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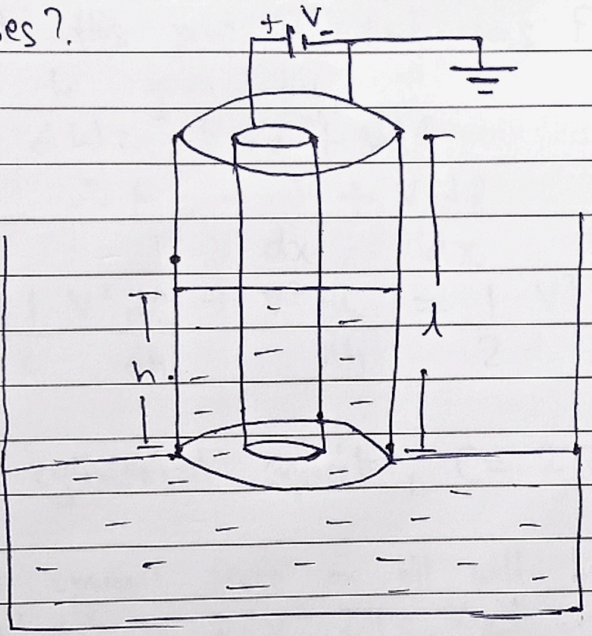
Q. Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at potential V, and outer one is grounded (Fig. 4.32). To what height (h) does the oil rise in the space between the tubes?



A. First find capacitance as a function of h.

Air part: $E = \dfrac{2\lambda}{4\pi\epsilon_0 r} \Rightarrow V = \dfrac{2\lambda}{4\pi\epsilon_0} \ln\left(\dfrac{b}{a}\right)$

Oil part: $D = \dfrac{2\lambda'}{4\pi r} \Rightarrow E = \dfrac{2\lambda'}{4\pi\epsilon r} \Rightarrow V = \dfrac{2\lambda'}{4\pi\epsilon} \ln\left(\dfrac{b}{a}\right)$

$\therefore \dfrac{\lambda}{\epsilon_0} = \dfrac{\lambda'}{\epsilon} \Rightarrow \lambda' = \dfrac{\epsilon}{\epsilon_0}\lambda = \epsilon_r \lambda$

$$Q = \lambda' h + \lambda(l - h) = \epsilon_r \lambda h - \lambda h + \lambda l = \lambda[(\epsilon_r - 1)h + l]$$
$= \lambda(\chi_e h + l)$, where l is total height.

$$\therefore C = \frac{Q}{V} = \frac{\lambda(\chi_e h + l)\, 4\pi\epsilon_0}{2\lambda \ln(b/a)} = \frac{2\pi\epsilon_0 (\chi_e h + l)}{\ln(b/a)}$$

Suppose if a downward force is applied on the oil to move it downwards by an infinitesimal distance dx.
$\because \lambda' > \lambda$, battery will have to do work to push charge dQ so as to maintain a potential difference of V between the two tubes at all points.
If the force that the capacitor was applying on the oil at this point in time was F, the total work done will be given by,

$$dW = -F\,dx + V\,dQ$$

$$\therefore F = -\frac{dW}{dx} + V\frac{dQ}{dx}$$

$$\therefore F = -\frac{1}{2}V^2\frac{dC}{dh} + V^2\frac{dC}{dh} = \frac{1}{2}V^2\frac{dC}{dh}$$

$\because$ for a cylindrical capacitor, $C = 2\pi\varepsilon_0 \dfrac{(\chi_e h + l)}{\ln(b/a)}$

$\therefore$ the net upward force on oil will be,

$$F = \frac{1}{2}V^2\frac{dC}{dh} = \frac{1}{2}V^2\frac{2\pi\varepsilon_0\chi_e}{\ln(b/a)}$$

Downwards gravitational force, $F = mg = \rho\pi(b^2 - a^2)hg$

$$\therefore h = \frac{\varepsilon_0\chi_e V^2}{\rho(b^2 - a^2)g\ln(b/a)}$$