Portfolio Optimization Using Financial Models and Machine Learning

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1. Introduction

Portfolio optimization is a crucial process for investors that involves maximizing returns while minimizing risks. However, in today's constantly changing financial markets, traditional portfolio optimization methods often struggle to keep up with dynamic conditions. This frequently leads to less-than-ideal investment outcomes, highlighting the need for more adaptive and sophisticated approaches that can bridge the gap between financial theory and practical implementation.

To address this need, our project aims to leverage advanced financial models and machine learning techniques. By doing so, we can provide investors with valuable insights to help them reshape their investment strategies and navigate the complexities of modern financial landscapes more effectively. Our approach to portfolio optimization builds on established theories such as Markowitz's Modern Portfolio Theory and Black-Litterman, while also incorporating machine learning algorithms to provide a more adaptive approach.

2. Data

The project utilizes historical financial data obtained from Yahoo Finance We have gathered historical data for a diverse set of assets, including popular stocks such as AAPL (Apple Inc.), MSFT (Microsoft Corporation), AMZN (Amazon.com Inc.), as well as others spanning various sectors. We retrieved adjusted close prices for each ticker, covering a timeframe from three years prior to the current date. Upon obtaining the adjusted close prices, we computed log returns, a crucial step in understanding the performance of each asset over time. These log returns serve as the basis for calculating key metrics such as mean returns and covariance matrices, essential components in portfolio optimization. We also fetched risk-free rates from the Federal Reserve Economic Data (FRED) API, enabling us to incorporate the time value of money into our optimization models.

3. Methods

Low-Risk Goal: Implemented the Markowitz model, also known as mean-variance optimization, which is based on the principle of diversification. This approach aims to construct portfolios that offer the highest expected return for a given level of risk. By considering historical returns, volatility, and covariance matrices of assets, the Markowitz model identifies the optimal allocation of assets to minimize portfolio risk while maximizing returns. We utilized the Sequential Least Squares Programming (SLSQP) optimization method to solve the Markowitz optimization problem, ensuring robust and efficient portfolio construction.

Medium-Risk Goal: Recognizing the importance of achieving a balance between risk and return, we iteratively tested multiple optimization methods to identify the one that maximized the Sharpe ratio, a key measure of risk-adjusted return. The Conjugate Gradient (CG) method, an iterative approach for unconstrained optimization, was employed to efficiently navigate the optimization landscape. Additionally, the Nelder-Mead algorithm, a direct search method that does not require derivatives, provided a robust alternative for optimizing portfolios without relying on gradient information. Limited-memory BFGS (L-BFGS-B) optimization, which maintains an approximation of the inverse Hessian matrix to navigate towards the optimal solution within bounded constraints, offered versatility and efficiency in optimizing portfolios. Trust Region Constrained optimization (trust-constr) and the Powell method, derivative-free optimization algorithms, were also considered for their ability to handle constraints and efficiently explore the solution space, respectively.

Among the various approaches tested, the Conjugate Gradient (CG) method emerged as the top performer in terms of maximizing the Sharpe ratio, a key metric for assessing risk-adjusted returns. The CG method, known for its efficiency in navigating the optimization landscape, proved to be instrumental in constructing portfolios that strike a balance between risk and return. By iteratively refining portfolio allocations, the CG optimization approach facilitated the generation of robust investment strategies tailored to medium-risk objectives.

High-Risk Goal: In scenarios where historical data alone may not accurately capture future expectations, the Black-Litterman model offers a unique approach to portfolio optimization. By blending quantitative market data with qualitative investor sentiment, this model adjusts portfolio allocations to reflect updated views on asset returns and correlations. The Black-Litterman model provides a systematic framework for incorporating subjective beliefs into portfolio optimization, enhancing decision-making in dynamic market environments.

Fig. 1: Black-Litterman model equation

$$\mu_{BL} = (\tau\Sigma)^{-1}(\tau\Sigma)^{-1}(\Pi + \Omega\tau)^{-1}\Pi + (\tau\Sigma)^{-1}$$

- μ_{BL}: Expected returns vector based on the Black-Litterman model.
- τ: Scaling factor reflecting the confidence in the equilibrium returns.
- Σ: Covariance matrix of asset returns.
- Π: Equilibrium excess returns vector.
- Ω: Diagonal matrix of the uncertainty in the investor's views.

