Classical Model of Diamagnetism

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June 30, 2022

1 Diamagnetism

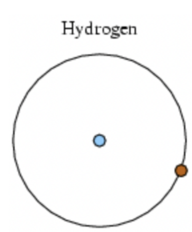
Diamagnetic materials are repelled by a magnetic field; an applied magnetic field creates an induced magnetic field in them in the opposite direction, causing a repulsive force. It is **universal** phenomenon.However, it is typically much weaker than paramagnetism, and is therefore observed mainly in atoms with even numbers of electrons, where paramagnetism is usually absent.

1.1 Hydrogen Atom

Diamagnetism is a quantam mechanical phenomenon but i will try to explain some of its properties using classical mechanics.
I have made a classical model inspired from langevin model of classical diamagnetism.

1.1.1 Assumptions

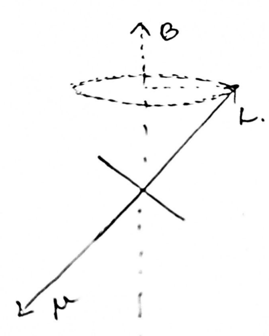
In this model a electron revolving around a proton, consider it like hydrogen. Here hydrogen is kept at fixed distance from proton which is Bohr radius (a). We are not considering interaction between any atom.



1.1.2 Finding magnetic moment

Initially H atoms do not have any magnetic moment because in total they cancel their magnetic moments. Now a uniform magnetic field ($\vec{B}$) is applied in $\vec{z}$ direction. As individual atom have a magnetic moment ($\vec{\mu}$), when magnetic field is applied it create torque.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



Larmor precession (named after Joseph Larmor) is the precession of the magnetic moment of an object about an external magnetic field. Torque is in side of plane. θ is angle between $\vec{B}$ and $\vec{L}$.

Now finding frequency of precession(ω).

$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

In time Δt horizontal component of angular momentum change direction by angle $\Delta\phi$. Magnitude of angular momentum will remain same. Only horizontal component changes its direction.

$$\Delta \vec{L} = L \sin\theta \Delta\vec{\phi}$$

$$\omega = \frac{\Delta\phi}{\Delta t}$$

$$\vec{\mu} \times \vec{B} = \frac{L \sin\theta \Delta\vec{\phi}}{\Delta t}$$

$$\mu B \sin\theta = L \sin\theta\omega$$

Using Gyromagnetic ratio.

$$\frac{\mu}{L} = \frac{e}{2m}$$

$$\omega = \frac{eB}{2m}$$

m is mass of electron.
The prcession of magnetic moment create new magnetic moment which is in opposite direction of magnetic field with effective area of radius $a \sin\theta$.
let new magnet be η.

$$\eta = IA$$

I is current. A is area.

$$I = q\frac{\omega}{2\pi}$$

$$I = -\frac{e^2 B}{4\pi m}$$

$$\eta = -\frac{e^2 a^2 B}{4m}(\sin\theta)^2$$

Every atom of hydrogen will have different θ. So we will take average of η. will take average of η using Maxwell-Boltzman statistics. We will also see temperature dependence of diamagnetic material. Maxwell-Boltzman statistics is applicable for high enough temperature. I will not derive Maxwell-Boltzman statistics here. I will take it as axiom. E is energy of magnetic moment.

$$E = -\eta \cdot B$$

$$E = \eta B$$

$$E = -\frac{e^2 a^2 B^2}{4m}(\sin\theta)^2$$

$$E_{min} = -\frac{e^2 a^2 B^2}{4m}$$

$$E_{max} = 0$$

η average in Maxwell-Boltzman statistics is as follow

$$<\eta>=\frac{\int_{E_{min}}^{E_{max}}\eta(E)e^{-\frac{E}{k_bT}}\,dE}{\int_{E_{min}}^{E_{max}}e^{-\frac{E}{k_bT}}\,dE}$$

$$<\eta>=\frac{E_0}{B(e^{\frac{-E_0}{k_bT}}-1)}+\frac{k_bT}{B}$$

$$E_0=|E_{min}|$$

We here got η as function of T.
Using Taylor expansion on $<\eta>$

$$<\eta>\approx-\frac{E_0}{2B}-\frac{E_0^2}{12Bk_bT}$$

Maxwell-Boltzman statistics is applicable for high enough temperature. $\frac{E_0^2}{k_b}\rightarrow 10^{-35}$. SO $<\eta>$ is approximately given below

$$<\eta>\approx-\frac{E_0}{2B}$$

Magnetic susceptibility $\chi=\frac{\mu_0 n<\eta>}{B}$ where n is number of particle. Value of χ for 1 mole of H atom using quantam mechanics is $-2.32*10^{-12}\frac{m^3}{mole}$(by Michigan state university) The value of Magnetic susceptibility that i got from classical model

$$\chi=-7.465*10^{-12}\frac{m^3}{mole}$$

1.2 Inference

- χ is negative.
- χ does not depend on temperature.
- Magnetic moment is opposite to applied magnetic field.

The above points are properties of diamagnetism that are proved by classical model.

1.3 Questions?

1. Hydrogen atom is paramagnetic why do I have χ diamagnetic and why i founded diamagnetism for paramagnetic material?
Diamagnetism is Universal phenomenon. Every object have χ diamagnetic, but for some object paramagnetic part dominates so they have positive χ.

2. So where is paramagnetic part of H atom, i was finding χ and hydrogen is paramagnetic so χ should be positive?
I will give reason in class with little glimpse of classical paramagnetic model.