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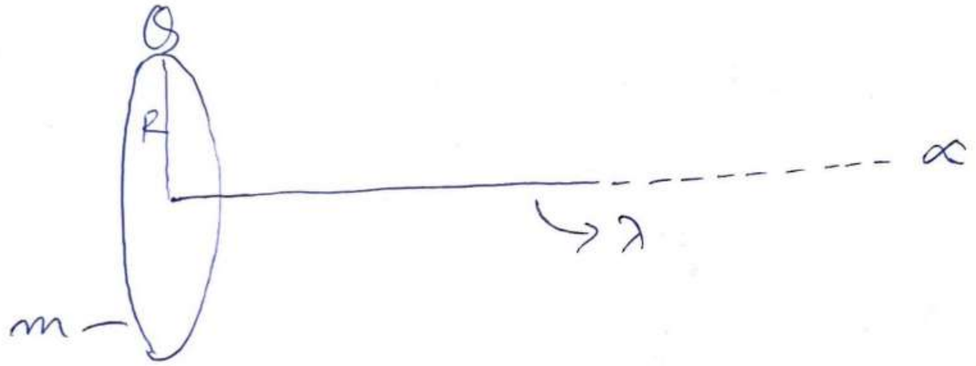
Roll-NO: 20221130 (B2, b)

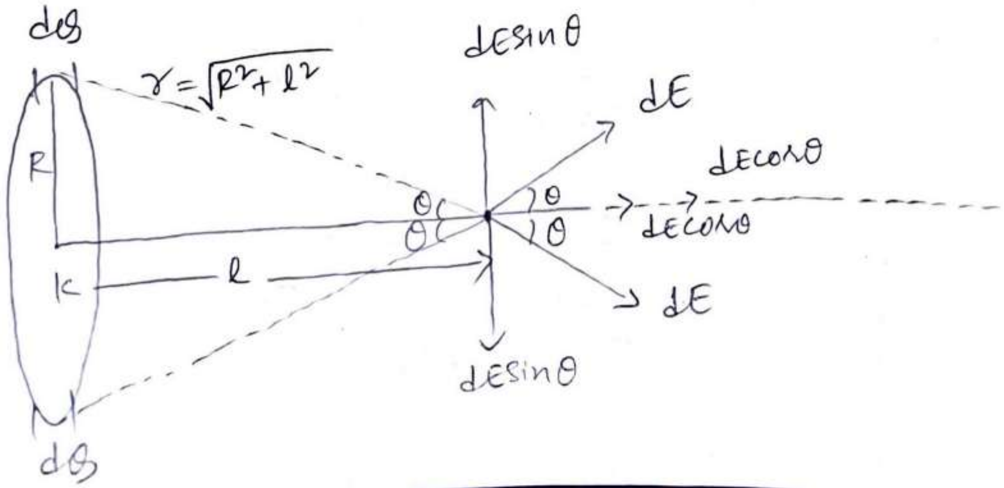
Q A fixed ring of mass 'm' and radius 'R' having uniform charge distribution Q is kept in space. A semi-infinite long wire passes through its centre having linear charge density 'λ'. Find the net force on wire due to the ring?

Soln:



Force on wire due to ring = ?

Let us approach this by fixing a point on the semi-infinite long wire at a distance 'l' from the centre of the ring.



cont:-

Now,

Let us find dE_{ring} at distance 'l'

$$\therefore \quad dE_{net} = 2dE\cos\theta$$

We know that, for dE due to $d\theta$ at a particular point say at distance l from centre can be written as

$$dE = \frac{k\,d\theta}{\left(\sqrt{R^2+l^2}\right)^2}$$

Therefore let us write the Net Electric field of Ring due to semi-infinite wire

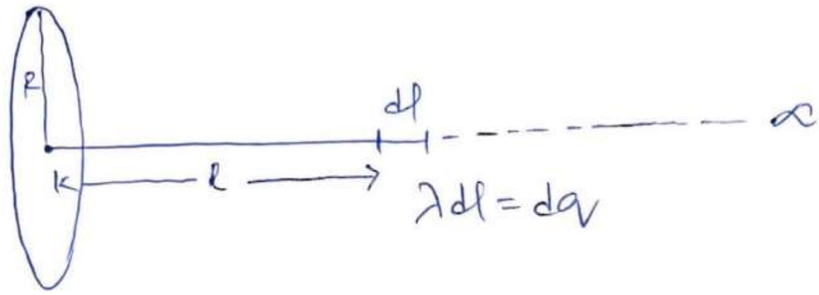
$$\int_0^{E_{ring}} dE_{net} = 2\int_0^{\theta} dE\cos\theta$$

$$E_{ring} = \int_0^{\theta} \frac{2k\,d\theta}{\left(\sqrt{R^2+l^2}\right)^2} \times \frac{l}{\sqrt{R^2+l^2}}$$

$$E_{ring} = \frac{2kl}{(R^2+l^2)^{3/2}} \int_0^{\theta} d\theta$$

$$E_{ring} = \frac{2k\theta l}{(R^2+l^2)^{3/2}}\,\hat{l}$$

Now



we also know that

$$\vec{F} = q\vec{E}$$

$\therefore$ force due to dl element; can be written as

$$d\vec{F} = dq\,\vec{E}$$

$$\int_0^{\vec{F}} d\vec{F} = \int_0^{\infty} \lambda\, dl \,\frac{2KQl}{(R^2+l^2)^{3/2}}\,\hat{l}$$

$$\vec{F} = \frac{KQ\lambda}{\cancel{Q}} \int_0^{\infty} \frac{2l\,dl}{(R^2+l^2)^{3/2}}\,\hat{l}$$

Let $R^2 + l^2 = t$ | when $l \to 0$ $t = R^2$

$dt = 2l\,dl$ | When $l \to \infty$ $t = \infty$

$$\therefore \quad \vec{F} = \frac{2KQ\lambda}{\cancel{Q}} \int_{R^2}^{\infty} \frac{dt}{t^{3/2}}\,\hat{l}$$

cont:-

$$\vec{F} = \frac{KQ\lambda}{\cancel{Q}}(-2) \quad t^{-1/2}\Big|_{R^2}^{\alpha} \hat{\ell}$$

$$\vec{F} = KQ\lambda(-2)\left[\cancel{\frac{1}{\sqrt{\alpha}}}^{\,0} - \frac{1}{\sqrt{R^2}}\right]\hat{\ell}$$

$$\vec{F} = (-2)KQ\lambda\left(-\frac{1}{R}\right)\hat{\ell}$$

$$\vec{F} = \frac{2KQ\lambda}{R}\hat{\ell} = \frac{\cancel{2}\lambda Q\hat{\ell}}{\cancel{4}_{2}\pi\epsilon_0 R} = \frac{\lambda Q\hat{\ell}}{2\pi\epsilon_0 R}$$

$\therefore$ force on ring due to wire is

$$\vec{F} = \frac{\lambda Q\hat{\ell}}{2\pi\epsilon_0 R}$$