29/6/22

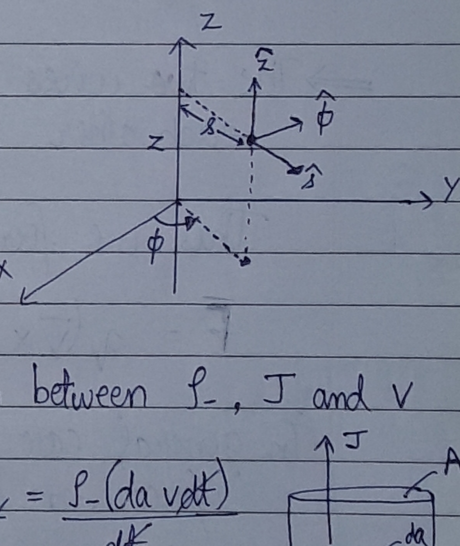
Physics Presentation

Q: It may have occured to you that since parallel currents attract, the current within a single wire should contract into a tiny concentrated stream along the axis. Yet in practice the current typically distributes itself quite uniformly over the wire. How do you account for this? If the positive charges (density ρ_+) are "nailed down", and the negative charges (density ρ_-) move at speed v (and none of these depends on the distance from the axis), show that $\boxed{\rho_- = -\rho_+ \gamma^2}$, where $\gamma = 1/\sqrt{1-(v/c)^2}$ and $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

If the wire as a whole is neutral, where is the compensating charge located?

Solution: Concepts to be used

1) Cylindrical coordinate system



$$(x, y, z) \equiv (s\cos\phi, s\sin\phi, z)$$

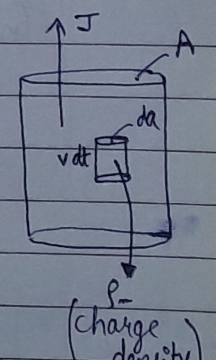
2) Gauss Law

$$\oint \bar{E}\cdot \bar{da} = \frac{Q_{in}}{\varepsilon_0}$$

3) Ampere's Law

$$\oint \bar{B}\cdot \bar{dl} = \mu_0 I_{enc}$$

4) Relation between ρ_-, J and v



$$dI = \frac{dq}{dt} = \frac{\rho_-(da\, v\cancel{dt})}{\cancel{dt}}$$

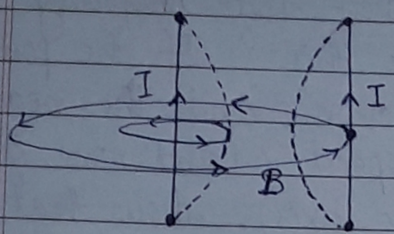
$$dI = \rho_- v\, da$$

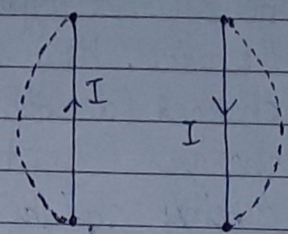
Integrating both sides,

$$I = \rho_- v \int da = (\rho_- v)A$$

$$\therefore \boxed{J = I/A = \rho_- v}$$

The first encounter with magnetic force was when two parallel current carrying wires felt a force of attraction or repulsion depending on direction of current.





Current in same direction
(The dotted lines represent the shape of wire when the current is turned on.)

$\Rightarrow$ The two wires attract each other

Current in opposite direction

(The dotted lines represent the shape of the wire)

$\Rightarrow$ The two wires repel each other.

This we found using the Lorentz force law which is

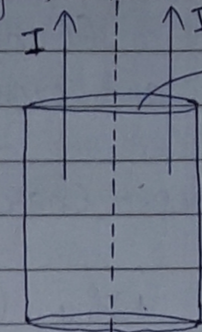
$$\bar{F} = q(\bar{v} \times \bar{B})$$

v is the vellocity of mobile charges

In general case where $\bar{E}$ and $\bar{B}$ both are present force on a moving charge q is

$$\boxed{F_{Net} = q[\bar{E} + (\bar{v} \times \bar{B})]}$$

After knowing that parallel current carrying wires attract each other we now try to analyze the situation inside a wire.



Here too we expect that the current carrying charges should collect near the axis

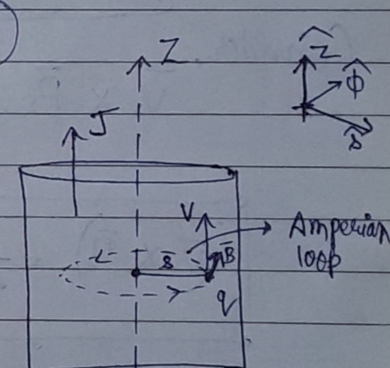
But after a few charges collected near the axis there is a net electric field generated which opposes the further acculmulation due to magnetic force.

