0.0361
7      6         0.0120
8      7         0.0034
9      8         0.0009
10     9         0.0002
11    10         0.0000
```

```
> ppois(0:10,2)                              # cumulative probabilities

 [1] 0.1353353 0.4060058 0.6766764 0.8571235 0.9473470 0.9834364 0.9954662
 [8] 0.9989033 0.9997626 0.9999535 0.9999917
```

Or

```
> P=data.frame(0:10,ppois(0:10,2))
> round(P,4)
   X0.10 ppois.0.10..2.
1      0         0.1353
2      1         0.4060
3      2         0.6767
4      3         0.8571
5      4         0.9473
6      5         0.9834
7      6         0.9955
8      7         0.9989
9      8         0.9998
10     9         1.0000
11    10         1.0000
```

Problem 3: Poisson distribution with parameter ‘2’

1. How to obtain a sequence from 0 to 10
2. Calculate P(0),P(1),...,P(10) when lambda =2 and Make the output prettier
3. Find P(x <= 6)
4. Sum all probabilities
5. Find P(Y>6)
6. Make a table of the first 11 Poisson probs and cumulative probs when # mu=2 and make the output prettier
7. Plot the probabilities Put some labels onthe axes and give the plot a title:

a. > 0:10 #sequence from 0:10

```
[1]  0  1  2  3  4  5  6  7  8  9 10
```

b. > round(dpois(0:10, 2), 3)

```
[1] 0.135 0.271 0.271 0.180 0.090 0.036 0.012 0.003 0.001 0.000 0.000
```

c. > ppois(6, 2) # Find P(x <= 6)

```
[1] 0.9954662
```

d. > sum(dpois(0:6, 2)) # Sum all probabilities

```
[1] 0.9954662
```

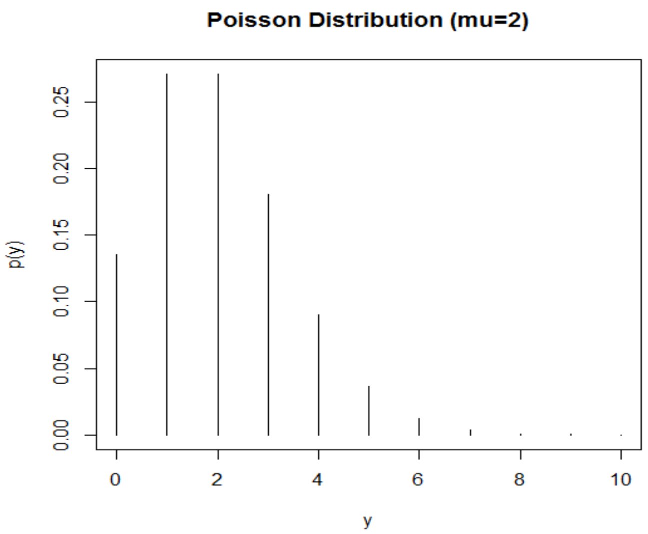
e. > 1 - ppois(6, 2) # Find P(Y>6)

```
[1] 0.004533806
```

f.

```
> round(cbind(0:10, dpois(0:10,2), ppois(0:10,2)), 3)
      [,1]  [,2]  [,3]
 [1,]    0 0.135 0.135
 [2,]    1 0.271 0.406
 [3,]    2 0.271 0.677
 [4,]    3 0.180 0.857
 [5,]    4 0.090 0.947
 [6,]    5 0.036 0.983
 [7,]    6 0.012 0.995
 [8,]    7 0.003 0.999
 [9,]    8 0.001 1.000
[10,]    9 0.000 1.000
[11,]   10 0.000 1.000
```

g. plot(0:10,dpois(0:10,2),type="h",xlab="y",ylab="p(y)",main="Poisson Distribution (mu=2)")



Practice Problems:-(Binomial distribution)

1. For a random variable X with a binomial(20,1/2) distribution, find the following probabilities.
   (i). Find Pr(X < 8)
   (ii). Find Pr(X > 12)
   (iii) Find Pr(8 <= X <= 12)

2. For a binomial(200,1/2) distribution:
   (i) Find Pr(X < 80)
   (ii) Find Pr(X > 120)
   (iii) Find Pr(80 <= X <= 120)

3. For a binomial(2000,1/2) distribution:
   Find Pr(X < 800)
   Find Pr(X > 1200)
   Find Pr(800 <= X <= 1200)

4. Let X be the number of heads in 10 tosses of a fair coin.
   1. Find the probability of getting at least 5 heads (that is, 5 or more).
   2. Find the probability of getting exactly 5 heads.
   3. Find the probability of getting between 4 and 6 heads, inclusive

5. Suppose our random variable X is Poisson with lambda = 12.33.

   1. What is the probability of 15 or fewer occurrences? P(X <= 15)
   2. What is the probability of EXACTLY 6 occurrences? P(X = 6)
   3. What is the probability of more than 15 occurrences? P(X > 15)
   4. What is the probability of 15 or more occurrences? P(X >= 15)
   5. What is the probability of 8, 9, or 10 occurrences? P(8 <= X <= 10)

Compare binomial distribution and Poisson distribution

6. Let X be the number of heads in 100 tosses of a fair coin.
7. Let X be the number of heads in 1000 tosses of a fair coin.

Challenging Experiments:

Problem 1: A recent national study showed that approximately 55.8% of college students have used Google as a source in at least one of their term papers. Let X equal the number of students in a random sample of size n = 42 who have used Google as a source:

1. How is X distributed?
2. Sketch the probability mass function (roughly).
3. Sketch the cumulative distribution function (roughly).
4. Find the probability that X is equal to 17.
5. Find the probability that X is at most 13.
6. Find the probability that X is bigger than 11.
7. Find the probability that X is at least 15.
8. Find the probability that X is between 16 and 19, inclusive
9. Give the mean of X, denoted IE X.
10. Give the variance of X.

11. Give the standard deviation of X.
12. Find IE(4X + 51:324)
13. Compare mean and variance

Problem 2: (Traffic accident problem)

The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 7.6.

1. Find the probability that less than three accidents will occur next month on this stretch of road.
2. Find the probability of observing exactly three accidents on this stretch of road next month.
3. Find the probability that the next two months will both result in four accidents each occurring on this stretch of road.
4. Check the mean and variance of the poisson distribution
5. Plot the Poisson distribution and compare with binomial distribution

LAB-7

Normal Distribution fitting and Plotting

AIM: ***Computing/plotting and visualising the following probability distributions***

About Normal Distribution:-

THE NORMAL DISTRIBUTION :

A random variable X is said to posses normal distribution with mean μ and variance σ^2 , if its probability density function can be expressed of the form,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad , \qquad -\infty < x < +\infty$$

The standard notation used to denote a random variable to follow normal distribution with appropriate mean and variance is, $X \sim N(\mu , \sigma^2)$

STANDARD NORMAL DISTRIBUTION :

If a random variable X follows normal distribution with mean μ and variance σ^2 , its transformation $Z = \frac{X-\mu}{\sigma}$ follows standard normal distribution (mean 0 and unit variance)

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad , \qquad -\infty < z < +\infty$$

The distribution function of the standard normal distribution

$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

R Synatax :-

R has four in built functions to generate normal distribution. They are described below.

