Physics Presentation
Batch-3

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1 Question 1

A particle of charge Q enters a region of width 'a' of uniform magnetic field B[pointing into the page]. The field deflects the particle a distance d above the original line of flight.[as shown in the figure1]

1.Determine the nature of charge Q.
2.Its position vector,velocity vector and momentum vector w.r.t. time
3.Find the change in the kinetic Energy of the particle
4.Magnitude of momentum based on Q,B and perimeter of apparatus[a and d]

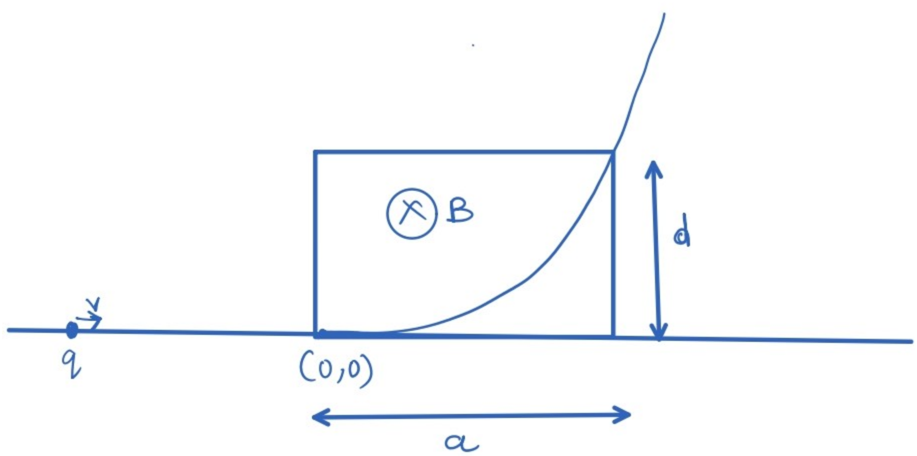

Figure 1: Setup of experiment

1.1 Solution

As per initial conditions given,
let

$$\vec{R_i} = initial\ position\ vector = 0\hat{i} + 0\hat{j} + 0\hat{k} \tag{1}$$

$$\vec{V_i} = initial\ velocity\ vector = v\hat{i} + 0\hat{j} + 0\hat{k} \tag{2}$$

$$\overrightarrow{R(t)} = x(t)\hat{i} + y(t)\hat{j}[as\ there\ is\ no\ force\ in\ z] \tag{3}$$

$$\overrightarrow{V(t)} = x'(t)\hat{i} + y'(t)\hat{j} \tag{4}$$

$$\overrightarrow{B(t)} = -B\hat{k} \tag{5}$$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x' & y' & 0 \\ 0 & 0 & -B \end{vmatrix} = B[x'\hat{j} - y'\hat{i}] \tag{6}$$

Now,

$$\overrightarrow{(F)_B} = Q[\vec{V} \times \vec{B}] = Q[Bx'\hat{j} - By'\hat{i}] \tag{7}$$

According to Newton's second law,

[as only magnetic force is acting on the particle]

$$\overrightarrow{F} = m[x''(t)\hat{i} + y''(t)\hat{j}] = QB[x'(t)\hat{j} - y'(t)\hat{i}] \tag{8}$$

from which we get,

$$mx'' = -QBy' \; and \; y'' = QBx' \tag{9}$$

if we consider $\omega = \frac{QB}{m}$ where ω=Cyclotron frequency,then

$$x'' = -\omega * y' \; and \; y'' = \omega * x' \tag{10}$$

Solving the above differential equations,

$$x(t) = C_1 cos(\omega * t) + C_2 sin(\omega * t) + C_3 \tag{11}$$

$$y(t) = C_2 cos(\omega * t) - C_1 sin(\omega * t) + C_4 \tag{12}$$

applying boundary conditions[at t=0],

$$x(t) = 0 \implies C_3 = 0 \tag{13}$$

$$y(t) = 0 \implies C_4 = 0 \tag{14}$$

$$x'(t) = v \implies C_2 = v \tag{15}$$

$$y'(t) = 0 \implies C_1 = 0 \tag{16}$$

So the solution is

$$\overrightarrow{R(t)} = V sin(\omega * t)\hat{i} + V cos(\omega * t)\hat{j} \tag{17}$$

$$\overrightarrow{V(t)} = V cos(\omega * t)\hat{i} - V sin(\omega * t)\hat{j} \tag{18}$$

$$\overrightarrow{P(t)} = m[V cos(\omega * t)\hat{i} - V sin(\omega * t)\hat{j}] \tag{19}$$

