Q. How can Magnetism be explained with the help of special relativity?

Ans: Before going through the answer, lets, first talk about some prerequisite knowledge:

1. $2^{nd}$ postulate of special theory of relativity
2. Length contraction
3. Lorentz Transformation.

$2^{nd}$ postulate: speed of light 'c' is constant, independent of the relative motion of the source.

Length contraction:

suppose an observer moves with some speed '$v_A$' (or even rest). when he calculates the speed of light he will find it to be 'c'.

suppose, another observer moving with a very high speed (compareable to the speed of light), when he will measure the speed of ~~the~~ light, he will find it to be 'c'

" According to $2^{nd}$ postulate of S.T.R, we can say the above things".

But, there are certain bizzarre consequences regarding this. Two ~~ob~~ observers in relative motion, may see that the distance betn points may shrink (length contraction) & the time period betn. diff. physical events may dilate (Time dilation) just to make sure the speed of lights remain constant.

# we are interested in knowing about the length contraction.

when we measure something, the measurement of the length "appears" to have contracted in the dirn of the relative motion wrt the observer in relative motion.

(*) Length contraction will only happen in the dirn of relative motion.

In order to derive the expression for Length contraction, we are going to use a set of transformation eq^ns known as Lorentz Transformation eq^ns. [This are eq^ns that connect the coordinate of space & time betn two observers in relative motion]

"I have not included the derivations of Length contraction using Lorentz transformation, otherwise the presentation will be too long".

~~Length~~ Contracted length $(\Delta L)$ = Original length $(\Delta L_0) \cdot \sqrt{1-\frac{v^2}{c^2}}$

i.e $\boxed{\Delta L = \Delta L_0 \sqrt{1-\frac{v^2}{c^2}}}$ where, $v \rightarrow$ vel. of observer/object

$c \rightarrow$ speed of light.

End of prerequisites

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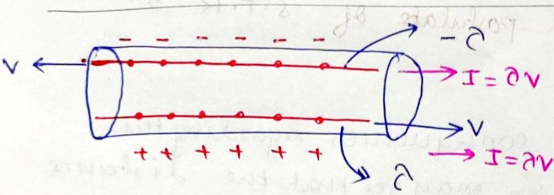
Now, let us explain ~~the~~ magnetism using ~~speci~~ length contraction from special theory of Relativity ~~using~~ by taking a simple case!

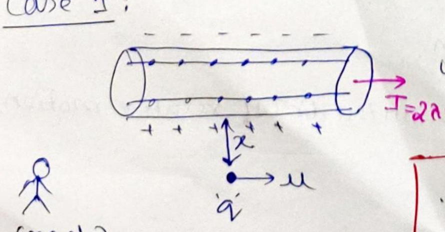



"For simplicity we have represented the case by considering two thin wires have equal charges ~~& opposite~~ but opposite sign.

Let, the big wire consists of two distinct wires consisting of +ve charges (moving to the right (with vel. $v$) & -ve charges moving to the left (with vel. $v$). The two wires are having linear charge density $+\lambda$ & $-\lambda$ respectively.

$\therefore$ Total current $= \lambda v + \lambda v = 2\lambda v$.

Case 1:




~~we know~~ Let, us ~~release~~ keep a point charge '$q$' moving to the right at velocity '$u$'

$\therefore$ The magnetic $\vec{B}$ produced by ~~an infinite~~ the cylindrical wire at a distance '$x$' from the axis of the wire:

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi x}}$$

[This can be derived using the ampere's circuital laws ie $\oint B \cdot dl = \mu_0 i_{inside}$] of the ampere's loop.

& the direction of magnetic field will be $\otimes$ [inside the copy].

$\therefore$ Force by the magnetic field on the moving charge

$$\Rightarrow q(\vec{u} \times \vec{B}) \Rightarrow quB \sin 90^\circ$$

$$\Rightarrow qu \frac{\mu_0 I}{2\pi x}.$$

$\therefore$ and the force will be towards the wire.

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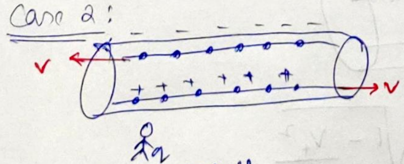
Now, lets keep all magnetic laws aside & only talk about electrostatics. [let us assume we know nothing about magnetism]

In case 1: when the observer is a rest & when he looks at the particle 'q', according to him the 'q' charge must feel zero electrostatic field [hence, zero electrostatic force], since the amt. of +ve & -ve charges in the wire are equal [linear charge ~~density~~ densities are equal & ~~opposite~~ opposite].

{Again, we are not assuming any magnetic laws}.

$\therefore$ the charge is not supposed to feel any kind of Electric force, but the net charge dif. is zero.

Case 2:



Now, the observer is also moving with the ~~particle~~ charge particle.

Now, using relativistic velocity transformations:

velocity of +ve charges wrt to the moving particle, $v_+ = \dfrac{v - u}{1 - \dfrac{vu}{c^2}}$.

