Q. There are two rectangular grounded conducting plates (Insulated with respect to each other) and a charged conducting sphere with its center at (a,b).Find the potential and electric field equation due to the charged conductor and induced charge on plates.(Use image method)

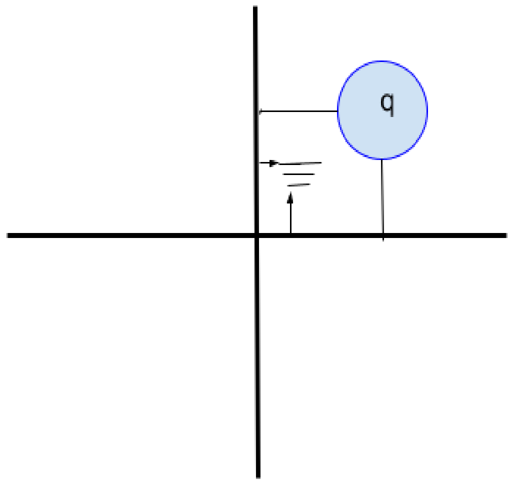

FIGURE - 1

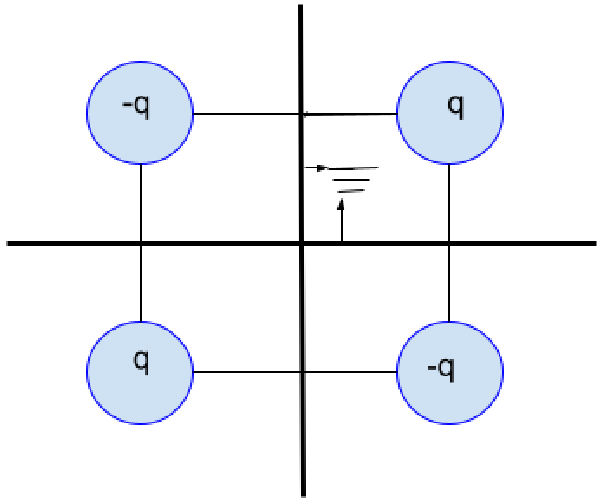

FIGURE - 2

Figure 1 represents diagrammatic representation of the question where there are two perpendicular conducting grounded plates at x =0 and y=0 and a charged spherical conductor with its center at (a,b).

Figure 2 represents diagrammatic representation of our assumed image charged conductor with there center at (-a,b) , (-a,-b) and (a,-b) and charge of (-q , q , -q) respectively.

Ans The expression for electrid field (due to sphere) can be obtained by Gaussian Law.

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\Rightarrow E(a) \times 4\pi a^2 = \frac{4\pi R^2 \sigma}{\epsilon_0} \qquad (a > R)$$

$$\Rightarrow E(a) = \frac{Q}{4\pi \epsilon_0 a^2} \qquad (\text{In radial direction})$$

So the expression for potential (due to sphere) can be obtained from $E(r)$,

$$V(a) = \frac{Q}{4\pi \epsilon_0 a}$$

Now, $E(a) = \dfrac{Q(\vec{r} - r_i)}{4\pi \epsilon_0 \|\vec{r} - \vec{r}_i\|^3}$

$V(a) = \dfrac{Q}{4\pi \epsilon_0 \|\vec{r} - \vec{r}_i\|}$

$r_i \Rightarrow$ centre of sphere
$r \Rightarrow$ any position vector
$\vec{a} = \vec{r} - \vec{r}_i$

- In our question we have our boundary conditions
  - $V = 0$ at $x = 0$ and $\vec{E}$ along x axis
  - $V = 0$ at $y = 0$ and $\vec{E}$ along y axis
  - $V = 0$ at $x = \infty$
  - $V = 0$ at $y = \infty$

- Now we are using Method of image and unique theorem of potential and electric field.

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- By some hit and trial and experience of doing charge and plate sytem in method of image, we arrange 3 more charges $(q, -q, -q)$ in the way arranged in the above figure.

There respective coordinates are =>

i	$\vec{r_i} = (x_i, y_i)$
1	(a, b)
2	$(-a, b)$
3	$(-a, -b)$
4	$(a, -b)$

Net $\vec{E}(x,y) = \vec{E}(x_1, y_1) + \vec{E}(x_2, y_2) + \vec{E}(x_3, y_3) + \vec{E}(x_4, y_4)$

$$\vec{E}(x,y) = \frac{Q}{4\pi\epsilon_0} \sum_{n=1}^{4} \frac{(-1)^{n-1}(\vec{r} - \vec{r_i})}{\|\vec{r} - \vec{r_i}\|^3}$$

$$\vec{E}(0,y) = \frac{Q}{4\pi\epsilon_0}\left[\frac{-2a}{(a^2+(y-b)^2)^{\frac{3}{2}}} + \frac{2a}{(a^2+(y+b)^2)^{\frac{3}{2}}}\right]\hat{i}$$

$$\vec{E}(x,0) = \frac{Q}{4\pi\epsilon_0}\left[\frac{-2b}{((x-a)^2+b^2)^{\frac{3}{2}}} + \frac{2b}{((x+a)^2+b^2)^{\frac{3}{2}}}\right]\hat{j}$$

Net $V(x,y) = V(x_1, y_1) + V(x_2, y_2) + V(x_3, y_3) + V(x_4 + y_4)$

$$V(x,y) = \frac{Q}{4\pi\epsilon_0} \sum \frac{(-1)^{n-1}}{\|\vec{r} - \vec{r_i}\|}$$

$$V(0,y) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{(a^2+(y-b)^2)^{\frac{1}{2}}} - \frac{1}{(a^2+(y-b)^2)^{\frac{1}{2}}} + \frac{1}{(a^2+(y+b))^{\frac{1}{2}}} - \frac{1}{(a^2+(y+b)^{\frac{1}{2}}}\right]$$

$$= 0$$

$$V(x,0) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{((x-a)^2+y^2)^{\frac{1}{2}}} - \frac{1}{((x-a)^2+y^2)^{\frac{1}{2}}} + \frac{1}{((x+a)^2+y^2)^{\frac{1}{2}}} - \frac{1}{((x+a)^2+y^2)^{\frac{1}{2}}}\right]$$

$$= 0$$

$$V(\infty, y) = \frac{Q}{4\pi\epsilon_0} \times 0$$

$$= 0$$

$$V(x, \infty) = \frac{Q}{4\pi\epsilon_0} \times 0$$

$$= 0$$

We see that the equation of $\vec{E}$ and V satisfies the boundary conditions and as $Q_{net} = 0 \Rightarrow \rho = 0 \Rightarrow V$ follows poisson equation and therefore by uniqueness theorem, only one equation of V is possible and we have that