

# HW1 Q2 K-means Arjun Singh

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K-Means algorithm. Please note that I have reused some of my old code from CSE 6040.

```
[1]: import numpy as np
import time
```

```
[32]: # Randomly selects k points from the input image
def init_centers(X, k):

    centers = np.random.choice(len(X), k, replace=False)
    # centers = np.random.choice(len(X), 1, replace=False)
    # return X[np.repeat(centers, k), :]
    return X[centers, :]
```

```
[55]: # Compute Euclidean distance
def compute_d2(X, centers):

    m = len(X)
    s=len(centers)

    S = np.empty((m,s))
    for i in range(m):
        S[i,:] = np.linalg.norm(X[i,:]-centers,ord=2,axis=1)**2

    return S
```

```
[56]: def assign_cluster_labels(S):
    return np.argmin(S,axis=1)
```

```
[57]: def update_centers(X, y):
    # X[:m, :d] == m points, each of dimension d
    # y[:m] == cluster labels
    m, d = X.shape
    k = max(y) + 1
    assert m == len(y)
    assert (min(y) >= 0)

    centers = np.empty((k, d))
    for j in range(k):
```

```

        # Compute the new center of cluster j,
        # i.e., centers[j, :d].
        centers[j,:] = np.mean(X[j==y],axis=0)
    return centers

```

```

[58]: # Calculate the within clusters sum of squares
def WCSS(S):
    return np.sum(np.amin(S,axis=1))

```

```

[23]: # The algorithm has converged if there is no change in the cluster centers in
      ↪ the next iteration
def has_converged(old_centers, centers):
    return set([tuple(x) for x in old_centers]) == set([tuple(x) for x in
      ↪ centers])

```

```

[24]: def kmeans(X, k,starting_centers=None,max_steps=np.inf):
        start = time.time()

        if starting_centers is None:
            centers = init_centers(X, k)
        else:
            centers = starting_centers

        converged = False
        labels = np.zeros(len(X))
        i = 1
        while (not converged) and (i <= max_steps):
            old_centers = centers
            S = compute_d2(X, old_centers)
            labels = assign_cluster_labels(S)
            centers = update_centers(X, labels)
            wc = WCSS(S)

            converged = has_converged(old_centers,centers)
            print ("iteration", i, "WCSS = ", WCSS (S))
            i += 1
        stop = time.time()
        print("Time taken = {:.2f} seconds".format(stop-start))
        return labels

```

```

[25]: from matplotlib.pyplot import imshow
      %matplotlib inline
def display_image(arr):
    """
    display the image
    input : 3 dimensional array
    """

```

```
arr = arr.astype(dtype='uint8')
img = Image.fromarray(arr, 'RGB')
imshow(np.asarray(img))
```

```
[26]: from PIL import Image
def read_img(path):
    """
    Read image and store it as an array, given the image path.
    Returns the 3 dimensional image array.
    """
    img = Image.open(path)
    img_arr = np.array(img, dtype='int32')
    img.close()
    return img_arr
```

```
[27]: def runKMeansAlgorithm(pixels,k):
    print("Running k-means algorithm...")
    # Read image
    image = read_img(pixels)
    r, c, l = image.shape
    # Flatten image
    img_resaped = np.reshape(image, (r*c, l), order="C")
    # Run k-means algorithm
    labels = kmeans(img_resaped, k, starting_centers=None)
    ind = np.column_stack((img_resaped, labels))
    centers = {}
    for i in set(labels):
        c = ind[ind[:,3] == i].mean(axis=0)
        centers[i] = c[:3]

    img_clustered = np.array([centers[i] for i in labels])
    r, c, l = image.shape
    img_disp = np.reshape(img_clustered, (r, c, l), order="C")

    print('Image with k = ' + str(k) + " clusters...")
    display_image(img_disp)
    print("The labels are: {}".format(labels.T))
    print("The cluster centers are {}".format(centers))
    return labels,centers
```

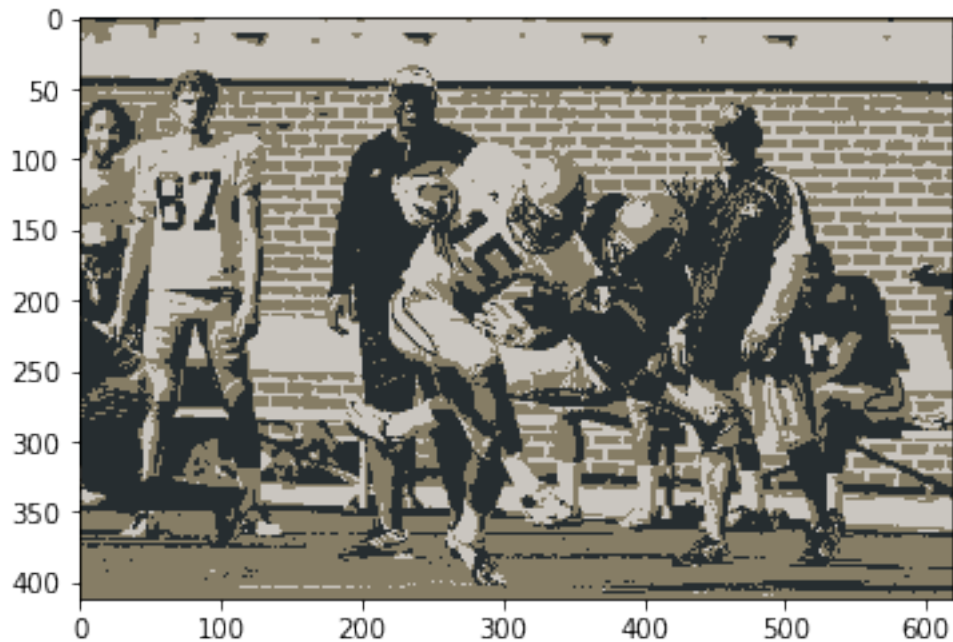
```
[36]: lab, cen = runKMeansAlgorithm("data/football.bmp",3)
```

```
Running k-means algorithm...
iteration 1 WCSS = 2456316508.0
iteration 2 WCSS = 954303493.6362097
iteration 3 WCSS = 671747803.6800143
iteration 4 WCSS = 639939317.5572674
```

```

iteration 5 WCSS = 635886999.0335318
iteration 6 WCSS = 634830313.722801
iteration 7 WCSS = 634448944.3235377
iteration 8 WCSS = 634294560.9102862
iteration 9 WCSS = 634226091.4041917
iteration 10 WCSS = 634195392.6893654
iteration 11 WCSS = 634182298.9958154
iteration 12 WCSS = 634178085.6680316
iteration 13 WCSS = 634175679.265678
iteration 14 WCSS = 634174624.9946772
iteration 15 WCSS = 634174059.4714675
iteration 16 WCSS = 634173882.9721487
iteration 17 WCSS = 634173783.4825951
iteration 18 WCSS = 634173756.5604436
iteration 19 WCSS = 634173755.1569637
Time taken = 133.51 seconds
Image with k = 3 clusters...
The labels are: [2 2 2 ... 0 0 0]
The cluster centers are {0: array([134.23584777, 125.34568766, 101.93988768]),
1: array([202.52108949, 198.84707504, 192.62337998]), 2: array([38.7890798 ,
45.35163548, 48.17869416])}

