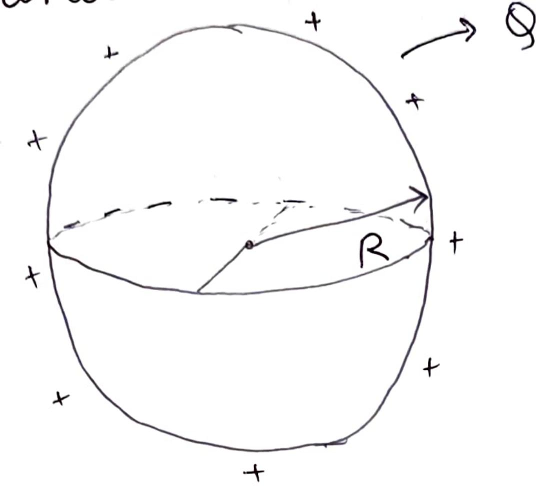
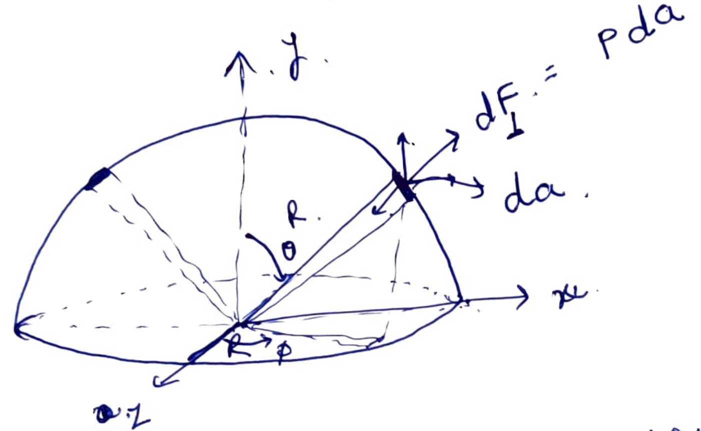
Physics - Problem

Q. A Metal sphere of radius R carries a total charge Q. What is the force of repulsion b/w the 'Northern' and Southern'



Sol^n



As sphere is conducting, so it will be uniformly charged with $\boxed{\sigma = \frac{Q}{4\pi R^2}}$

p (electrostatic Pressure on small area da) $= \frac{\sigma^2}{2\varepsilon_0}$

$dF_\perp$ (Force on 'da' area in perpendicular dir^n) $= p\,da = \frac{\sigma^2}{2\varepsilon_0} da.$

$$\underbrace{dF_\perp = \frac{\sigma^2 \, da}{2\varepsilon_0}}$$

- This force includes force of repulsion due to other hemisphere as well as due to other patches on itself. (But we need to calculate only due to ~~b~~ other remaining hemisphere so. Some how we need ~~so~~ remove extra force).

- This force $\boxed{dF_\perp = \frac{\sigma^2 \, da}{2\varepsilon_0}}$ is a component of net force perpendicular to surface. But, so because the hemisphere is conducting $dF_{\parallel}$ (parallel to surface) is 0.

Hence $dF = dF_\perp = \dfrac{\sigma^2 \, da}{2\varepsilon_0}$.

So, $d\vec{F} = \dfrac{\sigma^2 \, da}{2\varepsilon_0}\hat{r}$ (considering centre to be origin).

$da = R^2 \sin\theta \, d\theta \, d\phi$.

$$d\vec{F} = \frac{\sigma^2 \, da \, \hat{r}}{2\varepsilon_0} = d\vec{F}_{\text{hemisphere remain}} + \sum d\vec{F}_{da,n}$$

(Force by other hemisphere) which we need to cal.

