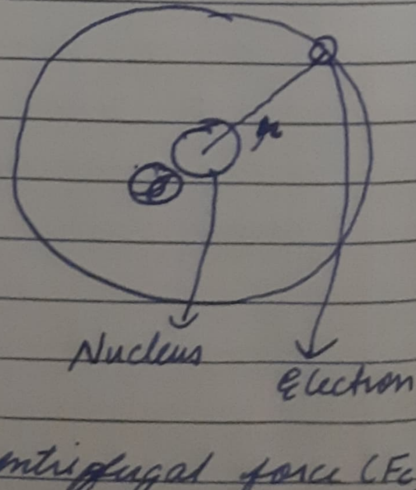
Question :- Find a general expression for frequency of radiation emitted when election goes from one orbit to another in a monoelectronic species. Also, comment on its quantization.

Introduction - Since the species is monoelectronic, it can be treated as ~~a~~ the atomic model proposed by Bohr. Thus, the observables (energy, momentum etc) are quantised but the system can be treated ~~a~~ classically.

Assumptions used :-

1) Elections move in fixed circular orbits or stationary states
2) The net force experienced by the election is zero.
3) Magnitude of angular momentum is quantised as $mvr = \frac{nh}{2\pi}$
4) When ~~elections~~ elections move from one stationary state to another, they absorb or emit a photon corresponding to the energy gain or ~~loss~~ loss. This emission or absorption follows Planck's equation, $E = h\nu$ (h = Planck's constant, ν = frequency of light)

Solution:



Let charge of election (q_e) be $-e$

Then, charge of nucleus (q_n) = Ze
where, Z = atomic number.

Forces acting on election are centrifugal force (F_c) and electrostatic force (F_e)

Since net force on election is zero,

$\therefore F_c + F_e = 0 \longrightarrow (1)$

$F_c = \frac{m_e v^2}{r}$ (m_e = mass of electron, v = velocity of electron, r = radius of orbit)

$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_e q_n}{r^2}$ (From Coulomb's Law)

On substituting values of q_e and q_n

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{(-e \times Ze)}{r^2} = \frac{-Ze^2}{4\pi\epsilon_0 r^2} \text{ or } \frac{-kZe^2}{r^2} \left(k = \frac{1}{4\pi\epsilon_0}\right)$$

$\therefore$ From equation 1,

$$\frac{m_e v^2}{r} - \frac{kZe^2}{r^2} = 0$$

$$\therefore m_e v^2 = \frac{kZe^2}{r} \longrightarrow (2)$$

Now, to calculate energy of electron, we use the ~~follow~~ following equation :-

$E = K + U$

E → Total energy, K → Kinetic energy, U → Potential Energy

Kinetic energy of electron is given by

$$K = \frac{m_e v^2}{2}$$

Potential energy of electron is from one source, electrostatic potential energy

$\therefore U_{elec} = q_e V$ (V = electrostatic potential)

Since potential at ∞ is 0, the following expression gives potential

$$\cancel{V = -\int} \quad \int_0^V dV = -\int_\infty^r \vec{E} \cdot d\vec{x}$$

$\vec{E}$ is electric field due to nucleus, which is given by :-

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Ze}{x^2} \hat{x} \text{ or } \frac{kZe}{x^2} \hat{x}$$ ($\hat{x}$ is direction radially outward)

$$\therefore \int_0^V dV = -\int_\infty^r \frac{kZe}{x^2} (\hat{x} \cdot d\vec{x}) = -\int_\infty^r \frac{kZe}{x^2} dx$$

$$\therefore V = \left. \frac{kZe}{x} \right]_\infty^r = \frac{kZe}{r}$$

$$\therefore U_{elec} = -e \times \frac{kZe}{r} = -\frac{kZe^2}{r}$$

$\therefore$ Total energy (E) can be written as

$$E = \frac{m_e v^2}{2} - \frac{kZe^2}{r}$$

but from equation 2.

$$E = \frac{kZe^2}{2r} - \frac{kZe^2}{r} = -\frac{kZe^2}{2r}$$

Radius of orbit can be deduced using the quantization of angular momentum

$$m_e v r = \frac{nh}{2\pi}$$

$$\therefore m_e^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\therefore m_e v^2 = \frac{n^2h^2}{4\pi^2 m_e r^2}$$

But from equation 2

$$\frac{kZe^2}{r} = \frac{n^2h^2}{4\pi^2 m_e r^2 r}$$

On rearrangement

$$r = \frac{n^2h^2}{4\pi^2 kZe^2}$$

On substituting in the total energy equation, we get

$$E = \frac{-kZe^2}{2\times n^2h^2} \times 4\pi^2 kZe^2 = \frac{-2\pi^2(kZe^2)^2}{n^2h^2}$$

This is expression for energy of electron at state n. If electron moves from state n_1 to state n_2, the energy absorbed ~~would~~ would be the difference in energy levels. But to find radiated energy (ΔE), we use the following expression:-

$$\Delta E = \underbrace{E_1}_{\text{Energy at state 1}} - \underbrace{E_2}_{\text{Energy at state 2}}$$

$$\cancel{\therefore \Delta E = -\frac{kZe^2}{2n_1^2h^2}} + kZe^2$$

$$\therefore \Delta E = -\frac{2\pi^2(kZe^2)^2}{n_1^2h^2} + \frac{2\pi^2(kZe^2)^2}{n_2^2h^2}$$

$$\Delta E = \frac{2\pi^2(kZe^2)}{h^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

The given expression shows energy of photon released when election goes from state 1 to state 2. Since the energy of this ~~pot~~ photon can be given by Planck's equation.

$$\therefore \Delta E = h\nu$$

$$\therefore h\nu = \frac{2\pi^2(kZe^2)^2}{h^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\therefore \nu = \frac{2\pi^2(kZe^2)^2}{h^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

This is the expression ~~foo~~ for frequency of light given out when the election changes from state 1 to 2. This frequency depends only on values of n_1 and n_2 as all the other terms (including Z) for a given species is constant.

~~The states n_1~~

Since n_1 and n_2 can only take integral values, frequency cannot take continuous values and will be discreet. Thus frequency will be quantized and can only take certain values, ~~of which are multiples of~~