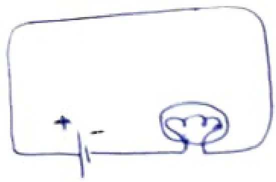
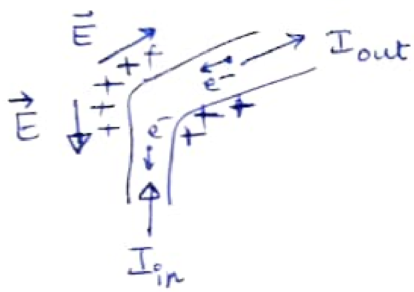
Induced Electric field

Electromotive force :-



Question: Current is nearly same in all parts of the loop. ~~charges~~
How is this possible when the driving force is only the battery?
Who pushes the current in the rest of the circuit.

Ans: If current is not the same (nearly same)
Then charges would pile up somewhere.



Due to piled up charge there is an electric field, such as to even out the flow.

The $\vec{E}$ opposes I_{in} and supports I_{out} until $I_{in} \approx I_{out}$

$\therefore$ There are two forces (per unit charge) involved in driving current around the circuit.

$f_s \rightarrow$ the source (per unit charge)

$E \rightarrow$ Electrostatic force (per unit charge)

$\therefore \vec{f} = \vec{f_s} + \vec{E}$; $\vec{f}$ = Total force per unit charge

→ we ~~define~~ line integral of $\vec{f}$ as $\mathcal{E}$.

$$\mathcal{E} = \oint \vec{f} \cdot dl$$

$$= \oint \vec{f_s} \cdot dl + \underbrace{\oint \vec{E} \cdot \vec{dl}}_{\text{zero}}$$

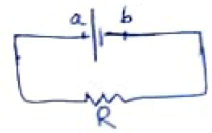
zero $\because \vec{E} \rightarrow$ purely electrostatic force per unit charge

$$\mathcal{E} = \oint \vec{f_s} \cdot dl$$

$\mathcal{E} \rightarrow$ was given the name electromotive force or emf.

→ P.T.O

In an ideal source of emf, net force on charges is zero.
↓
(resistanceless battery)



$$\vec{E} = -\vec{f_s}$$

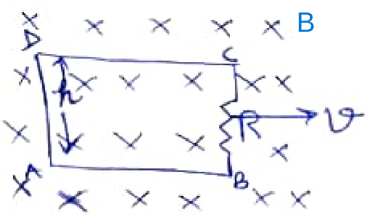
$$V = -\int_a^b \vec{E}\cdot d\ell = \int_a^b \vec{f_s}\cdot d\ell = \oint \vec{f_s}\cdot \vec{d\ell} = \mathcal{E}.$$

(because; $\vec{f_s} = \vec{0}$ outside the source)

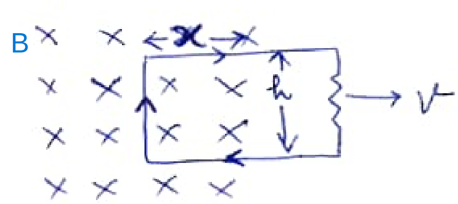
∴ function of battery ⟹ Establish and maintain Voltage difference equal to emf

$\mathcal{E} \rightarrow$ interpreted as work done per unit charge.

Motional emf:



At ~~any~~ some instant



$$\mathcal{E} = \oint f_s \cdot \vec{d\ell}$$

$$= \oint f_{mag} \cdot \vec{d\ell}$$

$F_{mag} = qvB \quad (v \perp B)$

$\Rightarrow f_{mag} = vB.$

$\mathcal{E} = \oint (vB)\, d\ell \quad (\because \vec{v}\times\vec{B} \parallel \vec{d\ell})$

$\mathcal{E} = vBh. \quad — (1)$

Current produced only due to motion of AD.

$$\Phi = \int \vec{B}\cdot\vec{da}$$

$$= \int B\, da \quad (\vec{da} \parallel \vec{B})$$

$$= B\int da = BA$$

$$= B\, xh. \quad (A = \text{area of loop inside } \vec{B})$$

$\dfrac{d\phi}{dt} = Bh\dfrac{dx}{dt} = -Bhv \; — (2) \; (x \text{ is decreasing})$

$\Rightarrow \mathcal{E} = -\dfrac{d\Phi}{dt} \rightarrow$ rule of flux / Faraday's law.

The proof is not rigourous.

~~Electromagnetic induction:~~

Faraday's law

Faraday's expt.

Exp 1: Pulled a loop ~~to~~ through a magnetic field

Exp 2: Keep loop still, & moved the magnet.

Exp 3: Kept both at rest, but change the strength of field.

In all 3 cases, he observed a current flow in the loop.

Exp 2 & 3 suggest that a changing magnetic field induces Electric field.

$$\text{Emf} = \varepsilon = \oint \vec{E}\cdot d\ell \; ; \quad \vec{E} = \text{Induced electric field.}$$

$\downarrow$ not electrostatic field.

$$= -\frac{d\phi}{dt}$$

$$\oint \bar{E}\cdot \vec{d\ell} = -\frac{d}{dt}\int \vec{B}\cdot \vec{da}$$

integral form of Faraday's law

$\vec{B}$ = changing magnetic field.

$$\oint \vec{E}\cdot \vec{d\ell} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot \vec{da}$$

$\vec{B}$ is function of time and some other variable like distance

$$\int_S (\vec{\nabla}\times\vec{E})\cdot\vec{da} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot \vec{da}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

$\rightarrow$ Faraday's law in differential form

P.T.O

Induced $\vec{E}$:

If $\vec{E}$ is pure faraday field (Exclusively due to changing $\vec{B}$; with $\rho = 0$)

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{and} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

$\downarrow$

($\vec{E}$ is not due to charge distribution but due to changing $\vec{B}$)

Analogue to

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{and} \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

$$= \frac{1}{4\pi} \int \frac{(\mu_0 \vec{J}) \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau$$

Analoguesly

$$\vec{E} = \frac{1}{4\pi} \int \frac{-\frac{\partial \vec{B}}{\partial t} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau$$

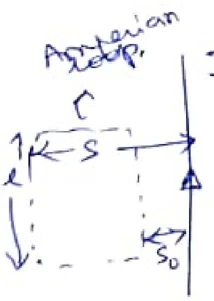
$$= -\frac{1}{4\pi} \frac{\partial}{\partial t} \int \frac{\vec{B} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau$$

> Question 1 continued...
>
> we observe as $s \to \infty$; $E \to \infty$.
>
> This occured due to overstepping of limit of quasistatic approx.
>
> → for large distance B not only depends of I but also on the current as it was at some earlier time

With this analogue, whatever symmetric exploitation we did using Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ can be done for

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}.$$

Q1: An infinitely long straight wire carries a slowly varying current $I(t)$. Determine induced electric field as a function of distance s from wire



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{a}\right)_s$$

$$(E(s_0) - E(s))\, l = \frac{-d}{dt} \int_{s_0} \frac{\mu_0 I}{2\pi s} l\, ds = -\frac{\mu_0 l}{2\pi} \frac{dI}{dt}(\ln(s) - \ln(s_0))$$

In general

$$\therefore E(s) = \left(\frac{\mu_0 l}{2\pi} \frac{dI}{dt} \ln s + K\right)$$