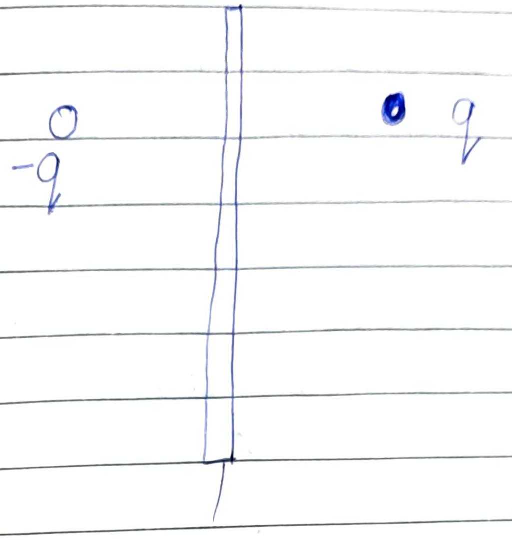
Q. A straight line charge with constant linear charge density λ is located perpendicular to the x-y plane in the first quadrant at (x_0, y_0). The intersecting planes $x=0, y\geq 0$ & $y=0, x\geq 0$ are conducting boundary surfaces held at zero potential. Consider the potential, fields & surface charges in the first quadrant.

Also the potential for an isolated line charge at (x_0, y_0) is $E(x,y) = \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{R^2}{r^2}$

where $r^2 = (x-x_0)^2 + (y-y_0)^2$ & R is a constant.

Determine the expression for the potential of the line charge in presence of these intersecting planes.

~~Verify that the tangential Electric field vanishes at the boundary surface~~



$x=0$ plan

Using the method of images we can build the image charges for this solution one plane at a time.

So if there were only a plane at $x=0$ then the line charge λ will generate an image charge $-\lambda$ at $(-x_0, y_0)$.

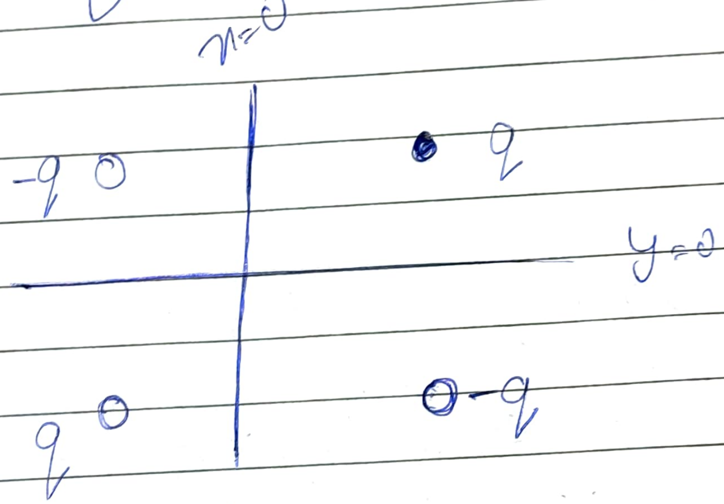
Now adding a plane at $y=0$ will result in the additional ~~imag~~ image charges for the above 2 charges.

The image of the initial charge λ at (x_0, y_0) will ~~be~~ give $-\lambda$ at $(-x_0, -y_0)$ &

$C @ (-x_0, y_0)$

the image of the initial image at $(-x_0, -y_0)$ will give λ at $(-x_0, y_0)$.

$\therefore$ The system will look something like this



Now the Electric field due to all these 4 charges at (x, y) in the 1st quadrant will be

$$E(x,y) = \frac{\lambda}{4\pi\varepsilon_0}\left(\ln \frac{R^2}{(x-x_0)^2+(y-y_0)^2} + \ln \frac{R^2}{(x+x_0)^2+(y+y_0)^2} - \ln \frac{R^2}{(x-x_0)^2+(y+y_0)^2} - \ln \frac{R^2}{(x+x_0)^2+(y-y_0)^2} \right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \ln \frac{\left[(x-x_0)^2+(y+y_0)^2\right]\left[(x+x_0)^2+(y-y_0)^2\right]}{\left[(x-x_0)^2+(y-y_0)^2\right]\left[(x+x_0)^2+(y+y_0)^2\right]}$$