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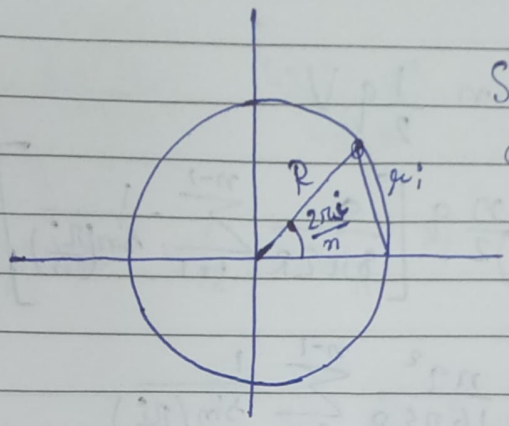
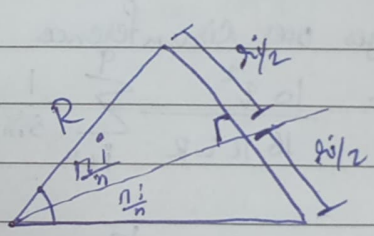
Q→ What is the minimum energy configuration for a system of N equal point charges placed on or inside a circle of radius R? Show that for N = 12 it is better to place 11 on the circumference & one at the centre.

Soln→



Suppose the n point charges are evenly spaced around the circle, with i^{th} particle at angle $\frac{2\pi i}{n}$.

We can get diff. charges on changing value of i.



$$\sin\theta = \frac{P}{H} \Rightarrow \sin\left(\frac{\pi i}{n}\right) = \frac{r_i}{2R}$$

$$r_i = 2R \sin\left(\frac{\pi i}{n}\right)$$

So, $$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)$$

$$W_n = n\frac{1}{2} q V$$

here, V is due to (n-1) other charges potential.

$$V = \frac{1}{4\pi\varepsilon_0} q \sum_{i=1}^{n-1} \frac{1}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0} q \sum_{i=1}^{n-1} \frac{1}{2R\sin\left(\frac{\pi i}{n}\right)}$$

$$V = \frac{q}{8\pi\varepsilon_0 R} \sum_{i=1}^{n-1} \frac{1}{\sin\left(\frac{\pi i}{n}\right)}$$

$$W_n = n \frac{1}{2} q V$$

$$= \frac{n}{2} q \left[\frac{q}{8\pi\varepsilon_0 R} \sum_{i=1}^{n-1} \frac{1}{\sin\left(\frac{\pi i}{n}\right)} \right]$$

$$W_n = \frac{n q^2}{16\pi\varepsilon_0 R} \sum_{i=1}^{n-1} \frac{1}{\sin\left(\frac{\pi i}{n}\right)}$$

For all charges over circumference

$$W_{10} = \frac{10 q^2}{16\pi\varepsilon_0 R} \sum_{i=1}^{9} \frac{1}{\sin\left(\frac{\pi i}{10}\right)} = 38.62 \frac{q^2}{4\pi\varepsilon_0 R}$$

$$W_{11} = \frac{11 q^2}{16\pi\varepsilon_0 R} \sum_{i=1}^{10} \frac{1}{\sin\left(\frac{\pi i}{11}\right)} = 48.57 \frac{q^2}{4\pi\varepsilon_0 R}$$

$$W_{12} = \frac{12 q^2}{16\pi\varepsilon_0 R} \sum_{i=1}^{11} \frac{1}{\sin\left(\frac{\pi i}{12}\right)} = 59.80 \frac{q^2}{4\pi\varepsilon_0 R}$$

For $(n-1)$ charges over circumference & n^{th} on centre.

$$W'_n = W_{n-1} + (n-1)\frac{q^2}{4\pi\varepsilon_0 R}$$

$$W'_{11} = W_{10} + (11-1)\frac{q^2}{4\pi\varepsilon_0 R}$$

$$= \frac{10 q^2}{16\pi\varepsilon_0 R} \sum_{i=1}^{9} \frac{1}{\sin\left(\frac{\pi i}{10}\right)} + \frac{10 q^2}{4\pi\varepsilon_0 R}$$

$$= \frac{10\,q^2}{4\pi\varepsilon_0 R}\left[\frac{10}{4}\sum_{i=1}^{9}\frac{1}{\sin\left(\frac{\pi i}{10}\right)} + 10\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0 R}\left[38.62 + 10\right]$$

$$W_{11}' = \frac{48.62\,q^2}{4\pi\varepsilon_0 R}$$

$$\boxed{W_{11}' > W_{11}}$$

$$W_{12}' = W_{11} + (12-1)\frac{q^2}{4\pi\varepsilon_0 R}$$

$$= \frac{11\,q^2}{16\pi\varepsilon_0 R}\sum_{i=1}^{10}\frac{1}{\sin\left(\frac{\pi i}{11}\right)} + \frac{11\,q^2}{4\pi\varepsilon_0 R}$$

$$= \frac{q^2}{4\pi\varepsilon_0 R}\left[\frac{11}{4}\sum_{i=1}^{10}\frac{1}{\sin\left(\frac{\pi i}{11}\right)} + 11\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0 R}\left[48.57 + 11\right]$$

$$W_{12}' = 59.57\,\frac{q^2}{4\pi\varepsilon_0 R}$$

$$\boxed{W_{12}' < W_{12}}$$

∴ it's to be noted that in 11 charges system the work done or energy is less when arranged on circumference while in 12 charges system the work done or energy is less when $(n-1)$ charges arranged on circumference and n^{th} charge on centre.

It's applicable for n greater than 12 only.