```
dnorm(x, mean, sd)
pnorm(x, mean, sd)
qnorm(p, mean, sd)
rnorm(n, mean, sd)
```

Following is the description of the parameters used in above functions:

- ***x*** *is a vector of numbers.*
- ***p*** *is a vector of probabilities.*
- ***n*** *is number of observations(sample size).*
- ***mean*** *is the mean value of the sample data. It's default value is zero.*
- ***sd*** *is the standard deviation. It's default value is 1*

(I) Normal distribution computations and graphs

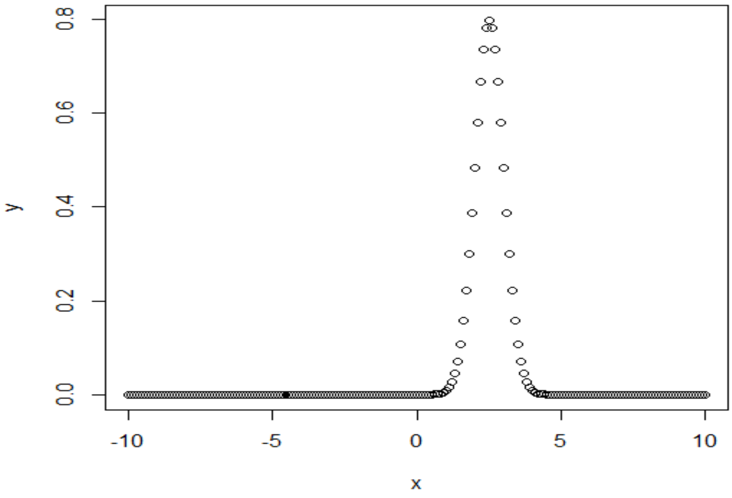
dnorm() :

This function gives height of the probability distribution at each point for a given mean and standard deviation.

CODE:-

```
Create a sequence of numbers between -10 and 10 incrementing by 0.1.
x <- seq(-10,10,by=.1)

# Choose the mean as 2.5 and standard deviation as 0.5.
y <- dnorm(x, mean= 2.5, sd = 0.5)
plot(x,y)
```

OUTPUT:



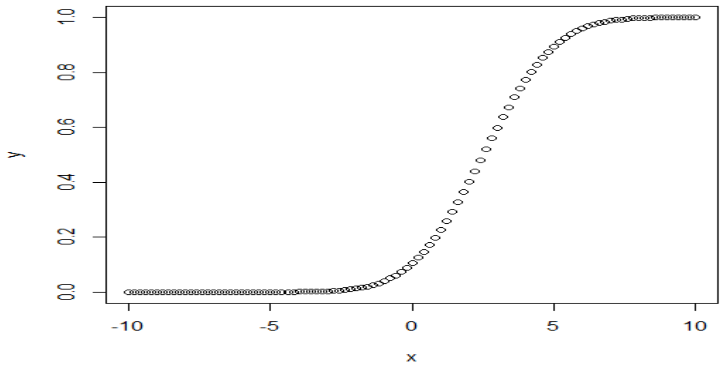
pnorm():

This function gives the probability of a normally distributed random number to be less that the value of a given number. It is also called "Cumulative Distribution Function".

CODE:-

```
# Create a sequence of numbers between -10 and 10 incrementing by 0.2.
 x <- seq(-10,10,by=.2)
# Choose the mean as 2.5 and standard deviation as 2.
 y <- pnorm(x,mean=2.5,sd = 2)
# Plot the graph.
plot(x,y)
```

OUTPUT:



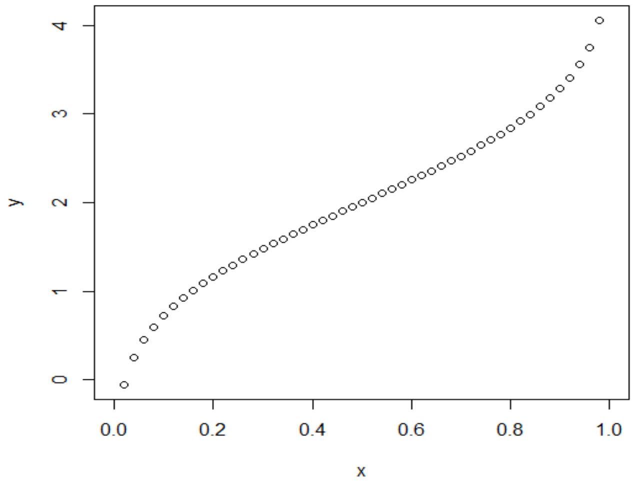
qnorm():-

This function takes the probability value and gives a number whose cumulative value matches the probability value.

CODE:-

```
# Create a sequence of probability values incrementing by 0.02.
x <- seq(0,1,by=0.02)
# Choose the mean as 2 and standard deviation as 3.
y <- qnorm(x,mean=2,sd=1)
# Plot the graph.
plot(x,y)
```

OUTPUT:-



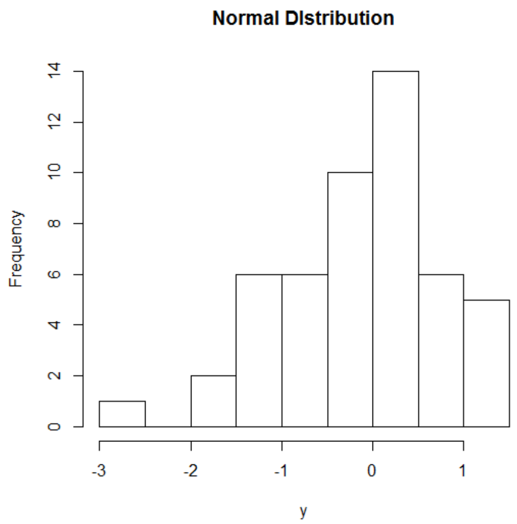
rnorm()

This function is used to generate random numbers whose distribution is normal. It takes the sample size as input and generates that many random numbers. We draw a histogram to show the distribution of the generated numbers.

CODE:-

