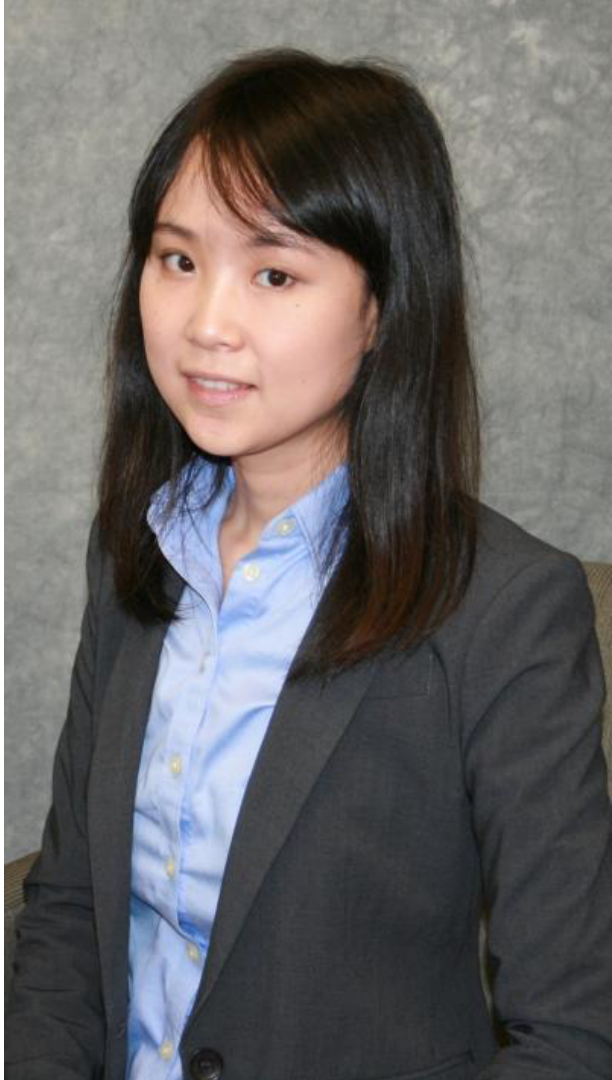
Computational Data Analysis

Machine Learning



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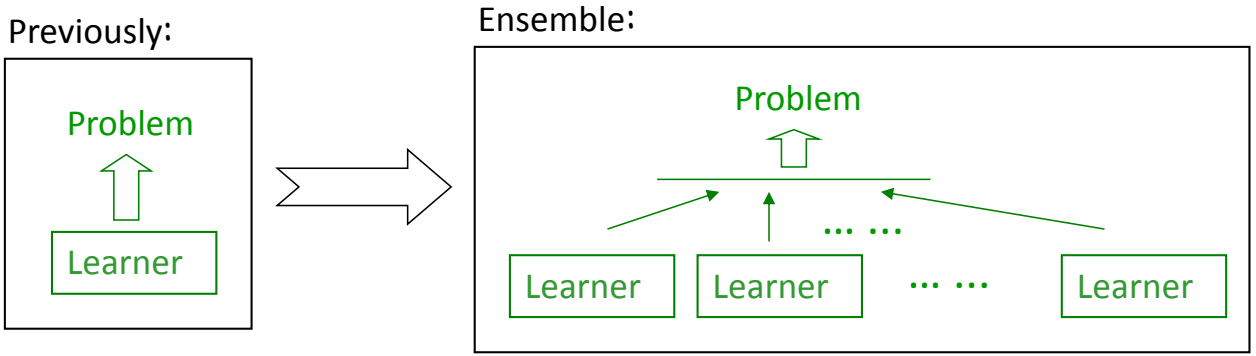
Boosting

Rationale: Combination of methods

- There is no algorithm that is always the most accurate
- We can select simple “weak” classification or regression methods and combine them into a single “strong” method
- Different learners use different
  - Algorithms, Hyperparameters, Representations (Modalities), Training sets, Subproblems



Email spam classification

- 4601 email messages
- Label: regular, spam
- Objective: design automatic spam detector that filter out spam before clogging users' mailboxes
- Bag-of-words model: 57 most commonly occurring words of spam email (example)

	george	you	your	hp	free	hpl	!	our	re	edu	remove
spam	0.00	2.26	1.38	0.02	0.52	0.01	0.51	0.51	0.13	0.01	0.28
email	1.27	1.27	0.44	0.90	0.07	0.43	0.11	0.18	0.42	0.29	0.01

