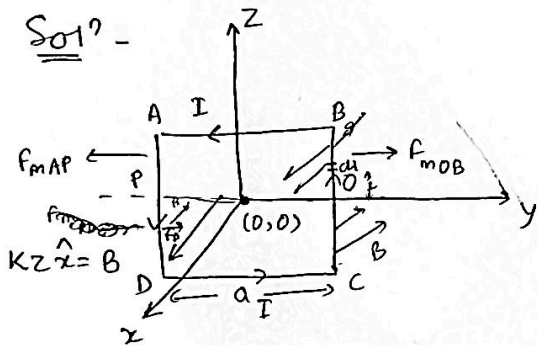


- ① Suppose that the magnetic field in some region has the form  $\vec{B} = Kx \hat{x}$ . (where  $K$  is a constant). Find the force on a square loop of side  $a$  lying on the  $yz$  plane and centered at the origin, if it carries a current  $I$ , flowing counterclockwise when you look down the  $x$  axis.

Sol<sup>n</sup> -

for force ~~on~~ on part AB -

$$\Rightarrow F_{m_{AB}} = \int I (d\vec{l} \times \vec{B}) \quad \text{here } B = K\left(\frac{a}{2}\right).$$

$$2) F_m = \left(\frac{Ka}{2}\right) \times aI = \frac{Ka^2I}{2} \hat{k}.$$

force

force on OB -

$$F_{m_{OB}} = \int I (d\vec{l} \times \vec{B}) = \int_0^{\frac{a}{2}} I x (d\vec{l} \times Kx \hat{x}) = \frac{Ka^2I}{4} \hat{j}.$$

on OC -

$$F_{m_{OC}} = -\frac{Ka^2I}{4} \hat{j}.$$

Similarly force on PD -  $F_{m_{PD}} = \frac{Ka^2I}{4} \hat{j}$

$$\text{on AP} = F_{m_{AP}} = -\frac{Ka^2I}{4} \hat{j}$$

and force on CD -

$$F_{m_{CD}} = \int I (d\vec{l} \times \vec{B}) = \left(\frac{Ka}{2}\right) \times aI = \frac{Ka^2I}{2} \hat{k}.$$

2) Total force on loop will be -

$$\begin{aligned} \vec{F}_{\text{Tot}} &= \vec{F}_{m_{AB}} + \vec{F}_{m_{BC}} + \vec{F}_{m_{CD}} + \vec{F}_{m_{DA}} + \vec{F}_{m_{AP}} + \vec{F}_{m_{OC}} \\ &= \frac{Ka^2I}{2} \hat{k} + \frac{Ka^2I}{2} \hat{k} \\ &= \frac{Ka^2I}{2} \hat{k} + \frac{Ka^2I}{2} \hat{k} \\ &= Ka^2I \hat{k}. \end{aligned}$$

$$\Rightarrow \boxed{F_{\text{Tot}} = Ka^2I \hat{k}}$$