Why is the curl of a gradient zero?

Mathematically speaking:
$\nabla \times \nabla f$ for any scalar field f, basically means that you are taking a cross product of two vectors in the same direction (one is a scaled version of the other), which inherently by definition is zero.

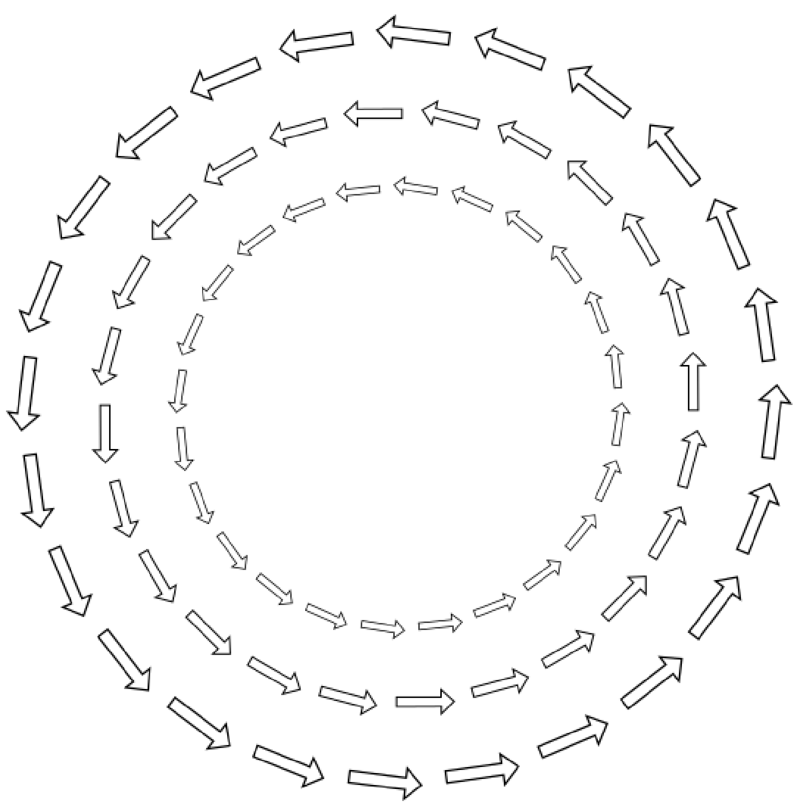
However, by brute force we can find this out:
$\nabla \times \nabla f$ = det({i,j,k},{∂/∂x , ∂/∂y , ∂/∂z},{∂f/∂x, ∂f/∂y , ∂f/∂z})
On computing this, we see that they cancel each other out completely to give a zero vector.

[∂/∂y(∂f/∂z) - ∂/∂z(∂f/∂y)]i - [∂/∂x(∂f/∂z) - ∂/∂z(∂f/∂x)]j +[∂/∂x(∂f/∂y) - ∂/∂y(∂f/∂x)]k = 0i - 0j +0k

Physically speaking:
Mathematically it makes sense, however without understanding why it is as such PHYSICALLY, we can only pretend to understand it.
To physically get the gist of it, lets see what happens if this is untrue, i.e.

$$\nabla \times \nabla f \neq 0$$

This means that we have a gradient that looks as such:



We know that conservative force can be expressed as the gradient of a scalar field (potential), So for better visualization, let's assume this hypothetical gradient to be that of the potential.

Side tangent:
To visualise a curl, it is effective to imagine a fan that is kept at the point the curl has to be calculated. This shows that if you keep a fan at some point, and it starts turning, assuming the surrounding field to behave as if water flowing, then the curl exists and the speed at which the fan turns can be an indication of the amount.

To visualise a gradient, place a ball in the space, assuming the scalar field value at any point to be represented by height. Thus a ball placed will fall in the direction opposite to the direction of the gradient.

Thus, this means, that a ball placed in this field would start circling a point and accelerating around it to return back to the same place, as the gradient is circular. This means that the ball would automatically start revolving around a point, and would return to the same point time and time again.

When we calculate the work done for the same, we can see that the work done by the ball for the motion it does is negative in magnitude and non-zero.
This is contradictory, however as we know that work done should only depend on the endpoints and not the path traveled for a conservative force, which is not the case here, as the same endpoints, should give zero work. Thus this scenario is not possible.

This can also directly be seen as a gradient means the direction of increase, thus a circular gradient means that the scalar quantity continuously increases and keeps on increasing which is contradictory to the laws of physics

What would happen if this wasn't the case:
Now that we have a fitting understanding of why the curl of a gradient is zero, let's find out what is happening in another universe where this is not the case.

In Tom's universe, The curl of a gradient is non-zero.
This means that the potential continuously keeps increasing in some special conditions. Thus a ball kept in this potential starts revolving and accelerating, increasing its tangential as well as angular velocity, without the addition of any force. The people of this universe use this fact to continuously generate energy, as their dynamo now has something that will revolve continuously without the input of any force. They now have infinite energy, which is a direct violation of the law of conservation of energy.