AMC Club Holiday Review 2019-2020

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“Students, warm-up! ” - Mr. Newton

How to use this review

This review contains several problems from various areas of math tested on the AMC exam.

Before beginning the problems in a section, look back at any notes you may have on that subject. Check our website (sem-amc-club.tk) to view old notes packets. For geometry, you may want to skim your notes from your actual geometry class last year.

Try out the warm-up in each section before starting the exercises, and make sure to check your answers. The warm-up contains some important formulas that you should either memorize or be able to derive in a few seconds. These are not the only formulas you will need to know however.

Attempt to solve each problem before looking at the solution. If you are stumped on a problem, take a break and look at it again. Don't jump to reading the solution.

Solve each problem on scratch paper, and be organized with your work, so you can stay organized on the real test.

Quick tips about the AMC 10

The Test: The AMC is made of 25 multiple-choice (5-answer) questions covering a range of topics from algebra, combinatorics, geometry, and number theory. You do not get a calculator on the exam.

Timing: You get 75 minutes for 25 questions, however it is unlikely that you will answer all 25 questions. Budget your time based on your ability (look at last year's scores to see how many questions you answered).

Scoring: You earn

- 6 points for each correct answer
- 1.5 points for each skipped question
- 0 points for each incorrect answer

This means you should only guess if you have eliminated at least two answers.

Algebra

Warm-up: What are the formulas for

- a. Sum of an arithmetic series with n terms, with first term a_1 and last term a_n?
- b. Sum of a geometric series with n terms, with first term a_1 and ratio r?
- c. Sum of a geometric series with ∞ terms, with first term a_1 and ratio $-1 < r < 1$?

Exercises

1. If m men can do a job in d days, then $m + r$ men can do the job in how many days?

   $$\text{(A) } d + r \quad \text{(B) } d - r \quad \text{(C) } \frac{md}{m+r} \quad \text{(D) } \frac{d}{m+r} \quad \text{(E) } \frac{(m+r)d}{m}$$

   Solution

2. From a group of boys and girls, 15 girls leave. There are then left two boys for each girl. After this 45 boys leave. There are then 5 girls for each boy. The number of girls at the start was

   $$\text{(A) } 40 \quad \text{(B) } 43 \quad \text{(C) } 29 \quad \text{(D) } 50 \quad \text{(E) } 55$$

   Solution

3. What is the sum of the reciprocals of the roots of the equation

   $$\frac{2003}{2004}x + 1 + \frac{1}{x} = 0?$$

   $$\text{(A) } -\frac{2004}{2003} \quad \text{(B) } -1 \quad \text{(C) } \frac{2003}{2004} \quad \text{(D) } 1 \quad \text{(E) } \frac{2004}{2003}$$

   Solution

4. Let $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$. What is $a + b + c + d$?

   $$\text{(A) } -5 \quad \text{(B) } -\frac{10}{3} \quad \text{(C) } -\frac{7}{3} \quad \text{(D) } \frac{5}{3} \quad \text{(E) } 5$$

   Solution

5. Find the value of x that satisfies

   $$25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}$$

   $$\text{(A) } 2 \quad \text{(B) } 3 \quad \text{(C) } 5 \quad \text{(D) } 6 \quad \text{(E) } 9$$

   Solution

6. There are 100 players in a single-elimination tennis tournament. Single-elimination means that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The tournament continues until only one player remains unbeaten. The total number of matches played is:

   $$\text{(A) prime} \quad \text{(B) divisible by 2} \quad \text{(C) divisible by 5} \quad \text{(D) divisible by 7} \quad \text{(E) divisible by 11}$$

   Solution

7. Two non-zero real numbers $a,\ b$ satisfy $ab = a - b$. What is the value of

   $$\text{(A) } -2 \quad \text{(B) } -\frac{1}{2} \quad \text{(C) } \frac{1}{3} \quad \text{(D) } \frac{1}{2} \quad \text{(E) } 2$$

   Solution

8. Al, Betty, and Claire split $\$1000$ among them to be invested in different ways. Each begins with a different amount. After one year, they have a total of $\$1500$. Betty and Claire have both doubled their money, whereas Al has managed to lose $\$100$. What was Al's original portion?

$$\mathrm{(A)}\ \$250 \qquad \mathrm{(B)}\ \$350 \qquad \mathrm{(C)}\ \$400 \qquad \mathrm{(D)}\ \$450 \qquad \mathrm{(E)}\ \$500$$

