Real Analysis I: Continuity

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1 Limits of Functions

- Let X and Y be metric spaces. Suppose $E \subset X$, f maps E into Y, and p is a limit point of E. Then, $\lim\limits_{x \to p} f(x) = q$ if there exists a point $q \in Y$ such that for every $\epsilon > 0$, there exists $\delta > 0$ such that $d_X(x,p) < \delta \implies d_Y(f(x),q) < \epsilon$, where $x \in E$. Restated in $\mathbb{R}$, $\lim\limits_{x \to p} f(x) = q$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|x-p| < \delta \implies |f(x) - q| < \epsilon$.
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2 Continuous Functions

3 Continuity and Compactness

4 Continuity and Connectedness

5 Discontinuities

6 Monotonic Functions

7 Infinite Limits and Limits at Infinity