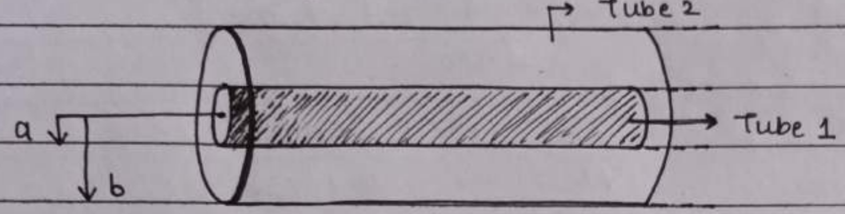
Physics Tutorial Problem

Q. Find the capacitance of per unit length of 2 co-axial metal cylindrical tubes, of radii a & b



Solution: Assume charge Q to be uniformly distributed along the surface of tube 1 (with radius 'a').

By Gauss law,

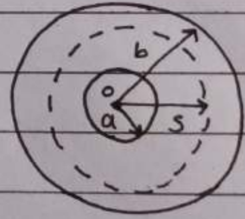
$$\int E \cdot da = \frac{Q_{enc}}{\epsilon_0} \quad -①$$

For some length 'L' of tube 2, with radius 's':-

$$\int E \cdot da = E \int da = E \cdot 2\pi s \cdot L \quad -②$$

equating ① & ②

$$\vec{E} = \frac{Q}{2\pi\epsilon_0 L}\left(\frac{1}{s}\right)\hat{s} \quad -(A)$$



(Front view of the system of co-axial cylinders)

Potential difference b/w cylinders is,

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l} \quad (\text{here } d\vec{l} = d\vec{s})$$

$$V(b) - V(a) = -\int_a^b \frac{Q}{2\pi\epsilon_0 L}\left(\frac{1}{s}\right)\hat{s} \cdot d\vec{s} \quad [\text{substitute } \vec{E} \text{ from } (A)]$$

$$V(b) - V(a) = \frac{-Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

Since Potential at 'a' is higher than that at 'b'

$\therefore$ Potential diff $= V = V(a) - V(b) = \dfrac{Q}{2\pi\epsilon_0 L}\ln\left(b/a\right)$ $-(B)$

Rearranging (B) $\Rightarrow$ $$\frac{V}{Q} = \frac{\ln(b/a)}{2\pi\epsilon_0 L}$$

Capacitance is given as

$$C = \frac{Q}{V}$$

$$\Rightarrow \quad C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

$\Rightarrow$ Capacitance per unit length of the cylinders

$$\Rightarrow \boxed{\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)}} \text{ — Ans.}$$