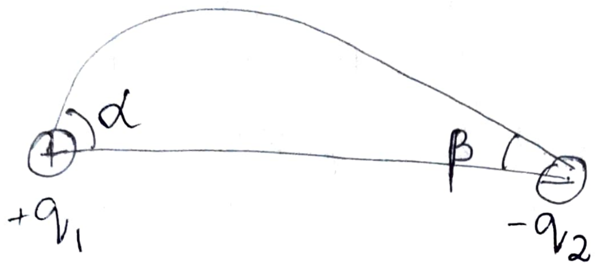
Q) There are 2 point charges $+q_1$ and $-q_2$ placed at a distance. An electric field line emerges from the $+q_1$ charge at an angle α to the straight line connecting it to the charge $-q_2$. At what angle will the field line end at the charge $-q_2$?

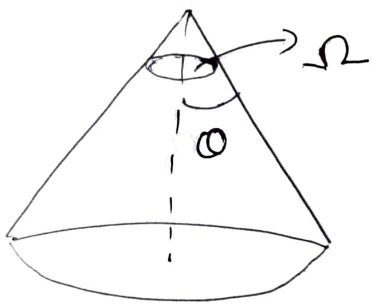


given: α

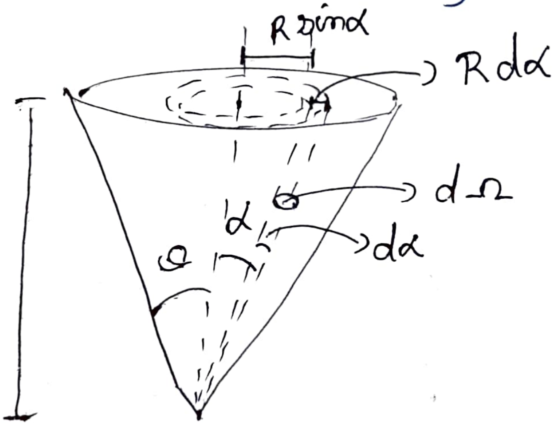
$\beta = ?$

solid angle $= \dfrac{\text{area}}{(\text{radius})^2}$ (By defn of solid angle)

Solid angle



$\Omega = 2\pi(1-\cos\theta)$

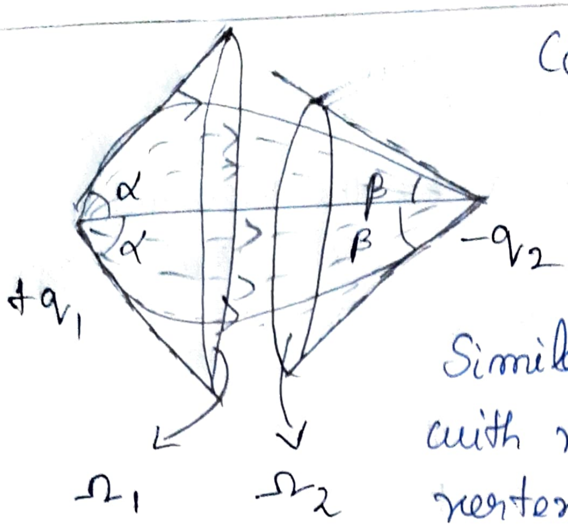


$$d\Omega = \frac{\text{area}}{(\text{radius})^2} = \frac{2\pi(R\sin\alpha)\,R\,d\alpha}{R^2}$$

$$d\Omega = 2\pi \sin\alpha \, d\alpha$$

$$\int d\Omega = \int_0^{\theta} 2\pi \sin\alpha \, d\alpha$$

$$\boxed{\Omega = 2\pi(1-\cos\theta)}$$

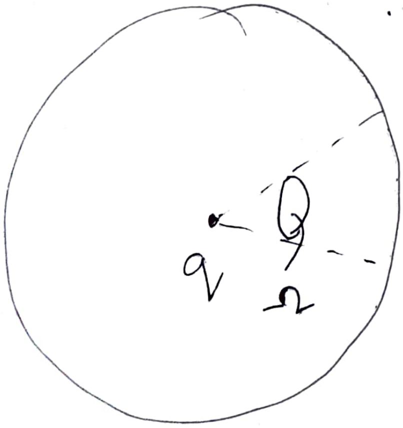


Consider a cone with vertex angle 2α and vertex at the point $+q_1$, cone A

Similarly consider a cone with vertex at $-q_2$ and vertex angle 2β, cone B

The number of electric field lines passing through the cone A = that passing into cone B

$\therefore$ Flux through the solid angle Ω_1 = that through solid angle Ω_2 — (1)



$$\phi = \oint \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

Solid angle of sphere $= 4\pi$

$\therefore$ flux through solid angle Ω

$$= \frac{\text{total flux}}{\text{total solid angle}} \times \Omega$$

$$= \frac{q}{\varepsilon_0} \times \frac{1}{4\pi} \times \Omega$$

$$\therefore (1) \equiv \frac{+q_1}{4\pi\varepsilon_0}\Omega_1 = \frac{q_2}{4\pi\varepsilon_0}\Omega_2$$

$$\Omega_1 = 2\pi(1-\cos\alpha)$$

$$\Omega_2 = 2\pi(1-\cos\beta)$$

$$\frac{q_1}{\cancel{4}\pi\varepsilon_0}\,\cancel{2}\pi(1-\cos\alpha) = \frac{q_2}{\cancel{4\pi\varepsilon_0}}\,2\cancel{\pi}(1-\cos\beta)$$

$$\cos\theta = 1 - 2\sin^2\theta/2$$

$$\therefore\quad q_1(\cancel{2}\sin^2\alpha/2) = q_2(\cancel{2}\sin^2\beta/2)$$

$$\sin^2\beta/2 = \frac{q_1}{q_2}\sin^2\alpha/2$$

$$\frac{\beta}{2} = \sin^{-1}\left(\sqrt{\frac{q_1}{q_2}\sin^2\frac{\alpha}{2}}\right)$$

$$\boxed{\beta = 2\sin^{-1}\left(\sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}\right)}$$

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