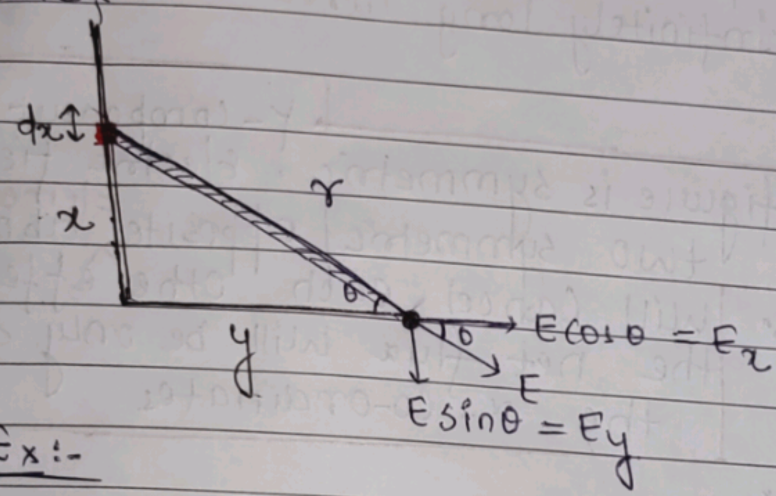
19). A Very long uniformly charged thread oriented Along the axis of a Circle of Radius 'R' rest On its Centre with one of the ends. The charge Of the thread per unit length is equal to λ. Find the flux of the Vector $\underline{E}$ across the Circle Area.

From the result described below we will proceed to find the flux of Vector E across the Circle area.

Semi Infinite



$r = \sqrt{x^2+y^2}$

$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$

$\boxed{r = \frac{y}{\cos\theta}}$

Electric field in X-dirn

E_x:-

$\tan\theta = \frac{x}{y}$

Differentiating w.r.t to 'θ' and 'x' with y = constant.

$\Rightarrow$ ~~Charge in x-~~

Charge density = 'λ'

$\boxed{dq = \lambda \cdot dx}$ Assuming charge distribution is uniform.

$dx = y \cdot \sec^2\theta \cdot d\theta$.

$\boxed{dq = \lambda y \sec^2\theta \cdot d\theta}$

For a continous charge distribution

$$E = \int \frac{k\,dq}{r^2}$$

$$E_x = \int_0^{\pi/2} \frac{k\lambda y \sec^2\theta \cdot d\theta}{\frac{y^2}{\cos^2\theta}} \cos\theta$$

$$E_x = \int_0^{\pi/2} \frac{k\lambda \cancel{y} \cancel{\sec^2\theta} \cdot d\theta \cdot \cancel{\cos^2\theta} \cdot \cos\theta}{y^{\cancel{2}}}$$

$$E_x = \int_0^{\pi/2} \frac{k\lambda}{y} \cos\theta \cdot d\theta$$

$$\Rightarrow \frac{k\lambda}{y} \int_0^{\pi/2} \cos\theta \cdot d\theta$$

$$\boxed{\begin{array}{c}\text{When } x \to \infty \\ \theta \to \pi/2\end{array}}$$

$$\Rightarrow \frac{k\lambda}{y} \sin\theta \left[\pi/2 - 0\right]$$

$$\Rightarrow \frac{k\lambda}{y}(+1) = \frac{+k\lambda}{y} \Rightarrow \frac{k\lambda}{y} \Rightarrow \boxed{\frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{y}}.$$

III

$$E_y = \int_0^{\pi/2} \frac{k \cdot \lambda \cancel{y} \cancel{\sec^2\theta} \cdot d\theta}{\frac{y^{\cancel{2}}}{\cancel{\cos^2\theta}}} \cdot (\sin\theta)$$

$$\Rightarrow \int^{\pi/2} \frac{k \cdot \lambda \sin\theta \cdot d\theta}{y}$$

$$\Rightarrow \frac{k\lambda}{y} \int_0 \sin\theta$$

$$\Rightarrow \frac{k\lambda}{y} \left[\cos(\pi/2) - \cos(0)\right]$$

$$\Rightarrow \boxed{\frac{-1}{4\pi\epsilon_0} \cdot \frac{\lambda}{y}}$$

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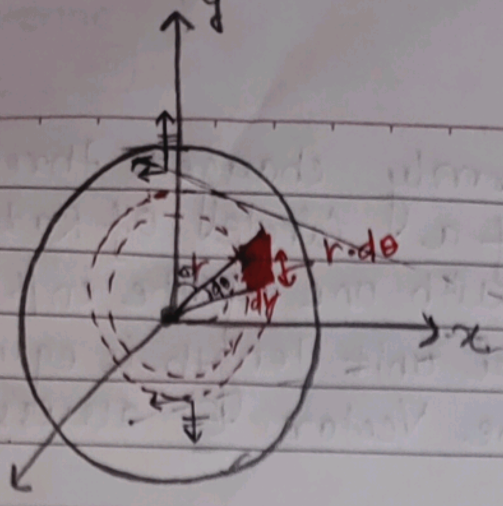

∞ Thread

*) $\boxed{\phi = \int \vec{E} \cdot \vec{da}}$ Normal way of finding flux.

*) Area ⇒ r·dθ·dr.

For a Infinitely long Thread

$$\vec{E} = \frac{k\lambda}{r}$$

* Since the figure is symmetric, Y-component of electric field at the two symmetric opposite Elemental Area about the X-Axis will cancel each other effect so the net flux will be only due to the x-co-ordinates.

$E_Y = 0$

$E_X = -2E_x$

$E_x = \frac{k\lambda}{r} \Rightarrow -\frac{2k\lambda}{r}$ Total electric field passing in x-dirn.

$$\boxed{E_x = \frac{-2k\lambda}{r}}$$

$da = dr.(r.d\theta)$

$$\boxed{\vec{da} \Rightarrow r \cdot dr d\theta}$$

$$\phi = \int \frac{-2k\lambda}{\cancel{r}} \cdot \cancel{r} \cdot dr \cdot d\theta$$

$$\Rightarrow -2k\lambda \int_0^{\pi} d\theta \int_0^{R} dr$$

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$$\Rightarrow \frac{-2k\lambda}{}(\pi-0)(R-0)$$

$$\Rightarrow \frac{-2\lambda}{4\cancel{\pi}\epsilon_0}(\cancel{\pi})(R)$$

$$\Rightarrow \frac{-\lambda R}{2\epsilon_0}$$

$$\boxed{\phi = \frac{-\lambda R}{2\epsilon_0}}$$

Appendix

$|E_x| = |E_y|$ Prooved below.