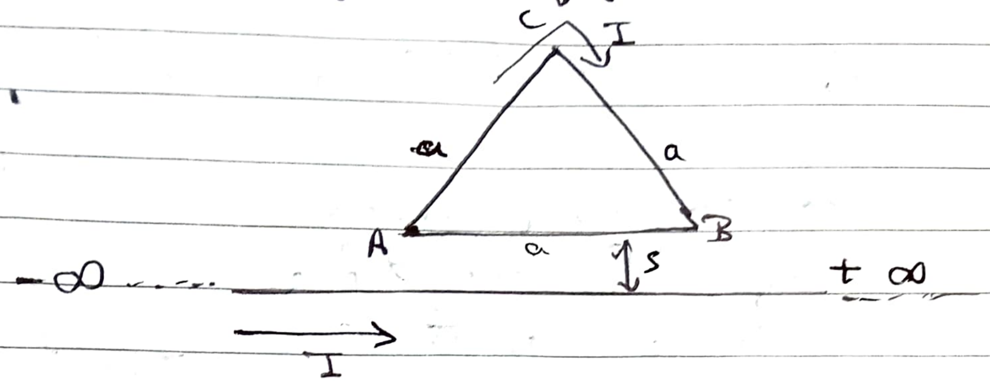
Q Find the force on the triangular loop in the given figure

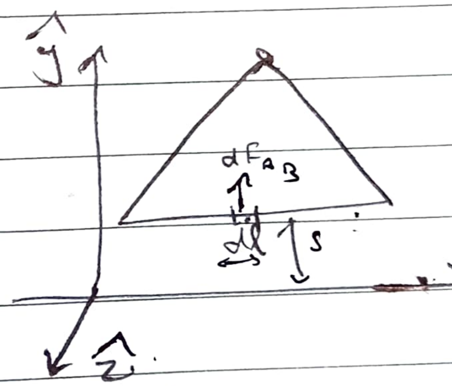


Solution

from Biot-Savart's law, we know that magnetic field around an infinit straigh current carrying conductor $B = \frac{\mu_0 I}{2\pi r}$,

where B is the magnetic field, I is the current and r is the distance from the conductor

We will find force on each side and then add them:



Force on side AB:

$$\vec{dl} = -dx\,\hat{x}.$$

$$\vec{dF_{BA}} = I(\vec{dl} \times \vec{B})$$

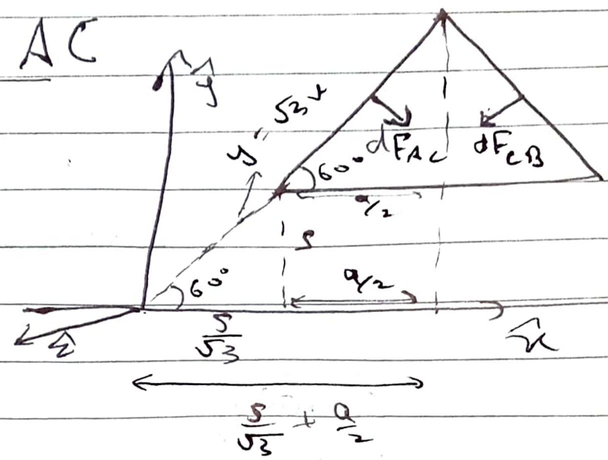
$$= I\left(-dx\,\hat{x} \times \frac{\mu_0 I}{2\pi s}\hat{z}\right)$$

$$\vec{dF_{BA}} = \frac{\mu_0 I^2}{2\pi s} dx\,\hat{y}$$

$$F_{BA} = \int dF_{BA} = \frac{\mu_0 I^2}{2\pi s} a \, \hat{y}$$

Force on BC and AC

The x-components of dF_{AC} and dF_{CB} cancel out.



Force in y direction

$$\cancel{2 \int I (d\vec{l} \times \vec{B})}$$

$$2 I \int (d\vec{l} \times \vec{B})$$

$$2 I \int (dx \, \hat{x} + dy \, \hat{y}) \times \left(\frac{\mu_0 I}{2\pi y} \hat{z} \right)$$

$$\cancel{2 I \int \frac{dx \, \hat{x}}{2\pi y}}$$

$$2 I \int \left(dx \, \hat{x} \times \frac{\mu_0 I}{2\pi y} \hat{z} \right) + \left(dy \, \hat{y} \times \frac{\mu_0 I}{2\pi y} \hat{z} \right)$$

$$= 2 I \int \frac{\mu_0 I \, dx}{2\pi y} (-\hat{y}) + \cancel{(\quad)} (\hat{x}) \to 0$$

because the x components cancels out

$$= -\frac{2\mu_0 I^2}{\sqrt{3} \, 2\pi} \int_{x=\frac{s}{\sqrt{3}}}^{x=\frac{s}{\sqrt{3}}+\frac{a}{2}} \frac{dx}{x} \, \hat{y}$$

$$= -\frac{2\mu_0 I^2}{2\sqrt{3}\pi} \ln\left(\frac{\frac{s}{\sqrt{3}} + \frac{a}{2}}{\frac{s}{\sqrt{3}}}\right)\hat{y}$$

$$= \frac{-2\mu_0 I^2}{2\sqrt{3}\pi} \ln\left(1 + \frac{\sqrt{3}a}{2s}\right)$$

adding F_{BA}, F_{AC}, F_{CB}, we get

$$\text{total force} = \frac{\mu_0 I^2}{2\pi}\left[\frac{a}{s} - \frac{2}{\sqrt{3}} \ln\left(1 + \frac{\sqrt{3}a}{2s}\right)\right]\hat{y}$$