A Paradox in Electrostatics

PAGE NO.
DATE :

↳ Assuming ideal conditions, a conducting spherical shell kept in a gravitational field will change its electron distribution so as to make the net field zero everywhere inside the conductor. Alternately, the electrons in the conductor are free to move, so they will settle slightly under the ~~tor~~ influence of gravity — this resulting excess of electrons in the bottom will result in an electric field ~~at the centre~~ in the interior which will be cancelled out by the gravitational field.

MATHEMATICAL PROOF:

Let; ϕ_e : potential energy of electron.

$$\therefore \phi_e = -e\phi_E + m_e \phi_G$$

where; ϕ_E = electrostatic pot.
ϕ_G = gravitational pot.

↳ Force on $(e^-) = \nabla\phi_e$.

* At the surface, the force must be normal to the surface as a tangential component will imply that the electron is not in equilibrium.

$\therefore \phi_e$ is constant on surface.

↳ In the interior;

$$\nabla^2\phi_E = \nabla^2\phi_G = 0.$$

* By uniqueness theorem,

ϕ_e = constant (in the interior)

$\therefore$ for an e^- in the interior, force $= \nabla\phi_e = 0$.

DATE :

\# Now suppose that we had a conductor which also had negative 'muons'.

$$\therefore \phi_e = -e\phi_E + m_e\phi_G ,$$
$$\phi_\mu = -e\phi_E + m_\mu\phi_G .$$

* We can now repeat the same argument :-
ϕ_e must be constant on surface.
& $\nabla^2\phi_e = 0$.

* By uniqueness theorem,
$$\phi_e = C_1 \quad \{C_1 : \text{constant}\}$$

\# Similarly,
$$\phi_\mu = C_2 .$$

→ However, this leads to :

$$\phi_E = \frac{m_e C_2 - m_\mu C_1}{e(m_\mu - m_e)} = \text{constant.}$$

$$\phi_G = \frac{C_2 - C_1}{m_\mu - m_e} = \text{constant.}$$

$\Rightarrow \nabla\phi_E = \nabla\phi_G = 0$ in the interior.

$\therefore$ The interior is shielded by both — electrostatic forces & gravitational forces! However, this is absurd.

PAGE NO.
DATE :

RESOLUTION OF PARADOX:

→ The flaw is that we've taken ϕ_e = constant on the surface to avoid tangential forces. But if a region of the surface is devoid of e^- electrons, the argument that ϕ_e = constant on surface has no basis.

→ A similar explanation can be given for (e^- + muon) paradox. On the surface, the e^- & muon clearly cannot be in equilibrium at the same point. Rather, the conduction muons will fall to positions lower than conduction electrons.