Computational Data Analysis

Machine Learning

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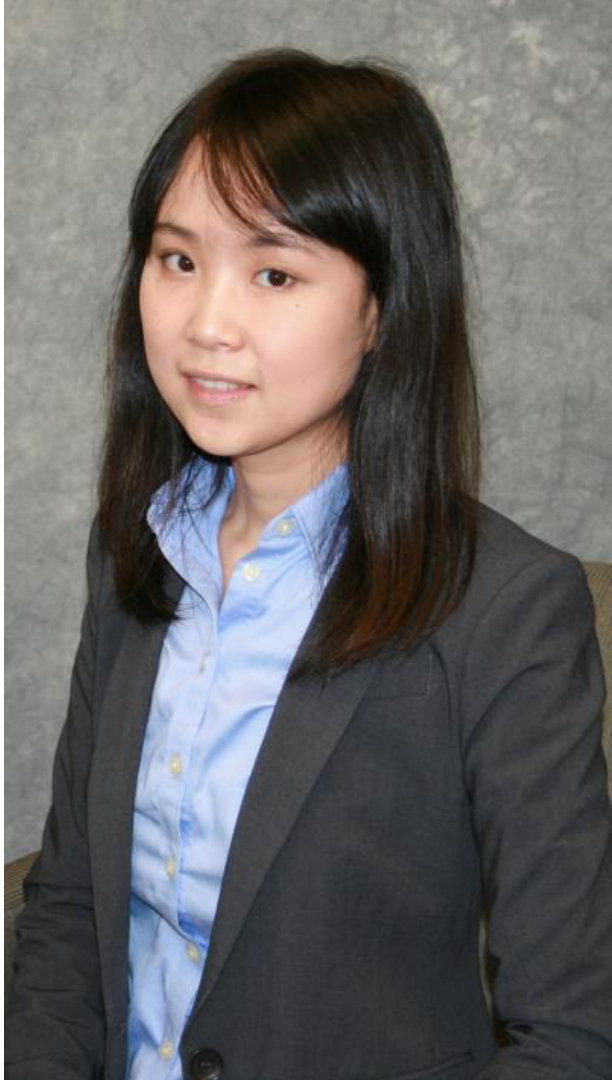
Introduction to Neural Networks

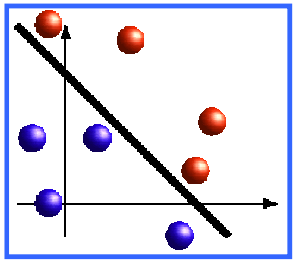


Main approaches to design classifiers

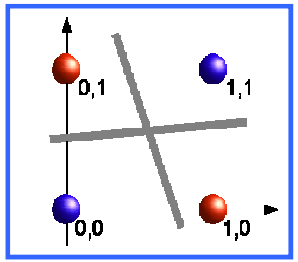
- Bayes rule, use simplifying assumption for $p(x|y=1)$
  - Assume $p(x|y=1)$ is Gaussian
  - Assume $p(x|y=1)$ is fully factorized
- Use geometric intuitions
  - k-nearest neighbor classifier
  - Support vector machine
- Directly go for the decision boundary $\mathrm{h(x)} = -\ln \frac{q_i(x)}{q_j(x)}$
  - Logistic regression
  - Neural networks

Learning nonlinear decision boundary

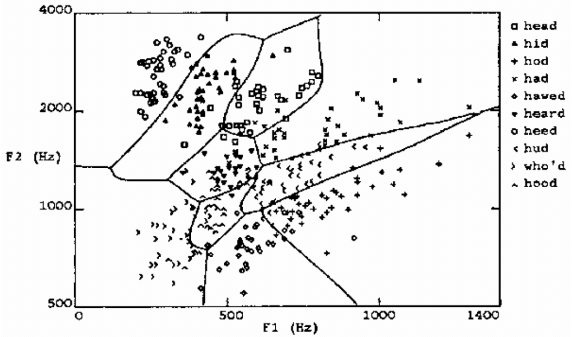
- Linearly separable



- Nonlinearly separable



The XOR gate

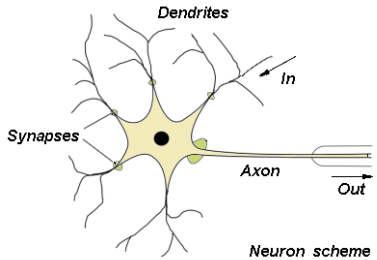


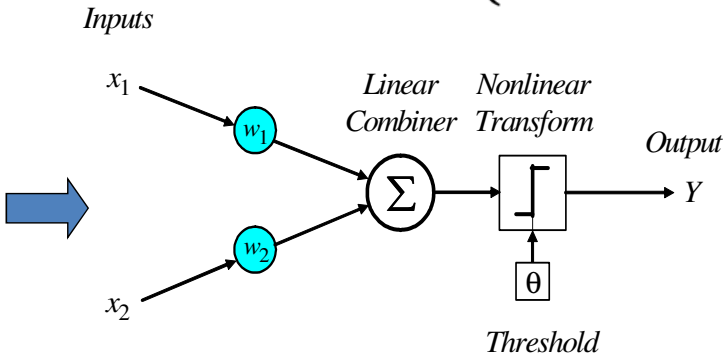
Speech recognition

Perceptron

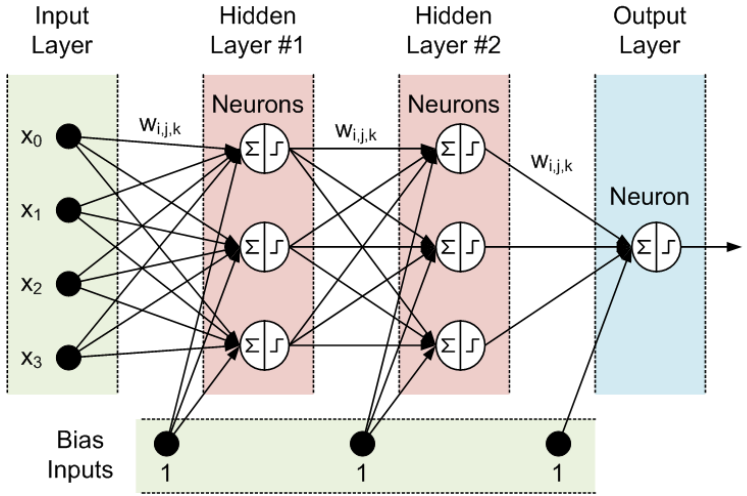
$$f(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{w} \cdot \mathbf{x} + b > 0, \\ 0 & \text{otherwise} \end{cases}$$

