Tushar presentation

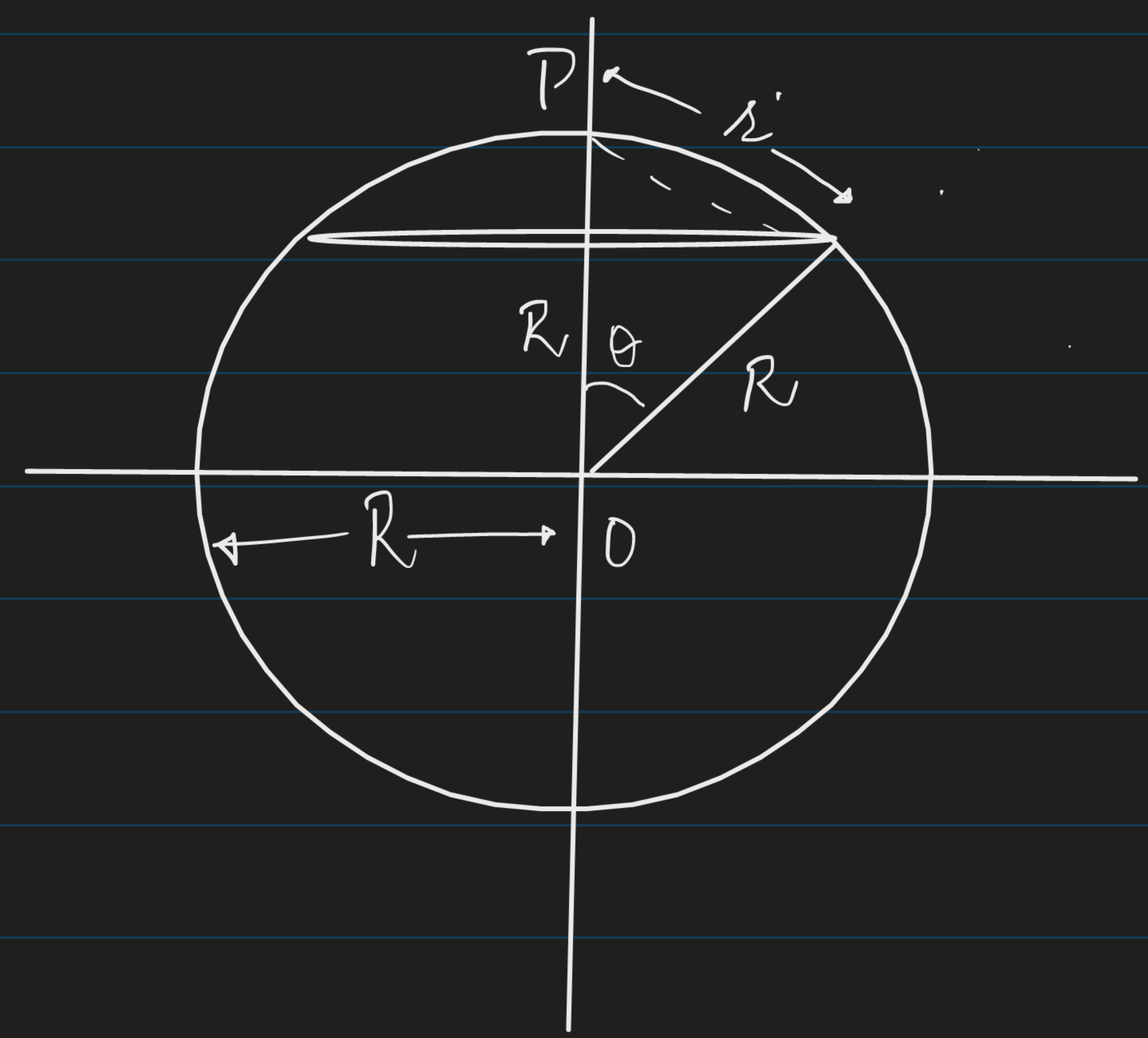
09 June 2022 13:21

Source : Griffiths : Introduction to Electrodynamics (Problem 2.48)

Problem 2.48 An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the "north pole" and the center. [*Answer:* $(R\sigma/2\epsilon_0)(\sqrt{2}-1)$]

#

To find: $V_p - V_0 = ?$



V_0 :

$$V_0 = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma da}{R}$$

$$= \frac{\sigma}{4\pi\varepsilon_0 R}\int da = \frac{\sigma}{4\pi\varepsilon_0 R}(2\pi R^2)$$

$$= \frac{\sigma R}{2\varepsilon_0}$$

V_p : From cosine rule, $\cos\theta = \dfrac{R^2+R^2-r^2}{2R^2} \Rightarrow r^2 = 2R^2(1-\cos\theta) \Rightarrow r = \sqrt{2}R(1-\cos\theta)^{1/2}$ [Considering an elemental ring]

$$V_p = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma da}{r} = \frac{\sigma}{4\pi\varepsilon_0}\int \frac{(R d\theta)(2\pi R\sin\theta)}{\sqrt{2}R\sqrt{1-\cos\theta}} = \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\int_0^1 \frac{dt}{\sqrt{t}} \qquad \left\{ \begin{array}{l} 1-\cos\theta = t \\ \sin\theta d\theta = dt \end{array} \right\}$$

$$= \frac{\sigma R}{\sqrt{2}\varepsilon_0}(\sqrt{1}-0) = \frac{\sigma R}{\sqrt{2}\varepsilon_0}$$

$$V_p - V_0 = \frac{\sigma R}{2\varepsilon_0}(\sqrt{2}-1) \quad \text{Ans}$$