```



```
[37]: lab, cen = runKMeansAlgorithm("data/football.bmp",16)
```

```

Running k-means algorithm...
iteration 1 WCSS = 389599008.0

```

iteration 2 WCSS = 194633966.26500764  
iteration 3 WCSS = 158409209.34289968  
iteration 4 WCSS = 148831373.6483631  
iteration 5 WCSS = 143824610.22942817  
iteration 6 WCSS = 139707579.16874346  
iteration 7 WCSS = 137296156.4037293  
iteration 8 WCSS = 135397899.4588457  
iteration 9 WCSS = 133925560.75748907  
iteration 10 WCSS = 132690377.51961173  
iteration 11 WCSS = 131667817.91402242  
iteration 12 WCSS = 130892740.08558576  
iteration 13 WCSS = 130364212.98891617  
iteration 14 WCSS = 129992012.59922427  
iteration 15 WCSS = 129693213.8722543  
iteration 16 WCSS = 129440147.52493048  
iteration 17 WCSS = 129184506.98292685  
iteration 18 WCSS = 128887669.48954152  
iteration 19 WCSS = 128557986.11477509  
iteration 20 WCSS = 128212061.16508226  
iteration 21 WCSS = 127908373.72405538  
iteration 22 WCSS = 127675522.41787711  
iteration 23 WCSS = 127514064.77921885  
iteration 24 WCSS = 127381072.11134267  
iteration 25 WCSS = 127296917.37258105  
iteration 26 WCSS = 127219626.15762621  
iteration 27 WCSS = 127135595.99641562  
iteration 28 WCSS = 127084561.22413416  
iteration 29 WCSS = 127031114.71142764  
iteration 30 WCSS = 126965027.98333591  
iteration 31 WCSS = 126903670.89468691  
iteration 32 WCSS = 126833696.83605571  
iteration 33 WCSS = 126778082.5017755  
iteration 34 WCSS = 126713484.00201091  
iteration 35 WCSS = 126660603.66072176  
iteration 36 WCSS = 126611707.37352398  
iteration 37 WCSS = 126570049.28539969  
iteration 38 WCSS = 126543355.6095167  
iteration 39 WCSS = 126516175.03414136  
iteration 40 WCSS = 126490316.327382  
iteration 41 WCSS = 126469858.9570937  
iteration 42 WCSS = 126447819.61642782  
iteration 43 WCSS = 126426702.5349611  
iteration 44 WCSS = 126404688.88800836  
iteration 45 WCSS = 126381547.31152752  
iteration 46 WCSS = 126355655.47235285  
iteration 47 WCSS = 126325482.90294963  
iteration 48 WCSS = 126291132.93700808  
iteration 49 WCSS = 126249281.98181044

iteration 50 WCSS = 126203599.13343744  
iteration 51 WCSS = 126146278.15400615  
iteration 52 WCSS = 126067165.09515291  
iteration 53 WCSS = 125947765.59821501  
iteration 54 WCSS = 125801496.48122527  
iteration 55 WCSS = 125618602.93741326  
iteration 56 WCSS = 125398772.76508854  
iteration 57 WCSS = 125168474.76176748  
iteration 58 WCSS = 124950028.84253912  
iteration 59 WCSS = 124713953.23097008  
iteration 60 WCSS = 124428617.59272456  
iteration 61 WCSS = 124121577.87629376  
iteration 62 WCSS = 123850386.9459919  
iteration 63 WCSS = 123617743.04269174  
iteration 64 WCSS = 123429225.64908963  
iteration 65 WCSS = 123278113.47015174  
iteration 66 WCSS = 123151183.77912122  
iteration 67 WCSS = 123039877.10813873  
iteration 68 WCSS = 122947117.21163964  
iteration 69 WCSS = 122872878.66243537  
iteration 70 WCSS = 122805320.62898089  
iteration 71 WCSS = 122743479.75850767  
iteration 72 WCSS = 122685088.12013635  
iteration 73 WCSS = 122635958.78536704  
iteration 74 WCSS = 122596086.12046702  
iteration 75 WCSS = 122561986.22790265  
iteration 76 WCSS = 122535067.16891102  
iteration 77 WCSS = 122509628.66869356  
iteration 78 WCSS = 122486993.20189187  
iteration 79 WCSS = 122467116.37805513  
iteration 80 WCSS = 122446064.64995241  
iteration 81 WCSS = 122427774.84981969  
iteration 82 WCSS = 122415317.02470523  
iteration 83 WCSS = 122402974.2085328  
iteration 84 WCSS = 122390120.51490752  
iteration 85 WCSS = 122378540.14667523  
iteration 86 WCSS = 122371670.88245495  
iteration 87 WCSS = 122366985.29189879  
iteration 88 WCSS = 122363660.70652373  
iteration 89 WCSS = 122358712.20760457  
iteration 90 WCSS = 122352572.05131795  
iteration 91 WCSS = 122345898.35536122  
iteration 92 WCSS = 122341346.20573317  
iteration 93 WCSS = 122338603.31101723  
iteration 94 WCSS = 122336909.27342421  
iteration 95 WCSS = 122335862.93781473  
iteration 96 WCSS = 122335002.74908623  
iteration 97 WCSS = 122334241.98644127

```
iteration 98 WCSS = 122332762.26002996
iteration 99 WCSS = 122331049.79412797
iteration 100 WCSS = 122329643.48998876
iteration 101 WCSS = 122327973.11487384
iteration 102 WCSS = 122325495.81644262
iteration 103 WCSS = 122323208.82716674
iteration 104 WCSS = 122321957.89813724
iteration 105 WCSS = 122321089.87948734
iteration 106 WCSS = 122319239.34875216
iteration 107 WCSS = 122317379.33636372
iteration 108 WCSS = 122316142.20912726
iteration 109 WCSS = 122315428.50033154
iteration 110 WCSS = 122315026.59455879
iteration 111 WCSS = 122314662.79439995
iteration 112 WCSS = 122314310.05917099
iteration 113 WCSS = 122314042.53917743
iteration 114 WCSS = 122313775.93435095
iteration 115 WCSS = 122313582.278637
iteration 116 WCSS = 122313464.33245645
iteration 117 WCSS = 122313011.4310619
iteration 118 WCSS = 122312377.52659644
iteration 119 WCSS = 122312000.4171061
iteration 120 WCSS = 122311730.82577868
iteration 121 WCSS = 122311498.76539102
iteration 122 WCSS = 122311168.37947495
iteration 123 WCSS = 122310871.68119371
iteration 124 WCSS = 122310591.64208212
iteration 125 WCSS = 122310375.34639943
iteration 126 WCSS = 122310279.15037306
iteration 127 WCSS = 122310225.40522182
iteration 128 WCSS = 122310183.55788857
iteration 129 WCSS = 122310158.0667483
iteration 130 WCSS = 122310129.83383945
iteration 131 WCSS = 122308552.96051888
iteration 132 WCSS = 122308233.74553815
iteration 133 WCSS = 122308197.40648106
iteration 134 WCSS = 122308176.44870742
iteration 135 WCSS = 122308147.07327214
iteration 136 WCSS = 122308144.55726838
iteration 137 WCSS = 122308141.03021088
iteration 138 WCSS = 122308140.55316299
```

