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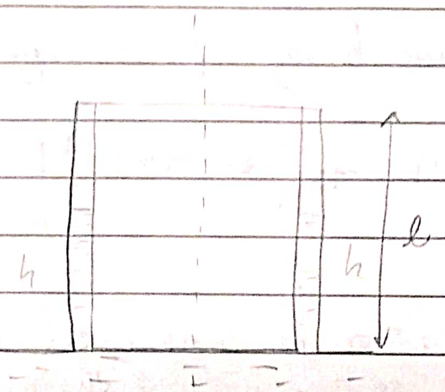
PH1213 → Tutorial Problem.

Q. A cylindrical capacitor connected to a DC Voltage source V touches the surface of water with its end. The separation 'd' between the cylindrical electrodes is substantially less than their mean radius (R >> d). Find a height 'h' to which the water level in the gap will rise. ~~The capillary effects are to be neglected.~~



Ans – The initial capacitance of this set up before water rises in it is :-

$$U_i = \frac{1}{2} C_0 V^2 \quad \text{where} \quad C_0 = \varepsilon_0 \frac{2\pi R l}{d}$$

Here
R → mean radius, d → gap b/w the plates; l = length/height of cylinder

Suppose liquid rises to a height h in it. Then the capacitance of the condensator is :-

$$C = \frac{\varepsilon \varepsilon_0 \, 2\pi R h}{d} + \frac{\varepsilon_0 (l-h) \, 2\pi R}{d} = \frac{\varepsilon_0 2\pi R}{d}\left(l + (\varepsilon - 1) h\right)$$

Now energy of the capacitor & the liquid on the whole is :-

$$E = \frac{1}{2} C V^2 + \rho g (2\pi R h d) \frac{h}{2}$$

$$= \frac{1}{2} \varepsilon_0 \frac{2\pi R}{d} (l + (\varepsilon - 1)h) + \rho g (2\pi R h d) \frac{h}{2}$$

Suppose we increase h by δh, Energy increases as :-

$$\delta E = \delta h \left(\frac{\varepsilon_0 2\pi R (\varepsilon - 1) V^2}{2d} + \rho g (2\pi R d) h \right)$$

Since the capacitor is connected to a battery, the entire potential energy of the system is to be minimised. This can happen when.

$$\frac{\varepsilon_0 2\pi R (\varepsilon - 1) V^2}{2d} = \rho g (2\pi R d) h$$

$$\Rightarrow \rho g d h = \frac{\varepsilon_0 (\varepsilon - 1) V^2}{2d}$$

$$\Rightarrow h = \frac{\varepsilon_0 (\varepsilon - 1) V^2}{2 \rho d^2 g}$$

Hence, h is the maximum height to which the water levels will rise.