Combinatorics Answer Key

Warm-up:

1. Permutations/Combinations formulas:
   - i. n^k
   - ii. $nPk = \frac{n!}{(n-k)!}$
   - iii. $\binom{c}{k} = nCk = \frac{n!}{k!(n-k)!}$
   - iv. $\binom{n+k-1}{n-1} = \binom{n+k-1}{k}$
2. A one-is-to-one correspondence is a mapping between two sets such that every element from the first set is mapped to exactly one element from the second set, and vice versa. For instance, you can form a one-is-to-one correspondence between the sets $\{Su, M, T, W, Th, F, S\}$ and $\{1, 2, 3, 4, 5, 6, 7\}$ using the mapping function which takes a day and outputs where it is in the week. Another example of a one-is-to-one correspondence is used in the derivation of the combinations with repetitions formula, where we form a correspondence between combinations of ice cream in a bowl allowing repetitions and permutations of arrows and stars in a line.

   If there is a one is to one correspondence $A \leftrightarrow B$ between A and B, then the size of A and the size of B are equal ($|A| = |B|$). This means that we can make a one-is-to-one correspondence between a set that is more difficult to count and a set which is easier to count, and then simplify a counting problem.

Exercises:

1. D
2. D
3. A
4. D
5. C
6. D
7. C
8. C
9. E
10. B
11. A