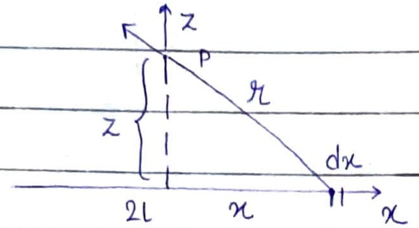
Q. Find the $\vec{E}$ at dist z above the midpoint of a straight line segment of length $2L$ that carries a uniform line charge λ



Now, we know λ signifies linear charge density

& $\lambda = \frac{dq}{dl}$

$dq = \lambda dl$

or $\lambda = \frac{dq}{dx}$ & $dq = \lambda dx$

Source point $r' = x\hat{x}$ Field point $r = z\hat{z}$ $dl' = dx$

Separation vector $\vec{\mathscr{r}} = r - r' = z\hat{z} - x\hat{x}$

$|\mathscr{r}| = \sqrt{z^2+x^2}$

$\therefore \hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\mathscr{r}|} = \frac{z\hat{z} - x\hat{x}}{\sqrt{z^2+x^2}}$

$\vec{E}$ for linearcharge distribution $= \int_{-L}^{L} \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{|\mathscr{r}|^2} \hat{\mathscr{r}}$

$$= \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{\lambda dx}{\sqrt{z^2+x^2}} \left(\frac{z\hat{z} - x\hat{x}}{\sqrt{z^2+x^2}} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\int_{-L}^{L} \frac{z\hat{z}\lambda dx}{(z^2+x^2)^{3/2}} - \int_{-L}^{L} \frac{x\hat{x}\lambda dx}{(z^2+x^2)^{3/2}} \right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\underbrace{z\hat{z} \int_{-L}^{L} \frac{dx}{(z^2+x^2)^{3/2}}}_{1} - \underbrace{\hat{x} \int_{-L}^{L} \frac{x\,dx}{(z^2+x^2)^{3/2}}}_{2} \right]$$

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In case of, $-\hat{x}\int_{-L}^{L}\frac{x\,dx}{(\sqrt{z^2+x^2})^3}$

Taking $z^2+x^2=u$
$2x\,dx=du$
$x\,dx=\frac{du}{2}$

$\therefore -\hat{x}\frac{1}{2}\int\frac{du}{u^{3/2}}$

$\Rightarrow -\hat{x}\times\frac{1}{2}\times\left(-2u^{-1/2}\right)_{-L}^{L}$

$\Rightarrow +\hat{x}\left(u^{-1/2}\right)_{-L}^{L}$

$\Rightarrow \hat{x}\left[\frac{1}{\sqrt{z^2+x^2}}\right]_{-L}^{L}$

$\Rightarrow \hat{x}\left[\frac{1}{\sqrt{z^2+L^2}}-\frac{1}{\sqrt{z^2-L^2}}\right]=0.$

For part 1 i.e., $\hat{z}\ z\hat{z}\int_{-L}^{L}\frac{1}{(z^2+x^2)^{3/2}}dx$

this turns out to be $\Rightarrow z\hat{z}\left(\frac{x}{z^2\sqrt{z^2+x^2}}\right)_{-L}^{L}$

$\Rightarrow z\hat{z}\left[\frac{L}{z^2\sqrt{z^2+L^2}}+\frac{L}{z^2\sqrt{z^2+L^2}}\right]\Rightarrow z\hat{z}\left[\frac{2L}{z^2\sqrt{z^2+L^2}}\right]$

$\therefore$ Putting the values in main eq^{n}

$$\frac{\lambda}{4\pi\varepsilon_0}\left[\cancel{z}\hat{z}\frac{2L}{z^2\sqrt{z^2+L^2}}+0\right]=\frac{2\lambda L}{4\pi\varepsilon_0 z\sqrt{z^2+L^2}}\hat{z}\ \text{(Ans)}$$

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