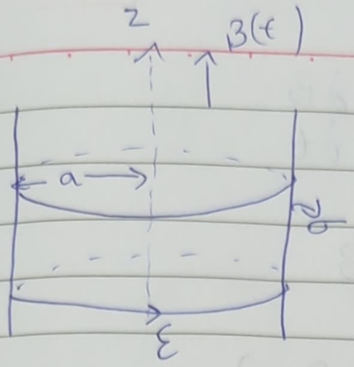
Q) The system is composed by a non-conducting cylindrical surface of height h and radius a, over which there is a net charge Q uniformly distributed with surface density $\sigma = \frac{Q}{2\pi a h}$. The cylindrical surface is free to rotate around its axis without friction and has moment of Inertia I per unit length. The system is at rest in the presence of an external uniform magnetic field B_{ext}, parallel to the system axis. Assume that boundary effects can be neglected.

Starting time $t=0$, the external magnetic field is reduced from its initial value $B_{ext} = B_0$ to zero, ~~at a time~~ according to some temporal law $B_{ext} = B_{ext}(t)$.

(a) Initially assuming that field generated by the motion of the charges on the cylinder is negligible, evaluate the angular velocity $\omega = \omega(t)$ of the cylinder as a function of time during the decay of B_{ext}, and the corresponding mechanical angular momentum L_c of the cylinder.

~~Why does the~~

(b) Does the law of conservation of angular momentum not hold in this case?



Ans:- E_{ind} $\Rightarrow$ Electric field induced ~~due to~~

We use cylindrical co-ordinates. E_{ind} will have azimuthal symmetry, $E_{ind} = E_\phi(r,t)\,\hat{\phi}$.

Applying faraday's law for a circle of radius r :-

$$E_\phi = -\frac{r}{2}\frac{\partial B}{\partial t}. \quad \left(E\cdot 2\pi r = \pi r^2 \times \frac{dB}{dt}\right)$$

On an infinitesimal surface element of cylinder surface $dS = a\,d\phi\,dz$ the induced electric field exerts a force

$$df = \hat{\phi}\,df = \hat{\phi}\,\sigma E_\phi(r=a)\,dS = -\hat{\phi}\,\sigma\frac{a}{2}\frac{\partial B}{\partial t}dS.$$

Torque $$d\tau = \hat{z}\,a\,df$$
$$= \sigma\frac{a^2}{2}\frac{dB}{dt}\,dS$$
$$\tau = \hat{z}\int \sigma\frac{a^2}{2}\frac{dB}{dt}\,dS =$$
$$= \int_0^h\int_0^\pi \sigma\frac{a^2}{2}\frac{dB}{dt}\,a\times d\phi\,dz$$
$$= \int_0^h\int_0^\pi \sigma\frac{a^3}{2}\frac{dB}{dt}\,d\phi\,dz$$
$$= -\pi a^3 h\sigma\frac{\partial B}{\partial t}.$$

$$I \frac{d\omega}{dt} = \tau = -\pi a^3 h \sigma \frac{dB}{dt}$$

$$\int_0^{\omega} I \, d\omega = \int_{B_0}^{0} -\pi a^3 h \sigma \, dB$$

$$I \omega = -\pi a^3 h \sigma (0 - B_0)$$

$$\omega = \pi a^3 h B_0 \times \frac{Q}{2\pi a h} \rightarrow \sigma$$

$$\omega = \frac{a^2}{2I} Q B_0 .$$

$$L_c = I\omega = \frac{a^2 Q B_0}{2} .$$

(b) We use the theorm: Whenever there is a flow of energy in any circumstance at all (field energy or any other kind of energy), the energy flowing through a unit area per unit time, when multiplied by $1/c^2$, is equal to the momentum per unit volume in the space.

We know that energy through unit area per unit time is the poynting vector S, $= \vec{E} \times \vec{B}$. ∴ The momentum per unit volume is $\frac{\vec{E} \times \vec{B}}{c^2}$. In this example

E is from the charges and B is from B_0. The energy is flowing in circles inside the cylinder along $\vec{E} \times \vec{B}$ and, ∴ momentum is stored in the space. When B becomes 0, all the momentum stored in the field gets converted to mechanical momentum. Similarly, we can associate angular momentum to be stored in the field as $L = \vec{r} \times \vec{P}$. ~~and~~ Hence, the initial angular momentum stored in the field gets converted to mechanical momentum, conserving the conservation of angular momentum.