Perceptron

CSCI 544 – Fall 2016
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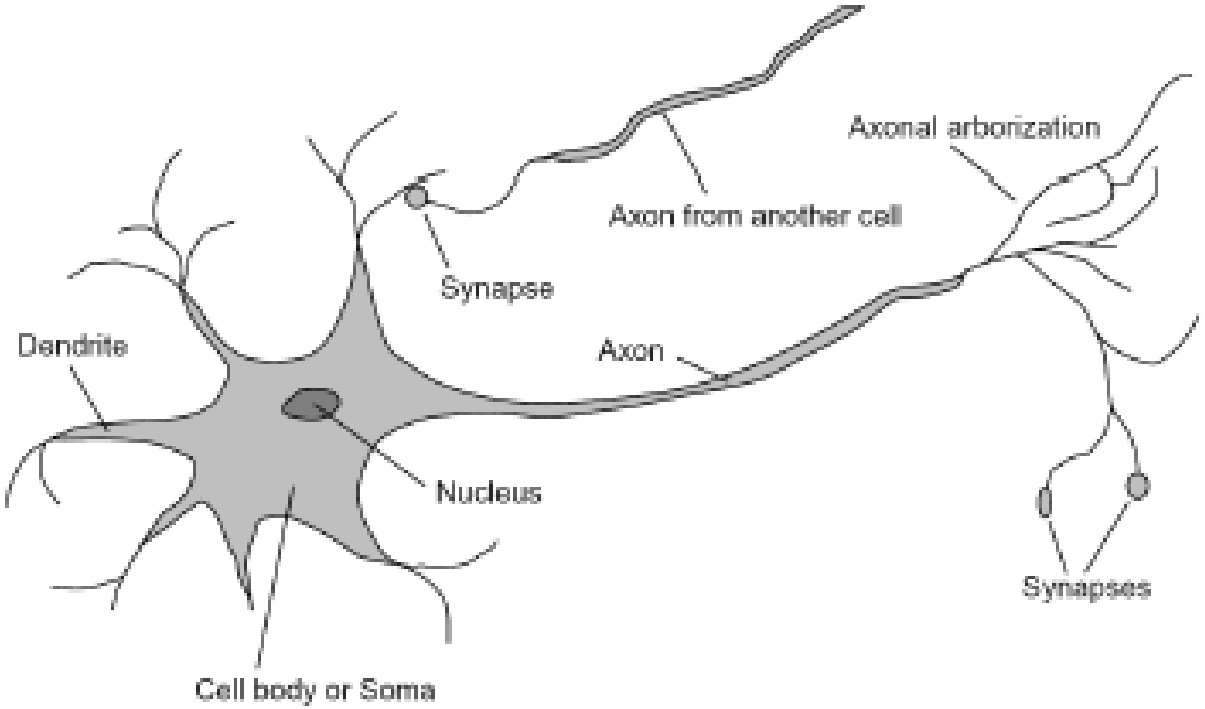
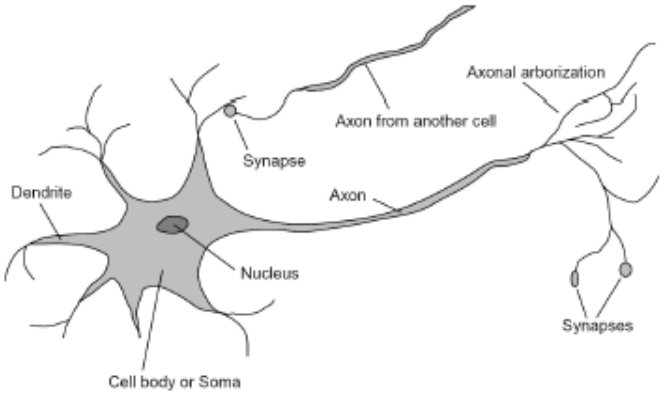
Human neurons – Very loose inspiration for the perceptron



Neurons send electrical signals to one another. The rate of firing tells us how activated a neuron is. A neuron may have many incoming neurons that fire at different rates (i.e., have different activations). Based on how much the incoming neurons are firing and how strong the neural connections are, the main neuron will decide how strongly it wants to fire.

