Ampere's law in Magnetised Material

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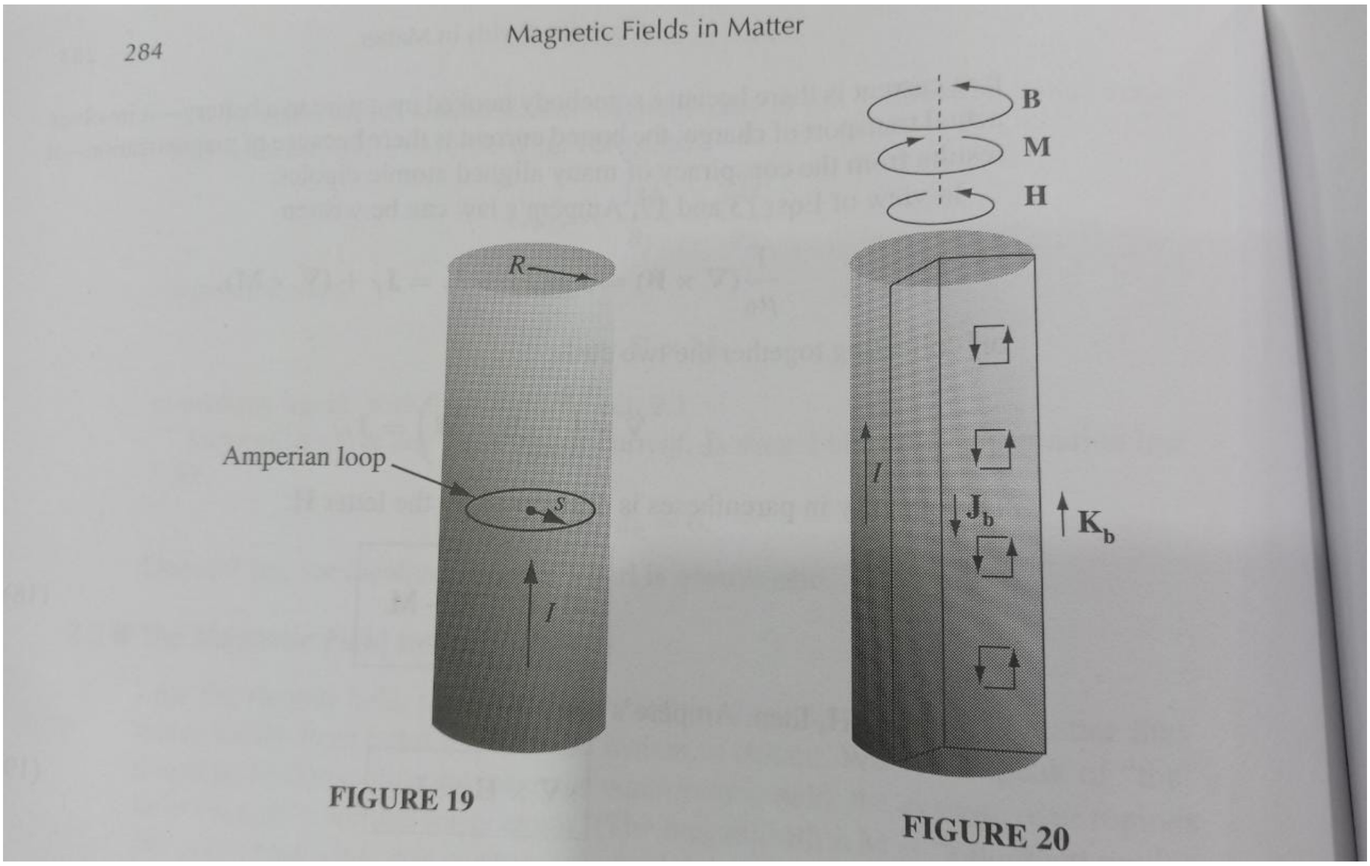
Question: *A long copper rod of radius R carries a uniformly distributed (free) current I. Find H inside and outside the rod.*



Solution:

Copper is weakly diamagnetic so the dipole will line up opposite to the field, this results in a bound current running *anti-parallel* to I within the wire and *parallel* to I along the surface. Just how great these bound currents will be. We are not in the position to say, but in order to calculate $\mathbf{H}$, it is sufficient to realise that all the currents are longitudinal, So, $\mathbf{B}$, $\mathbf{M}$ and therefore also $\mathbf{H}$ are circumferential.

Apply,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}}$$

to an Amperian loop of radius $s < R$.

$$\mathbf{H}(2\pi s) = I_{f_{enc}} = I\frac{\pi s^2}{\pi R^2}$$

so, inside the wire,

$$\mathbf{H} = \frac{I}{2\pi R^2} s\hat{\phi} \qquad (s \leq R)$$

and outside the wire,

$$\mathbf{H} = \frac{I}{2\pi s}\hat{\phi} \qquad (s \geq R)$$

In the latter region (as always, in empty space) $\mathbf{M} = 0$. So,

$$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi s}\hat{\phi} \qquad (s \geq R)$$

The same as for a *non-magnetised* wire. *Inside* the wire $\mathbf{B}$ cannot be determined at this stage, since we have no way of knowing $\mathbf{M}$ (though in practise, the magnetisation in copper is so slight that for most purposes we can ignore it altogether).
