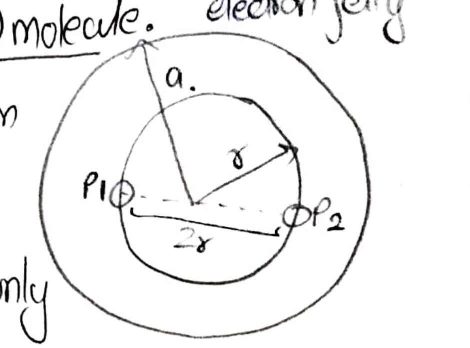
Electron Jelly Model

Q. Imagine a sphere of radius a filled with negative charge of uniform density, the total charge being equivalent to that of two electrons. Imbed in this jelly of negative charge two protons, and assume that, in spite of their presence, the negative charge distribution remains uniform. Where must the protons be located so that the force on each of them is zero?

A. Protons (each) will be in unstable equilibrium, since if slightly displaced, suppose towards outer negative charge or away from other proton, it will continue moving in that direction. (Similar to bond breaking)

<u>This is good representation of H_2 (H—H) molecule.</u>



The force on a proton, at radius r, from electron jelly is due to jelly inside radius r. Any electric field due to charge outside radius r will be only on outer side of surface.

The force due to jelly inside radius r will point towards center of sphere, since charge distributed is negative.

If net force on the proton is zero, force from other proton must point along force due to the jelly i.e., away from the center radially.

This means both protons must lie on the same diameter.

They must be the same distance r from the center because they feel the same force (magnitude) from each other and the jelly $\Rightarrow$ same value of r

$\because$ Volume $\propto r^3$

Negative charge inside shell $= q$

Total negative charge $= -2e$

$$q = \frac{-2er^3}{a^3}$$

Field at radius r due to the jelly is

$$E_n = +\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{-\cancel{2}er^{\cancel{3}}}{\cancel{2}\,4\pi\varepsilon_0 \cancel{r^2} a^3} = \frac{-er}{2\pi\varepsilon_0 a^3}$$

Field at one proton due to the other is

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{e}{(2r)^2} = \frac{e}{16\pi\varepsilon_0 r^2}$$

$\therefore$ At 1 proton $\quad E_p + E_n = 0$

$$\Rightarrow E_p = -E_n$$

$$\Rightarrow \frac{\cancel{e}}{8\,\cancel{16\pi\varepsilon_0} r^2} = \frac{\cancel{e}r}{\cancel{2\pi\varepsilon_0}a^3} \Rightarrow r^3 = \frac{a^3}{8} \Rightarrow \boxed{r = \frac{a}{2}}$$

If all $-2e$ electron charge were located at a point charge at the center, it would provide a force 8 times that of the force due to other proton. So forces balance if we reduce effective electron charge by a factor of 8.

$$q = \frac{(-2e)}{8} \quad \text{if} \quad r = \frac{a}{2}$$

$\because$ If $(-2e)$ at center, i.e., $\frac{r}{2}$ from 1 proton

$$E_n = \frac{-2e}{4\pi\varepsilon_0 r^2} \times 4 \qquad E_p = \frac{e}{4\pi\varepsilon_0 r^2}$$

$$E_n = 8\left(\frac{-e}{4\pi\varepsilon_0 r^2}\right)$$

$\therefore$ To make
$-E_n e = E_p e$
E_n needs to divided by 8.

This simple model of classical ~~mechanics~~ electrostatics produces results that are order-of-magnitude close to quantum calculations