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Problem - Using Laplace's Equations, find out the potential at any point between two plates in a

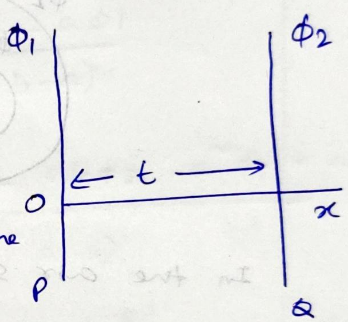
i) Parallel plate condenser

ii) Spherical condenser

iii) Coaxial cylinders

Solⁿ.

(i) Parallel Plate Condenser



Let P & Q be the plates with potentials ϕ_1 & ϕ_2 ($\phi_1 > \phi_2$).

If the plates are large enough to neglect the edge effects, the potential ϕ at a point between the plates will depend on its x-coordinate alone (as shown in fig.), i.e., ϕ will be independent of y & z. So, Laplace's eqⁿ in one dimension,

$$\frac{\partial^2 \phi}{\partial x^2} = 0$$

Integrating,

$$\frac{\partial \phi}{\partial x} = F$$

& $\phi = Fx + G$ where, F & G are arbitrary constants.

Applying boundary conditions, ~~x=0~~

$$\begin{cases} \phi = \phi_1 & \text{when } x = 0 \\ \phi = \phi_2 & \text{when } x = t \end{cases}$$

We get,

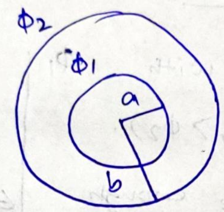
Subctituting,

$$G = \phi_1 \quad \& \quad F = -(\phi_1 - \phi_2)/t$$

$$\therefore \phi = \phi_1 - \frac{x}{t}(\phi_1 - \phi_2)$$

gives the potential at any point b/w the plates.

(ii) <u>Spherical condenser</u>



In the air space b/w the two spheres of radii a & b, (uniformly changed), by symmetry the potential ϕ will depend upon r alone. Then, Laplace's eq^n (in polar coordinates),

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = 0$$

Integrating

$$\frac{\partial \phi}{\partial r} = \frac{F}{r^2}$$

where, F & G are arbitary constants

$$\& \quad \phi = -\frac{F}{r} + G$$

Applying boundary conditions,

$$\begin{cases} \phi = \phi_1 & \text{when } r = a \\ \phi = \phi_2 & \text{when } r = b \end{cases}$$

we get by substituting,

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$$\phi_1 = -\frac{F}{a} + G$$

$$\phi_2 = -\frac{F}{b} + G$$

Solving,

$$a\phi_1 = -F + Ga$$

$$b\phi_2 = -F + Gb$$

$$(-)\quad a\phi_1 - b\phi_2 = G(a-b)$$

$$\therefore G = \frac{a\phi_1 - b\phi_2}{(a-b)} = \frac{b\phi_2 - a\phi_1}{(b-a)}$$

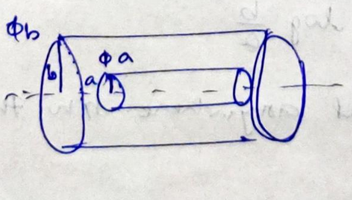
$$F = Ga - a\phi_1 = \frac{a^2\phi_1 - ab\phi_2 - a^2\phi_1 + ab\phi_1}{(a-b)}$$

$$\text{or,}\quad F = \frac{ab(\phi_1 - \phi_2)}{(a-b)} = \frac{-ab(\phi_1 - \phi_2)}{(b-a)}$$

$$\therefore \phi = \frac{ab(\phi_1 - \phi_2)}{(b-a)} + \frac{b\phi_2 - a\phi_1}{(b-a)}$$

gives the potential anywhere b/w the conductors.

(iii) <u>Coaxial cylinders.</u>



Neglecting the end effects, it is a two dimensional case & there being symmetry about the axis the potential ϕ is a func^n of r alone.

Using Laplace's eqn in cylindrical coordinates, i.e.,

$$\frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) = 0$$

Integrating, $\frac{\partial \phi}{\partial r} = \frac{F}{r}$ where, F & G are

& $\phi = F \log r + G$ arbitary consts.

Applying boundary conditions, we get,

$$\begin{cases} \phi = \phi_a & \text{when } r = a \\ \phi = \phi_b & \text{when } r = b \end{cases}$$

By substituting,

$$\phi_a = F \log a + G$$

$$\phi_b = F \log b + G$$

Solving, we will get,

$$F = -\frac{(\phi_a - \phi_b)}{\log(b/a)} \quad \& \quad G = \frac{(\phi_a \log b - \phi_b \log a)}{\log(b/a)}$$

$$\therefore \phi = \frac{\left(\phi_a \log \frac{b}{r} - \phi_b \log \frac{a}{r}\right)}{\log \frac{b}{a}}$$

gives the potential anywhere b/w the cylinders.