MUSIC Algorithm for Estimation of Parameters of Signals in Power System

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***Abstract*—This paper presents a method for the estimation of frequencies of spectral components in electric signals. Spectral components are an important factor for the estimation of power quality, i.e. *Total Harmonic Distortion* THD value. Parameters of very common switching transients were determined and analyzed using the MUSIC algorithm, representing a parametric method. Proposed method enabled accurate estimation of spectral components of simulated and measured power system signals, even in the presence of noise.**

***Keywords*—switching transients, power quality, MUSIC algorithm**

I. INTRODUCTION

The non-parametric methods are a wide group of methods including the wavelet transform, S transform, SFTF, Wigner-Wille transform and many more. Applying them requires no explicate knowledge of the signal model itself.

An opposite group is parametric methods. A parametric approach is substantially different as the a priori knowledge of the signal model is required. The Prony algorithm and SVD based methods are useful and commonly applied for PQ problems.

Waveforms of common distortions have spectral components in narrow bands or at discrete frequencies. Those are harmonic or interharmonic frequencies.

A sinusoidal model is most suitable for the characterisation of harmonics and interharmonics with a parametric method like MUSIC.

II. SINUSOIDAL MODEL OF A SIGNAL

A discrete time signal of length L has a model with K sinusoidal components buried in noise

$$x[n] = \sum_{k=1}^{K} a_k \cos(n\omega_k + \phi_k) + w[n] \tag{1}$$

where $a_k \geq 0$ is the amplitude, ϕ_k is the initial phase angle, $\omega_k = 2\pi f_k$ is the harmonic or interharmonic frequency and K the total number of sinusoids.

In this model the amplitude and frequency are assumed to be deterministic and unknown. The initial phase angle ϕ_k is random and uniformly distributed in $[-\pi, \pi]$.

An equivalent expression for the signal given in (1) is based on complex exponentials buried in noise, so called *harmonic model*

$$x[n] = \sum_{k=1}^{K} \underline{A}_k e^{jn\omega_k} + w[n] \tag{2}$$

where $A_k = |\underline{A}_k| e^{jn\omega_k}$ is the complex amplitude of the kth harmonic signal component.

If the voltage or current in a power system contains only odd harmonics beside the fundamental, then a signal model of the order K contains only *K-1* odd harmonicas beside the fundamental. On the contrary, the same model has *K-1* even harmonics and the fundamental if there are only even harmonics present in the power system.

In the signal model (2) the range of frequencies $\omega_k = 2\pi f_k$ is $[-\pi, \pi]$ therefore can be located at any nonharmonic frequency. There is no assumption about the frequency being a multiply of the fundamental component.

The frequency resolution of a method used for model's parameter estimation is quite high. The biggest advantage is the possibility to identify interharmonics. Anyway, the number of harmonics determined by the model order K must be known to avoid erroneous results.

III. MUSIC ALGORITHM

The MUSIC is a noise subspace method [1]. For a given parametric model (1) the MUSIC method computes corresponding frequencies and amplitudes of spectral components in the signal [2]. Whereas the frequencies are not necessarily harmonics, i.e. the integer multiplies of the fundamental.

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The autocorrelation matrix $\mathbf{R}_x$ of the signal $x[n]$ is a result of an operation on the signal samples

$$\hat{\mathbf{R}}_x = \frac{1}{N}\mathbf{X}^H\mathbf{X} \tag{3}$$

It is assumed that the signal $x[n]$ given in has the length $L = N + M - 1$. The autocorrelation matrix $\mathbf{R}_x$ has the size $M \times M$. The matrix $\mathbf{X}$ with the dimensions $N \times M$ is given as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}^T(0) \\ \mathbf{x}^T(1) \\ \vdots \\ \mathbf{x}^T(N-1) \end{bmatrix} = \begin{bmatrix} x(0) & x(1) & \cdots & x(M-1) \\ x(1) & x(2) & \cdots & x(M) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-1) & x(N) & \cdots & x(N+M-2) \end{bmatrix} \tag{4}$$

The superscript $(\ )^H$ denotes the Hermitian operator.

