Tutorial Problem

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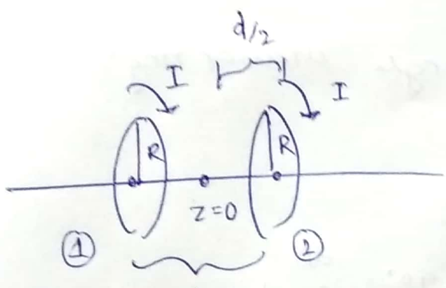


☑ Coil ① and ② are two coaxial circular ring carrying the same current I in the same direction, and kept parallel to each other.

☑ They are kept at a distance 'd' apart, and having radius 'R' each.

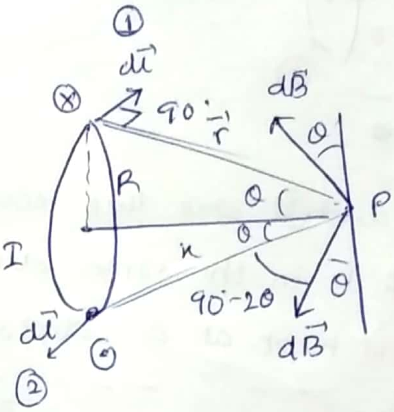
Q i) What is the nature of $\vec{B}$ at a point in the midpoint between them ($z=0$)

First, we have to derive the magnetic field for single circular ring:

Derivation and Analysis of magnetic field:

From Biot Savart Law:

$$|d\vec{B}| = \left| \frac{\mu_0}{4\pi} \frac{i(d\vec{l} \times \vec{r})}{r^3} \right|$$



$$dB = 2 \cdot \frac{\mu_0}{4\pi} \frac{idl \sin\theta}{r^2} \left[\begin{array}{l} \text{For two } d\vec{l} \text{ elements} \\ \theta = 90° \text{ ① and ②} \end{array} \right]$$

$$= 2 \cdot \frac{\mu_0}{4\pi} \frac{idl}{r^2}$$

$$\sin\theta = R/|\vec{r}|$$

Only $dB \sin\theta$ will be responsible for net magnetic field

$$B_{net} = \int_0^{\pi r} 2 \cdot \frac{\mu_0}{4\pi} \frac{idl}{r^2} \cdot \frac{R}{|\vec{r}|}$$

$$= 2 \cdot \frac{\mu_0}{4\pi} \frac{i \cdot \pi R^2}{r^3}$$

$$= \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$$

Features:

At $x \to \alpha$ and At $x = 0$

$B = 0$ $\quad B \to max$,

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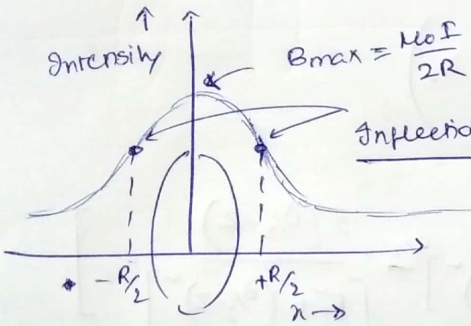
Graph:



Inflection point $\rightarrow$ where derivative of the function changes sign

$\rightarrow$ $\frac{dB}{\partial x}$ = constant ; $\frac{d^2 B}{\partial x^2} = 0$

$\rightarrow$ B linearly varies with x.

- Problem with the magnetic field $\rightarrow$ varies with distance
  $\downarrow$
  Not constant

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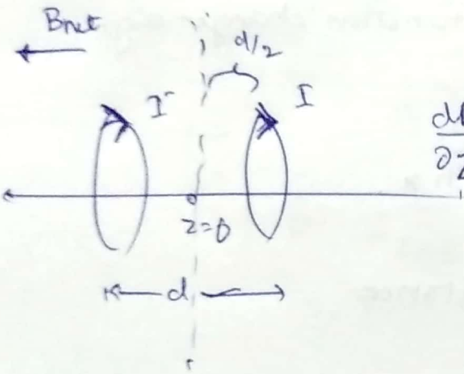
Now, Lets check for the system of two coaxial rings:

$$B_{net} = \frac{\mu_0 I R^2}{2}\left(\left[\frac{1}{\{R^2+(d/2+z)^2\}^{3/2}}\right] + \frac{1}{\{R^2+(d/2-z)^2\}^{3/2}}\right)$$



$$\frac{dB}{\partial z} = \frac{-3}{2}\,\frac{\mu_0 I R^2}{2}\left[\frac{2(d/2+z)}{[R^2+(d/2+z)^2]^{5/2}} - \frac{2(d/2-z)}{[R^2+(d/2-z)^2]^{5/2}}\right]$$

$\downarrow z=0$

$$= 0$$

Two possibilities:

The nature of B → • B is constant with z at the midpoint.