- Boosting algorithm decision rule

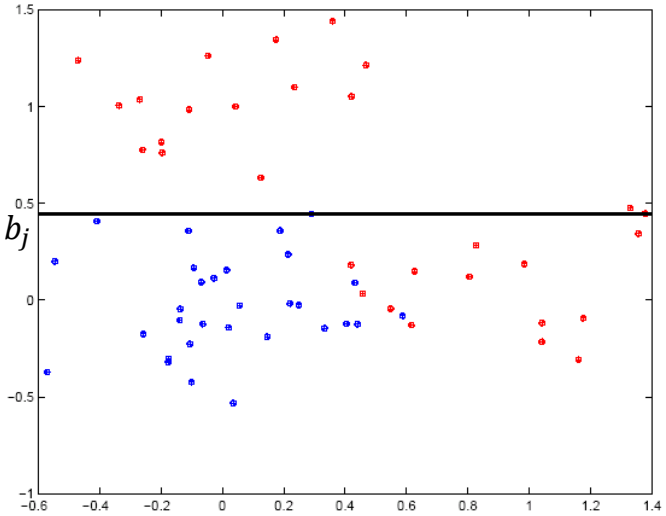
$$\text{if } (\%\texttt{george} < 0.6) \;\&\; (\%\texttt{you} > 1.5) \quad \text{then } \texttt{spam} \\ \text{else } \texttt{email}$$

Decision stump

- Let $y \in \{\pm 1\}$
- Decision stump:

$$h(x;\theta_j) = sign(w_j x_j + b_j)$$

- Each decision stump pays attention to only a single dimension of the input vector