Putting (2) into (4) results in

$$\hat{\mathbf{R}}_x = \mathbf{R}_s + \mathbf{R}_w = \mathbf{E}\mathbf{P}\mathbf{E}^H + \sigma_w^2\mathbf{I} \tag{5}$$

where

$$\mathbf{E} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \cdots \quad \mathbf{e}_k] \tag{6}$$

$$\mathbf{P} = diag\left\{|\underline{A}_1|^2 \quad |\underline{A}_2|^2 \quad \cdots \quad |\underline{A}_K|^2\right\} \tag{7}$$

The eigenvectors of $\mathbf{R}_s$ are

$$\mathbf{e}_l = \begin{bmatrix} 1 & e^{j\omega_l} & \cdots & e^{j(M-1)\omega_l} \end{bmatrix}^T \tag{8}$$

and $l = 1, 2, ..., K$.

The signal and noise subspace can be introduced. Given is $\hat{\mathbf{R}}_x$ of full rank *M*. The eigenvalues of $\hat{\mathbf{R}}_x$ are ordered decreasingly and the corresponding eigenvectors are $s_1, s_2, ..., s_M$, therefore

$$\hat{\mathbf{R}}_x\mathbf{s}_i = \lambda_i\mathbf{s}_i \tag{9}$$

The eigenvectors represent two groups. *K* eigenvectors matching the *K* biggest eigenvalues belong to the signal subspace. The rest of eigenvectors *(M-K)* belongs to the noise subspace. In the MUSIC method the noise subspace is used to determine the unknown harmonic frequencies ω_k . For that the pseudospectrum is computed

$$P_{music}\left(e^{j\omega}\right) = \frac{1}{\sum_{i=K+1}^{M}\left|\mathbf{e}^H\mathbf{s}_i\right|^2} \tag{10}$$

where $\mathbf{s}_i$ ($i = K+1, ..., M$) are eigenvectors associated with the noise subspace and orthogonal to the signal eigenvector $\mathbf{e}_l = \begin{bmatrix} 1 & e^{j\omega_l} & \cdots & e^{j(M-1)\omega_l} \end{bmatrix}^T$ and $\mathbf{e}^H$ denotes the complex-conjugate transpose.

As a result the denominator has zeros at the frequencies related to the signal eigenvectors. The plot of (10) shows no real spectrum. The peaks of the pseudospectrum coincide with frequencies of the outraging frequency components. Equation (10) can also be given in the Z-domain [3]:

$$P_{music}(z) = \left[\sum_{i=K+1}^{M} S_i(z)\, S_i^*\left(\frac{1}{z^*}\right)\right]^{-1} \tag{11}$$

where $S_i(z) = \sum_{m=0}^{M-1} s_i(m)\, z^{-m}$ and $s_i(m)$ is the m-th element in the i-th eigenvector.

Signal and noise subspace are orthogonal to each other, so the denominator in (11) is zero at the harmonic frequencies.

The frequencies ω_k of the components in the signal model (2) are detectable using the frequency locations corresponding to the *K* highest peaks in the pseudospectrum $P_{music}\left(e^{j\omega}\right)$. An alternative way is the use of the angles of *K* roots nearest the unit circle in Z-domain in (11).

Additionally to harmonic components the magnitudes and powers of components can be obtained. When ω_k is given, then $P_k = |\underline{A}_k|^2$ can be estimated by jointly solving the *K* equations

$$\sum_{k=1}^{K} P_k\left|\mathbf{e}_k^H\mathbf{s}_i\right|^2 = \lambda_i - \hat{\boldsymbol{\sigma}}_w^2 \tag{12}$$

where $i = 1, 2, ..., K$ and the noise is estimated by

$$\hat{\boldsymbol{\sigma}}_w^2 = \frac{1}{M-K}\sum_{i=K+1}^{M}\lambda_i \tag{13}$$

Due to the fact

$$\mathbf{e}_k^H\mathbf{s}_i = S_i\left(e^{j\omega_k}\right) = \sum_{m=0}^{M-1} s_i(m)\, e^{-jm\omega_k} \tag{14}$$

the equation (12) can be rewritten in the matrix notation

$$\begin{bmatrix} \left|S_1\left(e^{j\omega_1}\right)\right|^2 & \left|S_1\left(e^{j\omega_2}\right)\right|^2 & \cdots & \left|S_1\left(e^{j\omega_K}\right)\right|^2 \\ \left|S_2\left(e^{j\omega_1}\right)\right| & \left|S_2\left(e^{j\omega_2}\right)\right|^2 & \cdots & \left|S_2\left(e^{j\omega_K}\right)\right|^2 \\ \vdots & \vdots & \ddots & \vdots \\ \left|S_K\left(e^{j\omega_1}\right)\right| & \left|S_K\left(e^{j\omega_2}\right)\right|^2 & \cdots & \left|S_K\left(e^{j\omega_K}\right)\right|^2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_K \end{bmatrix} = \begin{bmatrix} \lambda_1 - \hat{\sigma}_w^2 \\ \lambda_2 - \hat{\sigma}_w^2 \\ \vdots \\ \lambda_K - \hat{\sigma}_w^2 \end{bmatrix} \tag{15}$$

Solving (15) returns the harmonic power P_K for $k = 1,2,...,K$ and the amplitudes in (2) are given as $\sqrt{2P_K}$.

