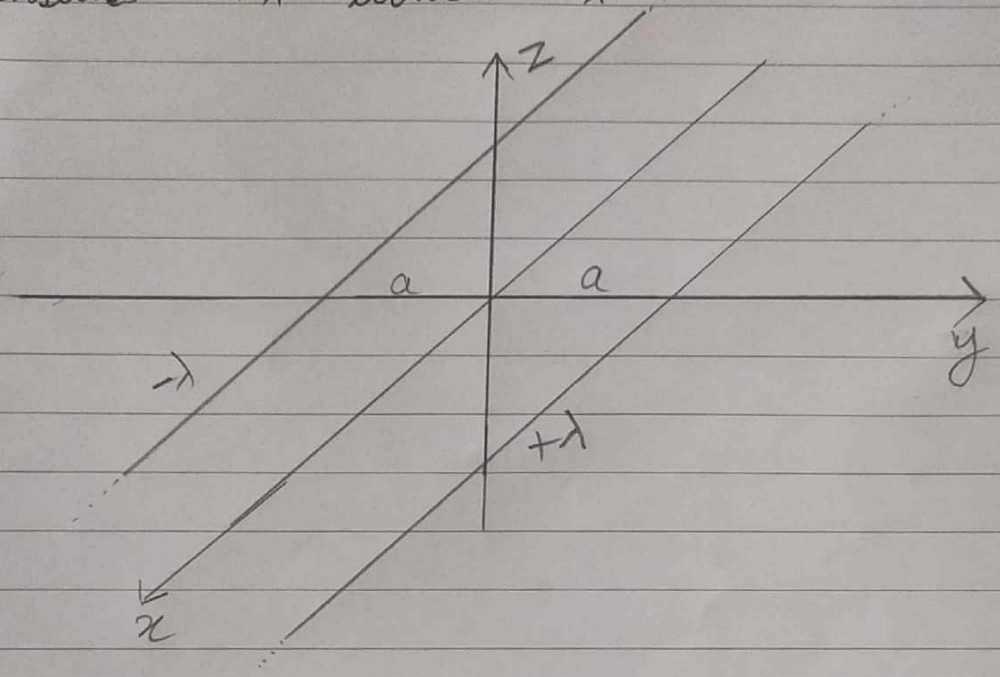
two infinitely long wires running parallel to the x axis carry uniform charge densities $+\lambda$ and $-\lambda$

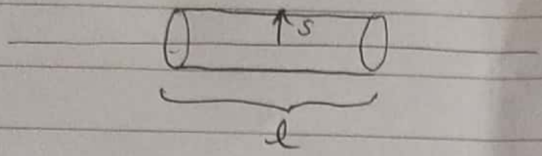


a) Find the potential at any point (x, y, z), using the origin as the reference

b) Show that the equipotential surfaces are circular cylinders and locate the axis and radius of the cylinder corresponding to a given ~~point~~ potential V_0.

electric flux

$$\phi = \oint E \cdot da$$



$$\phi = E \cdot 2\pi s \cdot l$$

gauss law, $\phi = \dfrac{Q_{enc}}{\varepsilon_0}$

$$\phi = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 s} \hat{s}$$

General case, we take reference point as a and point where we have to calculate potential as s.

$$V(a) - V(s) = + \int_a^s \frac{\lambda}{2\pi\varepsilon_0 s} \cdot ds$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s}{a}\right)$$

$$V(s) - V(a) = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s}{a}\right)$$

