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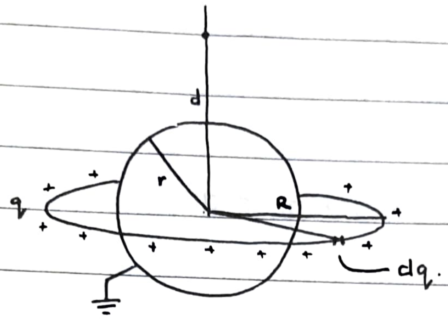
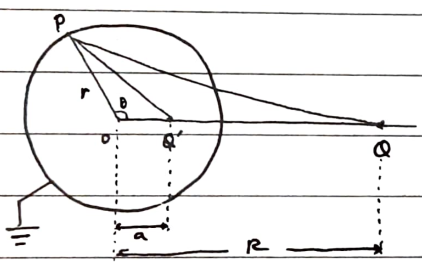
Question : A grounded conducting sphere of radius r is surrounded by a concentric ring of radius R ($R > r$). The ring carries a charge q uniformly distributed. Find the electric potential at a point on the axis of the ring at a distance d from its centre ($d > r$).



Answer :

First consider the system where a point charge Q is placed at a distance R from the centre of a grounded conducting sphere of radius r.



Place an image charge Q' on the line joining the centre O and Q at a distance a from O.

To find the magnitude & location of the image charge, we use the boundary conditions that the potential on the sphere & at infinity is zero.

So for any point P on the sphere, $V(P) = 0$:

$$V(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\text{dist. b/w P and Q}} + \frac{Q'}{\text{dist. b/w P \& Q'}}\right]$$

$$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\sqrt{r^2+R^2-2rR\cos\theta}} + \frac{Q'}{\sqrt{r^2+a^2-2ra\cos\theta}}\right]$$

(using cosine rule)

$$Q^2(r^2+a^2-2ra\cos\theta) = Q'^2(r^2+R^2-2rR\cos\theta)$$

This is true for any point P on the sphere (for any value of θ).

We get two equations :

$$Q^2(r^2+a^2) = Q'^2(r^2+R^2)$$

$$2Q^2 ra\cos\theta = 2Q'^2 rR\cos\theta .$$

$$Q' = -\sqrt{\frac{a}{R}}\, Q$$

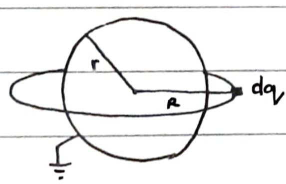
On substituting into the first equation:

$$Ra^2 - (r^2+R^2)a + Rr^2 = 0$$

$a = R$ (not possible) $\qquad a = r^2/R$

$\therefore$ We place an image charge $Q' = -\frac{r}{R} Q$ at a distance $\frac{r^2}{R}$ from the centre of the sphere.

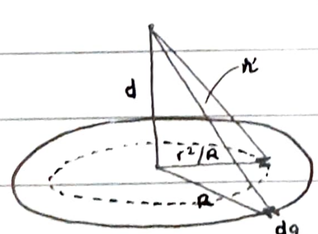
For the original problem, take an infinitesimal charge dq on the ring.



The image charge corresponding to dq is of magnitude $-\frac{r}{R} dq$ at $\frac{r^2}{R}$ from the centre.

Due to all of the infinitesimal charges dq around the ring, an 'image ring' of radius $\frac{r^2}{R}$ can be taken inside the sphere.

The problem simplifies to:



V at distance d due to the original ring

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\sqrt{d^2+R^2}}$$

Similarly, V due to the image ring $= \frac{1}{4\pi\varepsilon_0}\left(\frac{-rq}{R\sqrt{d^2+(r^2/R)^2}}\right)$

Total potential $= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{d^2+R^2}} - \frac{rq}{\sqrt{d^2R^2+r^4}}\right]$