IV. BLOCK BASED APPLICATION OF MUSIC

The signal model (2) assumes that the signal is stationary or at least statistically time invariant. In power systems signals often change over time. For non-stationary signals the proposed MUSIC method must be modified using the block-based signal modelling. In this approach the data is divided into blocs, within which the signal is approximately stationary. The blocs can be overlapped or non-overlapped. The parameters can be estimated for each data block separately. Therefore, the parameters will vary with time, as the estimations vary with sliding window.

The samples of a recorded signal $x[n]$ are divided into blocks of fixed length *L*. The size of blocks is determined empirically in order to regard the data within the blocks as stationary. The overlap of two adjacent blocks is denoted with K and $K < L$. The *m*-th sample in the *j*-th block is $x^{(j)}[m]$ where $m = 0,1,2,...,L-1$, $j = 1,2,...$ Then the data sample $x^{(j)}[m]$ in the *j*-th block is related to the original sample ordering n in the original signal

$$x^{(j)}[m] = x[m + (j-1)(L-K)] \tag{16}$$

The time index *m* is related to the time index *n* by

$$n = m + (j-1)(L-K) \tag{17}$$

The modified signal model given in (1) and used in the blocked approach is

$$x^{(j)}[m] = \sum_{k=1}^{K} a_k^{(j)} \cos\left(m\omega_k^{(j)} + \phi_k^{(j)} + w^{(j)}[m]\right) \tag{18}$$

For different windows the number of spectral components $K^{(j)}$ may be different.

V. ANALYSIS OF SIGNALS WITH MUSIC

Firstly, a signal containing odd and even harmonics has been analysed. The fundamental component has been deliberately omitted. The amplitudes of all components were equalized to better visualise the performance of the method. Harmonics from 2 to 8 were included. Each value settled to 10 per cent of the fundamental. Additional noise with mean zero and standard deviation 2 was added and then multiplied by 30. The signal is shown in Fig. 1 and Fig. 2. Parameters of the original clean signal and estimated from the signal with noise are given in Table 1.

The method performed satisfactory even for noisy signals. Similar tests were conducted for a signal with linearly changing initial phase of spectral components. There was no significant influence of the initial phase of harmonics on the accuracy of components' frequency estimation.

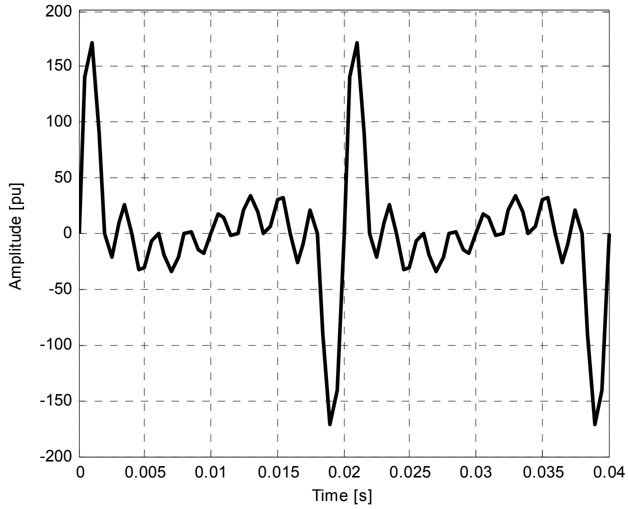

Fig. 1. Artificial signal with harmonics 2 to 8: phase angle of harmonics made zero, no noise

