

Appendix B1: Model

In this Appendix, we detail the ingredients of our model and walk through its solution.

B1.1 Productivity & Wages

We assume two levels of productivity, w_L and w_S , for the large and small metro respectively, where $w_L > w_S$. In this simplified setting with perfectly competitive labor markets, workers earn their marginal product so wages equal productivity. An example for such metros is New York and Indianapolis, respectively. We take productivity to be exogenous to abstract away from the underlying drivers of productivity. One limitation of this approach is that productivity responds endogenously to population density as documented by Glaeser and Gottlieb (2009) and Ciccone and Hall (1996). Because we are interested in qualitative predictions though, this assumption is justified since agglomeration will only strengthen the relationships found in the model. We further assume that productivity does not change when a worker is remote compared to non-remote. This assumption is uncertain but has some empirical basis – recent evidence indicates remote workers may even see a productivity boost (Barrero, Bloom, and Davis, 2021).

B1.2 Amenities

We define amenity levels across the four locations as follows:

- large metro city center = a_1
- large metro suburb = a_2
- small metro city center = a_3
- small metro suburb = a_4

We let $a_1 > a_2$ and $a_3 > a_4$. Alternate permutations of the amenity levels are useful to consider because many individuals value traditional suburban amenities like parks, neighborhood safety, and school quality over traditional city amenities like restaurants, bars, and tourist attractions.

B1.3 Rents

The final feature of the model is housing rents. We let rental costs have a constant elasticity with respect to population:

$$\log r_i = \alpha + \epsilon_i \log n_i \quad (4)$$

In reality, the functional form for this relationship depends on location-specific factors such as zoning, but we abstract away from this heterogeneity for analytical convenience.

B1.4 Spatial Equilibrium

We use the following clearing condition to derive the spatial equilibrium. Since agents are homogeneous, log utility will be equal across locations i.e. $u_i = u_j$ for all i, j . We use this condition to calculate the percent differences in population between areas under each WFH scenario.

B1.4.1 No work-from-home

The log utility levels for each of the four locations are given by:

- Large city: $u_1 = \beta w_1 - \theta + \gamma a_1 - \beta(\alpha + \epsilon n_1)$
- Large suburb: $u_2 = \beta w_1 - \theta x + \gamma a_2 - \beta(\alpha + \epsilon n_2)$
- Small city: $u_3 = \beta w_2 - \theta + \gamma a_3 - \beta(\alpha + \epsilon n_3)$
- Small suburb: $u_4 = \beta w_2 - \theta x + \gamma a_4 - \beta(\alpha + \epsilon n_4)$

Equating the log utilities pairwise yields the following equilibrium percent differences in population since we are operating in log space:

- $n_1 - n_2 = \frac{\theta(x-1) + \gamma(a_1 - a_2)}{\beta\epsilon}$
- $n_3 - n_4 = \frac{\theta(x-1) + \gamma(a_3 - a_4)}{\beta\epsilon}$

- $n_1 - n_3 = \frac{\beta(w_1 - w_2) + \gamma(a_1 - a_3)}{\beta\epsilon}$
- $n_2 - n_4 = \frac{\beta(w_1 - w_2) + \gamma(a_2 - a_4)}{\beta\epsilon}$

Observe that the *within*-metro difference (between city center and suburb) is pinned down by the commute cost term x and the relative amenity levels. Larger commute costs for the suburb drives more people to the city center. Furthermore, the *between*-metro difference (between the large metro and the small metro) is positively related to productivity differences and amenity differences. Larger productivity differences or amenity differences increase the total population in the larger metro area.

B1.4.2 Full work-from-home

Next, we consider the case of full-time work-from-home where commute costs go to zero in all locations. Note that no other parameters change in the model. With no commute costs, the only factor preventing full agglomeration on the large metro city center (which has the highest productivity and amenity levels) is housing rents.

The new log utility levels for each of the four locations are given by:

- Large city: $\bar{u}_1 = \beta w_1 + \gamma a_1 - \beta(\alpha + \epsilon n_1)$
- Large suburb: $\bar{u}_2 = \beta w_1 + \gamma a_2 - \beta(\alpha + \epsilon n_2)$
- Small city: $\bar{u}_3 = \beta w_2 + \gamma a_3 - \beta(\alpha + \epsilon n_3)$
- Small suburb: $\bar{u}_4 = \beta w_2 + \gamma a_4 - \beta(\alpha + \epsilon n_4)$

Equating the log utilities pairwise yields the following equilibrium percent differences in population since we are operating in log space:

- $n_1 - n_2 = \frac{\gamma(a_1 - a_2)}{\beta\epsilon}$
- $n_3 - n_4 = \frac{\gamma(a_3 - a_4)}{\beta\epsilon}$
- $n_1 - n_3 = \frac{\gamma(a_1 - a_3)}{\beta\epsilon}$
- $n_2 - n_4 = \frac{\gamma(a_2 - a_4)}{\beta\epsilon}$

Observe that the equilibrium population ratios are now solely pinned down by the relative amenities between locations. This makes sense as full-time WFH allows one to access the productivity level of the large metro, w_1 , from anywhere. We can now consider differences between the no work-from-home setting and the full-work-from-home setting. There are two differences to consider. First, the *within*-metro population ratios no longer have a commute cost term so the difference in population ratios falls. Since rents are purely a function of population, the difference in rents also narrows. Third, the *between*-metro population ratios no longer have a productivity term so the difference in population between the large metro and small metro falls. All that remains are differences from amenities. Importantly, the model takes amenities as exogenous. But as Diamond (2016) shows, amenities respond endogenously to economic agglomeration so in a less simplified WFH model, population differences from amenities may fall further.¹

B1.4.3 Hybrid work-from-home

Hybrid work-from-home is analytically similar to the baseline no work-from-home setting. The log utility level equations remain the same except the commute cost terms are now scaled down and are given by:

- Large city: $\tilde{u}_1 = \beta w_1 - \theta\pi + \gamma a_1 - \beta(\alpha + \epsilon n_1)$
- Large suburb: $\tilde{u}_2 = \beta w_1 - \theta x\pi + \gamma a_2 - \beta(\alpha + \epsilon n_2)$

¹ Such models assume consumption amenities like restaurants and nightlife respond endogenously to population. Natural amenities like access to water will remain generating some differences across locations in amenity levels. Empirical research confirms that such natural amenities lead to persistent effects on economic geography that are resistant to minor shocks like policies or natural disasters (Lee and Lin, 2018).

