LESSON 4 : REVIEW OF 2019 AMC 10A

- All problems are pulled from the 2019 AMC 10A that most of y'all took last week
- We will look over most of the questions and discuss strategies that could've been used to tackle them
- Make sure to take notes in class, but solutions written by the AoPS community are available for this test at: https://artofproblemsolving.com/wiki/index.php/2019_AMC_10A

Problem 2:

What is the hundreds digit of $(20! - 15!)?$

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

Problem 3:

Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is $n?$

$\textbf{(A) } 3 \qquad \textbf{(B) } 5 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 15$

Problem 4:

A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn$?$

$\textbf{(A) } 75 \qquad \textbf{(B) } 76 \qquad \textbf{(C) } 79 \qquad \textbf{(D) } 84 \qquad \textbf{(E) } 91$

Problem 5:

What is the greatest number of consecutive integers whose sum is $45?$

$\textbf{(A) } 9 \qquad \textbf{(B) } 25 \qquad \textbf{(C) } 45 \qquad \textbf{(D) } 90 \qquad \textbf{(E) } 120$

Problem 6:

For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

- a square
- a rectangle that is not a square
- a rhombus that is not a square
- a parallelogram that is not a rectangle or a rhombus
- an isosceles trapezoid that is not a parallelogram

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

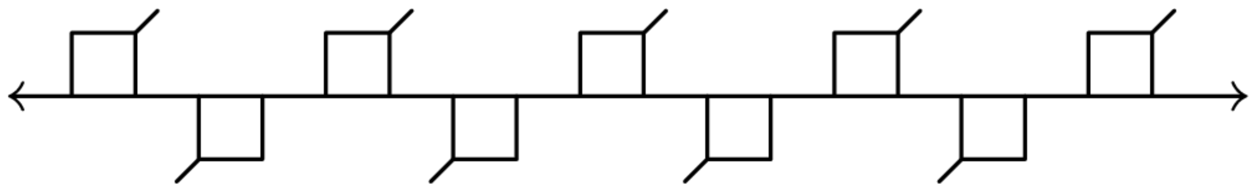
Problem 7:

Two lines with slopes $\dfrac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line $x+y=10?$

$\textbf{(A) } 4 \qquad \textbf{(B) } 4\sqrt{2} \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 8 \qquad \textbf{(E) } 6\sqrt{2}$

Problem 8:

The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point of line ℓ
- some translation in the direction parallel to line ℓ
- the reflection across line ℓ
- some reflection across a line perpendicular to line ℓ

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 9:

What is the greatest three-digit positive integer n for which the sum of the first n positive integers is $\underline{\mathit{not}}$ a divisor of the product of the first n positive integers?

$\textbf{(A)}\ 995 \qquad \textbf{(B)}\ 996 \qquad \textbf{(C)}\ 997 \qquad \textbf{(D)}\ 998 \qquad \textbf{(E)}\ 999$

Problem 10:

A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and the last tile, how many tiles does the bug visit?

$\textbf{(A) } 17 \qquad \textbf{(B) } 25 \qquad \textbf{(C) } 26 \qquad \textbf{(D) } 27 \qquad \textbf{(E) } 28$

Problem 11:

How many positive integer divisors of 201^9 are perfect squares or perfect cubes (or both)?

$\textbf{(A) } 32 \qquad \textbf{(B) } 36 \qquad \textbf{(C) } 37 \qquad \textbf{(D) } 39 \qquad \textbf{(E) } 41$

Problem 12:

Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, . . . , 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

$\textbf{(A) } \mu < d < M \qquad \textbf{(B) } M < d < \mu \qquad \textbf{(C) } d = M = \mu \qquad \textbf{(D) } d < M < \mu \qquad \textbf{(E) } d < \mu < M$

Problem 13:

Let ΔABC be an isosceles triangle with $BC = AC$ and $\angle ACB = 40^\circ$. Construct the circle with diameter $\overline{BC}$, and let D and E be the other intersection points of the circle with the sides $\overline{AC}$ and $\overline{AB}$, respectively. Let F be the intersection of the diagonals of the quadrilateral $BCDE$. What is the degree measure of $\angle BFC$?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 105 \qquad \textbf{(D)}\ 110 \qquad \textbf{(E)}\ 120$

