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BATCH - 1 (PH1213)

VECTOR POTENTIAL IS A "REAL" FIELD – A QUANTUM-MECHANICAL CASE STUDY

INTRODUCTION :

Real Field :

A "real" field does not act on a particle from a distance.
i.e., action-at-a-distance is not allowed.

Hence, a particle situated at point $\vec{r}$ (w.r.t an origin) is affected only by the field at that point ($\vec{r}$), if the field is real.

For a long time, it was believed that the vector potential $\vec{A}$ was not a "real" field.

Reason :

- Vector potential function has arbitrariness!

If $\vec{A}' = \vec{A} + \nabla\psi$ [ψ is a scalar]

Then, $\nabla \times \vec{A}' = \nabla \times (\vec{A} + \nabla\psi)$

$$\Rightarrow \nabla \times \vec{A}' = (\nabla \times \vec{A}) + \cancel{(\nabla \times \nabla\psi)}^{0}$$

$$\nabla \times \vec{A}' = \nabla \times \vec{A} = \vec{B} \quad \text{(Magnetic Field)}$$

Hence, only the curl of $\vec{A}$ is what matters for physics.

- Example of a solenoid :

In a long, tightly wound solenoid,

i) The $\vec{B}$ outside the solenoid, $\vec{B}_{ext} = \vec{0}$.

ii) The $\vec{B}$ inside the solenoid is non-zero.

iii) The vector potential $\vec{A}$ circulates around the solenoid inside and outside, i.e., it is non-zero.

Classically, the force depends only on $\vec{B}$, hence for a charged particle, to know the existence of $\vec{B}$, has to go inside the solenoid.

$\therefore$ $\vec{A}$ has no role though it is abundant outside!

DOUBLE SLIT EXPERIMENT (A CHANGE OF VIEWS):

Quantum Mechanical Approach :

- In Quantum mechanics, the ideas of classical force becomes un-important
- Energy and momenta are considerably of highest importance because :

i) We deal with probability amplitudes, rather than particle motion
ii) We are more interested in wavelengths associated with momenta and frequencies associated with energies
iii) Acceleration, associated with classical force, is not spoken about.

- Hence, naturally, $\vec{A}$ and ϕ [Vector potential & Scalar potential resp.] play more important role in Quantum mechanics, than $\vec{B}$ & $\vec{E}$.

- The Lorentz Force law,
$$\vec{F} = q(\vec{v} \times \vec{B})$$
is replaced by a "new" law that governs what happens to probability amplitudes under electromagnetic influences.

The New Law:

The phase of the amplitude to arrive via any trajectory is changed by the presence of a magnetic field by an amount equal to the integral of the vector potential along the whole trajectory times the charge of the particle over Planck's constant,

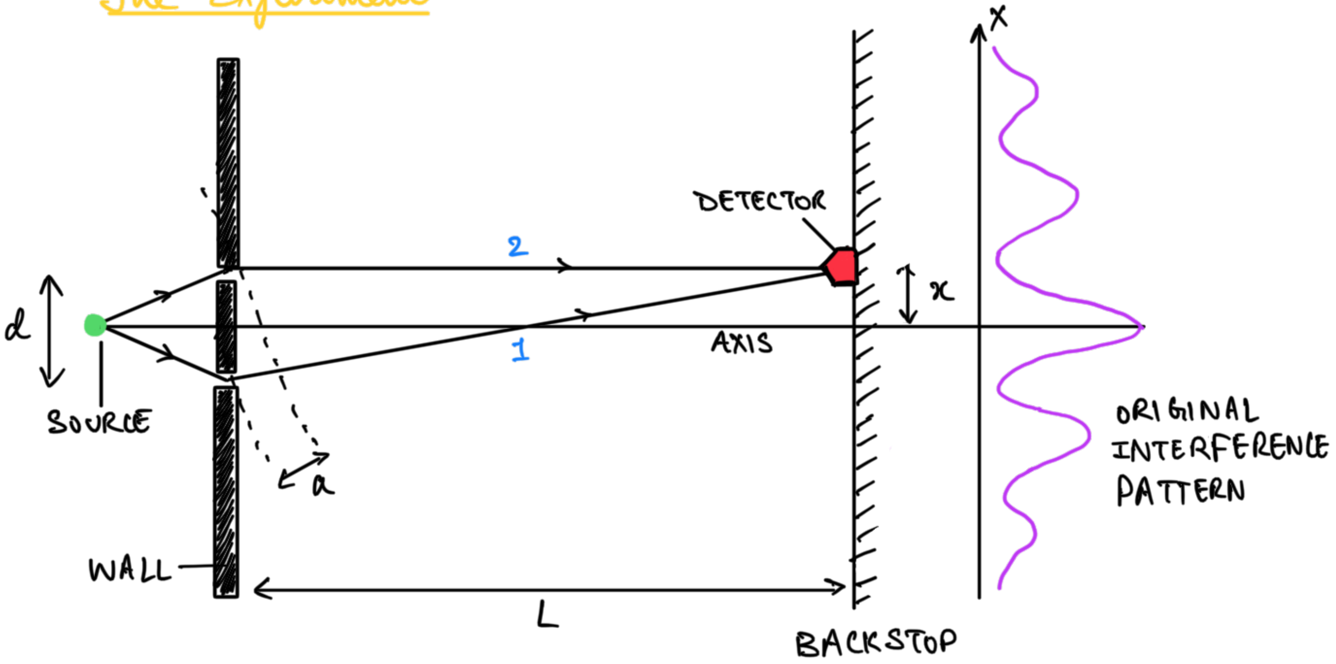
i.e.,
$$\Delta \psi_{mag} = \frac{q}{\hbar} \int_{P} \vec{A} \cdot \vec{dl} \qquad [P \equiv \text{Trajectory}]$$

