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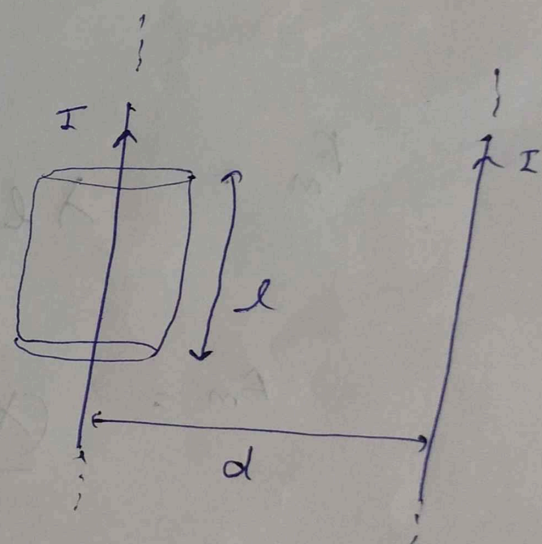
Q. B2 (b)
T3

Q. In the given systems, there are two infinitely long parallel wires carrying current I with charge density λ and separated by a distance d. Compare the forces on them due to electric and magnetic field. Infer from the result.

Sol. Electric force:-
between electrons of wires



$$F_e = q\vec{E}$$
$$= \lambda l E$$

λ = linear charge density
l = length of cylindrical gaussian surface

$$F = \lambda l \left(\frac{k(\lambda l)}{d^2} \right)$$

$$F_e = k \frac{(\lambda l)^2}{d^2}$$

we know that direction of $\vec{F}$ is the direction of $\vec{E}$, which is radially outwards in this case, which tends to push the other wire away from itself.

Magnetic force :-

$V =$ velocity of electrons

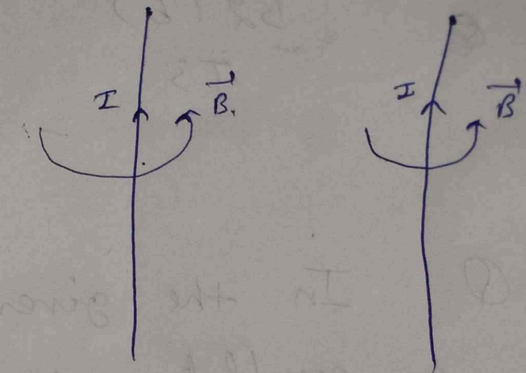
$F_m = q(\vec{V} \times \vec{B})$

$= \lambda l \, d \, |\vec{V}| \, |\vec{B}| \, |\sin\theta|$

$\because \theta = 90$

$\therefore$

$= \lambda l |\vec{V}| |\vec{B}|$



$= \cancel{\lambda l V}$

$\left[\because dB = \dfrac{\mu_0 I (dl \times \hat{r})}{4\pi r^2} \right.$ $\quad \because I = \dfrac{q}{t}$

$||^{ly}$, $dB = \dfrac{\mu_0 \frac{dq}{dt} (dl \times \hat{d})}{4\pi d^2}$

$\Rightarrow \left. B = \dfrac{\mu_0 \lambda l (\vec{V} \times \hat{d})}{4\pi d^2} \right]$

$F_M = \lambda l V \dfrac{\mu_0 \lambda l V}{4\pi d^2}$ $\quad \vec{V} \times \hat{d} = |\vec{V}| \sin\theta'$

$\theta' = 90$

$F_m = \dfrac{(\lambda l)^2 V^2 \mu_0}{4\pi d^2}$

~~Equating~~ ~~Magnetic~~

$\because$ direction of $\vec{F}_m$ will be in the direction of $(\vec{V} \times \vec{B})$. $\because \vec{V}$ in in y-axis (verticle) and $\vec{B}$ (caused due to another wire is inward the paper. the cross ~~product~~ of them will direct towards the wire itself.

Therefore, the wires will tend to pull each other due to magnetic force.

Equating Electric ~~field~~ force (Repulsive) and magnetic force (attractive) :-

$$F_e = F_m$$

$$\frac{k(\lambda l)^2}{d^2} = \frac{(\lambda l)^2 v^2 \mu_0}{4\pi d^2}$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda l}{d}\right)^2 = \left(\frac{\lambda l}{d}\right)^2 \frac{v^2\mu_0}{4\pi}$$

$$\frac{1}{\varepsilon_0} = v^2\mu_0$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$\because$ $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ is constant which is equal to the speed of light (c).

So, to balance these two forces and remain in stationary state, the speed of electrons have to be equal to c. Which is not possible at all.

Therefore, the electric force dominates and the net force is repulsive.

Electric force >> Magnetic force