Physics Presentation
Electric Field - Lines and Angles

Batch no.: 5
Name: Kinjal Dey
Reg. no.: 20211151

8 June 2022

1 Part I

An electric field line emerges from a positive point charge $+q_1$ at an angle α to the straight line connecting it to a negative point charge $-q_2$. The field line terminates at $-q_2$ at an angle β. Find the relationship between q_1, q_2, α, β. (*source: Krotov*)

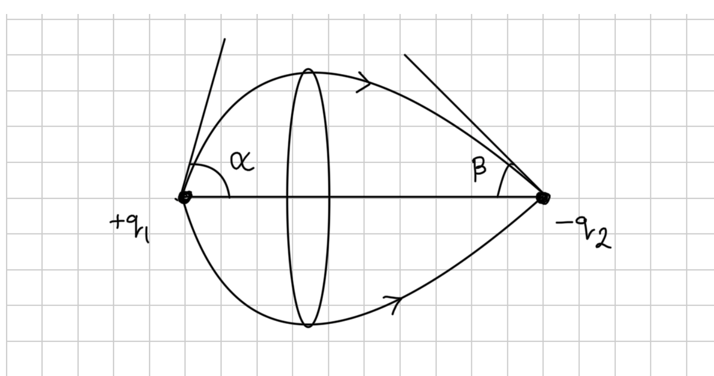

Fig. 1.1

Solution: Due to the symmetry in the system, all field lines coming out of q_1 with angle α, lie within a cone with half-vertex angle α. Similarly, all field lines terminating at $-q_2$ with an angle β, lie within a cone with half-vertex angle β.

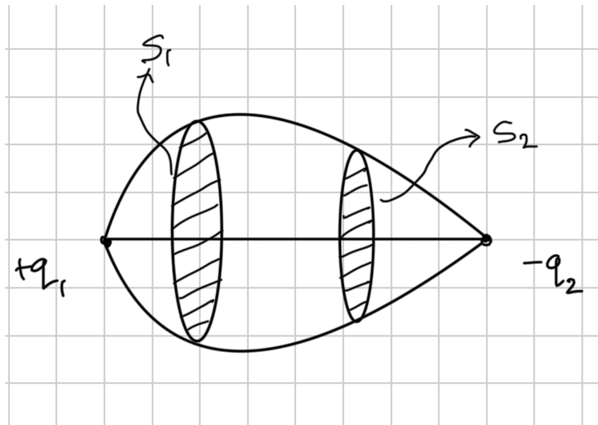

Fig. 1.2

The flux through any two vertical cross sections of the 3D volume in the diagram must be equal. To see why this is true, we take a Gaussian surface as shown in Fig. 1.2.

$$\oint \vec{E} \cdot da = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \int \vec{E} \cdot ds_1 + \int \vec{E} \cdot ds_2 = 0$$

(Note:The flux through the curved surface is zero because field lines cannot intersect.)

Note: The solid angle of a cone with half angle θ is

$$\Omega = 2\pi(1 - \cos\theta)$$

Using infinitesimally small spherical Gaussian surfaces centred at the point charges, The emergent flux,

$$\phi_E = \frac{\frac{q_1}{\epsilon_0}}{4\pi} 2\pi(1 - \cos\alpha) = \frac{q_1}{2\epsilon_0}(1 - \cos\alpha)$$

The terminating flux,

$$\phi_T = -\frac{\frac{q_2}{\epsilon_0}}{4\pi} 2\pi(1 - \cos\beta) = -\frac{q_2}{2\epsilon_0}(1 - \cos\beta)$$

Now,

$$\phi_1 = -\phi_2 \Rightarrow \frac{q_1}{q_2} = \frac{1 - \cos\beta}{1 - \cos\alpha}$$

2 Part II

There are two positive point charges q_1 and q_2 placed at a finite distance away from each other. The angle of emergence of an electric field line from q_1 is α. The tangent to this field line at infinity makes an angle β with the line joining the two point charges and intersects it somewhere between the two charges. Find the relationship between q_1, q_2, α and β.

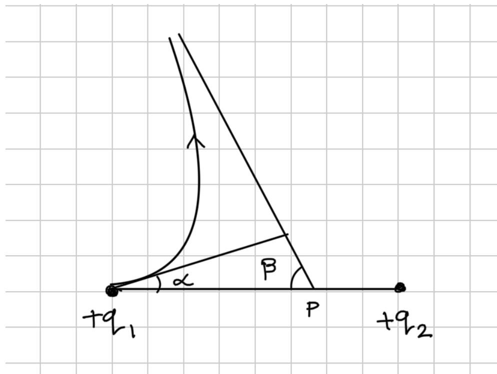

Fig. 2.1

Solution: We consider a spherical Gaussian surface centred at P and having infinite radius.

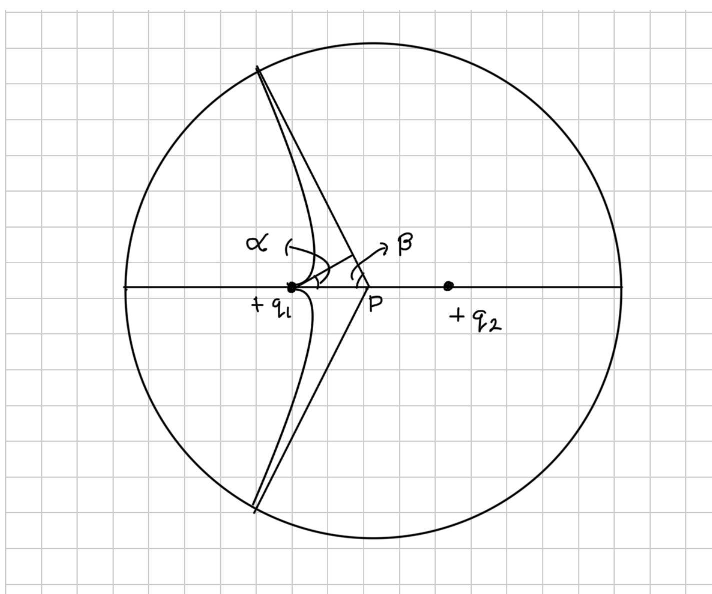

Fig. 2.2

We compute the flux through the area between the tangents to the left of P in two ways.

Note that the tangent lines are asymptotes for the set of field lines emerging from q_1 with angle α. Using an infinitesimally small spherical Gaussian surface centred at q_1, the flux due to q_1 is

$$\phi_{q_1} = \frac{\frac{q_1}{\epsilon_0}}{4\pi} 2\pi(1 - \cos(\pi - \alpha)) = \frac{q_1}{2\epsilon_0}(1 + \cos\alpha)$$

since, field lines emerging with an angle less than α will intersect this tangent and not come out of the specified surface. The flux due to q_1 and q_2 (both through the same surface) is

$$\phi_{total} = \frac{\frac{q_1+q_2}{\epsilon_0}}{4\pi}(1 - \cos\beta) = \frac{q_1 + q_2}{2\epsilon_0}(1 - \cos\beta)$$

Note that no field line emerging from q_2 can pass through the surface between the tangents, since electric field lines cannot intersect. Thus, we can equate both the fluxes. Hence,

$$q_1 = \frac{q_2(1 - \cos\beta)}{\cos\alpha + \cos\beta}$$