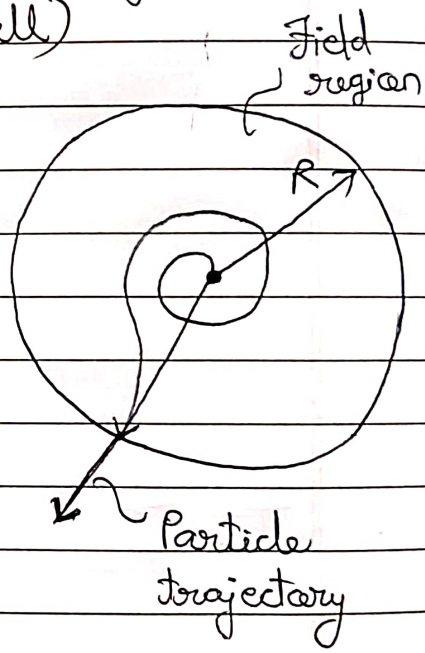
Physics Presentation Problem :-

Q. A Circularly Symmetrical magnetic field (B depends only on the distance from the axis), pointing perpendicular to the Page, occupies the shaded region as shown in figure below. If the total flux ($\int B \cdot da$) is zero, show that a charged particle that starts out at the center will emerge from the field region on a radial path (provided it escapes at all)



Ans : Since we need to show that Particle will exit the magnetic field region along the radial direction, this implies that at that instant of time, angular momentum of charged particle i.e. $[m(\vec{r} \times \vec{v})]$ would be zero as the velocity of particle would be along the radial direction.

$\therefore$ we calculate the total angular momentum acquired by the particle, using the Lorentz force law :-

Angular momentum acquired by Particle as it moves out from Center to the edge is

$$L = \int \frac{dL}{dt} \cdot dt = \int \tau \cdot dt$$

$$\tau = \vec{r} \times \vec{F}$$

$$\therefore \quad L = \int (\vec{r} \times \vec{F})\, dt$$

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Here, $\vec{F}$ = Magnetic force = $q(\vec{V}\times\vec{B})$

$$L = \int \vec{r}\times q(\vec{V}\times\vec{B})\,dt$$

$$L = q\int \vec{r}\times(\vec{dl}\times\vec{B})$$

where $\vec{V}\cdot dt = \vec{dl}$
&
$\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$$L = q\left[\int(\vec{r}\cdot\vec{B})\vec{dl} - \int\vec{B}(\vec{r}\cdot\vec{dl})\right]$$

$\because \vec{r}\perp\vec{B} \Rightarrow \vec{r}\cdot\vec{B} = 0$

$$\vec{r}\cdot\vec{dl} = (r\,.\,\hat{r})\cdot(dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi})$$

$\therefore \vec{r}\cdot\vec{dl} = r\,dr$

$$\therefore L = q\left[0 - \int\vec{B}(r\,dr)\right]$$

$$L = -\frac{q}{2\pi}\int\vec{B}(2\pi r\,dr) \qquad (\underbrace{d\vec{a}}_{\substack{\text{area of}\\ \text{elementary ring}\\ \text{at } r=r}} = 2\pi r\,dr)$$

$$\therefore L = -\frac{q}{2\pi}\int\vec{B}\cdot d\vec{a} \Rightarrow L = -\frac{q}{2\pi}\phi$$

$\phi = \int_0^R \vec{B}\cdot d\vec{a}$ = magnetic flux of the entire field region
= zero (given)

$$\therefore L = -\frac{q}{2\pi}(0) = 0$$

In particular, if $\phi = 0$, then $L = 0$, and the charge emerges with zero angular momentum, which means it is going along a radial line.

Note: on the reverse trajectory, a Particle fired at center from outside will hit its target (if it has sufficient energy), though it may follow a weird route getting there.