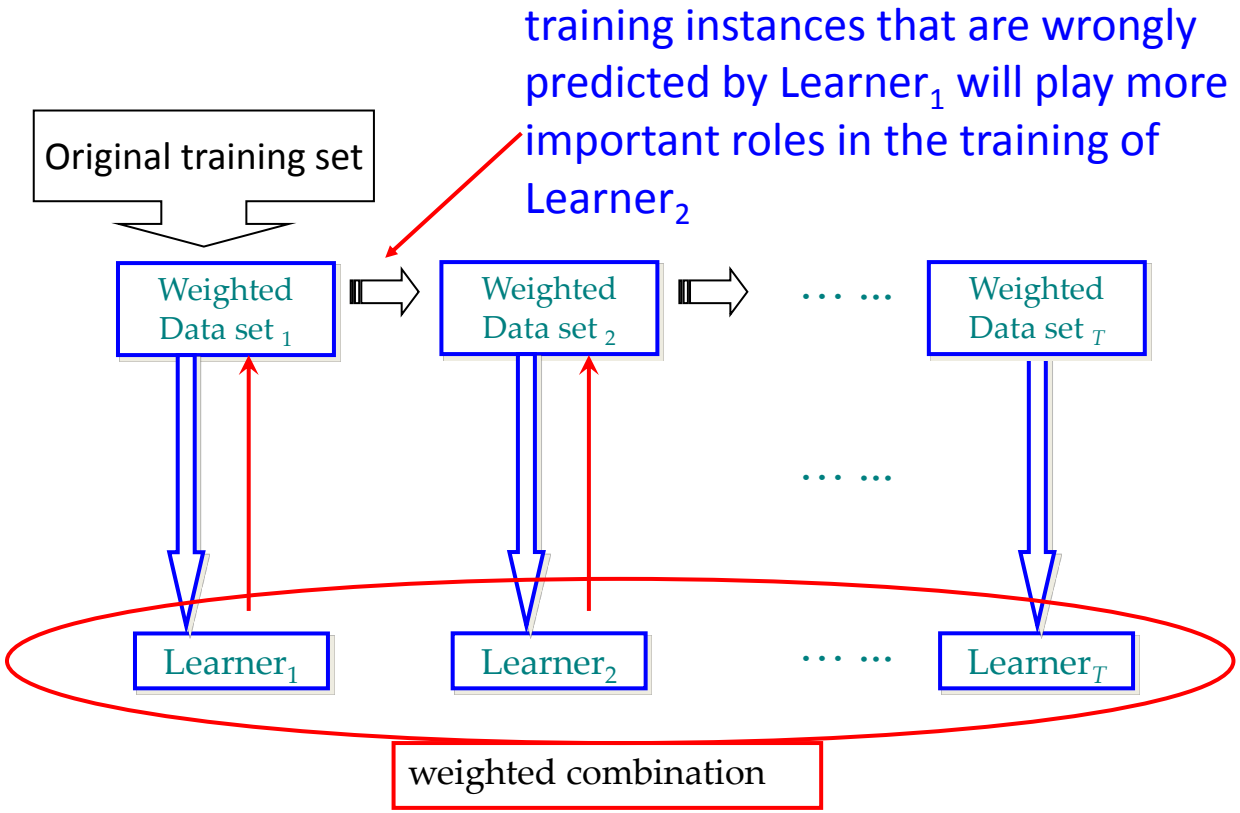
Boosting

- Boosting: general methods of converting rough rules of thumb (or weak classifier) into highly accurate prediction rule

- A family of methods which produce a sequence of classifiers
  - Each classifier is dependent on the previous one and focuses on the previous one's errors
  - Examples that are incorrectly predicted in the previous classifiers are chosen more often or weighted more heavily when estimating a new classifier.

- Questions:
  - How to choose "hardest" examples?
  - How to combine these classifiers?

Adaboost flow chart



AdaBoost

- constructing D_t:
  - $D_1(i) = 1/m$
  - given D_t and h_t: $\epsilon_t = \sum_{i=1}^{m} D_t(i) \mathbb{I}\{y_i \neq h_t(x_i)\}$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$

$$= \frac{D_t(i)}{Z_t} \exp(-\alpha_t \, y_i \, h_t(x_i))$$

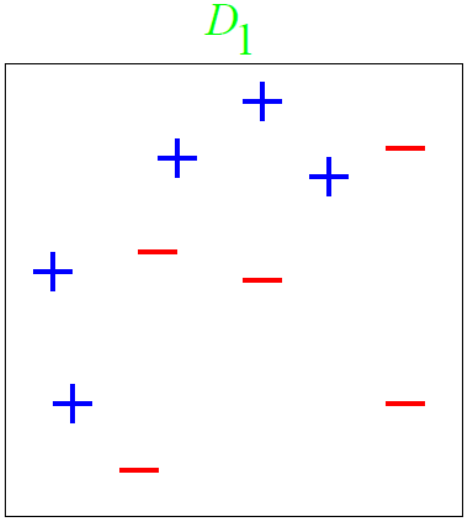
where $Z_t =$ normalization constant

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \epsilon_t}{\epsilon_t} \right) > 0$$

- final classifier:
  - $H_{\text{final}}(x) = \text{sign} \left(\sum_t \alpha_t h_t(x) \right)$

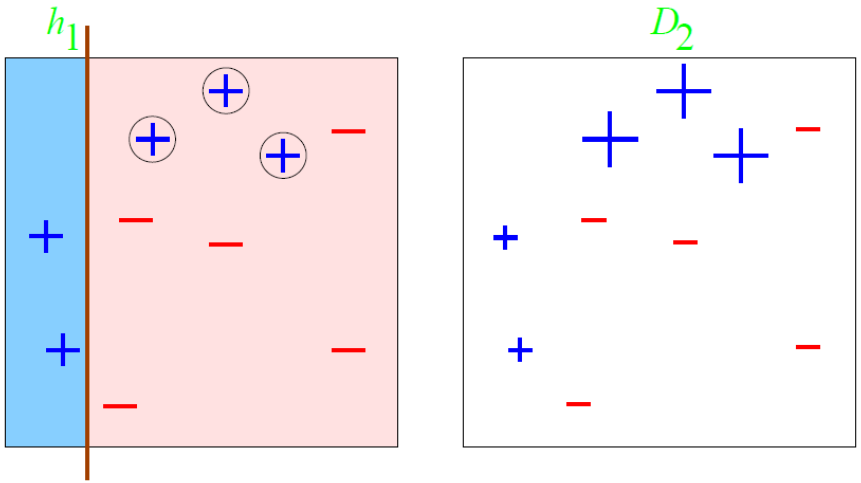
Toy Example

- Weak classifier (rule of thumb): vertical or horizontal half-planes (or decision stump)
- Let $y \in \{\pm 1\}$
- Uniform weights on all examples



