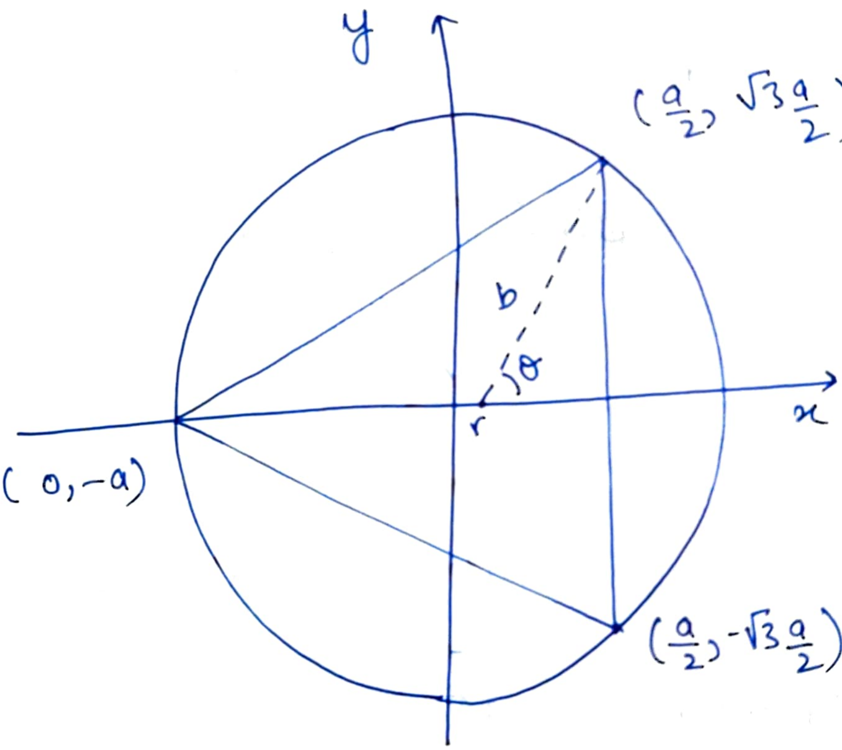
Consider a equilateral triangle, inscribed in a circle of radius a, with a point charge q at the vertex. The electric field is zero at the center, but there are three other points inside the triangle where the field is zero.

Where are they?

Soln



Taking one point on x-axis at $x = r$.

Here, the field is

$$E_x = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(a+r)^2} - \frac{2\cos\theta}{b^2}\right] = 0$$

$$\Rightarrow \quad \frac{2\cos\theta}{b^2} = \frac{1}{(a+r)^2}$$

Now,

$$\cos\theta = \frac{(a/2)-r}{b}$$

$$b^2 = \left(\frac{a}{2}-r\right)^2 + \left(\frac{\sqrt{3}a}{2}\right)^2$$

$$= (a^2 - ar + r^2)$$

$$\therefore \quad \frac{2[(a/2) - r]}{(a^2 - ar + r^2)^{3/2}} = \frac{1}{(a+r)^2}$$

simplifying $\frac{r}{a} \equiv u$

$$\frac{(1-2u)}{(1-u+u^2)^{3/2}} = \frac{1}{(1+u)^2}$$

$$(1-2u)^2 (1+u)^2 = (1-u+u^2)^3$$

Multiplying out each side,

$$1-6u^2 - 4u^3 + 9u^4 + 12u^5 + 4u^6$$
$$= 1-3u + 6u^2 - 7u^3 + 6u^4 - 3u^5 + u^6$$

or

$$3u - 12u^2 + 3u^3 + 3u^4 + 15u^5 + 3u^6 = 0$$

$u = 0$ is a solution (at center of the triangle)

factoring out $3u$, we get

$$1 - 4u + u^2 + u^3 + 5u^4 + u^5 = 0$$

→ Solving this, we get 2 complex roots & one negative roots.

→ 2 remaining solutions are $u = 0.2847$ & $u = 0.626$. The latter is outside the circle & is spurious.

→ So $\boxed{r = 0.2847a}$

→ Generalising for an n-sided regular polygon, there are evidently n points, lying on the radial spokes that bisect the sides

→ Their distance from the center appear to grow monotonically with

n: $r(3) = 0.285$

$r(4) = 0.547$

$r(5) = 0.689 \ldots$

→ At $n \to \infty$, they fill out a circle that coincides with the charge itself.