← Sol^n for Q1.

• or B is a maximum/minimum at point z=0

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Q2) Determine the condition which is important for the system of the two rings to be a helmholtz coil, and find the magnetic field at the pt. $z=0$.

→ Important property of helmholtz ring,

$$\frac{d^2B}{\partial z^2} = 0 \leftarrow \text{Inflection point}$$

at $z=0$ (midpoint)

$$\therefore \frac{\partial^2 B}{\partial z^2} = \frac{3\mu_0 I R^2}{2}\left[\frac{-1}{\left[R^2+\left(\frac{d}{2}+z\right)^2\right]^{5/2}} + \frac{-\left(\frac{d}{2}+z\right)\left(-\frac{5}{2}\right)2\left(\frac{d}{2}+z\right)}{\left[R^2+\left(\frac{d}{2}+z\right)^2\right]^{7/2}} + \frac{1}{\left[R^2+\left(\frac{d}{2}-z\right)^2\right]^{5/2}} + \frac{\left(\frac{d}{2}-z\right)\left(-\frac{5}{2}\right)2\left(\frac{d}{2}-z\right)(-1)}{\left[R^2+\left(\frac{d}{2}-z\right)^2\right]^{7/2}}\right]$$

$$= \overbrace{\frac{3\mu_0 I R^2}{\left[R^2+\left(\frac{d}{2}\right)^2\right]^{7/2}}}^{\text{constant}}\left(d^2-R^2\right) = 0$$

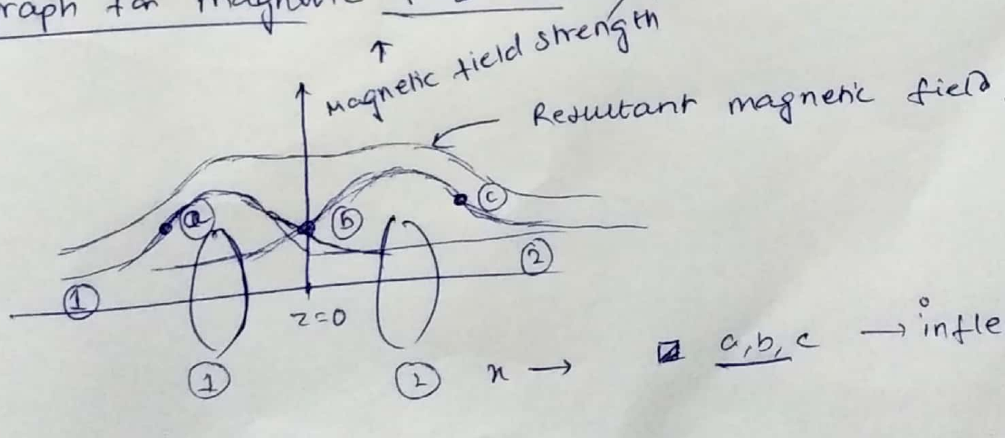
$$\therefore d^2 - R^2 = 0$$

$$\Rightarrow d = R \longrightarrow \text{necessary condition for helmholtz coil}$$

$$\therefore B(0) = \frac{\mu_0 I R^2}{2}\left[\frac{1}{\left[R^2+\left(\frac{R}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2+\left(\frac{R}{2}\right)^2\right]^{3/2}}\right]$$

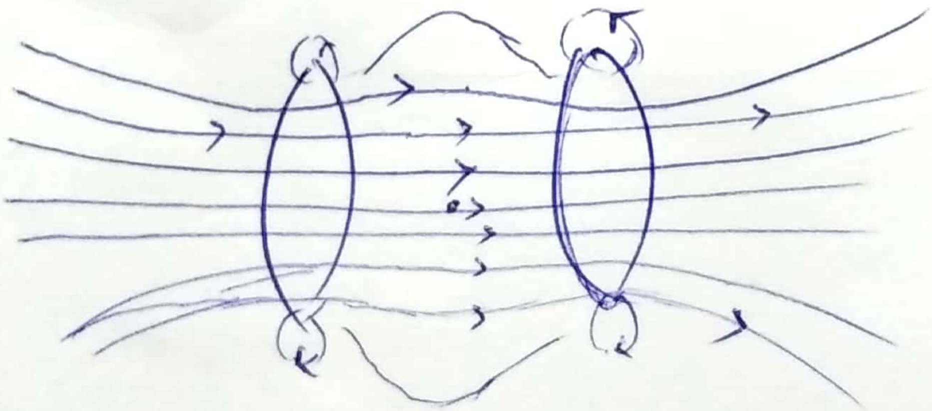
$$= \mu_0 I R^2 \frac{1}{\left(\frac{5R^2}{4}\right)^{3/2}} = \boxed{\frac{8\mu_0 I}{5\sqrt{5}R}}$$

