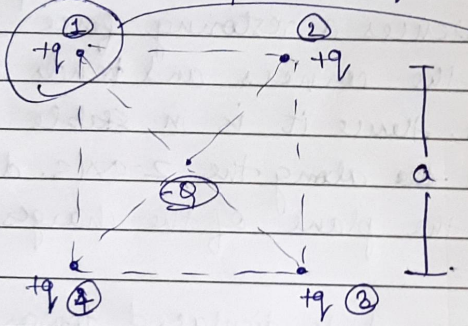
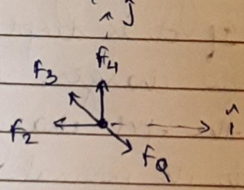
__/__/____

Q At each corner of a square is a particle with charge +q. Placed at the center of the square is a point charge of opposite sign, of magnitude Q. What value must Q have to make the total force on each of the 4 particles zero? Is the charge Q at the center of the square in stable or unstable equilibrium?





We can simply apply the principle of superposition to find the net force on any of the charges.

$$\vec{F}_{net} = \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_Q = \vec{0}$$

$$\vec{F}_2 + \vec{F}_4 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\hat{j} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\hat{i} = \frac{\sqrt{2}q^2}{4\pi\varepsilon_0 a^2}$$, 45° above, from the -ve x-axis

∴ F_3 & F_Q are also aligned at 45° from the axes.

$$\vec{F}_{net} = \frac{\sqrt{2}q^2}{4\pi\varepsilon_0 a^2} + \underbrace{\frac{q^2}{4\pi\varepsilon_0 (2a^2)}}_{(F_3)} \underbrace{- \frac{qQ}{4\pi\varepsilon_0 (a^2/2)}}_{(F_Q)} = 0$$

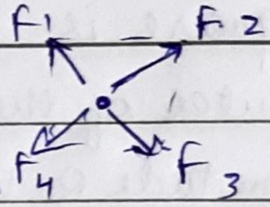
$$\frac{q^2}{4\pi\varepsilon_0 a^2}\frac{(2\sqrt{2}+1)}{2} = \frac{2qQ}{4\pi\varepsilon_0 a^2}$$

$$\Rightarrow q\left(\frac{2\sqrt{2}+1}{4}\right) = Q$$ (magnitude) [Q → is -ve charge]

Due to symmetry of the problem, similarly we can do for q_2, q_3 & q_4 to get the same value of $-Q$.

Now,

$-Q$ charge →

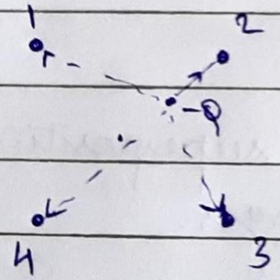


It experiences equal opposite forces, hence it is in equilibrium.

Regarding its stability (of equilibrium) –

1) If it is displaced in a direction $\perp^r$ to the plane of the charges, it experiences a restoring force from the +ve charges at the corners and tends to return to its original position. Hence it is in stable equilibrium wrt displacements ~~in the~~ along the z-axis dir^n.

2) If it is displaced in the plane of the charges –



The charge it is displaced towards (say 2) will now exert a stronger force because the force given by Coulomb's law has inverse square dependence to the distance, while the distance from the other three charges increases and hence the force due to them decreases. So $-Q$ will move towards the charge it is nearer to. Hence there is no "restoring force" tendency. So it is unstable equilibrium.