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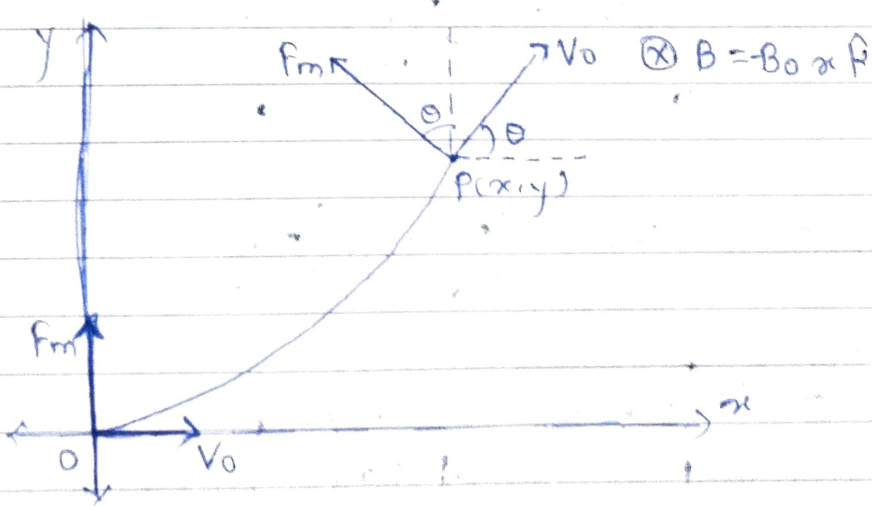
Q. A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = V_0 \hat{i}$ in a non-uniform magnetic field $B = -B_0 x \hat{k}$. Here, V_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x-coordinate of the particle during its motion.

→ Magnetic field is along negative z-direction. So in the coordinate axes shown in fig., it is perpendicular to paper inwards. (⊗) Magnetic force on the particle at origin is along positive y-direction. So, it will rotate in xy plane as shown. The path is not a perfect circle as the magnetic field is non-uniform. Speed of the particle in magnetic field remains constant. Magnetic force is always perpendicular to velocity. Let at point $P(x,y)$, its velocity vector makes an angle θ with positive x-axis. Then, magnetic force $\vec{F_m}$ will be at angle θ with positive y-direction. So,



At maximum x-displacement, $V_x = 0$ and the whole velocity is along +ve y-direction.

Now, $F_m = Bqv_0 \sin 90^\circ = Bqv_0$

$$a_y = \left(\frac{F_m}{m}\right)\cos\theta$$

$$\Rightarrow \frac{dv_y}{dt} = \frac{(B_0 x)(qv_0\cos\theta)}{m}$$

$$\Rightarrow \left(\frac{dv_y}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{B_0 q x}{m}\right)(v_0\cos\theta)$$

Here, $\frac{dx}{dt} = v_x = \cancel{v_0\cos\theta}\ v_0\cos\theta$.

$$\Rightarrow \frac{dv_y}{dx} = \left(\frac{B_0 q}{m}\right)x$$

$$\Rightarrow \int_0^{v_0} dv_y = \left(\frac{B_0 q}{m}\right)\int_0^{x_{max}} x\,dx$$

$$\Rightarrow v_0 = \left(\frac{B_0 q}{m}\right)\left(\frac{x_{max}^2}{2}\right)$$

$$\Rightarrow \boxed{x_{max} = \sqrt{\frac{2mv_0}{B_0 q}}}$$