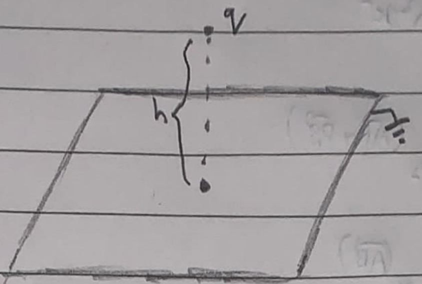
PHYSICS PROBLEM PRESENTATION

Q. A charge q is placed at height h from a grounded, large conducting plate. Find the radius of the circular area that contains charge $-q/3$ and has its centre at the foot of perpendicular from charge to plate.

Sol.



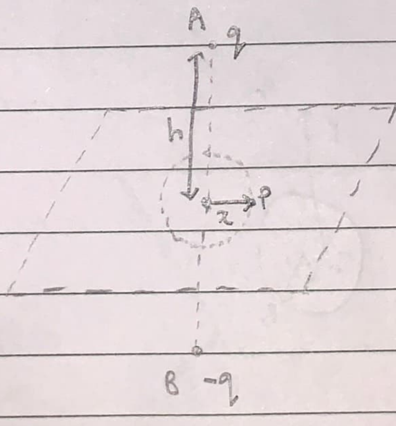
We shall use the method of images to solve this problem. We first find the Electric field on a ring of radius x from the foot of the perpendicular. We do this since we know the relation between charge density and electric field using Maxwell's equation or Gauss' Law.

Here, the boundary conditions are:

(i) The electric field at the plane should be normal to it.

(ii) The electric field as we move towards infinity should be zero

Both these conditions are satisfied if we remove the plate and put a negative charge $-q$ at distance h from the plane of the plate below it.



Consider a point P on a ring of radius x.

Electric field due to point charge q, $E_q = \dfrac{q}{4\pi\varepsilon_0 |\vec{AP}|^2} \dfrac{\vec{AP}}{|\vec{AP}|}$

Electric field due to point charge $-q$, $E_{-q} = \dfrac{q}{4\pi\varepsilon_0 |\vec{PB}|^2} \dfrac{\vec{PB}}{|\vec{PB}|}$

$\Rightarrow |\vec{AP}| = |\vec{PB}| = \sqrt{h^2+x^2}$

$$\Rightarrow \vec{E}_{net} = \frac{q}{4\pi\varepsilon_0 (h^2+x^2)^{3/2}} (\vec{AP}+\vec{PB})$$

$$\Rightarrow \vec{E}_{net} = \frac{q}{4\pi\varepsilon_0 (h^2+x^2)^{3/2}} (\vec{AB})$$

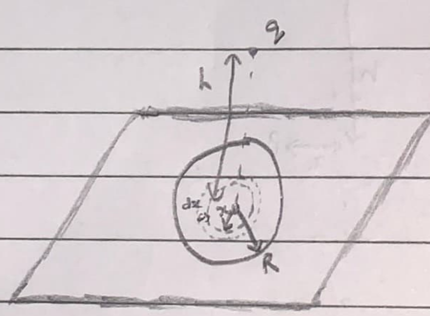
$$\Rightarrow \vec{E}_{net} = \frac{q}{4\pi\varepsilon_0 (h^2+x^2)^{3/2}} (2h) \text{ downwards}$$

Now, charge density at the point is given by

$$\sigma = \varepsilon_0 |\vec{E}|$$

$$\Rightarrow \sigma = \frac{qh}{2\pi (h^2+x^2)^{3/2}}$$

Now, we need to find the radius of the ~~ring~~ disc which contains $\frac{1}{3}$rd of the induced $-q$ charge on the plane. Let the radius of this ring ~~will~~ be R.



charge on ring with radius x and width dx is given by

$$dq = \sigma(2\pi x\, dx)$$

$$\Rightarrow -\frac{q}{3} = \int_0^R \frac{qhx}{(h^2+x^2)^{3/2}}\, dx$$

Let $h^2+x^2 = t$

$$\Rightarrow \frac{dt}{2} = x\,dx$$

$$\Rightarrow -\frac{q}{3} = \frac{1}{2}\int \frac{qh\, dt}{t^{3/2}}$$

$$\Rightarrow -\frac{q}{3} = qh\left(-t^{-1/2}\right)$$

$$\Rightarrow -\frac{q}{3} = qh\left[-(h^2+x^2)^{-1/2}\right]_0^R$$

$$\Rightarrow -\frac{1}{3} = \left(\frac{h}{\sqrt{h^2+R^2}} - 1\right)$$

$$\Rightarrow \frac{2}{3} = \frac{h}{\sqrt{h^2+R^2}}$$

$$\Rightarrow 4h^2 + 4R^2 = 9h^2$$

$$\Rightarrow \boxed{R = \frac{\sqrt{5}\,h}{2}}$$