Consider a spherical charge distribution of radius R that is centred at an origin O. The charge density $\rho(r)$ is defined by

$$\rho(r) = \begin{cases} \rho_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases} \quad (1)$$

where ρ_0 is a constant.

a) Use Gauss's Law to show that the electric field $E(r)$ is given by

$$E(r) = \begin{cases} \rho_0/3\epsilon_0 \, r & \text{for } r \leq R \\ \rho_0/3\epsilon_0 \left(R/r\right)^3 r & \text{for } r \geq R \end{cases} \quad (2)$$

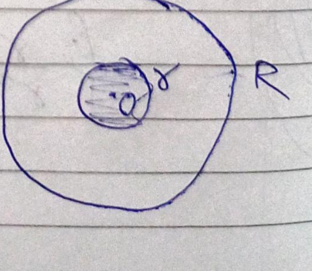
b) Hence show that the electric potential $V(r)$ is given by

$$V(r) = \begin{cases} \dfrac{\rho_0}{6\epsilon_0}(3R^2 - r^2) & \text{for } r \leq R \\ \dfrac{\rho_0}{3\epsilon_0}\dfrac{R^3}{r} & \text{for } r \geq R \end{cases} \quad (3)$$

Ans a) for $r \leq R$

(Gauss law

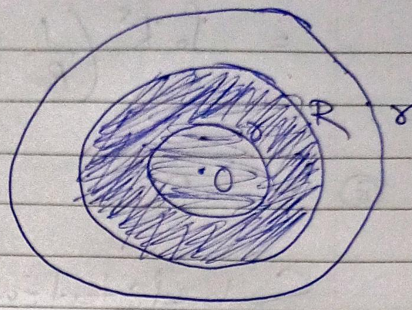
$E \times 4\pi r^2 = \frac{1}{\epsilon_0} \times q_{net}$



$= \frac{\rho_0}{\epsilon_0} \times \frac{4\pi r^3}{3}$

(a) for $r \geq R$

$$\frac{\rho_0}{\epsilon_0} \times \frac{4\pi R^3}{3}$$



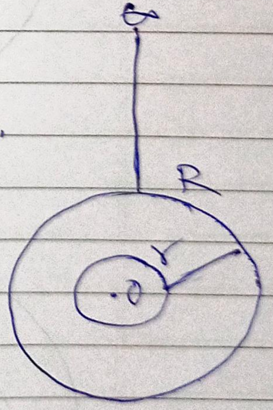
But this is (2), because $E(r) = E(r)\,\hat{r}$

b) Using the result $V_{ab} = V(b) - V(a)$

$$= -\int_a^b E \cdot dr$$

putting $V(\infty) = 0$ yield.

$$V(r) = -\int_\infty^r E \cdot dr \quad (4)$$



We consider each region separately.

(1) $r \leq R$

Substituting (2) into (4) gives

$$V(r) = -\int_\infty^R E\,dr - \int_R^r E\,dr$$

$$= -\frac{\rho_0 R^3}{3\epsilon_0}\int_\infty^R \frac{dr}{r^2} - \frac{\rho_0}{3\epsilon_0}\int_R^r r\,dr$$