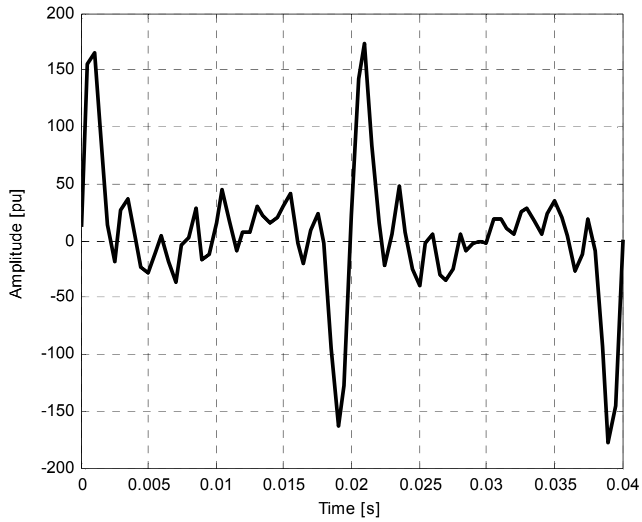

Fig. 2. Artificial signal with harmonics 2 to 8: phase angle of harmonics made zero, with noise

TABLE I. COMPUTATION RESULTS OF SPECTRAL COMPONENTS FOR ARTIFICIAL SIGNAL WITH HARMONICS 2 TO 8.

Frequencies of spectral components		
theoretical value [Hz]	computed, no noise [Hz]	computed, noise [Hz]
100	100	100.27
150	150	151.76
200	200	198.22
250	250	249.81
300	300	300.26
350	350	354.22
400	400	402.80

Compensation of reactive power with a capacitor bank is a common practice [5] and requires frequent switching operations for vast varying loads. A typical waveform following switching operation in a small wind generator (160 kW) equipped with an induction machine is shown in Fig. 3.

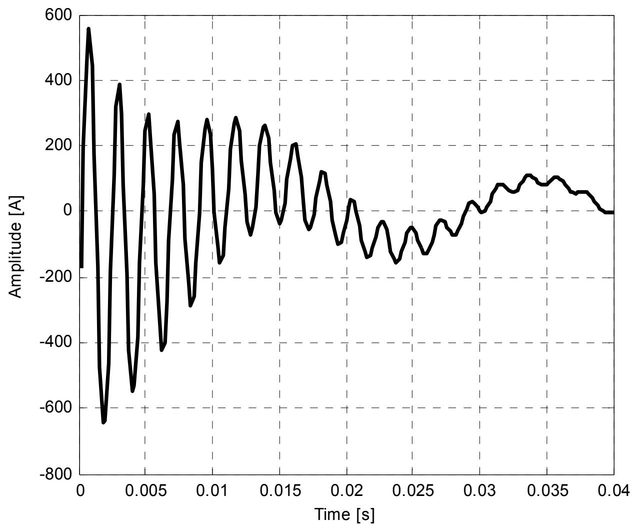

Fig. 3. Capacitor switching in a small wind generator with induction machine, no noise.

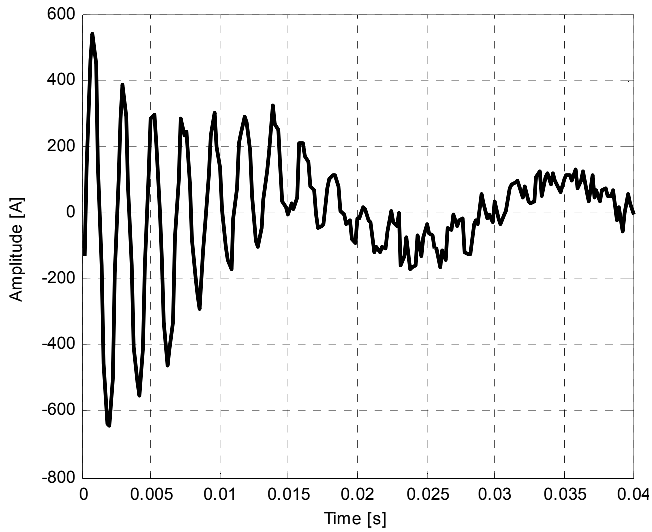

Fig. 4. Capacitor switching in a small wind generator, with induction machine, noise added

The results of spectral component estimation are given in Table 2. There are no theoretical values hence a real measured signal was analysed. The additional noise with mean zero and standard deviation 2 was added artificially and then multiplied by 30 to increase the amplitude of distortions. Fig. 4 presents the distorted signal.

As in the case of signal with high harmonic content, the frequency of a decaying component was detected adequately, even in the presence of noise.

TABLE II. COMPUTATION RESULTS OF SPECTRAL COMPONENTS FOR CAPACITOR SWITCHING OPERATION

Frequencies of spectral components		
theoretical value [Hz]	computed, no noise [Hz]	computed, with noise [Hz]
--	48.21	49.11
--	456.31	457.67

Another common phenomenon in the public distribution gird is switching of heavy loads. Especially in weak networks or in islanded networks it results in rapid increase of current amplitude overlaid with and decaying exponential component (Fig. 5).

