

Number Theory Exercises

1. A six place number is formed by repeating a three place number; for example, 256256 or 678678. Any number of this form is always divisible by

(A) 7 only (B) 11 only (C) 13 only (D) 101 (E) 1001

[Solution](#)

2. The largest number by which the expression $n^3 - n$ is divisible for all possible integral values of n , is:

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

[Solution](#)

3. In the base ten number system the number 526 means $5 \times 10^2 + 2 \times 10 + 6$. In the Land of Mathesis, however, numbers are written in the base r . Jones purchases an automobile there for 440 monetary units (abbreviated m.u). He gives the salesman a 1000 m.u bill, and receives, in change, 340 m.u. The base r is:

(A) 2 (B) 5 (C) 7 (D) 8 (E) 12

[Solution](#)

4. The number of solutions in positive integers of $2x + 3y = 763$ is:

(A) 255 (B) 254 (C) 128 (D) 127 (E) 0

[Solution](#)

5. The number of positive integers less than 1000 divisible by neither 5 nor 7 is:

(A) 688 (B) 686 (C) 684 (D) 658 (E) 630

[Solution](#)

6. A rectangular floor measures a by b feet, where a and b are positive integers and $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the whole floor. How many possibilities are there for the ordered pair (a, b) ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

[Solution](#)

7. Prove that the fraction $\frac{21n + 4}{14n + 3}$ is irreducible for every natural number n .

Note: this is an IMO problem! Try it!

[Solution](#)