Number Theory Answer Key

Warm-up:

1. See our notes on SFFT (Lesson 4) at sem-amc-club.tk/Week4.pdf

   Solutions are $(0, 3), (1, 1), (3, 0)$

2. Two numbers are coprime if they share no factors other than 1. A good rule of thumb is that a and b are coprime if and only if there exist integers p and q such that $ap + bq = 1$ (Bezout's Lemma)
   - Coprime ($\gcd(2, 3) = 1$ and $\underline{-1} \cdot 2 + \underline{1} \cdot 3 = 1$)
   - Not coprime ($\gcd(6, 8) = 2$)
   - Coprime ($\gcd(1, 9) = 1$ and $\underline{1} \cdot 1 + \underline{0} \cdot 9 = 1$)
   - Not coprime ($gcd(0, 5) = 5$)
3. See Euclidean Algorithm on AoPS.

   Note that $a \cdot b = \gcd(a, b) \cdot \text{lcm}(a, b)$.

Exercises:

1. E
2. E
3. D
4. D
5. B
6. B
7. See solution at https://bit.ly/2sAJ3V5