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20221257: Physics Presentation

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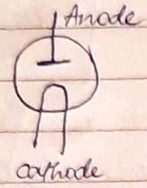
Presentation Topic:-

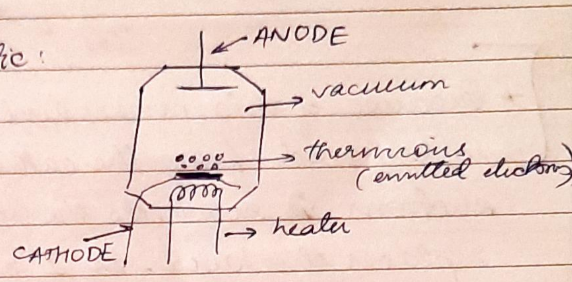
Vacuum Diode, its properties and Child Langmuir law.

device that only allows current to flow in one direction.

Vacuum Diode: [aka thermionic tube / fleming valve]

Invented in 1904 by Sir John Ambrose Fleming!

Symbol: 

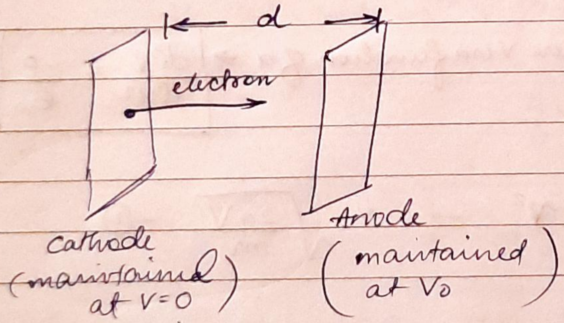
Schematic: 

Anode: ~~[illegible]~~ Mo/Ni

Cathode: Nickel covered in BaO and Sr.

[sometimes anode size > cathode size to enable rapid cooling of anode]

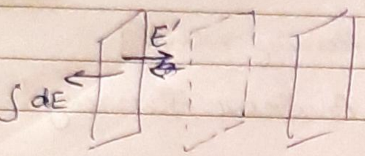
Working principle: thermionic emission ("boiling off of electrons from cathode")



Q. Explain this phenomenon:

"The cloud of electrons within the gap (called 'space charge') quickly builds up to point where it reduces the field at the surface of the cathode to zero. From there, a steady current I flows between the plates."

Answer: One can think of the space charge as being composed of thin sheets of plane charges, and thus the electric field ~~would be~~ by any of these layers is opposite to the direction of the electric field caused due to V_0 and 0 potential difference b/w the plates



$E' = -\int dE$

↳ during this stage, Electric field at the cathode becomes zero!

* Because of this nonexistent electric field, no new 'layer' of electrons is removed from the cathode, until a preexisting layer of electrons reaches the anode, at which point the new layer replaces this layer, so I remains constant

Q
b) Suppose the plates are relatively larger than the separation ($A \gg d^2$), find V, ρ, v of electrons as a function of x alone.

Something that relates V and ρ $\nabla^2 V = \frac{-\rho}{\epsilon_0}$

here V is a function of x ⇒ $\boxed{\frac{d^2V}{dx^2} = \frac{-\rho}{\epsilon_0}}$

Something that relates V and v ⇒ $\underbrace{qV}_{\text{as } V_{(0)}=0} = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2qV}{m}}$ — ①

Another thing that we know, another invariant is I.

$I = \frac{dq}{dt} = \frac{(\rho A\, dx)}{dt} = \rho A\left(\frac{dx}{dt}\right) = \rho A v$ — ②

$\rightarrow \rho = \frac{I}{Av}$

So, we can use ① and ② to convert everything into a differential equation in V.

$$\frac{d^2V}{dx^2} = \frac{-I}{Av\epsilon_0} = \frac{-I}{A\epsilon_0}\sqrt{\frac{m}{2qV}} = \frac{\beta}{\sqrt{V}}$$

$$\boxed{\frac{d^2V}{dx^2} = \beta V^{-1/2}}$$, where $\beta = \frac{-I}{A\epsilon_0}\sqrt{\frac{m}{2q}}$

$q>0$
$I<0$
$\rightarrow \beta > 0$

to solve this,

multiply by V' $\left(= \frac{dV}{dx}\right)$ on both sides.

$$\cancel{\frac{d^2V}{dx^2}} \qquad \frac{dV'}{dx} \times V' = \beta V^{-1/2} \times \frac{dV}{dx}$$

$$\int_{V_0'(x)}^{V'(x)} V' \, dV' = \beta \int_{V(0)}^{V(x)} V^{-1/2} \, dV$$

$$\frac{V'^2_{(x)} - V'^2_{(0)}}{2} = 2\beta\left(V^{1/2}_{(x)} - V^{1/2}_{(0)}\right)$$

But $V'_{(0)} = 0 \longrightarrow$ as field at cathode is 0
$V(0) = 0 \longrightarrow$ as $V_{(0)} = 0$ (in the exam).

$$\frac{V'^2_{(x)}}{2} = 2\beta V^{1/2}_{(x)}$$

$$\rightarrow \boxed{\left(\frac{dV}{dx}\right)^2 = 4\beta V^{1/2}}$$

$$\frac{dV}{dx} = 2\sqrt{\beta}\, V^{1/4} \rightarrow V^{-1/4} dV = 2\sqrt{\beta}\, dx$$

$$\downarrow$$

$$\frac{4}{3} V^{3/4} = 2\sqrt{\beta}\, x + C$$ $=0$, as $V(0)=0$

$$\boxed{V = \left(\frac{3}{2}\sqrt{\beta}\right)^{4/3} x^{4/3}}$$

(on substituting the value of β, $V = \left(\frac{81 I^2 m}{32 \epsilon_0^2 A^2 q}\right) x^{4/3}$)

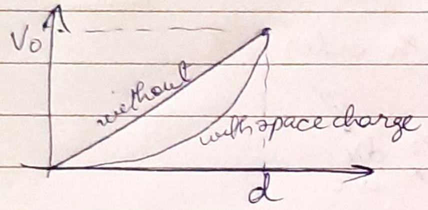
★ writing this in terms of $V_{(d)} = V_0$.

$$\rightarrow \quad V_0 = \left(\frac{3}{2}\sqrt{\beta}\right)^{4/3} d^{4/3}$$

$$\rightarrow \quad \boxed{V = V_0 \left(\frac{x}{d}\right)^{4/3}}$$

Without space charge, it would be similar to a capacitor.

$$V(x) = V_0 \left(\frac{x}{d}\right).$$



☆ Child Langmuir Law :

$$V_0 = \left(\frac{8I^2 m}{32\epsilon_0^2 A^2 q}\right)^{1/3} d^{4/3}$$

write relation between V_0 and I.

$$I = \left(\frac{4\epsilon_0 A}{9d^2}\sqrt{\frac{2q}{m}}\right) V_0^{3/2} \Rightarrow I = K V_0^{3/2}$$