Time taken = 977.91 seconds

Image with k = 16 clusters...

The labels are: [5 5 5 ... 1 1 1]

The cluster centers are {0: array([188.49935166, 195.5687889 , 203.2629668 ]),  
1: array([117.8208072 , 139.48794729, 87.18471614]), 2: array([ 25.10745839,  
69.52558044, 128.11177317]), 3: array([55.37723925, 51.19669001, 34.75188688]),  
4: array([184.08713364, 157.50175399, 136.74663347]), 5: array([85.42213893,

```
84.03601253, 64.75678497]], 6: array([17.83079513, 17.89143797, 17.12841091]),
7: array([236.34535561, 240.43735763, 245.13642116]), 8: array([153.1981332 ,
101.26696444, 79.546756 ]), 9: array([16.31240223, 40.05441341, 74.45296089]),
10: array([189.47646148, 177.33742699, 166.95047676]), 11: array([215.86818003,
220.53727157, 222.15947237]), 12: array([130.27638546, 142.86648429,
153.58656901]), 13: array([218.22310208, 200.73541057, 130.67877432]), 14:
array([ 95.24515007, 106.94637628, 117.70516105]), 15: array([169.78753541,
132.18570151, 105.5514388 ])]}
```



```
[38]: lab, cen = runKMeansAlgorithm("data/football.bmp",32)
```

```
Running k-means algorithm...
iteration 1 WCSS = 154156513.0
iteration 2 WCSS = 90854426.99404576
iteration 3 WCSS = 82374936.18956882
iteration 4 WCSS = 78979555.4186059
iteration 5 WCSS = 76874969.7167158
iteration 6 WCSS = 75376297.9050194
iteration 7 WCSS = 74181937.53857802
iteration 8 WCSS = 73510089.71580961
iteration 9 WCSS = 73042821.19014058
iteration 10 WCSS = 72611144.12092553
iteration 11 WCSS = 72173924.06588945
iteration 12 WCSS = 71681196.24227656
iteration 13 WCSS = 71134192.82620926
iteration 14 WCSS = 70540873.17103964
```



iteration 15 WCSS = 69924895.59256424  
iteration 16 WCSS = 69313949.72028396  
iteration 17 WCSS = 68802180.49051738  
iteration 18 WCSS = 68392598.33023295  
iteration 19 WCSS = 68090924.64142045  
iteration 20 WCSS = 67890058.1555307  
iteration 21 WCSS = 67739501.48248678  
iteration 22 WCSS = 67606630.3922924  
iteration 23 WCSS = 67473454.12835175  
iteration 24 WCSS = 67355732.49085784  
iteration 25 WCSS = 67255161.68398519  
iteration 26 WCSS = 67191316.78659722  
iteration 27 WCSS = 67139978.00513403  
iteration 28 WCSS = 67090993.50562282  
iteration 29 WCSS = 67051415.1236087  
iteration 30 WCSS = 67004459.00868036  
iteration 31 WCSS = 66960393.448468596  
iteration 32 WCSS = 66923387.652685605  
iteration 33 WCSS = 66886740.578007676  
iteration 34 WCSS = 66852165.68997419  
iteration 35 WCSS = 66818579.16168487  
iteration 36 WCSS = 66776739.065597385  
iteration 37 WCSS = 66724878.49203261  
iteration 38 WCSS = 66667391.04308212  
iteration 39 WCSS = 66612905.44686758  
iteration 40 WCSS = 66558704.06604199  
iteration 41 WCSS = 66512063.22512857  
iteration 42 WCSS = 66461844.02640541  
iteration 43 WCSS = 66416672.93931889  
iteration 44 WCSS = 66383996.182326905  
iteration 45 WCSS = 66357117.885337256  
iteration 46 WCSS = 66331327.83996085  
iteration 47 WCSS = 66309972.26288962  
iteration 48 WCSS = 66290467.90574199  
iteration 49 WCSS = 66273431.62863433  
iteration 50 WCSS = 66257536.00882024  
iteration 51 WCSS = 66240433.76244594  
iteration 52 WCSS = 66219918.11447064  
iteration 53 WCSS = 66197567.31849523  
iteration 54 WCSS = 66174297.33178489  
iteration 55 WCSS = 66148462.15137137  
iteration 56 WCSS = 66120285.63237766  
iteration 57 WCSS = 66091428.925723195  
iteration 58 WCSS = 66063793.93462781  
iteration 59 WCSS = 66039291.65311639  
iteration 60 WCSS = 66020576.33910455  
iteration 61 WCSS = 66005384.563282736  
iteration 62 WCSS = 65993470.60843236

