Linear Algebra: Systems of Linear Equations

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1 Introduction

- Consider a system of m linear equations with n unknowns: $\sum_{k=1}^{n} a_{1,k}x_k = b_1$, $\sum_{k=1}^{n} a_{2,k}x_k = b_2$, ..., $\sum_{k=1}^{n} a_{m,k}x_k = b_m$. Taking the coefficients from the equations, we can form a matrix $A = \begin{pmatrix} a_{1,1} & a_{1,2} & ... & a_{1,n} \\ a_{2,1} & a_{2,2} & ... & a_{2,n} \\ ... & ... & ... & ... \\ a_{m,1} & a_{m,2} & ... & a_{m,n} \end{pmatrix}$. Then the linear system can be written in the form of a matrix equation $Ax = b$, where $x = (x_1, x_2, ..., x_n)^T$ and $b = (b_1, b_2, ..., b_m)^T$.
- A here is the coefficient matrix of the system. If we join the coefficient matrix to the vector b, we get the augmented matrix $\begin{pmatrix} a_{1,1} & a_{1,2} & ... & a_{1,n} & b_1 \\ a_{2,1} & a_{2,2} & ... & a_{2,n} & b_2 \\ ... & ... & ... & ... & ... \\ a_{m,1} & a_{m,2} & ... & a_{m,n} & b_m \end{pmatrix}$, which contains all the information necessary to solve the system.

2 Solution of a Linear System, Echelon Form and Reduced Echelon Form

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3 Analyzing the Pivots

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4 Finding A^{-1} by Row Reduction

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5 Dimension

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6 Solution Set of a Linear System

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7 Fundamental Subspaces of a Matrix, Rank

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8 Represenation of a Linear Transformation in Arbitrary Bases

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