4. Results

Low Risk Outcome:
The application of the Markowitz model with SLSQP optimization yielded promising results for portfolios targeting low-risk. With a return of **40%** and portfolio volatility of **12%**, this approach enabled the construction of portfolios with relatively stable returns while minimizing risk exposure. (See Fig. 2)

Medium Risk Outcome:
For portfolios aiming for moderate risk levels, the utilization of the Markowitz model with CG optimization proved highly effective. Achieving an impressive return of **101%** with a portfolio volatility of **45%**, this method demonstrated its ability to generate significant returns while managing risk within acceptable bounds. (See Fig. 3)

High Risk Outcome:
Despite the incorporation of qualitative investor sentiment into the optimization process, the outcomes of the high-risk portfolio using the Black-Litterman model were less impressive compared to the low and medium-risk portfolios. The return of **17%** with a portfolio volatility of **22%** indicates a less favorable risk-return profile in comparison. This discrepancy underscores the challenges associated with integrating subjective views into the investment decision-making process. While the Black-Litterman model offers valuable insights by blending quantitative data with qualitative inputs, its effectiveness in high-risk scenarios may be limited by market dynamics' inherent complexity and variability. (See Fig. 4, Fig. 5)

5. Conclusion

In conclusion, this project demonstrates the effectiveness of various portfolio optimization methods in achieving different risk-level goals. By leveraging financial models and machine learning techniques, investors can tailor their portfolios to meet their specific objectives while effectively managing risk. The outcomes highlight the importance of selecting appropriate optimization methods based on the desired risk-return profile, ultimately enhancing investment decision-making processes.

Appendix

Fig. 2: Sharpe ratio (left) and Portfolio weights (right) using Markowitz model with SLSQP Optimization

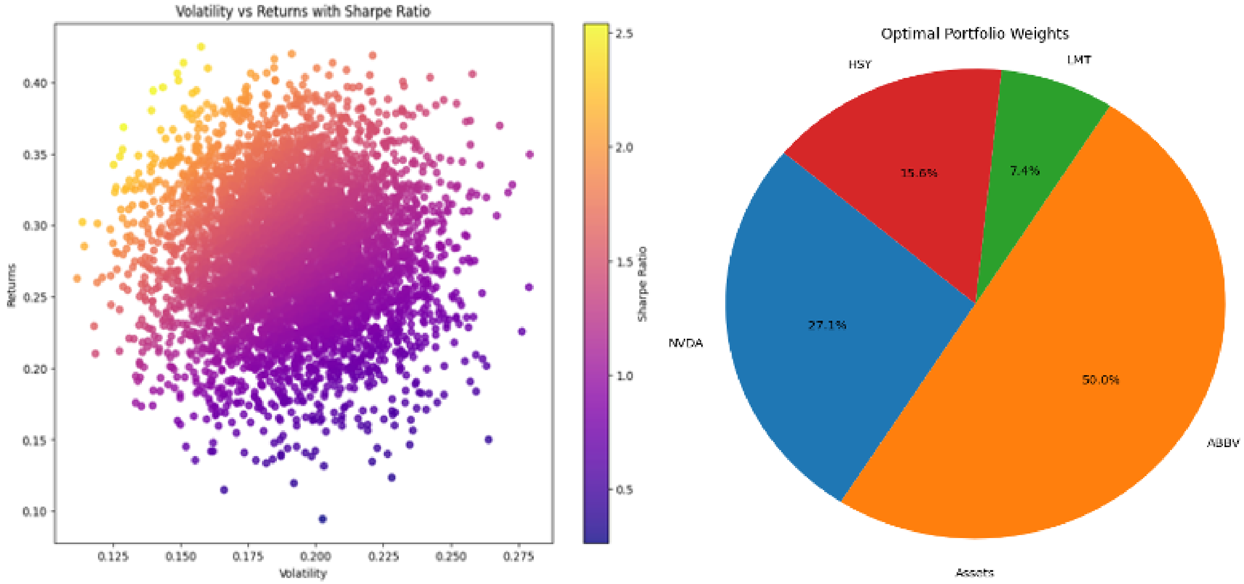

Fig. 3: Sharpe ratio (left) and Portfolio weights (right) using Markowitz model with CG Optimization

