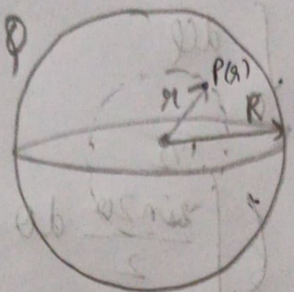
PH-1213 | Phy - Tutorial (T-6) | Shravan Rekhi 20221249

Q. Find the net force that northern hemisphere of a uniformly charged solid sphere experiences. Let the total charge be Q and radius R.

⇒



We know that $\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{q(r)}{\frac{4}{3}\pi r^3}$

Electric field at $P(r) = \vec{E}(r)$

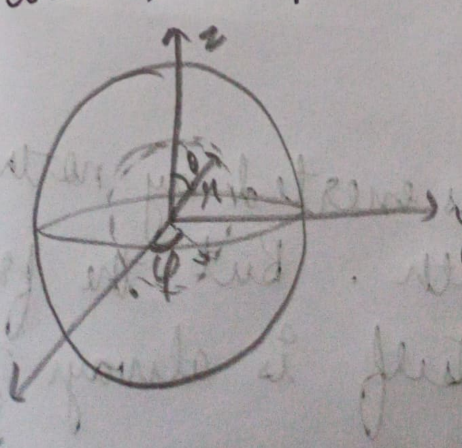
$$\Rightarrow \quad \vec{E}(r)\,(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_0}\,\hat{r}$$

$$\Rightarrow \quad \vec{E}(r) = \frac{\rho r}{3\epsilon_0}\,\hat{r} \quad \text{(radially outwards)}$$

Force ~~per~~ = qE

⇒ Force per unit volume = $\rho \cdot E(r)$

By symmetry, we can say that net force on the northern hemisphere would be in the z-dir^n.



$$F_z = \int F(\hat{r}) \cdot \hat{z} \, d\tau$$

$$= \int \frac{\rho^2 r}{3\epsilon_0} \cos\theta \times r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$F_z = \frac{\rho^2}{3\epsilon_0} \int_0^R r^3 \cdot dr \int_0^{\pi/2} \sin\theta\cos\theta \cdot d\theta \int_0^{2\pi} d\varphi$$

$$= \frac{3}{\epsilon_0}\left(\frac{q}{4\pi R^3}\right)^2 \left(\frac{R^4}{4}\right)(2\pi) \int_0^{\pi/2} \frac{\sin 2\theta}{2} \cdot d\theta$$

$$= \frac{3}{\epsilon_0} \frac{q^2}{16\pi^2 R^6} \frac{R^4}{4} \cdot 2\pi \cdot \left[\frac{-\cos 2\theta}{4}\right]_0^{\pi/2}$$

$$= \frac{3}{\epsilon_0} \frac{q^2}{16\pi^{\cancel{2}} \cancel{R^6}\, R^2} \cdot \frac{\cancel{R^4}}{4} \cdot \cancel{2\pi} \cdot \left(\frac{\cancel{2}}{\cancel{4}}\right)$$

$$\boxed{F_z = \frac{3q^2}{64\pi\epsilon_0}}$$

This is the net force the northern hemisphere experiences here.

This is the sum of the forces exerted by northern and southern hemispheres both. But the force on northern hemisphere by itself is always zero.