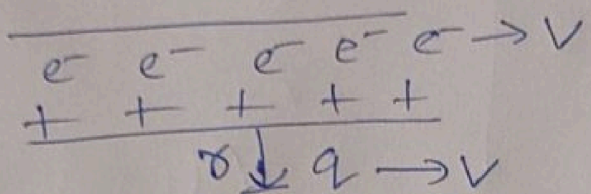
An electron is moving with a velocity v in a straight conducting wire. Outside the wire a charge q is moving with the same velocity v at some distance r. Analyze the motion of the system
(a) from ground frame
(b) with respect to the charge 'q'.

⇒ (a)



$l \rightarrow$ length of wire which have e^- or p^+ equal to q'

we know that Electric field outside the conducting wire is zero, since $\lambda_{proton} = \frac{q'}{l}$, & $\lambda_{electron} = \frac{-q'}{l}$.

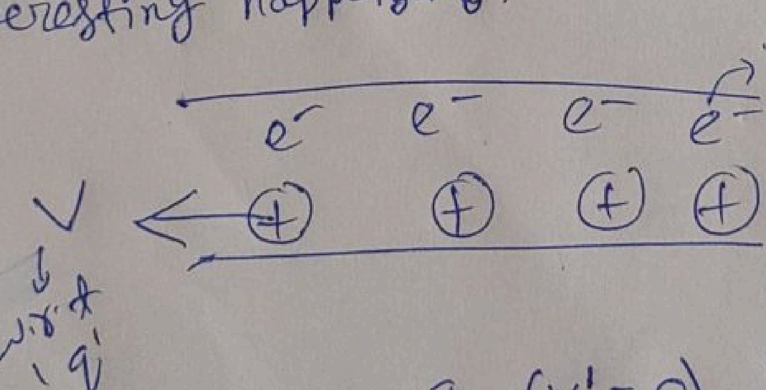
λ_{net} (net charge per unit length) $= 0$.

~~So~~ if $q_{net} = 0$, from gauss' law, $\vec{E} = 0$.

But at distance 'r' from wire we have '$\vec{B}$'.

, $|\vec{B}| = \frac{\mu_0 I}{2\pi r}$ ~~[illegible]~~, ~~so~~ therefore e^- will experience force and get deviated from it's initial path (~~[illegible]~~ which was parallel to wire).

(b) ~~[illegible]~~, In frame of charge, something interesting happens,



$q \; (v' = 0)$ $v' \rightarrow$ velocity with respect to wire.

Here, in this frame. $\lambda_{proton} = \frac{q'}{l}$ $\lambda_{e^-} = \frac{-q'}{l}$

$\lambda_{net} = 0$, wire is still neutral..

~~but~~ and magnetic field is still present due to the movement of proton.. we can say that $\vec{E} = 0$ & $\vec{B} \neq 0$ but $\vec{F} = q(\vec{v} \times \vec{B})$

and in frame of charge 'q', $\vec{v}$ is '0'.

therefore magnetic force is also zero.

Then, How can we explain ~~[illegible]~~ motion between charge 'q' & wire, since magnetic & Electric force both are zero in frame of 'q', But in part 'a' we had seen that it have relative motion w.r.t to the wire.

Now we will try special theory of Relativity to explain this motion

⇒ we have assumed that 'q'' is present in 'l' .. ~~In frame of charge 'q'~~

⇒ In ground frame.

In frame of charge 'q' we can say that q' ~~[illegible]~~ proton is present in some l', because proton is moving w.r.t to charge 'q' & it will show length contraction due to which λ_{proton} will be ~~not~~ $\neq \frac{q'}{l}$ ~~[illegible]~~, but λ_{e^-} is $\frac{q'}{l}$.

Here $l' = \frac{l}{\gamma}$ $\left(\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$

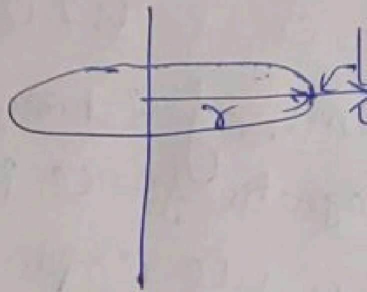
and $\boxed{\lambda_{proton} = \frac{q}{l}\gamma}$

We can see that according to Special theory of relativity, $\lambda_{proton} = \frac{q}{l}\gamma$.

$\therefore$ $\lambda_{net} = \frac{q}{l}(\gamma - 1)$, $\boxed{\lambda_{net} \text{ is not zero}}$

Now we can use

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}$$



$\vec{E} = \frac{\lambda_{net}}{2\pi\varepsilon_0 r}$

$$\boxed{|\vec{E}| = \frac{q(\gamma-1)}{l\, 2\pi\varepsilon_0 r}}$$

The force on charge 'q' is purely electric force, due to which it will show relative motion in it's frame (frame of q).

From this question we can say that SR fixes Electrodynamics, because it cannot explained with classical mechanics