- From biological neuron to artificial neuron (perceptron)





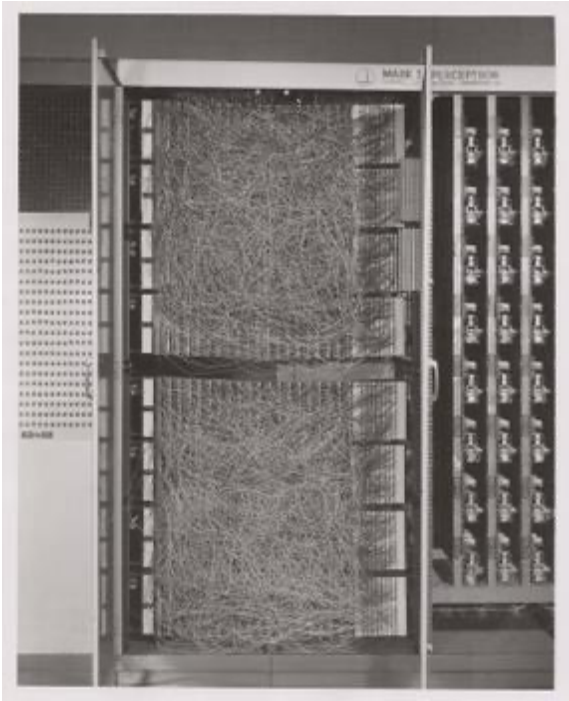
- Artificial neural nets (ANN)
  - Many neuron-like threshold switching units
  - Many weighted interconnections among units
  - Highly parallel, distributed processes



Early artificial intelligence

- The perceptron algorithm was first invented in 1958 at the Cornell Aeronautical Lab

- This machine was designed for image recognition: it had an array of 400 photocells, randomly connected to the "neurons".

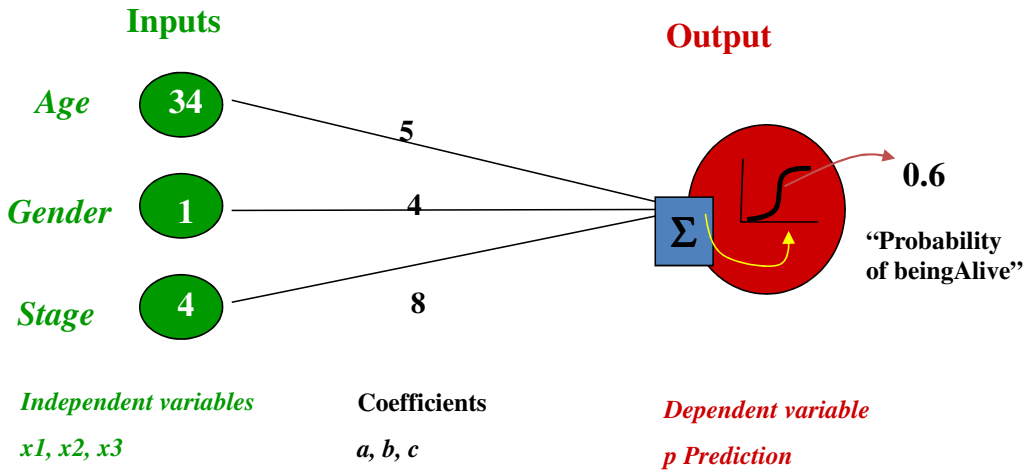


Mark I Perceptron machine, the first implementation of the perceptron algorithm. It was connected to a camera with 20×20 cadmium sulfide photocells to make a 400-pixel image. The main visible feature is a patch panel that set different combinations of input features. To the right, arrays of potentiometers that implemented the adaptive weights.

Logistic regression can be viewed as probabilistic perceptron

- Independent variable = input variable
- Dependent variable = output variable
- Coefficients = “weights”

Logistic Regression Model (the sigmoid unit) is a perceptron

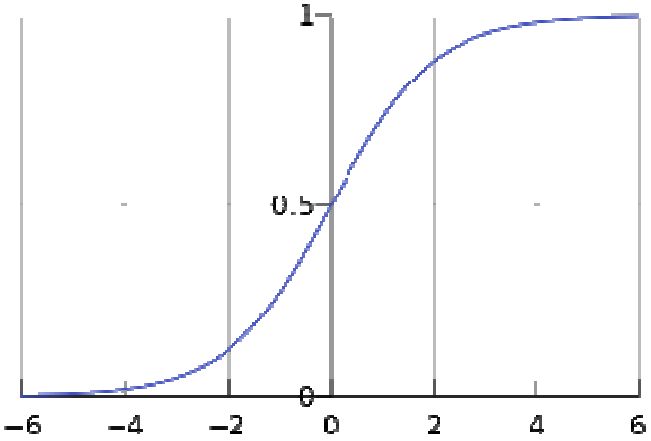


Logistic regression

- Assume that the posterior distribution $p(y = 1|x)$ take a particular form

$$p(y = 1|x, w) = \frac{1}{1 + \exp(-w^{\mathsf{T}} x)}$$

- Logistic function (or sigmoid function) $\sigma(u) = \frac{1}{1+\exp(-u)}$
  - $\frac{\partial \sigma(u)}{\partial u} = \sigma(u)\big(1 - \sigma(u)\big)$