At the end of region with magnetic field,

$$x(t) = a \; and \; y(t) = d \tag{20}$$

$$\implies sin(\omega * t) = \frac{a}{V} \; and \; cos(\omega * t) = \frac{b}{V} \; respectively \tag{21}$$

$$sin^2(\omega * t) + cos^2(\omega * t) = 1 \implies b^2 + a^2 = v^2 \tag{22}$$

now,

$$\vec{P} = m[d\hat{i} - a\hat{j}] \implies \quad \text{mod } \vec{P} = m * [a^2 + d^2]^{\frac{1}{2}} \tag{23}$$

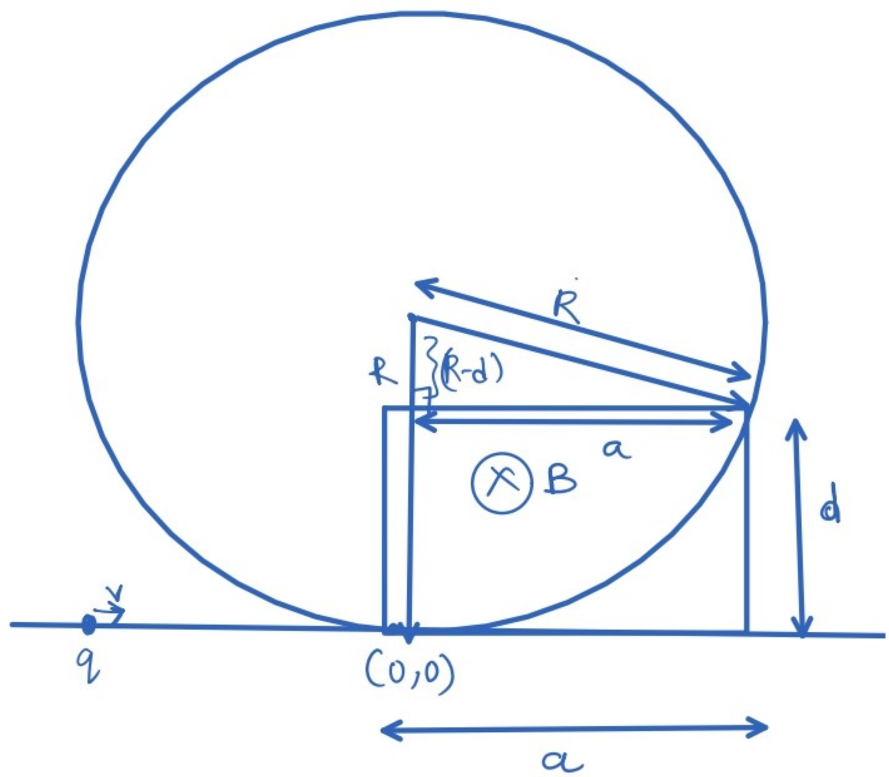

Figure 2: trajectory

from the geometry,

$$[R - d]^2 + a^2 = R^2 \implies R = \frac{a^2 + d^2}{2d} \tag{24}$$

for m, we know $mod \; F_c = mod \; F_B$

$$\implies \frac{m * v^2}{R} = QBV \implies m = \frac{QBR}{V} \tag{25}$$

$$mod\vec{P} = \frac{QB * [a^2 + d^2]^{\frac{1}{2}}}{2d} \tag{26}$$

1.1.1 Derivation of change in Kinetic Energy

Initial Kinetic Energy of the particle,

$$KE_i = \frac{1}{2} * m * v^2 \tag{27}$$

Final Kinetic Energy of the particle,

$$KE_f = \frac{1}{2} * m * mod[vcos(\omega * t)\hat{i} - vsin(\omega * t)\hat{j}] \implies KE_f = \frac{1}{2} * m * V^2 \tag{28}$$

So, the change in Kinetic Energy = 0
As per Work-Energy Theorem,
Work done=Change in Kinetic Energy = 0
Hence, Magnetic Force do NO work and also momentum of particle also get conserved.

1.2 Answers

1.Since, the particle deflects in the same way as it will do if the centripetal force was in the direction of $\vec{V} \times \vec{B}$ so, the charge on the particle is POSITIVE[+].

2.the time evolution is as follws,

$$\overrightarrow{R_t} = Vsin(\omega * t)\hat{i} + Vcos(\omega * t)\hat{j} \tag{29}$$

$$\overrightarrow{V_t} = Vcos(\omega * t)\hat{i} - Vsin(\omega * t)\hat{j} \tag{30}$$

$$\overrightarrow{P(t)} = m[Vcos(\omega * t)\hat{i} - Vsin(\omega * t)\hat{j}] \tag{31}$$

3.There is no net change in kinetic Energy as well as the magnitude of momentum.