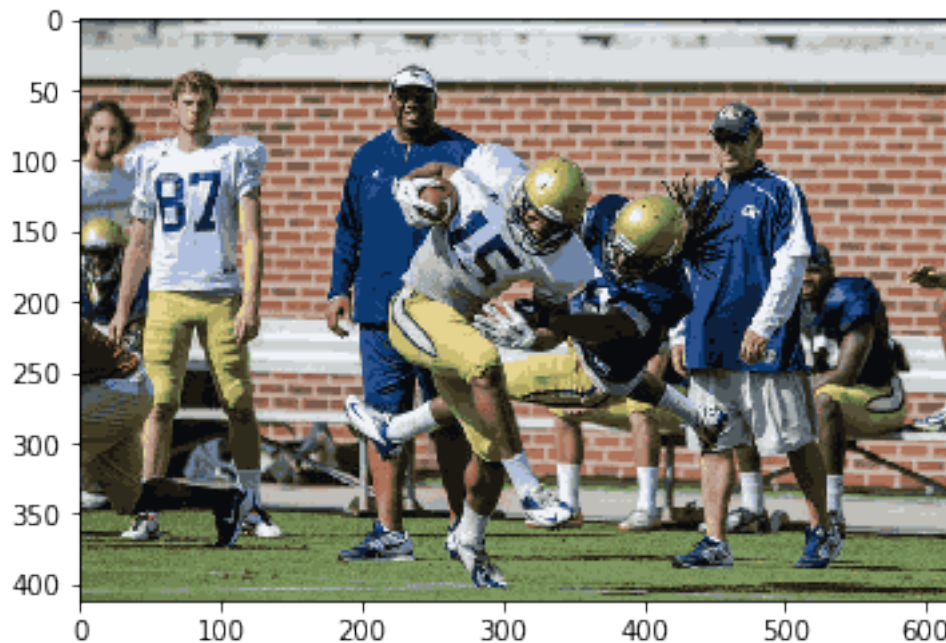
iteration 63 WCSS = 65984993.968146615  
iteration 64 WCSS = 65977775.38169028  
iteration 65 WCSS = 65971341.89105098  
iteration 66 WCSS = 65964897.213070914  
iteration 67 WCSS = 65958693.217298485  
iteration 68 WCSS = 65954604.293055214  
iteration 69 WCSS = 65950826.3306454  
iteration 70 WCSS = 65948188.39329451  
iteration 71 WCSS = 65945365.36959097  
iteration 72 WCSS = 65942145.552824594  
iteration 73 WCSS = 65939293.8508109  
iteration 74 WCSS = 65936503.16170342  
iteration 75 WCSS = 65934431.868236944  
iteration 76 WCSS = 65933017.755912535  
iteration 77 WCSS = 65931591.60981239  
iteration 78 WCSS = 65930179.809461065  
iteration 79 WCSS = 65928444.1488754  
iteration 80 WCSS = 65926988.283575125  
iteration 81 WCSS = 65925476.44836222  
iteration 82 WCSS = 65924130.256465636  
iteration 83 WCSS = 65922833.017187074  
iteration 84 WCSS = 65921666.39844116  
iteration 85 WCSS = 65920353.24527504  
iteration 86 WCSS = 65918798.59710905  
iteration 87 WCSS = 65917685.66757907  
iteration 88 WCSS = 65916766.63037999  
iteration 89 WCSS = 65916037.341308005  
iteration 90 WCSS = 65915276.57709618  
iteration 91 WCSS = 65914429.30425806  
iteration 92 WCSS = 65913942.53997429  
iteration 93 WCSS = 65913646.910160765  
iteration 94 WCSS = 65913327.305902086  
iteration 95 WCSS = 65912936.29789154  
iteration 96 WCSS = 65912669.35526211  
iteration 97 WCSS = 65912495.03862722  
iteration 98 WCSS = 65912367.35838308  
iteration 99 WCSS = 65912226.32508523  
iteration 100 WCSS = 65912109.62433027  
iteration 101 WCSS = 65911981.971656516  
iteration 102 WCSS = 65911754.81439332  
iteration 103 WCSS = 65911620.67678566  
iteration 104 WCSS = 65911509.611392036  
iteration 105 WCSS = 65911453.5028878  
iteration 106 WCSS = 65911380.97904354  
iteration 107 WCSS = 65911338.20146704  
iteration 108 WCSS = 65911317.31561838  
iteration 109 WCSS = 65911294.13023815  
iteration 110 WCSS = 65911241.64068819

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iteration 111 WCSS = 65911179.67890533
iteration 112 WCSS = 65911136.58227505
iteration 113 WCSS = 65911110.88510049
iteration 114 WCSS = 65911090.30270436
iteration 115 WCSS = 65911073.654683575
iteration 116 WCSS = 65911069.30501881
iteration 117 WCSS = 65911066.52864003
iteration 118 WCSS = 65911058.1734749
iteration 119 WCSS = 65911034.697925135
iteration 120 WCSS = 65911008.16123636
iteration 121 WCSS = 65910963.60735541
iteration 122 WCSS = 65910903.772770904
iteration 123 WCSS = 65910863.03522626
iteration 124 WCSS = 65910845.067064956
iteration 125 WCSS = 65910821.73000511
iteration 126 WCSS = 65910765.71878215
iteration 127 WCSS = 65910741.73739506
iteration 128 WCSS = 65910714.2128681
iteration 129 WCSS = 65910667.364258714
iteration 130 WCSS = 65910645.97710661
iteration 131 WCSS = 65910644.67183107
iteration 132 WCSS = 65910638.85006445
iteration 133 WCSS = 65910624.275976114
iteration 134 WCSS = 65910619.72531885
iteration 135 WCSS = 65910618.370981656
iteration 136 WCSS = 65910617.67236624
iteration 137 WCSS = 65910616.73920506
iteration 138 WCSS = 65910616.04407202
iteration 139 WCSS = 65910615.53114029
iteration 140 WCSS = 65910615.309055164
iteration 141 WCSS = 65910615.21597049
Time taken = 1025.84 seconds
Image with k = 32 clusters...
The labels are: [31 31 5 ... 9 20 20]
The cluster centers are {0: array([158.78160667, 99.65803336, 76.5679324 ]),
1: array([174.68212603, 115.90650766, 91.59231449]), 2: array([239.01673573,
242.75183296, 247.21883966]), 3: array([186.32316412, 158.03742602,
141.36934234]), 4: array([46.14246052, 42.86300538, 28.69772688]), 5:
array([55.35414145, 64.04634273, 68.70776414]), 6: array([13.47059673,
32.10981072, 61.07311082]), 7: array([85.25054693, 81.00885509, 56.88352953]),
8: array([ 42.47548121, 81.38817599, 142.42163153]), 9: array([127.76199978,
152.31770314, 97.35417359]), 10: array([176.65245255, 191.83657875,
222.98940104]), 11: array([122.8435745 , 90.90303931, 74.11446977]), 12:
array([100.62787424, 117.75113405, 69.49053652]), 13: array([ 12.06051948,
57.78662338, 112.86948052]), 14: array([109.92644295, 132.01369128,
167.12805369]), 15: array([191.69427896, 190.91432624, 185.39725768]), 16:
array([202.93570275, 190.7098546 , 115.18804523]), 17: array([229.66644137,
211.26225076, 144.17269348]), 18: array([176.54366094, 138.52563634,

```