Learning parameters in logistic regression

- Find θ, such that the conditional likelihood of the labels is maximized

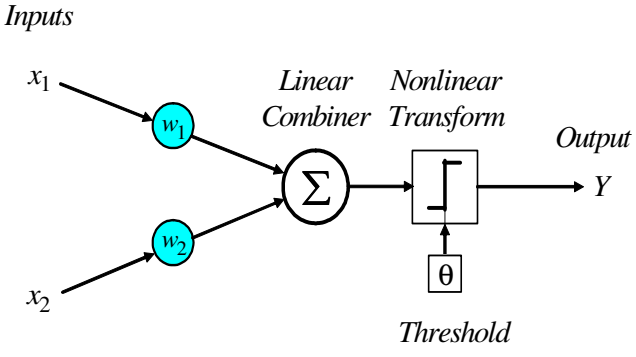
$$\max_{\theta} l(w) := \log \prod_{i=1}^{m} P\left(y^i \middle| x^i, w\right)$$

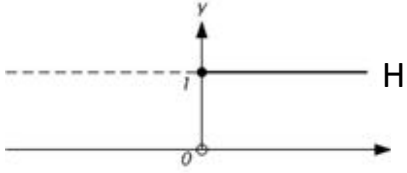
- Good news: $l(w)$ is concave function of w, and there is a single global optimum.

- Bad new: no closed form solution (resort to numerical method)

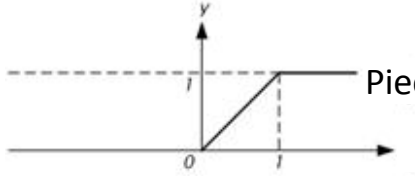
Other nonlinear neurons

- Use different nonlinear transformations $f(u)$
- Before that, perform weighted combination of inputs $u = w^{\top} x$

