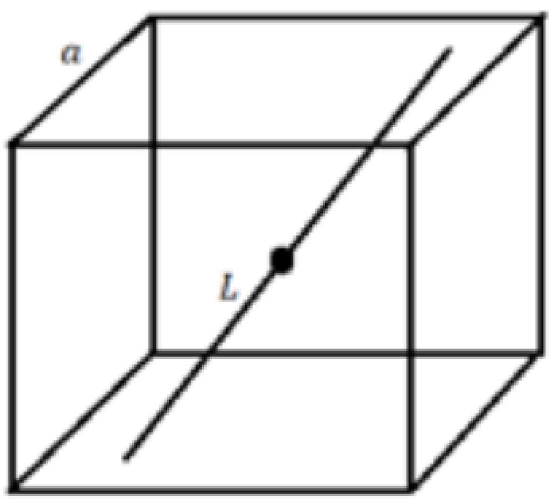
PH1213 tutorial presentation

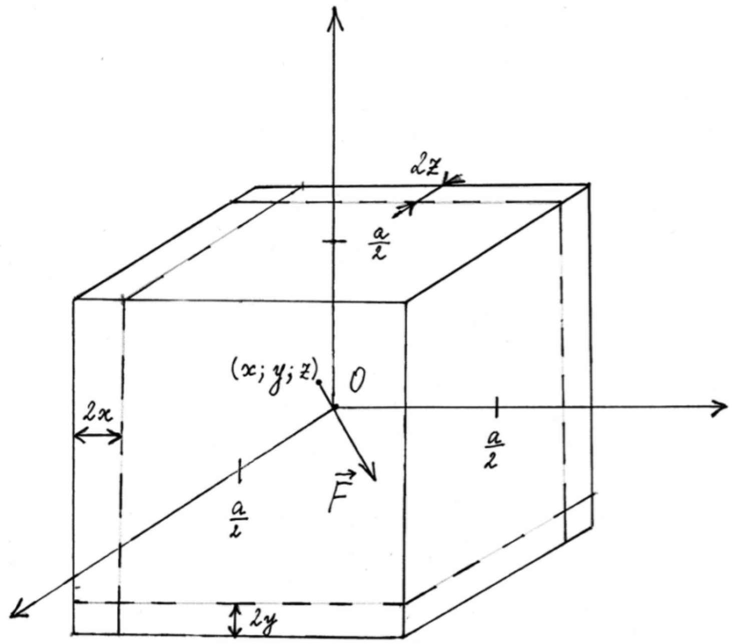
Name: Achyusman Samanta
Regt no. : 20211147
Batch: 5

A narrow straight channel passes through the center of a fixed cube of side a. The cube is uniformly charged of charge density ρ. The distance from the cube center to the point of intersection of the channel and a face is L. In the channel there is a particle of a mass *m* and charge *q* . Find the period of small oscillations of the particle near the center. The gravitational interaction of the particle and the cube can be neglected. The cube and the particle are oppositely charged.

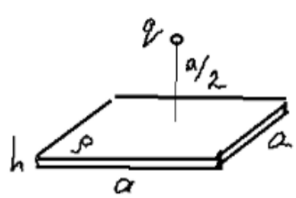


We will use a coordinate system with axes parallel to the cube's edges, with the origin set at the cube's center. Assuming the particle is at coordinates(x,y,z) $x << a$, $y << a$, $z << a$, we will find the force **F** acting on the particle, by splitting the cube into a rectangular cuboid of dimensions

$(a-2x) \times (a-2y) \times (a-2z)$ and three square plates of thickness $2x$, $2y$ and $2z$.



Now the particle is in the center of the cuboid, so there is no force from the cuboid. Hence we only need to find the force between a particle with a charge *q* and uniformly charged plate of small thickness *h* and edge length a. the plate's charge density is ρ, the particle is placed above the center of the plate at distance $a/2$.



due to symmetry and Gauss's law , the flux of the particle's electric field through the plate is

$$\phi = \frac{q}{6\epsilon_0}$$

Hence, the force

$$F = \sigma\phi = \frac{q\rho h}{6\epsilon_0}$$

Where $\sigma = \rho h$ is plate's surface charge density.

Three plates act on the particle with forces

$$\boldsymbol{F_1} = \frac{q\rho x}{3\epsilon_0}\hat{x}\ ,\ \ \boldsymbol{F_2} = \frac{q\rho y}{3\epsilon_0}\hat{y}\ ,\ \ \boldsymbol{F_3} = \frac{q\rho z}{3\epsilon_0}\hat{z}$$

Net force, $\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3} = \frac{q\rho x}{3\epsilon_0}\boldsymbol{r}$, where $\boldsymbol{r}$ is the position vector of the particle.

In SHM, $\boldsymbol{F} = -k\boldsymbol{x} \Rightarrow k = \frac{q(-\rho)}{3\epsilon_0}$

Time period in SHM , $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{3m\epsilon_0}{q(-\rho)}}$$