```
119.05516989]), 19: array([67.62414898, 60.51281538, 37.51621946]), 20:
array([114.66434635, 137.0098472 , 83.27410866]), 21: array([220.20979827,
224.79688761, 226.64616715]), 22: array([173.10397295, 163.40617075, 95.4103973
]), 23: array([103.39973615, 114.95705072, 125.09073585]), 24:
array([204.23231629, 209.35333047, 210.16347228]), 25: array([ 8.43815496,
10.64955854, 13.57909068]), 26: array([27.67181202, 24.02332562, 17.54746591]),
27: array([140.92210682, 117.52670623, 102.34254451]), 28: array([197.51942974,
174.29344196, 160.16211813]), 29: array([160.68984363, 170.2398664 ,
173.94899044]), 30: array([141.61348454, 147.20149138, 140.97296877]), 31:
array([81.60524017, 91.83435226, 95.6209607 ])]}
```



```
[39]: lab, cen = runKMeansAlgorithm("data/TechTower.jpg",2)
```

Running k-means algorithm...

```
iteration 1 WCSS = 404111557.0
iteration 2 WCSS = 185068009.59071818
iteration 3 WCSS = 170311286.1525032
iteration 4 WCSS = 169676642.10593435
iteration 5 WCSS = 169657307.86373463
iteration 6 WCSS = 169656916.80336154
iteration 7 WCSS = 169656891.6555998
iteration 8 WCSS = 169656890.81534788
```

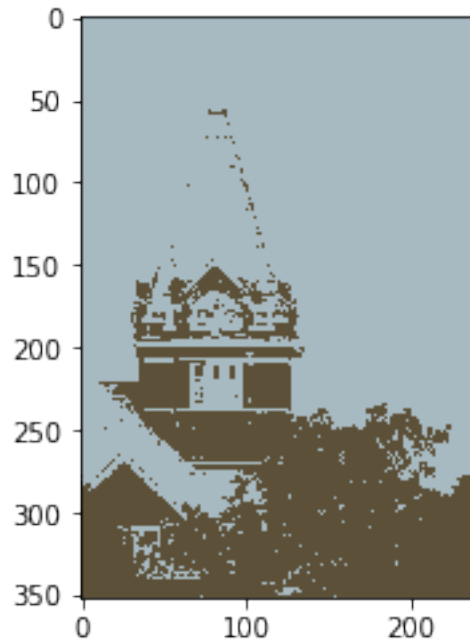
Time taken = 18.93 seconds

Image with k = 2 clusters...

The labels are: [1 1 1 ... 0 0 0]

The cluster centers are {0: array([92.84412979, 80.94779891, 56.73093098]), 1:

```
array([168.57808098, 186.45381739, 193.0552757 ])
```



```
[40]: lab, cen = runKMeansAlgorithm("data/TechTower.jpg",12)
```

Running k-means algorithm...

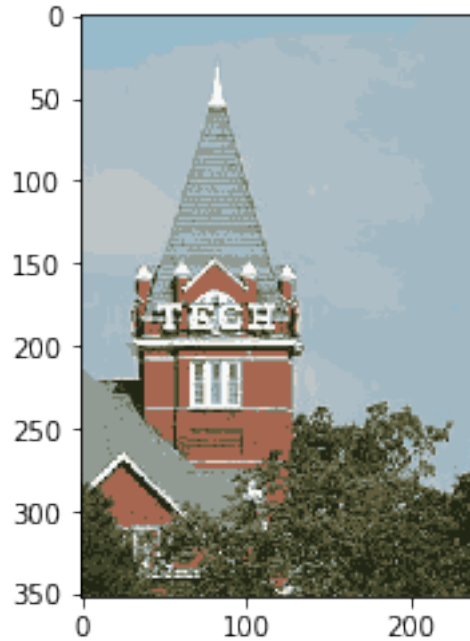
```
iteration 1 WCSS = 78081187.0
iteration 2 WCSS = 38936409.903284445
iteration 3 WCSS = 31357584.93121798
iteration 4 WCSS = 29227598.9107575
iteration 5 WCSS = 28650936.17738529
iteration 6 WCSS = 28411226.065431207
iteration 7 WCSS = 28192688.736099314
iteration 8 WCSS = 27965000.435471594
iteration 9 WCSS = 27583585.606403805
iteration 10 WCSS = 26785410.072711565
iteration 11 WCSS = 26346085.22983884
iteration 12 WCSS = 26048507.612918265
iteration 13 WCSS = 25797227.60573025
iteration 14 WCSS = 25566364.945790816
iteration 15 WCSS = 25338954.926837184
iteration 16 WCSS = 25118219.123322424
iteration 17 WCSS = 24830252.30974612
iteration 18 WCSS = 24360939.59759022
iteration 19 WCSS = 23748258.6143158
iteration 20 WCSS = 23169536.275939815
iteration 21 WCSS = 22679166.139020912
```

iteration 22 WCSS = 22317896.191743504  
iteration 23 WCSS = 22025442.517010275  
iteration 24 WCSS = 21802724.609557293  
iteration 25 WCSS = 21624312.162718233  
iteration 26 WCSS = 21480177.61772822  
iteration 27 WCSS = 21363760.94427106  
iteration 28 WCSS = 21276912.28350205  
iteration 29 WCSS = 21196653.6173141  
iteration 30 WCSS = 21124808.580648296  
iteration 31 WCSS = 21065369.621132564  
iteration 32 WCSS = 21015204.797175527  
iteration 33 WCSS = 20970273.4595458  
iteration 34 WCSS = 20931816.09624045  
iteration 35 WCSS = 20899758.778062407  
iteration 36 WCSS = 20875077.24558523  
iteration 37 WCSS = 20855216.874085985  
iteration 38 WCSS = 20838273.204672683  
iteration 39 WCSS = 20821200.45262719  
iteration 40 WCSS = 20806930.92696262  
iteration 41 WCSS = 20794085.365136594  
iteration 42 WCSS = 20774897.274009753  
iteration 43 WCSS = 20763147.170220234  
iteration 44 WCSS = 20748979.812213104  
iteration 45 WCSS = 20740097.265195515  
iteration 46 WCSS = 20726748.7160254  
iteration 47 WCSS = 20715893.216240577  
iteration 48 WCSS = 20705705.997667044  
iteration 49 WCSS = 20692077.80484623  
iteration 50 WCSS = 20682016.024851672  
iteration 51 WCSS = 20667964.003256697  
iteration 52 WCSS = 20655439.116513554  
iteration 53 WCSS = 20643727.054703187  
iteration 54 WCSS = 20633186.51691845  
iteration 55 WCSS = 20620871.853051785  
iteration 56 WCSS = 20606556.20117656  
iteration 57 WCSS = 20593357.742382225  
iteration 58 WCSS = 20579793.604372565  
iteration 59 WCSS = 20568208.0191338  
iteration 60 WCSS = 20556219.009414617  
iteration 61 WCSS = 20545535.895636324  
iteration 62 WCSS = 20538471.687272158  
iteration 63 WCSS = 20530973.459324382  
iteration 64 WCSS = 20523356.111371353  
iteration 65 WCSS = 20517006.505791984  
iteration 66 WCSS = 20510681.263629843  
iteration 67 WCSS = 20506576.851588584  
iteration 68 WCSS = 20502768.098947916  
iteration 69 WCSS = 20500118.385519367

