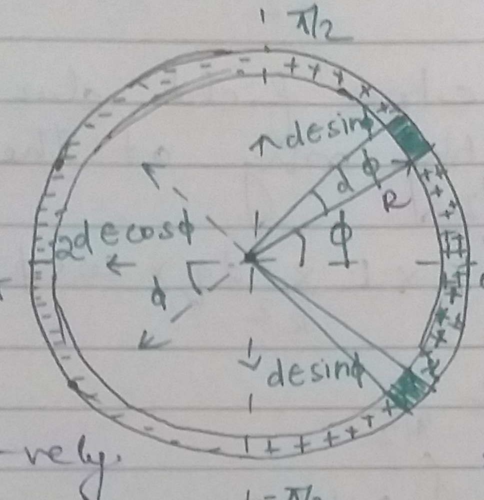
Physics presentation

A thin non-conducting ring of radius R has a linear charge density $\tau = \tau_0 \cos\phi$, where τ_0 is a constant, φ is the azimuthal angle. Find the magnitude of the electric field strength at the centre of the ring.

Since $\cos\phi$ is -ve in $\pi/2$ to $-\pi/2$ interval, the half part of the ring is -vely charged



$\tau = \tau_0 \cos\phi$

since the magnitude of $\cos\phi$ ↓ses from 0 to $\pi/2$, the charge density ↓ses from 0 to $\pi/2$, similarly ↑se from $\pi/2$ to π and ↓se from π and $-\pi/2$.

Here the $\tau = \tau_0 \cos\phi$ (given)

→ Let us find out the electric field de due to a small element on the ring $R\,d\phi$

Then $dq = R\,d\phi\,\tau_0 \cos\phi$

$$de = \frac{1}{4\pi\varepsilon_0} \times \frac{dq}{R^2}$$

$$de = \frac{k \times R\,d\phi\,\tau_0 \cos\phi}{R^2}$$

due to here the counter part element, the vertical component of the electric field i.e $de \sin\phi$ get's cancelled out and only the Horizontal component $de \cos\phi$ gets added up.

$$\therefore de = \frac{1}{4\pi\varepsilon_0} \times \frac{dq}{R^2} \times \cos\phi$$

And also -ve part of the ring points in the same direction and the vertical component of the electric field also get cancelled out due to the presence of counter part element

$$\therefore de = 2 \times k \times \frac{dq \times \cos\phi}{R^2}$$

$\therefore$ The electric field due to the whole non-conducting ring at the center is

$$\int de = \int_{-\pi/2}^{+\pi/2} 2 \times \frac{k \times dq}{R^2} \times \cos\phi$$

$$e = 2 \times k \int_{-\pi/2}^{\pi/2} \frac{R\,d\phi}{R^2} \lambda_0 \cos\phi \times \cos\phi$$

$$e = \frac{2k\lambda_0}{R} \int_{-\pi/2}^{\pi/2} \cos^2\phi \, d\phi$$

$$e = \frac{\lambda_0}{4\varepsilon_0 R}$$