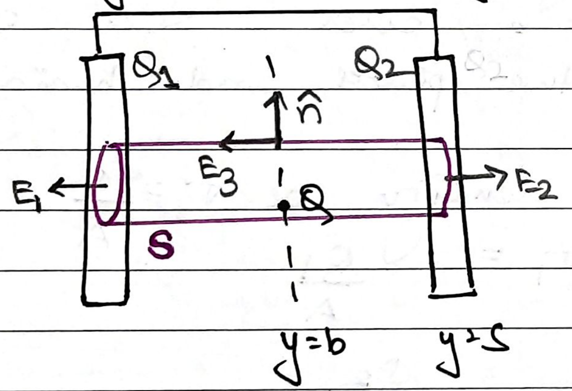
PHYSICS PRESENTATION

Q. Two parallel plates are connected by a wire so that they remain at the same potential. Let one plate coincide with the xz plane and the other be at $y = s$. s is much smaller than the dimensions of the plate. A point charge Q is located between the plates at $y = b$.

i) What is the magnitude of the total surface charge on the inner surface of both plates?

ii) What is the magnitude of the total surface charge on the inner surface of the individual plates?

iii) Analyze the result from part <ii> when $b \ll s$.



Let charge on the inner surface of the plates be Q_1 and Q_2, respectively.

Consider the Gaussian surface S

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\int \cancel{\vec{E_1} \cdot d\vec{a}}^{\,0} + \int \cancel{\vec{E_2} \cdot d\vec{a}}^{\,0} + \int \cancel{\vec{E_3} \cdot d\vec{a}}^{\,0} = \frac{Q + Q_1 + Q_2}{\varepsilon_0}$$

[$E_1 = E_2 = 0$, field inside a conductor $= 0$]

[$\vec{E_3} \perp^r d\vec{A}$]

$$Q_1 + Q_2 + Q = 0$$

$$\boxed{Q_1 + Q_2 = -Q} \quad \text{——— <i>}$$

Now, to calculate total charge on individual plates, we use the principle of superposition.
If we divide charge Q kept at $y = b$ into smaller charges, ~~plate~~ charge on each plate remains the same by superposition.

[To motivate this idea we can divide Q into $Q/2$ and $Q/2$, in the plane $y = b$. Each $Q/2$ will induce $Q_1/2$ and $Q_2/2$ on the respective plates. Total charge on plate 1 $= \frac{Q_1}{2} + \frac{Q_1}{2} = Q_1$. Similarly, for plate 2 it is Q_2.]

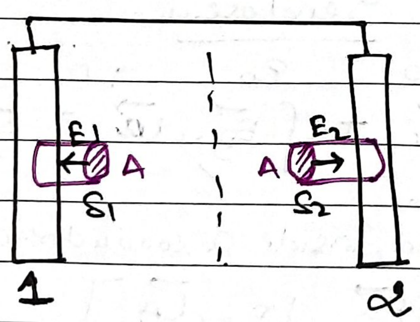
Thus, we can imagine spreading the charge Q uniformly in the plane $y = b$ over a sheet parallel to the plates and having the same area.

Let surface charge density be $\sigma = \frac{Q}{A}$.

For plate 1 $\quad \sigma_1 = \frac{Q_1}{A}$

For plate 2 $\quad \sigma_2 = \frac{Q_2}{A}$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{Q_1}{Q_2}$$



For 1

$$\oint E \cdot dA = \sigma_1 \frac{A}{\varepsilon_0}$$

$$= -E_1 \cancel{A} = \sigma_1 \frac{\cancel{A}}{\varepsilon_0} \Rightarrow E_1 = -\frac{\sigma_1}{\varepsilon_0}$$

Similarly for S_2,

$$E_2 = -\frac{\sigma_2}{\varepsilon_0}$$

But 1 and 2 are at the same potential

$$E_1(b) = E_2(s-b)$$

$$-\frac{\sigma_1}{\varepsilon_0}(b) = -\frac{\sigma_2}{\varepsilon_0}(s-b)$$

$$\frac{\sigma_1}{\sigma_2} = \frac{s-b}{b}$$

$$\Rightarrow \sigma_1 = \frac{s-b}{b} \cdot \sigma_2$$

$$\Rightarrow Q_1 = \frac{s-b}{b} \cdot Q_2$$

$$Q_1 + Q_2 = -Q$$

$$Q_2 \left\{ \frac{s-b}{b} + 1 \right\} = -Q$$

$$\boxed{\begin{aligned} Q_2 &= -\frac{b}{s} \cdot Q \\ Q_1 &= -\frac{(s-b)}{s} Q \end{aligned}}$$ —— ⟨ii⟩

for $b \ll s$

$$Q_2 \approx 0$$ —— ⟨iii⟩

$$Q_1 \approx -Q$$