PH1213 OM R. MAHAKAL

20211160

Tutorial Problem. Batch - 5

classmate

Date

Page

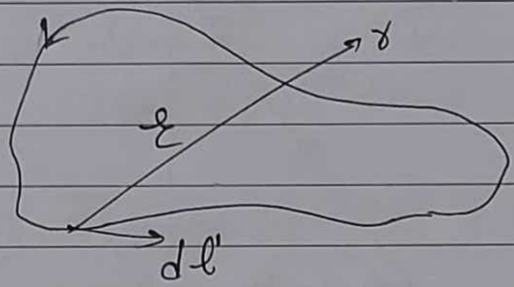
BIOT - SAVART LAW

Stationary charges produce constant electric fields: Electrostatics.

Steady currents produce constant magnetic fields: Magnetostatics.

The magnetic field of a steady line current is given by Biot-savart Law:

$$B(r) = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{r}}{r^2} \, dl' = \frac{\mu_0 I}{4\pi} \int \frac{dl' \times \hat{r}}{r^2}$$



The integration is along the current path, in the direction of the ~~of~~ flow; dl' is an element of length along wire, and r is the vector from the source to point r.

Question:

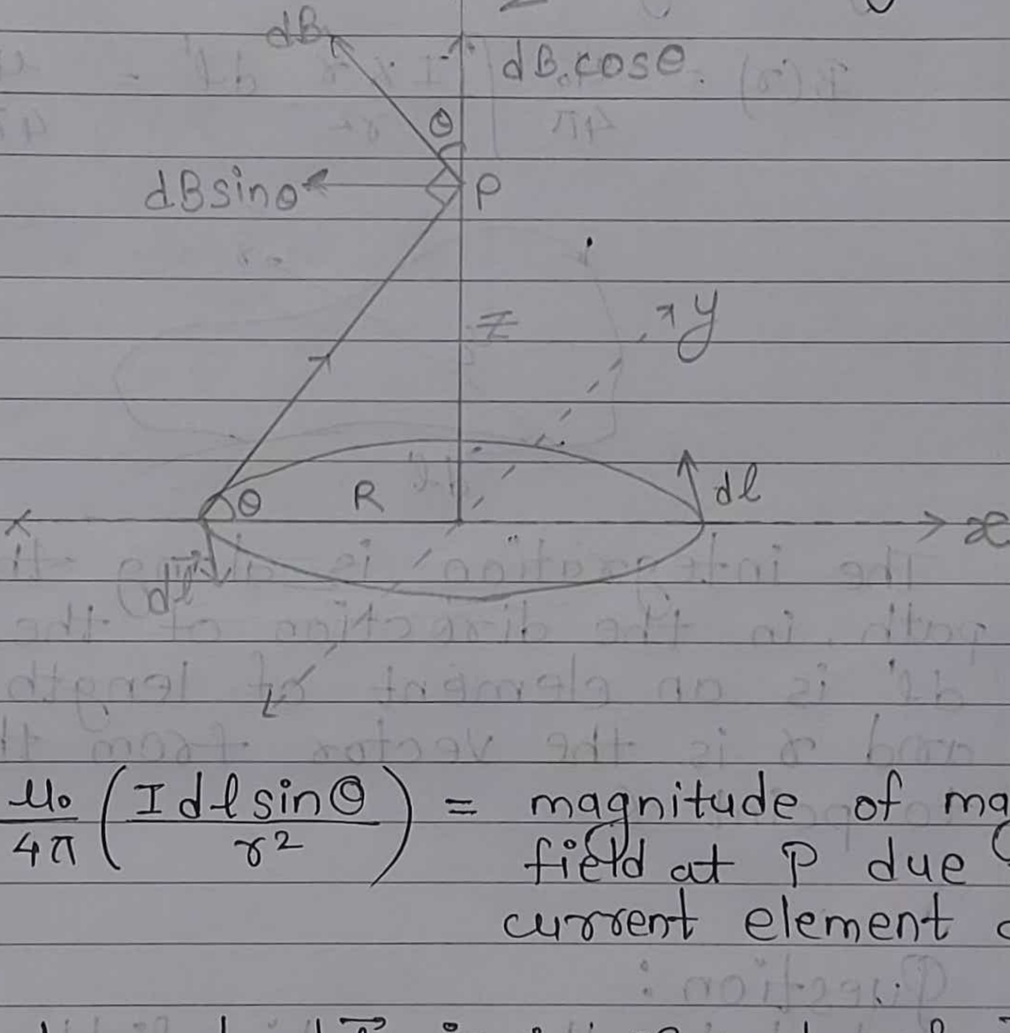
Calculate the magnetic field at the centre of a uniformly charged spherical shell, of radius R and total charge Q, spinning at constant angular velocity ω.

