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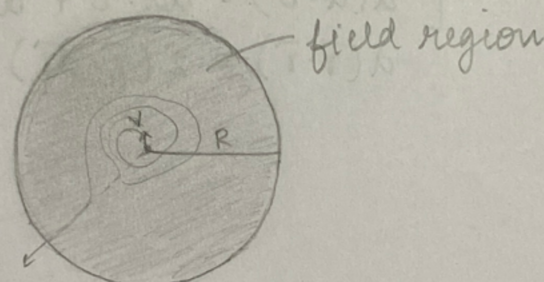
Batch 5

Q) A circularly symmetrical magnetic field (B depends only on the distance from the axis), pointing perpendicular to the page, occupies the shaded region. If total flux ($\int B \cdot da$) is zero, show that a charged particle that starts out at the center will emerge from the field region on a radial path. On the reverse trajectory, a particle fired at the center from outside will hit its target, though it may follow a weird route getting there.



→ Given: $\int \vec{B} \cdot da = 0$ (flux is 0)

charge particle starts out at the center.

To show: charge will emerge out the field on a radial path

soln: Charge is moving in magnetic field (B) where $B \perp V$

Force experienced by charge (F) = $q(V \times B)$

$\therefore F \perp B$ and $F \perp V$

As $F \perp V$, it changes only the direction of motion.

Particle emerging from the region radialy $\Leftrightarrow \vec{L} = \vec{r} \times m\vec{v} = 0$

(where $\vec{L}$ = angular momentum) (as $\vec{r}$ & $\vec{v}$ in same direction)

$$\vec{L} = \int \frac{d\vec{L}}{dt}\, dt$$

$$\vec{L} = \int \vec{N}\, dt \qquad (N = \text{Torque})$$

$$\vec{L} = \int (\vec{r} \times \vec{F})\, dt$$

$$\vec{L} = \int \vec{r} \times q(\vec{v} \times \vec{B})\, dt$$

$$\vec{L} = q \int \vec{r} \times (d\vec{l} \times \vec{B}) \qquad [\because \vec{v}\, dt = d\vec{l}]$$

$$\vec{L} = q \int (\vec{r} \cdot \vec{B})\, d\vec{l} - (\vec{r} \cdot d\vec{l})\, \vec{B}$$

$$\vec{L} = q \int -(\vec{r} \cdot d\vec{r})\, \vec{B} \qquad [\vec{r} \cdot \vec{B} = 0 \text{ as } \vec{r} \perp \vec{B}]$$

$$\vec{L} = q \int -\frac{1}{2}\, d(\vec{r} \cdot \vec{r})\, \vec{B} \qquad \left[\begin{array}{l} d(\vec{a} \cdot \vec{b}) = d\vec{a} \cdot \vec{b} + \vec{a} \cdot d\vec{b} \\ d(\vec{r} \cdot \vec{r}) = 2(\vec{r} \cdot d\vec{r}) \end{array} \right]$$

$$\vec{L} = q \int -\frac{1}{2}\, d(r^2)\, \vec{B}$$

$$\vec{L} = -\frac{q}{2} \int \vec{B}\, 2r\, dr$$

$$\vec{L} = -\frac{q}{2\pi} \int \vec{B}\, 2\pi r\, dr$$

$$\vec{L} = -\frac{q}{2\pi} \int \vec{B}\, da \qquad [da = 2\pi r\, dr]$$

a = area

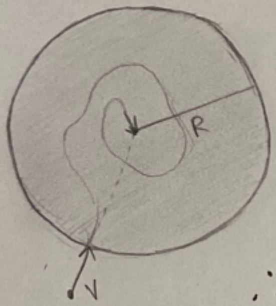
$$\vec{L} = -\frac{q}{2\pi} \Phi \qquad [\Phi = \int \vec{B}\, da = \text{flux}]$$

$$\vec{L} = 0 \qquad [\Phi = 0 \text{ given}]$$

$\vec{r} \times m\vec{v} = 0 \Rightarrow \vec{r}$ and $\vec{v}$ in same direction

$\therefore$ particle emerges from field region on a radial path.

Now, when the trajectory is reversed



When charged particle enters the field region it will experience force similer to the previous case i.e $F = q(v \times B)$ where $F \perp v$ & $F \perp B$

$\therefore$ Angular momentum $(\vec{L}) = \int \vec{r} \times \vec{F}\, dt = 0$

$\vec{L} = \vec{r} \times m\vec{v} = 0 \Rightarrow \vec{r}$ and $\vec{v}$ in same direction

i.e the particle will go along a radial line

$\therefore$ it will hit the center.

i.e it will successfully hit the target.