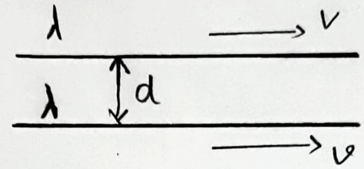
Q. Suppose you have two infinite straight line charges λ, a distance d apart, moving along at a constant speed v. How great would v have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number. Is this a reasonable sort of speed?



Solution: Let's consider the magnetic force per unit length.

$$\vec{f}_m = \frac{\vec{F}_m}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}\hat{d}$$

An infinite straight line charge moving with a velocity v produces a current, $I = \lambda v$, where λ is the linear charge density.

$$\therefore \; I_1 = I_2 = \lambda v$$

$$\Rightarrow |\vec{f}_m| = \frac{|\vec{F}_m|}{l} = \frac{\mu_0 \lambda^2 v^2}{2\pi d} \quad -(1)$$

Now let's consider the force due to electric field per unit length.

For that, Electric field, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 d}\hat{d}$

for an infinitely long charge

Force, $\vec{F}_e = q\vec{E}$

Force per unit length, $|\vec{f}_e| = \frac{|\vec{F}_e|}{l} = \lambda|\vec{E}|$

$$\therefore \; |\vec{f}_e| = \frac{\lambda^2}{2\pi\varepsilon_0 d} \quad -(2)$$

For the magnetic force to be equal to electric force, $\vec{F}_m = F_e$.

$$\Rightarrow f_m = f_e$$

$\therefore$ Equating ① and ②, we get

$$\frac{\mu_0 \cancel{\lambda^2} v^2}{\cancel{2\pi d}} = \frac{\cancel{\lambda^2}}{\cancel{2\pi} \varepsilon_0 \cancel{d}}$$

$$\Rightarrow \mu_0 v^2 = \frac{1}{\varepsilon_0}$$

$$\Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c}$$, speed of light. (3×10^8 m/s)

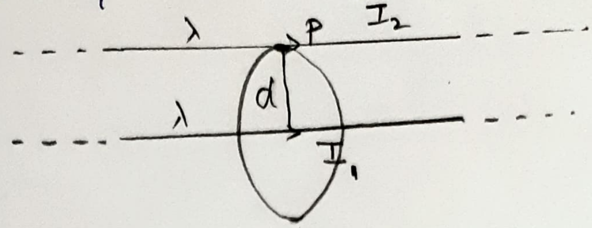
$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

Therefore,
for the magnetic force to be greater than the electric force, velocity of the system has to be greater than the speed of light, which is practically not possible.
Therefore, the electric force always dominates the magnetic force.

I Expression for magnetic force per unit length.

Consider an amperian loop around the straight moving charge. (straight current carrying wire):



According to ampere's law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow \quad |\vec{B}| \times 2\pi d = \mu_0 I_1$$

$$|\vec{B}| = \frac{\mu_0 I_1}{2\pi d}$$. This magnetic field acts on the other charge.

Force on the other wire, $\vec{F}_m = I_2 \int (d\vec{l} \times \vec{B})$

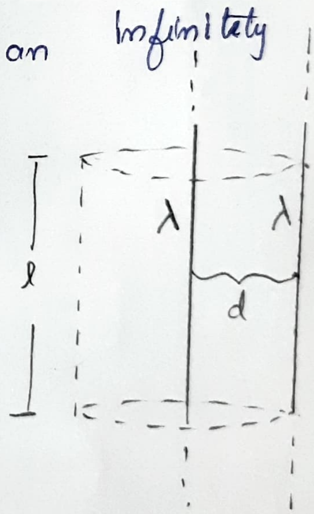
here $|\vec{F}_m| = I_2 \int |dl||B| \sin\theta$, $\theta = 90^\circ$

$$F_m = B I_2 l$$

$$f_m = \frac{F_m}{l} = B I_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Here, $I_1 = I_2 = I = \lambda v$

II Expression for electric force of an infinitely long charge by other.



Consider a cylindrical Gaussian surface of length l and radius d

According to Gauss's law,

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$|\vec{E}|\, 2\pi d\, l = \frac{\lambda l}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 d}\hat{d}$$

$$\vec{F}_e = q\vec{E} = \frac{\lambda q}{2\pi\varepsilon_0 d}\hat{d}$$

$$|f_e| = \frac{|\vec{F}_e|}{l} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$$