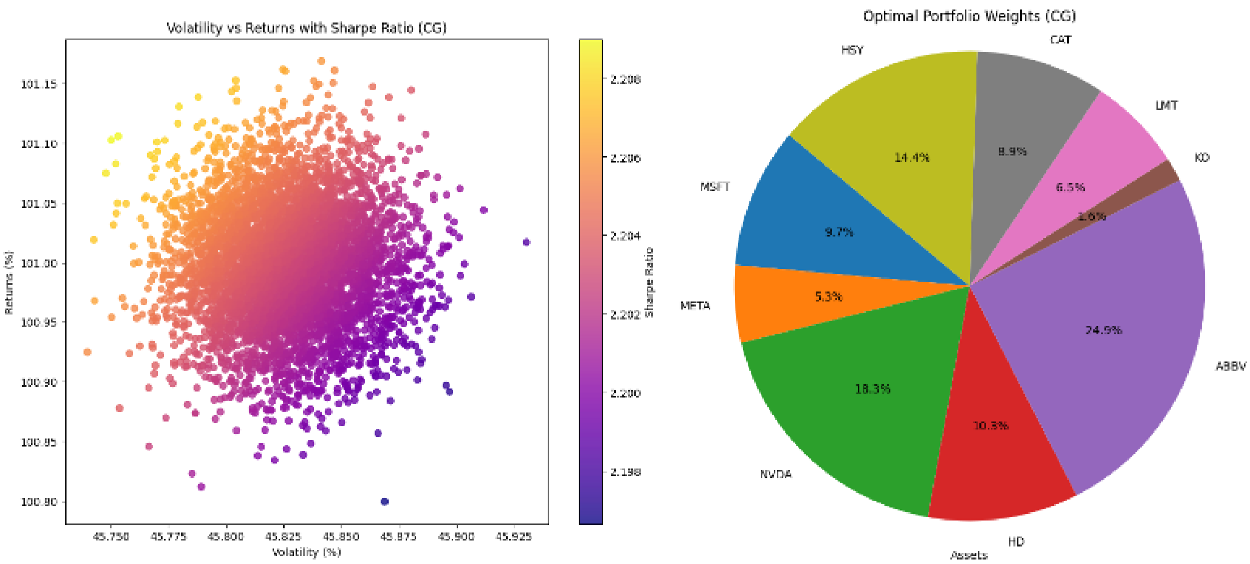

Fig. 4: Prior, Posterior, Views returns from Black-Litterman model

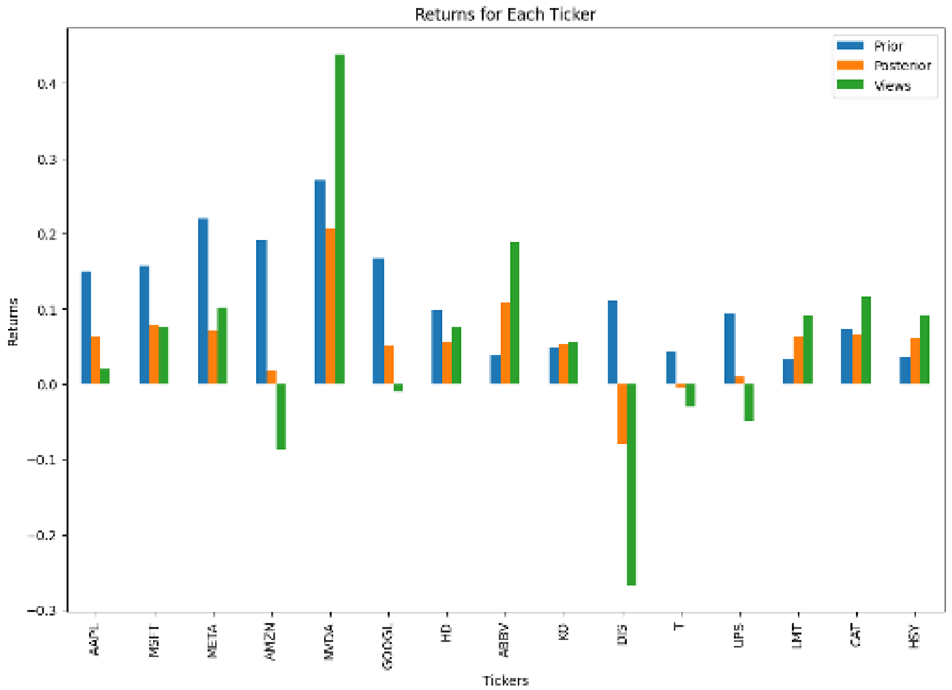

Fig. 5: Portfolio weights using Black-Litterman model