```
# Create a sample of 50 numbers which are normally distributed.
y <- rnorm(50)
# Plot the histogram for this sample.
hist(y, main = "Normal DIstribution")
```

OUTPUT:-



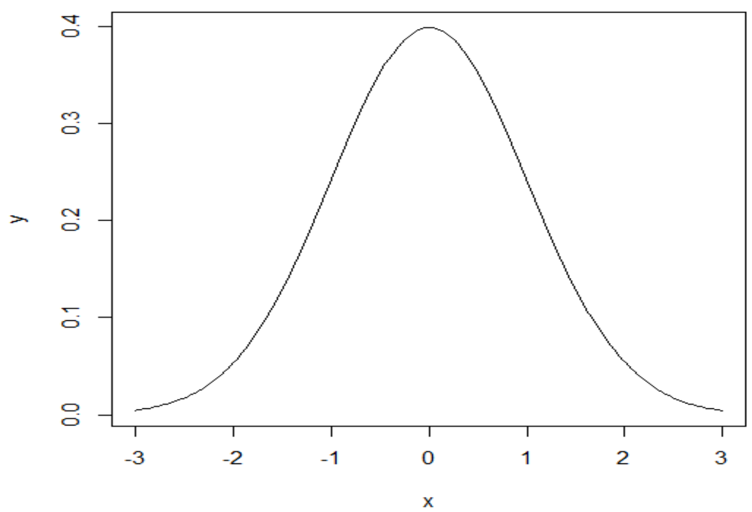
(II) Standard Normal Probability Distribution Plotting and Finding the Area:

CODE 1 :-

 # create a sequence of 200 numbers, beginning at x=-3 and ending at x=3

```
>x=seq(-3,3,length=200)
# use the dnorm command to compute the y-values of the standard normal
# probability density function (mean=0, standard deviation=1)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y)
>plot(x,y,type="l")
```

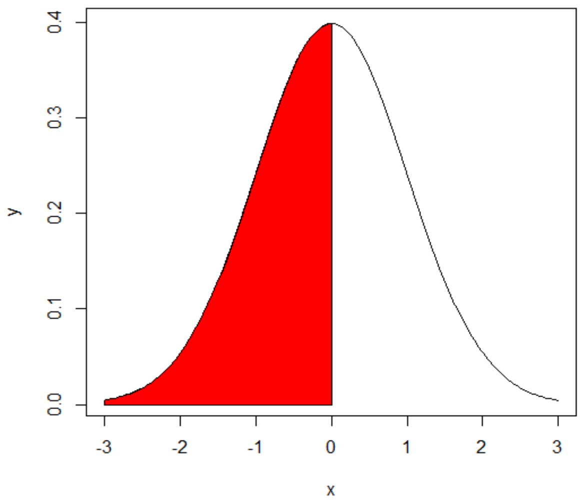
OUTPUT:-



CODE 2:

```
# Draw another normal curve, use a mean=50 and a standard
deviation=10.

>x=seq(20,80,length=200)
>y=dnorm(x,mean=50,sd=10)
>plot(x,y,type="l")

# Find the area under the curve to left of the mean

>x=seq(-3,3,length=200)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y,type="l")
>x=seq(-3,0,length=100)
>y=dnorm(x,mean=0,sd=1)
>polygon(c(-3,x,0),c(0,y,0),col="red")

# Find the area to the left of mean=0 (it should be 0.5)

>pnorm(0,mean=0,sd=1)
```

OUTPUT:-

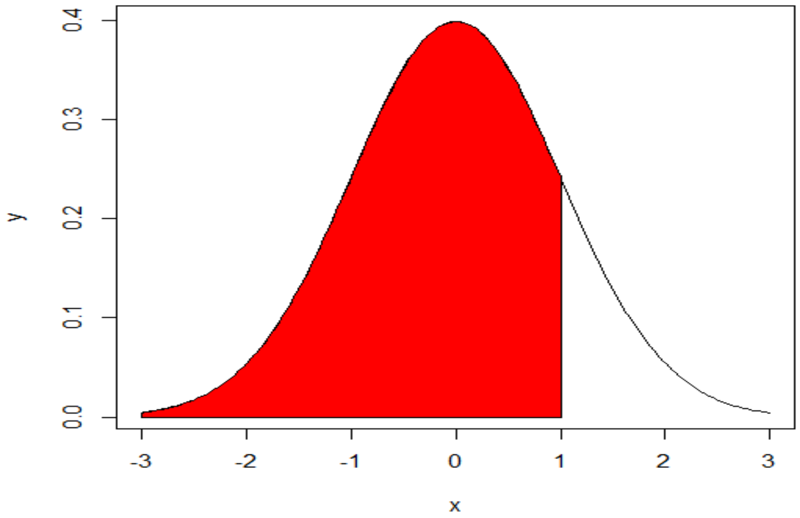


CODE 3:

 # Find the area to the left of 1. First, draw an image, then compute

```
>x=seq(-3,3,length=200)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y,type="l")
>x=seq(-3,1,length=100)
>y=dnorm(x,mean=0,sd=1)
>polygon(c(-3,x,1),c(0,y,0),col="red")
>pnorm(1,mean=0,sd=1)
```

OUTPUT:

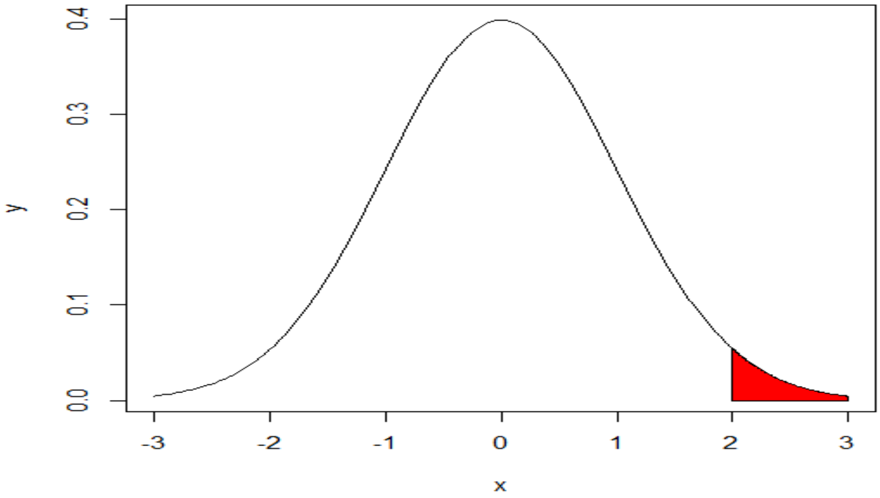


CODE 4:-

Get the area to the right of 2. First, draw an image, then compute

