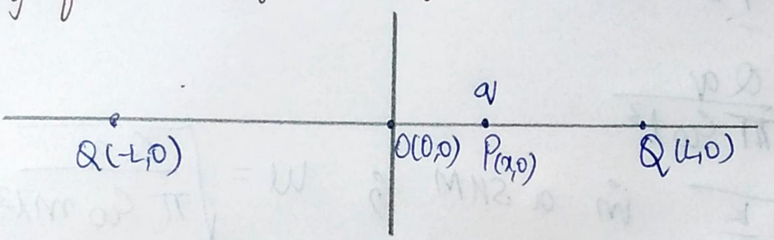
Osillation of charge on a line

Ref: Pg48 Prob-1.38 David J. Morin

2 positive charges 'Q' are located at points $(\pm l, 0)$
A particle with positive charge 'q' and mass 'm' is initially located midway between them and is then given a tiny kick. If it is constrained to move along the line joining the two charges Q, show that it is under SHM & find the frequency.



Force due to right side charge $= \dfrac{-Qq}{4\pi\epsilon_0 (l-x)^2}$ — sign indicates charge moving on left side

Force due to left side charge $= \dfrac{+Qq}{4\pi\epsilon_0 (l+x)^2}$

$$F_{net} = \frac{-Qq}{4\pi\epsilon_0}\left\{\frac{1}{(l-x)^2} - \frac{1}{(l+x)^2}\right\}$$ By binomial expansion

$$= -\frac{Qq}{4\pi\epsilon_0 l^2}\left\{\frac{1}{1-\frac{2x}{l}} - \frac{1}{1+\frac{2x}{l}}\right\} \quad (l >> x)$$

$$= \frac{-Qq}{4\pi\epsilon_0 l^2}\left\{\frac{1+2x/l - 1+2x/l}{1-\left(\frac{2x}{l}\right)^2}\right\}$$

$$= -\frac{Qq}{4\pi\epsilon_0 l^2}\,\frac{4x}{l}$$

$$F_{net} = -\frac{Qqx}{\pi\epsilon_0 l^3}$$

For a particle to move in SHM.

i) Force $\propto$ displacement

ii) $F = -\left(\frac{Qq}{\pi\epsilon_0 l^3}\right) x$

(ii) <u>Acceleration $a = -\omega^2 x$</u>

$F = m\ddot{x} = -\frac{Qq}{\pi\epsilon_0 l^3} x \Rightarrow \ddot{x} = \frac{-Qq}{\pi\epsilon_0 l^3 m} x$ — ①

from hooke's law,

$$F = -kx$$

$$k = \frac{Qq}{\pi\epsilon_0 l^3}$$

$\omega = \sqrt{\frac{k}{m}}$ in a SHM ; $\omega = \sqrt{\frac{Qq}{\pi\epsilon_0 m l^3}}$

acceleration from eqn. ① can be rewritten as

$$\ddot{x} = -\omega^2 x$$

$\therefore$ The particle is in SHM.

$\omega = 2\pi\nu$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{Qq}{\pi\epsilon_0 m l^3}}$$