Consider a uniformly charged circular disc of radius 'a' with a negligible thickness carrying a total charge 'q'. Suppose the disc lies in the x-y plane of Cartesian co-ordinates and is centred on the origin.

a) Show that the electric potential at an arbitrary point P on the symmetry axis of the disc is given by

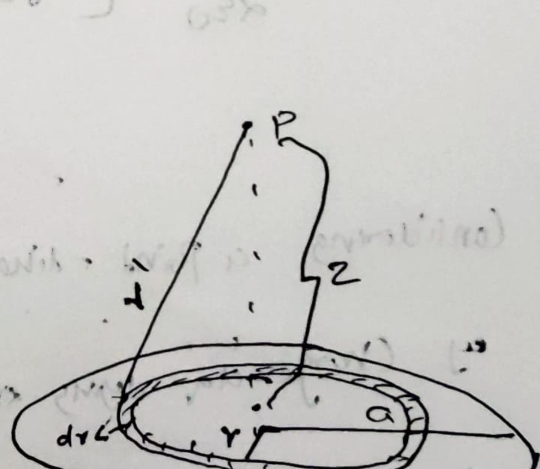
$$\phi(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2} \mp z\right)$$

where the upper (lower) sign is for $z>0$ $(z<0)$ & $\sigma = q/\pi a^2$.

b) Show that the electric potential at an arbitrary point P on the circumference of the disc is given by

$$\phi_P = \frac{\sigma a}{\pi\varepsilon_0}$$

ans:



Consider 2 concentric circles having radii 'r' & $r+dr$ (here $r \le a$) lying in the plane of disc. The charge 'dq' b/w these circles is located at a distance "r'" from P where

$dq = \sigma \times (2\pi r)(dr)$ & $r' = \sqrt{r^2+z^2}$ by Pythagoras.

$r'^2 = r^2 + z^2$ (differentiating)

$2r'dr' = 2r\,dr$ ($z \to$ constant).

$\therefore \phi_p = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r'} = \frac{\sigma 2\pi}{4\pi\varepsilon_0}\int_0^a \frac{r\,dr}{r'}$

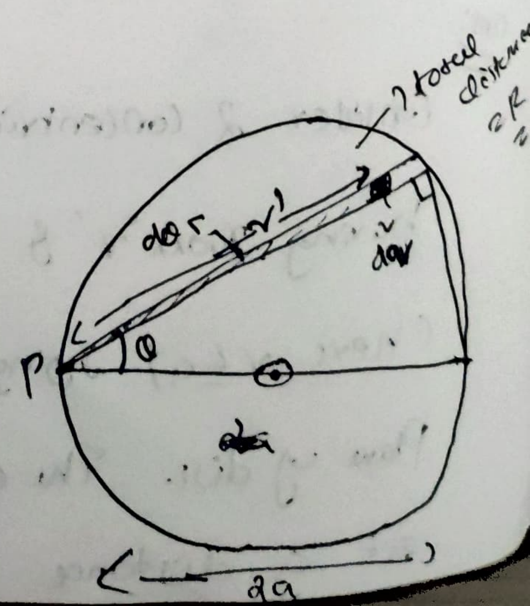
$dq = \sigma \times 2\pi r\,dr$

when $r=0$, $r'=z$ & when $r=a$, $r' = \sqrt{z^2+a^2}$.

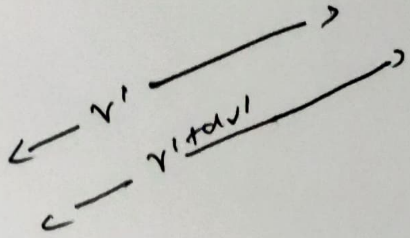
So $\phi_p = \frac{\sigma}{2\varepsilon_0}\int_{|z|}^{\sqrt{a^2+z^2}} dr$

$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+z^2} \mp z\right]$ //

b) Considering a point-like element of charge dq lying inside the shaded wedge at a distance r' from P.



So $dq = \sigma \times (r' d\theta) \times dr'$



$$d\phi_p = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r'} =$$

$$= \frac{\sigma\, d\theta}{4\pi\epsilon_0}\int_0^R dr' = \frac{\sigma R\, d\theta}{4\pi\epsilon_0}$$

where $R = 2a\cos\theta$

$\left(\cos\theta = \frac{R}{2a}\right)$, so $d\phi_p = \frac{\sigma a \cos\theta\, d\theta}{2\pi\epsilon_0}$

hence integrating the small wedge to the whole circle

$$\phi_p = \frac{\sigma a}{2\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{\sigma a}{2\pi\epsilon_0}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\sigma a}{\pi\epsilon_0} \quad //$$