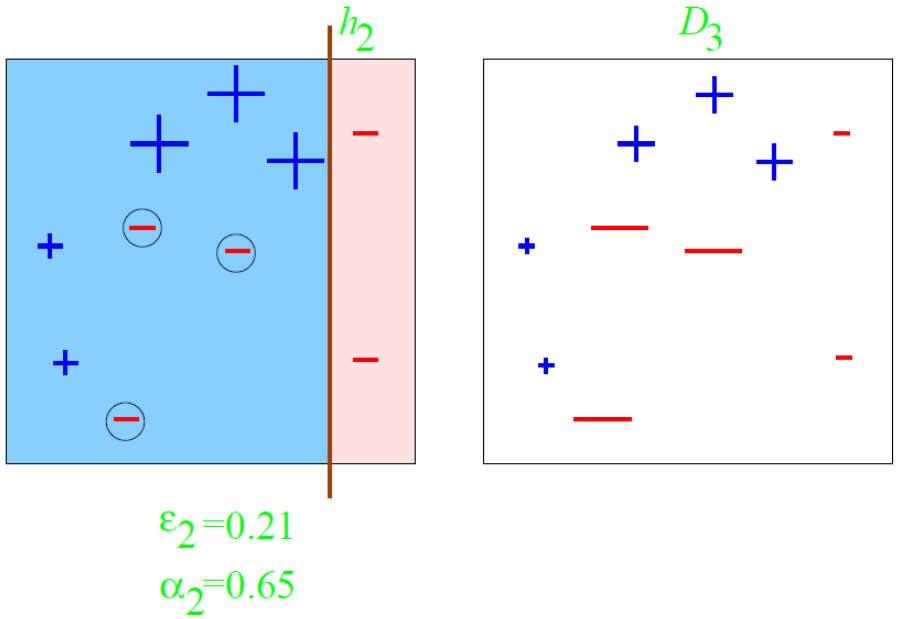
Boosting round 1

- Choose a decision stump (weak classifier)
- Some data points obtain higher weights because they are classified incorrectly



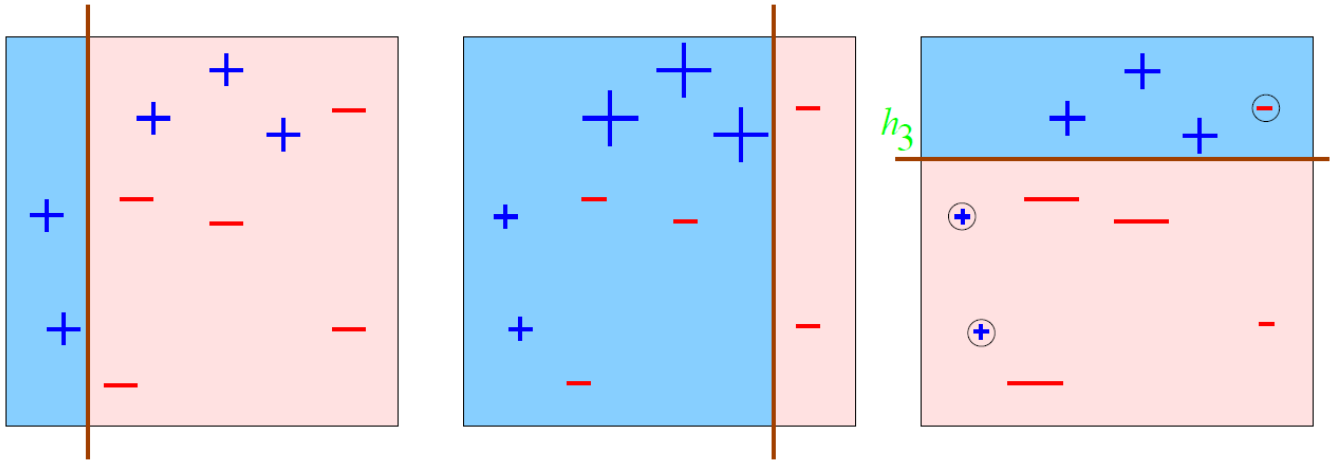
Boosting round 2

- Choose a new decision stump
- Reweight again. For incorrectly classified examples, weight increased



