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1] Prove the green's reciprocity theorem, for charge distribution $\rho_1(r)$ and $V_1(r)$ & $\rho_2(r)$ and $V_2(r)$ that,

$$\int_{\text{all space}} V_1 \rho_2 \, d\tau = \int_{\text{all space}} \rho_1 V_2 \, d\tau$$

Ans 1] Proof :- Let us evaluate,

$$\int_{\text{all space}} \vec{E_1} \cdot \vec{E_2} \, d\tau = \int_{\text{all space}} \vec{E_2} \cdot \vec{E_1} \, d\tau$$

$$\int (\nabla V_1) \cdot (\nabla V_2) \, d\tau = \int (\nabla V_2) \cdot (\nabla V_1) \, d\tau$$

W.K., $\cancel{(\nabla V_1)} \cdot \nabla \cdot (V_1 \nabla V_2) = (\nabla V_1)\cdot(\nabla V_2) + V_1 \nabla^2 V_2$

$$\nabla \cdot (V_2 \nabla V_1) = (\nabla V_2) \cdot (\nabla V_1) + V_2 \nabla^2 V_1$$

$$\therefore \int \nabla \cdot (V_1 \nabla V_2) \, d\tau - \int V_1 \nabla^2 V_2 \, d\tau$$

$$= \int \nabla \cdot (V_2 \nabla V_1) \, d\tau - \int V_2 \nabla^2 V_1 \, d\tau$$

Using Poisson's Equation, And Green's theorem,

$$\oint_S V_1 \vec{E_2} \, d\vec{a} + \frac{1}{\varepsilon_0} \int V_1 \rho_2 \, d\tau$$

$$= \oint_S V_2 \vec{E_1} \, d\vec{a} + \frac{1}{\varepsilon_0} \int V_2 \rho_1 \, d\tau$$

Since ~~($\vec{E_1}$ & $\vec{E_2}$ is)~~ V_1 & V_2 at infinite surface, is zero,

$$\frac{1}{\varepsilon_0} \int_{\text{all space}} V_1 \rho_2 \, d\tau = \frac{1}{\varepsilon_0} \int_{\text{all space}} V_2 \rho_1 \, d\tau$$

$$\boxed{\int_{\text{all space}} V_1 \rho_2 \, d\tau = \int_{\text{all space}} V_2 \rho_1 \, d\tau}$$