The phase of the amplitude to arrive via any trajectory is changed by the presence of a electric field by an amount equal to the negative of the time integral of the scalar potential times the charge of the particle over Planck's constant,

i.e.,
$$\Delta \psi_{elec} = -\frac{q}{\hbar} \int \phi \, dt$$

Together, these 2 Equations give correct result for "any" electromagnetic field (static or dynamic).

The Experiment :



- e^- (same energy) leave the source and travel equal distance to reach narrow slits in a wall.
- Beyond wall, there is a backstop with movable detector, which measures rate of e^- flux (I) at distance x from axis.

$$I \propto \text{Probability of } e^- \text{ reaching } x$$

- Probability distribution is shown by interference pattern determined by the phase difference of of the 2 interfering amplitudes.

$$\delta = \Phi_1 - \Phi_2 \qquad \left[P_1 = C_1 e^{i\Phi_1} ; \; P_2 = C_2 e^{i\Phi_2} \right]$$

- Now,

$$\frac{\delta}{2\pi} = \frac{a}{\lambda} \qquad [a \equiv \text{path difference}]$$

$$a = d\sin\theta \sim d\tan\theta \qquad [\theta \text{ is very small}]$$

$$\tan\theta = \frac{x}{L}$$

$$\therefore \; a = \frac{dx}{L}$$

Hence, $\delta = \frac{dx}{L} \cdot \frac{2\pi}{\lambda} = \left(\frac{x}{L}\right)\frac{d}{\lambdabar} \qquad \left[\lambdabar = \frac{\lambda}{2\pi} \right]$

When $x = 0$, $\delta = 0 \longrightarrow$ "in phase" $\longrightarrow$ Maxima

$\delta = \pi \longrightarrow$ "out of phase" $\longrightarrow$ Minima

Application of the "New Law":

Let Φ_1 = phase of wave along trajectory 1
& Φ_2 = phase of wave along trajectory 2

$$\Phi_1 = \Phi_1(\vec{B}=\vec{0}) + \frac{q}{\hbar}\int_1 \vec{A}\cdot\vec{dl}$$

[in presence of some $\vec{A}$ & $\vec{B}$]

$$\Phi_2 = \Phi_2(\vec{B}=\vec{0}) + \frac{q}{\hbar}\int_2 \vec{A}\cdot\vec{dl}$$

$$\delta = \Phi_1 - \Phi_2$$

$$= \Phi_1(\vec{B}=\vec{0}) - \Phi_2(\vec{B}=\vec{0}) + \frac{q}{\hbar}\oint_{1-2} \vec{A}\cdot\vec{dl}$$

Now,
we should not worry about arbitrariness of $\vec{A}$.

Let $\vec{A}' = \vec{A} + \nabla\psi$

Then, $\displaystyle \oint_{1-2} \vec{A}'\cdot\vec{dl} = \oint_{1-2} \vec{A}\cdot\vec{dl} + \oint_{1-2} \nabla\psi\cdot\vec{dl}$

$$= \oint_{1-2} \vec{A}\cdot\vec{dl} + \int_S \overbrace{(\nabla\times\nabla\psi)}^{0}\cdot d\vec{a}$$