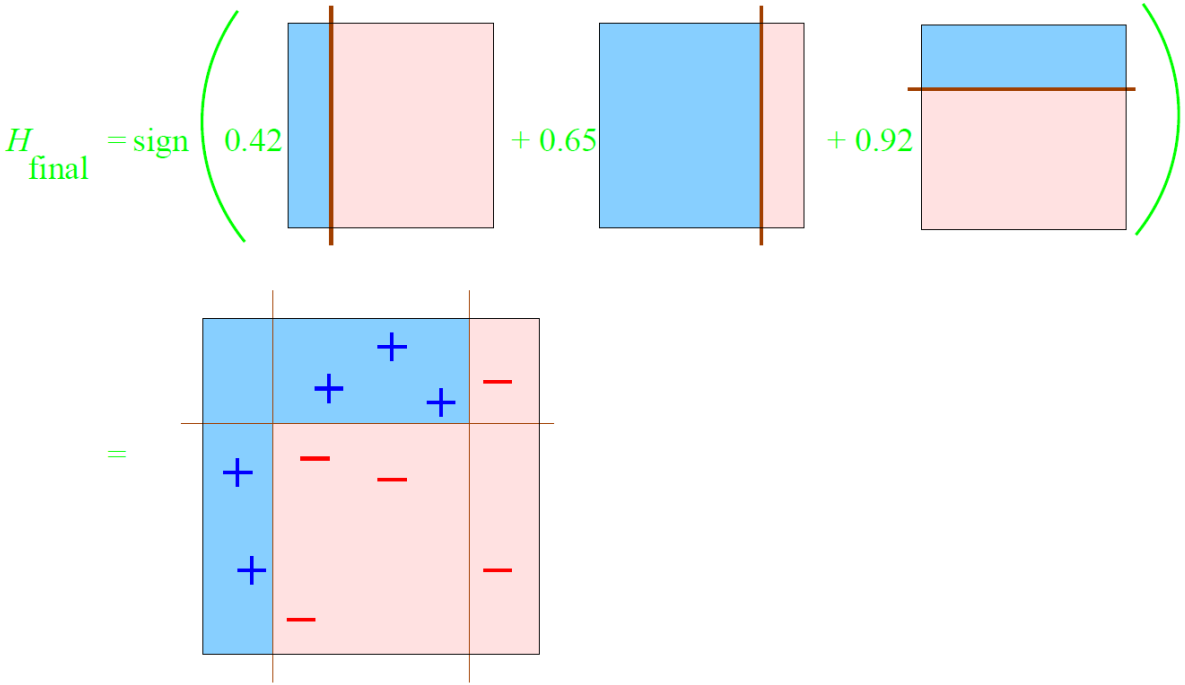
Boosting round 3

- Repeat the same process
- Now we have 3 classifiers



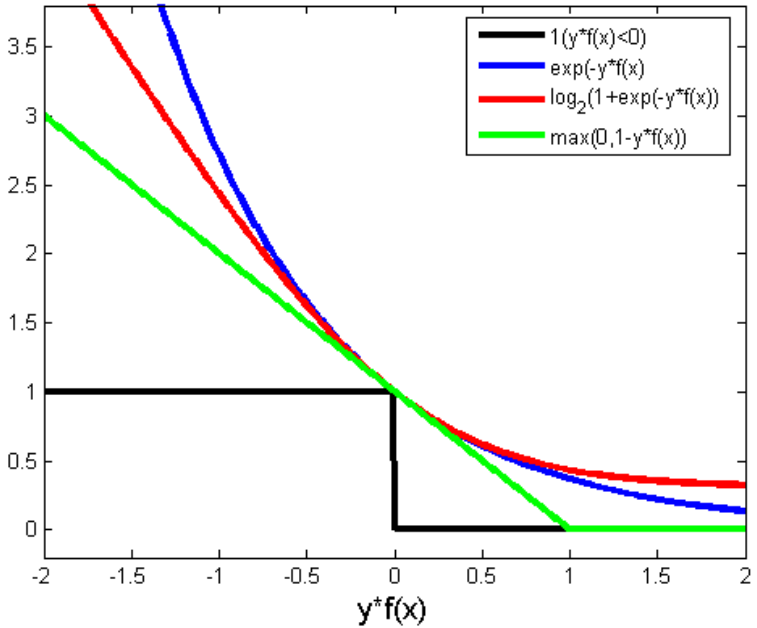
Boosting aggregate classifier

- Final classifier is weighted combination of weak classifiers



Loss functions

- Loss function: $l(y, f(x))$
- Eg., $\mathbb{I}\{y * f(x) < 0\}$



Understanding boosting: exponential loss

- Combined classifier

$$f_t(x) = \alpha_1 h_1(x;\theta_1) + \cdots + \alpha_t h_t(x;\theta_t)$$

- Exponential loss

$$\hat{L}(f) = \frac{1}{m}\sum_{i=1}^{m} \exp\left(-y^i f_t\left(x^i\right)\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m} \exp\left(-y^i\left(f_{t-1}\left(x^i\right) + \alpha_t h_t(x;\theta_t)_i\right)\right)$$

$$= \frac{1}{m}\sum_{i=1}^{m} \exp\left(-y^i f_{t-1}\left(x^i\right)\right)\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right)$$

Understanding boosting: exponential loss

- Define weighting on data point (we can prove the following by recursion)

$$D_t(i) = \exp\left(-y^i f_{t-1}\left(x^i\right)\right)/Z_t$$

- Then loss in the i-th iteration

$$\hat{L}(f) = \frac{1}{m}\sum_{i}^{m} D_t(i)\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right)\cdot Z_t$$

- The combined classifier f_{t-1} based on $t-1$ iterations defines a weighted loss criterion for the next simple classifier to add

- each training sample is weighted by its "classifiability" (or difficulty) seen by the classifier we have built so far

Adding $h(x;\theta_n)$ sequentially

- $\hat{L}(f) = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\exp\left(-y^i f_{t-1}\left(x^i\right)\right)}_{D_t(i)} \exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right) \cdot Z_t$
