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Physics Tutorial Problem - By ANAND KARTHIK (2021093)

Q) Consider 2 electric fields in different co-ordinate spaces. Only one of these is an electrostatic field while the other one is not. Find the respective types and further find potential for the applicable case using origin as reference point. Check your answer by computing ∇V.

(k is suitable constant)
$$\begin{bmatrix} \vec{E}_1 = k(xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}) \\ \vec{E}_2 = k(y^2\,\hat{x} + (2xy+z^2)\,\hat{y} + 2yz\,\hat{z}) \end{bmatrix} \text{—(i)}$$

Ans/

One of the properties of an electrostatic field (produced by stationary charges) is that the curl of the electric field is zero. This implies that a potential exists. So, taking the curl for both cases.

$$\vec{\nabla}\times\vec{E}_1 = k\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}(3xz) - \frac{\partial}{\partial z}(2yz)\right) + \hat{y}\left(\frac{\partial}{\partial z}(xy) - \frac{\partial}{\partial x}(3xz)\right) + \hat{z}\left(\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial y}(xy)\right)$$

$$\therefore \vec{\nabla}\times\vec{E}_1 = k(-2y\,\hat{x} - 3z\,\hat{y} - x\,\hat{z}) \neq 0$$

$\therefore \vec{E}_1$ is not an electrostatic field (Maybe produced by moving charges)

Then

$$\therefore \vec{\nabla}\times\vec{E}_2 = k\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy+z^2 & 2yz \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}(2yz) - \frac{\partial}{\partial z}(2xy+z^2)\right) + \hat{y}\left(\frac{\partial}{\partial z}(y^2) - \frac{\partial}{\partial x}(2yz)\right) + \hat{z}\left(\frac{\partial}{\partial x}(2xy+z^2) - \frac{\partial}{\partial y}(y^2)\right)$$

$$\cancel{= \hat{x}(2y - 2y}$$

$$= k(\hat{x}(2z - 2z) + \hat{y}(0 - 0) + \hat{z}(2y - 2y))$$

$$= 0\hat{x} + 0\hat{y} + 0\hat{z} = 0$$

$\therefore \vec{\nabla} \times \vec{E_2} = 0$

Hence $\vec{E_2}$ is an electrostatic field. So, a potential exists.

$\Rightarrow V = -\int \vec{E_2} \cdot \vec{dl}$

where $\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$$V = -\left[k\int\left(y^2 dx + (2xy + z^2) dy + 2yz\,dz\right)\right]$$

$$V = -k\left[xy^2 + \frac{\cancel{2}xy^2}{\cancel{2}} + yz^2 + \frac{\cancel{2}yz^2}{\cancel{2}}\right] + c$$

$$\boxed{V = -2\lambda\left[xy^2 + yz^2\right]} \quad \text{where } 2\lambda = k$$

This is the expression for potential at a point (x, y, z) in the co-ordinate space of $\vec{E_2}$.

Now, to verify, wkt $E = -\nabla V$

(Since $\vec{\nabla} \times \vec{E} = 0$, $\vec{E}$ is a gradient of some scalar V)

$$\therefore E = -(-2\lambda)\left[\frac{\partial}{\partial x}(xy^2 + yz^2)\hat{i} + \frac{\partial}{\partial y}(xy^2 + yz^2)\hat{j} + \frac{\partial}{\partial z}(xy^2 + yz^2)\hat{k}\right]$$

$$= 2\lambda\left[y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\,\hat{k}\right]$$

$$\vec{E_2} = k\left(y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\,\hat{k}\right) - \text{(ii)}$$

So, eqn (i) = (ii) $\therefore$ Result is verified