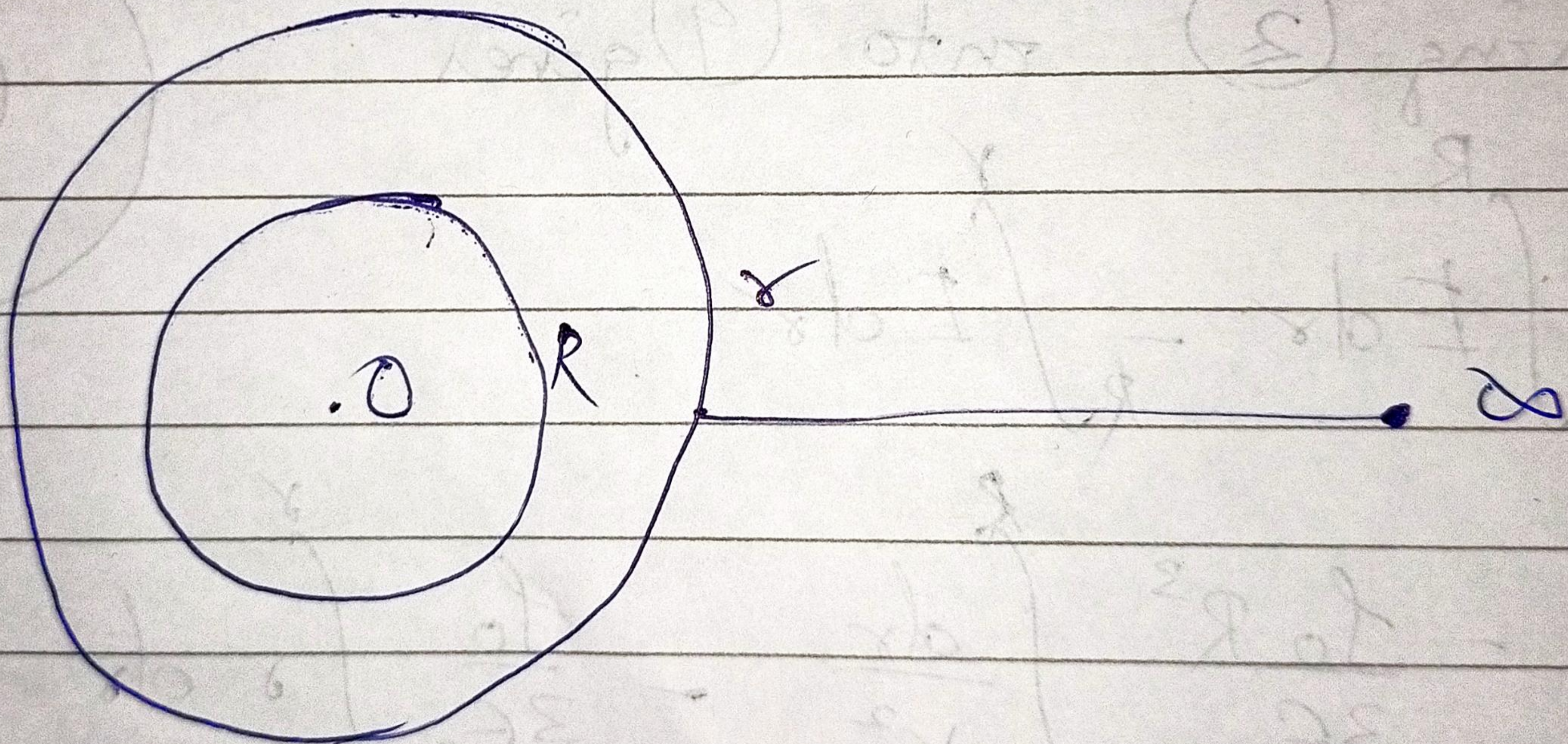
$$= \frac{\rho_0 R^2}{3\epsilon_0}\left[\frac{1}{r}\right]_\infty^R - \frac{\rho_0}{3\epsilon_0}\left[\frac{r^2}{2}\right]_R^r$$

$$= \frac{\rho_0 R^3}{3\epsilon_0}\left(\frac{1}{R} - \frac{1}{\infty}\right) - \frac{\rho_0}{6\epsilon_0}(r^2 - R^2)$$

(ii) $r \geq R$

Substituting (2)$_2$ into (4) gives

$$V(r) = -\int_{\infty}^{r} E\, dr = -\frac{\rho_0 R^3}{3\epsilon_0}\int_{\infty}^{r}\frac{dr}{r^2} = \frac{\rho_0 R^3}{r}$$



O
R
r
∞