PHYSICS ROLL.No - 20211169

Q. A hollow spherical shell carries charge density

$$\rho = \frac{k}{r^2} \quad (k - \text{is constant})$$

in the region $a \leq r \leq b$. Find the electric field in the three regions (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$.
Plot $|E|$ as a function of r

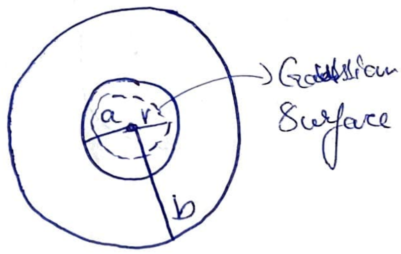
Solution :-

Given $\rho = \frac{k}{r^2}$ (k is constant) for $a \leq r \leq b$

(i) For $r < a$

Draw a spherical surface of radius $r < a$, Consider it to be a Gaussian surface.

From Gauss law



$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{enc}}{\epsilon_0}$$

$Q_{enc} = 0$ ($\because \rho = 0$ in this region)

$$\Rightarrow \vec{E} = 0$$

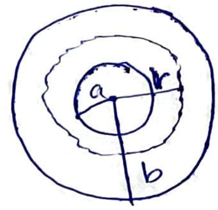
(ii) For $a < r < b$

Consider a spherical surface of radius $a < r < b$, let it be our Gaussian surface

From Gauss' law $\oint \vec{E} \cdot \vec{da} = \frac{Q_{enc}}{\epsilon_0}$

$$Q_{enc} = \int \rho \, d\tau = \int_0^{2\pi}\int_0^{\pi}\int_a^{r} \frac{k}{(r')^2} \cdot (r')^2 \, dr' \sin\theta \, d\theta \, d\phi$$

$$Q_{enc} = 4\pi k (r-a)$$



$$\oint \vec{E} \cdot \vec{da} = \frac{4\pi k (r-a)}{\epsilon_0}$$

$$|\vec{E}| \int da = \frac{4\pi k (r-a)}{\epsilon_0}$$

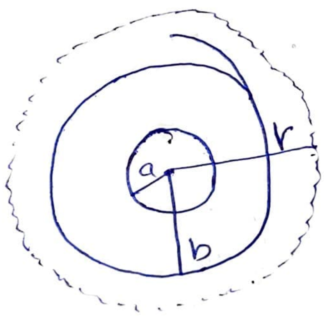
$$|\vec{E}| \, 4\pi r^2 = \frac{4\pi k (r-a)}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{k(r-a)}{r^2 \epsilon_0} \hat{r}}$$

(iii) $r > b$ Draw a spherical surface of radius $r > b$
Consider it to be a Gaussian surface

→ By Gauss' law

$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{enc}}{\epsilon_0}$$



$$Q_{enc} = \int \rho \, d\tau$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_a^{b} \frac{k}{(r')^2} \cdot (r')^2 \, dr' \sin\theta \, d\theta \, d\phi$$

$$= 4\pi k (b-a)$$

$$\oint \vec{E} \cdot \vec{da} = \frac{4\pi k (b-a)}{\epsilon_0}$$

$$|\vec{E}| \cdot 4\pi r^2 = \frac{4\pi k (b-a)}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{k(b-a)}{r^2 \epsilon_0} \hat{r}}$$