```

iteration 70 WCSS = 20497376.686463825
iteration 71 WCSS = 20495799.64377183
iteration 72 WCSS = 20494220.044422064
iteration 73 WCSS = 20492874.263305224
iteration 74 WCSS = 20491847.578377612
iteration 75 WCSS = 20490746.616709426
iteration 76 WCSS = 20490197.50945997
iteration 77 WCSS = 20489837.242113557
iteration 78 WCSS = 20489570.67735978
iteration 79 WCSS = 20489210.12731411
iteration 80 WCSS = 20488900.940064684
iteration 81 WCSS = 20488739.237768378
iteration 82 WCSS = 20488630.46174035
iteration 83 WCSS = 20488589.671364233
iteration 84 WCSS = 20488581.27814218
iteration 85 WCSS = 20488572.305760473
iteration 86 WCSS = 20488553.176570028
iteration 87 WCSS = 20488508.172715124
iteration 88 WCSS = 20488428.128104635
iteration 89 WCSS = 20488366.354039375
iteration 90 WCSS = 20488291.20517655
iteration 91 WCSS = 20488260.062870495
iteration 92 WCSS = 20488240.21080901
iteration 93 WCSS = 20488216.908208635
iteration 94 WCSS = 20488209.89345389
iteration 95 WCSS = 20488204.266885314
iteration 96 WCSS = 20488183.404505007
iteration 97 WCSS = 20488144.810690735
iteration 98 WCSS = 20488132.606662843
iteration 99 WCSS = 20488127.732815232
iteration 100 WCSS = 20488123.090184085
iteration 101 WCSS = 20488122.38633112
iteration 102 WCSS = 20488122.02670927
Time taken = 238.52 seconds
Image with k = 12 clusters...
The labels are: [1 1 1 ... 2 2 2]
The cluster centers are {0: array([63.64358452, 68.14533023, 42.10503346]), 1:
array([151.77939198, 187.2055421 , 203.47807372]), 2: array([34.08600237,
38.20897588, 21.03459866]), 3: array([118.91362764, 123.92098528,
100.57165707]), 4: array([167.08229295, 102.05820292, 79.61790047]), 5:
array([173.130089 , 190.73404658, 199.55259421]), 6: array([145.54917355,
155.75096419, 148.69669421]), 7: array([96.03782121, 93.43612016, 62.94299674]),
8: array([162.78061891, 185.26378981, 195.52436451]), 9: array([222.50398936,
216.93484043, 204.95212766]), 10: array([189.8633851 , 197.95051329,
199.44827586]), 11: array([247.74863719, 249.86977589, 248.30284676])}

```



```
[54]: lab, cen = runKMeansAlgorithm("data/beach.bmp",32)
```

Running k-means algorithm...

