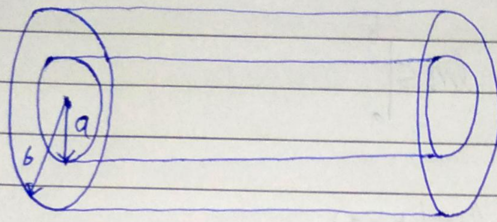
Q. Find the Capacitance per unit length of two coaxial metal cylinders tube of radius a and b.

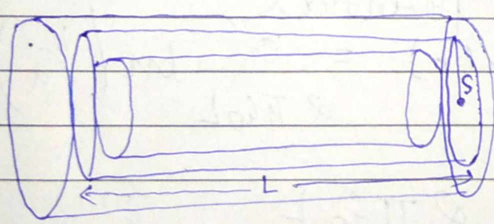


Sol:- We know that,

$$C = \frac{Q}{V}$$

So, we have to find potential b/w two cylinders but for that we first need $\vec{E}$.

Let us assume Gaussian Surface, with radius s & length L



Applying Gauss law,

$$\int \vec{E} \cdot \vec{da} = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi s L = \frac{Q}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{Q}{\epsilon_0 \cdot 2\pi s L} \hat{s}$$

now, potential difference b/w two cylinders is given by

$$V(b) - V(a) = -\int_a^b E \cdot dl$$

$$\Rightarrow \Delta V = -\frac{Q}{2\pi \varepsilon_0 L} \int_a^b \frac{1}{S} \cdot dS$$

$$= \frac{-Q}{2\pi \varepsilon_0 L} \ln S \Big|_a^b$$

$$= \frac{-Q}{2\pi \varepsilon_0 L} \ln (b-a)$$

$$= \frac{-Q}{2\pi \varepsilon_0 L} \ln \left(\frac{b}{a}\right)$$

$$= \frac{Q}{2\pi \varepsilon_0 L} \ln \left(\frac{a}{b}\right)$$

here, a is at higher potential,

so, $V = V(a) - V(b) = \frac{Q}{2\pi \varepsilon_0 L} \ln \left(b/a\right)$

$$\Rightarrow C = \frac{Q}{V} = \frac{2\pi \varepsilon_0 L}{\ln (b/a)}$$

Capacitance per unit length $= \frac{2\pi \varepsilon_0}{\ln (b/a)}$.