The additional noise with mean zero and standard deviation 2 was added artificially and then multiplied by 30 to increase the amplitude of distortions. Fig. 6 presents the distorted signal.

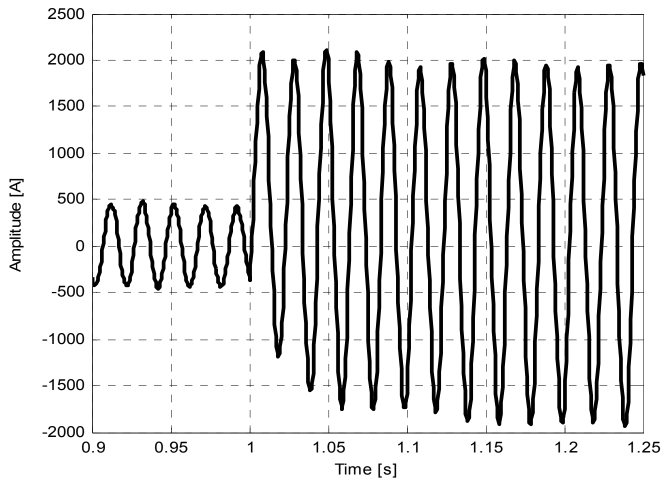

Fig. 5. Switching on a heavy load

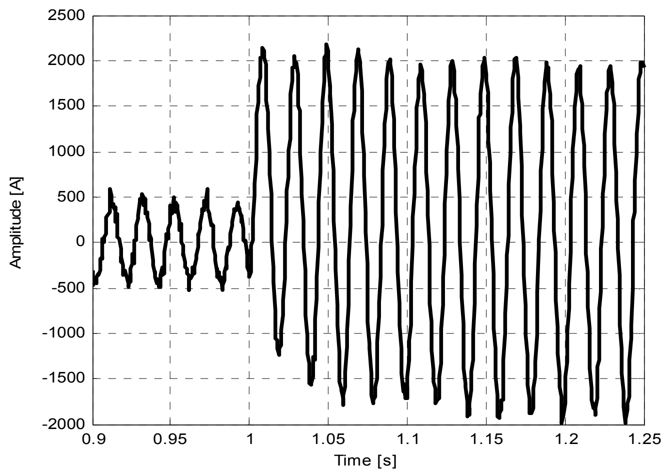

Fig. 6. Switching on a heavy load, with noise

Frequency estimation results are given in Table 3.

TABLE III. COMPUTATION RESULTS OF SPECTRAL COMPONENTS FOR SWITCHING OF A HEAVY LOAD

Frequencies of spectral components		
theoretical value [Hz]	computed, no noise [Hz]	computed, with noise [Hz]
--	50.15	50.22
--	49.03	48.53

VI. CONCLUSION

Presented method is an appropriate tool for the estimation of frequencies of spectral components in signals. The method is relatively immune against distortion by noise. Decaying exponential component or varying amplitude was also not deteriorating the computation of frequencies. Spectral components are an important factor for the estimation of power quality, i.e. *Total Harmonic Distortion* THD value, and are crucial for the proper computation of active and reactive powers for nonsinusoidal voltage and current waveforms. Therefore, the MUSIC method is regarded as an important verifying tool in the identification of spectral components' frequencies in distorted signals.

REFERENCES

[1] Schmidt R.O, "Multiple Emitter Location and Signal Parameter Estimation", IEEE Trans. Antennas Propagation, Vol. AP-34, March 1986, pp.276-280.

[2] Monson H. Hayes, "Statistical digital signal processing and modeling", John Wiley & Sons, 1996

[3] B.D. Rao, K.V.S. Hari, "Performance Analysis of Root-Music, IEEE Transactions on Acoustics, Speech and Signal Processing", Vol. 37, No. 12, 1989, pp.1939-1949

[4] Z. Leonowicz, "Parametric Methods for time frequency analysis of electric signals", Oficyna Politechniki Wroclawskiej, Wroclaw, 2006

[5] W. Hofmann, J. Schlabbach, W. Just, "Reactive Power Compensation. A practical guide.", John Wiley & Sons, 2012.

[6] Z. Lubosny, "Wind Turbine Operation in Electric Power Systems". Springer Verlag, Berlin Heidelberg (Germany), 2003

[7] M.H.J. Boolen, I.Y.H. Gu, "Signal Processing of Power Quality Disturbances", John Wiley & Sons, 2006