Potential at $\lambda +$

$$V_+ = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_+}{a}\right)$$

Potential at $\lambda -$,

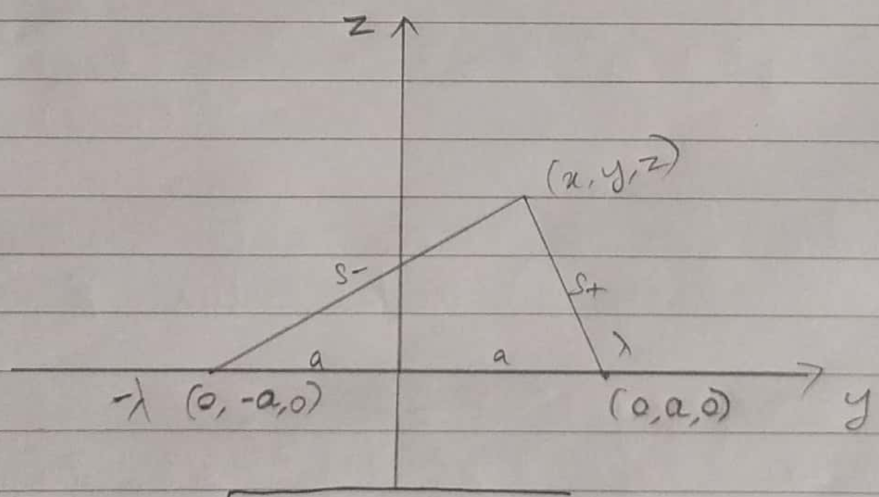
$$V_- = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_-}{a}\right)$$

s_+ - distance from $\lambda +$

s_- - distance from $\lambda -$

$$V = (V_+) + (V_-)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_-}{s_+}\right)$$



$$s_+ = \sqrt{(y-a)^2 + (z)^2}$$

$$s_- = \sqrt{(y+a)^2 + z^2}$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\sqrt{\frac{(y+a)^2+z^2}{(y-a)^2+z^2}}\right)$$

Equipotential surface - potential remains constant at all points on an equipotential surface.

$$\cancel{K} \; e^{2\pi\varepsilon_0 V_0/\lambda} = \sqrt{\frac{(y+a)^2+z^2}{(y-a)^2+z^2}}$$

$$K = e^{4\pi\varepsilon_0 V_0/\lambda} = \frac{(y+a)^2+z^2}{(y-a)^2+z^2}$$

$$y^2 + 2ay + a^2 + z^2 = Ky^2 + Ka^2 + Kz^2 - 2Kay$$

$$(K-1)y^2 + (K-1)a^2 + (K-1)z^2 - 2ay(K+1) = 0$$

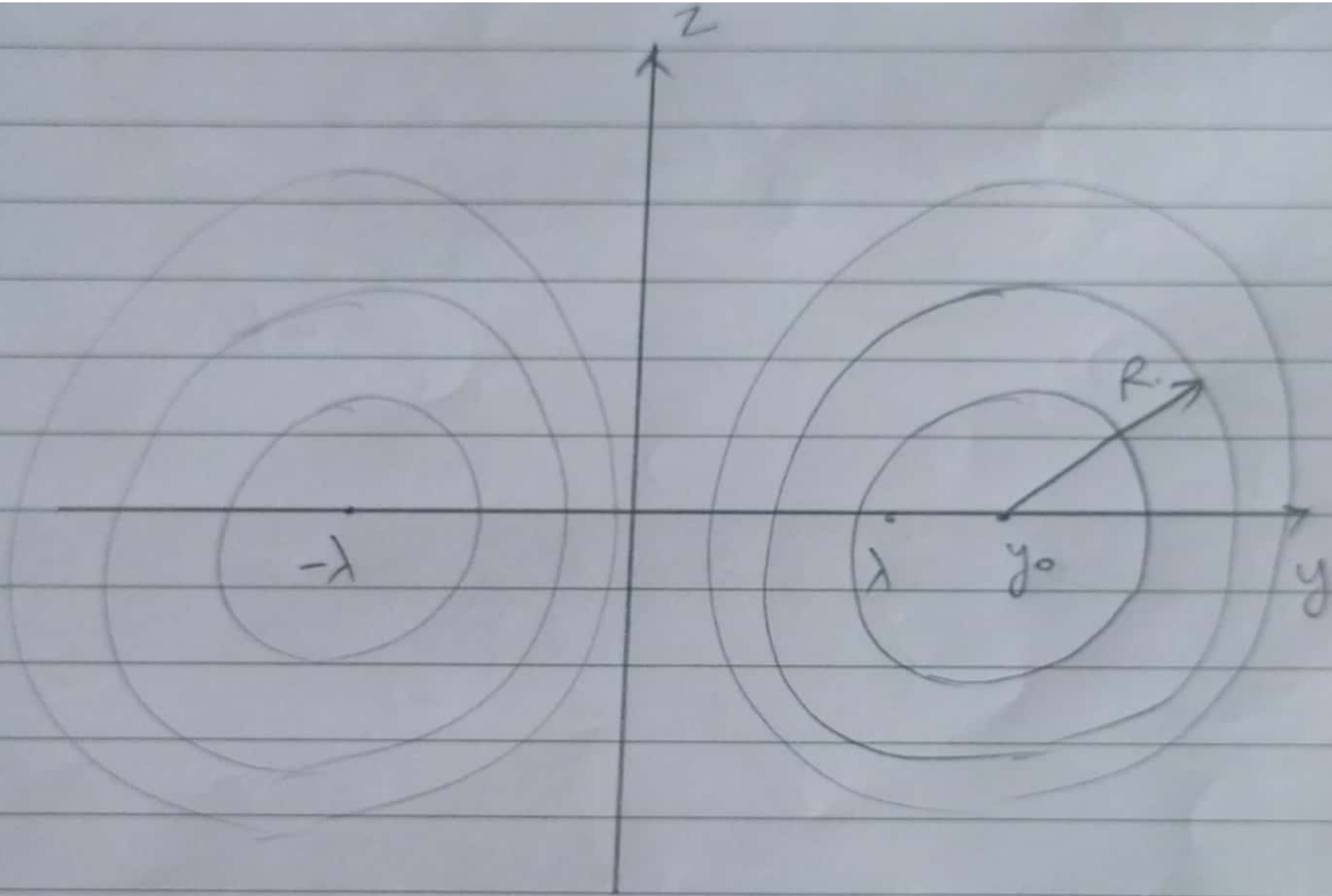
$$y^2 + a^2 + z^2 - 2ay\frac{(K+1)}{(K-1)} = 0 \quad \rightarrow \text{eqn } ①$$

