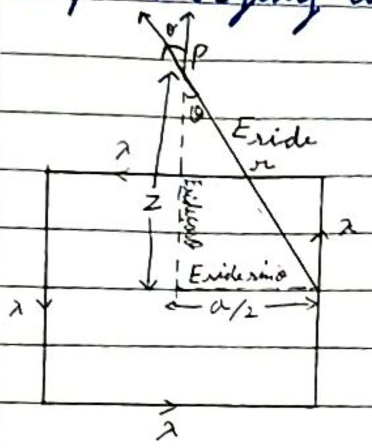
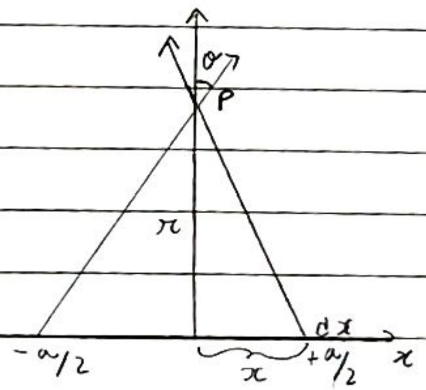
Q Find the electric field a distance z above the center of a square loop carrying uniform line charge λ.

Sol^n:



~~Find~~ Firstly we need to find ~~the~~ E_{side} (Electric field at P due to one side of the square loop.



$$dE = 2E\cos\theta$$

$$dE = 2\frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda dx}{r^2+x^2}\right)\cos\theta\,\hat{z}$$

$\cos\theta = \dfrac{r}{\sqrt{r^2+x^2}}$ and x runs from 0 to $a/2$

$$\vec{E}_{side} = \frac{1}{4\pi\varepsilon_0}\int_0^{a/2}\frac{2\lambda r}{(r^2+x^2)^{3/2}}dx\,\hat{z}$$

$$= \frac{2\lambda r}{4\pi\varepsilon_0}\left[\frac{x}{r^2\sqrt{r^2+x^2}}\right]_0^{a/2}\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda a/2}{r\sqrt{r^2+a^2/4}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda a}{\sqrt{z^2+a^2/4}\sqrt{z^2+a^2/4+a^2/4}}\hat{z}$$

So we have ~~Eside~~ electric field at P due to one side of the square loop.

~~There are 4 sides in a square. So we will multiply it to~~

Now E side has two components, Vertical (E side $\cos\theta$) and horizontal (E side $\sin\theta$)

since, horizontal components are equal and opposite, so they get ~~cancled~~ cancelled.

4 vertical components are equal and in same direction. So they get added up.

$$\cos\theta = \frac{z}{\sqrt{z^2+a^2/4}}$$

So, $\vec{E}_P = 4\vec{E}_{side}\cos\theta$

$$= 4 \times \frac{1}{4\pi\varepsilon_0}\frac{\lambda a}{\sqrt{z^2+a^2/4}\sqrt{z^2+a^2/2}} \times \frac{z}{\sqrt{z^2+a^2/4}}\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4\lambda a z}{(z^2+a^2/4)\sqrt{z^2+a^2/2}}\hat{z}$$

1) When $z \gg a$

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda a}{z^2}$$

Thus. The loop behaves like a point charge in this condition

2) When $a \gg z$

~~$E_P = \frac{1}{4\pi\varepsilon_0}$~~ $E_P = 0$

Because there is no vertical component.