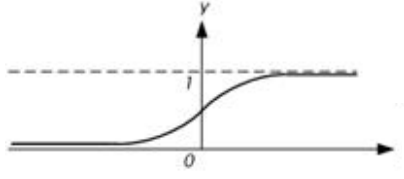




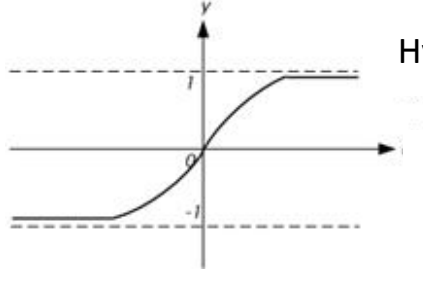
Hard limiter



Piecewise linear



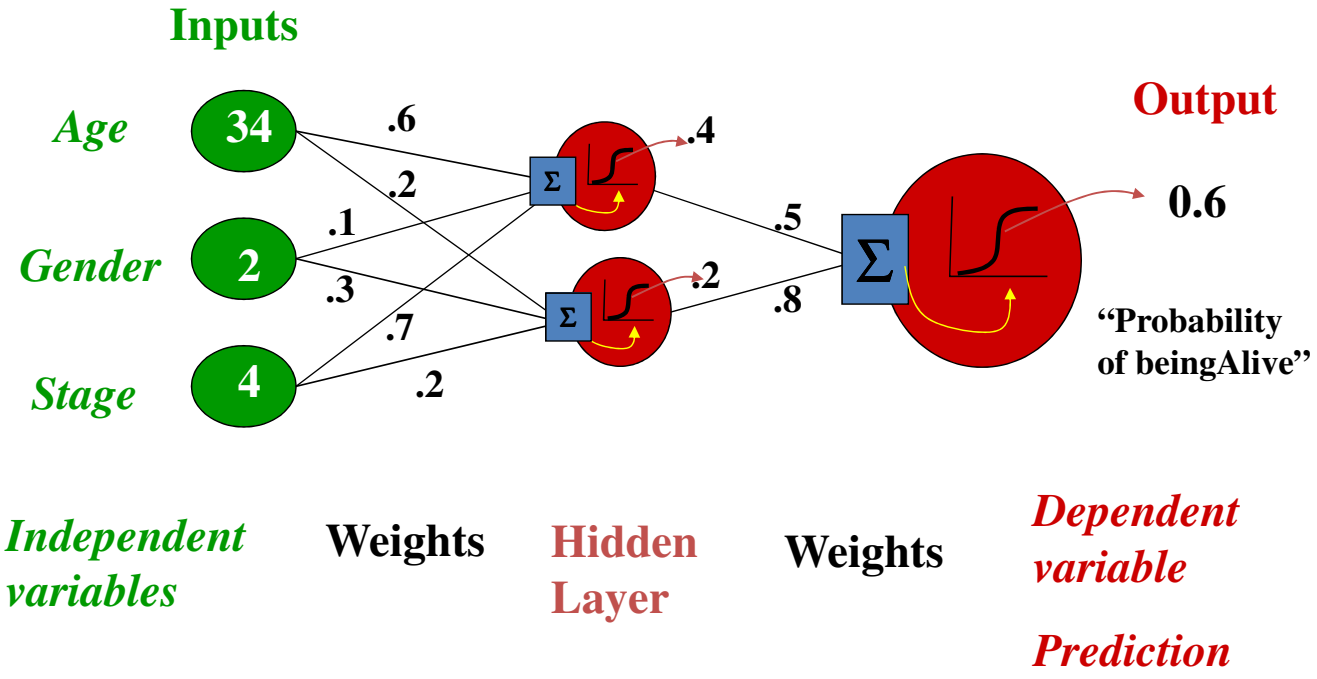
Sigmoid



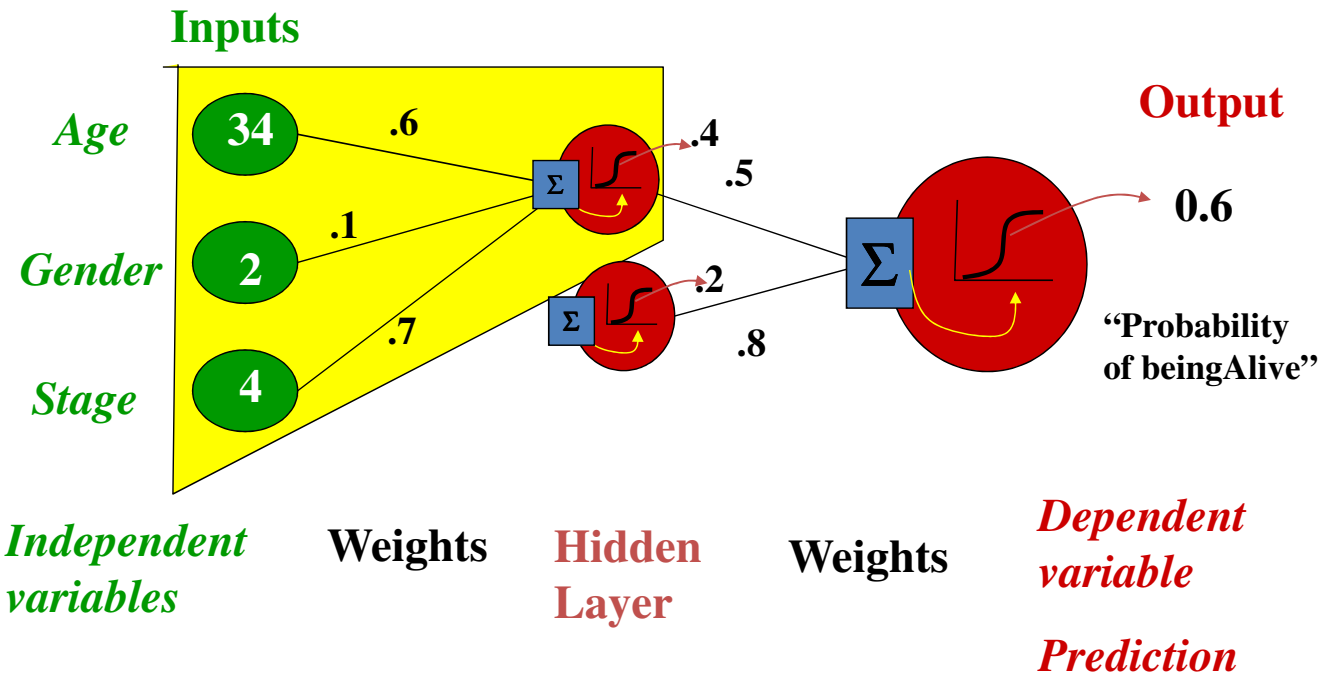
Hyperbolic tangent

Deep Neural Network Model

Multiple layers of interconnected perceptrons



“Combined logistic models”



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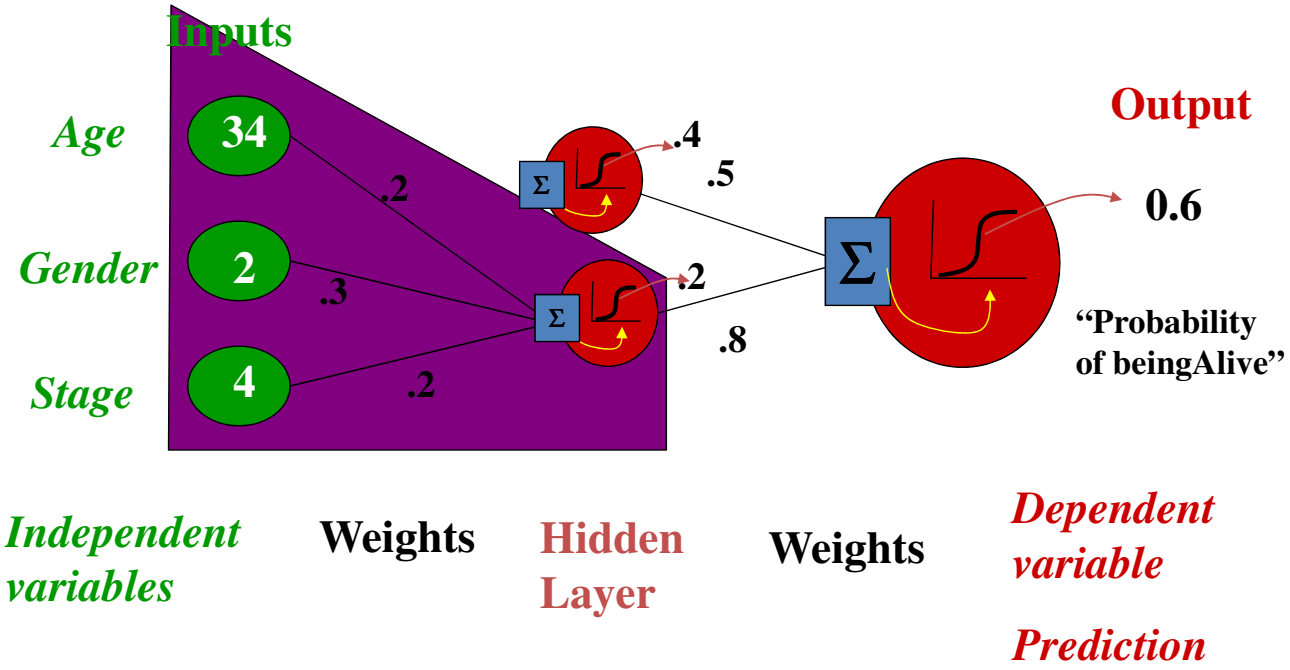

