Physics Presentation

# Preliminary results in Electrostatics

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## Result I

We seek to prove that static electric fields are conservative in nature. - Laws and definitions used:

1. Coulomb's Law: $\mathbf{E} = \frac{kq\hat{\mathbf{r}}}{r^2}$
2. Principle of superposition
3. $\mathbf{r} = r\hat{\mathbf{r}}$

We start with an arbitrary static electric field and a test charge placed within it,to compute the line integral along some path. It is safe to assume that the static field is generated by a set of stationary charges. Therefore, let us start by computing the integral for the field corresponding to just one charge. Then by superposition we can extend it to the set of charges.

$$W = \int_a^b \mathbf{E} \cdot \mathbf{dr}$$

By coulomb's law we get,

$$W = \int_a^b \frac{kq\hat{\mathbf{r}}}{r^2} \cdot \mathbf{dr}$$

Note: $\hat{\mathbf{r}} \cdot \mathbf{dr} = \hat{\mathbf{r}} \cdot \hat{\mathbf{r}} dr + \hat{\mathbf{r}} \cdot r d\hat{\mathbf{r}}$
But $\hat{\mathbf{r}} \cdot d\hat{\mathbf{r}} = 0 \; \forall \mathbf{r} \Rightarrow \hat{\mathbf{r}} \cdot d\mathbf{r} = dr$
Therefore, we get,

$$W = \int_a^b \frac{kq}{r^2} dr = [\frac{-kq}{r}]_a^b$$

Therefore, the electric field due to point charge is conservative . Then by superposition the field due to set of charges is also conservative.

Comment: This is true in general for any central force.

# Result II

Find average potential over the surface of a sphere in free space having arbitrary static electric field given that potential at the centre of the sphere is $V(O)$.
- Laws and Definitions used in the solution:

1. Gauss' Law: $\oint \mathbf{E} \cdot \mathbf{da} = \frac{Q_{enc}}{\epsilon_0}$
2. $V(\mathbf{r}) - V(\mathbf{O}) = -\int_{\mathbf{O}}^{\mathbf{r}} \mathbf{E} \cdot \mathbf{dl}$

We assume a sphere of radius R in free space centred at O. The average of potential on the surface is given by

$$V_{avg} = \frac{\oint_S V da}{\oint_S da}$$

By using the definition of $V$, we get,

$$V(\mathbf{R}) = V(\mathbf{O}) - \int_{\mathbf{O}}^{\mathbf{R}} \mathbf{E} \cdot \mathbf{dl}$$

$$V_{avg} = \frac{\oint_S [V(\mathbf{O}) - \int_{\mathbf{O}}^{\mathbf{R}} \mathbf{E} \cdot \mathbf{dl}] da}{\oint_S da}$$

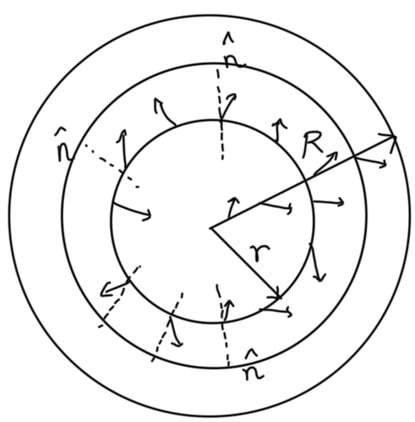


Fig. 1

The double integral physically means to compute dot product of $E$ on all points along a radial line for all the radial lines. So, instead of summing in this manner we could sum $E \cdot dl$ for all points on a smaller shell, since, the area vector is along the radial direction. Thus, the integrals get swapped in some manner and we get,

$$V_{avg} = V(\mathbf{O}) - \frac{\int_{\mathbf{O}}^{\mathbf{R}} [\oint_S \mathbf{E} \cdot \mathbf{dl}] da}{\oint_S da}$$

But, by Gauss' Law we know that all of the shells will get the value 0 for the surface integral since, none of them contain any charge within. Therefore,

$$V_{avg} = V(\mathbf{O})$$

$$-\nabla \times \nabla-$$