MAT422 Notes

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1 Basic Definitions

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2 Convergence of Sequences

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3 Open Sets and Interior Points

- Let $x \in X$, $r > 0$. The open sphere/ball/disc centered at x with radius r, denoted $S(x, r)$ is $\{y \in X : d(y, x) < r\}$.
- Alternate definition of convergence:
  $(x_n) \to x \Leftrightarrow \forall \epsilon > 0, \exists M : n \geq M \implies d(x_n, x) < \epsilon$
  $\implies \forall \epsilon > 0, \exists M : n \geq M \implies x_n \in S(x, \epsilon)$.
- Let $a \in X$, $A \subseteq X$. a is said to be an interior point of A if and only if $\exists r > 0 : S(a, r) \subseteq A$.
- The set of all interior points of A is called the interior of A, denoted $\text{Int}(A)$ or A°.
- Let $b \in X$. $N \subseteq X$ is said to be a neighborhood of b if and only if $b \in N^\circ$.
- **$A^\circ \subseteq A$.** *Proof:* Let $a \in A^\circ$. Then $\exists r > 0 : S(a, r) \subseteq A$. As $a \in S(a, r)$, $a \in A$. ■
- **$A \subseteq B \implies A^\circ \subseteq B^\circ$.** *Proof:* Let $a \in A^\circ$. Then $\exists r > 0 : S(a, r) \subseteq A \subseteq B$. So $a \in B^\circ$. ■
- **Let $A \subseteq B$, and A be a neighborhood of a. Then B is a neighborhood of a.** *Proof:* $a \in A^\circ \implies a \in B^\circ \implies B$ is a neighborhood of A. ■
- **$(A^\circ)^\circ = A^\circ$.** *Proof:* We know that $(A^\circ)^\circ \subseteq A^\circ$. So let $a \in A^\circ$. Then $\exists r > 0 : S(a, r) \subseteq A$.

4 Closed Sets and Limit Points

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