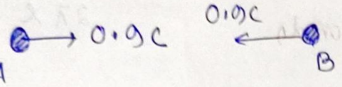
velocity of -ve charges " " " " , $v_- = \dfrac{v + u}{1 + \dfrac{vu}{c^2}}$.

~~FAQ~~

Why, we are taking $V_+ = \frac{V-u}{1-\frac{vu}{c^2}}$ also for $V_-$, not not considering

$V_+ = V - u$?

→ ~~Becau~~ Because, when we are dealing with objects which are moving with very high velocities, we can't use normal relative velocity eqns.

eg: 

$\therefore$ Vel. of A w.r.t B

$V_{AB} = V_A - (V_B)$

$= 0.9 - (-0.9c)$

$= 1.8c$. which is not possible!

Hence, ~~we are using~~ relativistic velocity transformations are necessary.

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~~FAQ~~: ~~In~~

~~# In real life~~

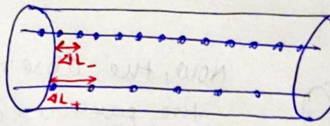
Now, clearly we can see $V_- > V_+$

Consequences: The charge density of the +ve & -ve wire won't be same any more.

Now, bcz of this diff. in velocities, the distance betn the +ve & the -ve charges are going to experience length contraction.

→ Hence, one of the wire will be much more denser [linear charge density will be higher]

$$\boxed{\Delta L = \Delta L_0 \sqrt{1-\frac{V^2}{c^2}}}$$



$$\Delta L_- = \Delta L \sqrt{1-\frac{V_-^2}{c^2}}, \quad \Delta L_+ = \Delta L \sqrt{1-\frac{V_+^2}{c^2}}.$$

$$\therefore \lambda^+ = \frac{\lambda_0}{\sqrt{1-\frac{V_+^2}{c^2}}}, \quad \lambda^- = \frac{\lambda_0}{\sqrt{1-\frac{V_-^2}{c^2}}}$$

Clearly the density of the -ve charge are now greater than the ~~-ve~~ +ve charges.

$\therefore$ The cross section of the wire has non-zero net charge.

→ As a result, the external charged particle would now experience some net force.

" w.r.t. to the ground observer, there should be no force, but w.r.t to a moving observer there should be force".

But we can show that, the moment we do tranformation from moving ~~charge~~ to the rest observer, it will be exactly the same as we know about the magnetic field.

Now, Net charge density:

$$\lambda_{net} = \lambda_+ - \lambda_- = \frac{\lambda_0}{\sqrt{1-\frac{v_+^2}{c^2}}} - \frac{\lambda_0}{\sqrt{1-\frac{v_-^2}{c^2}}}$$

put, $v_+ = \dfrac{v-u}{1-\frac{vu}{c^2}}$ & $v_- = \dfrac{v+u}{1+\frac{vu}{c^2}}$

On solving we get,

$$\boxed{\lambda_{net} = \frac{-2\lambda_0 vu}{c^2\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}}$$

→ net ~~total~~ charge density w.r.t the moving observer. Clearly, the net charge density is a non-zero term.

* And, the force the wire will exert to the particle will be toward the wire.

$\therefore$ $\vec{E}$ due to a line charge density at a distance '$x$' from the wire:

$$\vec{E}' = \frac{\lambda_{net}}{2\pi\varepsilon_0 x}$$

[can be derived using gauss law. $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ of the gaussian surface.]

$\therefore$ Force experienced by the particle, when the observer is in moving frame

$$F' = qE'$$

$$= \frac{-2q\lambda vu}{2\pi\varepsilon_0 x c^2\sqrt{1-\frac{u^2}{c^2}}}$$

& $c^2 = \dfrac{1}{\mu_0\varepsilon_0}$ [we know]

permeability of free space

On solving, we get

$$\boxed{F' = \frac{-q\lambda v u \mu_0}{\pi x\sqrt{1-\frac{u^2}{c^2}}}}$$

Now, using relativistic force transformation:

Force experienced by the charge when obs. is in moving frame $(F')$ = $\dfrac{\text{Force experienced by the charge when the observer is in ground } (F)}{\sqrt{1-\frac{u^2}{c^2}}}$.

i.e $\boxed{F' = \frac{F}{\sqrt{1-\frac{u^2}{c^2}}}}$

$$\therefore F = F' \times \sqrt{1-\frac{u^2}{c^2}} = \frac{-q\lambda v u \mu_0}{\pi x\sqrt{1-\frac{u^2}{c^2}}} \times \sqrt{1-\frac{u^2}{c^2}}.$$

$$= \frac{-q\lambda v u \mu_0}{\pi x}. \quad [\because I = 2\lambda v],$$

$$= -qu\left(\frac{\mu_0 I}{2\pi x}\right)$$

$$= -quB$$

Clearly, we found a force acting on the particle having a magnitude $\left(-qu.\frac{\mu_0 I}{2\pi x}\right)$ and direction toward the wire.

This, exactly looks like the force exerted by the magnetic field on the particle.

Hence, using Relativity we explained the Magnetism!