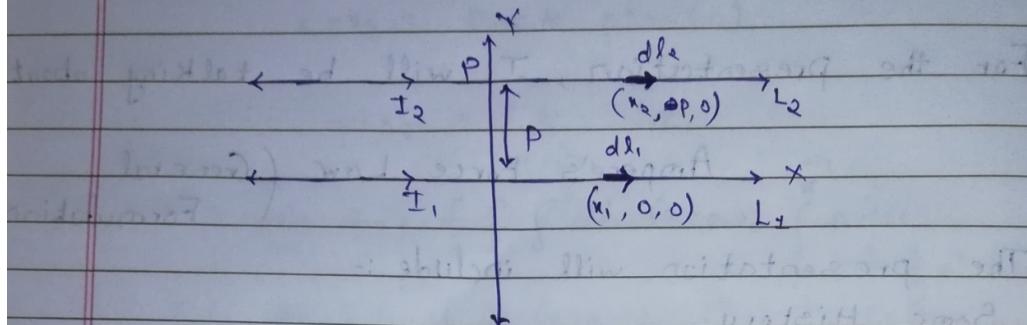


### Case of parallel wires:-



Consider two parallel wires, one placed along  $x$ -axis ( $L_1$ ) and other ( $L_2$ ) along set  $y = p$  parallel to  $L_1$ .

Differential (infinitesimal) element of  $L_1$  at  $(x_1, 0, 0)$  and that of  $L_2$  at  $(x_2, p, 0)$ .

$$\therefore dl_1 \equiv (dx_1, 0, 0) \text{ and } dl_2 \equiv (dx_2, p, 0)$$

$$\hat{r}_{12} = \frac{\pm}{\sqrt{(x_1 - x_2)^2 + p^2}} (x_1 - x_2, p, 0)$$

$$|r_{12}| = \sqrt{(x_1 - x_2)^2 + p^2}$$

The force on line wire 1 due to wire 2 is given by (According to Ampere's force law):

$$F_{12} = \frac{\mu_0 I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} dl_1 \times (dl_2 \times \hat{r}_{12})$$

$$\therefore F_{12} = \frac{\mu_0 I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} (dx_1, 0, 0) \times [(dx_2, p, 0) \times (x_1 - x_2, p, 0)] \left[ (x_1 - x_2)^2 + p^2 \right]^{3/2}$$

The cross product is,  $(dx_2, 0, 0) \times (x_1 - x_2, p, 0)$  is,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx_2 & 0 & 0 \\ x_1 - x_2 & p & 0 \end{vmatrix} = \frac{\hat{i}(0) - \hat{j}(0)}{(pdx_2)} + \frac{\hat{k}(pdx_2)}{(pdx_2)} \\ = (0, 0, pdx_2)$$

The cross product  $(dx_1, 0, 0) \times (0, 0, pdx_2)$  is,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx_1 & 0 & 0 \\ 0 & 0 & pdx_2 \end{vmatrix} = \frac{\hat{i}(0) - \hat{j}(pdx_1 dx_2)}{(-pdx_1 dx_2)} + \frac{\hat{k}(0)}{(-pdx_1 dx_2)} \\ = (0, -pdx_1 dx_2, 0)$$

$$\therefore F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \int_{L_1} \int_{L_2} \frac{dx_1 dx_2 (0, -p, 0)}{[(x_1 - x_2)^2 + p^2]^{3/2}}$$

$$\therefore F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \int_{L_1} dx_1 \frac{(0, -p, 0)}{[(x_1 - x_2)^2 + p^2]^{3/2}} \int_{-\infty}^{\infty} \frac{dx_2}{(z^2 + p^2)^{3/2}}$$

Put,

$$x_1 - x_2 = z$$

$$\therefore -dx_2 = dz$$

$$\therefore -dx_2 = -dz$$

$$\therefore F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \int_{L_1} dx_1 \frac{(0, -p, 0)}{(z^2 + p^2)^{3/2}} \int_{-\infty}^{\infty} \frac{dz}{(z^2 + p^2)^{3/2}}$$

Consider,

$$A = \int_{-\infty}^{\infty} \frac{dz}{(z^2 + p^2)^{3/2}}$$

put;  $z = p \tan u$

$$\therefore dz = p \sec^2 u du$$

$$\therefore A = \int_{-\pi/2}^{\pi/2} p du - \int_{-\pi/2}^{\pi/2} \cos^2 u [p^2 (1 + \tan^2 u)]^{3/2}$$

$$\therefore A = \int_{-\pi/2}^{\pi/2} \frac{p du}{\cos^2 u} p^2 (\sec^2 u)^{3/2}$$

$$\therefore A = \frac{1}{p^2} \int_{-\pi/2}^{\pi/2} \cos^2 u du$$

$$\therefore A = \frac{1}{p^2} [\sin u]_{-\pi/2}^{\pi/2}$$

$$\therefore A = \boxed{\frac{2}{p^2}}$$

$$\therefore F_{12} = \frac{\mu_0 I_1 I_2}{2\pi p} \int_{L_1} dx, (0, -p, 0)$$

$$\therefore F_{12} = \frac{\mu_0 I_1 I_2}{2\pi p} p (0, -1, 0) \int_{L_1} du,$$

$$\therefore F_{12} = \frac{\mu_0 I_1 I_2}{2\pi p} \int_{L_1} du, (0, -1, 0)$$

Since,  $L_1$  is infinite  $\Rightarrow$  Force will be infinite

$\therefore$  Force per unit length is,

$$\boxed{\frac{F_{12}}{L} = \frac{\mu_0 I_1 I_2}{2\pi p} (0, -1, 0)} \quad \begin{array}{l} \text{i.e. Force is along } \hat{j} \\ \text{i.e. Attractive} \end{array}$$