

# Algebra

---

**Warm-up:** What are the formulas for

- a. Sum of an arithmetic series with  $n$  terms, with first term  $a_1$  and last term  $a_n$ ?
- b. Sum of a geometric series with  $n$  terms, with first term  $a_1$  and ratio  $r$ ?
- c. Sum of a geometric series with  $\infty$  terms, with first term  $a_1$  and ratio  $-1 < r < 1$ ?

## Exercises

1. If  $m$  men can do a job in  $d$  days, then  $m + r$  men can do the job in how many days?

(A)  $d + r$  (B)  $d - r$  (C)  $\frac{md}{m+r}$  (D)  $\frac{d}{m+r}$  (E)  $\frac{(m+r)d}{m}$

[Solution](#)

2. From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this 45 boys leave. There are then 5 girls for each boy. The number of girls at the start was

(A) 40 (B) 43 (C) 29 (D) 50 (E) 55

[Solution](#)

3. What is the sum of the reciprocals of the roots of the equation

$$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

(A)  $-\frac{2004}{2003}$  (B)  $-1$  (C)  $\frac{2003}{2004}$  (D)  $1$  (E)  $\frac{2004}{2003}$

[Solution](#)

4. Let  $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$ . What is  $a + b + c + d$ ?

(A)  $-5$  (B)  $-\frac{10}{3}$  (C)  $-\frac{7}{3}$  (D)  $\frac{5}{3}$  (E)  $5$

[Solution](#)

5. Find the value of  $x$  that satisfies

$$25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$$

(A) 2 (B) 3 (C) 5 (D) 6 (E) 9

[Solution](#)

6. There are 100 players in a single-elimination tennis tournament. Single-elimination means that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The tournament continues until only one player remains unbeaten. The total number of matches played is:

(A) prime (B) divisible by 2 (C) divisible by 5 (D) divisible by 7 (E) divisible by 11

[Solution](#)

7. Two non-zero real numbers  $a$ ,  $b$  satisfy  $ab = a - b$ . What is the value of

(A)  $-2$  (B)  $-\frac{1}{2}$  (C)  $\frac{1}{3}$  (D)  $\frac{1}{2}$  (E)  $2$

[Solution](#)

8. Al, Betty, and Claire split \$1000 among them to be invested in different ways. Each begins with a different amount. After one year, they have a total of \$1500. Betty and Claire have both doubled their money, whereas Al has managed to lose \$100. What was Al's original portion?

(A) \$250 (B) \$350 (C) \$400 (D) \$450 (E) \$500

[Solution](#)

9. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

(A) 250 (B) 300 (C) 350 (D) 400 (E) 500

[Solution](#)

10. The first four terms in an arithmetic sequence are  $x + y$ ,  $x - y$ ,  $xy$ , and  $x/y$  in that order. What is the fifth term?

(A)  $-\frac{15}{8}$  (B)  $-\frac{6}{5}$  (C) 0 (D)  $\frac{27}{20}$  (E)  $\frac{123}{40}$

[Solution](#)