Physics Presentation

Vaibhav Anand
20211053

June 15, 2022

Question: A line segment of length 2l and having a uniform charge density λ is kept with its center at the origin, what is the shape of equipotential curves?

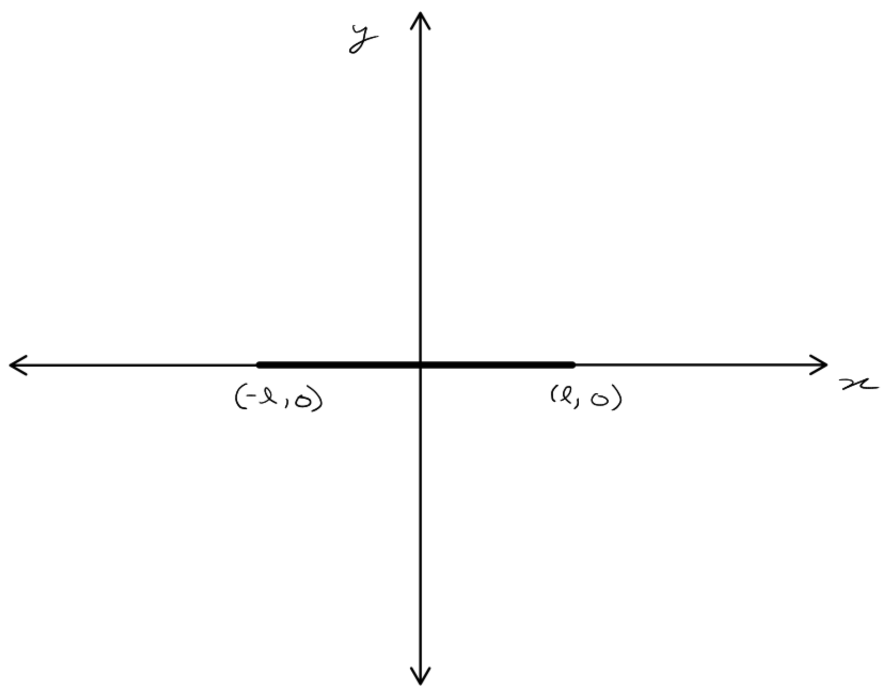

Figure 1: Charge Configuration

Solution: We know that the potential at a point can be found out using the following formula (The symbols have their usual meaning)

$$V(r) = \int \frac{1}{4\pi\epsilon_0} \frac{\rho(r')}{|r - r'|} d\tau'$$

In our situation, we obtain the following the figure

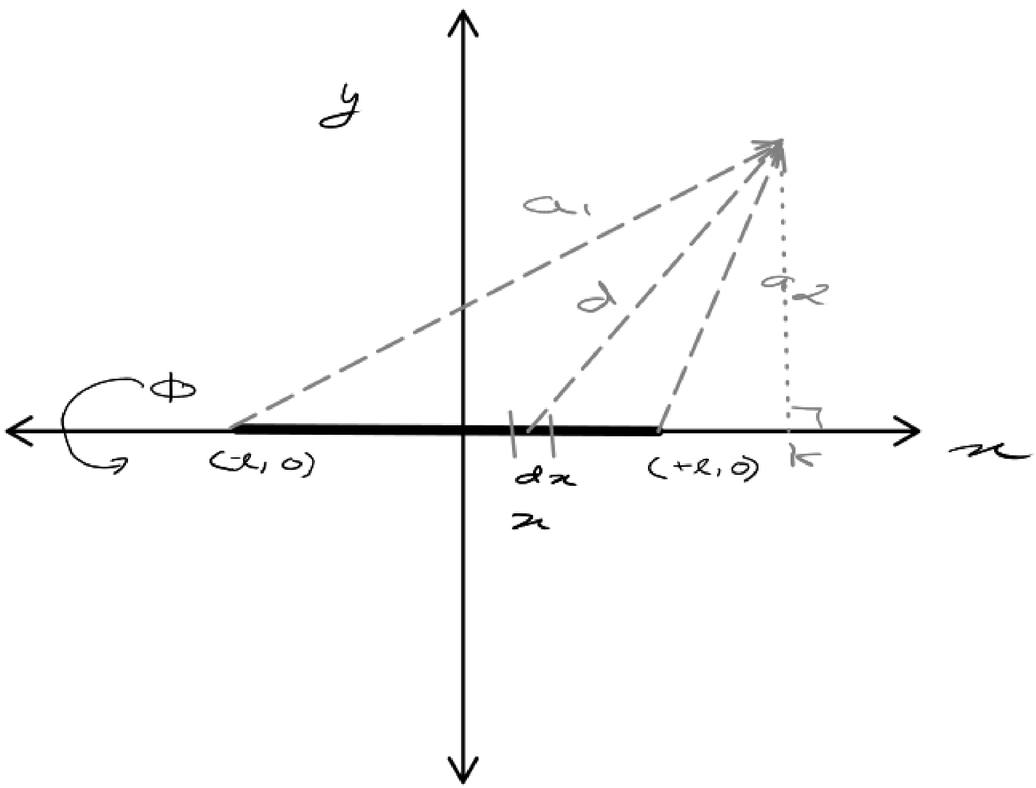

Figure 2: Charge Configuration

The electric potential at an point P is

$$V(P) = \frac{1}{4\pi\epsilon_0}\int_{-l}^{+l}\frac{\lambda}{d}dx = \frac{1}{4\pi\epsilon_0}\int_{-l}^{+l}\frac{\lambda}{\sqrt{(x-k)^2+y^2}}dx$$

