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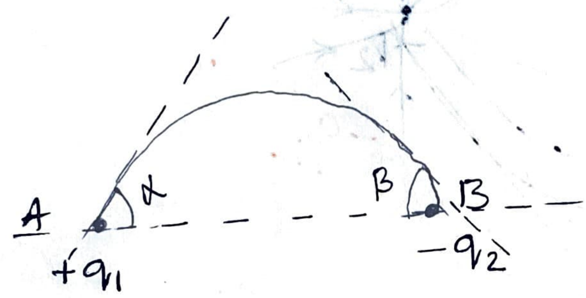
B-2 20221132

Q. Two charges $+q_1$ and $-q_2$ are placed at points A & B as shown. A (~~th~~) line of force Emerges out from q_1 at angle α with line AB. Find the angle β at which it terminates on $-q_2$.



Sol. For Given α & β

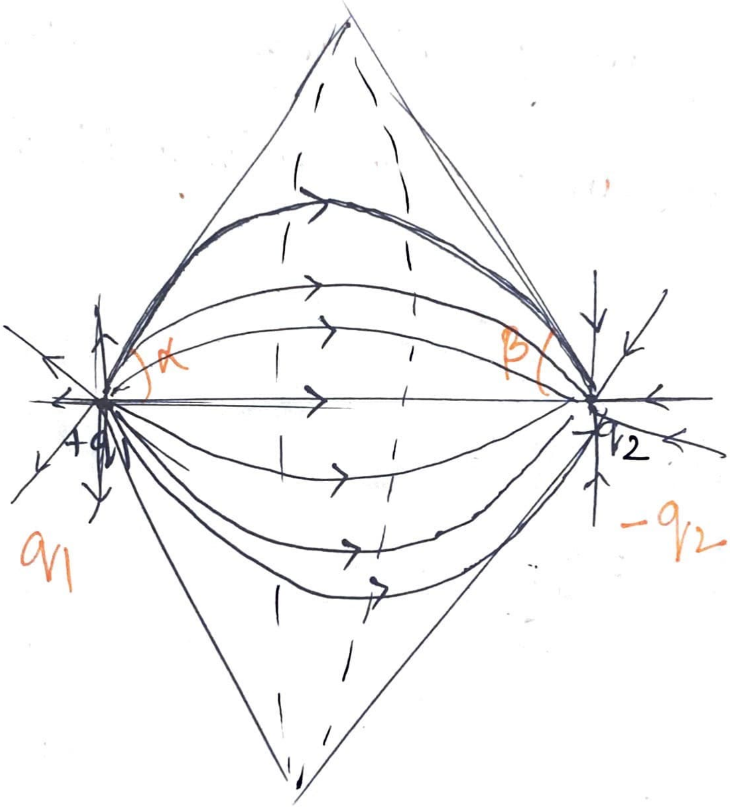
If the charges were Equal in magnitude then the system would be Symmetrical and the angles would be Equal

$\therefore \alpha = \beta$

But here $|q_1| \neq |q_2|$ they are not Equal

$\Rightarrow$

we will consider a cone in which all fields originating with half angle of the cone as α. Then all field lines will terminate at cone with half angle B.



we know from Gaus law that total flux originating at

$q_1 \Rightarrow$

$$\phi_{q_1} = \frac{q_1}{\varepsilon_0}$$

$q_2 \Rightarrow$

$$\phi_{q_2} = \frac{q_2}{\varepsilon_0}$$

Now the amount of flux coming through half angle is given by ↳ within the cone.

$$\phi_1 = \frac{\phi q_1}{4\pi}\left(\overbrace{2\pi(1-\cos\alpha)}^{\text{solid angle of cone with half angle }\alpha}\right) \rightarrow (1)$$

↳ solid angle of sphere $(\theta = \pi)$

$$\phi_2 = \frac{\phi q_2}{4\pi}\left(2\pi(1-\cos\beta)\right) \rightarrow (2).$$

Equating (1) & (2)

$$\frac{\phi q_1}{\cancel{4\pi}}(1-\cos\alpha)\cancel{2\pi} = \frac{\phi q_2}{\cancel{4\pi}}(1-\cos\beta)\cancel{(2\pi)}$$

$$\Rightarrow \frac{q_1}{\cancel{\varepsilon_0}}(1-\cos\alpha) = \frac{q_2}{\cancel{\varepsilon_0}}(1-\cos\beta)$$

$$1-\cos\beta = \frac{q_1}{q_2}(1-\cos\alpha)$$

$$\Rightarrow \quad \beta = \cos^{-1}\left[1-\frac{q_1}{q_2}(1-\cos\alpha)\right]$$

∴ The angle β is determined.

Bonus.

If $q_1 = q_2$

$$\beta = \cos^{-1}\left(1 - \frac{q_2}{q_1}(1-\cos\alpha)\right)$$

$$\beta = \cos^{-1}\left(1-1+\cos\alpha\right)$$

$$\beta = \cos^{-1}(\cos\alpha)$$

$$\boxed{\alpha = \beta}$$

which is Expected due to symmetry