```
>x=seq(-3,3,length=200)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y,type="l")
>x=seq(2,3,length=100)
>y=dnorm(x,mean=0,sd=1)
>polygon(c(2,x,3),c(0,y,0),col="red")
>1-pnorm(2,mean=0,sd=1)
```

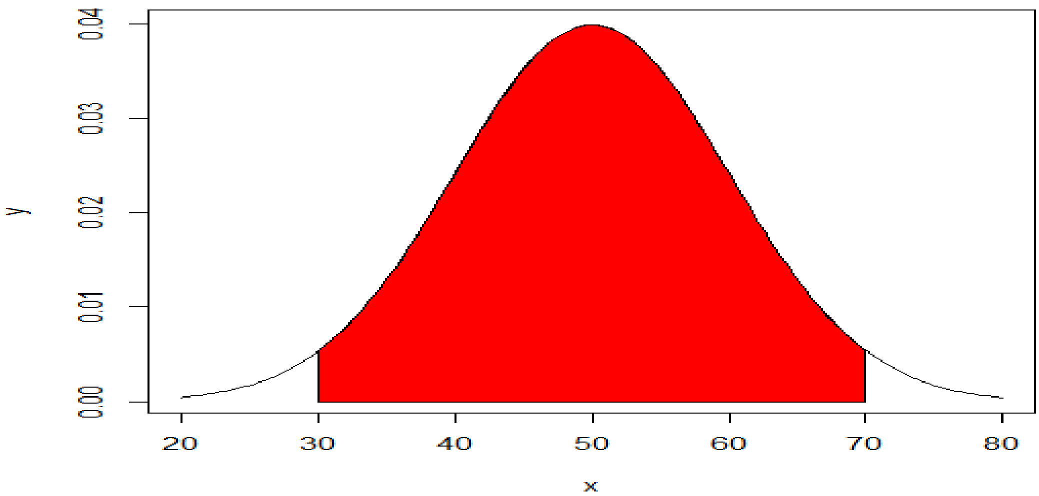
OUTPUT:-



CODE 5:

```
# Use the pnorm command to find areas under the normal density curve,
# regardless of the mean and standard deviation values.
# Example, mean=50 and standard deviation=10.

> x=seq(20,80,length=200)
>y=dnorm(x,mean=50,sd=10)
>plot(x,y,type="l")
>x=seq(30,70,length=100)
>y=dnorm(x,mean=50,sd=10)
>polygon(c(30,x,70),c(0,y,0),col="red")
>pnorm(70,mean=50,sd=10)-pnorm(30,mean=50,sd=10)
```

OUTPUT:

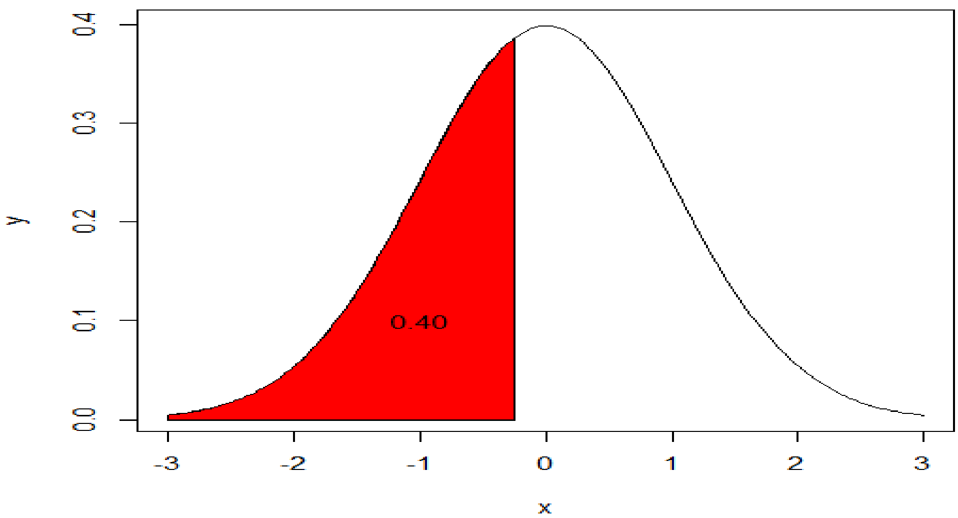


CODE 5:

Find the Quantile (Percentile) - i.e., reverse the process.
That is, given the area, find the value of x.

```
>x=seq(-3,3,length=200)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y,type="l")
>x=seq(-3,-0.2533,length=100)
>y=dnorm(x,mean=0,sd=1)
>polygon(c(-3,x,-0.2533),c(0,y,0),col="red")
>text(-1,0.1,"0.40")
>qnorm(0.40,mean=0,sd=1)
```

OUTPUT:-



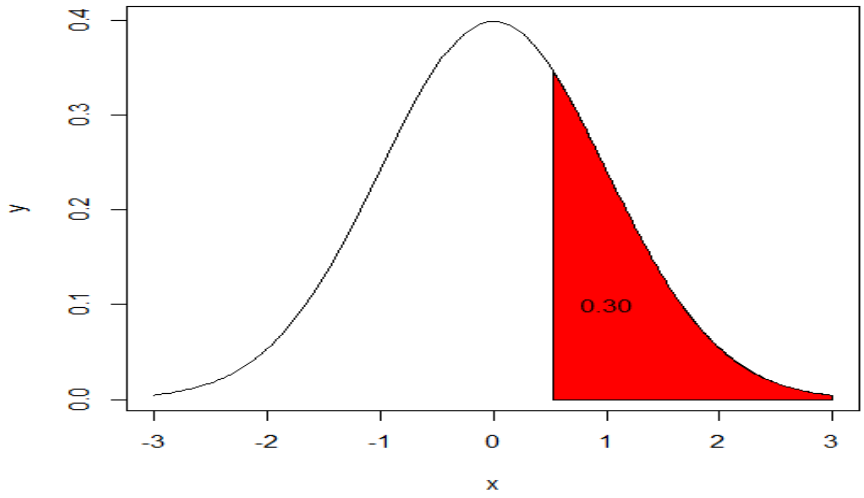
CODE 6:

density curve to the right of x is the given area.

