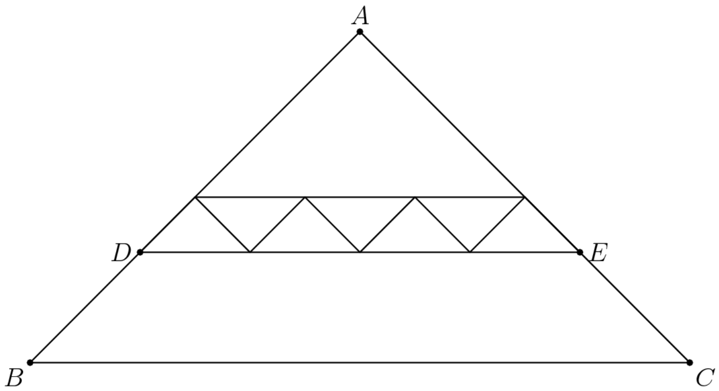
Geometry Exercises

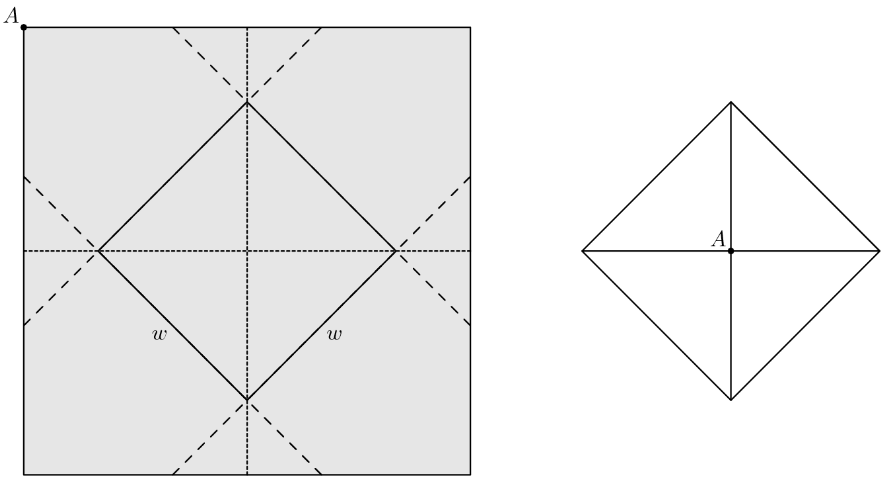
1. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?



$$\textbf{(A) } 16 \qquad \textbf{(B) } 18 \qquad \textbf{(C) } 20 \qquad \textbf{(D) } 22 \qquad \textbf{(E) } 24$$

Solution

2. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?



$$\textbf{(A) } 2(w+h)^2 \qquad \textbf{(B) } \frac{(w+h)^2}{2} \qquad \textbf{(C) } 2w^2+4wh \qquad \textbf{(D) } 2w^2 \qquad \textbf{(E) } w^2h$$

Solution

3. A circle has a chord of length 10, and the distance from the center of the circle to the chord is 5. What is the area of the circle?

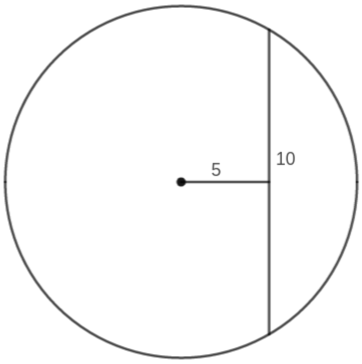


Diagram by Arjun

$$\text{(A) } 25\pi \quad \text{(B) } 50\pi \quad \text{(C) } 75\pi \quad \text{(D) } 100\pi \quad \text{(E) } 125\pi$$

Solution

4. Let $O = (0,0)$ be the origin. Points A and B are selected on the graph of $y = -\frac{1}{2}x^2$ so that triangle ABO is equilateral. Find AB.

$$\text{(A) } 2 \quad \text{(B) } 2\sqrt{3} \quad \text{(C) } 4 \quad \text{(D) } 4\sqrt{3} \quad \text{(E) } 4\sqrt{5}$$

Solution

5. The lengths of the sides of a triangle with positive area are $\log_{10} 12$, $\log_{10} 75$, and $\log_{10} n$, where n is a positive integer. Find the number of possible values for n.

Hint: you will need to use Triangle Inequality and some Log Properties

$$\text{(A) } 642 \quad \text{(B) } 813 \quad \text{(C) } 893 \quad \text{(D) } 900 \quad \text{(E) } 917$$

Solution

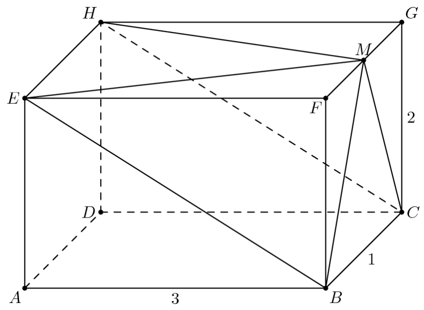
6. Side $\overline{AB}$ of $\triangle ABC$ has length 10. The angle bisector of A meets $\overline{BC}$ at D, and $CD = 3$. The set of all possible values of AC is an open interval (m,n). What is $m+n$?

Hint: you will need to use Triangle Inequality and Angle Bisector Theorem

$$\text{(A) } 16 \quad \text{(B) } 17 \quad \text{(C) } 18 \quad \text{(D) } 19 \quad \text{(E) } 20$$

Solution

7. In the rectangular parallelepiped shown, $AB = 3,\ BC = 1,\ CG = 2$. Point M is the midpoint of FG. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?



$$\textbf{(A) } 1 \qquad \textbf{(B) } \frac{4}{3} \qquad \textbf{(C) } \frac{3}{2} \qquad \textbf{(D) } \frac{5}{3} \qquad \textbf{(E) } 2$$

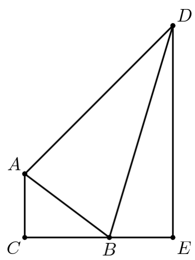
Solution

8. Right triangle $\triangle ABC$ has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

$$\textbf{(A) } 5 \qquad \textbf{(B) } 9 \qquad \textbf{(C) } 12 \qquad \textbf{(D) } 13 \qquad \textbf{(E) } 15$$

Solution

9. Triangle ABC has a right angle at C, $AC = 3$ and $BC = 4$. Triangle ABD has a right angle at A and $AD = 12$. Points C and D are on opposite sides of $\overline{AB}$. The line through D parallel to $\overline{AC}$ meets $\overline{CB}$ extended at E. If $\frac{DE}{DB} = \frac{m}{n}$, where m and n are relatively prime positive integers, then $m + n =$



$$\textbf{(A) } 25 \qquad \textbf{(B) } 128 \qquad \textbf{(C) } 153 \qquad \textbf{(D) } 243 \qquad \textbf{(E) } 256$$

Solution (look at Solution 2, especially note the author!)

10. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

$$\textbf{(A) } \sqrt{2} \qquad \textbf{(B) } \frac{3}{2} \qquad \textbf{(C) } \frac{5}{3} \qquad \textbf{(D) } \sqrt{3} \qquad \textbf{(E) } 2$$

Solution