Griffiths problem 1.12

The height of a certain hill (in feet) is given by
$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$,
where y is the distance (in miles) north, x the distance east of South Hadley.

(a) Where is the top of the hill located?
(b) How high is the hill?
(c) How steep is the slope (in feet per mile) at a point one mile north and one mile east of South Hadley? In what direction is the slope steepest, at that point?

Solution:

The top of the hill is an extremism for h(x, y), which corresponds to the gradient being zero (Similarly to df/dx=0 is max/min for the scalar function f(x)).

del h / del x = 10(2y-6x-18) ; del h / del y = 10(2x-8y+18)
∇ h = 10{ (2y-6x-18)icap + (2x-8y+18)jcap }

By solving..
Two equations and two unknowns (x, y)
2y-6x-18 & 2x-8y+18
We got x = -2 & y = 3

(a) Top of hill location at (2 miles west, 3 mile north) coordinate at which ∇ h = 0 (if coordinates are more than one you have to check by putting the value in h(x, y) and choose the coordinate which gives maximum value.)

(b) By putting x = -2 & y = 3 in h(x, y) we get a max height of hill that's 720 feets.
$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$
$= 10\{2(-2)3 - 3(-2)^2 - 4(3)^2 + 18(-2) + 28(3) + 12\}$
$= 720$

(c) By putting x = 1 & y = 1 in ∇ h we get the slope at that point.
∇ h = 10{ (2-6-18)icap + (2-8+18)jcap }
= 10(-22 icap + 22 jcap)
= 220(-icap + jcap)
= Approximately 311 fit/mile northwest.