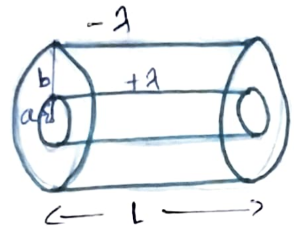
Q) Two long, co-axial cylindrical conductors are separated by vacuum. The inner cylinder has radius a and linear charge density $+\lambda$. The outer cylinder has inner radius b and linear density $-\lambda$?

1) Find Electric Field between two conductors i.e. in the region $a<r<b$.

2) Find the potential difference between two conductors.

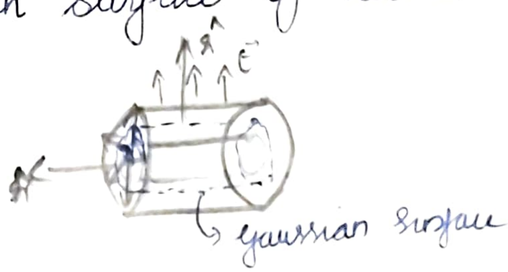
3) Find the capacitance per unit length for this capacitor.



The inner cylinder has radius a and the linear charge density is $+\lambda$, L is the length of the cylinders

1) The Electric Field between the two conductors is given by Gauss's law,

Consider a cylindrical gaussian surface of radius r where $a<r<b$.



$$\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$$

$$E\, 2\pi r L = \frac{+\lambda L}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{r}$$

2) potential difference between the two conductors

$$V(b) - V(a) = -\int_a^b E \cdot dl$$

$$= \frac{-\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r}\, dr$$

$$= \frac{-\lambda}{2\pi\varepsilon_0} \ln\left[\frac{b}{a}\right]$$

here, a is at higher potential,

$$\therefore V = V(a) - V(b)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left[\frac{b}{a}\right]$$

3] capacitance for this capacitor, $C = \frac{Q}{V}$

capacitance per unit length $= \frac{C}{L} = \frac{Q}{VL} = \frac{\lambda}{V}$

$$= \frac{2\pi\varepsilon_0}{\ln\left[\frac{b}{a}\right]}$$