PH1213

TUTORIAL

PROBLEM

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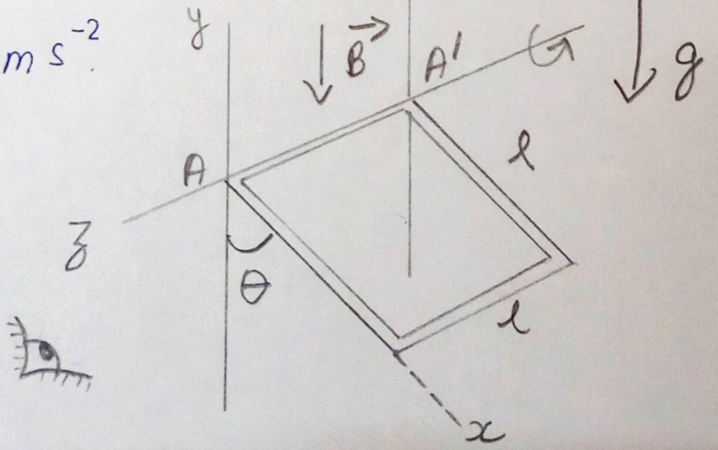
TORQUE AND FORCE:

A square loop of wire, of length l, on each side, and mass 'm', pivots about an axis AA' that corresponds to a horizontal side of the square, as shown in the figure below. The external magnetic field $\vec{B}$ of magnitude B is directed vertically downward, and uniformly fills the region in the vicinity of the loop. A current I flows around the loop. The gravitational torque on the loop and the magnetic torque on the loop sum to zero when the loop makes an angle θ with y axis. The magnitude of gravitational field is $g = 9.8\ m s^{-2}$



Questions:

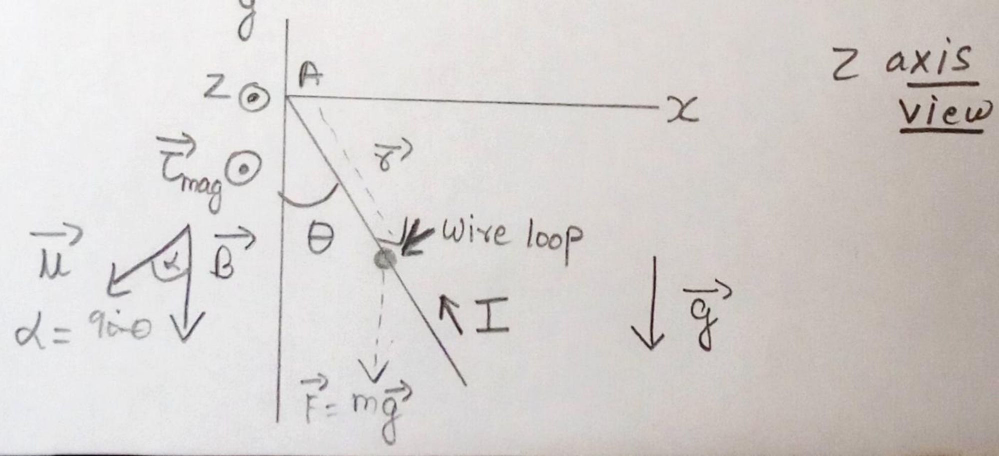
a) In what direction does the current need to flow in order that the magnetic torque acts in an opposite direction from gravitational torque?

b) Calculate magnitude of magnetic torque on this loop of wire in terms of the quantities given.

c) Suppose that mass of loop $m = 0.4\,kg$ and length of side $(l = 1.0\,m)$. Suppose that when current in the loop is $I = 2.0\,A$, the torques on the loop balance when $\theta = 45°$. What is the magnitude of the magnetic field?

SOLUTIONS:

a) The system can be represented in 2-D as follows:



a) $\vec{\tau}_{grav}$ is given by $\vec{r} \times \vec{F}$: $\vec{r}$ is the Position vector parallel to x-y Plane and $\vec{F} = m\vec{g}$ Pointing in $-y$ direction.

By right-hand rule, $\vec{\tau_{grav}}$ Points in $-z$ direction.

Therefore the magnetic torque must act in the Positive z direction.

Magnetic torque is given by expression

$$\vec{\tau}_{mag} = \vec{\mu} \times \vec{B}.$$

$\vec{B}$ Points in $-y$ direction, ie, $\vec{B} = -B\hat{j}$

The magnetic dipole moment is given by the direction of current in any loop by the right hand rule.

$\Rightarrow$ Clockwise or Counter-clockwise current will lead to magnetic moment which is Parallel to x-y Plane.

$\Rightarrow$ $\vec{\mu}$ has the following form.

$$\vec{\mu} = \mu_x \hat{i} + \mu_y \hat{j}$$

From the 2D diagram, $\vec{\mu}$ should point out in third quadrant:

Such that both components are negative.

i.e, $\mu_x < 0$ and $\mu_y < 0$.

This is possible only when current flows in clockwise direction when viewed from above.

b) We know that $\vec{\tau}_{mag} = \vec{\mu} \times \vec{B}$

and $|\vec{\mu}|$ for any loop is IA.

$\Rightarrow |\vec{\mu}| = I \cdot A = I l^2$

where $A = l^2$ = Area of square loop.

$\vec{\mu} = I\vec{A}$,

$\vec{A}$ vector denotes the area vector given by right hand rule.

$\Rightarrow |\vec{\tau}_{mag}| = \mu B \sin\alpha$

where α is the angle between them.

$$\Rightarrow |\vec{\tau}_{mag}| = \mu B \sin(90-\theta)$$

$$= \mu B \cos\theta$$

$$= I l^2 B \cos\theta.$$

c) The loop is pivoted along one side.

So we consider $\vec{\tau}_{grav}$ on bottom leg and two side legs.

$\vec{\tau}_{grav}$ for each arm is calculated

and net $\vec{\tau}_{grav}$

$= \left(\frac{m}{4}\right) g l \sin\theta \, (-\hat{k}) + 2\left(\frac{m}{4}\right)\left(\frac{l}{2}\right) \sin\theta \, (-\hat{k})$

$= \frac{mgl}{2} \sin\theta \, (-\hat{k})$

$\vec{\tau}_{mag}$ acts opposite to $\vec{\tau}_{grav}$

$\therefore$ $\vec{\tau}_{mag}$ in $\hat{k}$ direction

$= I l^2 B \cos\theta \, \hat{k}$

For the given case.

$I l^2 B \cos\theta = \left(\frac{m}{2}\right) g l \sin\theta$

$I = 2A, \; m = 0.4 \, kg, \; \theta = 45^\circ, \; g = 9.8 \, m s^{-2}$

$l = 1.0 \, m$

$\Rightarrow B = \frac{mgl\sin\theta}{2 I l^2 \cos\theta} = \frac{mg\tan\theta}{2 I l}$

$= \frac{(0.4)(9.8)(1)}{2(2.0)(1)}$

$= 0.98 \, T$

$\simeq 1 T$