Physics presentation

Agneya Srinivasan
20211206.

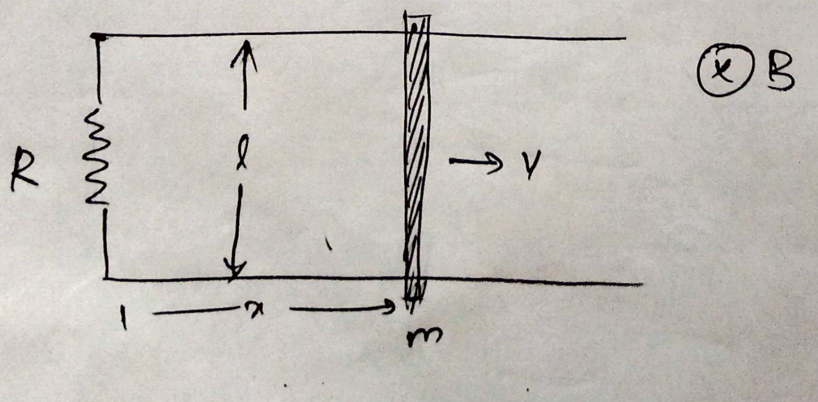
A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page, fills the entire region.

(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

(b) What is the magnetic force on the bar? In what direction?

(c) If the bar starts out with speed v_0 at time $t=0$ and is left to slide, what is its speed at a later time t?

(d) The initial kinetic energy of the bar was $\frac{1}{2}mv_0^2$. Check that energy delivered to the resistor is exactly $\frac{1}{2}mv_0^2$.



Solution

(a) Magnetic flux $\phi_B = \int B dA$.

Magnetic field is a constant

$$\phi_B = B\int dA$$

$$\phi_B = Blx$$

$$\frac{d\phi_B}{dt} = Bl\frac{dx}{dt}$$

$$\frac{d\phi_B}{dt} = Blv$$

Because of the change in magnetic flux, there is an induced emf.

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

$$\mathcal{E} = -Blv$$

$$\mathcal{E} = IR$$

$$I = -\frac{Blv}{R}$$

$(\vec{v} \times \vec{B})$ is upwards. i.e. the force through the rod is upwards. Therefore the current is upward direction. Forming a loop, the current moves in the anticlockwise direction. The current through the resistor R is downwards.

(b) Magnetic force on the current carrying rod

$$F = \int \vec{I}(d\vec{l} \times \vec{B})$$

Current is uniform.

So $F = I\int \vec{dl}\times\vec{B}$; $\vec{I}$ and $\vec{dl}$ are in the same direction.

$\vec{dl}$ and $\vec{B}$ are perpendicular. ~~$\vec{I}$ and $\vec{dl}$~~

$\therefore F = I\int \vec{B}\,\vec{dl}$

Putting intervals

$$F = I\int_0^l B\,dl$$

$$F = IBl$$

$$I = -\frac{Blv}{R}$$

$$\therefore F = -\frac{B^2l^2v}{R}$$

$(\vec{dl}\times\vec{B})$ gives a direction to the left.

The magnetic force is to the left.

The force is opposing the motion of the rod.

(c) $F = ma$

$$m\frac{dv}{dt} = -\frac{B^2l^2v}{R}$$

$$\frac{dv}{dt} = -\frac{B^2l^2}{mR}v$$

Let $k = \frac{B^2l^2}{mR}$

$$\frac{dv}{dt} = -kv$$

$$\int_{v_0}^{v}\frac{dv}{v} = -k\int_0^t dt$$

$$\ln\frac{v}{v_0} = \cancel{-kt} -kt$$

$$v = v_0 e^{-kt}$$

$$\boxed{v = v_0 e^{-\frac{B^2 l^2}{mR} t}}$$

(d) The initial k.E of the bar is $\frac{1}{2} m v_0^2$.
The energy goes as heat in the resistor.
Power delivered to the resistor is
$P = I^2 R$.

$P = \frac{dW}{dt}$ W → Work done

$$\frac{dW}{dt} = I^2 R$$

$$\frac{dW}{dt} = \frac{B^2 l^2 v^2 \cancel{R}}{R^{\cancel{2}}}$$

$$= \frac{B^2 l^2}{R} v_0^2 e^{-2kt}$$

where $k = \frac{B^2 l^2}{2mR}$

$km = \frac{B^2 l^2}{R}$

$$\frac{dW}{dt} = km\, v_0^2 e^{-2kt}$$

$$W = \int_0^\infty km\, v_0^2 e^{-2kt}$$

$$= mk v_0^2 \int_0^\infty e^{-2kt}$$

$$= m\cancel{k} v_0^2 \left. \frac{e^{-2kt}}{-2\cancel{k}} \right|_0^\infty$$

$$= -\frac{1}{2} m v_0^2 \left(e^{-\infty} - e^{0}\right)$$

$$= -\frac{1}{2} m v_0^2 (0-1) = \frac{1}{2} m v_0^2$$

Work done is change in ~~kin~~ k.E.