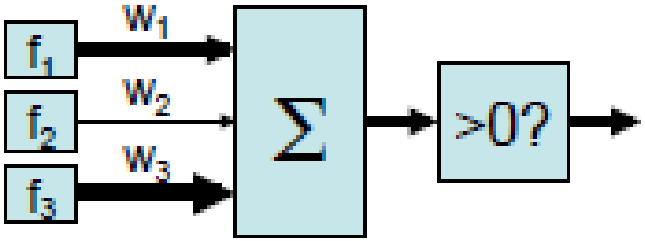
Perceptron

- Inputs are feature weights
- Each feature has a weight
- Sum is the activation



$$activation_w(x) = \sum_i w_i f_i(x)$$

- If the activation is
  - positive, output +1
  - 0 or negative, output -1



Perceptron bias

- No bias

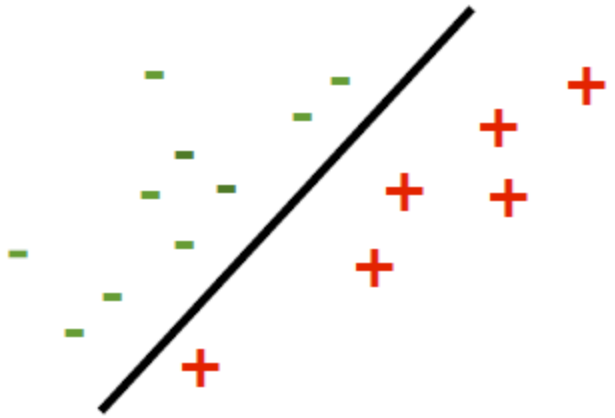
  $$\alpha \leftarrow \sum_{d=1}^{D} w_d x_d$$

- With bias

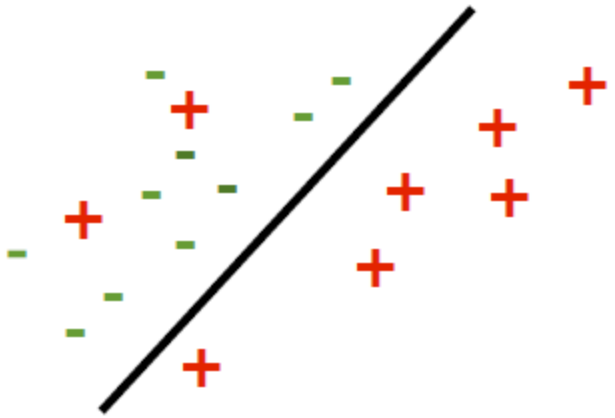
  $$\alpha \leftarrow \sum_{d=1}^{D} w_d x_d + b$$

- A positive bias means more examples should be classified as positive
- A negative bias means more examples should be classified as negative

Linear separability



linearly separable



not linearly separable

The perceptron works well for linearly separable problems only

Perceptron (binary classification)

Training

Training data: T = {(**x**, y)}, **x**: vector of input data features, y: output label

Number of iterations: MaxIter, Number of features: D

$w_d \leftarrow 0$, for all d = 1…D //initialize weights

$b \leftarrow 0$ //initialize bias

for iter = 1…MaxIter

  for all (**x**,y) $\in$T

    $\alpha \leftarrow \sum_{d=1}^{D} w_d x_d + b$ //compute activation for current example

    if $y\alpha \leq 0$ //wrong prediction

      $w_d \leftarrow w_d + y\, x_d$, for all d = 1…D

      $b \leftarrow b + y$

    endif

  endfor

endfor

return $w_1, \ldots, w_D$, b

Test

Test data: T = {**x**}, **x**: vector of features

calculate $\alpha \leftarrow \sum_{d=1}^{D} w_d x_d + b$

if $\alpha > 0$ then output +1 otherwise output -1

Example (training, iteration 1)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 0 + 0 + 0 = 0$

$y\alpha = 1 \times 0 = 0 \leq 0$

(we need to change the weights and bias)

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	0	0	0	0	0	0	0	0	0	0	0	0

bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

w = **w** + y **x**

b = b + y = 0 + 1 = 1

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	0	0	0	0	0	0	0	0	0

bias b = 0

updated bias b = 1

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 0 + 0 + 1 = 1$
$y\alpha = (-1) \times 1 = -1 \leq 0$
(we need to change the weights and bias)