(Force on 'da' due to other 'n' patches 'n → ∞').

$$d\vec{F} = \frac{\sigma^2}{2\varepsilon_0}\, da\, \hat{r} = dF_{\text{hemisphere, remain}} + \sum_{n=1}^{\infty} dF_{da,n}$$

Now, integrate

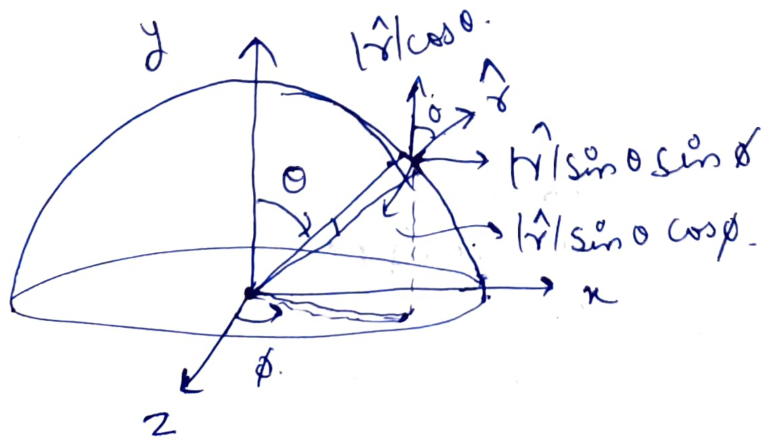
$$\int \left(dF_{\text{hemisphere, remain}} + \sum_{n=1}^{\infty} dF_{da,n} \right) = \int \frac{\sigma^2}{2\varepsilon_0}\, da\, \hat{r}$$

$$\int dF_{\text{hemisphere, remain}} + \underbrace{\int \sum_{n=1}^{\infty} dF_{da,n}}_{\text{I.}} = \int \frac{\sigma^2}{2\varepsilon_0}\, da\, \hat{r}$$

I. When we integrate it, due to ~~[illegible]~~ Newton's Third Law it will form action reaction pairs and got cancelled.

$$F_{\text{hemisphere, remain}} = \int \frac{\sigma^2}{2\varepsilon_0}\, da\, \hat{r}$$

$$F_{\text{repulsive}} = \int \frac{\sigma^2}{2\varepsilon_0} \left(R^2 \sin\theta \, d\theta \, d\phi \right) \hat{r}$$



$\hat{r}_\theta \neq \cancel{\quad} \cos\theta \, \hat{j} +$

$$\hat{r} = (\sin\theta \sin\phi)\hat{i} + (\cos\theta)\hat{j} + (\sin\theta \cos\phi)\hat{k}$$

$$\vec{F}_{repuslive} = \int \frac{\sigma^2}{2\varepsilon_0}\left(R^2 \sin\theta \, d\theta \, d\phi\right)\left\{ \hat{r} \right\}$$

from symmetry, integral will have $\hat{i}$ and $\hat{k}$ component as 0.

$$\vec{F}_{repulsive} = \int_0^{\pi/2}\int_0^{2\pi} \frac{\sigma^2}{2\varepsilon_0}\left(R^2 \sin\theta \, d\theta \, d\phi\right)(\cos\theta \, \hat{j})$$

$$= \int_0^{\pi/2}\int_0^{2\pi} \left(\frac{\sigma^2}{2\varepsilon_0}\right)\left(R^2 \sin\theta \cos\theta \, d\theta \, d\phi\right)\hat{j}$$

$$\frac{\sigma^2}{2\varepsilon_0} \times R^2 \int_0^{\pi/2} (\sin\theta \cos\theta \, d\theta)(2\pi)$$

$$\frac{\pi\sigma^2}{2\varepsilon_0} R^2 \int_0^{\pi/2} \sin 2\theta \, d\theta$$

$$\frac{\pi}{2\varepsilon_0} \sigma^2 R^2 \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$F_{repulsive} = \frac{\pi}{2\varepsilon_0} \frac{\sigma^2 R^2}{2} \left[2 \right]$$

$$= \frac{\pi \sigma^2 R^2}{2\varepsilon_0} \qquad \left\{ \sigma = \frac{Q}{4\pi R^2} \right\}$$

$$= \frac{\pi \times Q^2 \quad R^2}{16\pi^2 R^4 \times 2\varepsilon_0}$$

$$= \frac{1}{32\pi\varepsilon_0} \frac{Q^2}{R^2}$$

$$\boxed{F_{repulsive} = \frac{1}{32\pi\varepsilon_0} \times \frac{Q^2}{R^2}}$$