- At iteration t, Suppose $f_{t-1}(x)$ is already given to you
- Define weighting on data point $D_t(i)$, then

$$\hat{L}(f) = \frac{1}{m}\sum_{i}^{m} D_t(i) \exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right) \cdot Z_t$$

- Try to find α_t and h_t

Find h_t Linearization of loss function

- Assuming α_t is small, the empirical loss criterion reduces to

$$\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right) \approx 1 - y_i \alpha_t h_t\left(x^i;\theta_t\right)$$

- We could choose a new component classifier to optimize this weighted agreement

$$\frac{1}{m}\sum_i^m D_t(i)\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right)\cdot Z_t$$

$$\approx \frac{1}{m}\sum_i^m D_t(i)\left(1 - y^i \alpha_t h_t\left(x^i;\theta_t\right)\right)\cdot Z_t$$

$$= \frac{1}{m}\sum_i^m D_t(i) - \frac{1}{m}\alpha_t \sum_i^m D_t(i)\, y^i h_t\left(x^i;\theta_t\right)\cdot Z_t$$

Find h_t : a possible algorithm

- At stage t we find θ_t that maximize (or at least give a sufficiently high) weighted agreement:

$$\theta_t \leftarrow \text{argmax}_\theta \frac{1}{m} \sum_{i=1}^{m} D_t(i)\, y^i h_t\left(x^i; \theta\right)$$

- Note that $y^i \in \{\pm 1\}$, and $h_t(x^i) \in \{\pm 1\}$

- $y^i h_t\left(x^i; \theta\right) \in \{\pm 1\}$

Find α_t

Then we go back and find the "votes" α_t associated with the new classifier by minimizing the **original** weighted (exponential) loss

$$L(\alpha_t) = \frac{1}{m}\sum_{i}^{m} D_t(i)\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right)$$

Set derivative to 0 and solve for α_t

$$\frac{\partial L}{\partial \alpha_t} = -\frac{1}{m}\sum_{i}^{m} D_t(i)\exp\left(-y^i \alpha_t h_t\left(x^i;\theta_t\right)\right) y^i h_t\left(x^i;\theta_t\right) = 0$$

