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20211110
BATCH - 4

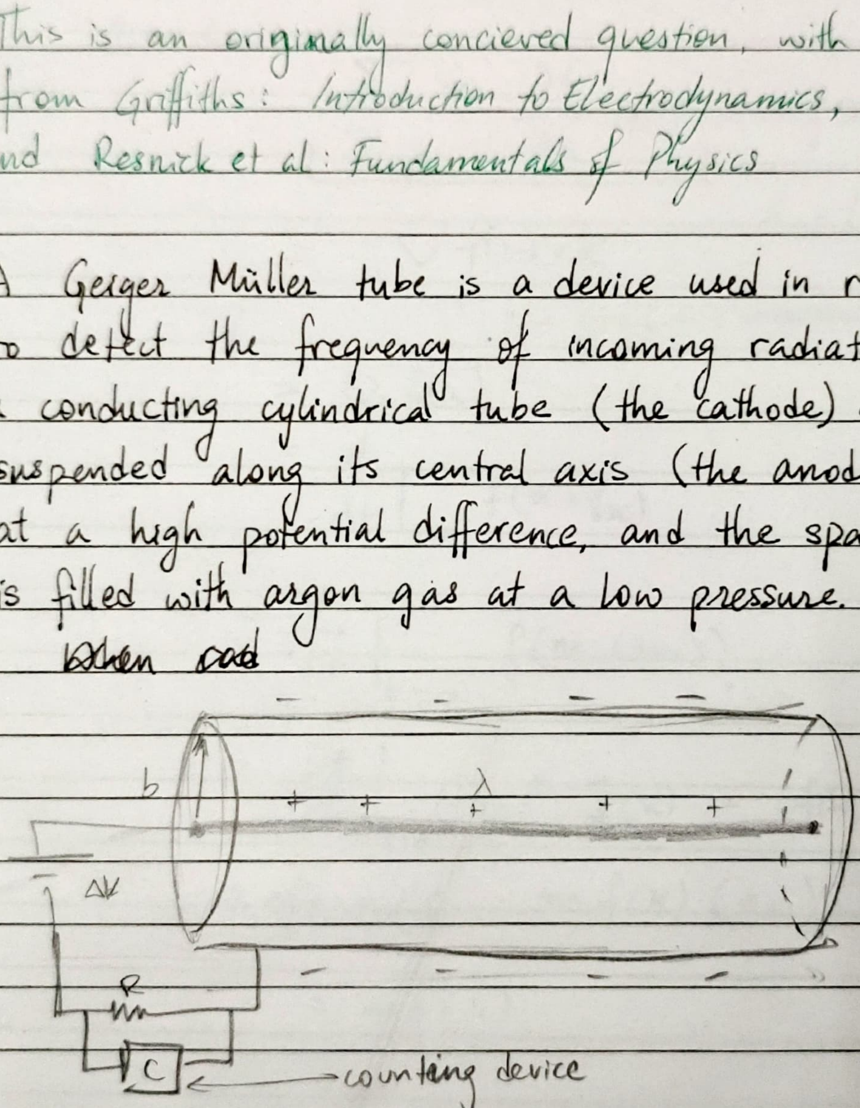
~~Prob~~

TUTORIAL PRESENTATION PROBLEM

This problem explores the working of a Gieger-Muller tube, a device used in radiation detection. ~~dev~~
This is an originally concieved question, with inspiration taken from Griffiths: *Introduction to Electrodynamics*,
and Resnick et al: *Fundamentals of Physics*

A Geiger Müller tube is a device used in radiation detectors, to detect the frequency of incoming radiation. It consists of a conducting cylindrical tube (the cathode) and a metal wire suspended along its central axis (the anode). They are held at a high potential difference, and the space between them is filled with argon gas at a low pressure.

~~When rad~~



b = radius of tube
a = radius of wire
L = length of tube

When radiation incidents on an argon atom, it gets ionized, sending the electron accelerating towards the anode at the center (under the influence of the electric field). When the electron gets enough energy, it can ionize another argon atom. This sets off a chain reaction called electron avalanche. When the large

number of electrons reaches the anode, it creates a pulse of current, which is then picked up by a counting device.
The charged particles then neutralize via the connecting wires, and the tube is ready to pick up another count.

a) Given that the battery maintaining the potential difference supplies 500 V, and the radii of the cylinder and tube are $b = 10^{-2}$ m and $a = 10^{-4}$ m, find the charge per length of the inner wire.

b) Use the result of the previous part to find how many levels of the electron avalanche occurs, if the initial electron was ionised close to the ~~outh~~ outer tube (i.e., at a distance b from the center). Given: the first ionisation energy of argon is 1520.6 J/mol

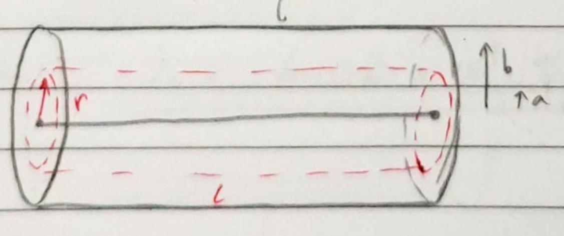
ANSWERS

a) Given: $\Delta V = V_a - V_b = 500V$

$a = 10^{-4}$ m, $b = 10^{-2}$ m

Need to find the charge per length (λ) of the inner wire

We start with taking a Guassian Surface as:



According to Guass Law, $\int \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$

Here, $\vec{E}$ is always parallel to $d\vec{a}$ on the curved surface, and is equal in magnitude by symmetry

