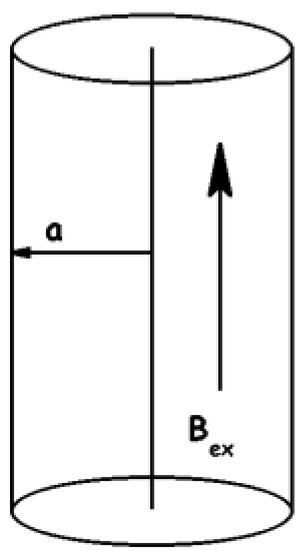
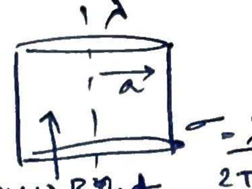
An infinitely long wire with linear charge density $-\lambda$ lies along the z-axis. An insulating cylindrical shell of radius a and moment of inertia I per unit length is concentric with the wire and can rotate freely about the z-axis. The areal charge density on the cylinder is $\sigma = \lambda/2\pi a$ and is uniformly distributed. The cylinder is immersed in an external magnetic field $B_{ex}\hat{z}$, and is initially at rest. Starting at $t = 0$ the external magnetic field is slowly reduced to zero over a time $T \gg a/c$, where c is the speed of light. What is the final angular velocity ω of the cylinder?



Given, $\sigma = \frac{\lambda}{2\pi a}$; radius $= a$; length is infinite

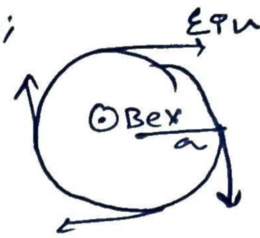


As we know that,

$$\bar{\nabla} \times \bar{E} = -\frac{dB}{dt} \quad ; \text{ Faradays law. Bind}$$

$$\int E \cdot dl = \frac{-d\phi}{dt} ; \quad \phi = \int B \cdot dA \text{ ; lenz law.}$$

Now for a circular loop of radius a;



$$E_{in} \cdot 2\pi a = \frac{-d(B \cdot \pi a^2)}{dt}$$

$$E_{in} = -\frac{dB}{dt} \cdot \frac{a}{2} \quad \ldots\ldots (1)$$

As we know;

Force per unit length $=$ charge per unit $\times E_{in}$

$$F_L = (\sigma \cdot 2\pi a) \times E_{in} ,$$

$$\text{Torque} = \vec{a} \times \vec{F}$$

$$\tau_L = \vec{a} \times \vec{F_L}$$

$$= \sigma \cdot 2\pi a \times E_{in} \times a$$

$$I \frac{d\omega}{dt} = \sigma \, 2\pi a^2 \left(-\frac{dB}{dt}\right) \frac{a}{2} ,$$

given $\sigma = \frac{\lambda}{2\pi a}$

$$\therefore \quad I \frac{d\omega}{dt} = \frac{\lambda}{\cancel{2\pi a}} \frac{a^{\cancel{3}} \times \cancel{2\pi}}{2} \left(-\frac{dB}{dt}\right)$$

integration on both sides

$$\int I \cdot \frac{d\omega}{dt} = -\frac{\lambda a^2}{2} \int \frac{dB}{dt}$$

$$I \cdot \int \frac{d\omega}{dt} = -\frac{\lambda a^2}{2} \int \frac{dB}{dt}$$

$$\int \frac{d\omega}{dt} = -\frac{\lambda a^2}{2I} \int \frac{dB}{dt}$$

$$(\omega_{final} - \cancel{\omega_{int}}^{\,0}) = \frac{\lambda a^2}{2I} - (B_{final} - B_{int})$$

$$\omega_{final} = \frac{\lambda a^2}{2I} (B_{int} - B_{final});$$

in general people assume B_{final} shall be "zero" but not; because, at the time $B_{induced}$ becomes zero but the cylinder rotates with an angular velocity ω; can cause an magnetic field:

Induced current I; per unit length I_L is

$$\text{current } I_L = \frac{\text{charge per unit length}}{\text{time period}}$$

$$\text{current } I_L = \frac{\sigma \cdot \cancel{2\pi} a}{\cancel{2\pi}} \cdot \omega, \quad \because T = \frac{2\pi}{\omega}$$

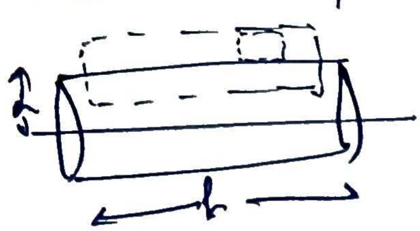
$$\text{current } I_L = \sigma a \omega; \quad \sigma = \frac{\lambda}{2\pi a}$$

$$\text{current } I_L = \frac{\lambda}{2\pi} \omega; \quad \omega \text{ is } \omega_{final} \text{ at } B_{induced} = 0$$

$\therefore$ magnetic field due to cylinder B_{final}.

Assume an Ampere

$$B_{final} \cdot L = \mu_0 \, I_L^{\text{current}} \cdot L$$



$$B_{final} = \frac{\mu_0 \lambda \omega_{final}}{2\pi}$$

$$\omega_f = \frac{\lambda a^2}{2I}\left(B_{ext} - \frac{\mu_0 \lambda \omega_{final}}{2\pi} \right)$$

$$\omega_f = \frac{\frac{\lambda a^2}{2F}(B_{int})}{1 + \frac{\mu_0 \lambda^2 a^2}{4\pi I}}$$