TOPIC - CONDUCTORS

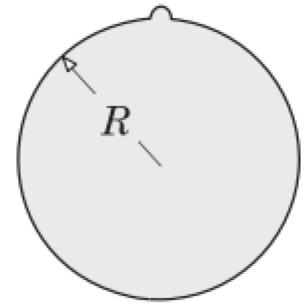
Concepts to be covered:

Charge distribution

Symmetric cases (classic)

Asymmetric/ irregular surfaces

Cause of charge concentration at irregularities



Using these concepts → solve a simple case of an irregular charge distribution.

Q) Due to non-uniform mechanical strength. When a conducting sphere is given a charge Q. the electrostatic forces make a small hemispherical bulge in its surface, The radius of the bulge is $r<<R$ where R is the radius of the sphere.

1. What is the qualitative distribution of charge like?
2. Estimate the charge q on the bulge
3. Consequently how does the capacitance of the sphere change due to this bulge?

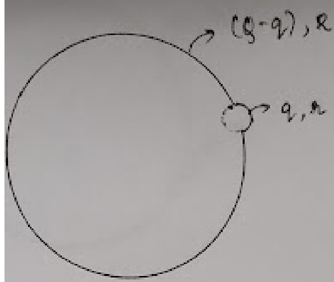
<u>Explanation/ answers</u>:

- We know that conductors are equipotential surfaces, meaning their potential is constant throughout the surface.
- This idea can be confirmed, as the presence of any potential difference should imply that there's a flow of charge and production of electric current.
- By extension/ in practice this means that, if we connect a wire with a bulb across two points on the conductor, it should glow.
- However, this doesn't happen, we know that there is no tangential flow of electrons on the surface of a conductor.
- This idea is easy to digest in the case of regular conductors, which don't assume any distorted shape. I can bet half of you imagined a spherical object as soon as I said conductor.
- So if I don't specify any regular shape for this conductor, and allow it to assume any arbitrary shape, then what can we say about the potential?

- In the case of a sphere, a uniform charge distribution was analogous to uniform potential, what about an irregular conductor?
- As the lack of tangential currents is true for all conductors, it should be true even in this case. Convince yourself about this fact.
- Now, what exactly does the potential depend on? If you recall, it depends on the charge as well as the radius of curvature as $V=kq/r$ alternately it can be written in terms of surface charge density as $V=k\sigma r$ (where σ is the surface charge density).
- Does this ring a bell? In cases of irregularities where there is a sharp peak, the radius of curvature is small, very small. If you recall from our discussion right now, you know that the conductor remains at a constant potential. Thus, the charge has to compensate for this, to maintain the constant ratio.
- If the radius decreases, the corresponding charge density increases.
- Therefore, charge tends to get concentrated at the sharp corners, or places with small radii of curvature
- The charge on this bulge accumulates as far as possible from the center of the sphere, and gets concentrated there. Thus, this charge can be effectively treated as a point charge

<u>ANSWERING THE QUESTION</u>

1. No, the charge distribution is no longer uniform. There is a higher charge density in the region where the curvature is small.
2. The charge in this region can be calculated by equating the potentials as they should be equal



As the potential is constant over the whole surface $V_r = V_R$

Total charge $= Q$

Let charge on bulge $= q$

$\therefore$ Charge remaining on the big sphere.

Potential of the larger sphere :

as q is also significant

$$V_R = \frac{K(Q-q)}{R}$$

This is approximately a sphere as $R >> r$.

$$V_r = \frac{Kq}{r}$$

assuming this to be a point sphere/charge (given in question).

equating $V_R = V_r$,

$$\frac{\cancel{K}(Q-q)}{R} = \frac{\cancel{K}q}{r}$$

$$r(Q-q) = qR.$$

$$rQ - qr = qR.$$

$$q(R+r) = Qr$$

$$q = \frac{Qr}{(R+r)}$$

3. Since the bulge is negligibly small, the change in the capacitance is also very small. Moreover, the total charge is constant and thus, we can say that the fractional increase in the capacitance is equal to the fractional decrease in potential.