Physics Problem

Q. Magnetostatics treats the "source current" (the one that sets up the field) & the "receipt current" (the one that experiences the force) so everyone assymetrically that it is by no means obivous that the magnetic force between two current loops is consistent with Newton's third law. Show, starting with the Biot-Savart law & the lorentz force law, that the force on loop 2 due to loop 1 can be written as

$$F_2 = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\hat{r}}{r^2} dI_1 \cdot dI_2 .$$

In this form, it is clear that $F_2 = -F_1$ since $\hat{r}$ changes direction when the roles of 1 & 2 are interchanged. (If you seem to be getting as 'extra' term, it will help to note that $dI_2 \cdot \hat{r} = dr$.)

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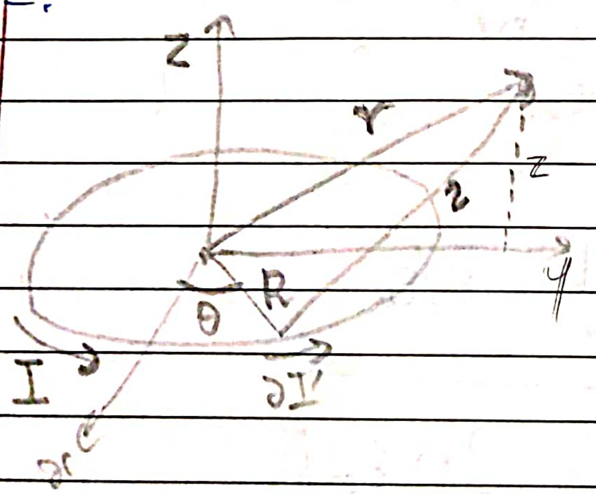

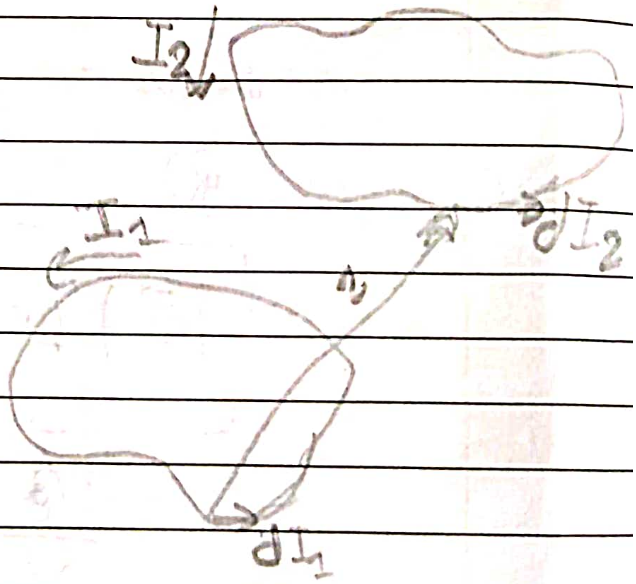

Solution -: From biot-Savart's law, the field of loop 1

$$B = \frac{\mu_0 I_1}{4\pi} \oint_1 \frac{dI_1 \times \hat{r}}{r^2},$$ the force on loop 2 is given by

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$$F = I_2 \oint_2 dI_2 \times B = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{dI_2 \times (dI_1 \times \hat{r})}{r^2},$$

Now $dI_2 \times (dI_1 \times \hat{r}) = dI_1 (dI_2 \cdot \hat{r}) - \hat{r}(dI_1 \cdot dI_2)$ so

$$F = -\frac{\mu_0}{4\pi} I_1 I_2 \left\{ \oint \oint \frac{\hat{r}}{r^2} (dI_2 \cdot dI_1) - \oint dI_1 \oint \frac{(dI_2 \cdot \hat{r})}{r^2} \right\}$$

The first term is what we want, that term we got, now its time to show that remaining term is zero.

$$r = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (z_2 - z_1)\hat{z}$$

now $\nabla_2 (1/r) = \frac{\partial}{\partial x_2} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]^{-1/2} \hat{x}$

$+ \frac{\partial}{\partial y_2} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]^{-1/2} \hat{y} + \frac{\partial}{\partial z_2} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]^{-1/2} \hat{z}$

$$= -\frac{(x_2 - x_1)}{r^3}\hat{x} - \frac{(y_2 - y_1)}{r^3}\hat{y} - \frac{(z_2 - z_1)}{r^3}\hat{z}$$

$$= -\frac{r}{r^3} = -\frac{\hat{r}}{r^2}$$

so $\oint \frac{\hat{r}}{r^2} \cdot dI_2 = -\oint \nabla_2 \left(\frac{1}{r}\right) \cdot dI_2 = 0$ by (corollary)