Q) Consider a uniform charged circular disc of radius 'a' with a negligible thickness carrying a total charge 'q'. Suppose the disc lie on the x-y plane of cartesian coordinate and is centred on the origin

a) show that the electric potential at an arbitrary point P on the symmetric axis of the disc is given by

$$\phi(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2} \mp z\right)$$

where the upper (lower) sign is for $z>0$ $(z<0)$

& $\sigma = \frac{q}{\pi a^2}$

(b) Show that electric potential at an arbitrary point P on the circumference of the disc is given by

$$\phi P = \frac{\sigma a}{\pi \varepsilon_0}$$

ans) Consider 2 concentric circles having radius 'r' & r+dr (here r<a) lying in the plane of disc. The charge 'dq' between these circles is located at a distance r' from P where

$dq = \sigma\, 2\pi r\, dr$ & $r' = \sqrt{r^2 + z^2}$ (Pythagoras)

$$r'^2 = r^2 + z^2$$

$$2r'\,dr' = 2r\,dr$$

$$d\phi_p = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'} = \int \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, 2\pi \cancel{r}\,\cancel{dr}\; dr'}{\cancel{r}\,\cancel{dr}}$$

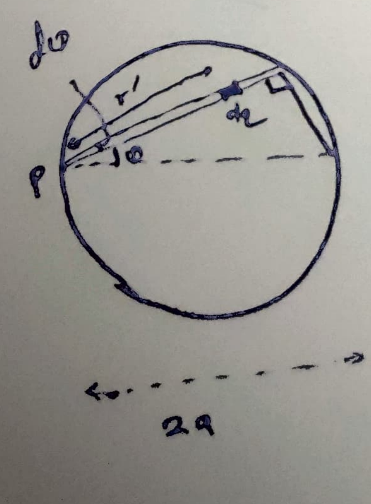
$$\phi_1 = \int_{z}^{\sqrt{a^2+z^2}} \frac{\sigma\, 2\pi\; dr'}{\cancel{4\pi}\varepsilon_0} =$$

when
$r = 0$
$r' = z$
$r = a$
$r' = \sqrt{a^2+z^2}$

$$= \frac{\sigma}{2\varepsilon_0}\Big[r'\Big]_{z}^{\sqrt{a^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2} - z\right)$$

(b) considering a point . line element of charge 'dq' lying inside the shaded wedge at a distance r' from P



so $dq = \sigma \times (r' d\theta)\, dr'$

$$d\phi_p = \frac{1}{4\pi\epsilon_0}\,\frac{dq}{r'}$$

$$\cancel{R = 2a\cos\theta}$$

$$\cancel{\cos\theta = \frac{R}{2a}}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{dq}{r'} = \frac{\sigma R\, d\phi}{4\pi\epsilon_0}$$

where $R = 2a\cos\theta$

$$\cos\theta = \frac{R}{2a}$$

so $d\phi_p = \dfrac{\sigma a\cos\theta\, d\phi}{2\pi\epsilon_0}$

$$\phi_p = \frac{\sigma a}{2\pi\epsilon_0}\int_{-\pi/2}^{\pi/2} \cos x\, d\theta = \frac{\sigma a}{\cancel{4}\pi\epsilon_0}\cancel{2}$$

$$= \frac{\sigma q}{\pi\epsilon_0}$$