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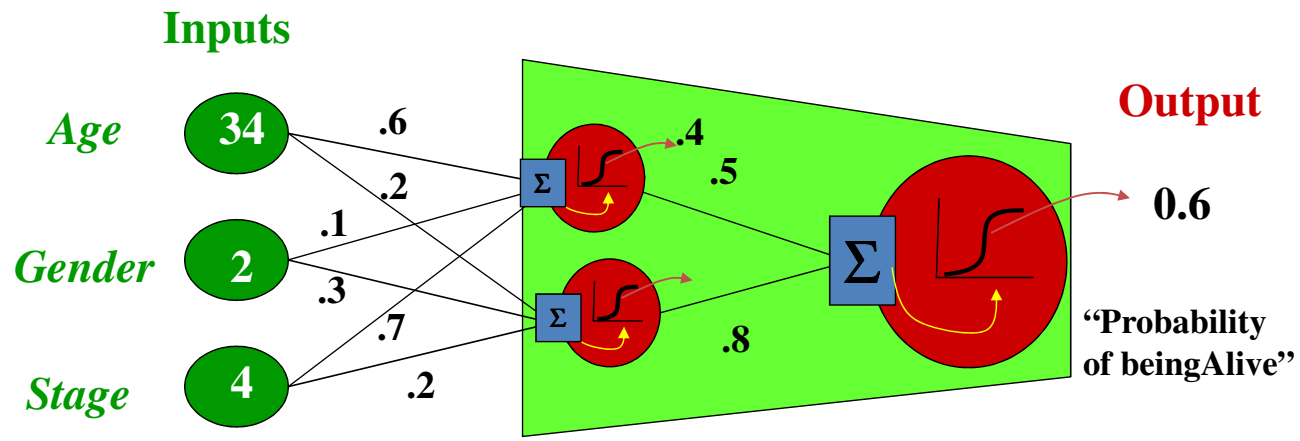
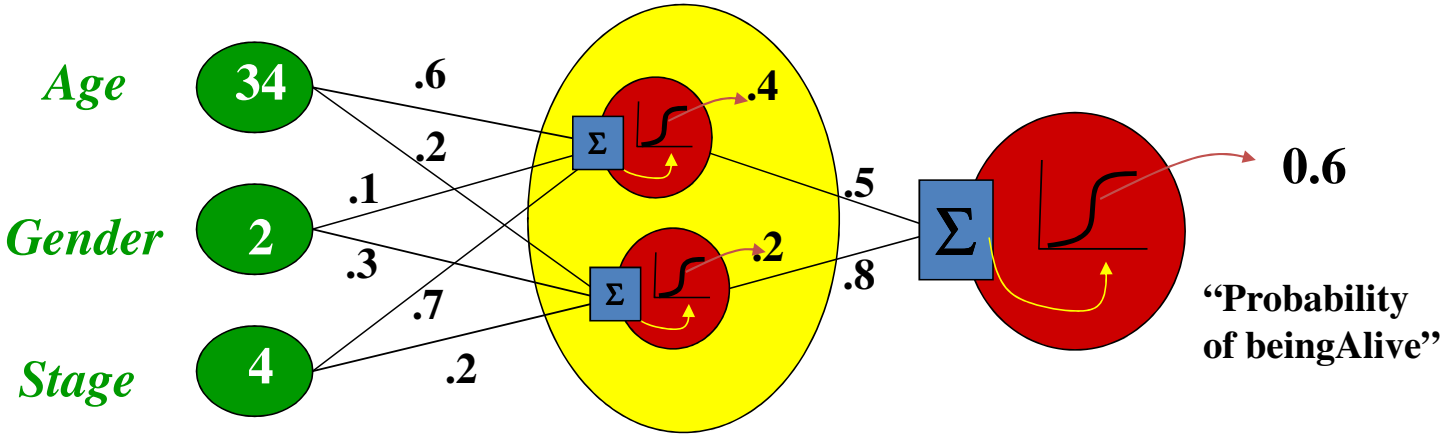


Overall structure

Turn weights and thresholds such that the neural networks will give desired prediction results given input.



Expressive Capabilities of Neural Networks

- Boolean functions:
  - Every Boolean function can be represented by network with single hidden layer
  - But might require exponential (in number of inputs) hidden units

- Continuous functions:
  - Every bounded continuous function can be approximated with arbitrary small error, by network with one hidden layer [Cybenko 1989; Hornik et al 1989]
  - Any function can be approximated to arbitrary accuracy by a network with two hidden layers [Cybenko 1988].

Training/fitting neural networks: Backpropagation

Find θ, α, β, such that the value of the output unit are close to the actual labels (0 or 1)

$$\min_{w,\alpha,\beta} l(w, \alpha, \beta) := \sum_{i}^{n} \left(y^i - \sigma\left(w^\top z^i\right)\right)^2$$

where (use sigmoid function σ)

$$z_1^i = \sigma\left(\alpha^\top x^i\right)$$
$$z_2^i = \sigma(\beta^\top x^i)$$

Non-convex objective function

Use gradient decent to find a local optimum

The gradient of $l(w, \alpha, \beta)$

$l(w, \alpha, \beta) := \sum_{i=1}^{n} \left(y^i - \sigma\left(w^\top z^i\right)\right)^2$

where $z_1^i = \sigma\left(\alpha^\top x^i\right), z_2^i = \sigma(\beta^\top x^i)$

Let $u^i = w^\top z^i$

Gradient

$$\frac{\partial l(w, \alpha, \beta)}{\partial w} = -\sum_i 2\left(y^i - \sigma\left(u^i\right)\right)\sigma\left(u^i\right)\left(1 - \sigma\left(u^i\right)\right)z^i$$

z^i is computed using α and β from previous iteration

$$z_1^i = \sigma\left(\alpha^\top x^i\right), z_2^i = \sigma(\beta^\top x^i)$$

The gradient with respect to α, β

Use chain rule of derivatives (backpropagation)

