Physics Presentation

Friday, 10 June 2022 10:02 AM

2.41 *A sphere between planes* **

A spherical shell with radius R and surface charge density σ is sandwiched between two infinite sheets with surface charge densities $-\sigma$ and σ, as shown in Fig. 2.46. If the potential far to the right at $x = +\infty$ is taken to be zero, what is the potential at the center of the sphere? At $x = -\infty$?

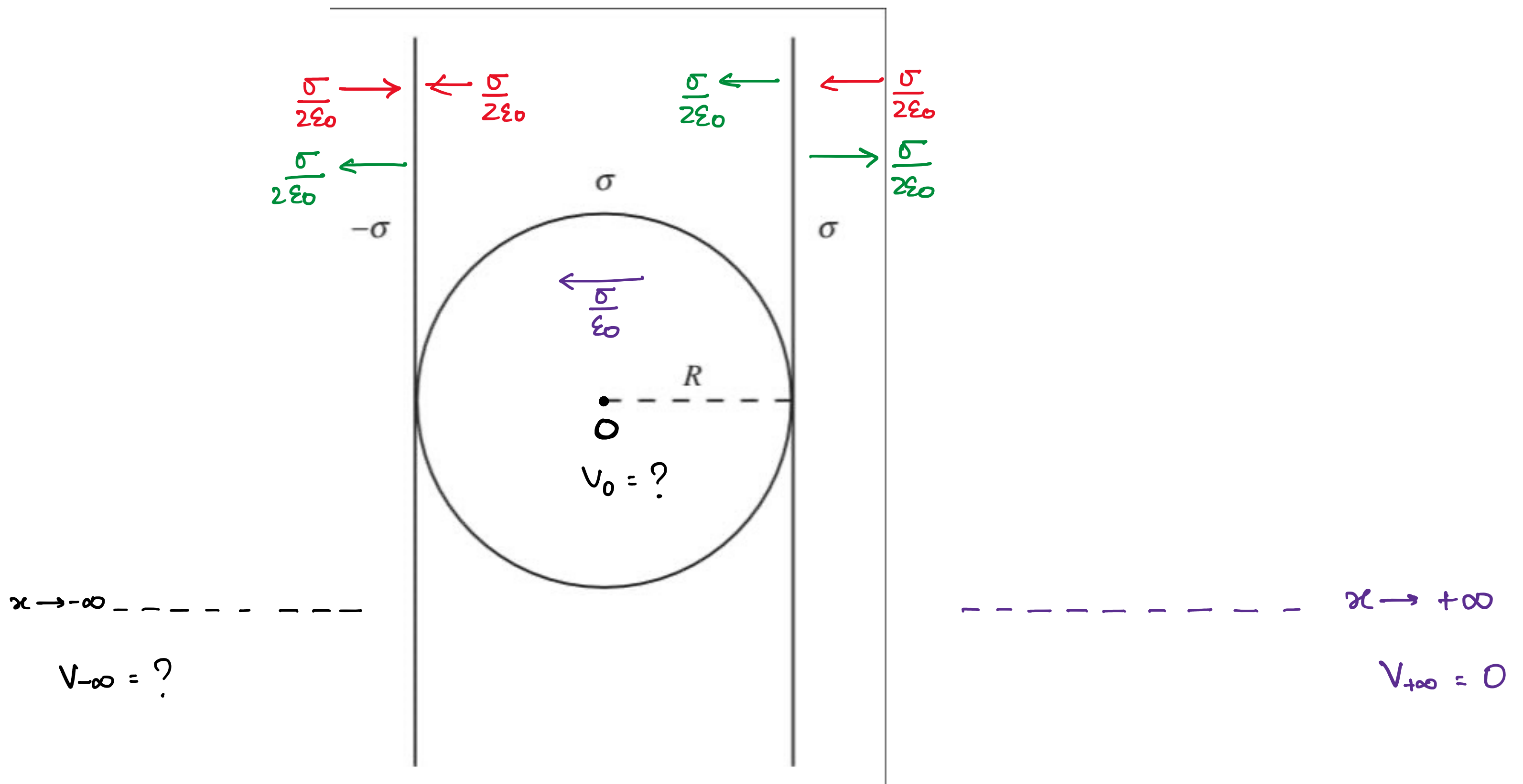

Figure 2.46.

We know the electric field due to a charge sheet of charge density $+\sigma$; $E = \frac{+\sigma}{2\varepsilon_0}$

$-\sigma$; $E = \frac{-\sigma}{2\varepsilon_0}$

Thus $\vec{E}$ outside the plate is $\frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$

b/w the plate is $\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Thus the $\vec{E}$ due to sheets is non-zero only b/w them, i.e the $\vec{E}$ on the right of the system is solely due to the sphere in between.

Now the potential at the point where the sphere touches the right sheet is $\frac{kQ_{sphere}}{R} = \frac{Q_{sphere}}{4\pi\varepsilon_0 R} = \frac{\sigma \cdot A_{sphere}}{4\pi\varepsilon_0 R} = \frac{\sigma \cdot 4\pi R^2}{4\pi\varepsilon_0 R} = \frac{\sigma R}{\varepsilon_0}$

The sphere has no internal electric field $\because$ $q_{enc} = 0 \Rightarrow \phi = \frac{Q_{enc}}{\varepsilon_0} = 0$ But $\phi = \oint \vec{E} \cdot d\vec{A}$

$0 = E \oint dA \cos\theta \Rightarrow \vec{E} = 0$

The field in its interior is only due to the sheets. i.e it takes the value $\frac{\sigma}{\varepsilon_0}$ pointing leftwards.

The potential difference between the surface of the sphere and its center is $-\frac{\sigma R}{\varepsilon_0}$, with the center at lower potential.

(The $\vec{E}$ inside a hollow conducting sphere is 0, so the potential remains constant at the value it reaches the surface.)

The total potential at the center relative to $x = +\infty$ is therefore $\frac{\sigma R}{\varepsilon_0} - \frac{\sigma R}{\varepsilon_0} = 0$

The potential at the point where the sphere touches the left sheet, relative to center of sphere $= -\frac{\sigma R}{\varepsilon_0}$

The potential at $x = -\infty$ relative to the contact point on left sheet $= \frac{kQ_{sphere}}{R} = -\frac{\sigma A_{sphere}}{4\pi\varepsilon_0 R} = -\frac{4\pi R^2 \sigma}{4\pi\varepsilon_0 R} = -\frac{\sigma R}{\varepsilon_0}$ due to the $\vec{E}$ of the sphere. ($\because$ $\vec{E}$ due to the sheet $= 0$ outside the plates)

The total potential at $x = -\infty$ relative to center (which has same potential as $x = +\infty$, i.e 0) is $-\frac{\sigma R}{\varepsilon_0} - \frac{\sigma R}{\varepsilon_0} = -\frac{2\sigma R}{\varepsilon_0}$