$$= \oint \vec{A}\cdot\vec{dl}$$

Hence, we have,

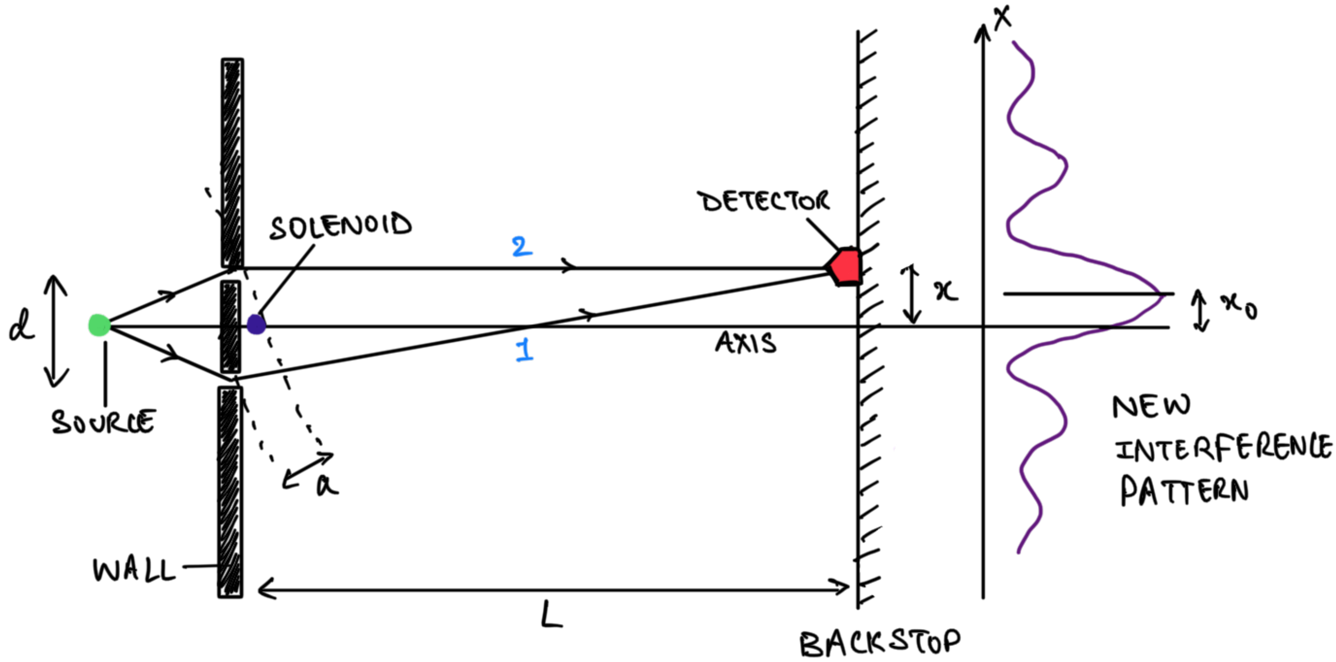
$$\delta = \delta(\vec{B}=\vec{0}) + \frac{q}{\hbar}\oint_{1-2} \vec{A}\cdot\vec{dl}$$

$$= \delta(\vec{B}=\vec{0}) + \frac{q}{\hbar}\underbrace{\int_S \vec{B}\cdot d\vec{a}}$$

Flux of $\vec{B}$ between 1 & 2

Tweaking the Experiment:

- Suppose we put a very long solenoid of very small diameter ($<<d$) behind & between the 2 slits. (The e^- will not touch the solenoid due to its closeness to the wall)



- When current in solenoid is turned off, we have no $\vec{B}$ or $\vec{A}$
  $\therefore$ we get original interference pattern.

- When current in solenoid is turned on, we have build-up of $\vec{B}$ inside solenoid and $\vec{A}$ outside.

- Since flux of $\vec{B}$ through any 2 pairs of trajectories is same, so is the circulation of $\vec{A}$

$$\therefore \quad \delta = \delta(\vec{B}=\vec{0}) + \frac{q}{\hbar}\underbrace{\int_S \vec{B}\cdot d\vec{a}}$$

constant for any 1 & 2

$$\delta = \delta(\vec{B}=\vec{0}) + \frac{q}{\hbar}\oint_{1-2} \vec{A}\cdot d\vec{l}$$

For x_n [point of Maxima]

$$\delta = 0 \implies \delta(\vec{B} = \vec{0}) = -\frac{q}{\hbar} \oint_{1-2} \vec{A} \cdot d\vec{l}$$

i.e.,
$$\frac{x_0 d}{L \lambda\!\!\!^{-}} = \frac{-q}{\hbar} \oint_{1-2} \vec{A} \cdot d\vec{l}$$

$$x_0 = -\frac{L}{d} \frac{\lambda\!\!\!^{-} q}{\hbar} \oint_{1-2} \vec{A} \cdot d\vec{l} = -\frac{L}{d} \frac{\lambda\!\!\!^{-} q}{\hbar} \int_S \vec{B} \cdot d\vec{a}$$

- The whole interference pattern shifts up by a distance x_0. Hence, theoretically, the new interference pattern should look as shown above.

- The experiment was carried out in 1960's using thin iron crystals, which approximately behaved as long, thin solenoids.

- The theoretically predicted displacement of the interference pattern of e^- was observed!!

CONCLUSION :

- The solenoid was kept very close to the wall and was much smaller in diameter than 'd'. Hence, the probability of finding an e^- at the location of the solenoid is exceedingly small.

- Hence, $\vec{B}$, which we know is a "Real" field, has apparently "acted" at a distance on the e^-, thereby affecting the phases of their probability amplitudes.

- But this is not acceptable for a real field. Therefore, it must be the work of the $\vec{A}$, the Vector Potential!!

- This simple experiment proves that the vector potential too is a "Real" Field.

- In Quantum mechanics, what matters is the interference of probability amplitudes and this is only affected by the derivatives of $\vec{A}$,
  i.e, how much $\vec{A}$ changes from point-to-point.

- However, $\vec{A}$ along with ϕ appears to give more insight of the physics than $\vec{E}$ or $\vec{B}$!

- In Modern Physics, $\vec{A}$ & ϕ replace $\vec{E}$ & $\vec{B}$ as the fundamental quantities for the set of laws that govern Quantum Electrodynamics.