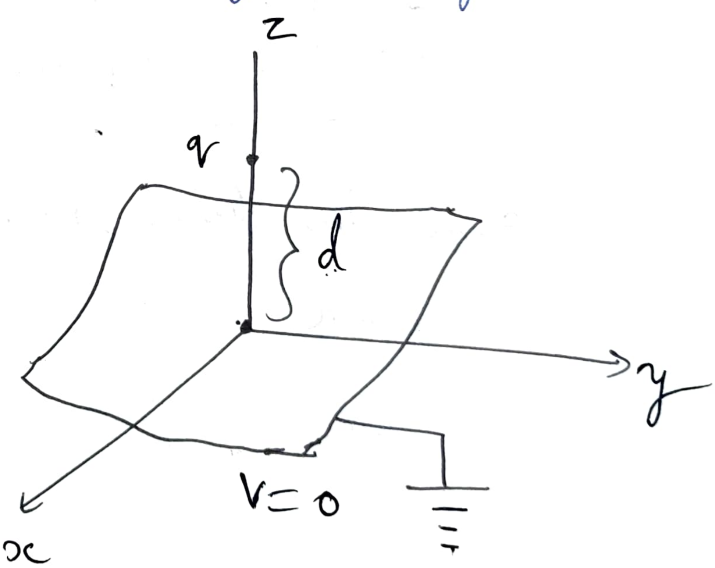
The Method of Images

We assume a point charge q above an infinitely grounded conducting plane as shown in figure below. We need to find the $V(\vec{r})$ where $z > 0$
↳ Potential.

Boundary Conditions

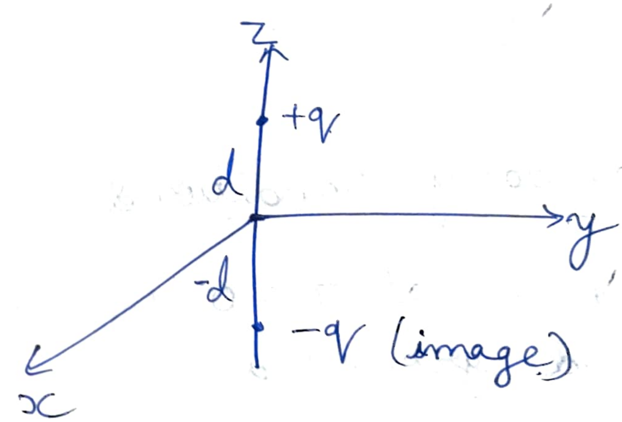
1. $V = 0$ at $z = 0$ (since the conducting plane is grounded

2. $V \longrightarrow 0$ far from the charge $(x^2 + y^2 + z^2 >> d^2)$



The first uniqueness theorem (actually, its corollary) guarantees that there is only one function that meets these requirements.

Trick → Consider a new configuration consisting of two point charges, $+q$ at $(0,0,d)$ and $-q$ at $(0,0,-d)$ and no conducting plane. For this configuration, I can easily write down the potential



$$\boxed{V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}}\right]}$$

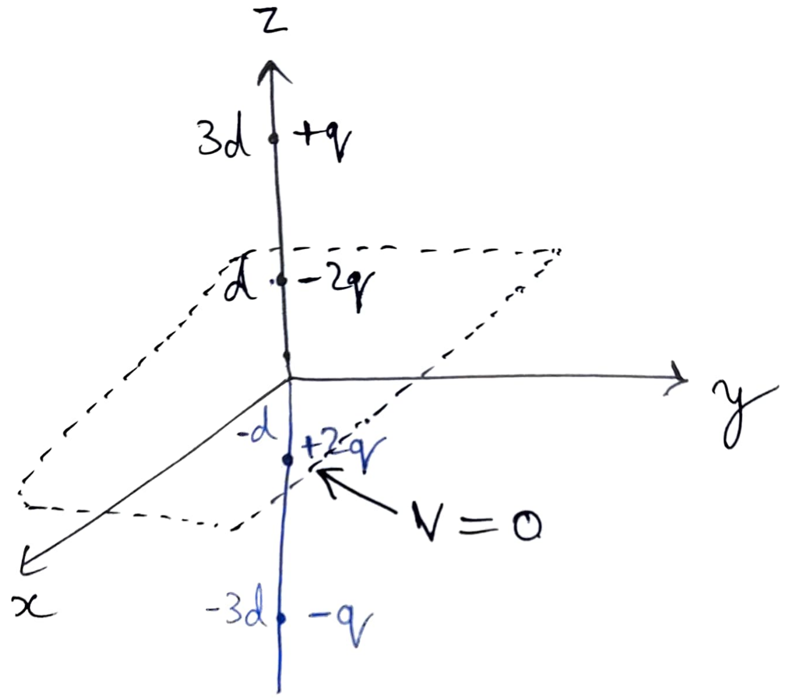
This follows the boundary conditions:-

1.) $V=0$ when $z=0$

2.) $V \to 0$ for $x^2+y^2+z^2 \gg d^2$;

• The second configuration happens to produce the same potential as the first configuration in the upper region $z \geq 0$.

Q) Find the force on the charge $+q$ in the given figure. (The xy plane is a grounded conductor).



Sol^n

Place image charges $+2q$ at $z = -d$ and $-q$ at $z = -3d$

Total force on $+q$ is :-

$$F_{net} = \frac{q}{4\pi\varepsilon_0}\left[\frac{-2q}{(2d)^2} + \frac{2q}{(4d)^2} + \frac{-q}{(6d)^2}\right]\hat{z}$$

$$= \frac{q^2}{4\pi\varepsilon_0 d^2}\left(\frac{-1}{2} + \frac{1}{8} - \frac{1}{36}\right)\hat{z}$$

$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{29q^2}{72d^2}\right)\hat{z} \quad \text{Ans}$$