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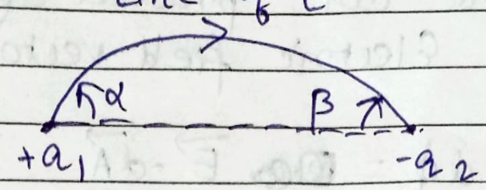
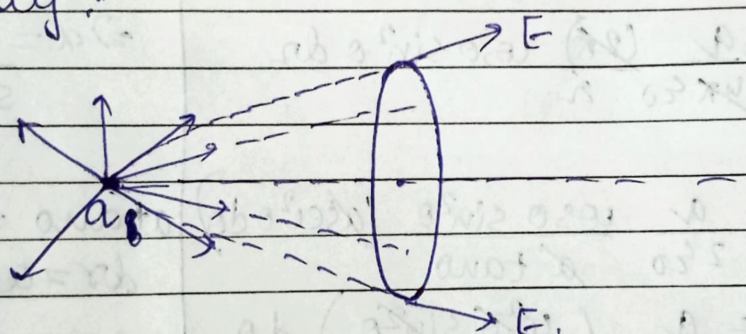
PHYSICS TUTORIAL QUESTION

(Q) An electrostatic field line leaves at angle α from point charge q_1 and connects with point charge $-q_2$ at angle β (see figure). Then find the relationship between α and β.



Ans:-

As we know Electric field lines for a point charge is radial in nature, so to solve the above problem we think in the following way:-



Let us consider a thin disc and a charge q. Now, as Electric field lines are radial in nature, some number of field lines will pass through the disc.

The measure of the no of field lines passing through the disc can be given by Electric flux.

Now, from Gauss's law.

$$\phi = \int \vec{E} \cdot d\vec{A}$$

So, first we try to find the electric flux due to charge q.

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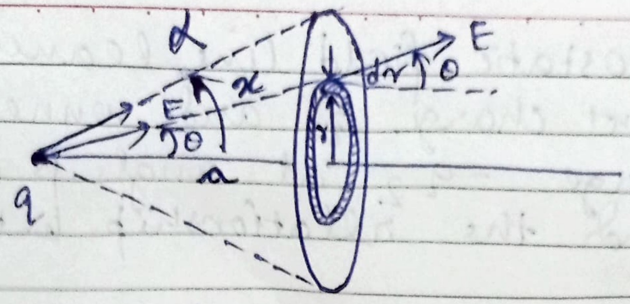

Flux is the dot product of Area vector and Electric field vector.

$$\therefore\ d\phi = \vec{E}\cdot\vec{dA}$$
$$= |\vec{E}||\vec{dA}|\cos\theta$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}(2\pi r\,dr)\cos\theta$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}(2\pi r)\cos\theta\sin^2\theta\,dr$$

$$\Rightarrow d\phi = \frac{q}{2\cancel{4\pi}\varepsilon_0}\frac{(\cancel{2\pi})}{r}\cos\theta\sin^2\theta\,dr$$

$$\Rightarrow d\phi = \frac{q}{2\varepsilon_0}\frac{\cos\theta\sin^2\theta}{\cancel{a}\tan\theta}(\cancel{a}\sec^2\theta\,d\theta)$$
$$= \frac{q}{2\varepsilon_0}\left(\frac{\cancel{\cos^2\theta}\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\cancel{\cos^2\theta}}\right)d\theta$$

$$\Rightarrow d\phi \Rightarrow \frac{q}{2\varepsilon_0}\sin\theta\,d\theta$$

$$\Rightarrow \int d\phi = \frac{q}{2\varepsilon_0}\int_0^{\alpha}\sin\theta\,d\theta$$
$$= \frac{q}{2\varepsilon_0}\left(-\cos\alpha-(-1)\right)$$

$$\boxed{\phi = \frac{q}{2\varepsilon_0}(1-\cos\alpha)}$$

$$dA = 2\pi r\,dr$$
$$\sin\theta = \frac{r}{x}$$
$$\Rightarrow x = \frac{r}{\sin\theta}$$
$$a\tan\theta = r$$
$$dr = a\sec^2\theta\,d\theta$$

Now, we come back to the given question.

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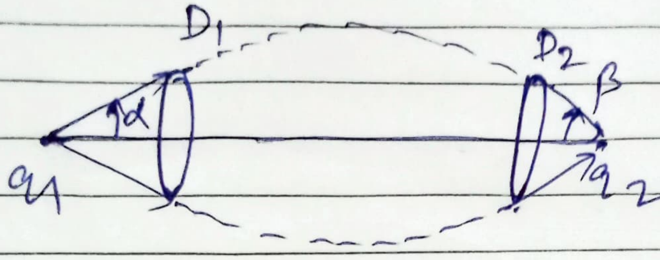
Electric field lines will diverge from q_1 and will converge to q_2.



We consider a very small disc in front of q_1 and q_2.

As the field lines are originally radial, it will remain radial for very small distance ~~but~~ from the charges.

As, Two EF lines never cross each other and As, all the EF lines will emerge from q_1 and then converge at q_2 we can say that the no of field lines passing through disc D_1 and disc D_2 are the same.

And we know that number of field lines passing through an area gives us the idea of flux.

So, we can say that.

$$\phi_{\text{through } D_1} = \phi_{\text{through } D_2}$$

$$\Rightarrow \frac{q_1}{2\varepsilon_0}(1-\cos\alpha) = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$$

$$\Rightarrow \frac{q_1}{2}\left(2\sin^2\frac{\alpha}{2}\right) = \frac{q_2}{2}\left(2\sin^2\frac{\beta}{2}\right)$$

$$\Rightarrow \boxed{\sin\frac{\beta}{2} = \sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}}$$

Ans