☑ The graph for magnetic field intensity:



☑ a, b, c → inflection points

☑ Magnetic field lines through helmholtz coil:



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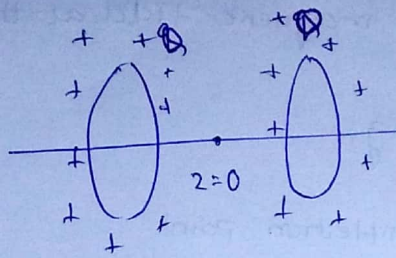
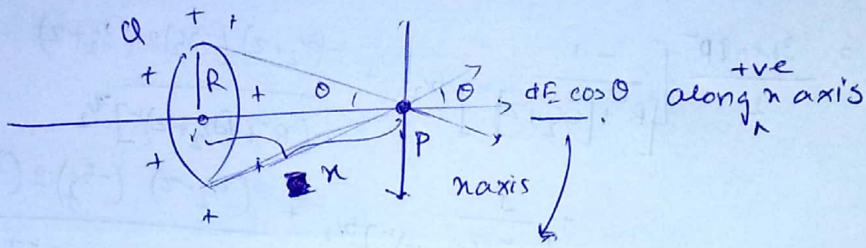
Q3> ~~Find~~ Depict the situation in Electrostatic ~~condition~~ ~~in terms~~ Qualitatively.



Net $\vec{E}$ for single ring:



$$\frac{KQx}{(x^2+R^2)^{3/2}}$$

In order to find the maxima/ minima,

$$\frac{\partial B}{\partial x} = 0$$

$$\Rightarrow (x^2+R^2)^{1/2}\left[x^2+R^2-3x^2\right] = 0$$

$\downarrow$ +ve value

$\downarrow$ $R^2 - 2x^2 = 0$

$$\Rightarrow x = \pm \frac{R}{\sqrt{2}}$$

Now,

$E_{\parallel}$ along x axis, at $x = R/\sqrt{2} \rightarrow$ +ve value of E ↗ Maximum value

$x = -R/\sqrt{2} \rightarrow$ -ve value of E $\downarrow$ Minimum value

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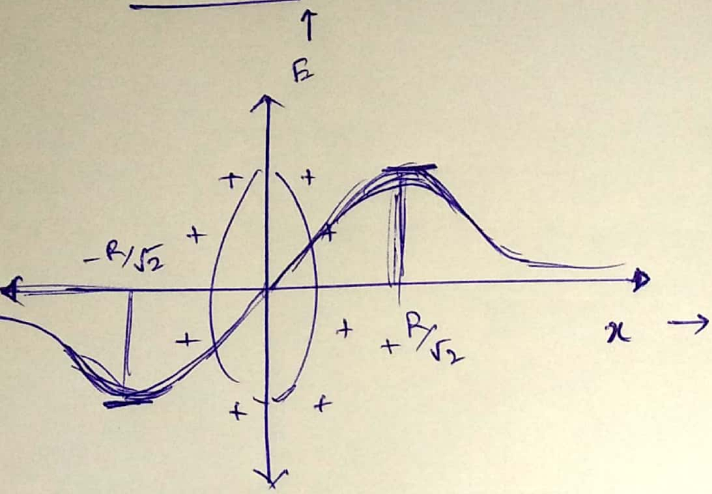
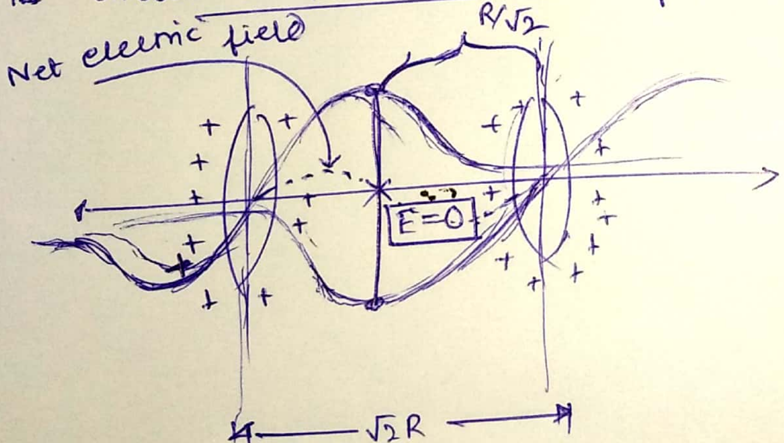
- Graph for E:



- ~~Situatn~~ Graph for two ring system:



- In the electrostatic situation, the net $\vec{E}$ in the axis at the midpoint of the two ~~rings~~ coaxial rings kept at a distance $\sqrt{2}R$ is 0. whereas, ~~the rings~~ the magnetic field was evidently constant at the midpoint.

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