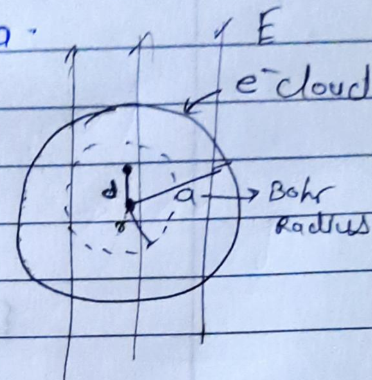
Page No.
Date

According to quantum Mechanics, the e^- cloud for a hydrogen atom in the ground state has a charge density

$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a}$$

where q is the charge of e^- and a is the Bohr radius. Find the atomic Polarisability of such atom [Hint: first calculate the electric field of the electon cloud $E_e(r)$; then expand the exponential, assuming $r \ll a$.



$$\vec{P} = \alpha \vec{E}$$
$$\vec{E} = \frac{1}{\alpha} \vec{P}$$

Given
$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a}$$

by Gauss's law:
$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \int \rho \, d\tau = \frac{q}{\pi a^3} \int e^{-2r/a} \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{q}{\pi a^3} \int_0^r r^2 e^{-2r/a} dr \cdot \underbrace{\int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi}_{4\pi}$$

$$q_{enc} = q\left[1 - e^{-2r/a}\left(1 + 2\frac{r}{a} + 2\frac{r^2}{a^2}\right)\right]$$

$$E \cdot (4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} \rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\left[1 - e^{-2r/a}\left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right)\right]$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}\left[1 - e^{-2r/a}\left(1 + 2\frac{r}{a} + 2\frac{r^2}{a^2}\right)\right]$$

~~electron cloud~~ s Proton inside shift by D &
electron cloud will move on other side

tom shift by D

we are looking for electric field D

$r = d$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\left[1 - e^{-2d/a}\left(1 + \frac{2d}{a} + \frac{2d^2}{a^2}\right)\right]$$

$$; e^{-2d/a} = 1 + \left(-\frac{2d}{a}\right) + \frac{1}{2!}\left(\frac{2d}{a}\right)^2 + \frac{1}{3!}\left(\frac{2d}{a}\right)^3$$

$$= 1 - \frac{2d}{a} + 2\left(\frac{d}{a}\right)^2 - \frac{4}{3}\left(\frac{d}{a}\right)^3 + \cdots$$

$$d \ll a.$$

$$E \cong \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\left(\frac{4}{3}\frac{d^3}{a^3}\right) \Rightarrow \frac{1}{3\pi\varepsilon_0 a^3} \cdot qd.$$

$$E = \frac{1}{3\pi\varepsilon_0 a^3}\vec{P} \quad \left[E = \frac{1}{\alpha}\vec{P}\right]$$

$$\therefore \boxed{\alpha = 3\pi\varepsilon_0 a^3}$$