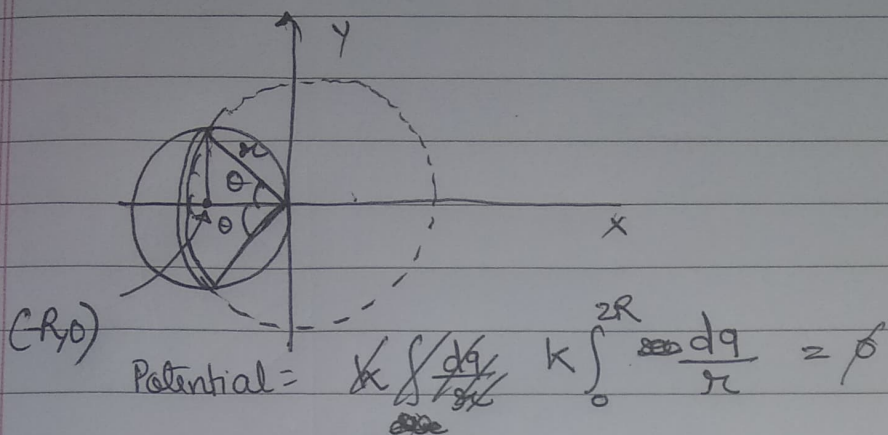
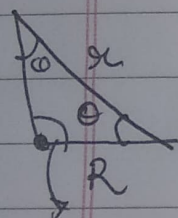


Find the potential ϕ at the edge of a disk (a thin disc) of radius R , carrying uniformly distributed charge of surface density σ



$$\frac{dq}{r} = \frac{\sigma dA}{r} \quad dA = \cancel{2\pi r dr} \quad \sigma \frac{2\pi r dr}{r} = 2\sigma \pi dr$$



when $x = 0$, $\phi = k \int_0^{2R} 2\sigma \pi dr$

$$\pi - 2\theta \quad \frac{\sin(\pi - 2\theta)}{r} = \frac{\sin \theta}{R} \Rightarrow r = 2R \cos \theta$$

$$dr = 2R(-\sin \theta) d\theta$$

when $x = 0$, $\theta = \pi/2$

when $x = 2R$, $\theta = 0$

$$\phi = k \int_{\pi/2}^0 2\sigma \pi (-2R \sin \theta) d\theta = -4R\sigma k \int_{\pi/2}^0 \sin \theta d\theta$$

using integration by parts $\int_{\pi/2}^0 \sin \theta d\theta = \left[-\cos \theta \right]_{\pi/2}^0 = [-1 - 0] = -1$

$$\phi = -4R\sigma k [-1] = 4R\sigma k = \frac{\sigma R}{\epsilon_0}$$