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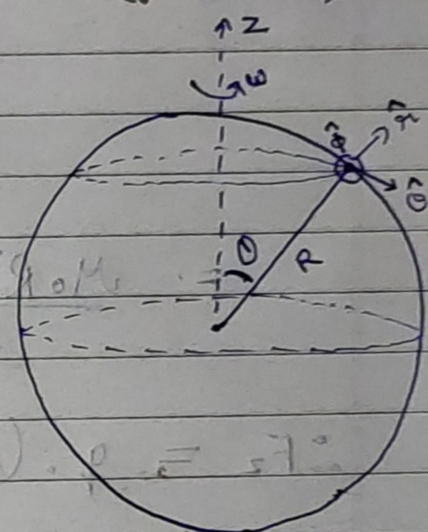
Q. Consider a sphere of radius R with uniform area charge density σ. This sphere is spinning about z-axis with v velocity. Calculate the magnetic force of attraction between the northern & Southern hemispheres of this spinning charged shell.

(Vector potential for the following conditions are given:-

$$\vec{A}_{in} = \left(\frac{1}{3}\cdot \mu_0 R \omega \sigma\right) r \sin\theta \,\hat{\phi} \quad (\text{for } r \le R)$$

$$\vec{A}_{out} = \left(\frac{1}{3}\cdot \mu_0 R^4 \omega \sigma\right) \frac{\sin\theta}{r^2} \,\hat{\phi} \quad (\text{for } r \ge R))$$



Solution:-

As we know, $\vec{B} = \nabla \times \vec{A}$

$$\therefore \vec{B} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, A(\phi))\right]\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}(r A(\phi))\hat{\theta}$$

$$\Rightarrow \vec{B}_{in} = \nabla \times \vec{A}_{in}$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(r\sin^2\theta \cdot \frac{\mu_0 R\omega\sigma}{3}\right)\right)\hat{r}$$

$$- \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r^2\sin\theta \cdot \frac{\mu_0 R\omega\sigma}{3}\right)\right)\hat{\theta}$$

$$\therefore \vec{B}_{in} = \frac{2}{3}\mu_0 R\omega\sigma\,(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}).$$

Similarly,

$$\vec{B}_{out} = \frac{2}{3}\mu_0 R^4\omega\sigma\left(\cos\theta\,\hat{r} + \frac{\sin\theta}{2}\hat{\theta}\right).$$

$$\therefore \vec{B}_{surface} = \left.\left(\frac{\vec{B}_{in} + \vec{B}_{out}}{2}\right)\right|_{r=R}$$

$$= \frac{1}{2}\left[\mu_0 R\omega\sigma\left(2\cos\theta\,\hat{r} - \frac{\sin\theta}{2}\hat{\theta}\right)\right]$$

$$= \frac{\mu_0 R\omega\sigma}{3}\left(2\cos\theta\,\hat{r} - \frac{\sin\theta}{2}\hat{\theta}\right)$$

As we know, $F_{mag} = q(\vec{V}\times\vec{B})$.

where $q = \sigma\, da = \sigma\cdot(R^2\sin\theta\cdot d\theta\cdot d\phi)$.

$$B_{avg} = \frac{\mu_0 R\omega\sigma}{3}\left(2\cos\theta\,\hat{r} + \frac{\sin\theta}{2}\,\hat{\theta}\right).$$

$$V = (R\sin\theta)\omega\cdot\hat{\phi}$$

$$\therefore (\vec{V}\times\vec{B}) = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ 0 & 0 & (R\sin\theta)\omega \\ \left(\frac{\mu_0 R\omega\sigma}{3}\right)\cdot 2\cos\theta & \left(\frac{\mu_0 R\omega\sigma}{6}\right)\sin\theta & 0 \end{vmatrix}$$

$$= \frac{\mu_0 R^2\omega^2\sigma}{3}\left(\frac{\sin^2\theta}{2}\,\hat{r} + 2\sin\theta\cos\theta\,\hat{\theta}\right)$$

$$\therefore F_z = q\cdot(\vec{V}\times\vec{B})_z = \int \sigma\cdot da\,(\vec{V}\times\vec{B})_z$$

$$= \int \sigma\cdot(R^2\sin\theta\cdot d\theta\cdot d\phi)\cdot(\vec{V}\times\vec{B})_z$$

Here, $$(\vec{V}\times\vec{B})_z = \frac{\mu_0 R^2\omega^2\sigma}{3}\left(\frac{\sin^2\theta}{2}\cos\theta\,\hat{z} - 2\sin^2\theta\cos\theta\,\hat{z}\right)$$

$$= \frac{\mu_0 R^2\omega^2\sigma}{3}\left(-\frac{3}{2}\sin^2\theta\cos\theta\,\hat{z}\right).$$

$$= -\frac{\mu_0 R^2\omega^2\sigma}{2}\left(\sin^2\theta\cos\theta\,\hat{z}\right).$$

$$\therefore F_z = \int \sigma\cdot(R^2\sin\theta\cdot d\theta\cdot d\phi)\cdot\left(-\frac{\mu_0 R^2\omega^2\sigma}{2}\right)\left(\sin^2\theta\cos\theta\,\hat{z}\right).$$

$$= -\mu_0\frac{(R^2\omega\sigma)^2}{2}\int_0^{2\pi}\int_0^{\pi/2}\sin^3\theta\cdot\cos\theta\cdot d\theta\cdot d\phi.$$

$$= -\frac{\mu_0}{2}(R^2\omega\sigma)^2\cdot 2\pi\int_0^{\pi/2}\sin^3\theta\cdot\cos\theta\cdot d\theta.$$

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Considering $\sin\theta = t$ & $\cos\theta \cdot d\theta = dt$

$$\therefore \int \sin^3\theta \cdot \cos\theta \cdot d\theta = \int t^3 \cdot dt = \frac{t^4}{4}$$

$$\therefore \int_0^{\pi/2} \sin^3\theta \cdot \cos\theta \cdot d\theta = \frac{1}{4} \sin^4\theta \Big|_0^{\pi/2} = \frac{1}{4}$$

$$\therefore F_z = -\frac{\mu_0 \pi}{4} (R^2 \omega \sigma)^2 \hat{z}.$$