```
>x=seq(-3,3,length=200)
>y=dnorm(x,mean=0,sd=1)
>plot(x,y,type="l")
>x=seq(0.5244,3,length=100)
>y=dnorm(x,mean=0,sd=1)
>polygon(c(0.5244,x,3),c(0,y,0),col="red")
>text(1,0.1,"0.30")
```

>qnorm(0.70,mean=0,sd=1)

OUTPUT:-



CODE 7:

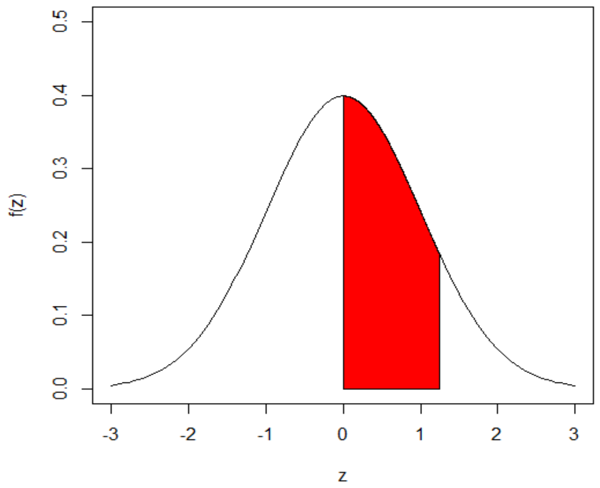
1. Find $P(0 < Z < 1.24)$

```
> pnorm(1.24) - pnorm(0)
[1] 0.3925123
```

Normal Probability Shape :

