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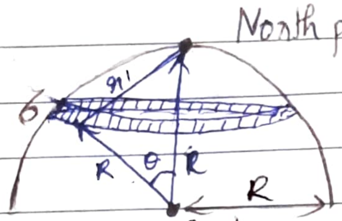
Q. An inverting hemispherical conducting bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the north pole and the centre. If another hemisphere is brought to complete the sphere, how will this potential difference change? What is special about this set-up?



$$V_{centre} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma \, da}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sigma}{R} \int da$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sigma}{R} \times 2\pi R^2$$

$$= \frac{\sigma R}{2\varepsilon_0}$$

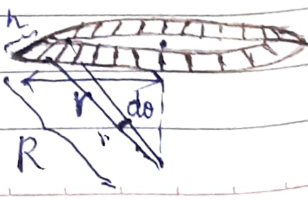
To find V at pole, consider a ring as cross section with each point on the ring being at a distance of r' from the pole.

Applying the law of cosines,

$$r'^2 = R^2 + R^2 - 2R^2\cos\theta$$
$$= 2R^2(1-\cos\theta)$$

To find da,

$$da = 2\pi r h$$



$r = R\sin\theta$
$h = Rd\theta$.

$\therefore da = 2\pi R^2 \sin\theta \, d\theta$.

Substituting in,

$$V_{pole} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma}{r'} da .$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{\pi/2} \frac{\sigma}{\sqrt{2R^2(1-\cos\theta)}} 2\pi R^2 \sin\theta \, d\theta .$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \frac{(2\pi R^2)}{\sqrt{2}R} \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0} \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}} \quad \text{——— (1)}$$

Let $1-\cos\theta = t \Rightarrow \sin\theta \, d\theta = dt$.

$$\Rightarrow V_{pole} = \frac{\sigma R}{2\sqrt{2}\varepsilon_0} \int_0^1 \frac{dt}{\sqrt{t}}$$

$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0} \left[2\sqrt{t} \right]_0^1$$

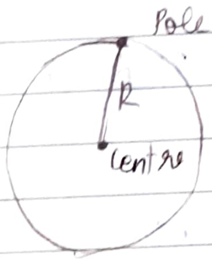
$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0} \cdot 2 .$$

$$= \frac{\sigma R}{\sqrt{2}\varepsilon_0}$$

$$\Rightarrow V_{pole} - V_{centre} = \frac{\sigma R}{\sqrt{2}\,\varepsilon_0} - \frac{\sigma R}{2\varepsilon_0}$$

$$= \frac{\sigma R}{2\varepsilon_0}\left(\sqrt{2}-1\right)$$

With 2 hemispheres forming spherical shell,



$$V_{centre} = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma}{R}\int da$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma \times 4\pi R^2}{R}$$

$$= \frac{\sigma R}{\varepsilon_0}$$

For V_{pole}, change limits in eqn (1), as (0 to π)

$$V_{pole} = \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\int_0^{\pi} \frac{\sin\theta\, d\theta}{\sqrt{1-\cos\theta}}$$

Let $1-\cos\theta = t$, $\Rightarrow \sin\theta\, d\theta = dt$

$$\Rightarrow V_{pole} = \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\int_0^{2} \frac{dt}{\sqrt{t}}$$

$$= \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\left[2\sqrt{t}\right]_0^2$$

$$= \frac{\sigma R}{2\sqrt{2}\,\varepsilon_0}\cdot 2\sqrt{2} = \frac{\sigma R}{\varepsilon_0}$$

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$\Rightarrow V_{pole} - V_{centre} = 0$

Since $E = -\frac{dV}{dr}$, $E = 0$ inside the shell.

We know that $\vec{E}$ inside a closed conducting shell is always 0. This is because the electrons on the surface rearrange in the presence of an external electric field, generating a second $\vec{E}$ of their own to create net $\vec{E}$ inside shell as 0.

This makes such a set up useful for protecting sensitive electronic equipment from external radio frequency interference.
→ Faraday's cage (Works on electromagnetic shielding)