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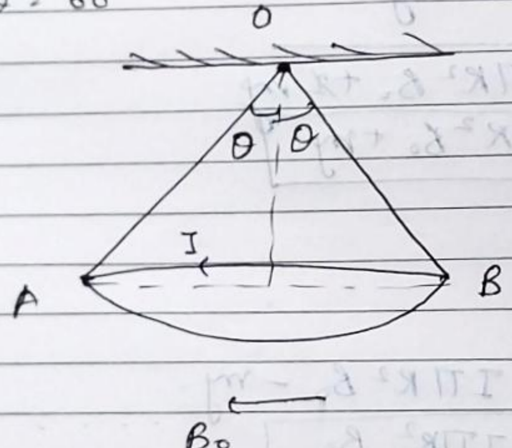
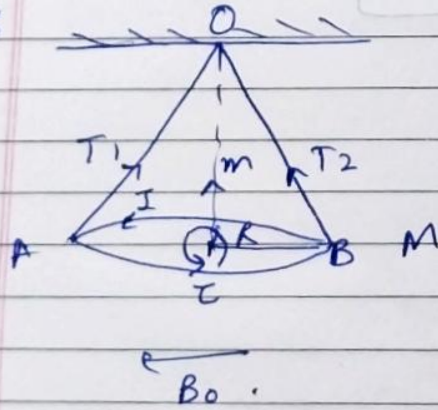
Q A uniform ring of mass m and Radius R carries a uniform current I. The ring is suspended using two identical strings OA & OB. There exists a uniform magnetic field B_0, parallel to the diameter AB of the ring. Calculate the tension in the two strings?

$\theta = 60^\circ$.



Sol:



Here, the tensions won't be same in both the strings.

By Right Hand rule, m points upwards. [curl in direction of I.]

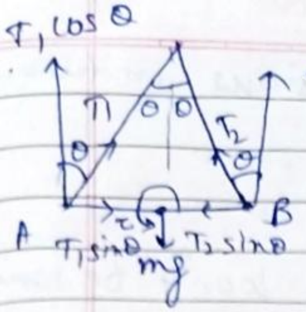
$\vec{m} \times \vec{B_0} \Rightarrow$ outwards $\Rightarrow$ anti-clockwise.

Because of Mag. field B_0 & current I in the loop there will be a torque on the ring.

$$\vec{\tau} = \vec{M} \times \vec{B}.$$

$$= MB \sin 90^\circ$$

$$= nIAB_0 = IAB_0 = I\pi R^2 B_0.$$



As the system is suspended & stationary i.e. Rotational equilibrium &

$$\Sigma \tau = 0.$$

$$\tau_{T_1} + \tau_{T_2} + \tau_{mg} + \tau_{B_0} = 0. \quad (1)$$

Here τ_{T_1} & $T_1 \sin\theta$ ~~cancels out~~ do not produce any torque because it passes through the axis & the point from where we are measuring it

$$\Rightarrow \tau_{T_1} = R T_1 \cos\theta$$

$\Rightarrow$ produces $\Rightarrow$ clockwise torque $\Rightarrow$ +ve.

Same way $\tau_{T_2} = R T_2 \cos\theta$ $\Rightarrow$ anticlockwise torque –ve

$\tau_{mg} = 0$ (as passing through point) zero.

$\tau_{B_0} = I \pi R^2 B_0$. $\Rightarrow$ anticlockwise $\Rightarrow$ –ve

Put in (1)

$$\cancel{R} T_1 \cos\theta - \cancel{R} T_2 \cos\theta + 0 - I\pi R^2 B_0 = 0.$$

$$T_1 \cos\theta - T_2 \cos\theta = I\pi R^2 B_0.$$

$$(T_1 - T_2) \cos\theta = I\pi R^2 B_0.$$

$$(T_1 - T_2) \frac{1}{2} = I\pi R^2 B_0$$

$$T_1 - T_2 = 2I\pi R^2 B_0 \quad \text{(i)}$$

As there is also translational equilibrium. (vertical)

$$\Rightarrow \Sigma F = 0.$$

$$T_1 \cos\theta + T_2 \cos\theta = mg.$$

$$(T_1 + T_2)\cos\theta = mg$$

$$T_1 + T_2 = 2mg \quad -(ii)$$

From (i) & (ii) ,

$$T_1 - \cancel{T_2} = 2I\pi R^2 B_0$$

$$T_1 + \cancel{T_2} = 2mg$$

$$2T_1 = 2I\pi R^2 B_0 + 2mg$$

$$\boxed{T_1 = I\pi R^2 B_0 + mg}$$

Put in (ii) ,

$$T_2 = 2mg - I\pi R^2 B_0 - mg$$

$$\boxed{T_2 = mg - I\pi R^2 B_0}$$