Physics Tutorial

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1 Questions

Consider an arbitrary distribution of charges having density $\rho(\vec{r})$. Let v be a spherical region of space of radius r_0 centered at origin. the average electric field $\bar{E}$ inside v is given by

$$\bar{E} = \frac{1}{\frac{4}{3}\pi r_0^3}\int_v \vec{E}(\vec{r})dv = \bar{E}_{int} + \bar{E}_{ext}$$

. Find $\bar{E}_{int}$ and $\bar{E}_{ext}$

2 Solution

Substituting $\vec{E} = -\vec{\nabla}V$, we see that $\bar{E} = -\frac{1}{\frac{4}{3}\pi r_0^3}\oint_{\partial v} V(\vec{r})da$. By substituting $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r})}{|\vec{r}-\vec{r'}|}dv'$, we get $\bar{E} = -\frac{1}{\frac{4}{3}\pi r_0^3}\frac{1}{4\pi\epsilon_0}\int_v \rho(\vec{r'})\left(\oint_{\partial v}\frac{da}{|\vec{r}-\vec{r'}|}\right)dv'$ which then simplifies to 2 cases, when $r \leq r_0$ and $r > r_0$ which are respectively the internal and external electric fields. So we get that

$$\bar{E}_{int} = -\frac{\vec{\mathbf{p}}}{4\pi\epsilon_0 r_0^3}$$

$$\bar{E}_{ext} = -\frac{1}{4\pi\epsilon_0}\int_{\tilde{V}}\frac{\rho(\vec{r'})\vec{r'}}{r'^3}dv'$$

Where $\vec{\mathbf{p}} := \int_v \vec{r'}\rho(\vec{r'})$ and $\tilde{V} := \mathbb{R}^3/v$