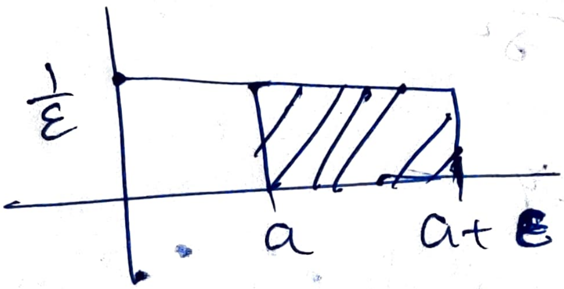
$$\delta x = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

→ $\delta x$ = generalized function (or) distribution.



Let $\delta(x) = \frac{1}{\varepsilon}$ if $a \leq x \leq a+\varepsilon$

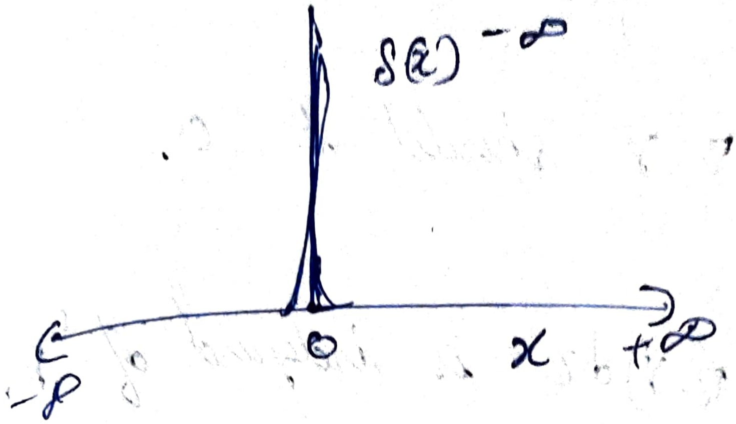
$= 0$ if otherwise

$$\int_{-\infty}^{\infty} \delta(x)\, dx = \int_{a}^{a+\varepsilon} \delta(x)\, dx$$

$$= \frac{1}{\varepsilon}\, \varepsilon = 1$$

→ $\int_{a}^{a+\varepsilon} \delta(x)\, dx = 1$ is independent on '$\varepsilon$'.

as $\varepsilon \to 0$ ; $\int_{-\infty}^{\infty} \delta(x)\, dx = 1$




→ Spike function.

Q: Why divergence of $\frac{\hat{r}}{r^2} = 0$.

Sol: $\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{1}{r^2}\right)$

$= \frac{1}{r^2} \frac{\partial}{\partial r}(1) = 0$

$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 0$ ?

But
$\oint (\nabla \cdot v)\, d\tau = \oint v \cdot da$

$= \int \left(\frac{\hat{r}}{r^2}\right) \cdot \left(r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}\right)$

$= \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = 4\pi$

~~Sol~~ So $\int(\nabla\cdot\vec{v})\,d\tau = 4\pi$

But $\nabla\cdot\vec{v}$ should be 0.

Also $\int(\nabla\cdot\vec{v})\,d\tau$ is independ of 'r'.

→ Here Bizzarity is at $x=0$,

$\frac{\hat{r}}{r^2} \Rightarrow \infty$.

→ So, this is Dirac Delta func.

$$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\,\delta^3(\vec{r})$$

$$\int \nabla\cdot\left(\frac{\hat{r}}{r^2}\right)d\tau = \int 4\pi\,\delta^3(\vec{r})\,d\tau = 4\pi\int \delta^3(\vec{r})\,d\tau$$

$$= 4\pi$$

→ Charge density, point mass, point charge are also dirac delta functions.