Concept : Ampere's law

The equation for the curl of B,

$$\boxed{\nabla \times B = \mu_0 J}$$

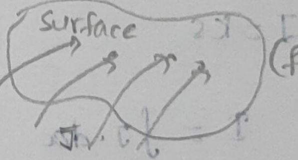
is called Ampere's law.

It can be converted to integral form by the usual device of applying one of the fundamental theorems - in this case Stokes theorem :

$$\int (\nabla \times B) \cdot da = \oint B \cdot dl = \mu_0 \int J \cdot da$$

Now, $\int J \cdot da$ is the total current passing through the surface (fig), which we call I_{enc} (the current enclosed by Amperian loop). Thus

$$\boxed{\oint B \cdot dl = \mu_0 I_{enclosed}}$$

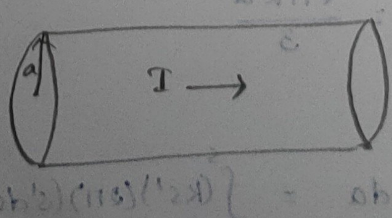


(fig)

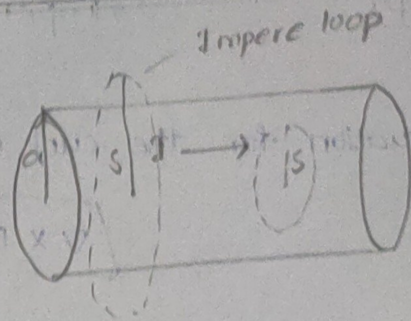
problem: A steady current I flows down a long cylindrical wire of radius a (fig). Find the magnetic field, both inside and outside the wire, if

a) The current is uniformly distributed over the outside surface of the wire

b) The current is distributed in such a way that J is proportional to s, the distance from the axis.



Solution:



a) Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = B\int dl$$

$$= B\int s\,d\phi$$

$$= B(2\pi s)$$

$$= \mu_0 I_{enclosed}$$

$I_{enclosed} = I$

$$\vec{B} = \begin{cases} \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & \text{for } s>a \text{ (outside)} \\ 0 & \text{for } s<a \text{ (inside)} \end{cases} \qquad \boxed{I_{enc}=0}$$

b) $J \propto s$

$J = ks$

$$I = \int J\cdot da$$

$$= \int (ks)(s\,ds\,d\phi)$$

$$= k\int_0^a s^2\,ds \int_0^{2\pi} d\phi$$

$$= (k)\left(\frac{a^3}{3}\right)(2\pi)$$

$$I = \frac{2\pi k a^3}{3} \rightarrow k = \frac{3I}{2\pi a^3}$$

outside $(s>a)$

$$I_{enclosed} = \frac{2\pi k a^3}{3}$$

inside $(s<a)$

$$I_{enc} = \int_0^s J\cdot da = \int_0^s (ks')(2\pi)(s'ds') = k\int_0^s 2\pi\, s'^2 ds' = \frac{2\pi k s^3}{3} = I\frac{s^3}{a^3}$$

$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi s}\,\hat{\phi} & \text{for } s > a \quad I_{enclosed} = I \\[2ex] \dfrac{\mu_0 I}{2\pi a^3}\, s^2\,\hat{\phi} & \text{for } s < a \quad I_{enc} \neq I. \end{cases}$$