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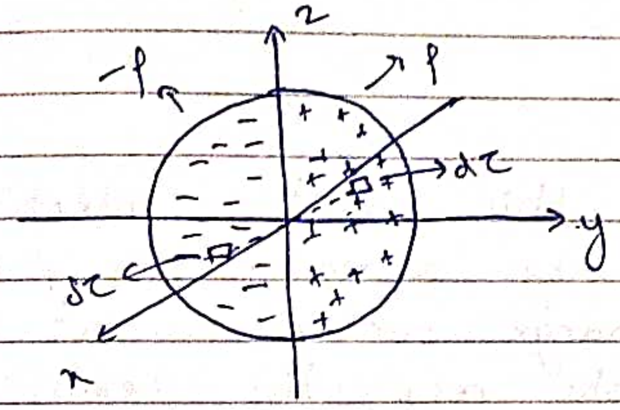
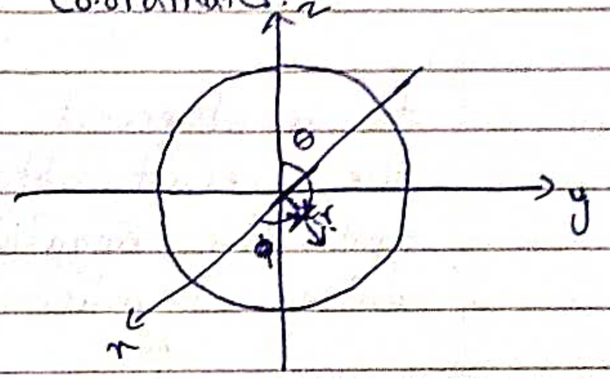
PH1213 - Question Presentation.

Q. Assume a spherical uniform charge distribution with one of the hemispheres having positive charge (ρ) and the other having a negative charge ($-\rho$). Calculate the dipole moment of such a sphere with Radius R.



Polar Coordinates:



$$d\vec{p} = dq \times d\hat{r} = \rho\, d\tau \times 2r\,\hat{r}$$

$$d\tau = r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$\hat{r} = \cancel{r\cos\phi\,\hat{i}} + r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$$

$$\therefore d\vec{p} = 2r\rho\,(r^2\sin\theta\, dr\, d\theta\, d\phi)(r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z})$$

$$= 2\rho r^3(\sin^2\theta\cos\phi\,\hat{x} + \sin^2\theta\sin\phi\,\hat{y} + \sin\theta\cos\theta\,\hat{z})\, dr\, d\theta\, d\phi$$

$$\therefore \vec{p} = 2\rho \int_0^{\pi}\int_0^{\pi}\int_0^{R} r^3(\sin^2\theta\cos\phi\,\hat{x} + \sin^2\theta\sin\phi\,\hat{y} + \sin\theta\cos\theta\,\hat{z})\, dr\, d\theta\, d\phi$$

$$= \frac{2\rho R^4}{4}\int_0^{\pi}\int_0^{\pi}(\sin^2\theta\cos\phi\,\hat{x} + \sin^2\theta\sin\phi\,\hat{y} + \sin\theta\cos\theta\,\hat{z})\, d\theta\, d\phi$$

Now, $\int_0^{\pi}\sin^2\theta\int_0^{\pi}\cos\phi\, d\phi\, d\phi = 0$ as $\int_0^{\pi}\cos\phi\, d\phi = 0$

and $\int_0^{\pi}\sin\theta\cos\theta\, d\theta = 0$

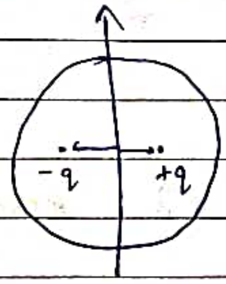
$$\therefore \vec{p} = \frac{\rho R^4}{2}\int_0^{\pi}\int_0^{\pi} \sin^2\theta \sin\phi \,\cancel{\hat{y}}\, d\theta\, d\phi \cdot \hat{y}$$

$$= \frac{\rho R^4}{2}\cdot 2\int_0^{\pi} \sin^2\theta\, d\theta \;\hat{y} = \rho R^4 \int_0^{\pi} \sin^2\theta\, d\theta$$

$$\vec{p} = \frac{\rho R^4 \pi}{2}\,\hat{y}$$

It is observed that this result is identical to the result obtained by assuming the positive and the negative charge concentrated at the centre of mass of the respective hemispheres.

$\therefore$



C.O.M of solid hemisphere $= \frac{3R}{8} = a$

$$q = \rho\cdot\frac{2}{3}\pi R^3$$

$$\therefore \quad \vec{p} = 2\times a\times q = \cancel{2}\times\frac{\cancel{3}R}{\cancel{8}\,4}\times\rho\times\frac{\cancel{2}\pi R^3}{\cancel{3}}$$

$$= \frac{\rho\pi R^4}{2}$$

This result can be extended to any such symmetrical systems to find the dipole moment, like for example two cones sharing a base, a cube, a cuboid, a cylinder, etc.