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Physics Presentation

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Batch-3

10/06/22

Insertion of Dielectric in a Parallel Plate Capacitor (PPC)

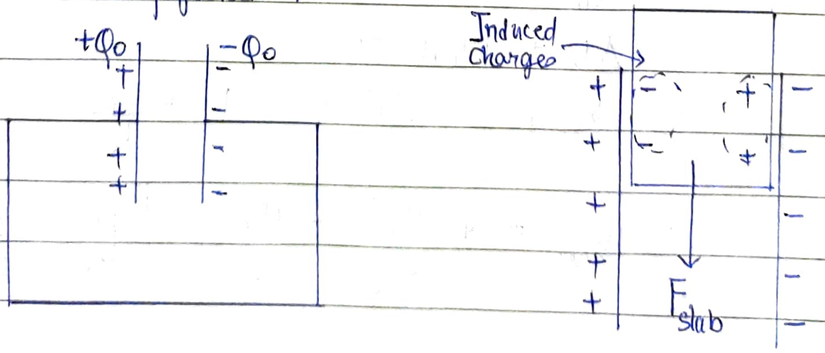
(of equal length & area)

Q) a) When a dielectric is inserted in a charged PPC, which is not connected to a cell, the energy stored in the PPC reduces from U_0 to U_0/k [k = dielectric constant]. Where does this energy go? Find out this difference of energy in terms of Q_0, k & C_0.

Q_0 = Initial charge
C_0 = Initial capacitance.

{Assume frictionless surfaces & no loss due to heat}

Ans:- When a dielectric is inserted in a charged PPC, it is always inserted "slowly" (i.e., $\Delta KE = 0$), as elsewise it would undergo some oscillatory motion & would not give us our configuration.



Thus, an external force (F_{ext}) is applied on the slab opposite to the direction of motion. This $|\vec{F}_{ext}|$ should be just slightly less than

$|\vec{F}_{slab}|$, so that "slow" process is maintained.

$$|\vec{F}_{ext}| \approx |\vec{F}_{slab}|$$

Now, Work done due to this ext. force $= W_e$
$= \vec{F} \cdot d\vec{s}$
$= -ve$

Here $-ve$ work $\Rightarrow$ Work is obtained by the ext. agent.

$\therefore$ The energy difference $= U_0 - \frac{U_0}{K}$ is lost to the ext. agent.

Now, Applying Work Energy theorem:

$$W_{\text{all forces}} = \cancel{\Delta KE}^{\,0}$$

$$\Rightarrow W_{\substack{PPC \\ \text{on slab}}} + W_{ext} = 0$$

$$\Rightarrow \Delta U + W_{ext} = 0$$

$$\Rightarrow W_{ext} = -\Delta U = -U_0 + \frac{U_0}{K}$$

$$\Rightarrow W_{ext} = +\frac{Q_0^2}{2KC_0} - \frac{Q_0^2}{2C_0}$$

$$U_0 = \frac{Q_0^2}{2C_0}$$

$$= \frac{Q_0^2}{2C_0}\left[\frac{1}{K} - 1\right]$$

③

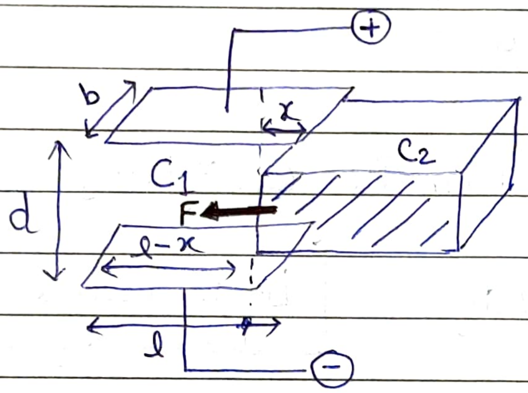
$\therefore$ Energy diff. $= \Delta U = \dfrac{Q_0^2}{2C_0^2}\left(1-\dfrac{1}{K}\right)$

[Dimensions of PPC are ~~of~~ same fitting as dielectric]

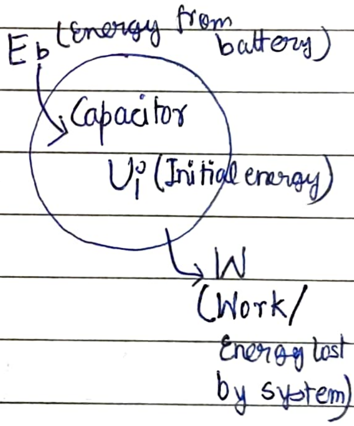
b) Find the force on the slab in a different scenario : When the PPC is connected to the cell & a dielectric is inserted.

{Assume no energy lost due to friction & heat}

Ans:-



Here "V" is constant.