```
iteration 1 WCSS = 45019968.0
iteration 2 WCSS = 27767575.689218067
iteration 3 WCSS = 23898738.075360753
iteration 4 WCSS = 22352898.928599153
iteration 5 WCSS = 21484255.380953792
iteration 6 WCSS = 20815592.421394795
iteration 7 WCSS = 20220248.45025015
iteration 8 WCSS = 19626269.678854804
iteration 9 WCSS = 19087538.242243454
iteration 10 WCSS = 18596517.368090734
iteration 11 WCSS = 18083614.717021115
iteration 12 WCSS = 17596459.88655153
iteration 13 WCSS = 17157083.371140227
iteration 14 WCSS = 16711904.483522398
iteration 15 WCSS = 16296609.056771155
iteration 16 WCSS = 15964166.881750261
iteration 17 WCSS = 15709637.325473953
iteration 18 WCSS = 15471680.80901358
iteration 19 WCSS = 15204757.67213304
iteration 20 WCSS = 14880927.562267441
iteration 21 WCSS = 14500658.953930428
iteration 22 WCSS = 14194685.86238497
iteration 23 WCSS = 13976880.939191196
```



iteration 24 WCSS = 13838267.36840708  
iteration 25 WCSS = 13738525.530358797  
iteration 26 WCSS = 13667216.899612563  
iteration 27 WCSS = 13609941.36692111  
iteration 28 WCSS = 13557017.495982805  
iteration 29 WCSS = 13508680.635023192  
iteration 30 WCSS = 13453754.620556723  
iteration 31 WCSS = 13391642.920642532  
iteration 32 WCSS = 13326003.705468126  
iteration 33 WCSS = 13258539.167323949  
iteration 34 WCSS = 13181681.684857016  
iteration 35 WCSS = 13121533.884225918  
iteration 36 WCSS = 13072574.633198025  
iteration 37 WCSS = 13024967.759802537  
iteration 38 WCSS = 12979457.397783194  
iteration 39 WCSS = 12938028.339539427  
iteration 40 WCSS = 12893932.894749936  
iteration 41 WCSS = 12853989.8608272  
iteration 42 WCSS = 12816882.375179872  
iteration 43 WCSS = 12783196.339708138  
iteration 44 WCSS = 12752950.188900907  
iteration 45 WCSS = 12728497.258673282  
iteration 46 WCSS = 12708456.502574416  
iteration 47 WCSS = 12693461.341536349  
iteration 48 WCSS = 12679884.31734222  
iteration 49 WCSS = 12666346.084923895  
iteration 50 WCSS = 12654851.480446061  
iteration 51 WCSS = 12643597.608039401  
iteration 52 WCSS = 12632437.932018476  
iteration 53 WCSS = 12621036.359780535  
iteration 54 WCSS = 12610957.200396188  
iteration 55 WCSS = 12602623.40389106  
iteration 56 WCSS = 12594842.284573963  
iteration 57 WCSS = 12587285.053839942  
iteration 58 WCSS = 12580624.025513945  
iteration 59 WCSS = 12573782.713509941  
iteration 60 WCSS = 12569531.367298346  
iteration 61 WCSS = 12564912.087189432  
iteration 62 WCSS = 12561972.234068686  
iteration 63 WCSS = 12559028.771523768  
iteration 64 WCSS = 12556365.315948239  
iteration 65 WCSS = 12553658.957864676  
iteration 66 WCSS = 12551113.593543626  
iteration 67 WCSS = 12547916.846415263  
iteration 68 WCSS = 12544753.563985482  
iteration 69 WCSS = 12540657.294862336  
iteration 70 WCSS = 12536371.203422928  
iteration 71 WCSS = 12530310.772718543

iteration 72 WCSS = 12521658.383171786  
iteration 73 WCSS = 12510027.980022606  
iteration 74 WCSS = 12494904.039293844  
iteration 75 WCSS = 12476450.053978875  
iteration 76 WCSS = 12455106.31783842  
iteration 77 WCSS = 12432651.74851378  
iteration 78 WCSS = 12413832.968960686  
iteration 79 WCSS = 12395922.285882289  
iteration 80 WCSS = 12376816.994168688  
iteration 81 WCSS = 12358248.545607967  
iteration 82 WCSS = 12340274.380309071  
iteration 83 WCSS = 12315343.834254948  
iteration 84 WCSS = 12292036.912821546  
iteration 85 WCSS = 12269577.209209476  
iteration 86 WCSS = 12249175.374485949  
iteration 87 WCSS = 12228867.636745503  
iteration 88 WCSS = 12211966.266162788  
iteration 89 WCSS = 12195395.544180887  
iteration 90 WCSS = 12182709.356720056  
iteration 91 WCSS = 12173153.922856841  
iteration 92 WCSS = 12164247.407576885  
iteration 93 WCSS = 12154601.109877376  
iteration 94 WCSS = 12146379.77442271  
iteration 95 WCSS = 12139747.243896605  
iteration 96 WCSS = 12133019.465266364  
iteration 97 WCSS = 12127325.836457692  
iteration 98 WCSS = 12122217.621295631  
iteration 99 WCSS = 12118095.249393994  
iteration 100 WCSS = 12115061.937037438  
iteration 101 WCSS = 12112725.576144315  
iteration 102 WCSS = 12110957.989098338  
iteration 103 WCSS = 12109251.112020874  
iteration 104 WCSS = 12107594.093575923  
iteration 105 WCSS = 12105480.474864507  
iteration 106 WCSS = 12103114.791187843  
iteration 107 WCSS = 12101205.29481378  
iteration 108 WCSS = 12098852.070808975  
iteration 109 WCSS = 12096795.020683402  
iteration 110 WCSS = 12094439.687005566  
iteration 111 WCSS = 12091584.712140342  
iteration 112 WCSS = 12088315.001579128  
iteration 113 WCSS = 12085773.359141104  
iteration 114 WCSS = 12083413.599208545  
iteration 115 WCSS = 12080896.27714626  
iteration 116 WCSS = 12078720.31485486  
iteration 117 WCSS = 12077292.383412044  
iteration 118 WCSS = 12076233.799854983  
iteration 119 WCSS = 12074633.330525931

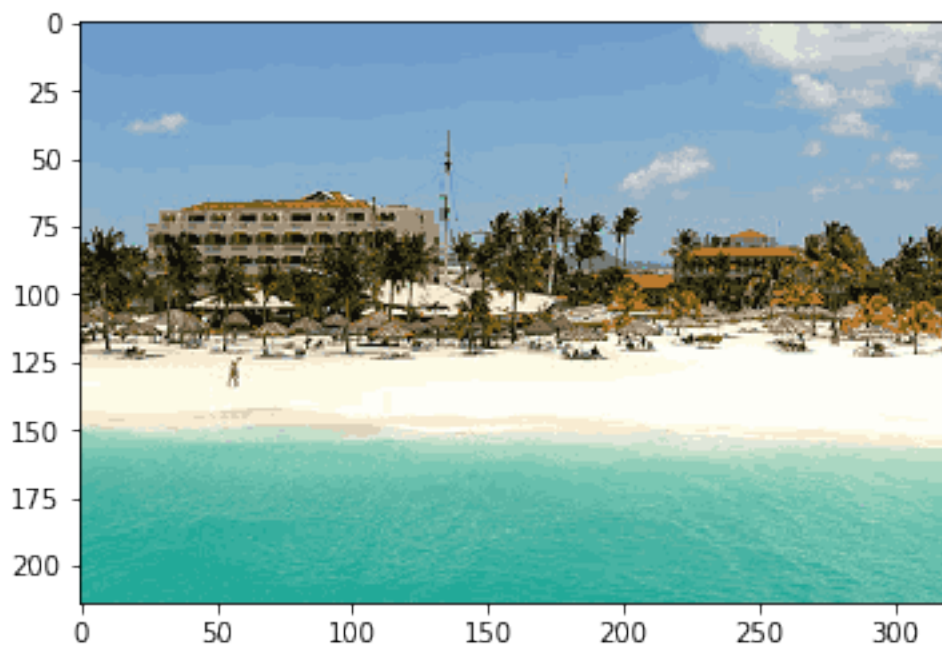
iteration 120 WCSS = 12072864.15654355  
iteration 121 WCSS = 12071634.495801235  
iteration 122 WCSS = 12070079.262982232  
iteration 123 WCSS = 12067935.096918315  
iteration 124 WCSS = 12065411.869487232  
iteration 125 WCSS = 12062118.600492494  
iteration 126 WCSS = 12058068.066725899  
iteration 127 WCSS = 12053955.1898843  
iteration 128 WCSS = 12050161.897327507  
iteration 129 WCSS = 12045587.005010098  
iteration 130 WCSS = 12040835.827572871  
iteration 131 WCSS = 12036011.03797824  
iteration 132 WCSS = 12030041.637581395  
iteration 133 WCSS = 12024608.989578852  
iteration 134 WCSS = 12018160.343562547  
iteration 135 WCSS = 12011181.83984205  
iteration 136 WCSS = 12004893.777427336  
iteration 137 WCSS = 11999183.757419096  
iteration 138 WCSS = 11992626.871406361  
iteration 139 WCSS = 11986127.531601181  
iteration 140 WCSS = 11980651.05377796  
iteration 141 WCSS = 11976176.189491648  
iteration 142 WCSS = 11972315.458984416  
iteration 143 WCSS = 11969092.549933357  
iteration 144 WCSS = 11966930.44005058  
iteration 145 WCSS = 11965469.449237531  
iteration 146 WCSS = 11964295.841543075  
iteration 147 WCSS = 11963100.489264412  
iteration 148 WCSS = 11962625.516585309  
iteration 149 WCSS = 11962181.211487884  
iteration 150 WCSS = 11960957.652884718  
iteration 151 WCSS = 11960122.652047219  
iteration 152 WCSS = 11959487.474610144  
iteration 153 WCSS = 11958502.777246289  
iteration 154 WCSS = 11957772.83604165  
iteration 155 WCSS = 11957111.627938602  
iteration 156 WCSS = 11956766.545048788  
iteration 157 WCSS = 11956194.64685188  
iteration 158 WCSS = 11955522.094592327  
iteration 159 WCSS = 11954949.914927276  
iteration 160 WCSS = 11954595.92319678  
iteration 161 WCSS = 11954322.64258102  
iteration 162 WCSS = 11954027.633079138  
iteration 163 WCSS = 11953756.897055842  
iteration 164 WCSS = 11953561.359836362  
iteration 165 WCSS = 11953394.150561273  
iteration 166 WCSS = 11953143.744293556  
iteration 167 WCSS = 11952872.0460485