Find α_t

Note that $y^i \in \{\pm 1\}$, and $h_t(x^i) \in \{\pm 1\}$, $y^i h_t(x^i;\theta) \in \{\pm 1\}$

$$\frac{\partial L}{\partial \alpha_t} = -\frac{1}{m}\sum_{i=1}^{m} D_t(i)\exp\left(-y^i \alpha_t h_t(x^i;\theta_t)\right) y^i h_t(x^i;\theta_t)$$

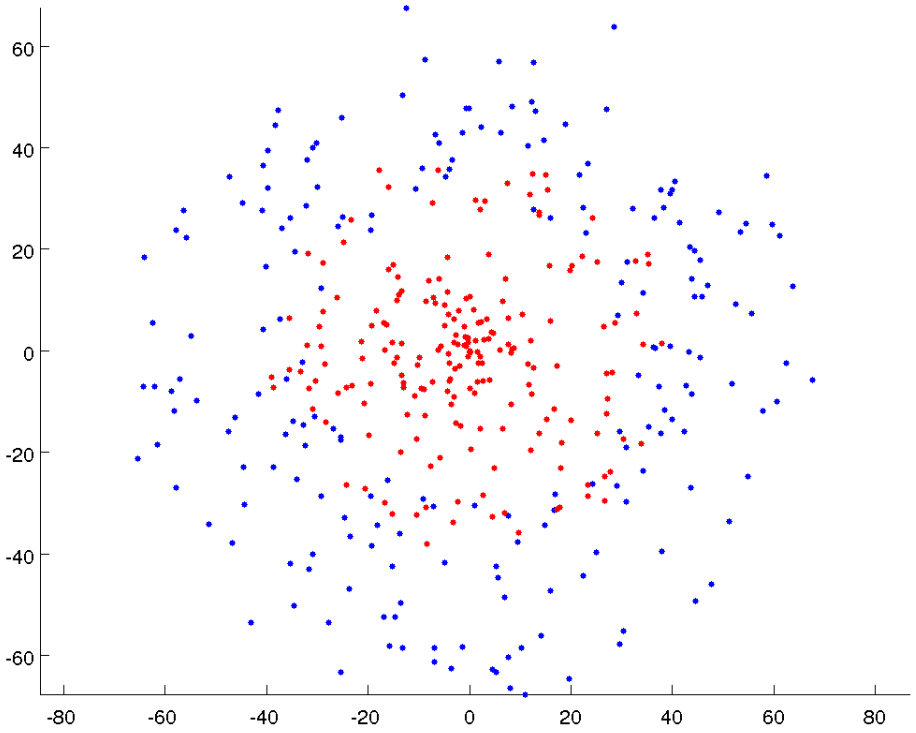
$$= -\frac{1}{m}\sum_{i:h_t(x^i)=y^i} D_t(i)\exp(-\alpha_t) + \frac{1}{m}\sum_{i:h_t(x^i)\neq y^i} D_t(i)\exp(\alpha_t)$$

$$= \frac{1}{m}(\epsilon_t - 1)\exp(-\alpha_t) + \frac{1}{m}\epsilon_t \exp(\alpha_t) = 0$$

$$\epsilon_t := \sum_{i=1}^{m} D_t(i)\, I\{y^i \neq h_t(x^i;\theta)\}$$

$$\Rightarrow \frac{1-\epsilon_t}{\epsilon_t} = \exp(2\alpha_t) \Rightarrow \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) = \alpha_t$$

Two rings dataset (demo)



Overall comparison

TABLE 10.1. *Some characteristics of different learning methods. Key:* ▲ *= good,* ◆ *=fair, and* ▼ *=poor.*

Characteristic	Neural Nets	SVM	Trees	MARS	k-NN, Kernels
Natural handling of data of "mixed" type	▼	▼	▲	▲	▼
Handling of missing values	▼	▼	▲	▲	▲
Robustness to outliers in input space	▼	▼	▲	▼	▲
Insensitive to monotone transformations of inputs	▼	▼	▲	▼	▼
Computational scalability (large N)	▼	▼	▲	▲	▼
Ability to deal with irrelevant inputs	▼	▼	▲	▲	▼
Ability to extract linear combinations of features	▲	▲	▼	▼	◆
Interpretability	▼	▼	◆	▲	▼
Predictive power	▲	▲	▼	◆	▲


SUMMARY