Solution

9. Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

$$\mathrm{(A)}\ 250 \qquad \mathrm{(B)}\ 300 \qquad \mathrm{(C)}\ 350 \qquad \mathrm{(D)}\ 400 \qquad \mathrm{(E)}\ 500$$

Solution

10. The first four terms in an arithmetic sequence are $x+y$, $x-y$, xy, and x/y in that order. What is the fifth term?

$$\mathrm{(A)}\ -\frac{15}{8} \qquad \mathrm{(B)}\ -\frac{6}{5} \qquad \mathrm{(C)}\ 0 \qquad \mathrm{(D)}\ \frac{27}{20} \qquad \mathrm{(E)}\ \frac{123}{40}$$

Solution

Algebra Answer Key

Warm-up:

- a. $n \cdot (\frac{a_1+a_n}{2})$
- b. $a_1 \cdot \frac{1-r^n}{1-r}$
- c. $a_1 \cdot \frac{1}{1-r}$

Exercises:

1. C
2. A
3. B
4. B
5. B
6. E
7. E
8. C
9. C
10. D

Combinatorics

Warm-up: What are the formulas for (n items, k slots)

- a. Permutations with reputation
- b. Permutations without reputation
- c. Combinations without repetition
- d. Combinations with repetition

Exercises

1. A fair die is rolled six times. The probability of rolling at least a five at least five times is

   $$\text{(A) } \frac{2}{729} \quad \text{(B) } \frac{3}{729} \quad \text{(C) } \frac{12}{729} \quad \text{(D) } \frac{13}{729} \quad \text{(E) None of these}$$

   Solution

2. Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

   $$\text{(A) } \frac{1}{8} \quad \text{(B) } \frac{1}{4} \quad \text{(C) } \frac{3}{8} \quad \text{(D) } \frac{1}{2} \quad \text{(E) } \frac{3}{4}$$

   Solution

3. Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

   $$\text{(A) } \frac{1}{9} \quad \text{(B) } \frac{1}{8} \quad \text{(C) } \frac{1}{6} \quad \text{(D) } \frac{2}{11} \quad \text{(E) } \frac{1}{5}$$

   Solution

4. Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

   $$\text{(A) } \frac{1}{4} \quad \text{(B) } \frac{3}{8} \quad \text{(C) } \frac{4}{7} \quad \text{(D) } \frac{5}{7} \quad \text{(E) } \frac{3}{4}$$

   Solution

5. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

   $$\text{(A) } 202 \quad \text{(B) } 223 \quad \text{(C) } 224 \quad \text{(D) } 225 \quad \text{(E) } 234$$

   Solution

6. A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

   $$\text{(A) } 10^4 \cdot 26^2 \quad \text{(B) } 10^3 \cdot 26^3 \quad \text{(C) } 5 \cdot 10^4 \cdot 26^2$$
   $$\text{(D) } 10^2 \cdot 26^4 \quad \text{(E) } 5 \cdot 10^3 \cdot 26^3$$

   Solution

7. Henry's Hamburger Haven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties and any collection of condiments. How many different kinds of hamburgers can be ordered?

$$\text{(A) } 24 \quad \text{(B) } 256 \quad \text{(C) } 768 \quad \text{(D) } 40,320 \quad \text{(E) } 120,960$$

Solution

8. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park?

$$\text{(A) } \frac{11}{20} \quad \text{(B) } \frac{4}{7} \quad \text{(C) } \frac{81}{140} \quad \text{(D) } \frac{3}{5} \quad \text{(E) } \frac{17}{28}$$

Solution

9. Compute the number of 7-digit positive integers that start <u>or</u> end (or both) with a digit that is a (nonzero) composite number.

$$\text{(A) } 3 \cdot 10^6 \quad \text{(B) } 6 \cdot 10^6 \quad \text{(C) } 9 \cdot 10^6 \quad \text{(D) } 10 \cdot 10^6 \quad \text{(E) } 12 \cdot 10^6$$

Solution

10. Each of 2010 boxes in a line contains a single red marble, and for $1 \leq k \leq 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

$$\text{(A) } 45 \quad \text{(B) } 63 \quad \text{(C) } 64 \quad \text{(D) } 201 \quad \text{(E) } 1005$$

Solution

Combinatorics Answer Key

Warm-up:

- a. n^k
- b. $nPk = \frac{n!}{(n-k)!}$
- c. $\binom{c}{k} = nCk = \frac{n!}{k!(n-k)!}$
- d. $\binom{n+k-1}{n-1} = \binom{n+k-1}{k}$

Exercises:

1. D
2. A
3. D
4. C
5. D
6. C
7. C
8. E
9. B
10. A