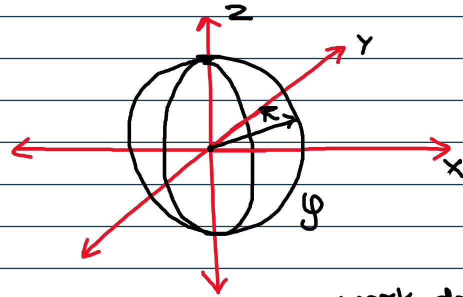
Physics Problem Tut.

19 May 2022 19:33

★ Calculate the Self-Energy of a spherical charge distribution of uniform volume charge density ρ, bound by a radius R containing a total charge Q

(A)



The $\vec{E}$ of a sphere for $r < R$;

$$\vec{E}_{in} = \frac{KQ}{R^3}\vec{r}$$

for $r > R$;

$$\vec{E}_{out} = \frac{KQ}{r^3}\vec{r}$$

work done to make a charge distrub

We know that $W = \frac{1}{2}\int \rho V \, d\tau$

where V is the existing potential.

$$\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}$$

⇒ SE is the energy required to bring the infinitesimal charges from ∞ to form the charge distrub

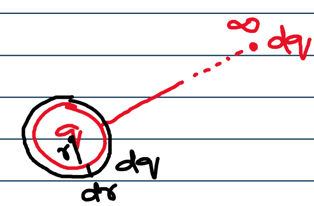
$$\rho = \varepsilon_0 (\vec{\nabla}\vec{E})$$

$$W = \frac{\varepsilon_0}{2}\int_V (\vec{\nabla}\vec{E}) V \, d\tau$$

(simplify using $\vec{\nabla}(\vec{E}V) = \vec{\nabla}V\,\vec{E} + V\vec{\nabla}\vec{E}$)

$$= \frac{\varepsilon_0}{2}\oint_V E^2 \, d\tau$$

↳ all space



$$W_{net} = SE = \frac{\varepsilon_0}{2}\int_0^r \left(\frac{KQ}{R^3}r\right)^2 4\pi r^2 \, dr + \frac{\varepsilon_0}{2}\int_r^\infty \left(\frac{KQ}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{\varepsilon_0}{2}\left[\int_0^R \frac{K^2Q^2}{R^6} 4\pi r^2 r^2 \, dr + \int_R^\infty \frac{K^2Q^2}{r^2} 4\pi \, dr\right]$$

$$= \frac{KQ^2}{2}\left[\int_0^R \frac{r^4}{R^6} dr + \int_R^\infty \frac{1}{r^2} dr\right]$$

$$= \frac{KQ^2}{2}\left[\frac{1}{5R} + \left[\frac{1}{R}\right]\right]$$

$$= \frac{3KQ^2}{5}$$

⇒ We observe that we have used only the $\vec{E}$ to calculate the energy of the charge distribution