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	0	0	0	0	0	0	0	0	0

bias b = 1

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

w = **w** + y **x**
b = b + y = 1 - 1 = 0

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0

bias b = 1 updated bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 1 + 0 + 0 = 1$
$y\alpha = 1 \times 1 = 1$ (no change)

	click	for	**pharmacy**	free	time	today	**online**	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0

bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 - 1 - 1 + 0 = -2$
$y\alpha = (-1) \times (-2) = 2$ (no change)

	click	for	pharmacy	**free**	**time**	today	online	link	**no**	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0

bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
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¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 + 0 + 1 + 0 = 0$
$y\alpha = (-1) \times 0 = 0 \leq 0$
(we need to change the weights and bias)

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0

bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

w = **w** + y **x**
b = b + y = 0 - 1 = -1

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	0	-2	-1	-1	0	0	0	-1	0	0

bias b = 0

updated bias b = -1

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 - 2 + 0 - 1 = -3$
$y\alpha = 1 \times (-3) = -3 \leq 0$
(we need to change the weights and bias)

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	0	-2	-1	-1	0	0	0	-1	0	0

bias b = -1

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

w = **w** + y **x**
b = b + y = -1 + 1 = 0

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = -1 updated bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 - 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	**for**	pharmacy	free	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 + 0 + 0 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	for	pharmacy	free	**time**	today	online	link	no	good	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 + 1 + 1 + 0 = 3$

$y\alpha = 1 \times 3 = 3$ (no change)

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 - 1 - 1 + 0 = -3$
$y\alpha = (-1) \times (-3) = 3$ (no change)

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 1 + 1 + 0 = 2$
$y\alpha = 1 \times 2 = 2$ (no change)

	click	for	**pharmacy**	free	time	today	**online**	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
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¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 - 1 - 1 + 0 = -2$
$y\alpha = (-1) \times (-2) = 2$ (no change)

	click	for	pharmacy	**free**	**time**	today	online	link	**no**	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 - 1 + 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 + 1 + 0 = 1$
$y\alpha = 1 \times 1 = 1$ (no change)

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 - 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	**for**	pharmacy	free	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 + 0 + 0 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	for	pharmacy	free	**time**	today	online	link	no	good	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (model)

Final model

	click	for	pharmacy	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (test)

Test
pharmacy for pharmacy

if $\alpha = \mathbf{w} \bullet \mathbf{x} + b > 0$ then SPAM else $\neg$SPAM

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 + 1 + 1 + 0 = 3 > 0$
thus this is SPAM

Final model

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Example (test, cont.)

Test
money is good

if $\alpha = \mathbf{w} \bullet \mathbf{x} + b > 0$ then SPAM else ¬SPAM

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 0 - 1 + 0 = -1 \leq 0$
thus this is ¬SPAM

Final model

	click	for	pharmacy	free	time	today	online	link	no	**good**	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0

bias b = 0

Problem with standard perceptron

- The standard perceptron algorithm is very sensitive to the order of training examples
- A solution to this problem is to **randomize the order of the training examples** in each iteration
- Another solution is to **use average weights (averaged perceptron) together with randomizing the order of the training examples** in each iteration
- Overfitting happens when the perceptron is trained for too many iterations

Averaged perceptron (binary classification)

Training

Training data: T = {(**x**, y)}, **x**: vector of input data features, y: output label

Number of iterations: MaxIter, Number of features: D

$w_d \leftarrow 0$, for all d = 1…D, $b \leftarrow 0$ //initialize weights and bias

$u_d \leftarrow 0$, for all d = 1…D, $\beta \leftarrow 0$ //initialize averaged weights and bias

$c \leftarrow 1$ //initialize counter

for iter = 1…MaxIter

  for all (**x**,y) $\in$T

    $\alpha \leftarrow \sum_{d=1}^{D} w_d x_d + b$ //compute activation for current example

    if $y\alpha \leq 0$ //wrong prediction

      $w_d \leftarrow w_d + y\, x_d$, for all d = 1…D

      $b \leftarrow b + y$

      $u_d \leftarrow u_d + y\, c\, x_d$, for all d = 1…D

      $\beta \leftarrow \beta + y\, c$

    endif

    $c \leftarrow c + 1$

  endfor

endfor

$u_d \leftarrow w_d - (1/c)\, u_d$, for all d = 1…D, $\beta \leftarrow b - (1/c)\, \beta$

return $u_1, \ldots, u_D$, β

Averaged perceptron (binary classification)

Test

Test data: T = {**x**}, **x**: vector of features

calculate $\alpha \leftarrow \sum_{d=1}^{D} u_d x_d + \beta$

if $\alpha > 0$ then output +1 otherwise output -1

Example (training, iteration 1)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 0 + 0 + 0 = 0$

