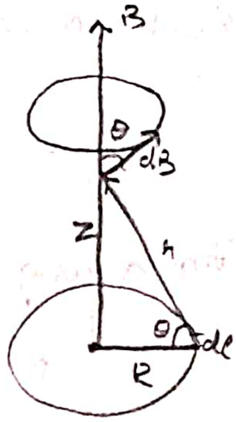
Physics Presentation.

Concept : Biot - Savart's law - Magnetostatics.

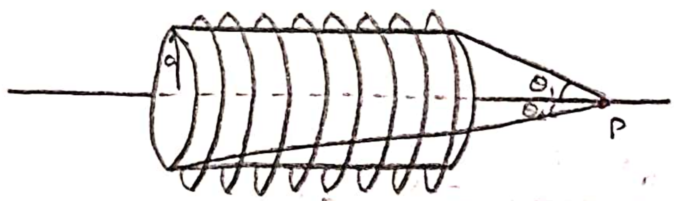
The magnetic field at a distance z above the centre of a circular loop (ring) of radius R, which carries a steady current is,



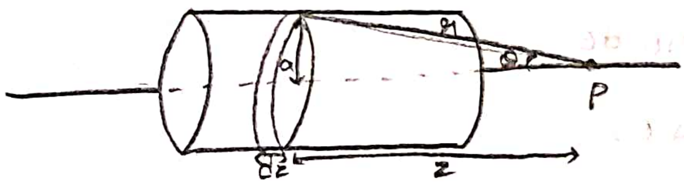
$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}.$$

Question :

Find the magnetic field at point 'P' on the axis of a tightly wound solenoid consisting of n- turns per unit length wrapped around a cylindrical tube of radius a and carrying current I. what is the field on the axis of an infinite solenoid ?



Answer:



Consider a particle (ring) of width dz, with a distance 'z' from P.

The solenoid has 'n' turns, hence $I_s = nI$

Using trigonometry,

$$\cot\theta = \frac{z}{a}$$

$$z = a\cot\theta.$$

On differentiation, $dz = -a\,\mathrm{cosec}^2\theta.$

$$= \frac{-a}{\sin^2\theta} \longrightarrow (1)$$

We know that magnetic field of a circular loop of radius R which carries a steady current I at a distance 'z' is,

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

For a ring of width dz (solenoid),

$$B = \frac{\mu_0 I_s}{2} \cdot \int \frac{a^2}{(a^2+z^2)^{3/2}} dz$$

$$= \frac{\mu_0 n I}{2} \int \frac{a^2}{(a^2+z^2)^{3/2}} dz$$

$$(a^2+z^2)^{3/2} = (a^2 + a^2\cot^2\theta)^{3/2}$$

$$= [a^2(1+\cot^2\theta)]^{3/2} = [a^2 \operatorname{cosec}^2\theta]^{3/2}$$

$$= a^3 \operatorname{cosec}^3\theta$$

$$= \frac{a^3}{\sin^3\theta}$$

$$B = \frac{\mu_0 n I}{2} \int \frac{a^2}{\frac{a^3}{\sin^3\theta}} \cdot \frac{-a}{\sin^2\theta} d\theta$$

$$= \frac{\mu_0 n I}{2} \int \sin\theta \, d\theta$$

θ follows from $\theta_1 \longrightarrow \theta_2$.

$$\Rightarrow B = -\frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\mu_0 n I}{2} [\cos\theta]_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 I . n}{2} [\cos\theta_2 - \cos\theta_1]}$$

For an infinite solenoid, $\theta_1 = \pi$, $\theta_2 = 0$.

$$\Rightarrow B = \frac{\mu_0 I n}{2} (1-(-1))$$

$$\Rightarrow \boxed{B = \mu_0 n I}$$