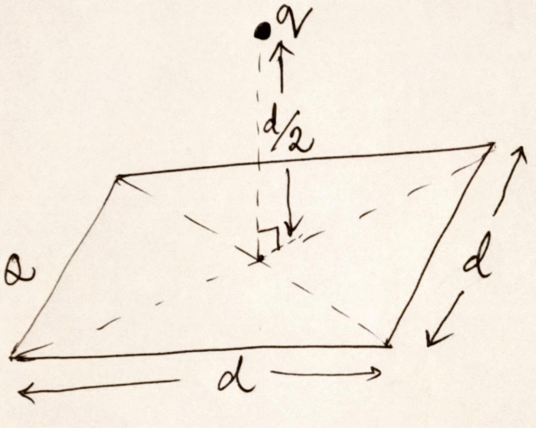
Q. A square of side 'd', made from a thin insulating plate, is uniformly charged and carries a total charge of Q. A point charge 'q' is placed on the symmetrical normal axis of the square at a distance of '$d/2$' from the plate. How large is the force acting on the point charge?



Solution :- According to Newton's 3^{rd} law, the insulating plate acts on the point charge with a force of the same magnitude (but opposite direction) as the point charge does on the plate. We calculate the magnitude of this latter force.

Divide the plate (notionally) into small pieces, and denote the area of the ith piece by ΔA_i.

Because of the uniform charge distribution, the charge on this small piece is,

$$\Delta Q_i = \frac{Q}{d^2} \cdot \Delta A_i.$$

So, electric force acting on it is, $F_i = E_i \Delta Q_i$, where E_i is the magnitude of the electric field produced by q at the position of the small piece.

The force acting on the insulating plate, as a whole, can be calculated as the vector sum of the forces acting on the individual pieces of the plate.

Because of the axial symmetry, the net force is perpendicular to the plate, and so it is sufficient to sum the perpendicular components of the forces:

$$F = \sum_i F_i \cos\theta_i = \sum_i E_i \Delta Q_i \cos\theta_i$$

$$= \sum_i E_i \frac{Q}{d^2} \Delta A_i \cos\theta_i$$

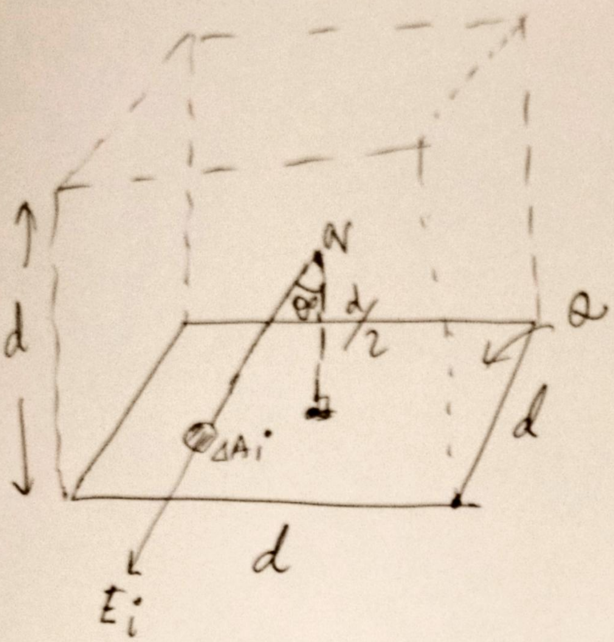
$$= \frac{Q}{d^2} \sum_i E_i \Delta A_i \cos\theta_i$$

where, θ_i is the angle between the normal to the plate and the line that connects the point charge to the ith piece of it.

The sum in the given expression is nothing other than the electric flux through the square sheet produced by 'q'.

$$\Psi = \sum_i E_i \Delta A_i \cos\theta_i$$

$$\therefore \quad F = \frac{Q}{d^2} \cdot \Psi \quad \text{——} \quad (1)$$



Let us imagine that a cube of edge 'd' is constructed symmetrically around the point charge. Then, the distance of the point charge from each side of the cube is just 'd/2'. According to Gauss' law, total flux through the six sides $= \frac{q}{\varepsilon_0}$

$\therefore$ Flux through one single side, $\varphi = \frac{q}{6\varepsilon_0}$

From ① $\Rightarrow$

$$F = \frac{Q}{d^2} \cdot \frac{q}{6\varepsilon_0} = \frac{Qq}{6\varepsilon_0 d^2}$$ //