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20211190 Batch - 6

classmate

Date

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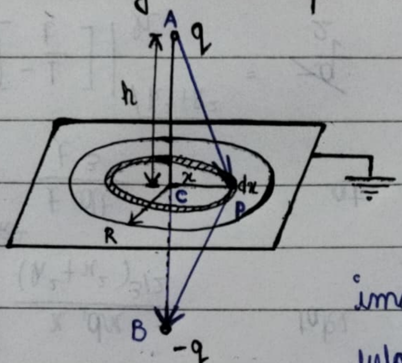
Ques) A charge q is placed at hight h from a grounded, large conducting plate. Find the radius of the circular area that contains charge $-q/2$ and has its centre at foot of perpendicular from the charge to the plate.



(Image charge)

Steps to be followed -

I) $-q$ charge will be induced on full plate

II) we can take a $-q$ point image charge at a distance h below centre point C.

III) let area of plate with radius $r = R$ enclosed the required induced charge of $-q/2$

IV) compute $\vec{E}$ at a point (say P) which is at a distance x from centre

V) compute σ by $\epsilon_0 \times \vec{E}$ as func^n of variable dist. x

VI) take the ring of radius $r = x$ and thickness dx and find out its charge by $\sigma \times dA$

VII) integrate the ring of thickness dx from 0 to R to get the charge on the desired circular plate and equate this charge to $-q/2$.

Computation →

→ IV) field due to charge A on P $= \dfrac{q}{4\pi\epsilon_0} \dfrac{\vec{AP}}{(AP)^3}$

field due to img charge B on P $= \dfrac{q}{4\pi\epsilon_0} \dfrac{\vec{PB}}{(PB)^3}$

Net field at P $= \vec{E_P} = \dfrac{q(\vec{AP} + \vec{PB})}{4\pi\epsilon_0 (AP)^3} = \dfrac{q(\vec{AB})}{4\pi\epsilon_0 (AP)^3}$

$(AP = PB)$

$AP = (h^2 + x^2)^{1/2}$ and $\overrightarrow{AB} = 2h\ \hat{AB}$

$$\vec{E_P} = \frac{q(2h)}{4\pi\epsilon_0 (h^2+x^2)^{3/2}}\ \hat{AB}$$

⇒ v) surface charge density (as func^n of x) $= \sigma = \epsilon_0 \times E$

$$\sigma = -\epsilon_0 \times \frac{q(2h)}{4\pi\epsilon_0 (h^2+x^2)^{3/2}}$$

(-ve charge as $-q$ charge is induced on the plate)

⇒ vi) charge on ring $= \sigma\, dA = \sigma (2\pi x\, dx)$

$$= \frac{-\cancel{\epsilon_0}\, q(\cancel{2}h)}{\cancel{4\pi\epsilon_0}(h^2+x^2)^{3/2}} \times \cancel{2\pi} x\, dx$$

$$= \frac{-q h x\, dx}{(h^2+x^2)^{3/2}}$$

⇒ vii) charge upto radius $R = -\frac{q}{2} = \int_0^R \frac{-q h x\, dx}{(h^2+x^2)^{3/2}}$

$$-qh \int_0^R \frac{x\, dx}{(h^2+x^2)^{3/2}}$$

take $h^2 + x^2 = t^2$

$x\, dx = t\, dt$

$$-qh \int_h^{\sqrt{h^2+R^2}} \frac{t\, dt}{t^3}$$

at $x = 0$ $t = h$

$x = R$ $t = (h^2+R^2)^{1/2}$

$$\Rightarrow +\cancel{q}h \left[-\frac{1}{t}\right]\Bigg|_h^{\sqrt{h^2+R^2}} = \frac{\cancel{q}}{2}$$

$$\Rightarrow h\left[\frac{1}{h} - \frac{1}{\sqrt{h^2+R^2}}\right] = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{h}{\sqrt{h^2+R^2}} = \frac{1}{2} \Rightarrow \frac{1}{4} = \frac{h^2}{h^2+R^2}$$

ie. $R^2 = 3h^2 \Rightarrow \boxed{R = \sqrt{3}h}$

Hence, at a radius distance of $\sqrt{3}h$ from center, the circular plate will contain an induced charge of $-q/2$.