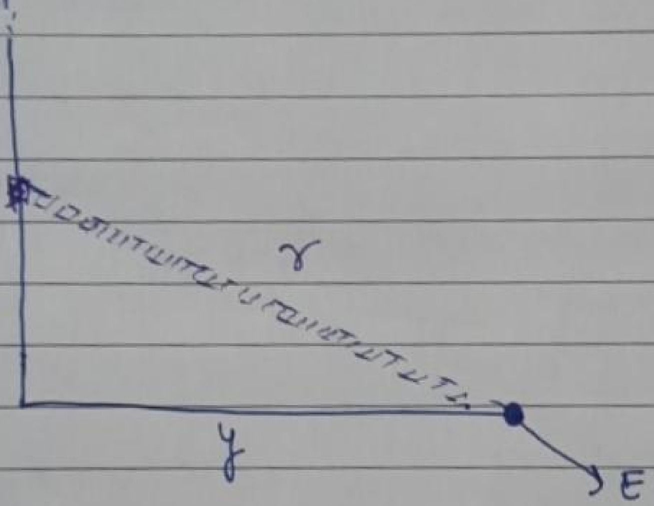
Q. A very long straight uniformly charged thread carries a charge λ per unit length. Find the magnitude and direction of the electric field strength at a point which is at a distance y from the ~~the~~ thread & lies on the perpendicular passing through one of the thread ends.

Semi Infinite



$r = \sqrt{x^2+y^2}$

$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$

$$\boxed{r = \frac{y}{\cos\theta}}$$

electric field in x-direⁿ

Ex :-

$\tan\theta = \frac{x}{y}$

differentiating w.r.t to 'θ' and 'x' with y = constant

charge density = 'λ'

$\boxed{dq = \lambda \cdot dx}$ Assumming charge distribution is uniform.

$dx = y \cdot \sec^2\theta \cdot d\theta$.

$dq = \lambda y \sec^2\theta \cdot d\theta$.

For a continuous charge distribution

$$E = \int \frac{K\,dq}{r^2}$$

$$E_x = \int_0^{\pi/2} \frac{K\lambda y \sec^2\theta \cdot d\theta}{\frac{y^2}{\cos^2\theta}} \cos\theta.$$

$$E_x = \int_0^{\pi/2} \frac{K\lambda \cancel{y} \cancel{\sec^2\theta} \cdot d\theta \cdot \cancel{\cos^2\theta} \cdot \cos\theta}{y^{\cancel{2}}} \qquad \boxed{\because \cos\theta = \frac{1}{\sec\theta}}$$

$$E_x = \int_0^{\pi/2} \frac{K\lambda}{y} \cos\theta \cdot d\theta$$

$$= \frac{K\lambda}{y} \int_0^{\pi/2} \cos\theta \cdot d\theta. \qquad \boxed{\begin{array}{l} \text{Where } x \to \infty \\ \theta = \frac{\pi}{2} \end{array}}$$

$$= \frac{K\lambda}{y} \sin\left[\pi/2 - 0\right]$$

$$= \frac{K\lambda}{y}(+1) = +\frac{K\lambda}{y} \Rightarrow \frac{K\lambda}{y} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{y}.$$

Similarly,

$$E_y = \int_0^{\pi/2} \frac{K \cdot \lambda \cancel{y} \cancel{\sec^2\theta} \cdot d\theta}{\frac{y^{\cancel{2}}}{\cancel{\cos^2\theta}}} (\sin\theta)$$

$$= \int \frac{K \cdot \lambda \sin\theta \, d\theta}{y}$$

$$= \frac{R\lambda}{y}\int_0^{\pi/2} \sin\theta$$

$$= \frac{R\lambda}{y}\left[\cos(\pi/2) - \cos(0)\right)$$

$$\boxed{E_y = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{y}}$$

Net $\vec{E} = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\,k\lambda}{y}$