$y\alpha = 1 \times 0 = 0 \leq 0$

(we need to change the weights and bias)

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	0	0	0	0	0	0	0	0	0	0	0	0
avg weights (u)	0	0	0	0	0	0	0	0	0	0	0	0

bias b = 0 avg bias β = 0 count c = 1

Example (training, iteration 1, cont.)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

w = **w** + y **x**

u = **u** + y c **x**

b = b + y = 0 + 1 = 1

$\beta = \beta + y\ c = 0 + 1 \times 1 = 1$

c = c + 1

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	0	0	0	0	0	0	0	0	0
avg weights (u)	1	1	1	0	0	0	0	0	0	0	0	0

bias b = 0

updated bias b = 1

avg bias $\beta = 0$

updated avg bias $\beta = 1$

count c = 1

updated count c = 2

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 0 + 0 + 1 = 1$
$y\alpha = (-1) \times 1 = -1 \leq 0$
(we need to change the weights and bias)

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	0	0	0	0	0	0	0	0	0
avg weights (u)	1	1	1	0	0	0	0	0	0	0	0	0

bias b = 1 avg bias $\beta = 1$ count c = 2

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

w = **w** + y **x**
u = **u** + y c **x**
b = b + y = 1 - 1 = 0
$\beta = \beta + y\ c = 1 + (-1) \times 2 = -1$

c = c + 1

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0
avg weights (u)	1	1	1	-2	-2	-2	0	0	0	0	0	0

bias b = 1
updated bias b = 0

avg bias $\beta = 1$
updated avg bias $\beta = -1$

count c = 2
updated count c = 3

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 1 + 0 + 0 = 1$
$y\alpha = 1 \times 1 = 1$ (no change)

c = c + 1

	click	for	**pharmacy**	free	time	today	**online**	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0
avg weights (u)	1	1	1	-2	-2	-2	0	0	0	0	0	0

bias b = 0

avg bias β = -1

count c = 3
updated count c = 4

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

α = **w•x** + b = 0 - 1 - 1 + 0 = -2
yα = (-1) x (-2) = 2 (no change)

c = c + 1

	click	for	pharmacy	**free**	**time**	today	online	link	**no**	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0
avg weights (u)	1	1	1	-2	-2	-2	0	0	0	0	0	0

bias b = 0

avg bias β = -1

count c = 4
updated count c = 5

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 + 0 + 1 + 0 = 0$
$y\alpha = (-1) \times 0 = 0 \leq 0$
(we need to change the weights and bias)

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	1	-1	-1	-1	0	0	0	0	0	0
avg weights (u)	1	1	1	-2	-2	-2	0	0	0	0	0	0

bias b = 0 avg bias β = -1 count c = 5

Example (training, iteration 1, cont.)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

w = **w** + y **x**

u = **u** + y c **x**

b = b + y = 0 - 1 = -1

$\beta = \beta + y\ c = -1 + (-1) \times 5 = -6$

c = c + 1

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	0	-2	-1	-1	0	0	0	-1	0	0
avg weights (u)	1	1	-4	-7	-2	-2	0	0	0	-5	0	0

bias b = 0 avg bias β = -1 count c = 5

updated bias b = -1 updated avg bias β = -6 updated count c = 6

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 - 2 + 0 - 1 = -3$
$y\alpha = 1 \times (-3) = -3 \leq 0$
(we need to change the weights and bias)

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	0	-2	-1	-1	0	0	0	-1	0	0
avg weights (u)	1	1	-4	-7	-2	-2	0	0	0	-5	0	0

bias b = -1 avg bias β = -6 count c = 6

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

w = **w** + y **x**
u = **u** + y c **x**
b = b + y = -1 + 1 = 0
β = β + y c = -6 + 1 x 6 = 0

c = c + 1

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = -1
updated bias b = 0

avg bias β = -6
updated avg bias β = 0

count c = 6
updated count c = 7

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 - 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

c = c + 1

	click	**for**	pharmacy	free	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 7
updated count c = 8

Example (training, iteration 1, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

α = **w•x** + b = -1 + 0 + 0 + 0 = -1
yα = (-1) x (-1) = 1 (no change)

	click	for	pharmacy	free	**time**	today	online	link	no	good	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias β = 0 count c = 8
updated count c = 9

