Algebra II: Euclidean Domains, Principal Ideal Domains and Unique Factorization Domains

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Created: 4th April 2022
Last updated: 4th April 2022

1 Euclidean Domains

- Let R be an integral domain. Any function $N : R \to \mathbb{Z}^+ \cup \{0\}$ with $N(0) = 0$ is called a norm on R. If $N(a) > 0$ for $a \neq 0$, N is called a positive norm.
- An integral domain R is said to be a Euclidean Domain if there exists a norm N on R such that the division algorithm holds, i.e, for any two elements $a, b \in R$, there exist $q, r \in R$ such that $a = qb + r$, where $r = 0$ or $N(r) < N(b)$.
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