```
> plot.new()
> curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab = "z", ylab="f(z)")
> zleft = 0
> zright = 1.24
> x = c(zleft, seq(zleft, zright, by=.001), zright)
> y = c(0, dnorm(seq(zleft, zright, by=.001)), 0)
> polygon(x, y, col="red")
```

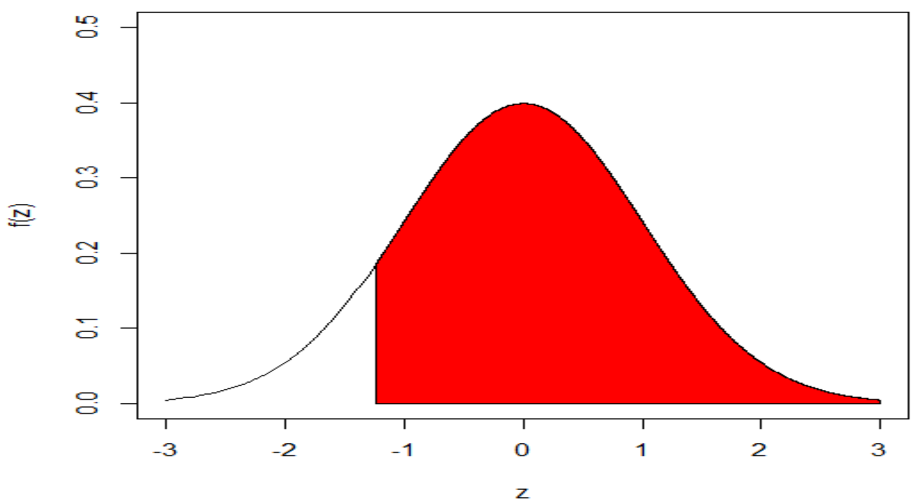
Output:-



2. Find $P(Z > -1.24)$

>1 - pnorm(-1.24)
> plot.new()
>curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab = "z", ylab="f(z)")
>z = -1.24
>x = c(z, seq(z, 3, by=.001), 3)
>y = c(0, dnorm(seq(z, 3, by=.001)), 0)
>polygon(x, y, col="red")

Output:-

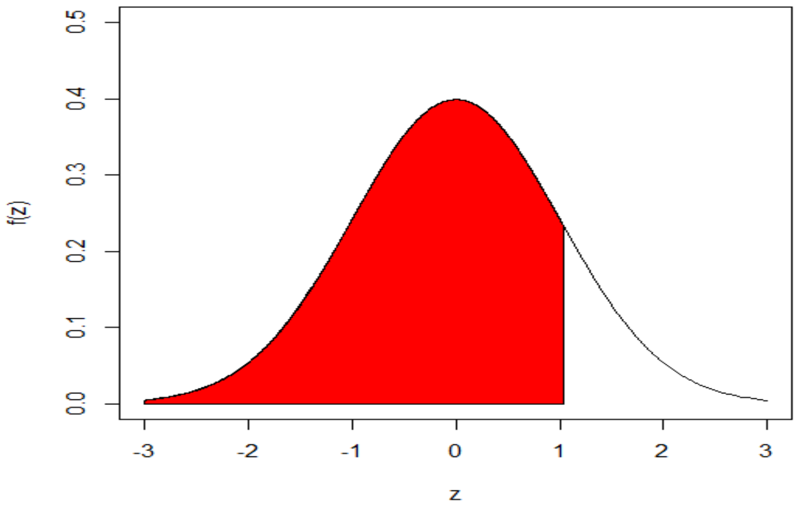


3. Find P_{85}, the 85_{th} percentile of the standard normal “z” distribution.

> qnorm(0.85)
[1] 1.036433
> plot.new()
> curve(dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab = "z", ylab="f(z)")

> prob = 0.85
> x = c(-3, seq(-3, qnorm(prob), by=.001), qnorm(prob))
> y = c(0, dnorm(seq(-3, qnorm(prob), by=.001)), 0)
> polygon(x, y, col="red")

Output:-

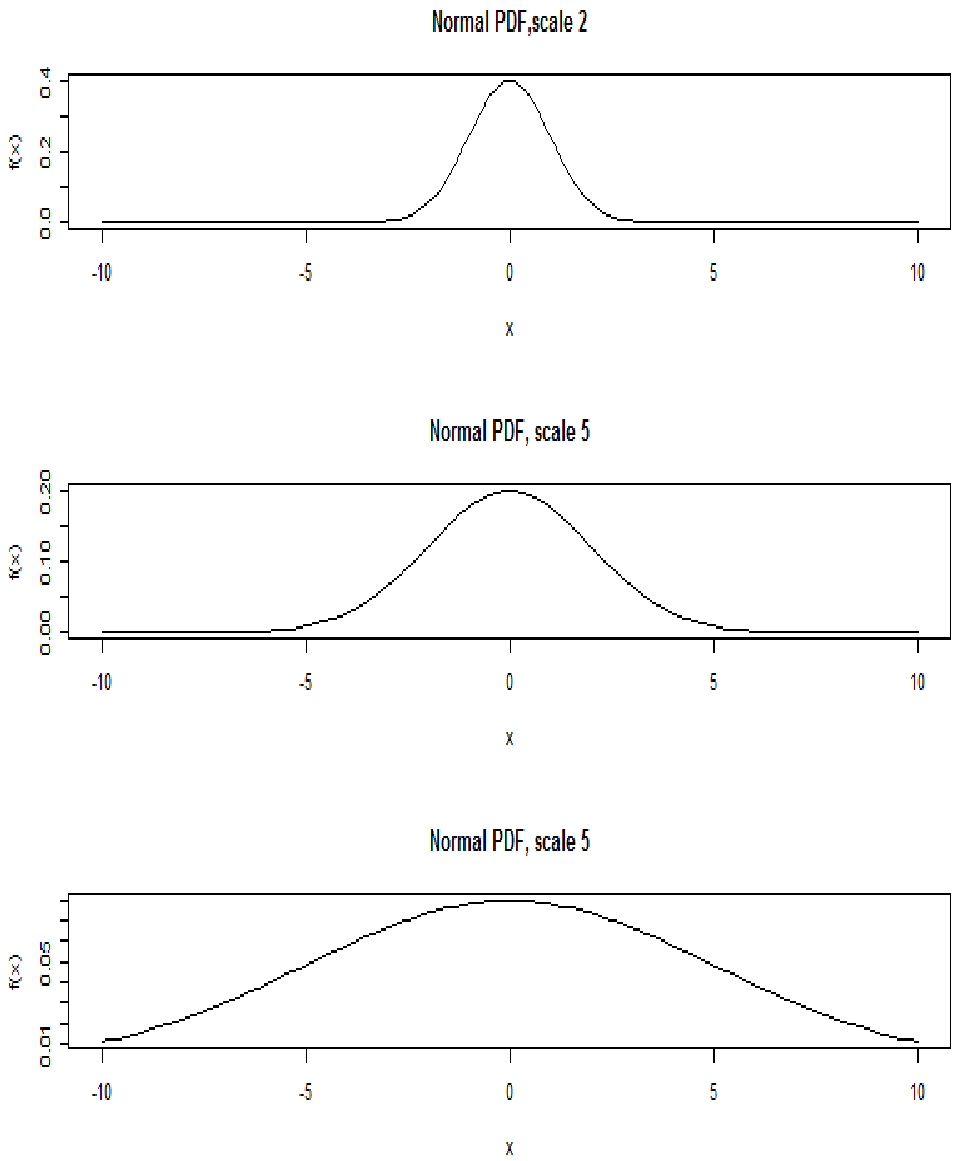


CODE 8:

(a) Plot a normal density for a range of x from –10 to 10 with mean 0 and standard deviation 1:{This Problem Explains types of kurtosis by changing standard deviation}

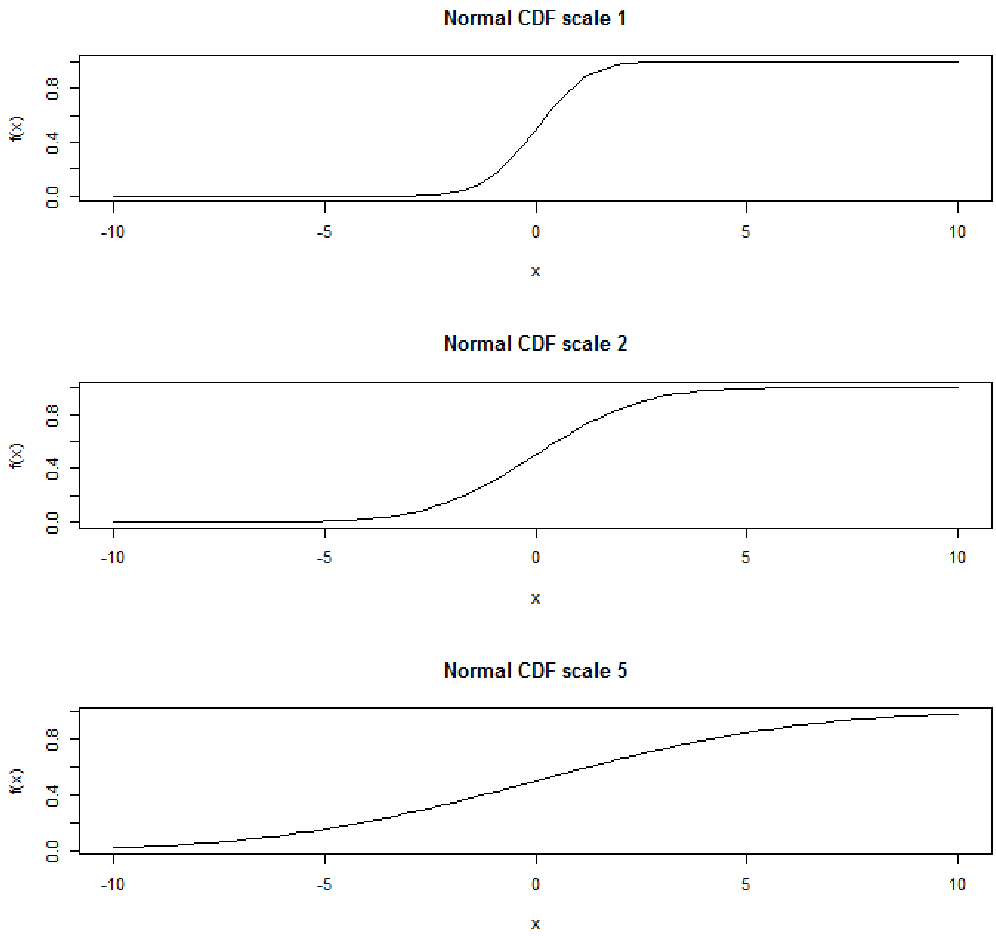
```
> x<-seq(-10,10,length=100)
> plot(x,dnorm(x,0,1),xlab="x", ylab="f(x)", type='l', main="Normal PDF")
> par(mfrow=c(3,1))
> plot(x,dnorm(x,0,1),xlab="x",ylab="f(x)", type='l', main="Normal PDF,scale 2")
> plot(x,dnorm(x,0,2),xlab="x",ylab="f(x)", type='l',main="Normal PDF, scale 5")
> plot(x,dnorm(x,0,5),xlab="x",ylab="f(x)", type='l',main="Normal PDF, scale 5")
```

OUTPUT:-



(b) Normal distribution Cumulative Distribution Function with different scale parameters