Example (training, iteration 2)

Documents with labels

SPAM	click for pharmacy
¬SPAM	free time today
SPAM	online pharmacy link
¬SPAM	no free time
¬SPAM	free good pharmacy
SPAM	pharmacy free link
¬SPAM	for time today
¬SPAM	time is money

SPAM label y = 1

¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 + 1 + 1 + 0 = 3$

$y\alpha = 1 \times 3 = 3$ (no change)

c = c + 1

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0

avg bias $\beta = 0$

count c = 9

updated count c = 10

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 - 1 - 1 + 0 = -3$
$y\alpha = (-1) \times (-3) = 3$ (no change)

$c = c + 1$

	click	for	pharmacy	**free**	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 10
updated count c = 11

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 0 + 1 + 1 + 0 = 2$
$y\alpha = 1 \times 2 = 2$ (no change)

c = c + 1

	click	for	**pharmacy**	free	time	today	**online**	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 11
updated count c = 12

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

α = **w•x** + b = 0 - 1 - 1 + 0 = -2
yα = (-1) x (-2) = 2 (no change)

c = c + 1

	click	for	pharmacy	**free**	**time**	today	online	link	**no**	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias β = 0 count c = 12
updated count c = 13

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 - 1 + 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

c = c + 1

	click	for	**pharmacy**	**free**	time	today	online	link	no	**good**	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 13
updated count c = 14

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 + 1 + 0 = 1$
$y\alpha = 1 \times 1 = 1$ (no change)

c = c + 1

	click	for	**pharmacy**	**free**	time	today	online	**link**	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0

avg bias $\beta = 0$

count c = 14
updated count c = 15

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = 1 - 1 - 1 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

	click	**for**	pharmacy	free	**time**	**today**	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 15
updated count c = 16

Example (training, iteration 2, cont.)

Documents with labels

SPAM click for pharmacy
¬SPAM free time today
SPAM online pharmacy link
¬SPAM no free time
¬SPAM free good pharmacy
SPAM pharmacy free link
¬SPAM for time today
¬SPAM time is money

SPAM label y = 1
¬SPAM label y = -1

$\alpha = \mathbf{w} \bullet \mathbf{x} + b = -1 + 0 + 0 + 0 = -1$
$y\alpha = (-1) \times (-1) = 1$ (no change)

c = c + 1

	click	for	pharmacy	free	**time**	today	online	link	no	good	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 16
updated count c = 17

Example (model)

	click	for	pharmacy	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	1	1	2	-1	-2	-2	0	6	0	-5	0	0

bias b = 0 avg bias $\beta = 0$ count c = 17

$\mathbf{u} = \mathbf{w} - (1/c)\ \mathbf{u}$

$\beta = b - (1/c)\ \beta$

Final model

	click	for	pharmacy	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	0.94	0.94	0.88	-0.94	-0.88	-0.88	0	0.65	0	-0.71	0	0

avg bias $\beta = 0$

Example (test)

Test
pharmacy for pharmacy

if $\alpha = \mathbf{u} \bullet \mathbf{x} + \beta > 0$ then SPAM else $\neg$SPAM

$\alpha = \mathbf{u} \bullet \mathbf{x} + \beta = 0.88 + 0.94 + 0.88 + 0 = 2.7 > 0$
thus this is SPAM

Final model

	click	**for**	**pharmacy**	free	time	today	online	link	no	good	is	money
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	0.94	0.94	0.88	-0.94	-0.88	-0.88	0	0.65	0	-0.71	0	0

avg bias $\beta = 0$

Example (test, cont.)

Test
money is good

if $\alpha = \mathbf{u} \bullet \mathbf{x} + \beta > 0$ then SPAM else $\neg$SPAM

$\alpha = \mathbf{u} \bullet \mathbf{x} + \beta = 0 + 0 - 0.71 + 0 = -0.71 \leq 0$
thus this is $\neg$SPAM

Final model

	click	for	pharmacy	free	time	today	online	link	no	**good**	**is**	**money**
weights (w)	1	1	1	-1	-1	-1	0	1	0	-1	0	0
avg weights (u)	0.94	0.94	0.88	-0.94	-0.88	-0.88	0	0.65	0	-0.71	0	0

avg bias $\beta = 0$

References

- Hal Daumé III. A course in machine learning. http://ciml.info/dl/v0_9/ciml-v0_9-ch03.pdf