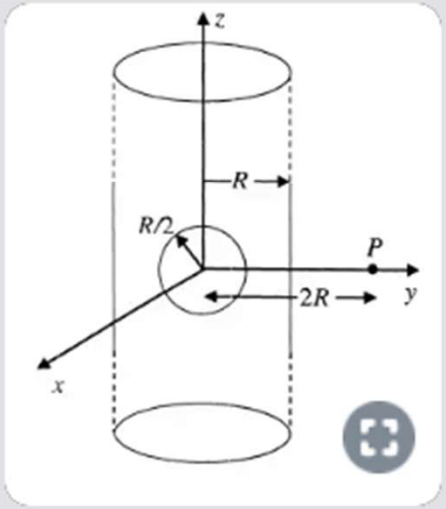
ELECTROSTATICS

Question

An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. it has a spherical cavity of radius R/2 with its center on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\epsilon_0}$. The value of k is:



Solution

Verified by Toppr

Correct option is A)

The given system of cylinder with cavity can be expressed as superposition of Infinite cylinder with charge density $+\rho$ and a sphere with charge density $-\rho$.

Field due to infinite cylinder is given by $E_{cyl} = \frac{\lambda}{2\pi d\epsilon_0}$

Here, λ is charge per unit length $\lambda = \rho \times A = \pi R^2 \rho$

and $d = 2R$

Thus, $E_{cyl} = \frac{\pi R^2 \rho}{2\pi(2R)\epsilon_0} = \frac{\rho R}{4\epsilon_0}$

Field due to sphere is given by $E_{sph} = \frac{1}{4\pi\epsilon_0}\frac{Q}{d^2}$

Here, Q is the total charge in the sphere. $Q = -\rho \times V = -\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho$

and $d = 2R$

Thus, $E_{sph} = -\frac{4\pi\left(\frac{R}{2}\right)^3 \rho}{3 \times 4\pi\epsilon_0(2R)^2} = -\frac{\rho R}{96\epsilon_0}$

Thus, the net electric field is $E = \frac{\rho R}{\epsilon_0}\left(\frac{1}{4} - \frac{1}{96}\right) = \frac{23\rho R}{96\epsilon_0}$

Thus $16k = 96 \Rightarrow k = 6$