```
> par(mfrow=c(3,1))
> plot(x,pnorm(x,0,1),xlab="x",ylab="f(x)", type='l', main="Normal CDF scale 1")
> plot(x,pnorm(x,0,2),xlab="x", ylab="f(x)", type='l', main="Normal CDF scale 2")
> plot(x,pnorm(x,0,5),xlab="x", ylab="f(x)", type='l', main="Normal CDF scale 5")
```



CODE 9:

Problem : In a photographic process the developing times of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation 0.12 second. Find the probability that it will take

(i) *Atleast 16.20 seconds to develop one of the prints;*
(ii) *atmost 16.35 seconds to develop one of the prints*

Ans) *Atleast 16.20 seconds to develop one of the prints;*
Print developing time : X ~ N(16.28 , $(0.12)^2$)

(Solution). (i) Required event : [X ≥ 16.20]

$$P[X \geq 16.20] = P\left[\frac{X - 16.28}{0.12} \geq \frac{16.20 - 16.28}{0.12}\right] = P[Z \geq -0.6667]$$

$$P[X \geq 16.20] = 0.7486 (\text{Dotted area})$$

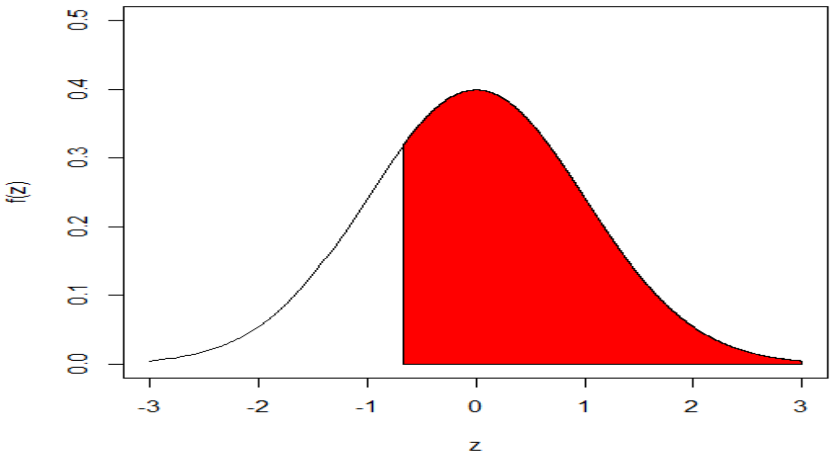
R Code:-

(i). $P[Z \geq -0.6667]$

> (1-pnorm (-0.6667))

[1] 0.7475181

```
> 1 - pnorm((-0.6667))
> plot.new()
> curve (dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab = "z", ylab="f(z)")
> z = -0.6667
> x = c(z, seq(z, 3, by=.001), 3)
> y = c(0, dnorm(seq(z, 3, by=.001)), 0)
> polygon(x, y, col="red")
```



(ii) *Required event : [X ≤ 16.35]*

$$P[X \leq 16.20] = P\left[Z \leq \frac{16.35 - 16.28}{0.12}\right]$$

$$= P[X \leq 0.5833] \text{(dotted area)}$$

$$= 0.5 + 0.2190 = 0.7190$$

$$P[X \leq 16.35] = 0.7190$$

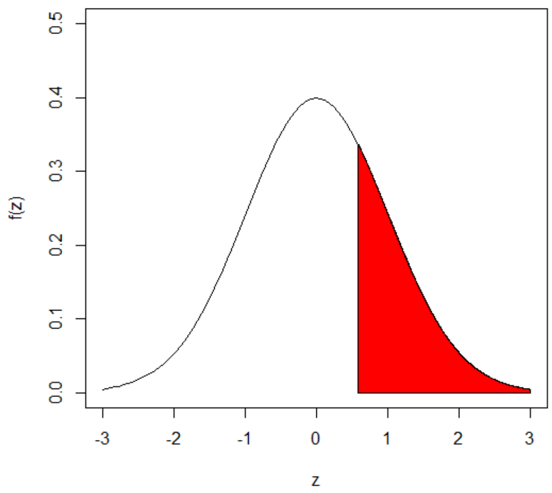
R code:-

> pnorm(0.5833)

[1] 0.7201543

```
> pnorm(0.5833)
> plot.new()
> curve (dnorm, xlim = c(-3, 3), ylim = c(0, 0.5), xlab = "z", ylab="f(z)")
> z = 0.5833
> x = c(z, seq(z, 3, by=.001), 3)
> y = c(0, dnorm(seq(z, 3, by=.001)), 0)
> polygon(x, y, col="red")
```

Output:-



(III) GENERAL NORMAL PROBABILITY DISTRIBUTIONS

1. *Suppose X is normal with mean 527 and standard deviation 105. Compute P(X ≤ 310).*

   > pnorm(310,527,105)
   [1] 0.01938279

2. *If* $X \sim N\left(\mu = 100 \text{ pts.}, \sigma = 15 \text{ pts.}\right)$

(i) Find $P\left(80 \text{ pts.} < X < 95 \text{ pts.}\right)$

> pnorm(95, mean=100, sd=15) - pnorm(80, mean=100, sd=15)
[1] 0.2782301

(ii) Find $P(X > 125 \text{ pts.})$.

```
> 1 - pnorm(125, mean=100, sd=15)
[1] 0.04779035
```

(iii) Find P_{75}, the 75_{th} percentile of the above distribution. This is the same as the 0.75 quantile.

