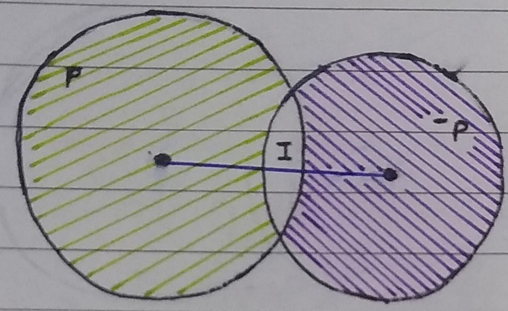
Gauss theorem: $\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$

Q. Find the electric field intensity E in the region of intersection of two spheres uniformly charged by unlike charges with the volume densities ρ and $-\rho$, if the distance between the centres of the sphere is determined by Vector I.



→ Using Gauss Theorem, we can show the electric field intensity within an uniformly charged sphere.

$$E \oint dS = \frac{Q_{enc}}{\varepsilon_0}$$ ... using Gauss Theorem

To Find Q_{enc}

→ $$Q_{enc} = \int_V \rho \, d\tau = \rho 2(2\pi)\frac{r^3}{3} = \frac{4}{3}\pi r^3 \rho$$

$$\therefore \ E(4\pi r^2) = \frac{\frac{4}{3}\pi r^3 \rho}{\varepsilon_0}$$ .... (Substituted the value of Q_{enc})

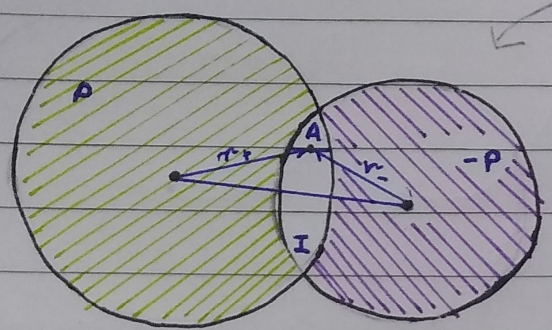
$$\therefore \ E = \frac{1}{3}\frac{r\rho}{\varepsilon_0}(\hat{r})$$

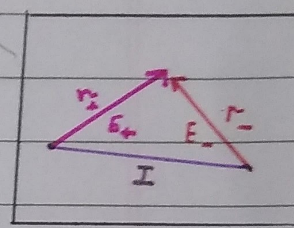
* In the above obtained equation, $\hat{r}$ is the direction of r.

The field in the region of interaction of the spheres can be considered as the superposition of the field of two uniformly charged spheres.

$\therefore$ At an arbitrary point A,

$$E = E_+ + E_-$$





$$E_+ = \frac{1}{3}\frac{r_+ P}{\varepsilon_0}\hat{r} \quad \text{and} \quad E_- = \frac{1}{3}\frac{-r_- P}{\varepsilon_0}\hat{r}$$

$$\therefore \quad E = \frac{1}{3}\frac{r_+ P}{\varepsilon_0}\hat{r} + \frac{1}{3}\frac{-r_- P}{\varepsilon_0}\hat{r}$$

$$= \frac{P}{3\varepsilon_0}(r_+ - r_-)$$

$$= \frac{P(I)}{3\varepsilon_0}$$

$\therefore$ In the region of intersection of spheres, the field is uniform.