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Q. Since parallel currents attract, the current within a single wire should contract into a tiny concentrated stream along the axis. Yet in practice the current typically distributes itself quite uniformly over the wire. How do we account for this? If the positive charges (density $\rho_+$) are at rest, and the negative charges (density $\rho_-$) move at speed $v$ (and none of these depends on the distance from the axis), show that $\rho_- = -\rho_+ \gamma^2$, where $\gamma \equiv 1/\sqrt{1-(v/c)^2}$ and $c^2 = 1/\mu_0 \varepsilon_0$.

If the wire as a whole is neutral, where is the compensating charge located?

→ The mobile charges do pull in towards the axis, but the resulting concentration of (negative) charge sets up an electric field that repels away further accumulation. Equilibrium is reached when the electric repulsion on a mobile charge 'q' balances the magnetic attraction:

$$F = q[E + (v \times B)] = 0$$

$$\therefore \frac{0}{q} = [E + (v \times B)]$$

$$\therefore \vec{E} = -(\vec{v} \times \vec{B}) \quad \text{—— (1)}$$

say the current is in the z-direction:

$J = \rho_- v \hat{z}$ ( $\rho_-$ & $v$ are both negative)

From Ampère's Law:

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$A$ = Area of cross section

$A = \pi s^2$

$(I = J A_\perp$

As $B$ is const,

$$B \oint dl = \mu_0 J . A$$

$$B . 2\pi s = \mu_0 J \pi s^2$$

$$B = \frac{\mu_0 \rho_- v s}{2} \hat{\phi}$$

From Gauss Law,

$$\oint E . da = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} (\rho_+ + \rho_-) \pi s^2 l$$

$$E \, 2\pi s l = \frac{1}{\varepsilon_0} (\rho_+ + \rho_-) \pi s^2 l$$

$$E = \frac{1}{2\varepsilon_0} (\rho_+ + \rho_-) s \hat{s} \quad \text{——— } (2)$$

$$\vec{v} \times \vec{B} = v\hat{z} \times \frac{\mu_0 \rho_- v s}{2}\hat{\phi}$$

$$= -\frac{\mu_0}{2}\rho_- v^2 s\,\hat{s}$$

From equ ① & ②.

$$\frac{1}{2\varepsilon_0}(\rho_+ + \rho_-)s\hat{s} = \frac{\mu_0}{2}\rho_- v^2 s\,\hat{s}$$

$$(\rho_+ + \rho_-) = \rho_- \cdot \mu_0\varepsilon_0 v^2$$

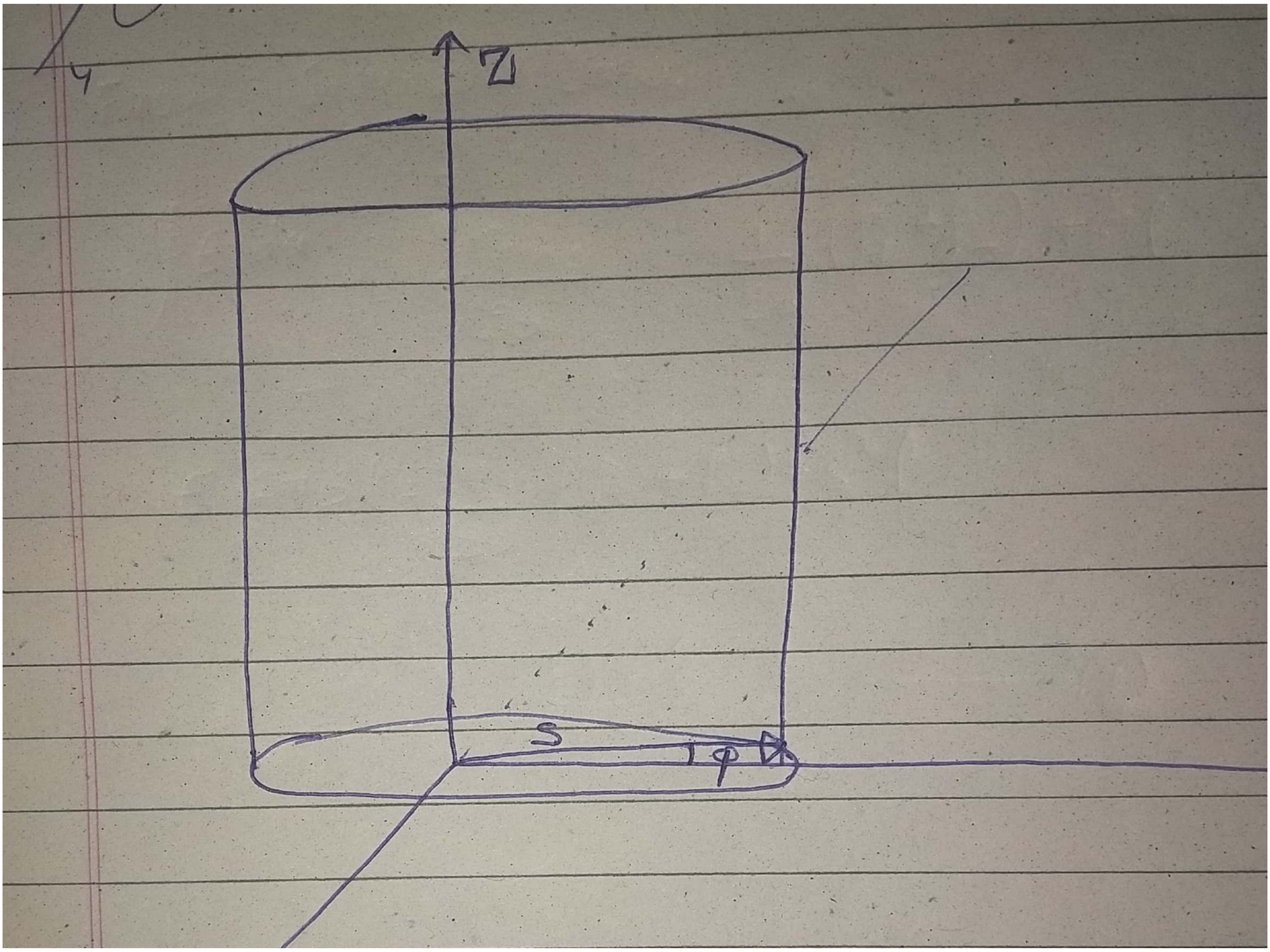
$$\rho_+ + \rho_- = \rho_- \times \left(\frac{v^2}{c^2}\right) \quad \text{— as } c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\therefore \rho_+ = \rho_- - \rho_-\left(\frac{v^2}{c^2}\right)$$

$$\rho_+ = \rho_-\left(1 - \frac{v^2}{c^2}\right)$$

$$\rho_+ = \frac{\rho_-}{\gamma^2}$$

- From this model we can conclude that mobile negative charges fill a smaller inner cylinder, leaving a shell of positive (stationary) charge at the outside. But since $v \ll c$, the effect is extremely small.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