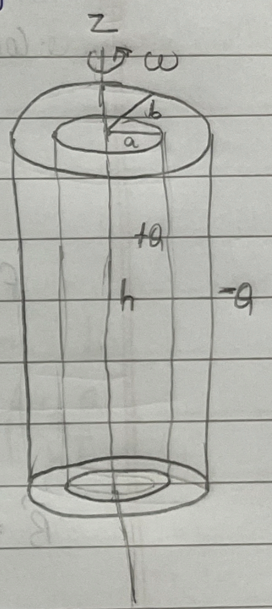
QUESTION

The concentric shells of a cylindrical capacitor have radii a and b ($b > a$) and height $h >> b$. The capacitor charge is Q; with $+Q$ on the ~~outer~~ inner shell and $-Q$ on the ~~inner~~ outer shell. The whole capacitor rotates about its axis with angular velocity $\omega = 2\pi/T$. Boundary effects are negligible.

a) Evaluate the magnetic field $\vec{B}$ generated by the rotating capacitor over the whole space.

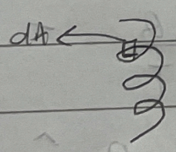
b) Evaluate the magnetic forces on the charges of the two rotating cylindrical shells, and compare them to electrostatic forces.



a) Finding the current due to the system:

$$i = \frac{dq}{dt}$$

For this; we can take the rotating cylinder to be a solenoid with infinitesimal thickness wires. Let thickness of the wires be dx. Then, if we consider some ~~current~~ area element for current:



The element dA has height dx.

Current will be amount of charge dq that passes in time dt. But, since angular velocity is constant;

$$\frac{dq}{dt} = \frac{Q'}{T}$$

where $T = 2\pi/\omega$ is the time period of rotation of the cylinder.

(uniform charge density ⇒ σ is const.)

$$Q' = \sigma A$$

$$= \frac{Q}{2\pi a h} \times 2\pi a \, dx$$

$$= \frac{Q \cdot dx}{h}$$

Substituting;

$$i = \frac{Q}{hT}\, dx$$

As it doesn't depend on radius, the magnitude of current is same for the other cylinder (it will be in opposite direction).

Let n be no. of turns per unit length for a solenoid. Then; $n = \frac{1}{dx}$

For a solenoid; $\vec{B} = \mu_0 n i \hat{z}$ inside the solenoid; and 0 outside (since $h >> b$)

Consider the outer cylinder.

$$0 < r < b; \quad \vec{B}_b = -\mu_0 \frac{Q}{hT} \hat{z}$$

$$r > b; \quad \vec{B}_b = \vec{0}$$

For the inner cylinder;

$$0 < r < a; \quad \vec{B}_a = \frac{\mu_0 Q}{hT} \hat{z}$$

$$r > 0; \quad \vec{B}_a = \vec{0}$$

$$\vec{B} = \vec{B}_a + \vec{B}_b$$

$$0 < r < a; \quad \vec{B} = \left(\frac{\mu_0 Q}{hT} - \frac{\mu_0 Q}{hT}\right)\hat{z} = \vec{0}$$

$$a < r < b; \quad \vec{B} = -\frac{\mu_0 Q}{hT} \hat{z}$$

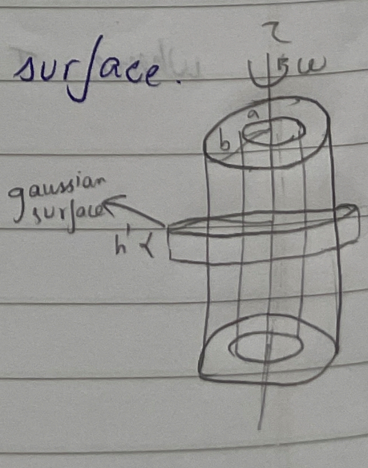
$$r > b; \quad \vec{B} = \vec{0}$$

b) ~~From~~ Finding electric field:

Consider a cylinder-shaped gaussian surface.

For $r > b$ and $r < a$; $\vec{E} = \vec{0}$

Since we are using symmetry to assume $\vec{E}$ is only radially outward.



For $a < r < b$;

$$E \cdot 2\pi r h = \frac{Q}{2\pi a h \varepsilon_0} \cdot 2\pi a h$$

$$E = \frac{Q}{2\pi r h \varepsilon_0} \hat{r}$$

Now, force on one shell by another (consider outer):

f^e = force per unit area (electrostatic)

$$\cancel{f^e = \sigma_b \frac{E(b)}{2}}$$

~~(E is due to contributions of both outer &~~

$$\vec{f}^e = \sigma_b E(b)$$

$$= -\hat{r} \frac{Q}{2\pi b h} \times \frac{Q}{2\pi b h \varepsilon_0}$$

$$= \frac{-Q^2}{4\pi^2 b^2 h^2 \varepsilon_0} \hat{r}$$

The magnetic force per unit area, f^b is given by

$$\vec{f}^b = \sigma_b \vec{v} \times \vec{B} = +\hat{r} \left(\frac{+Q}{2\pi b h}\right) \times \left(\frac{2\pi b}{T}\right) \times \left(\frac{\mu_0 Q}{h T}\right)$$

$$= \hat{r} \frac{Q^2 \mu_0}{h^2 T^2}$$

The ratio $\frac{f^m}{f^s}$ = $\frac{Q^2 \mu_0}{h^2 T^2} \times \frac{4\pi^2 b^2 h^2 \varepsilon_0}{Q^2}$ (only seeing magnitude)

$$= \left(\frac{2\pi b}{T}\right)^2 \times \frac{1}{c^2}$$

$$= \frac{v_b^2}{c^2}$$

This ratio is very small for most values of v_b.