The charge densities in a current-carrying wire & the PINCH EFFECT

Introd^n → All magnetic fields and magnetic forces acting on an electrical charge can be understood in terms of electric fields & the forces felt by the charge while in its own rest frame. [The rest frame of a body is the reference frame in which its own velocity is zero even though if it is actually not at rest]. This idea helps us to describe magnetic phenomena as relativistic effects.

- We know that the electromagnetic force experienced by any charged particle can be stated using the Lorentz force law i.e. $\boxed{\bar{F} = q[\bar{E} + \bar{v} \times \bar{B}]}$ (1)

→ We just need to know the $\bar{E}$ & $\bar{B}$ vector fields acting on this charge q to get the force F.

Theory → In a current-carrying wire, we usually view/model a superposition of 2 uniform charge densities : one positive (the rigid ion lattice) and one negative (the mobile conduction electrons).

→ By transforming these 2 charge densities into the rest frame of a test charge q moving relative to the wire, it is possible to show that the magnetic force felt by q in the rest frame of the positive ions of the wire, is actually equivalent to an electrostatic force in the rest frame of q.

- It may have occured to you that if 2 currents running in parallely-placed wires attract, the current within a single wire should contract itself into a tiny concentrated stream along the axis. Yet in practice the current typically distributes itself quite uniformly over the wire. How does that happen ???

→ Well the answer to this lies in the fact that the mobile charges i.e. negative electrons get concentrated around the axis & that generates an electric field which repels any further accumulation of these electrons.

→ In order to establish equilibrium, the electric repulsion on a mobile electron e must balance the magnetic attraction produced due to motion of these charges.

$$\therefore \quad \bar{F}_{net} = q[\bar{E} + \bar{v} \times \bar{B}] = -e[\bar{E} + \bar{v} \times \bar{B}] = 0 \quad \Rightarrow \quad \bar{E} = -(\bar{v} \times \bar{B})$$

- If we consider a 3-dimensional wire, the distribution of charge with radius inside this wire is not uniform & the wire has separate surface & volume charge densities.

And this non-uniformity in the radial charge distribution in a current-carrying wire is a proof of the same fact discussed above - the mobile charge carriers concentrate towards

Date: / /
Page No.

The centre of the wire. In the rest frame of the positively charged ion lattice of the wire ~~electrons that make up the current~~, this phenomenon is described as the "pinch effect" or a "self-induced Hall effect".

- By simple mathematics, we can show that if ρ_+ is the charge density of the positive charges and ρ_- is the charge density of the negative charges, then $\rho_- = -\rho_+\gamma^2$, where $\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}}$, where v: the speed at which the negative charges are moving & c: speed of light

- Consider a cylinderical wire of radius a made up of rigid positive ion lattice and a nonrigid distribution of electrons which constitute the current.

→ In the reference frame of the rigid positive ion lattice (S), this electron distribution appears to be contracted by a factor $\gamma = \sqrt{1-V_d^2/c^2}$ where V_d: drift velocity

→ On the other hand, in the reference frame of the moving electrons (S'), the positive ion lattice also appears contracted by exactly the same factor.

Quantitative Analysis → Let ρ_+ be the charge density of positive ion lattice & ρ_- be that of the mobile electrons

In S', $\rho_+' = \gamma\rho_+$ ∵ The length of the lattice has got contracted. This is called Lorentz Contraction

Now, in order for the net volume charge density in S' to be zero,

$$\rho_-' = -\rho_+' = -\gamma\rho_+ \quad \text{——— ②}$$

Here, $\rho_- = \gamma\rho_-' = -\gamma^2\rho_+$ [∵ in S, the negative charge density has enhanced due to Lorentz contraction]

Net charge density in S $= \rho = \rho_+ + \rho_- = \rho_+(1-\gamma^2)$

$$\therefore \rho = -\rho_+\gamma^2\beta_d^2, \quad \text{where } \beta_d = V_d/c$$

Result → Thus, in this naive model, the mobile negative charges fill a smaller inner cylinder, leaving a shell of positive (stationary) charge at the outside. But since $v \ll c$, the effect is extremely small.

That's why in normal current-carrying wires with typical velocities, this "pinch effect" is not very significant.

Date : / /
Page No.

Add-ons → However, in electric charge distributions where both the positive and negative charges are capable of moving, the situation is different.

One of the most exciting examples of this is perhaps the 4^{th} state of matter – **PLASMA** where both positive & negative charges can move & thus the pinch effect is very significant. The theory regarding what happens in plasmas, while keeping in mind both the electrical & magnetic fields, was given by W.H. Bennett in his work : Magnetically self-focusing Streams (1934).

Acknowledgements →

1) David J. Griffiths : 'Introduction to Electrodynamics', ~~4~~ 4^{th} edition, Problem-5.40 Pg-256

2) Richard P. Feynman, R.B. Leighton, M. Sands : 'The Feynman Lectures of Physics' Vol.2, Sec-13.6

3) E.M. Purcell : 'Electricity & Magnetism' pp. 172-178

4) M.A. Matzek & B.R. Russell, "On the transverse electric field within a conductor carrying a steady current", Am. J. Phys. 36, 905-907 (1968)

5) P.C. Peters, "In what frame is a current-carrying conductor neutral?", Am. J. Phys. 53, 1165-1169 (1985)

6) 'The Bennett Pinch revisited', J.E. Allen, 37^{th} EPS conference on Plasma Physics.

THANK YOU