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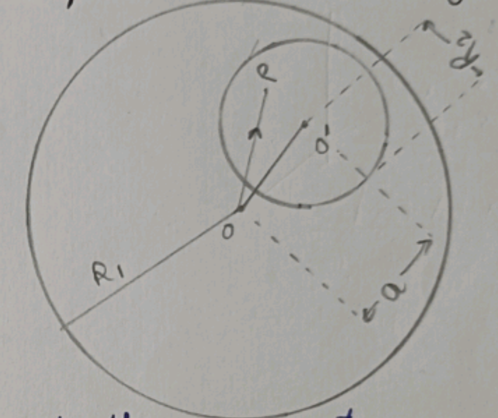
Batch 1 Physics

Q] A sphere of radius R_1 has charge density ρ uniformly distributed within its volume, except for a small spherical hollow region of radius R_2 located a distance a from the centre.

ⓐ Find the field E in the cavity of the hollow sphere.

ⓑ Find the potential at the same place P as shown in diagram.



→ Pre requisite : As discussed last week,

$$E_{sphere} = \frac{\rho \vec{r}}{3\epsilon_0} \quad r < R$$

$$= \frac{\rho R^3}{3\epsilon_0 r^3} \vec{r} \quad r > R$$

ⓐ

To solve this problem, we need to think of the given system as, a solid sphere of radius R_1 & charge density ρ & another small solid sphere of radius R_2 & charge density $-\rho$ superimposed on it.

This will be consistent with our previous/original system. It makes calculations easy. Its a mathematical trickery.

So, Electric field at any point in the cavity (say P) will be the sum of electric fields by the sphere of charge density ρ & sphere of charge density $-\rho$. (Principle of superposition)

$$OP = r \quad , \quad O'P = r' \quad , \quad OO' = a \quad , \quad r' = r - a$$

$$E = E_1 + E_2 = \frac{\rho}{3\epsilon_0}\vec{OP} + \frac{(-\rho)}{3\epsilon_0}\vec{O'P} = \frac{\rho}{3\epsilon_0}\left(\vec{r} - \vec{r'}\right) = \boxed{\frac{\rho}{3\epsilon_0}\vec{a}}$$

In the final result, there is no r or r' term. This means that the field does not depend on the position inside the cavity. The field is uniform throughout the cavity.

(b) To find potential, we will use the same mathematical trickery. First assume charges are distributed throughout the sphere of radius R_1. Let V_1 be the potential at the centre O' of the hollow sphere/cavity. If this cavity is replaced by a small sphere of uniform density $-\rho$ of radius R_2 in the hollow region, let the potential at O' be V_2

Now, Suppose the potential taken to be zero at infinite point.

$$V = -\int_{\infty}^{r} E \cdot dl = -\left(\int_{\infty}^{R} + \int_{R}^{r}\right) E \cdot dl = \left(\int_{r}^{R} + \int_{R}^{\infty}\right) E \cdot dl$$

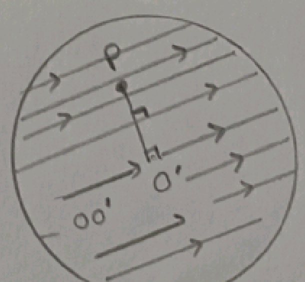
& E as we know is
$$E = \begin{cases} \dfrac{\rho r}{3\epsilon_0} & r < R \\ \dfrac{\rho R^3}{3\epsilon_0 r^3} r & r > R \end{cases}$$

• $$V_1 = \int_{a}^{R_1} \frac{\rho}{3\epsilon_0} r\, dr + \int_{R_1}^{\infty} \frac{\rho R_1^3}{3\epsilon_0 r^2} dr = \frac{\rho}{3\epsilon_0}\left[\frac{r^2}{2}\Big|_{a}^{R_1} + R_1^3\left(\frac{-1}{r}\right)\Big|_{R_1}^{\infty}\right]$$
$$= \frac{\rho}{3\epsilon_0}\left[\frac{R_1^2 - a^2}{2} + R_1^3\left(0 + \frac{1}{R_1}\right)\right]$$

$$\therefore V_1 = \frac{\rho}{3\epsilon_0}\left[\frac{3R_1^2 - a^2}{2}\right]$$

• $$V_2 = \int_{0}^{R_2} \frac{-\rho}{3\epsilon_0} r\, dr + \int_{R_2}^{\infty} \frac{-\rho R_2^3}{3\epsilon_0 r^2} dr = \frac{-\rho}{3\epsilon_0}\left[\frac{r^2}{2}\Big|_{0}^{R_2} + R_2^3\left(-\frac{1}{r}\right)\Big|_{R_2}^{\infty}\right]$$
$$= \frac{-\rho}{3\epsilon_0}\left[\frac{R_2^2}{2} + R_2^3\left(0 + \frac{1}{R_2}\right)\right]$$

$$\therefore V_2 = -\frac{\rho}{3\epsilon_0}\left[\frac{3R_2^2}{2}\right]$$



Hence, $$\boxed{V = V_1 + V_2 = \frac{\rho}{6\epsilon_0}\left[3(R_1^2 - R_2^2) - a^2\right]}$$ //

This is potential at O'. This will be the same as the potential at P because we will be moving ⊥ar to field when we go from O' to P. So, field potential will remain the same.