The "error" at layer k depends on the error at the next layer k+1

Note $z_1^i = \sigma(\alpha^\top x^i), z_2^i = \sigma(\beta^\top x^i)$

Let $v^i = \alpha^\top x^i$

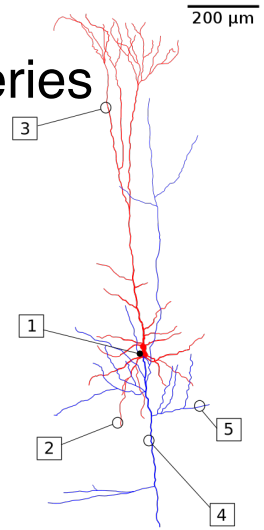
Gradient

$$\frac{\partial l(w,\alpha,\beta)}{\partial \alpha} = \frac{\partial l(w,\alpha,\beta)}{\partial z_1^i}\frac{\partial z_1^i}{\partial \alpha}$$

$$= -\sum_i 2\left(y^i - \sigma(u^i)\right)\sigma(u^i)\left(1-\sigma(u^i)\right)w_1\sigma(v^i)\left(1-\sigma(v^i)\right)x^i$$

Some common neural network structures

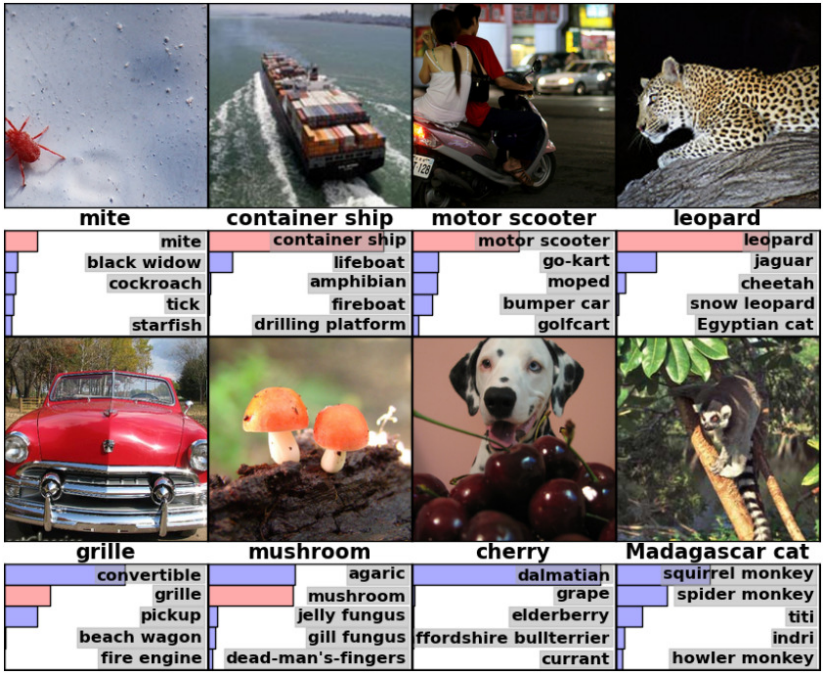
- Full connected (all nodes in the k-th layer are connected to all nodes in the (k+1)-th layer (not practical)
- Convolutional neural networks (nodes in one layer are only ”locally” connected to nodes in the next layer)
- Recursive neural network (RNN): good for sequential / time series data; neural network structure ”repeats” itself.
- Residual neural networks
- Generative adversarial neural networks (GAN)



ImageNet

Image classification with 1.3M color images and 1000 classes

Need large scale nonparametric methods



https://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

Georgia Tech

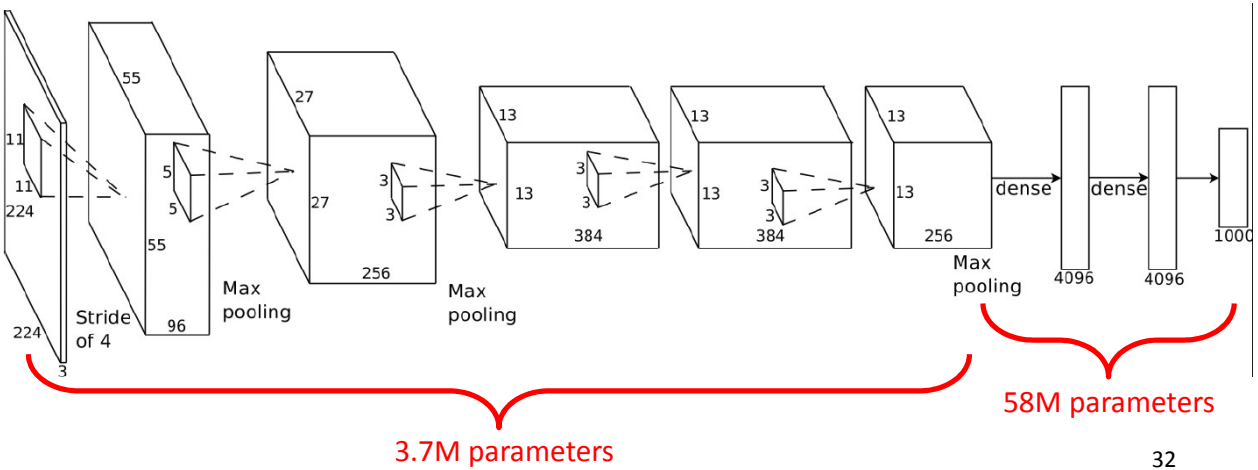
Convolutional Neural Network (CNN)

8 layer convolution neural network [Krizhevsky12], Achieved state-of-the-art result (beating the second place by 10%)

