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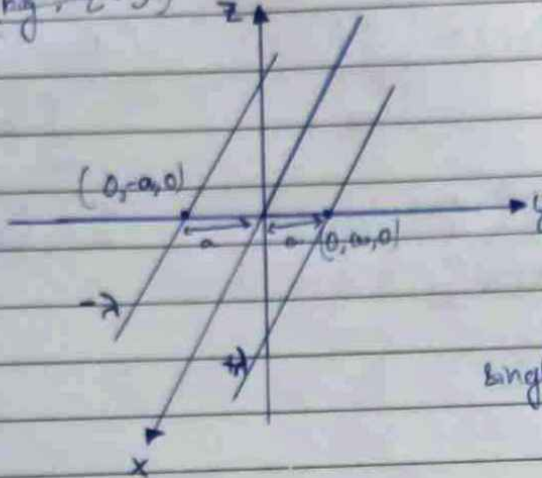
PROBLEM 2.52: Two infinitely long wires running parallel to the x-axis carry uniform charge densities $+\lambda$ and $-\lambda$ ~~(Fig. 2.5)~~



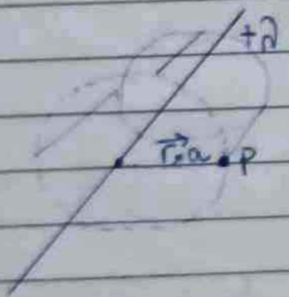

(a) Find the potential at any point $(x,y,z)$.

~~First dera~~

(b)

First derive potential at any point due to a single uniformly charged $\infty$ wire.

Solution: For solving this question, let us calculate the potential of a uniformly charged infinite wire.




Finding potential at point P which is $\vec{r}=a$ units radially away from wire.

Note: Here we cannot consider potential to be 0 at $\infty$ ~~at~~ as there wire extends to $\infty$.

Therefore we redefine our boundary condition and assume,

$$V=0 \quad \text{at} \quad \vec{r}=1$$

We are taking 1 as it make computation easy.

In reality we can assume potential to be zero at any point convenient to us.

$\boxed{r = \text{radial distance from the wire}}$

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Now, consider a gaussian cylindrical surface.

$$\therefore \quad \vec{E} \cdot (2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$ (From Gauss law, $\int \vec{E} \cdot \vec{da} = \frac{q_{enc}}{\varepsilon_0}$)

$$\Rightarrow \vec{E} = \frac{\lambda \cancel{l}}{2\pi r \cancel{l} \varepsilon_0} \hat{r}$$ $l$ = length of cylinder

$$V = -\int_{a}^{1} \vec{E} \cdot dr$$

$$V = -\int_{1}^{a} \frac{\lambda \cancel{l}}{2\pi r \cancel{l} \varepsilon_0} dr$$

$$V_+ = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{a}{1}\right)$$

$\therefore$ for wire with uniform charge density $+\lambda$

Potential, $V_+ = \frac{-\lambda}{2\pi\varepsilon_0} \ln(a_+)$ --- (1) where $a_+$ is radial distance from wire.

Illy. Potential of wire with linear charge density,

$$V_- = \frac{\lambda}{2\pi\varepsilon_0} \ln(a_-) \quad (2)$$ $a_-$ is radial distance from wire

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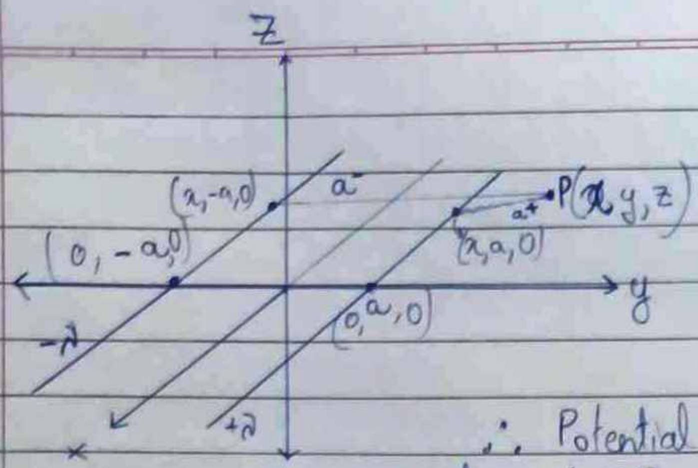


$\therefore$ Potential at point P from equations 1 & 2 are:

$$\boxed{V_P = V_+ + V_- = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{a^-}{a^+}\right)}$$

Now, $a^- = \sqrt{(y+a)^2 + z^2}$

$a^+ = \sqrt{(y-a)^2 + z^2}$

$\therefore$ Potential at point P, $V_P = \dfrac{\lambda}{2\pi\varepsilon_0} \ln\left(\sqrt{\dfrac{(y+a)^2 + z^2}{(y-a)^2 + z^2}}\right)$

$$V_P = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{(y+a)^2 + z^2}{(y-a)^2 + z^2}\right)$$