PH1213 Presentation

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The Question

A stationary electric dipole p $= p\hat{z}$ is situated at the origin. A positive point charge q (mass m) executes circular motion (radius s) at constant speed in the field of the dipole. Characterize the plane of the orbit. Find the speed and the angular momentum of the charge.

The Solution

For the point charge to undergo circular motion in a plane, it must be below the dipole. And, for the path to be symmetric, the motion should be perpendicular to the z-axis. This means that the z-component of the electric field must be zero. This is not possible for $z > 0$.

We have,

$$\vec{E}_{dipole} = \frac{p}{4\pi\epsilon_0 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$$

Let θ be the angle made by $\vec{E}$ with the z-axis and $\alpha = \pi - \theta$. Since the z-component of $\vec{E}$ is zero,

$$|\vec{E}|(2cos\alpha.sin(\frac{\pi}{2} - \alpha)) = |\vec{E}|sin^2\alpha$$
$$|\vec{E}|(2cos^2\alpha) = |\vec{E}|sin^2\alpha$$

$$\Rightarrow tan^2\alpha = 2$$

Since $\alpha < \frac{\pi}{2}$, the negative root can be ignored.

$$tan\alpha = \sqrt{2}$$
$$\Rightarrow tan\theta = -\sqrt{2}, cos\theta = -\frac{1}{\sqrt{3}}, sin\theta = \sqrt{\frac{2}{3}}$$

From figure 2, the plane of the orbit is

$$z = -s.cot\alpha$$
$$\therefore z = -\frac{s}{\sqrt{2}}$$

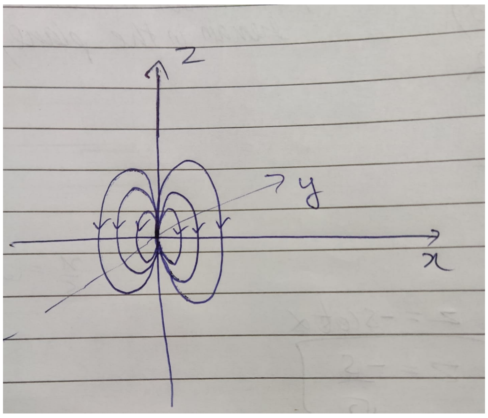

Figure 1: Field of the dipole

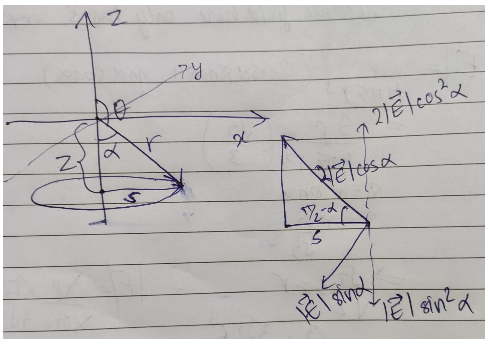

Figure 2:

For computing the magnitude of the electric field,

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}(2cos\alpha.sin\alpha + sin\alpha.cos\alpha)$$
$$= \frac{p}{4\pi\epsilon_0 r^3}(3.\frac{\sqrt{2}}{3})$$

$$\because r = s.cosec\alpha = s\sqrt{\frac{3}{2}}$$

$$|\vec{E}| = \frac{p}{3\sqrt{3}\pi\epsilon_0 s^3}$$

$$\therefore \vec{E} = \frac{p}{3\sqrt{3}\pi\epsilon_0 s^3}\hat{s}$$

To calculate the speed(v),

$$|\vec{F}_{centrifugal}| = |\vec{F}_{electric}|$$
$$\Rightarrow \frac{mv^2}{s} = q|\vec{E}|$$

$$\therefore v = \sqrt{\frac{qp}{3\sqrt{3}\pi\epsilon_0 m s^2}}$$

To calculate the magnitude of angular momentum(L),

$$L = mvs$$

$$\therefore L = \sqrt{\frac{qpm}{3\sqrt{3}\pi\epsilon_0}}$$

The direction of angular momentum depends upon the direction of the velocity of the point charge.