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And, it it perpendicular to $d\vec{a}$ on the flat circular surfaces

Hence, the equation becomes :

$$E\,(2\pi r L) = \lambda L / \varepsilon_0$$

from this we get

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{r}$$

Now, since we know the potential difference

$$-E \cdot dr = dV$$

$$-\frac{\lambda}{2\pi\varepsilon_0}\frac{\hat{r}}{r} \cdot dr = dV$$

$$\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}\,dr = \int_{V_a}^{V_b} dV$$

$$-\frac{\lambda}{2\pi\varepsilon_0}\ln\left(b/a\right) = V_b - V_a = -(V_a - V_b)$$

So, $$\lambda = \frac{(V_a - V_b)\,(2\pi\varepsilon_0)}{\ln\left(b/a\right)}$$

Substituting : $$\lambda = \frac{500\text{ V} \times 2 \times 3.14 \times 8.85 \times 10^{-12}}{\ln\left(\frac{10^{-2}}{10^{-4}}\right)}$$

We get, $$\lambda = 6.03 \times 10^{9}\ C/m$$

b) Given: Ionization energy, E_i of Argon $= 1520.6 \times 10^3$ J/mol

We assume that the ionized free eletron start accelerating from rest. When it reaches a distance y from the center, we can find its kinetic energy:

$$\text{Since } \Delta E = q \cdot \Delta V$$
$$= e\left(\frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{y}\right)\right)$$

And if the electron is to ionize another atom, its energy must equal the E_i of one Argon atom:

$$\frac{-e\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{y}\right) = \frac{E_i\,(Ar)}{N_A}$$ ← per atom

$$\ln\left(\frac{y}{b}\right) = \frac{E_i\,(Ar) \cdot 2\pi\varepsilon_0}{N_A\, e\, \lambda}$$

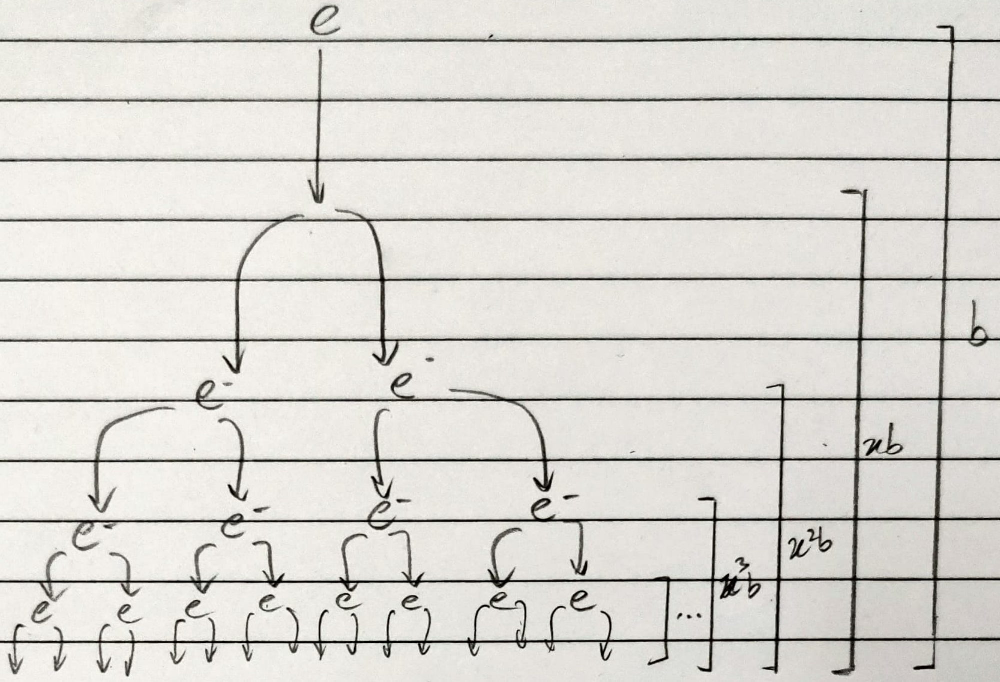
Substituting: $$\ln\left(\frac{y}{b}\right) = \frac{1.5206 \times 10^6 \times 6.28 \times 8.85 \times 10^{-12}}{6.022 \times 10^{23} \times 1.6 \times 10^{-19} \times 6.03 \times 10^{-9}}$$

$$\ln\left(\frac{y}{b}\right) = -0.1454$$

So $$\frac{y}{b} = e^{-0.1454} = 0.865$$

This is an interesting result: that the distance that the electron has to trave before attaining ionization energy, is a fixed fraction of its distance from the center.

We can use this fact to take the same fraction of the remaining distance ~~each~~ after each ionization.



Let us call the fraction of distance remaining each time, x.

So after the nth ionization, the distance remaining will be, $x^n b$ where $x = \frac{y}{b}$, from the result previously.

For the avalanche to end, the electrons have to reach the anode wire, whose radius is a.

So, remaining distance $= a$

$$x^n b = a$$

$$x^n = \frac{b}{a}$$

$$n \ln x = \ln\left(\frac{b}{a}\right)$$

$$n = \frac{\ln\left(\frac{b}{a}\right)}{\ln x}$$

$$n = \frac{\ln\left(10^{-2}\right)}{\ln(0.865)}$$

$$n = 31.754\ldots$$

So, there will be 31 levels in the cascade be fore the electrons reach the anode. For every 1 electron that begins the cascade, 2^{31} electrons reach the anode. That gives the device its sensitivity. ($2^{31} \approx 2$ billion).