4.Momentum in terms of apparatus dimension is given by,

$$mod\vec{P} = \frac{QB * [a^2 + d^2]^{\frac{1}{2}}}{2d} \tag{32}$$

2 Question 2

In 1897 J.J.Thompson discovered the electron by measuring the charge to mass ratio of cathode rays (stream of electrons)
a. First he passed the beam through uniform crossed electric and magnetic fields E and B (mutually perpendicular to each other and both perpendicular to the beam) and adjusted the electric field until he got zero deflection. What then was the speed of particles in terms of E and B?
b. Then he turned off the electric field, and measured the radius of curvature, R, of the beam, as deflected by the magnetic field alone. In terms of E, B, and R, what is the charge-to-mass ratio of the particles?

2.1 Cathode ray tube[CRT] with Electric field and no deviation of particles

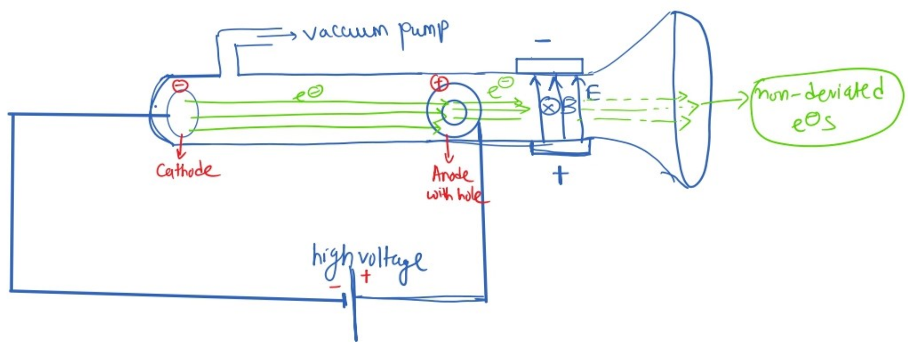

Figure 3: CRT-1

magnetic field, $\vec{B} = -B\hat{k}$
Electric field, $\vec{E} = E\hat{j}$
charge on electron=-e[let] and mass of electron=m[let]
velocity of electrons=v$\hat{i}$
$\vec{a} = 0$ [as there is no deviation in path]
$\implies$ F=0
According to Lorentz force [force on a charge particle moving in $\vec{E}$ and $\vec{B}$ is,
$\vec{F} = Q * [\vec{E} + \vec{V} \times \vec{B}] = 0$
Considering only magnitudes, eE=eVB[as V,B,E are perpendicular to each other and $\vec{V} \times \vec{B}$ is opposite to E

$$\implies V = \frac{E}{B} \tag{33}$$

So the speed of electrons is the ratio between Electric field and magnetic field

2.2 Cathode ray tube[CRT] without Electric Field

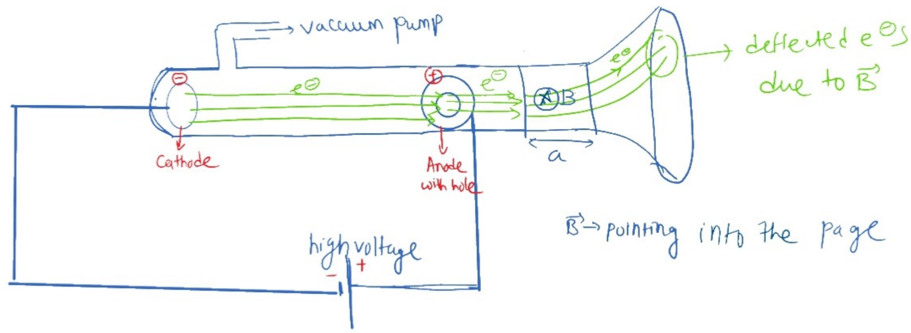

Figure 4: CRT-2

If the Electric field is switched off ,

$$\overrightarrow{F_E} = -e\vec{E} = 0 \tag{34}$$

So, the net force on the particle is the magnetic force,

$$\vec{F} = \overrightarrow{F_B} = Q[\vec{V} \times \vec{B}] = -eVB\hat{\jmath} \tag{35}$$

Since, the force is perpendicular to direction of motion[as$\vec{F_B}.\vec{V} = 0$],so the force provide centripetal acceleration,

$$\frac{m * V^2}{R} = -eBV \tag{36}$$

considering only magnitude

$$\implies V = \frac{eB}{m * R} \tag{37}$$

from equation 33 and equation 37,

$$\frac{e}{m} = \frac{E}{B^2 * R} \tag{38}$$

in this way if we measure radius of curvature of deviated electrons[while electric field is not applied] and then balance the magnetic force by electric force and measure strength of electric and magnetic field we can find specific charge of electron.
1.Since, The charge and mass are the fundamental properties of a particle and since atom is neutral is an evident fact so a paricle present in every type of matter with an unique specific charge showed the fact that matter was consisting of charged particles with unique specific charge which can be separated from it.
2.since, The CRT experiment yield same value of specific charge irrespective of the gas taken inside the tube so it proved that the fundamental particle[electron] was a fundamental component of every matter.