Hence, we see that equilibrium will be attained when the net force on a charge q inside the wire will be zero.

$$\therefore \; F_{Net} = q\left[\bar{E} + (\bar{V} \times \bar{B})\right] = 0$$

$$\therefore \; \boxed{\bar{E} = -\bar{V} \times \bar{B}} \longrightarrow (1)$$

Consider the following figure,



Using Ampere's Law,

$$\oint \bar{B} \cdot \bar{dl} = \mu_0 I_{enc} = \mu_0 (\bar{J} \pi s^2)$$

$$\therefore \; \bar{B} \cdot 2\pi s = \mu_0 \bar{J} \pi s^2$$

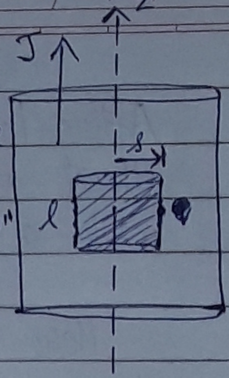
$$\therefore \; \bar{B} = \frac{\mu_0 J s}{2}$$

$$\boxed{\bar{B} = \frac{\mu_0 \rho_- V s}{2} \hat{\phi}} \longrightarrow (2)$$

Earlier we derived that
$J = \rho_- V$
from the diagram we say that $\boxed{\bar{J} = (\rho_- V)\hat{z}}$

Now to calculate electric field at a distance "s" from the axis.

Consider a cylindrical shell of length "l" and cross section area (πs^2)



$\therefore$ Charge enclosed $(Q_{in}) = \rho \times V$

$$= (\rho_+ + \rho_-)(\pi s^2 l)$$

Using Gauss Law,

$$\oint \bar{E} \cdot d\bar{a} = \underbrace{E(2\pi s l)}_{\text{curved surface area}} = \frac{Q_{in}}{\varepsilon_0}$$

Since, $\bar{E}$ is along $\hat{s}$ direction, the flux through top and bottom is zero

$$\therefore E(2\pi s l) = \frac{(\rho_+ + \rho_-)\pi s^2 l}{\varepsilon_0}$$

$$\therefore \boxed{\bar{E} = \frac{1}{2\varepsilon_0}(\rho_+ + \rho_-)\, s\, \hat{s}} \longrightarrow (3)$$

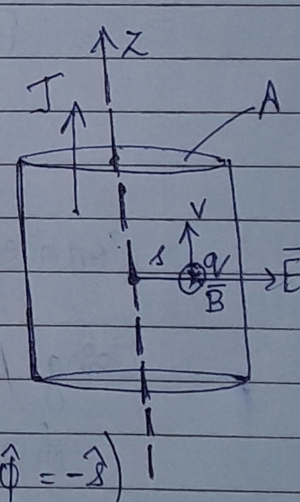
Consider,

$$\therefore \bar{V} \times \bar{B} = \left[(v\,\hat{z}) \times \left(\frac{\mu_0 \rho_- v s}{2}\right)\hat{\phi}\right]$$

(from (2))

$$\boxed{(\bar{V} \times \bar{B}) = -\frac{\mu_0 \rho_- v^2 s}{2}\,\hat{s}} \longrightarrow (4)$$

(since, $\hat{z} \times \hat{\phi} = -\hat{s}$)



Using (1), (3) & (4)

$$\frac{1}{2\varepsilon_0}(\rho_+ + \rho_-)\, s = \frac{\mu_0 \rho_- v^2 s}{2}$$

$$\therefore \rho_+ + \rho_- = (\mu_0 \varepsilon_0) v^2 \rho_- = \left(\frac{v}{c}\right)^2 \rho_- \quad \left(\because c^2 = \frac{1}{\mu_0 \varepsilon_0}\right)$$

$$\therefore \rho_+ = \left[\left(\frac{v}{c}\right)^2 - 1\right]\rho_- = -\left[1 - \left(\frac{v}{c}\right)^2\right]\rho_-$$

Let, $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ $\Rightarrow$ $\gamma^2 = \frac{1}{1-(v/c)^2}$

$\therefore$ $\rho_+ = -\frac{\rho_-}{\gamma^2}$ $\Rightarrow$ $\boxed{\rho_- = -\gamma^2 \rho_+}$ $\longrightarrow$ (5)

Since by definition $\gamma > 1$
from equation (5) we see that $|\rho_-| > |\rho_+|$

Hence, some negative mobile charge does get accumulated near the axis

Since, we assumed that the wire as a whole is neutral so the amount of positive and negative charge must be same.

From $\boxed{\rho_- = -\gamma^2 \rho_+}$ we can conclude that the mobile negative charges fill a smaller inner cylinder, leaving a shell of positive (stationary) charges at the outside.

★ **NOTE:-** Since the drift velocity (v) is very small compared to c (i.e. $v \ll c$) the effect is extremely small but real.