Q. A thick spherical shell carries charge density

$$\rho = \frac{k}{r^2} \quad (a \le r \le b)$$

- Find the electric field in three regions a) $r < a$
- also plot $|E|$ as function of r. b) $a < r < b$
c) $r > b$

Solutions :- Gauss law states =

$$\boxed{\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}}$$

where Q_{enc} is the enclosed charge within gaussian surface.

a) $r < a$, we select the surface just before a.
. We observe $Q_{encl} = 0$
$\therefore E = 0$

b) $a < r < b$

here $E \cdot (\text{area}) = \frac{Q_{enclosed}}{\varepsilon}$

$Q_{enc} = \int \rho \, d\tau$ $\quad d\tau = r^2 \sin\theta \, d\theta \, d\phi \, dr$

$$= \int \frac{k}{r^2} \cdot r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= k \int_a^r dr \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$-(r-a)(2\pi)(\cos\theta)\Big|_0^{\pi} \times k$$

$$-(r-a)(2\pi)(-1-1) \times k$$

$$= k(r-a)(4\pi)$$

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{k(4\pi)(r-a)}{\varepsilon_0} \quad -(1)$$

Area of sphere $= 4\pi r^2$

$$E \cdot 4\pi r^2 = \frac{k(4\pi)(r-a)}{\varepsilon_0}$$

$$\boxed{E = \frac{k(r-a)}{\varepsilon_0 (r^2)} \hat{r}}$$

c) ~~r=b~~ $r > b$

~~here even though r=b the charges are from spherical shell from a to b~~ even though $r > b$ the charge distribution is from a to b.

∴ here the integration changes

$$Q_{enc} = k \int_a^b dr \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{k}{\varepsilon_0} (b-a)(4\pi)$$

$$\therefore E(4\pi r^2) = \frac{k(b-a)(4\pi)}{\varepsilon_0 (4\pi r^2)}$$

since here the surface area will be of radius r $(r > b)$

$$= \boxed{E = \frac{k(b-a)}{\varepsilon_0 r^2} \hat{r}}$$