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Class/Batch: 3

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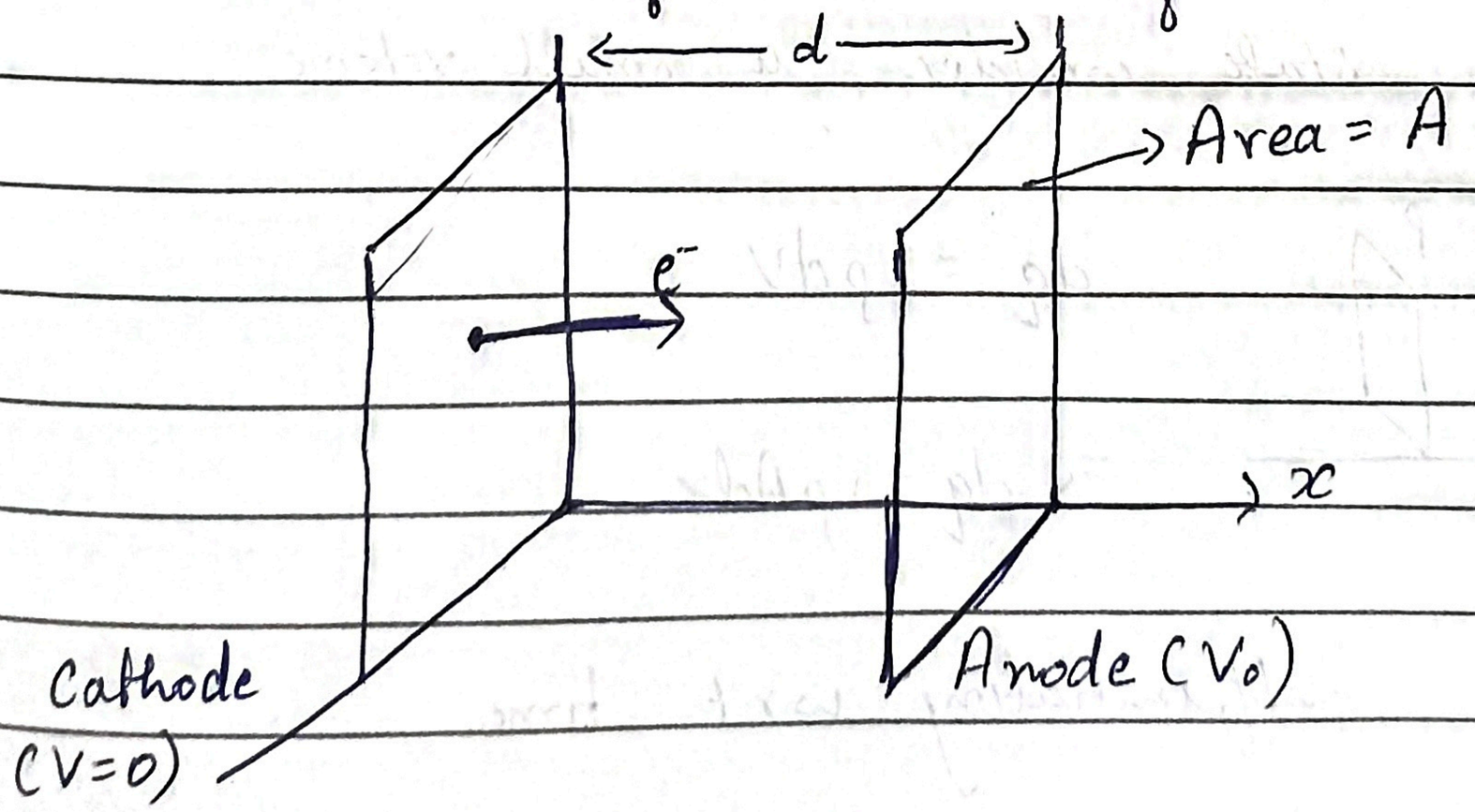
Presentation Problem and Solution

→ Source: Griffiths: Introduction to Electrodynamics
Problem - 2.53

Problem:

→ In a vacum diode, electrons are "boiled" off a hot cathode, at potential zero and accelarated accross a gap to the anode, which is held at positive potential V_0. The cloud of moving electrons within the gap (called space charge) quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then on, a steady current I flows between the plates.

Suppose the plates are large relative to the separation ($A >> d^2$) so that edge effects can be neglected. Then V, ρ and v (the speed of e^-) are all the functions of x alone.



(a) Write the Poisson's equation for the region between the plates.

(b) In the steady state, I is independent of x. What, then, is the relation between ρ and v.

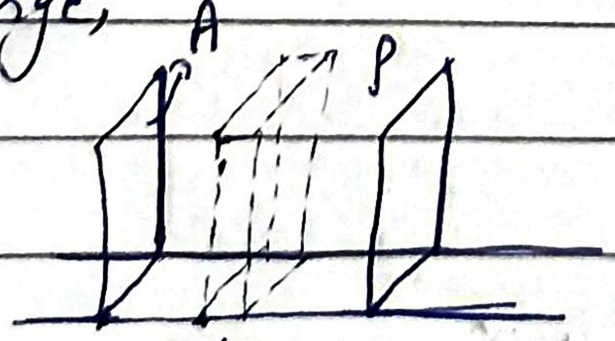
Answers

→ Note: We are given I = current
ρ = charge density
v = speed of electrons
V = voltage

(a) The voltage is V, and charge density is ρ, so simply,

$$\nabla^2 V = -\rho/\varepsilon_0$$

(b) In steady state consider a small volume charge,



$$dq = \rho dV$$

$$= dq = \rho A dx$$

differentiating w.r.t time,

$$\frac{dq}{dt} = \rho A \frac{dx}{dt}$$

$$\frac{dq}{dt} = I \quad , \frac{dx}{dt} = v$$

$$\boxed{I = \rho A v}$$