We know the solution of integrals of this form,

$$\int\frac{dx}{\sqrt{(x-x_0)^2+y^2}} = -\ln(2\sqrt{(x-x_0)^2+r^2}+2(x-x_0))+C$$

Using this and substituting our values we get,

$$V(P) = \frac{1}{4\pi\epsilon_0}\lambda\ln\left[\frac{\sqrt{(l+k)^2+y^2+}+k+l}{\sqrt{(k-l)^2+y^2}+k-l}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{2l}\ln\left[\frac{a_1+k+l}{a_2+k-l}\right]$$

Where, $a_1 = \sqrt{(l+k)^2+y^2}$ and $a_2 = \sqrt{(k-l)^2+y^2}$ are the distances of point P from the end points of the line and Q is the total charge distributed over a length 2l On using eliptic coordinates u and v such that ,

$$u = \frac{a_1+a_2}{2l} \text{ and } v = \frac{a_1-a_2}{2l}$$

$$a_1 = l(u+v) \text{ and } a_2 = l(u-v)$$

$$uv = \frac{a_1^2 - a_2^2}{4} = \frac{z}{l}$$

After substituting this into the equation for potential we get,

$$V(P) = \frac{1}{4\pi\epsilon_0}\frac{Q}{2l}\ln\left[\frac{l(u+v)+luv+l}{l(u-v)+luv-l}\right] = \frac{1}{4\pi\epsilon_0}\frac{Q}{2l}\ln\left[\frac{(u+1)(v+1)}{(u-1)(v+1)}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{2l}\ln\left[\frac{u+1}{u-1}\right]$$

The last expression tells us that the electric potential depends only on the coordinate u and is constant on curve $u =$ constant , u being constant impiles that the sum of distance of the point from the two end points of the line segment is constant, the surface therefore, is an ellipse. We also know that ellipise's and hyperbola's sharing the same foci are perpendicular to each other at their intersections just as electric field is perpendicular to the equipotential curves, this leads us to conclude that $v =$ constant gives us the electric field lines which form a hyperbola

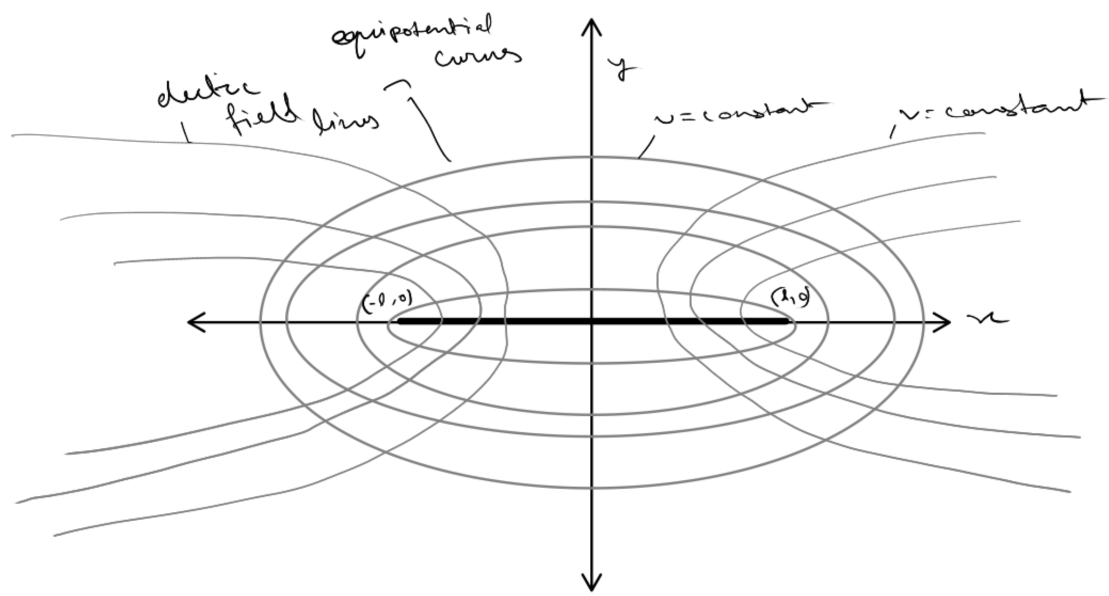

Figure 3: Equipotential curves and field lines