First 5 layers: convolution + max pooling

Next 2 layers: fully connected nonlinear neurons

Last layer: multiclass logistic regression



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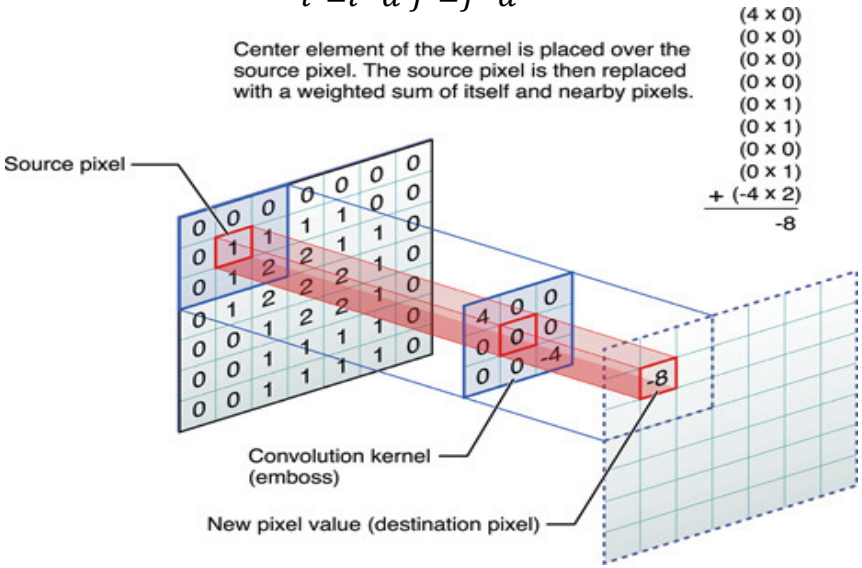
Convolution layer

- Source image *I*, destination image *O*, convolving image with a kernel *K* of size *2d+1*

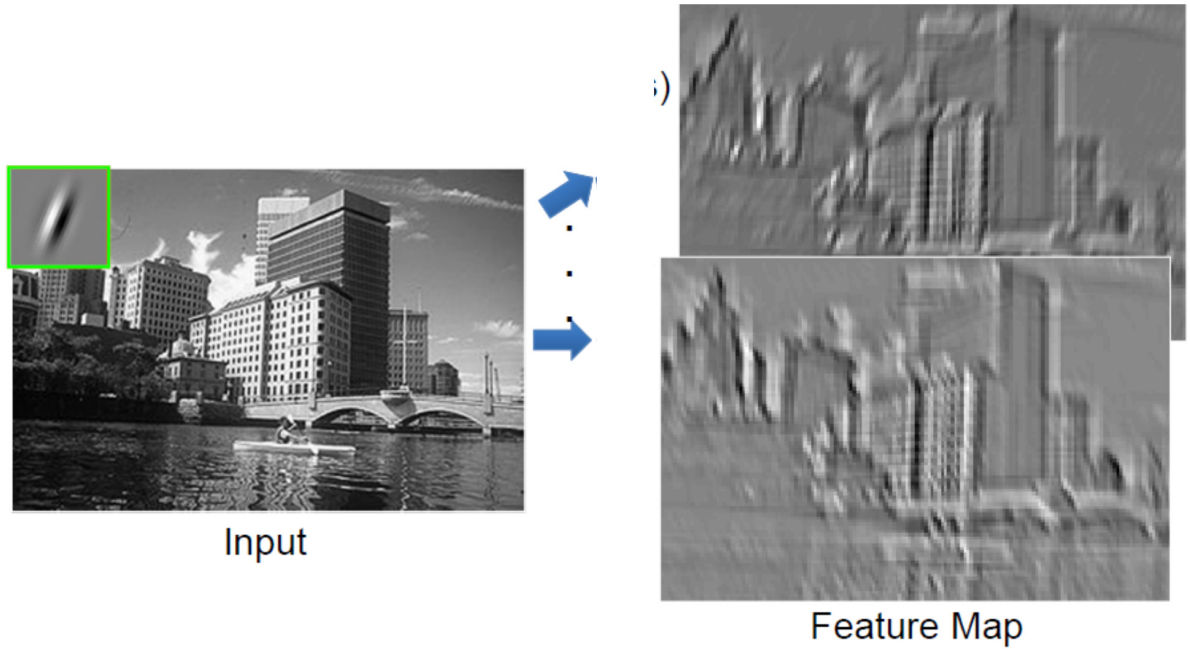
$$O(i,j) = \sum_{i'=i-d}^{i+d} \sum_{j'=j-d}^{j+d} I(i',j')K(i',j')$$



https://www.embedded-vision.com/sites/default/files/technicalarticles/ARMFFT/2014Article/kernel_convolution.jpg

Convolutional feature

Convolution extract local image features

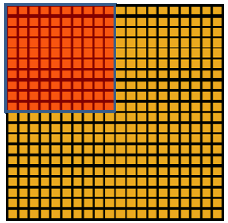


https://i.imgur.com/TqTJCrz.png

Convolutional Neural Network

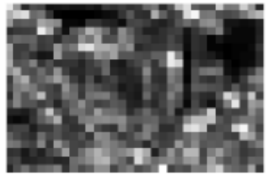
Spatial pooling

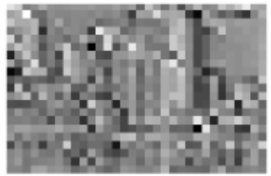
- Sum or max (usually max)
- Overlapping or non-overlapping regions
- Invariant to small translations



