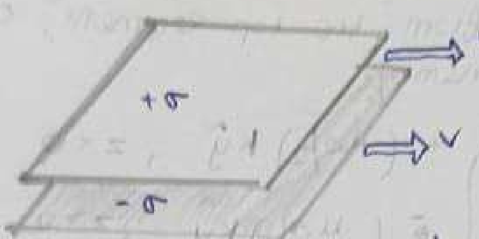
Q. A large parallel plate capacitor with uniform surface charge σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v as shown in figure.



a) Find the magnetic field between the plates & also above & below them

b) Find the magnetic force per unit area on the upper plate, including its direction

c) At what speed v ~~should~~ would the magnetic force balance the electrical force

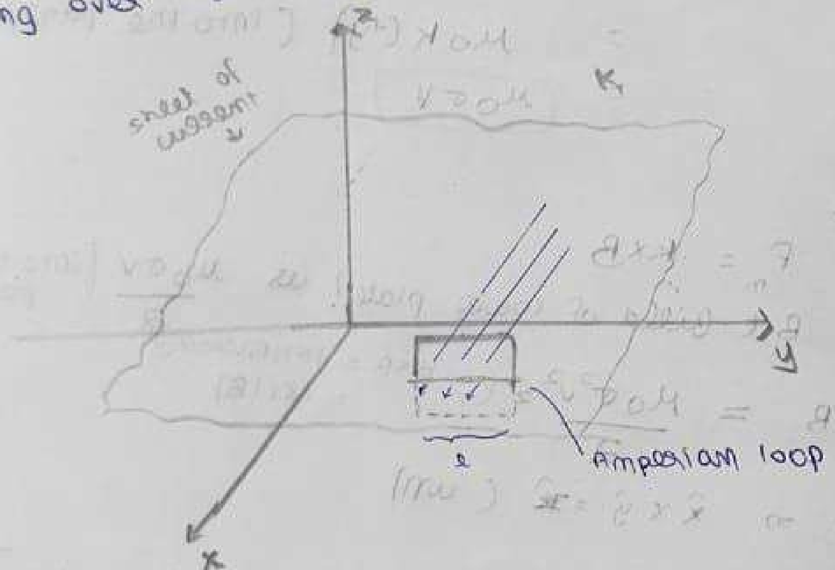
Solution

Magnetic field of an infinite uniform surface current $K = K\hat{x}$ flowing over the x-y plane

$K \rightarrow$ surface current density

$$K = \frac{dI}{dl_{\perp}}$$

current per unit width perpendicular to the flow



Finding the direction of $\vec{B}$. $\vec{B}$ cannot ~~be in~~ have a component in $\hat{x}$ direction as $\vec{B} \perp K$ [from Biot Savart's law]

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{K(r') \times \hat{\mathscr{r}}\, da'}{\mathscr{r}^2}$$

$\bar{B}$ cannot have a $\vec{z}$-component as any contribution from a filament at $+y$ is cancelled by the filament at $-y$

$\therefore$ B only has a y-component

[using right hand rule / vector product we get direction of B

left $(-\hat{y}) \rightarrow$ above plate $(\hat{x} \times \hat{z})$

right $(+\hat{y}) \rightarrow$ below plate $(\hat{x} \times -\hat{z})$

with this in mind, we draw a rectangular amperian loop as shown in figure which extends an equal distance above below the surface. Applying Ampere's law

$$\oint B \cdot dl = B2L = \mu_0 I_{enc} = \mu_0 K L$$

[One BL comes from the top segment, other BL from bottom segment]

$$\therefore B = \begin{cases} +(\mu_0/2) K \hat{y} & , z<0 \\ -(\mu_0/2) K \hat{y} & , z>0 \end{cases}$$

coming to part (a) of question

(i)

a) Top plate produces a field $-\frac{\mu_0}{2} k\hat{y}$ for points above it & $+\frac{\mu_0}{2} k\hat{y}$ for points below it. The bottom plate produce a field $+\frac{\mu_0 k}{2}\hat{y}$ for points above it & $-\frac{\mu_0 k}{2}\hat{y}$ for points below it

Point Above and below plate two fields cancel $\Rightarrow \boxed{B=0}$

Points between plates $\Rightarrow + \frac{\mu_0 k}{2}\hat{y} + \frac{\mu_0 k}{2}\hat{y}$

$$= \mu_0 K (-\hat{y}) \text{ (into the page)}$$

$$= \boxed{\mu_0 \sigma v}$$

b) Lorentz force Law

$$F = \int (K \times B)\, da$$

force per unit area $F_m = K \times B$

here $K = \sigma v$ and B (field of lower plate) is $\frac{\mu_0 \sigma v}{2}$ (into the page)

$$\therefore F_m = K \times B = \frac{\mu_0 \sigma^2 v^2}{2} \hat{z} \qquad \begin{aligned} K \times B &= |K||B| \sin 90 \\ &= |K||B| \end{aligned}$$

Direction $\Rightarrow \hat{x} \times \hat{y} = \hat{z}$ (up)

c) The electric field of the lower plate is $\frac{\sigma}{2\epsilon_0}$ [from electrostatics]

$\therefore$ Electric ~~field~~ force per unit area on upper plate (fe) $= \frac{\sigma^2}{2\epsilon_0}$ (down)

$$F_e = F_m$$

$$\frac{\sigma^2}{2\epsilon_0} = \frac{\mu_0 \sigma^2 v^2}{2}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$v = 1/\sqrt{\mu_0 \epsilon_0} = c \text{ (speed of light)}$$