Algebra I: Quotient Groups and Homomorphisms

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1 Definitions

- **Let $f : G \to H$ be a homomorphism. Then, $\mathbf{Im}(f)$ is a subgroup of H.** *Proof:* Since $e_H \in \mathrm{Im}(f)$, it is nonempty. Suppose $x, y \in \mathrm{Im}(f)$. Then there exist $a, b \in G$ such that $f(a) = x$ and $f(b) = y$. So $f(ab^{-1}) = f(a)f(b)^{-1} = xy^{-1} \in \mathrm{Im}(f)$. Therefore $\mathrm{Im}(f) \leq H$. ■
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2 Cosets and Lagrange's Theorem

3 Isomorphism Theorems

4 Composition Series and the Holder Program

5 Transpositions and the Alternating Group