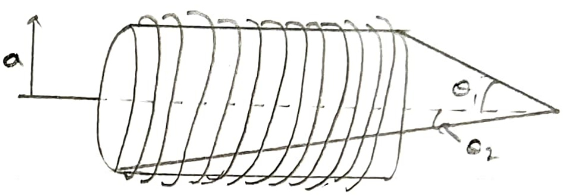
Dharavath Anil,
20211225,
Batch - 3.

Q. Find the magnetic field at point P on the axis of a tightly wound solenoid (helical coil) consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I. Express your answer in terms of θ_1 and θ_2. Consider the turns to be essentially circular, ~~and. use~~ what is the field on the axis of an infinite solenoid (infinite in both directions)?



Ans:

Radius of a cylindrical tube = a

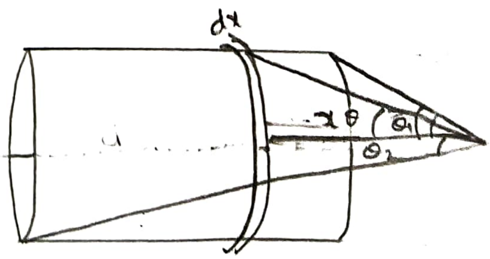
No. of turns per unit length = n

Magnetic field of a circular ring $\vec{B}_{ring} = \dfrac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$ → For 1 turn

let dx be the width of differential ring element.

let N be total no. of turns,

$N = n\,dx$



let the the ring is at distance of d

Then,

$I' = NI = nI\,dx$

Now,

$$\vec{dB} = \frac{\mu_0 n I \, dx \, a^2}{2 (x^2+a^2)^{6/2}}$$

$R = a$

$I' = nI dx$

So, to get magnetic field, we need to integral

$$B_{net} = \int \vec{dB} = \mu_0 n I a^2 \int \frac{dx}{(x^2+a^2)^{3/2}}$$

$$\vec{B}_{net} = \frac{\mu_0 n I a^2}{2} \left(- \int \frac{a \ \operatorname{cosec}^2\theta}{a^3 \ \operatorname{cosec}^3\theta} d\theta \right)$$

$$= - \frac{\mu_0 n I a^3}{2 a^3} \int \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}^3\theta} d\theta$$

$$= - \frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$\boxed{\vec{B}_{net} = \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)}$$

From $\triangle$le,

$\tan\theta = \frac{a}{x}$.

$x = a\cot\theta$

$$\boxed{dx = a \operatorname{cosec}^2\theta \, d\theta}$$

$\therefore$ Magnetic field at P from finite length $= \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)$

Now, $\vec{B}$ on the axis of an infinite solenoid

$\theta_1 = \pi$, and $\theta_2 = 0$

$$\vec{B}_{net} = \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)$$

$$= \frac{\mu_0 n I}{2} (1 - (-1))$$

$$\boxed{\vec{B}_{net} = \mu_0 n I}$$