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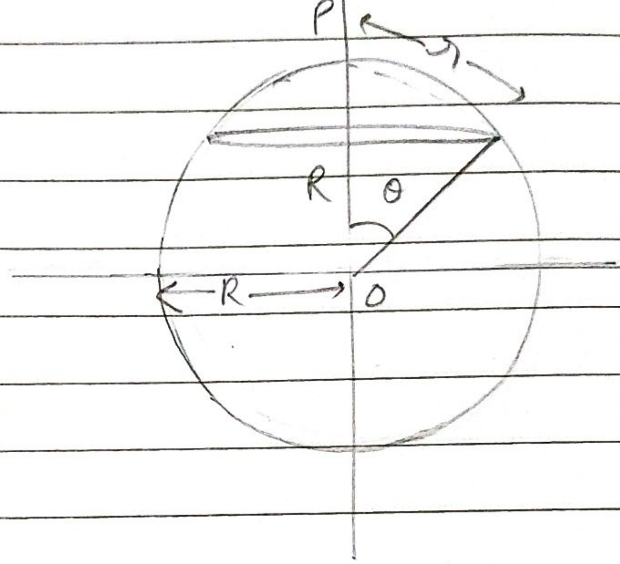
(20211219)

Source : Griffiths : Introduction to electrodynamics

Problem: An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the "north pole" and the center. [Answer: $(R\sigma/2\epsilon_0)(\sqrt{2}-1)$]



To find: $V_p - V_0 = ?$

V_0:

$$V_0 = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma\, da}{R}$$

$$= \frac{\sigma}{4\pi\epsilon_0 R}\int da = \frac{\sigma(2\pi R^2)}{4\pi\epsilon_0 R}$$

$$= \frac{\sigma R}{2\epsilon_0}$$

V_p: From cosine rule $\cos\theta = \frac{R^2+R^2-r^2}{2R^2} \Rightarrow r^2 = 2R^2(1-\cos\theta)$
$r = \sqrt{2}R(1-\cos\theta)^{1/2}$ [considering an elementary ring]

$$V_p = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma\, da}{r} = \frac{\sigma}{4\pi\epsilon_0}\int\frac{(R\,d\theta)(2\pi R\sin\theta)}{\sqrt{2}R\sqrt{1-\cos\theta}} = \frac{\sigma R}{2\sqrt{2}\epsilon_0}\int_0^{\pi/2}\frac{\sin\theta\, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma R}{2\sqrt{2}\epsilon_0}\int_0^1\frac{dt}{\sqrt{t}}$$

$$\left[\begin{array}{l} 1-\cos\theta = t \\ \sin\theta\, d\theta = dt \end{array}\right]$$

$$= \frac{\sigma R}{\sqrt{2}\epsilon_0}(\sqrt{1}-0) = \frac{\sigma R}{\sqrt{2}\epsilon_0}$$

$$\boxed{V_p - V_0 = \frac{\sigma R}{2\epsilon_0}(\sqrt{2}-1)}$$