[The shell shall be broken into stacked rings.]

Solution:

Let us first derive magnetic field at a distance z above the centre of a circular loop of radius R, which carries a stedy current I and use the result subsequently for solving the problem further.

Consider a circular loop in the x-y plane with its centre at the origin 'o'.



$$dB = \frac{\mu_0}{4\pi}\left(\frac{I\,dl\sin\theta}{r^2}\right)$$ = magnitude of magnetic field at P due to current element dl.

The element $d\vec{l}$ is in x-y plane & $\vec{r} \perp d\vec{l}$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

$d\vec{B}$ is $\perp$r to plane formed by the $d\vec{l}$ and $\vec{r}$.

vertical components contribute to net field whereas horizontal components cancel out.

$$\Rightarrow |\vec{B}| \text{ at point P.} = \int dB \cos\theta$$

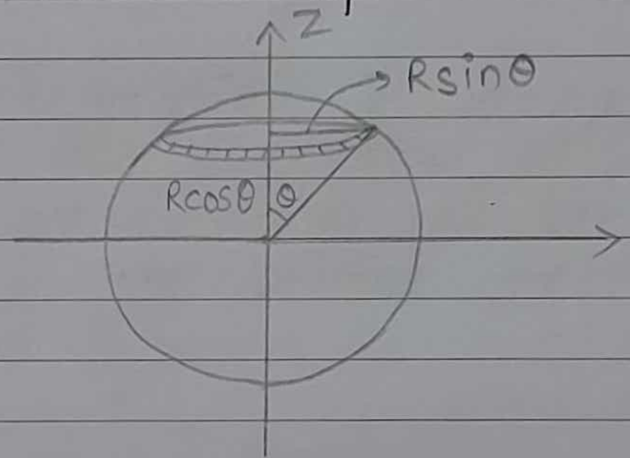
$$= \int \frac{\mu_0 I dl}{4\pi r^2} \cos\theta$$

$$= \frac{\mu_0 I \cos\theta}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi r^2} \frac{R}{\sqrt{R^2+z^2}} (2\pi R)$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}} \hat{k} \qquad r^2 = R^2 + z^2$$

Now consider the spherical shell :-



from the previous result, field at centre of sphere due to ring at θ,

$$dB = \frac{\mu_0 dI}{2} \frac{(R\sin\theta)^2}{[(R\sin\theta)^2 + (R\cos\theta)^2]^{3/2}} = \frac{\mu_0}{2R} \sin^2\theta \, dI$$

$dI = KRd\theta$, where K is surface current density

$K = \sigma v$ where 'σ' is surface charge density and 'v' is velocity of moving charges.

$$\sigma = \frac{\text{charge}}{\text{Area}} = \frac{Q}{4\pi R^2} \qquad v = \omega(R\sin\theta) = \omega R \sin\theta$$

$$\Rightarrow dI = \frac{Q}{4\pi R^2} \omega R \sin R\, d\theta = \frac{Q\omega}{4\pi} \sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 Q \omega}{8\pi R} \int_0^{\pi} \sin^3\theta\, d\theta = \frac{\mu_0 Q\omega}{8\pi R}\left(\frac{4}{3}\right)$$

$$\boxed{\vec{B} = \frac{\mu_0 Q \omega}{6\pi R}\hat{z}}$$