```
> qnorm(0.75, mean=100, sd=15)
[1] 110.1173
```

3. *The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 with S.D of Rs 5.Estimate the number of workers whose weekly wages will be*
   - *(i) Between Rs 69 and Rs 72*
   - *(ii) Less than Rs 69*
   - *(iii) More than Rs 72*

(i) Between Rs 69 and Rs 72

```
>( pnorm(72, mean=70, sd=5) - pnorm(69, mean=70, sd=5))*1000
 [1] 234.6815
```

The number of workers whose wages lies between Rs.69 and Rs.72 is 234

(ii) Less than Rs 69

```
> (pnorm(69, mean=70, sd=5))*1000
[1] 420.7403
```

The number of workers whose wages is less than Rs.69 is 421

(iii) More than Rs 72

```
> (1 - pnorm(72, mean=70, sd=5))*1000
[1] 344.5783
```

The number of workers whose wages is More than Rs.72 is 345

4. *In a test on 2000 Electric bulbs ,it was found that the life of particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for*
   - *(i) More than 2150 hours*
   - *(ii) Less than 1950 hours*
   - *(iii) More than 1920 hours but less than 2160 hours*
   - *(iv) More than 2150 hours*

R CODE:-

```
>(1 - pnorm(2150, mean=2040, sd=60))*2000
 [1] 66.75302
```

The number of bulbs expected to burn for more than 2150 hours is 67 (approximately)

(i) *Less than 1950 hours*

```
>(pnorm(1950, mean=2040, sd=60))*2000

[1] 133.6144
```

The number of bulbs expected to burn for less than 1950 hours is 134 (approximately)

(ii) *More than 1920 hours but less than 2160 hours*

```
>( pnorm(2160, mean=2040, sd=60) - pnorm(1920, mean=2040,
   sd=60))*2000
[1] 1908.999
```

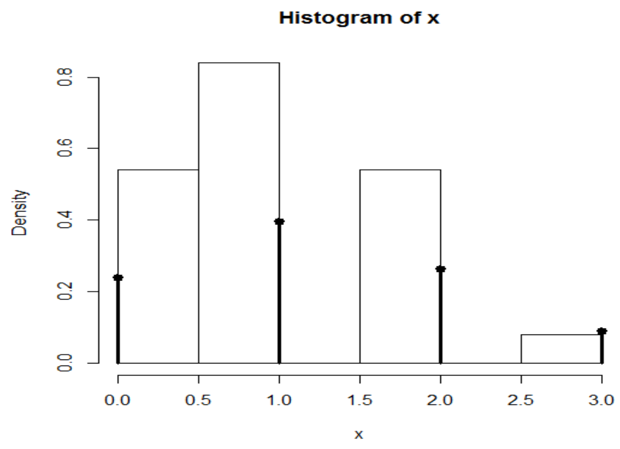
The number of bulbs expected to burn more than 1920 hours but less than 2160 is 1909 (approximately)

(IV) BINOMIAL DISTRIBUTION TENDS TO NORMAL DISTRIBUTION AS 'n' TENDS TO INFINITY:

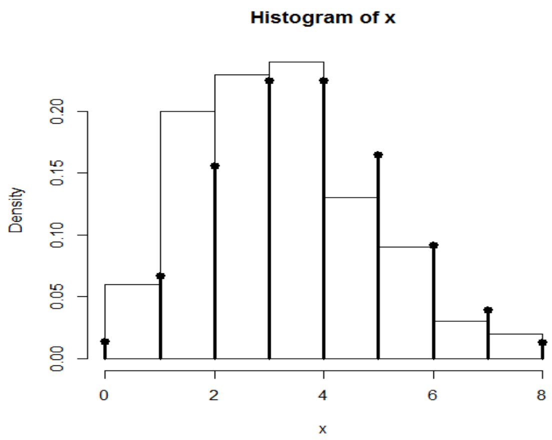
Binomial distribution tends to Normal distribution :------

```
> n=5;p=.25
> x=rbinom(100,n,p)
> hist(x,probability=TRUE,)
> ## use points, not curve as dbinom wants integers only for x
> xvals=0:n;points(xvals,dbinom(xvals,n,p),type="h",lwd=3)
> points(xvals,dbinom(xvals,n,p),type="p",lwd=3)
```

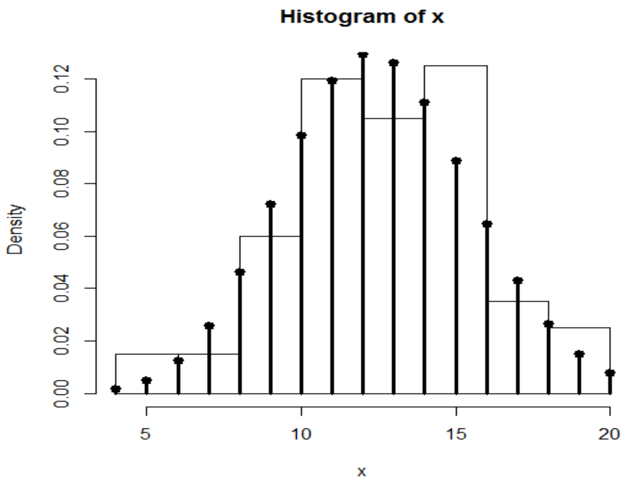
Output:-



n=15



n=50



(Note ;- In Statistics any probability distribution follows normal distribution as ‘n’ tends to ‘infinity’ with certain assumptions)

Practice Experiments:-

Standard Normal distribution:-

1. *Find (i)* $P(0.8 \le Z \le 1.5)$ *(ii)* $P(Z \le 2)$ *(iii)* $P(Z \ge 1)$
   Find These probability values and Plot the graph .

   General Normal Distribution.

2. *If mean=70 and Standard deviation is 16*

   i) $P(38 \le X \le 46)$ *ii)* $P(82 \le X \le 94)$ *iii)* $P(62 \le X \le 86)$

   Find the Probability values and Plot the graph with text.

   (***Standard Normal distribution or General Normal Distribution.)***

3. *1000 students had Written an examination the mean of test is 35 and standard deviation is 5.Assumning the to be normal find*
   i) How many students Marks Lie between 25 and 40
   ii) How many students get more than 40
   iii) How many students get below 20
   iv) How many students get 50