Problem 14:

For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 21$

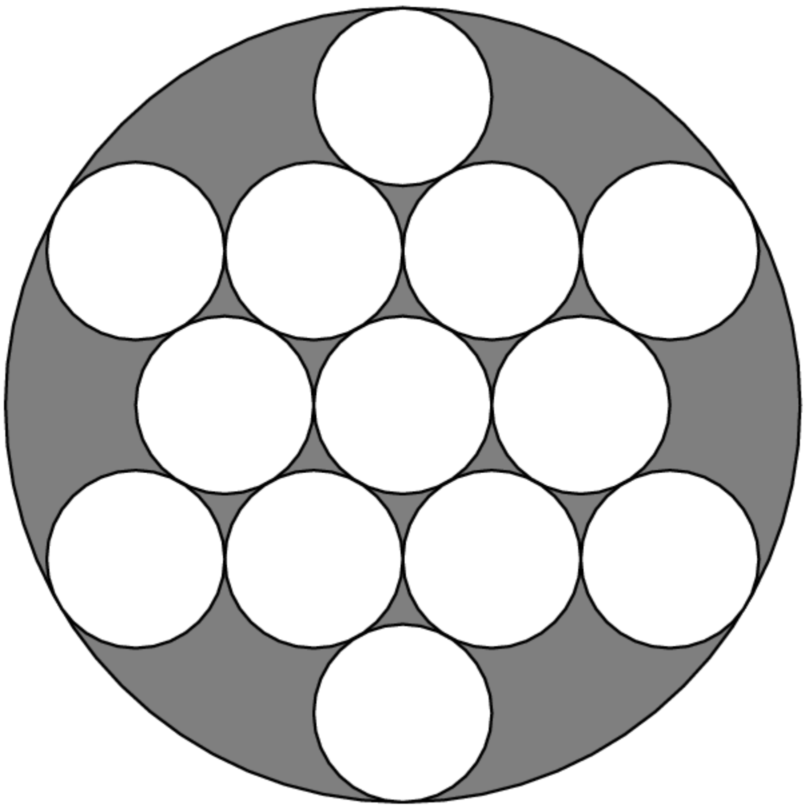
Problem 15:

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and $a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$ for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p+q$?

$\textbf{(A)}\ 2020 \qquad \textbf{(B)}\ 4039 \qquad \textbf{(C)}\ 6057 \qquad \textbf{(D)}\ 6061 \qquad \textbf{(E)}\ 8078$

Problem 16:

The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



$\textbf{(A)}\ 4\pi\sqrt{3} \qquad \textbf{(B)}\ 7\pi \qquad \textbf{(C)}\ \pi(3\sqrt{3}+2) \qquad \textbf{(D)}\ 10\pi(\sqrt{3}-1) \qquad \textbf{(E)}\ \pi(\sqrt{3}+6)$

Problem 17:

A child builds towers using identically shaped cubes of different color. How many different towers with a height 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 288 \qquad \textbf{(C)}\ 312 \qquad \textbf{(D)}\ 1,260 \qquad \textbf{(E)}\ 40,320$

Problem 19:

What is the least possible value of $(x+1)(x+2)(x+3)(x+4)+2019$ where x is a real number?

$\textbf{(A)}\ 2017 \qquad \textbf{(B)}\ 2018 \qquad \textbf{(C)}\ 2019 \qquad \textbf{(D)}\ 2020 \qquad \textbf{(E)}\ 2021$

Finally, my favorite question I found while looking over the test. If you've taken BC and were there for the Vieta's lecture try this one as a challenge!

Problem 24:

Let p, q, and r be the distinct roots of the polynomial $x^3 - 22x^2 + 80x - 67$. It is given that there exist real numbers A, B, and C such that

$$\frac{1}{s^3 - 22s^2 + 80s - 67} = \frac{A}{s-p} + \frac{B}{s-q} + \frac{C}{s-r}$$ for all $s \not\in \{p,q,r\}$. What is $\tfrac1A+\tfrac1B+\tfrac1C$?

$\textbf{(A)}\ 243 \qquad \textbf{(B)}\ 244 \qquad \textbf{(C)}\ 245 \qquad \textbf{(D)}\ 246 \qquad \textbf{(E)}\ 247$