Final energy of PPC

$\therefore U_i + E_b = W + U_f$

$\Rightarrow E_b = (U_f - U_i) + W$

$\Rightarrow E_b = \Delta U + W$

In differential form $\Rightarrow dE_b = dU + dW$

$\Rightarrow \int dE_b = \int dU + \int dW \longrightarrow ①$

Now, Capacitance at a time when it has moved "x" distance $= C = C_1 + C_2$ [Parallel combination]

Now, $C_1 = \frac{\varepsilon_0 A_1}{d}$; $C_2 = \frac{k\varepsilon_0 A_2}{d}$

$= \frac{\varepsilon_0 (l-x) b}{d}$; $= \frac{k\varepsilon_0 x b}{d}$

$\therefore C = \frac{\varepsilon_0 b}{d}\left[l - x + kx\right]$

& $\frac{dc}{dx} = \frac{\varepsilon_0 b}{d}\left[k-1\right] \longrightarrow (2)$

Now, Energy from the cell $= \int dE_b$

$= \int V dq$ [Let us assume dq charge delivered by the cell]

$= \int V^2 dc \longrightarrow (3)$

& $q = CV$

$\Rightarrow dq = dc\, V$

Also, $U = \frac{Q^2}{2C} = \frac{1}{2} CV^2$

$\Rightarrow dU = \frac{V^2}{2} dC \longrightarrow (4)$

Using (1), (3) & (4), we get

$\int dE_b = \int dW + \int dU$

$\Rightarrow \int V^2 dc = \int dW + \int \frac{V^2}{2} dc$

$\Rightarrow \int dW = \int \frac{V^2}{2} dc \longrightarrow (5)$

⑤

Now, Work done by the system $= \int \vec{F} \cdot d\vec{x}$

$\int dW = \int F\,dx$ [$\because \vec{F}$ & $d\vec{x}$ are in same dir^n]

$\therefore \int F\,dx = \int \frac{V^2}{2}\,dC$ [From ⑤]

Using ②,

$$\int F\,dx = \int \frac{V^2}{2} \frac{\varepsilon_0 b}{d}[K-1]\,dx$$

On same limits $\Rightarrow$ $\boxed{|\vec{F}| = \frac{\varepsilon_0 b}{2d} V^2 (K-1)}$

$\Rightarrow$ Independent of "x"

c) In part ⓑ, due to the force, there would be an oscillatory motion (not an SHM !!!). Find the time period of this motion.

Ans:-

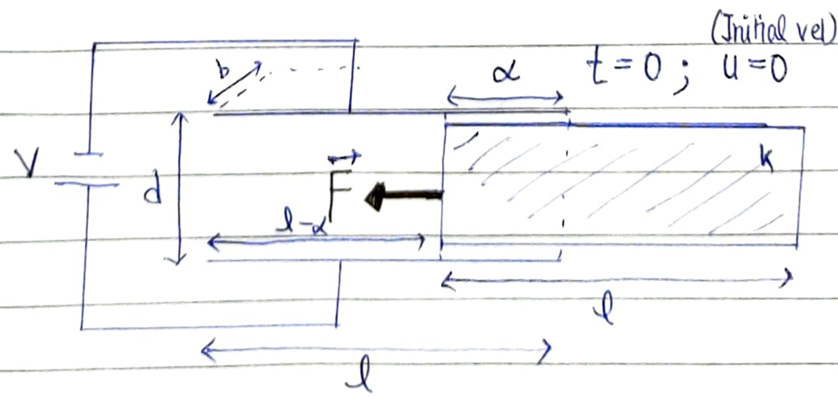


Fig 1

Here, at $t=0$; $|\vec{F}| = \frac{\varepsilon_0 b}{2d} V^2 (K-1)$

Now, $|\vec{a}| = \dfrac{|\vec{F}|}{m} = \dfrac{\varepsilon_0 b V^2 (k-1)}{2dm}$

Lets say after some time $t=t$; the dielectric completely covers the area of PPC.

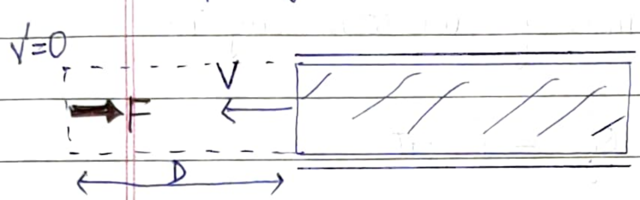


Fig 2

Using eq^n of motion, as $\vec{a}$ is constant :

$$\text{Dist. trav.} = S = \cancel{ut}^{\,0} + \frac{1}{2}at^2$$

$$\Rightarrow l - x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2(l-x)}{a}} \longrightarrow (1)$$

Now, in Fig 2; All the work done by the PPC on the dielectric has been converted to its K.E.

$$\therefore \frac{1}{2}mv^2 = F(l-x) \longrightarrow (2)$$

Now, after ~~some time~~ t', the some part of the dielectric pops out of the PPC; the dir^n of force reverses. &

However, the dielectric keeps on moving till $V \longrightarrow V' = 0$.

Let us say, it went till D

Now, Work done here $= -FD = -\frac{1}{2}mv^2 \longrightarrow (3)$

(as all KE got converted)

From (2) & (3), we get $\underline{\underline{D = l - \alpha}}$.

Thus, the dielectric pops out that much only as much as it was popped out in initial case.

Thus, from symmetry we can say that

Time period of oscillation $= T = 4t$

$$= 4\sqrt{\frac{2(l-\alpha)}{a}} \qquad [\text{From } (1)]$$

$$= 4\sqrt{\frac{2(l-\alpha)\,2dm}{\varepsilon_0 b V^2 (K-1)}}$$

$$\boxed{T = 8\sqrt{\frac{(l-\alpha)\,dm}{\varepsilon_0 b V^2 (k-1)}}}$$