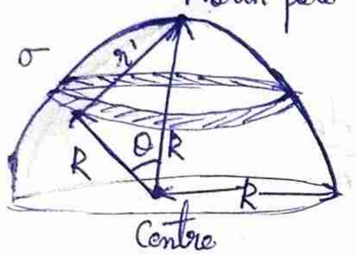
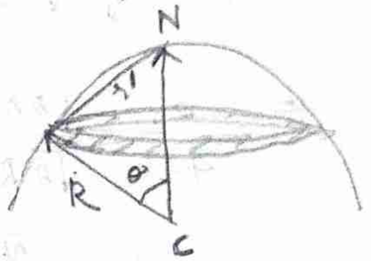
Q. An inverting hemispherical conducting bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the north pole and the centre. If another hemisphere is brought to complete the sphere, how will this potential difference change? What is special about this set up?

Ans:





$$V_{centre} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma \, da}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma}{R}\int da$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma}{R} \times 2\pi R^2$$

$$= \frac{\sigma R}{2\varepsilon_0}$$

To find V at pole, consider a ring as cross-section with each point on the ring being at a distance of r' from the pole.

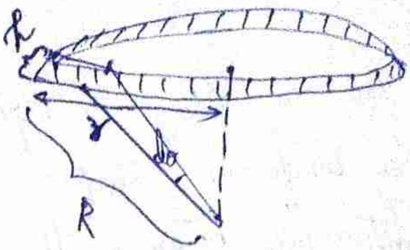
Applying law of cosines,

$$r'^2 = R^2 + R^2 - 2R^2\cos\theta$$

$$= 2R^2 - 2R^2\cos\theta$$

$$= 2R^2(1-\cos\theta)$$

To find da, $da = 2\pi r h$



$r = R\sin\theta$

$h = R\,d\theta$

$da = 2\pi R^2 \sin\theta \, d\theta$

$$\therefore V_{pole} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma \, da}{r'}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_0^{\pi/2} \frac{\sigma \; 2\pi R^2 \sin\theta \, d\theta}{\sqrt{2R^2(1-\cos\theta)}}$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\frac{(2\pi R^2)}{\sqrt{2}R}\int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}} \longrightarrow (1)$$

Put $1-\cos\theta = u$

$\sin\theta \, d\theta = du$.

$$\therefore V_{pole} = \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\int_0^1 \frac{du}{\sqrt{u}}$$

$$\frac{u^{-1/2+1}}{-1/2+1} \quad \frac{u^{1/2}}{1/2}$$

$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\left[2\sqrt{u}\right]_0^1$$

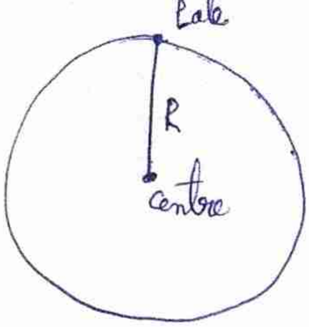
$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0}(2)$$

$$= \frac{\sigma R}{\sqrt{2}\varepsilon_0}.$$

$$\therefore V_{pole} - V_{centre} = \frac{\sigma R}{\sqrt{2}\varepsilon_0} - \frac{\sigma R}{2\varepsilon_0}$$

$$= \frac{\sigma R}{2\varepsilon_0}\left(\sqrt{2} - 1\right)$$

With 2 hemispheres forming spherical shell,



$$V_{centre} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, da}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma}{R}(4\pi R^2)$$

$$= \frac{\sigma R}{\varepsilon_0}$$

For V_{pole}, Consider ①

$$V_{pole} = \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\int_0^{\pi} \frac{\sin\theta\, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma}{2\sqrt{2}}\frac{R}{\varepsilon_0}\int_0^{\pi}\frac{du}{\sqrt{u}}$$

$1-\cos\theta = u$
$\sin\theta\, d\theta = du$

$$= \frac{\sigma}{2\sqrt{2}}\frac{R}{\varepsilon_0}\left(2\sqrt{u}\right)_0^{2}$$

$$= \frac{\sigma}{2\sqrt{2}}\frac{R}{\varepsilon_0}\left(2\sqrt{2}\right)$$

$$= \frac{\sigma R}{\varepsilon_0}$$

$\therefore V_{pole} - V_{centre} = 0.$

Since $E = \frac{-dV}{dr}$, $E = 0$ inside the shell.

$\vec{E}$ inside a closed ~~sph~~ conducting shell is always zero. This is because the electrons on the surface rearrange in the presence of an external electric field, generating an electric field of their own to create $\vec{E}_{net} = 0$.

- This set up is useful for protecting sensitive electronic equipment from external radio frequency interference.

→ Faraday cage (Works on electromagnetic shielding)