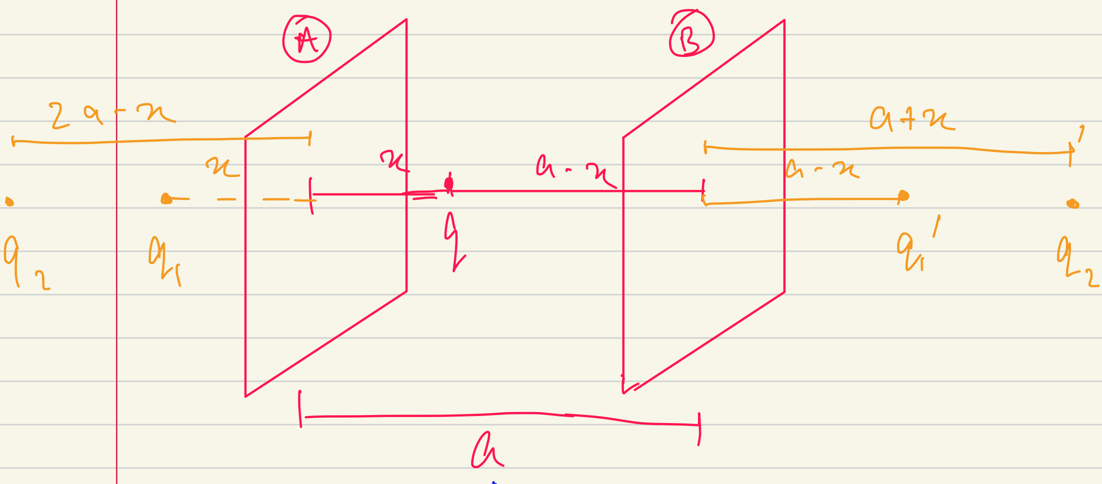
Physics Presentation

Two infinite parallel grounded conducting planes are held a distance *a* apart. A point charge *q* is placed in the region between them, a distance *r* from one plate. Find the force on *q*.



Taking inspiration from geometrical optics, we can come up with a trial solution as shown. There will be an infinite number of image charges formed.

To satisfy boundary condition, we check if $V = 0$ at plates A & B at any point.

$$V = \frac{q}{\sqrt{x^2+y^2}} + \frac{q_1}{\sqrt{x^2+y^2}} + \frac{q_1'}{\sqrt{(2a+x)^2+y^2}} + \frac{q_2}{\sqrt{(2a-x)^2+y^2}} + \frac{q_2'}{\sqrt{(2a+x)^2+y^2}}$$

$$+ \frac{q_3}{\sqrt{(2a+x)^2+y^2}} + \frac{q_3'}{\sqrt{(4a-x)^2+y^2}} + \frac{q^4}{\sqrt{(4a-x)^2+y^2}} \; - - - -$$

This sol^n fits the eq^n.

$q_1 = q_1' = -q$ $\quad$ $q_3 = q_3' = -q_2 = -q$

$q_2 = q_2' = -q_1 = +q$

$q_{2n} = q_{2n}' = -q \quad [n = 0, 1, 2 \ldots]$

$q_{2n+1} = q_{2n+1}' = +q \quad [n = 0, 1, 2, \ldots]$

$$V = \frac{q}{\sqrt{x^2+y^2}} + \frac{-q}{\sqrt{x^2+y^2}} + \frac{-q}{\sqrt{(2a-x)^2+y^2}} + \frac{q}{\sqrt{(2a-x)^2+y^2}} \; - - -$$

$$= 0$$

Similarly, we can see that for plate B, at any point,

$V = 0$ by subsitituting

$x \longrightarrow a - x$

$q_i' \longrightarrow q_i$

the same solution fits.

To calculate force of charge q:

$$(\vec{F})_A = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{1}{(2a-x)^2} + \frac{1}{(2a+x)^2} - \frac{1}{(4a-x)^2} - - \ldots\right]$$

Force on q due to image charges behind A.

$$(\vec{F})_B = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2} + \frac{1}{(3a-x)^2} - \frac{1}{(3a+x)^2} \ldots\right]$$

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B$$