Prove that a stable equilibrium cannot be established by electrostatic forces alone.

A particle is in stable equilibrium iff it experiences no net force and upon being displaced by a small distance, returns to the equilibrium position due to the restoring force.

Let q_+, a positive charge, rest at point P_0 (arbitrary) in stable equilibrium.

$$\Rightarrow \vec{F}_q = q\vec{E} \quad \text{for any } q.$$

By definition, if q_+ is displaced in some neighbourhood V from P_0, a restoring force with corresponding field $\vec{E}_r$ acts towards P_0.

By Gauss' Law;

$$\oint_{S=\partial V} \vec{E}_r \cdot \vec{n} = \frac{q_+}{\epsilon_0}$$

But as $\vec{n}$ is directed away from P_0 as per convention,

$$\vec{E} \cdot \vec{n} < 0$$

$$\Rightarrow \oint_{S=\partial V} \vec{E}_r \cdot \vec{n} < 0$$

$$\Rightarrow \frac{q_+}{\epsilon_0} < 0, \text{ which is false.}$$

Hence, electrostatic forces alone cannot establish stable equilibrium. □