Convolved feature

Pooled feature





https://yoyoinwanderland.github.io/Deep-Learning-1/

ImageNet

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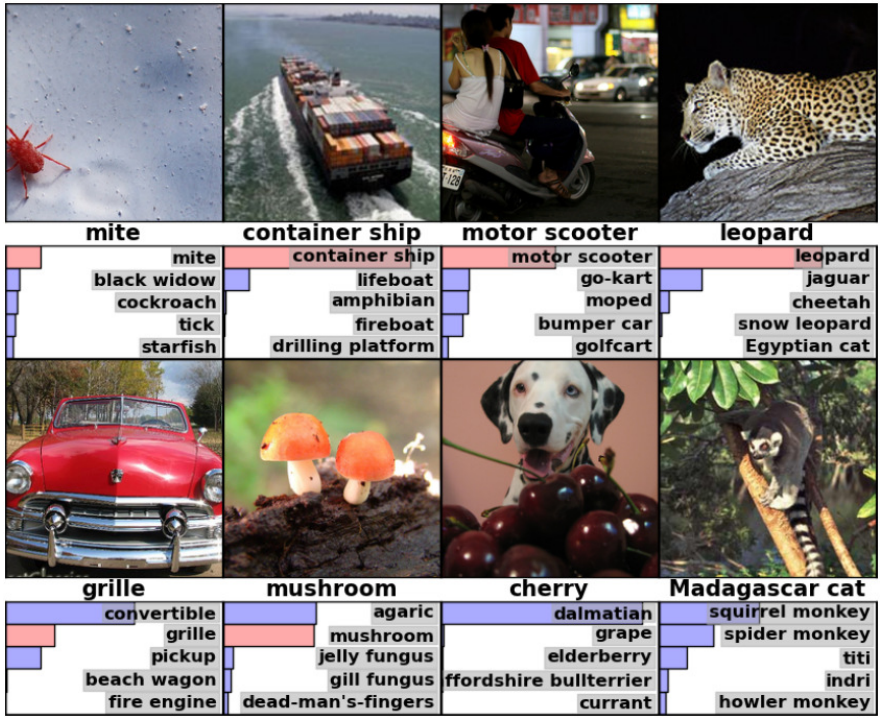
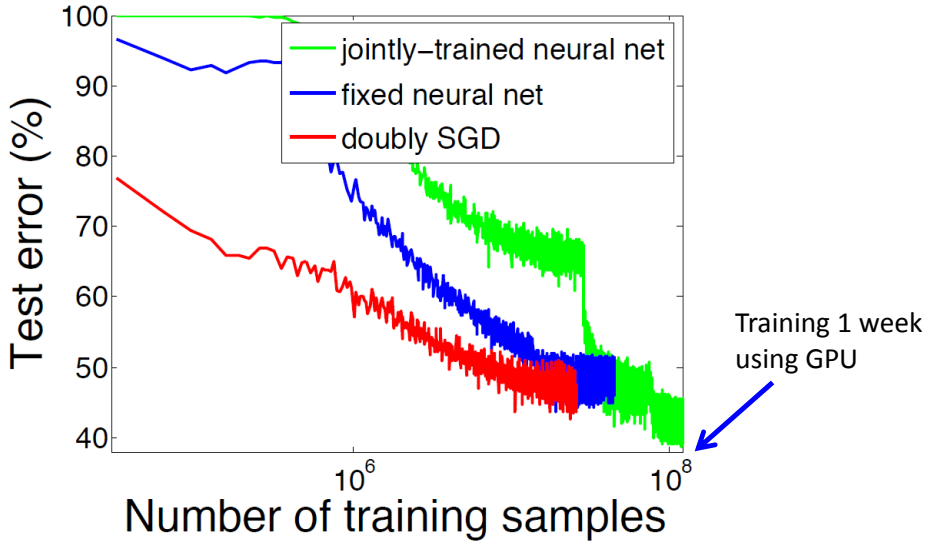


ImageNet

Millions of training points, 1000 class classification task



https://github.com/zixu1986/Doubly_Stochastic_Gradients

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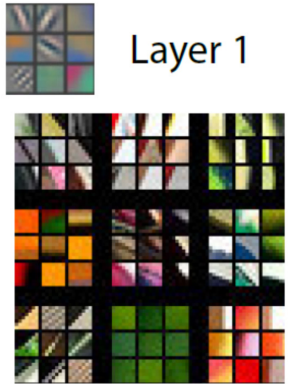
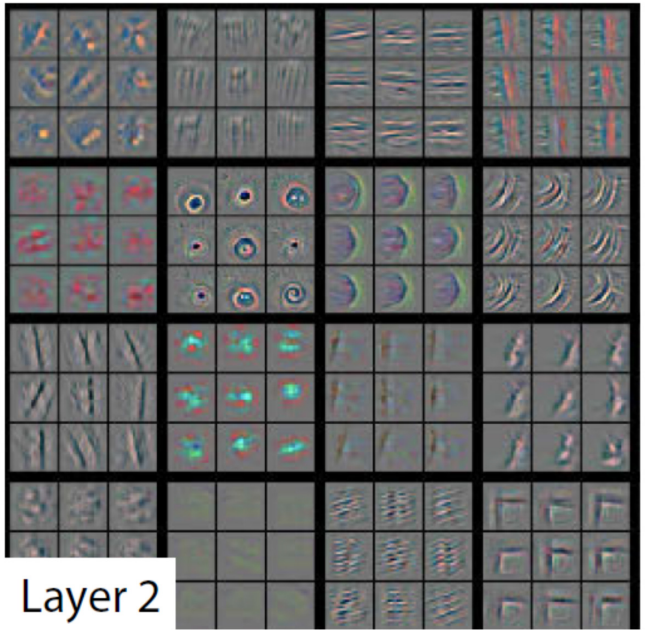
Convolutional Neural Networks

Visualize learned filters

- Lower layer learns more primitive features
- Higher layer learns more complex features





https://arxiv.org/pdf/1311.2901v3.pdf

Convolutional Neural Networks

TABLE 11.1. *Test set performance of five different neural networks on a handwritten digit classification example (Le Cun, 1989).*

	Network Architecture	Links	Weights	% Correct
Net-1:	Single layer network	2570	2570	80.0%
Net-2:	Two layer network	3214	3214	87.0%
Net-3:	Locally connected	1226	1226	88.5%
Net-4:	Constrained network 1	2266	1132	94.0%
Net-5:	Constrained network 2	5194	1060	98.4%

Ending words

- Deep learning is the ”hot-topic” in “hot-topics”
- Rapidly evolving
- Dedicated course on neural networks
- Many engineering efforts, hugely successful in various domain: image, speech etc.
- Still fairly nascent in other more complex domains: healthcare, logistics, finance.
- Additional references
- NeurIPS 2015 tutorial on deep learning: https://aiatadams.files.wordpress.com/2016/02/bengio-lecun-20151207-deep-learning-tutorial-nips.pdf
- NeurIPS 2016 tutorial on GAN: https://media.nips.cc/Conferences/2016/Slides/6202-Slides.pdf


SUMMARY