Equation for a circle with centre at $(y_0, 0)$ and radius R.

$$(y-y_0)^2 + z^2 = R^2$$

eqn ② $\rightarrow$ $$y^2 + z^2 + y_0^2 - R^2 - 2yy_0 = 0$$

comparing eqn ① and ②,



z
R
$-\lambda$
λ
y_0
y

$$y_0 = \frac{a(K+1)}{(K-1)}$$

$$a^2 = y_0^2 - R^2$$

$$R^2 = y_0^2 - a^2$$

$$R^2 = \frac{a^2(K+1)^2}{(K-1)^2} - a^2$$

$$= \frac{a^2K^2 + a^2 + 2a^2K - (a^2K^2 + a^2 - 2a^2K)}{(K-1)^2}$$

$$= \frac{4a^2K}{(K-1)^2}$$

$$R = \frac{2a\sqrt{K}}{|K-1|}$$

$$K = e^{4\pi\varepsilon_0 V_0/\lambda}$$

$$y_0 = \frac{a\left(e^{4\pi\varepsilon_0 V_0/\lambda} + 1\right)}{\left(e^{(4\pi\varepsilon_0 V_0/\lambda)} - 1\right)}$$

$$R = \frac{2a \; e^{2\pi\varepsilon_0 V_0/\lambda}}{e^{4\pi\varepsilon_0 V_0/\lambda} - 1}$$

$$= \frac{2a}{e^{2\pi\varepsilon_0 V_0/\lambda} - e^{-2\pi\varepsilon_0 V_0/\lambda}}$$

circles in the z-y plane is proved from this.

$\therefore$ in 3-D equipotential surfaces occur in circular cylinder in $\mathbb{R}^3$.

radius of cylinder - R

axis location - y_0

$$R = \frac{2a}{e^{2\pi\varepsilon_0 V_0/\lambda} - e^{-2\pi\varepsilon_0 V_0/\lambda}}$$

$$y_0 = \frac{a\left(e^{4\pi\varepsilon_0 V_0/\lambda} + 1\right)}{\left(e^{4\pi\varepsilon_0 V_0/\lambda} - 1\right)}$$