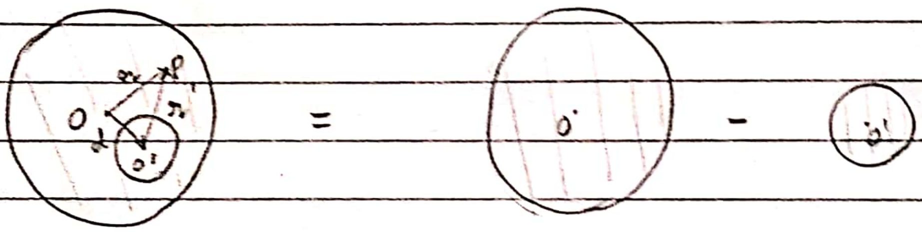
Q A non-conducting sphere of radius R centered at O contains a spherical cavity of radius R' centered at O'. Let d be the displacement of O' relative to O. Throughout the sphere there is a uniform charge density ρ_0 (except inside the cavity, which is uncharged). Assume that the permittivity of the sphere has the vacuum value ε_0. Write down an expression for $E(r)$ everywhere using principle of superposition and Gauss' law.

soln.



Here all the 3 spheres have the same charge density (ρ_0)

Let P be any arbitrary point in space such that,

(1) — E_P due to a sphere with a spherical cavity of radius R' centered on O' = E_P due to a solid sphere of radius R centered on O − E_P due to a smaller solid sphere of radius R' centered on O'

⇒ $r \le R$ with P outside the cavity

NOTE :- $\oint E.da = \frac{1}{\varepsilon_0}\int \rho(r)\,dV = \frac{1}{\varepsilon_0} q_{enclosed}$

$$E \times 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0} = \begin{cases} \frac{\rho_0}{\varepsilon_0} \times \frac{4}{3}\pi r^3, & r \le R \\ \\ \frac{\rho_0}{\varepsilon_0} \times \frac{4}{3}\pi R^3, & r \ge R \end{cases}$$

$$E(r) = \begin{cases} \dfrac{\rho_0 r}{3\varepsilon_0} , & r \leq R \\ \dfrac{\rho_0}{3\varepsilon_0}\left(\dfrac{R}{r}\right)^3 r , & r \geq R \end{cases}$$

Here,
the RHS of equation 1 becomes,

$$E(r) = \frac{\rho_0}{3\varepsilon_0}\left(r - \left(\frac{R'}{r'}\right)^3 r'\right)$$

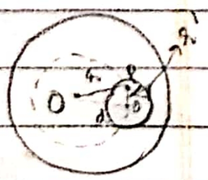
$R' \rightarrow$ radius of the cavity
$r' \rightarrow$ dis. b/w O' & r

$$= \frac{\rho}{3\varepsilon_0}\left(r - \frac{R'^3}{(r^2+d^2-2rd)^{3/2}}(r-d)\right) \quad \text{--- (2)}$$

$\overline{OO'} = d$, $\overline{OP} = r$ & $\overline{O'P} = r'$ $\quad \therefore d = r - r'$, $r' = r - d$
$\therefore r' = \sqrt{r^2 - 2rd + d^2}$

$\Rightarrow r \leq R$ with P inside the cavity

$$E(r) = \frac{\rho_0}{3\varepsilon_0}(r - r') = \frac{\rho_0}{3\varepsilon_0} d \quad \text{--- (3)}$$



~~Since R' = r'~~ Since $E(r)$ inside a non-conducting sphere @ r' is $\frac{\rho}{3\varepsilon_0} r'$

$\Rightarrow r \geq R$

$$E(r) = \frac{\rho_0}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3 r - \left(\frac{R'}{r'}\right)^3 r'\right]$$

$$= \frac{\rho}{3\varepsilon_0}\left[\frac{R^3}{r^3} r - \frac{R'^3}{(r^2+d^2+2rd)^{3/2}}(r-d)\right] \quad \text{--- (4)}$$