Q) Find the energy of a uniformly charged spherical shell of total charge q and radius R.

$$W = \frac{1}{2}\int \sigma V da$$

Potential at surface of this sphere is

$$\cancel{\frac{1}{4}\pi\varepsilon_0} \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{R}$$

$$W = \frac{1}{8\pi\varepsilon_0}\frac{q}{R}\int \sigma da = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}$$

Sol2 →

Inside the sphere $E = 0$ outside

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{so}$$

$$E^2 = \frac{q^2}{(4\pi\varepsilon_0)^2 r^4}$$

$$W_{tot} = \frac{\varepsilon_0}{2(4\pi\varepsilon_0)^2}\int_{outside}\left(\frac{q^2}{r^4}\right)(r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

$$= \frac{1}{32\pi^2\varepsilon_0} q^2 4\pi \int_R^\infty \frac{1}{r^2} dr = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}$$

A perplexing inconsistency

$$W = \frac{\varepsilon_0}{2}\int E^2 d\tau \quad \text{--- (1)}$$

This equation clearly implies that energy of a stationary charge distribution is always positive.

whereas

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i) \quad \text{--- (2)}$$

this equation suggests that it can be both positive or negative.

Energy of two eq but opposite charges ~~co~~ at distance r apart is $\left(-\frac{1}{4\pi\varepsilon_0}\right)\frac{q^2}{r}$

What's gone wrong.

Answer is that both are correct.

Equation 2 does not take into account the work necessary to make the point charges in the first place; we started with point charges and simply found the work required to bring them together.

Eq (1) indicates that energy of a point charge is infinite. This is more appropriate when you're dealing with point charges.

But in this case this self energy does not play much role because if we compare charge on a point with respect to all other charges on that continuous charge distribution it would be insignificant.

Significant difference between energies via the two formula can be seen in the following case.

Eg → We have a positive & a negative charge of equal mag at d distance apart so here if

If we ~~take~~ calculate energy from the formula (2) it gives us $\cancel{\frac{-kq_1 r^2}{r}}$ $\frac{-kq^2}{r}$ whereas from formula (1) we get infinity so here we because we have point charge the distinction becomes significant.

$$W = \frac{\varepsilon_0}{2(4\pi\varepsilon_0)^2} \int \left(\frac{q^2}{r^4}\right) (r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

$$= \frac{q^2}{8\pi\varepsilon_0} \int_0^\infty \frac{1}{r^2} dr = \infty$$