Let The distance between charge q_2 and ~~chest~~ center of sphere be 'b'

$$\therefore b = a - d$$

$$= \frac{q_1^2 d}{q_1^2 - q_2^2} - d = \frac{q_2^2 d}{q_1^2 - q_2^2}$$

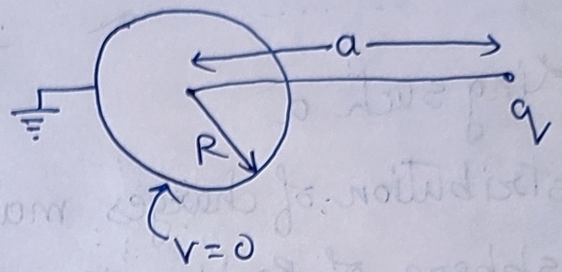
also $\frac{R^2}{a} = \frac{q_2^2 d}{q_1^2 - q_2^2}$

$$\therefore \boxed{b = \frac{R^2}{a}}$$

Now, $\frac{R}{a} = \frac{q_2}{q_1}$

$$\therefore \boxed{q_2 = \frac{R}{a} q_1}$$

Q) A point charge q is situated a distance a from The centre of The grounded conducting sphere of radius R. Find The Potential outside The sphere?



Solution :-

The boundary condition for The given question is

① $V=0$ for The given conducting sphere.

② $V=0$ for a Point $P(x,y,z)$ such That $x^2+y^2+z^2 \gg a^2$

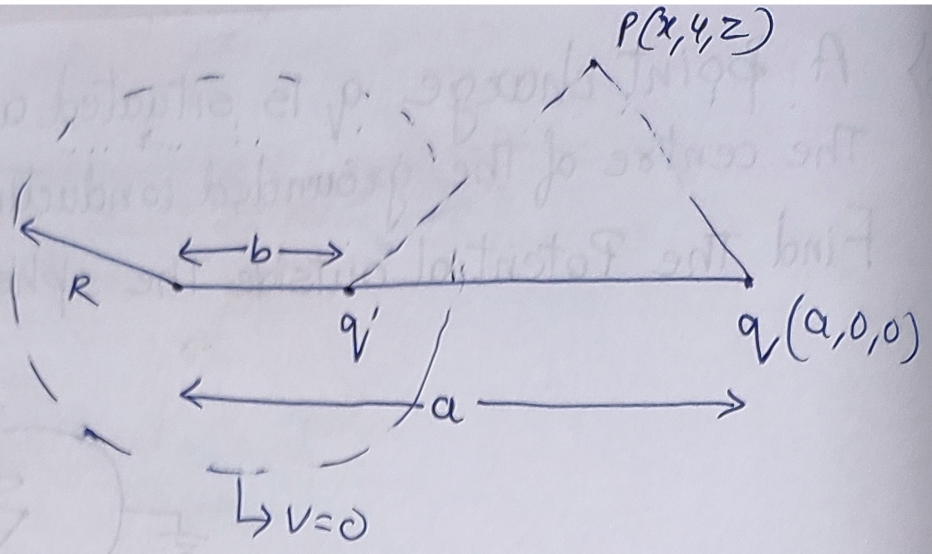
Now consider a totally different construction of charges. Such That, There are two Point charges q and q'

$$\boxed{q' = -\frac{R}{a} q}$$

and. q is Placed at a distance Right of The sphere and.

q' is placed at b distance Right of The sphere.

$$\boxed{b = \frac{R^2}{a}}$$



Taking such a distribution of charges makes the Potential of a sphere of Radius R equal to zero.

Thus This distribution of charges satisfy The initial boundary conditions of The question.

Therefore by using The uniqueness theorem, This is the only way to satisfy The initial boundary conditions.

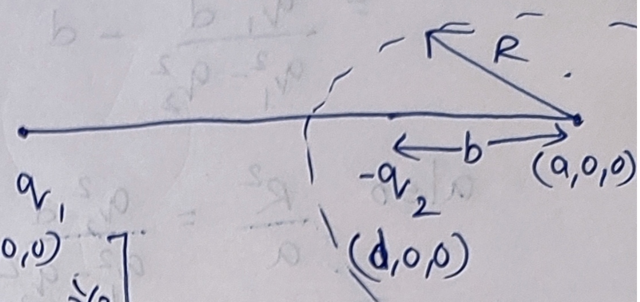
Therefore, The question can be solved by Taking The given distribution of charges.

$$\therefore V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{\left((x-a)^2+y^2+z^2\right)^{1/2}} + \frac{1}{4\pi\epsilon_0}\frac{q'}{\left((x-(a-b))^2+y^2+z^2\right)^{1/2}}$$

for The region outside The conducting sphere.

Now, how did we got The value of q' and b.

for This consider two charges q_1 and $-q_2$ on x-axis d distance away.



$$V(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{(x^2+y^2+z^2)^{1/2}} - \frac{q_2}{((x-d)^2+y^2+z^2)^{1/2}}\right]$$

For a equipotential surface with $V=0$.

$$\frac{q_1}{(x^2+y^2+z^2)^{1/2}} - \frac{q_2}{((x-d)^2+y^2+z^2)^{1/2}} = 0$$

$$q_1^2((x-d)^2+y^2+z^2) = q_2^2(x^2+y^2+z^2)$$

$$(x^2+y^2+z^2)(q_1^2-q_2^2) - 2q_1^2 d\,x + q_1^2 d^2 = 0$$

$$x^2+y^2+z^2 - \frac{2q_1^2 d}{q_1^2-q_2^2}x + \frac{q_1^2 d^2}{q_1^2-q_2^2} = 0 \quad \text{— (1)}$$

Now comparing eq (1) with,

$$(x-a)^2+y^2+z^2 = R^2$$

$$\boxed{a = \frac{q_1^2 d}{q_1^2-q_2^2}} \quad \text{and} \quad \boxed{R = \frac{q_1 q_2 d}{q_1^2-q_2^2}}$$

clearly $a>d$.