PH-1213 PRESENTATION

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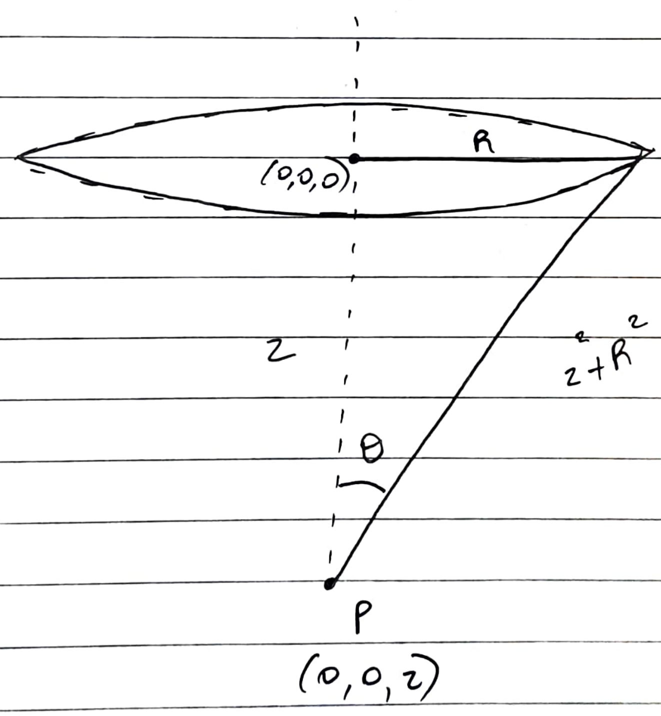
Q. For the magnetic induction at point P with coordinate 'z' produced by an increment of current $I\,dl'$ at z', show explicitly that for a closed loop carrying current I the magnetic induction at P is,

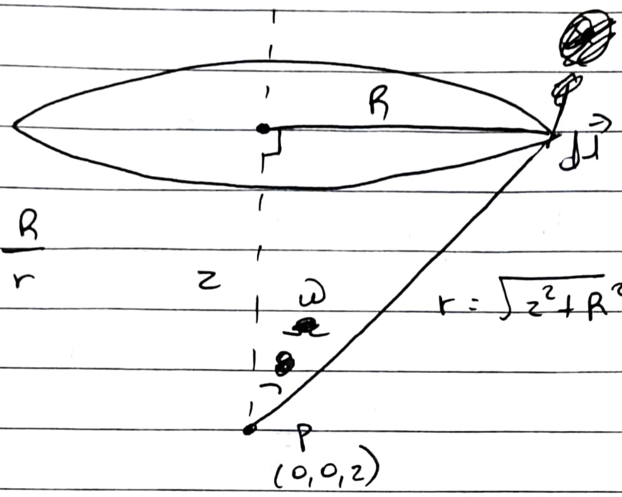
$$B = \frac{\mu_0 I \nabla\Omega}{4\pi}$$

where 'Ω' is the solid angle subtended by loop at that point P.



Consider the following figure wherein we have taken a current carrying closed loop with radius 'R'. The centre of the ring is situated at (0,0,0). 'θ' is the angle between the axis & a part of ring.

now, first we will find magnetic field outside the current carrying ring



$\sin\theta = \frac{R}{\sqrt{z^2+R^2}} = \frac{R}{r}$

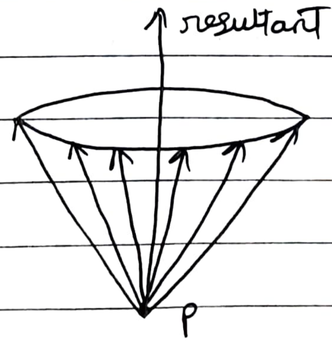
According to Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

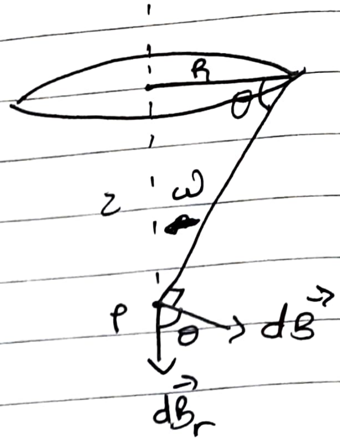
$$= \frac{\mu_0}{4\pi} \frac{I\,dl \sin 90^\circ}{z^2+R^2}$$

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,dl}{z^2+R^2}$$

Here, every single point will have its own ~~direction~~ vector of $d\vec{B}$ on that loop. Hence, the resulting ~~dir~~ vector will be along the axis which is 'z' axis.



& Hence the resulting magnetic field is going to be along the axis now here, $\cos\theta = \dfrac{dB_r}{dB}$



$\therefore dB_r = dB\cos\theta$

now,

$$d\vec{B}_r = \frac{\mu_0}{4\pi}\frac{I\,dl}{z^2+R^2}\cdot\cos\theta$$

$$d\vec{B}_r = \frac{\mu_0}{4\pi}\frac{I\,dl}{z^2+R^2}\times\frac{R}{\sqrt{z^2+R^2}}$$

$\therefore$ Total field is integral of above eq^n

$$\therefore B = \frac{\mu_0}{4\pi}\frac{IR}{(\sqrt{z^2+R^2})^3}\int dl$$

$$\therefore B = \frac{\mu_0}{4\pi}\frac{IR}{(\sqrt{z^2+R^2})^{3/2}}\times 2\pi R$$

$$\therefore B = \frac{\mu_0}{\cancel{2}\cancel{4}\cancel{\pi}}\frac{\cancel{2}IR^2\cancel{\pi}}{(\sqrt{z^2+R^2})^{3/2}}$$

$$\therefore B = \frac{\mu_0}{2}\frac{IR^2}{(\sqrt{z^2+R^2})^{3/2}} \quad - \cancel{B}\ ①$$

Now, from the above diagram,

$$\cos\omega = \frac{z}{\sqrt{z^2+R^2}}$$

now, solid angle is given by expression

$$\Omega = 2\pi(1-\cos\omega)$$

$$\Omega = 2\pi\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

Taking divergence of above eq^n

i.e. $$\nabla\Omega = (2\pi)\left[\frac{\sqrt{z^2+R^2}-\dfrac{z^2}{\sqrt{z^2+R^2}}}{z^2+R^2}\right]$$

$$\nabla\Omega = 2\pi\left[\frac{R^2}{(\sqrt{z^2+R^2})^{3/2}}\right]$$

$$\therefore \frac{\nabla \Omega}{2\pi} = \frac{R^2}{\left(\sqrt{z^2+R^2}\right)^{3/2}} \quad - (2)$$

putting this in eq^n (1)

$$\therefore B = \frac{\mu_0}{4\pi} I \nabla \Omega$$