- Small city: $\tilde{u}_3 = \beta w_2 - \theta\pi + \gamma a_3 - \beta(\alpha + \epsilon n_3)$
- Small suburb: $\tilde{u}_4 = \beta w_2 - \theta x\pi + \gamma a_4 - \beta(\alpha + \epsilon n_4)$

Equating the log utilities pairwise yields the following equilibrium percent differences in population since we are operating in log space:

- $n_1 - n_2 = \frac{\theta\pi(x-1) + \gamma(a_1 - a_3)}{\beta\epsilon}$
- $n_3 - n_4 = \frac{\theta\pi(x-1) + \gamma(a_3 - a_4)}{\beta\epsilon}$
- $n_1 - n_3 = \frac{\beta(w_1 - w_2) + \gamma(a_1 - a_3)}{\beta\epsilon}$
- $n_2 - n_4 = \frac{\beta(w_1 - w_2) + \gamma(a_2 - a_4)}{\beta\epsilon}$

The equilibrium percent differences have the same form as the differences from the no WFH case. The sole difference is the commute costs term is multiplied by π in the *within*-metro percent differences. The model predicts that the population percent differences will decrease. This means the city center to suburb population difference will decrease for both metros in the hybrid WFH world. Interestingly, the *between*-metro population percent difference does not change because it does not depend on commute costs. Thus, the model predicts that though there is reallocation of population (and therefore real estate demand) *within* metro areas, there is zero reallocation *between* metro areas.

B1.5 Comparative statics

Combining the results from the previous equilibrium solutions, we derive the following comparative statics.

B1.5.1 Within metro reallocation

No telework vs full telework:

$$\begin{aligned}
 \Delta n_1 - n_2 + n_3 - n_4 &= \frac{4\theta(x-1)}{\beta\epsilon} && \text{Commute costs} \\
 &+ \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
 &- \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
 &= \frac{4\theta(x-1)}{\beta\epsilon} && \text{Reallocation} \tag{5}
 \end{aligned}$$

Observe that there is a positive amount of within-metro reallocation as long as $x > 1$. Furthermore, recall that all variables are in log space so we interpret the reallocation term as a percent change in population from the city centers to the suburbs.

No telework vs partial telework:

$$\begin{aligned}
 \Delta n_1 - n_2 + n_3 - n_4 &= \frac{4\theta(x-1)}{\beta\epsilon} && \text{Commute costs} \\
 &+ \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
 &- \frac{4\theta\pi(x-1)}{\beta\epsilon} && \text{Relative commute costs}
 \end{aligned}$$

$$\begin{aligned}
& - \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
& = \frac{(1 - \pi)4\theta(x - 1)}{\beta\epsilon} && \text{Reallocation} \quad (6)
\end{aligned}$$

Similar to the full telework case above, observe that there is positive within-metro reallocation as long as $x > 1$. Importantly, the extent of this reallocation is inversely proportional to the share of days that are worked in the office, π . Thus, when WFH increases, π decreases, leading to more reallocation. Comparing the two comparative statics, we see that within-metro reallocation is greater under full telework compared to partial telework. Equality is achieved when $\pi = 0$ or the share of days done at home is 1.

$$\Delta \text{ within-metro full WFH} = \frac{4\theta(x - 1)}{\beta\epsilon} > \frac{(1 - \pi)4\theta(x - 1)}{\beta\epsilon} = \Delta \text{ within-metro hybrid WFH} \quad (7)$$

B1.5.2 Between metro reallocation No telework vs full telework:

$$\begin{aligned}
\Delta n_1 - n_2 + n_3 - n_4 &= \frac{2(w_1 - w_2)}{\epsilon} && \text{Commute costs} \\
&+ \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
&- \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
&= \frac{2(w_1 - w_2)}{\epsilon} && \text{Reallocation} \quad (8)
\end{aligned}$$

The relative metro-level populations with full telework no longer include the productivity term so there is sizable between-metro reallocation proportional to the productivity difference.

No telework vs partial telework:

$$\begin{aligned}
\Delta n_1 - n_2 + n_3 - n_4 &= \frac{2(w_1 - w_2)}{\epsilon} && \text{Commute costs} \\
&+ \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
&- \frac{2(w_1 - w_2)}{\epsilon} && \text{Relative commute costs} \\
&- \frac{\gamma(a_1 - a_2 + a_3 - a_4)}{\beta\epsilon} && \text{Relative amenities} \\
&= 0 && \text{Reallocation} \quad (9)
\end{aligned}$$

The relative metro-level populations are pinned down solely by productivity and amenities which do not change with partial telework. Therefore, there is no between-metro reallocation. Comparing the two comparative statics it is easy to see that there is a substantial amount of between-metro reallocation with full WFH proportional to underlying productivity differences, but there is no between-metro reallocation under hybrid WFH. In general, hybrid WFH generates qualitative predictions that better match the patterns seen in the USPS and Zillow home price index data than does full-time WFH.

