Physics Presentation

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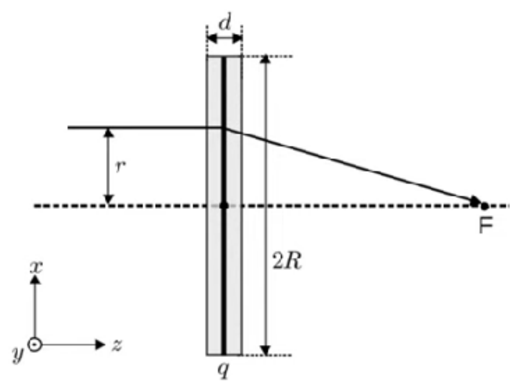
Problem Statement:

Charged Ring as an Electrostatic Lens

One wants to build a device to focus electrons—an electrostatic lens. Let us consider the following construction. The ring is situated perpendicularly to the z-axis, as shown in Figure. We have a source that produces on-demand packets of non-relativistic electrons. Kinetic energy of these electrons is $E = mv^2/2$ (v is velocity) and they leave the source at precisely controlled moments. The system is programmed so that the ring is charge-neutral most of the time, but its charge becomes q when electrons are closer than a distance $d/2$ ($d \ll R$) from the plane of the ring (shaded region in Figure, called "active region"). Assume that charging and de-charging processes are instantaneous and the electric field "fills the space" instantaneously as well. One can neglect magnetic fields and assume that the velocity of electrons in the z-direction is constant. Moving electrons do not perturb the charge distribution on the ring.

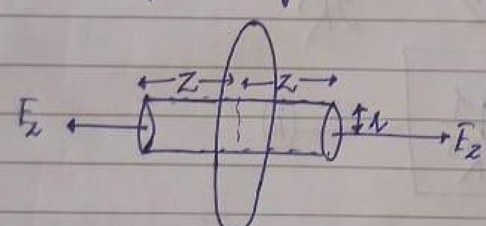


Determine the focal length f of this lens. Assume that $f \gg d$. Assume that before reaching the "active region" the electron packet is parallel to the z-axis and $r \ll R$. The sign of q is such so that the lens is focusing.

(*Source: IPho 2021 Physics Question Paper)*

Solution:

Looking at the ring sideways,



Consider a gaussian surface, pill-box of length '2z' and radius 'r'.
($z << R$ & $r << R$)

We know, the electric field due to a charged ring

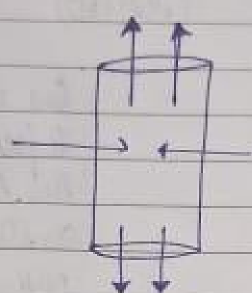
$$|\vec{E_z}| = \frac{Kqz}{(z^2+R^2)^{3/2}} = \frac{Kqz}{R^3}\left(1+\frac{z^2}{R^2}\right)^{-3/2} = \frac{Kqz}{R^3}\left(1-\frac{3}{2}\frac{z^2}{R^2}\right)$$

As $z << R$, we say

$$|\vec{E_z}| = \frac{Kqz}{R^3} \quad \text{— (i)}$$

Consider the pill-box.

(Considering charge on ring to be +ve)



For this surface,

$Q_{net} = 0$ {as no charge is enclosed}

$$\therefore \quad |\phi_z| = |\phi_r|$$

{ϕ_r = Flux due to radial electric field}

$$2(E_z)(\pi r^2) = (E_r)(2\pi r)(2z)$$

$$\Rightarrow \quad E_r = \frac{(E_z)r}{2z} = \frac{Kqr}{2R^3}$$

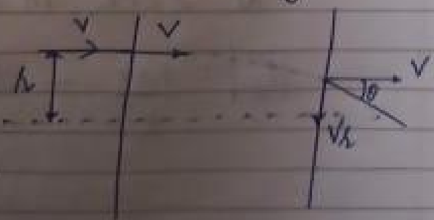
$$\Rightarrow \quad \boxed{E_r = \frac{Kqr}{2R^3}} \quad \text{— (ii)}$$

As the lens is converging nature, so the radial force on electron must be inwards i.e. E_r must be radially outward

$\therefore$ For $Q_{net} = 0$, the charge on ring must be $\underline{-ve}$.

$a_r = \dfrac{Kqre}{2R^3 m}$

Active Region

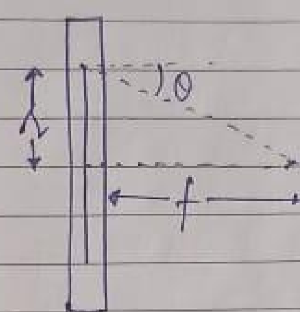


$v_r = (a_r)(t) = (a_r)\left(\dfrac{d}{v}\right)$

The deflection θ is very small.

$\tan\theta \approx \theta = \dfrac{v_r}{v}$ — (iii)

Also, from the original set-up



$\tan\theta = \dfrac{r}{f}$ — (iv)

Equating both of them, (iii) & (iv)

$\dfrac{r}{f} = \dfrac{(v_r)}{v}$

$$\Rightarrow f = \frac{(v)(t)}{(v_1)} = \frac{(vt)}{(a_1)\left(\frac{d}{v}\right)} = \frac{v^2 t}{\left(\frac{Kqte}{2R^3 m}\right)(d)} = \frac{2R^3 m v^2}{Kqed}$$

$$\Rightarrow f = \frac{4R^3\left(\frac{1}{2} m v^2\right)}{\left(\frac{qed}{4\pi\varepsilon_0}\right)} = \frac{16\pi\varepsilon_0 R^3 E}{qed} \qquad \left\{E = \frac{1}{2} m v^2\right\}$$

given

(Ans)