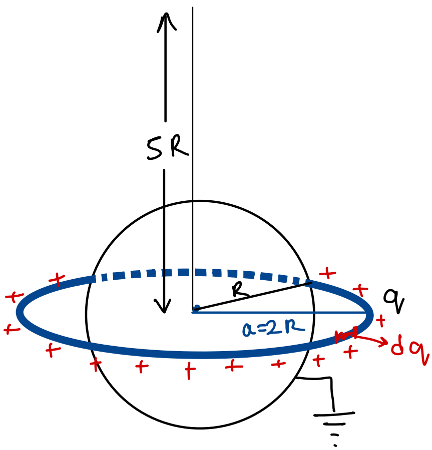
A grounded metallic sphere of radius R is surrounded by a concentric ring of radius 2R. The ring carries a charge q uniformly distributed on its length. Find the electric potential at a point on the axis of the ring at a distance 5R from the centre.



Boundary conditions :-

$V=0$ at R

$V \to 0$ at ∞

A negative charge will be induced over the sphere. Let dq be an infinitesimal charge on the ring. There will be an image charge corresponding to dq at a distance b from the centre of the sphere.

Formula for an image charge on sphere corresponding to a charge q at a distance a from the sphere

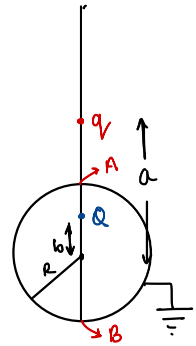
$$\boxed{\text{Image charge} = -\frac{R}{a}q}$$

$\Rightarrow$ (Q) Image charge $= -\frac{\cancel{R}}{2\cancel{R}}dq = \frac{-dq}{2}$

Let b be the distance of image charge Q from the centre $\boxed{b = \frac{R^2}{a}}$

$\Rightarrow b = \frac{R^{\cancel{2}}}{2\cancel{R}} = R/2$

Point A: At the boundary $V = 0$ $\because$ $k = \frac{1}{4\pi\epsilon_0}$



$$V_A = \frac{kQ}{R-b} + \frac{kq}{a-R} = 0$$

$$\frac{kQ}{R-b} = \frac{-kq}{a-R} \Rightarrow Q(a-R) = -q(R-b)$$

$$\Rightarrow Qa - QR = qb - qR$$

Point B: $$V_B = \frac{kQ}{b+R} + \frac{kq}{a+R} = 0$$

$$\Rightarrow \frac{Q}{b+R} = \frac{-q}{a+R} \Rightarrow Qa + QR = -qb - qR \text{ —— } ②$$

① + ②

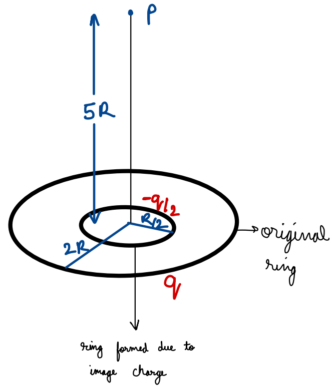
$$Qa - Q\cancel{R} = -qR + q\cancel{b}$$
$$Qa + \cancel{Q}R = -qR - \cancel{q}b$$
$$\cancel{2}Qa = -\cancel{2}qR \Rightarrow \boxed{Q = \frac{-qR}{a}}$$

① − ②

$$Q\cancel{a} - QR = -q\cancel{R} + qb$$
$$-\cancel{Q}a - QR = \cancel{+}qR + qb$$
$$-\cancel{2}QR = \cancel{2}qb \Rightarrow b = -\frac{QR}{q}$$

$$\Rightarrow b = \frac{\cancel{q}R}{a} \times \frac{R}{\cancel{q}} \Rightarrow \boxed{b = \frac{R^2}{a}}$$

⇒ We can imagine our image charge as a ring (formed by summation of dq) with a radius R/2



Potential due to original ring $= \dfrac{q}{4\pi\epsilon_0\sqrt{(5R)^2+(2R)^2}}$

$= \dfrac{q}{4\pi\epsilon_0 R\sqrt{29}}$

Potential due to image charge $= \dfrac{(-q/2)}{4\pi\epsilon_0\sqrt{(5R)^2+(R/2)^2}}$

$= \dfrac{-q}{4\pi\epsilon_0\sqrt{100R^2+R^2}}$

$= \dfrac{-q}{4\pi\epsilon_0 R\sqrt{101}}$

⇒ Resultant potential at P $= \dfrac{q}{4\pi\epsilon_0 R}\left(\dfrac{1}{\sqrt{29}} - \dfrac{1}{\sqrt{101}}\right)$

$$\boxed{\text{Resultant potential at P} = \frac{q}{4\pi\epsilon_0 R}\,\frac{\sqrt{101}-\sqrt{29}}{\sqrt{2929}}}$$