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BATCH 1
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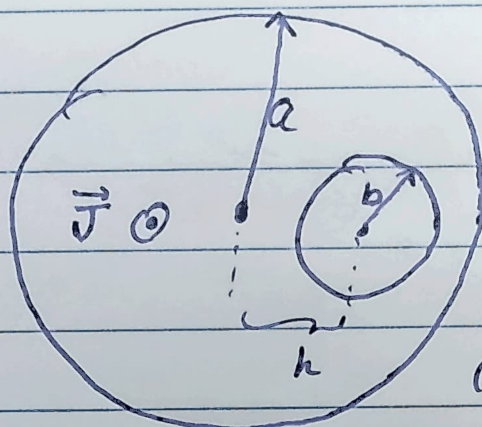
Physics Presentation:

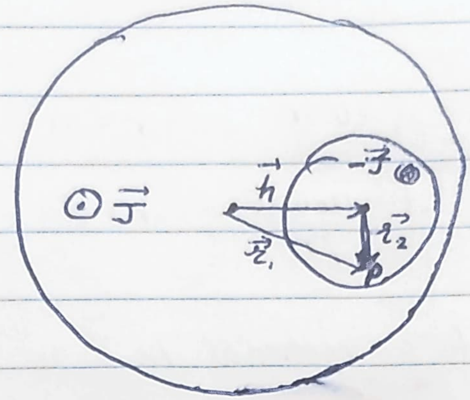
Q: CYLINDRICAL CONDUCTOR WITH AN OFF-CENTER CAVITY!



(radius a)

→ an infinite cylindrical conductor has a cylindrical cavity of radius b bored parallel to, and centered at a distance $h < a-b$ from the cylindrical axis as shown in figure. The current density is uniform over the cross-section (and perpendicular to it), excluding the cavity. Evaluate the magnetic field $\vec{B}$, & hence show that it is uniform inside the cavity.

SOLUTION:



→ according to superposition, the given set-up is equivalent to a field generated by a cycdr uniform cylindrical wire of section with radius a, & charge density J, superimposed with a uniform cylidrical wire of section with radius b & charge density −J running opposite to +J.

★ $\vec{B}$ in uniform cylindrical wire of radius a:

→ I define the direction of J as $+\hat{z}$, then the −J is in the $-\hat{z}$ direction. Also let the axis of the wire coincide with the $\hat{z}$-axis. Using cylindrical co-ordinates, $\vec{B}$ is then defined in the $\hat{\phi}$ direction.

→ $\vec{B} = B_\phi(r)\,\hat{\phi}$ can be evaluated using Ampere's Law.

↳ consider a circle of radius r.

(i) inside circle: $\oint \vec{B}(r)\,dl = \mu_0 I_{enc}$

magnetic field is uniform, ie:

$$\vec{B}(r)\oint dl = \mu_0 I_{enc}$$

$$\Rightarrow 2\pi r\,\vec{B}(r) = \mu_0 \int \vec{J}\cdot d\vec{s}$$

$$\Rightarrow \vec{B}_\phi(r) = \frac{\mu_0 J \pi r^2}{2\pi r} = \frac{\mu_0 J r}{2}$$

& similarly, for $r > a$, $B_\phi(r) = \frac{\mu_0 a^2 J}{2r}$.

in summary: $B_\phi(r) = \begin{cases} \frac{\mu_0 J r}{2} & r < a \\ \frac{\mu_0 J a^2}{2r} & r > a \end{cases}$ — (I)

converting to vector form using standard cylindrical coordinates:

$$\vec{B}_w(r : a) = \begin{cases} \frac{\mu_0}{2} \vec{J} \times \vec{r} & \text{for } r < a \\ \frac{\mu_0}{2} \frac{a^2}{r^2} \vec{J} \times \vec{r} & \text{for } r > a \end{cases} \quad \text{— (II)}^*$$

c) Consider now the problem at hand:
↳ magnetic field at point P is sum of magnetic fields generated by wire of radius a with current $\vec{J}$ & wire of radius b with current $-\vec{J}$, with the distance between the two axes = $\vec{h}$. & the distances b/w point P and the centres = $\vec{r}_1$ & $\vec{r}_2$ respectively. we then have $\vec{r}_1 - \vec{r}_2 = \vec{h}$.

↳ at P, $\vec{B}(P) = \vec{B}_w(\vec{r}_1 : a) + \vec{B}_w(\vec{r}_2 : b)$.
→ particularly inside the cavity: $r_1 < a$ & $r_2 < b$

$\therefore$ $\vec{B}_w(\vec{r}_1 : a) = \frac{\mu_0}{2} \vec{r}_1 \times \vec{J} \hat{\phi}$ & $\vec{B}_w(r_2, b) = \frac{\mu_0}{2} \vec{r}_2 \times (\vec{J}) \hat{\phi}$

$\therefore$ $\vec{B}(P) = \frac{\mu_0((\vec{r}_1 - \vec{r}_2) \times \vec{J})}{2} = \frac{\mu_0}{2} h \times \vec{J} \hat{\phi}$ which is a constant

Addendum: $\star$ $\vec{J} \times \vec{r}$ is simply $rJ\hat{\phi}$

Proof: $\vec{J} = J\hat{z}$ & $\vec{r} = r\hat{r}$

then $\vec{J} \times \vec{r} = \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ 0 & 0 & J \\ r & 0 & 0 \end{vmatrix}$

$$\Rightarrow \vec{J} \times \vec{r} = 0\hat{r} - (-rJ)\hat{\phi} + 0\hat{z}$$

$$= rJ\hat{\phi}$$

& thus we may convert I $\longrightarrow$ II