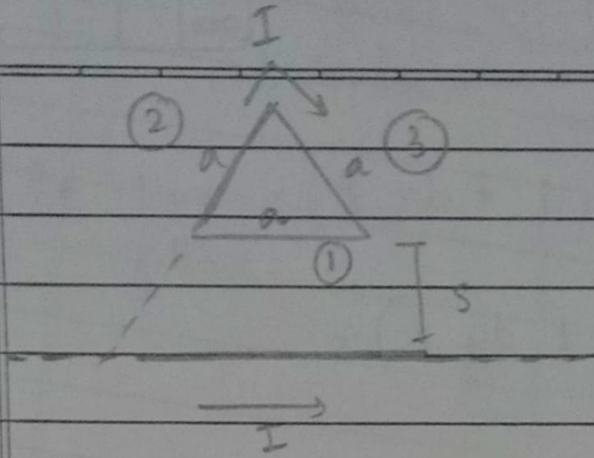
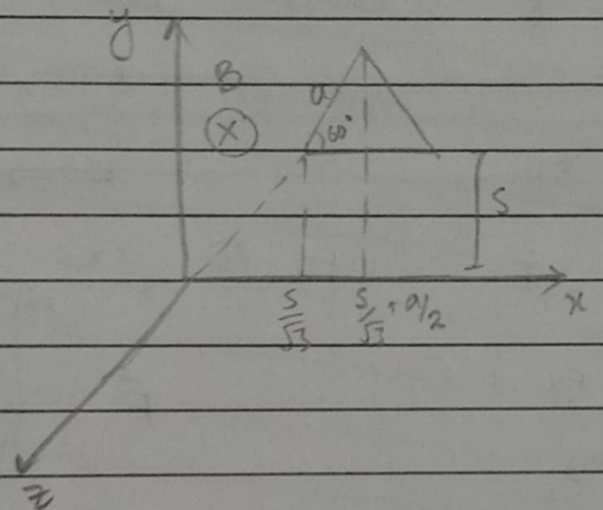
Q.] 

Find the force on the triangular loop.

Ans For wire ①:



$$\int B \cdot dl = \mu_0 I$$

B points into the page

$$\Rightarrow \quad B = -\frac{\mu_0 I}{2\pi s}\hat{z}$$

and $\vec{I} = \vec{I}\,\hat{x}$

$$dF_1 = (\vec{I} \times \vec{B})\, dl$$
$$= \left((\vec{I}\,\hat{x}) \times \left(\frac{-\mu_0 I}{2\pi s}\hat{z} \right) \right)$$

$$F_1 = \frac{\mu_0 I^2}{2\pi s} a\, \hat{y}$$

For wire ② & ③:

$$dF_2 = (I \times B)\, dl = (dl \times B)\, I$$
$$dF_2 = ((dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}) \times B)\, I$$

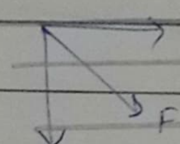
Now, $B = -\frac{\mu_0 I}{2\pi y}\hat{z}$

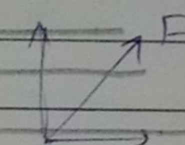
$$= I\left(\left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right) \times \left(\frac{\mu_0 I}{2\pi y}\,\hat{z}\right)\right)$$

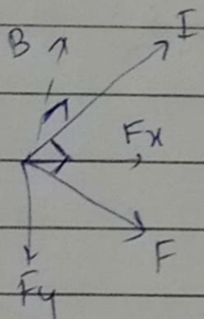
$$= I\,\frac{\mu_0 I}{2\pi y}\left(-dx\,\hat{y} + dy\,\hat{x}\right)$$

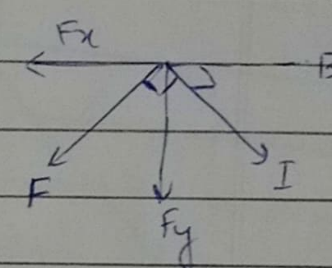
$$= \frac{-\mu_0 I^2}{2\pi y}\,dx\,\hat{y}$$

the x component of F cancels out

~~as For (3):~~ 

and (2): 

as for (2): 

For (3): 

Thus, $dF_2 = \frac{-\mu_0 I^2}{2\pi y}\,dx\,\hat{y}$

we see the ratio of $y : x$ is always $\sqrt{3}$
i.e. $y = \sqrt{3}x$

$$\therefore \quad dF_2 = \frac{-\mu_0 I^2}{2\pi\sqrt{3}x}\,dx\,\hat{y}$$

$$\Rightarrow \quad F_2 = -\frac{\mu_0 I^2}{2\pi\sqrt{3}}\int_{s/\sqrt{3}}^{s/\sqrt{3}+a/2} \frac{1}{x}\,dx\,\hat{y}$$

$$= -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(\frac{s/\sqrt{3} + a/2}{s/\sqrt{3}}\right)\hat{y}$$

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$$F_2 + F_3 = -\frac{\mu_0 I^2}{\sqrt{3}\pi} \ln\left(\frac{s/\sqrt{3} + a/2}{s/\sqrt{3}}\right)\hat{y}$$

$$F_{net} = F_1 + F_2 + F_3$$

$$= \frac{\mu_0 I^2 a}{2\pi s}\hat{y} - \frac{\mu_0 I^2}{\sqrt{3}\pi} \ln\left(\frac{s/\sqrt{3} + a/2}{s/\sqrt{3}}\right)\hat{y}$$

$$= \frac{\mu_0 I^2}{\pi}\left(\frac{a}{2s} - \frac{1}{\sqrt{3}} \ln\left(\frac{s/\sqrt{3} + a/2}{s/\sqrt{3}}\right)\right)\hat{y}$$