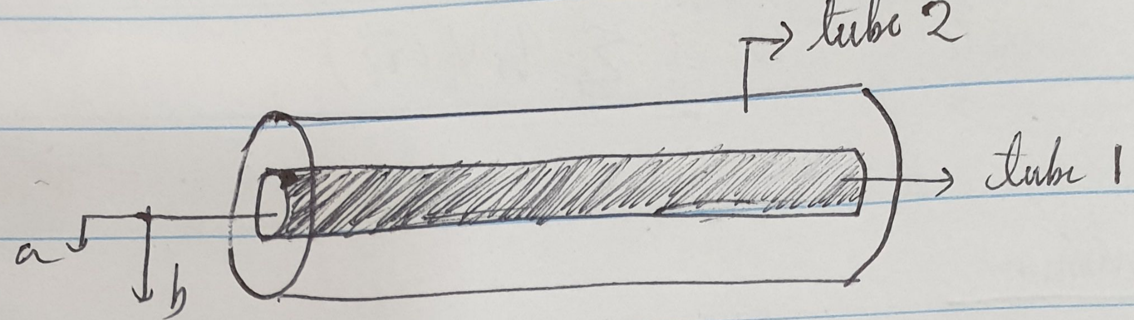
PRESENTATION



find the capacitance per unit length of 2 co-axial metal cylindrical metallic tubes of radii a & b.

⇒ Assume charge Q to be distributed uniformly along the surface of tube 1 (with radius 'a')

By Gauss law,

$$\int E \cdot da = \frac{Q_{enc}}{\varepsilon_0} \quad —(1)$$

for some length 'L' of the tube 2, with radius 's' :-

$$\int E \cdot da = E \int da = E \cdot 2\pi s \cdot L \quad —(2)$$

equating ① & ②

$$\vec{E} = \frac{Q}{2\pi \varepsilon_0 L} \cdot \left(\frac{1}{s}\right) \hat{s} \quad —(A)$$

potential difference between cylinders is,

$$V(b) - V(a) = -\int_a^b \vec{E}\cdot\vec{dl}$$

(here $\vec{dl} = \vec{ds}$)

~~(check~~

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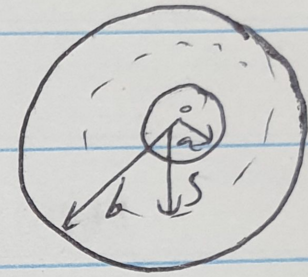

$$= -\int_a^b \frac{Q}{2\pi\epsilon_0 L}\left(\frac{1}{s}\right)\hat{s}\cdot\vec{ds}$$

front view of the system of co-axial cylinders

$$= \frac{-Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

sub. $\vec{E}$ from (A)

Since potential at 'a' is higher than that at 'b'

$\therefore$ potential diff $= V = V(a) - V(b)$

$$= \frac{Q}{2\pi\epsilon_0 L}\ln(b/a) \quad \text{—(B)}$$

$\therefore$ potential diff $= V = V(a) - V(b)$

$$= \frac{Q}{2\pi\epsilon_0 L} \ln(b/a) \quad \text{—(B)}$$

rearranging B, $\frac{V}{Q} = \frac{\ln b/a}{2\pi\epsilon_0 L}$

Capacitance is given as

$$C = \frac{Q}{V}$$

$$\therefore C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Capacitance per unit length of the cylinder

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)} \rightarrow \text{ans}$$