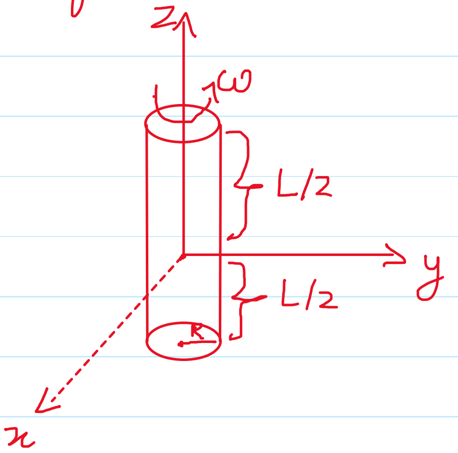
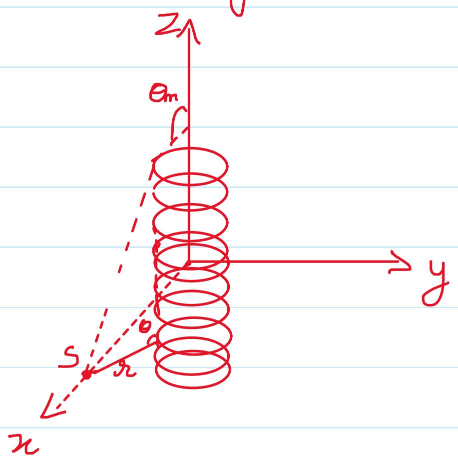
* A rod of radius R and length L carries a uniform surface charge σ. It is set spinning about its axis, at an angular velocity ω. Find the magnetic field at a distance $s \gg R$ from the axis in the xy plane.



→ The first thing, we might think of to approach this problem is by considering infinitesimally small area elements and then considering their magnetic fields and integrating.
But, there's a better way to solve this question.
We can consider this cylinder to be a stack of dipoles.



We consider s to be on the x-axis, (we just do this so as to keep our calculations and $\vec{B}$ in the xz plane, we can consider s to be anywhere on the xy plane it would give us the same answer.

Magnetic field due to a dipole at origin & in the xz plane,

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) = \frac{\mu_0}{4\pi}\frac{m}{r^3}\left[2\cos\theta(\sin\theta\,\hat{x} + \cos\theta\,\hat{z}) + \sin\theta(\cos\theta\,\hat{x} - \sin\theta\,\hat{z})\right]$$

$$\therefore B = \frac{\mu_0}{4\pi}\frac{m}{r^3}\left[3\sin\theta\cos\theta\,\hat{x} + (2\cos^2\theta - \sin^2\theta)\,\hat{z}\right]$$

We have same number of dipoles above and below the xy plane,
So, the x components cancel out.

$$\therefore B = \frac{\mu_0}{4\pi} 2M \int_0^{L/2} \frac{(3\cos^2\theta - 1)}{r^3}\,dz\,\hat{z} \; \ldots \left[\begin{aligned} m &= I \cdot \pi R^2 = (\sigma v h)\pi R^2 \\ &= \sigma \omega R \pi R^2 h \\ M &= \frac{m}{h} = \pi\sigma\omega R^3 \end{aligned}\right]$$

Now, $\sin\theta = \frac{s}{r}$, so $\frac{1}{r^3} = \frac{\sin^3\theta}{s^3}$

$$z = -s\cot\theta \Rightarrow dz = \frac{s}{\sin^2\theta}\,d\theta$$

Hence,

$$B = \frac{\mu_0}{2\pi}(\pi\sigma\omega R^3)\,\hat{z} \int_{\pi/2}^{\theta_m} (3\cos^2\theta - 1)\frac{\sin^3\theta}{s^3}\frac{s}{\sin^2\theta}\,d\theta$$

$$= \frac{\mu_0\sigma\omega R^3}{2s^2}\,\hat{z} \int_{\pi/2}^{\theta_m} (3\cos^2\theta - 1)\sin\theta\,d\theta$$

$$= \frac{\mu_0\sigma\omega R^3}{2s^2}\,\hat{z}\,(-\cos^3\theta + \cos\theta)\Big|_{\pi/2}^{\theta_m}$$

$$= \frac{\mu_0\sigma\omega R^3}{2s^2}\cos\theta_m \sin^2\theta_m\,\hat{z}$$

Here, $\sin\theta_m = \frac{s}{\sqrt{s^2 + (L/2)^2}}$, $\cos\theta_m = \frac{-(L/2)}{\sqrt{s^2 + (L/2)^2}}$

So,

$$\boxed{B = -\frac{\mu_0\sigma\omega R^3 L}{4[s^2 + (L/2)^2]^{3/2}}\,\hat{z}}$$