Physics Presentation Problem

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Tutorial Batch: B3

Q. An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the "north pole" & the center.

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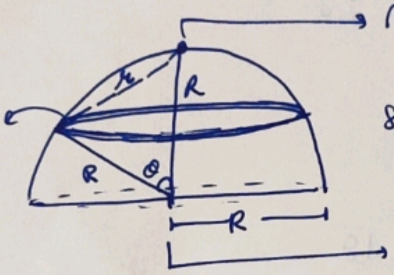
North pole

small area element 'da'



surface charge density = σ

center

For a uniformly charged surface, we know that

$$V_{center} = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma}{r}\,da$$

For this perticular hemisphere,

$$V_{center} = \frac{1}{4\pi\epsilon_0}\frac{\sigma}{\cancel{R}}\int da = \frac{1}{4\pi\epsilon_0}\frac{\sigma}{\cancel{R}}\int_0^R \frac{2\pi}{r}\, r\,dr$$

~~[illegible]~~

$$\therefore\ V_{center} = \frac{\sigma R}{2\epsilon_0}$$

Finding V at pole:

Consider a small area element 'da' on the sphere as shown in the diagram. We will integrate da over θ from $\underbrace{0 \text{ to } \pi/2}$.

↳ Reason being we are considering only the northern hemisphere

~~Using polar coordinates~~.

$da = \cancel{2R} R^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$ — spherical coordinates

$r^2 = \cancel{2R^2} \, 2R^2(1-\cos\theta)$ — cosine rule

Now

$\cancel{V_{pole} = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma}{r^2} da = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma \cdot 2R^2(1-\cos\theta)}{\sqrt{2R^2(1-\cos\theta)}}}$

$$V_{pole} = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma}{r} da = \frac{\sigma}{4\pi\epsilon_0}\int_0^{\pi/2}\int_0^{2\pi} \frac{R^2 \sin\theta \, d\theta \, d\phi}{\sqrt{2R^2(1-\cos\theta)}}$$

$$= \frac{\sigma}{4\pi\epsilon_0}\int_0^{\pi/2}\int_0^{2\pi} \frac{R^2 \sin\theta \, d\theta \, d\phi}{R\sqrt{2(1-\cos\theta)}}$$

$$= \frac{\sigma R}{4\pi\epsilon_0} \cdot \frac{2\pi}{\sqrt{2}} \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma R}{2\sqrt{2}\,\epsilon_0}\left[2\sqrt{1-\cos\theta}\right]_0^{\pi/2}$$

$$= \frac{\sigma R}{2\sqrt{2}\,\epsilon_0}(2-0)$$

$$V_{pole} = \frac{\sigma R}{\sqrt{2}\,\epsilon_0}$$

To find the potential difference:

$$\Delta V = V_{pole} - V_{center} = \frac{\sigma R}{\sqrt{2}\,\epsilon_0} - \frac{\sigma R}{2\epsilon_0}$$

$$\therefore \boxed{\Delta V = \frac{\sigma R}{2\epsilon_0}(\sqrt{2}-1)}$$