```

iteration 168 WCSS = 11952726.840225533
iteration 169 WCSS = 11952607.951913094
iteration 170 WCSS = 11952485.861721046
iteration 171 WCSS = 11952399.74689186
iteration 172 WCSS = 11952253.03103106
iteration 173 WCSS = 11952022.830360595
iteration 174 WCSS = 11951854.329699213
iteration 175 WCSS = 11951421.764006669
iteration 176 WCSS = 11950859.074740596
iteration 177 WCSS = 11950242.422732463
iteration 178 WCSS = 11949566.615759084
iteration 179 WCSS = 11949180.753478503
iteration 180 WCSS = 11949017.025349438
iteration 181 WCSS = 11948675.964173885
iteration 182 WCSS = 11948249.065074338
iteration 183 WCSS = 11947859.993417114
iteration 184 WCSS = 11947490.178907802
iteration 185 WCSS = 11947088.728614189
iteration 186 WCSS = 11946867.108135125
iteration 187 WCSS = 11946631.074624576
iteration 188 WCSS = 11946334.08826181
iteration 189 WCSS = 11946171.448187334
iteration 190 WCSS = 11946089.118143868
iteration 191 WCSS = 11946064.301593367
iteration 192 WCSS = 11946045.449806698
iteration 193 WCSS = 11946000.293923449
iteration 194 WCSS = 11945959.450258758
iteration 195 WCSS = 11945928.799474988
iteration 196 WCSS = 11945911.698249673
iteration 197 WCSS = 11945905.71659477
iteration 198 WCSS = 11945899.023637032
iteration 199 WCSS = 11945897.552198278
Time taken = 391.71 seconds
Image with k = 32 clusters...
The labels are: [11 11 11 ... 29 29 29]
The cluster centers are {0: array([117.14167813, 123.39477304, 121.93122421]),
1: array([14.69779853, 13.80386925, 8.8305537 ]), 2: array([248.48988064,
236.99844318, 210.39543332]), 3: array([170.81076389, 215.015625 ,
194.26822917]), 4: array([253.40602786, 251.17401194, 230.14444129]), 5:
array([45.48518851, 43.66337522, 19.502693 ]), 6: array([134.17755573,
113.59569562, 84.44504228]), 7: array([223.9772118 , 212.23458445,
188.26541555]), 8: array([138.56654306, 177.5039143 , 205.5986815 ]), 9:
array([149.07262731, 178.81105324, 198.67332176]), 10: array([127.92953822,
171.22651274, 206.40067675]), 11: array([111.18968583, 156.69176052,
202.94694724]), 12: array([224.78610603, 231.79616088, 218.34277879]), 13:
array([167.67966102, 185.94237288, 200.81949153]), 14: array([187.21122112,
122.51320132, 46.10066007]), 15: array([144.50282087, 206.01622003,
185.27433004]), 16: array([188.38696109, 196.21556257, 202.21451104]), 17:

```

```
array([121.65080875, 94.39105614, 25.77069458]), 18: array([ 54.11405836,
174.60515347, 158.47063282]), 19: array([204.72897196, 209.70560748,
214.70794393]), 20: array([ 71.50782313, 179.78707483, 163.26054422]), 21:
array([197.35307517, 224.03075171, 202.8166287 ]), 22: array([94.89716546,
88.19644034, 66.18193804]), 23: array([121.8874669 , 199.42365402,
179.94042365]), 24: array([ 34.42066077, 169.17543043, 152.22056771]), 25:
array([160.83688833, 145.4523212 , 122.42346299]), 26: array([73.32458126,
67.9932096 , 30.1611589 ]), 27: array([119.11422992, 163.84159141,
204.86331462]), 28: array([103.72065889, 191.21551132, 173.6273164 ]), 29:
array([ 90.03983516, 182.20192308, 165.75549451]), 30: array([254.13774717,
254.47145079, 248.99022439]), 31: array([190.88261351, 174.28460687,
150.12403101])}
```



[ ]: