Magnetism and Electromagnetic Induction

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Example 7.8
Introduction to Electrodynamics
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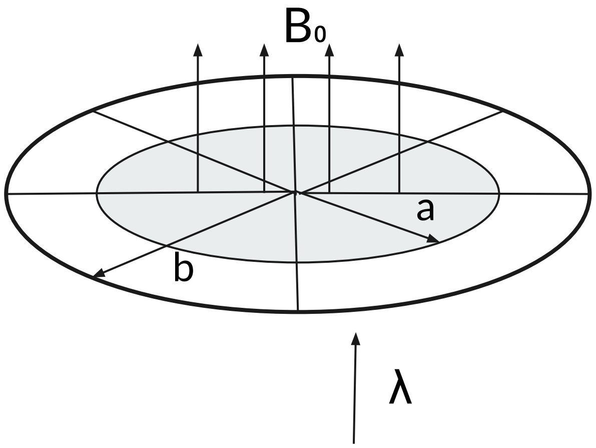
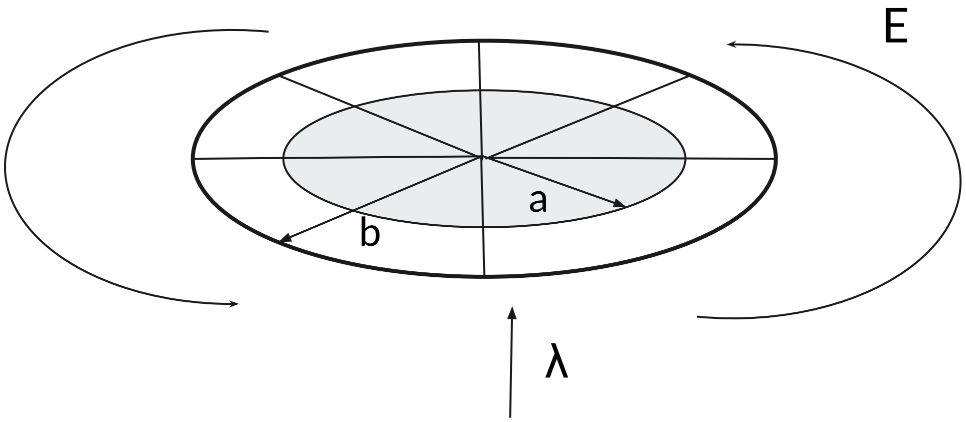
A line charge λ is glued onto the rim of a wheel of radius b, which is then suspended horizontally as shown in the figure, so that it is free to rotate. The spokes are made of non-conducting material, wood. In the central region out to radius a there is a uniform magnetic field B_0 pointing upwards. Now someone turns the field off. What happens?





As Faraday's law of Induction states; Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. Likewise, if the conductor circuit is closed, a current is induced, which is called Induced Current.

Now, since in our problem, the magnetic field has changed it will induce an electric field, curling around the axis of the wheel. The electric field exerts a force on the charges at the rim, and the wheel starts to turn. The direction of rotating wheel can be determined by Lenz's law. It will rotate in such a direction that its field tends to restore the upward flux. This motion, then, is counterclockwise, as viewed from above.

$$\oint E.dl = -\frac{d\phi}{dt} = -\pi a^2 \frac{dB}{dt}$$

$$E \cdot 2\pi b = -\pi a^2 \frac{dB}{dt}$$

$$E = -\frac{a^2\, dB}{2b\, dt}$$

$$Force = qE$$

$$= \lambda 2\pi b \left(-\frac{a^2 dB}{2b\, dT}\right)$$

$$= -\lambda \pi a^2 \left(\frac{dB}{dT}\right)$$

$$Torque\ (N) = \vec{r} \times \vec{F}$$

$$= b \times -\lambda \pi a^2 \left(\frac{dB}{dt}\right)$$

$$Angular\ Momentum\ (L) = \int N.dt = -\int \lambda 2 \pi a^2 b \left(\frac{dB}{dt}\right) dt$$

$$= -a^2 b \lambda \pi \int_{B_0}^{0} dB$$

$$= a^2 b \lambda \pi B_0$$

The above equation also shows that it does not matter how fast or slow we turn off the field, ultimately only the initial and final magnetic field is considered.

We clearly saw that at any point of time there was no magnetic field at the rim (where charge was present), as the magnetic field inside the ring of radius a was turned off. The changing flux induced an electric field curling around the rim and started the rotation of the wheel. The rotation is in such a way so that upward flux is restored. All the rotating of wheel is purely caused by the Electric Field.

Thank You