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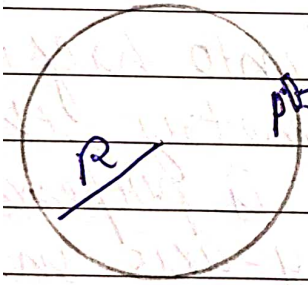
BATCH 6 (B4)

A conducting sphere with total charge Q is cut into half. What force must be used to hold the halves together?

Solution:

The charge is entirely distributed over the surface with a surface charge density of $\sigma = \frac{Q}{4\pi R^2}$, where R is the radius of the sphere.

Now, we consider a point p inside the sphere close to an element area ds. The charge dq on this area element will produce at the point p an electric field which is approximately that due to a uniformly charged infinite plate



$$\vec{E}_{\text{on p due to ds}} = -\frac{\sigma}{2\varepsilon_0}\hat{n},$$

where n is a unit vector normal to ds in the outward direction.

As we know Electric field is zero inside the sphere. Hence, electric field due to other charges on the spherical surface except element ds,

$$E_{\text{Total}} = \vec{E}_{\text{on p due to ds}} + \vec{E}_{\text{on p due to rest}} = 0$$

$$\vec{E}_{\text{on p due to rest}} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

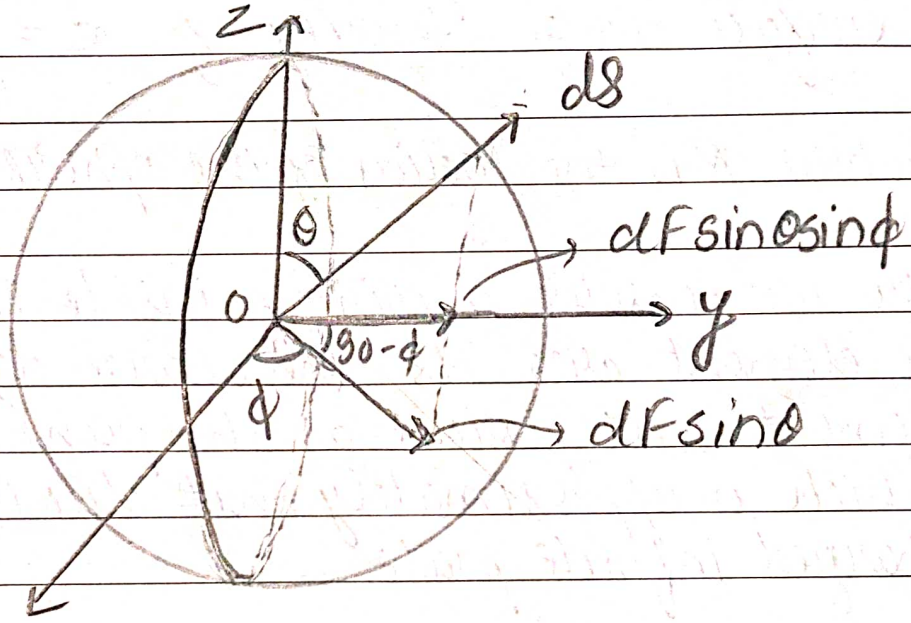
Hence, the force acting on ds element is

$$dF = dq \times E \text{ on } p \text{ due to rest others}$$

$$\vec{dF} = dq \times \frac{\sigma}{2\epsilon_0}\hat{n}$$

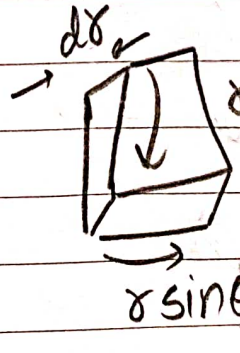
$dq = \sigma ds$, so

$$\vec{dF} = \frac{\sigma^2}{2\epsilon_0}\hat{n}$$



The plane where the sphere is cut into half is taken to be the xoz plane. Due to symmetry, the repulsive force between the two half-sphere is perpendicular to the xoz plane and along the y-axis on the right half. The magnitude of the resultant force is

$$F = \int dF\sin\theta\sin\phi = \frac{\sigma^2}{2\epsilon_0}\int \sin\theta\sin\phi\, ds$$



$$ds = r\sin\theta\, d\phi\, r\, d\theta$$
$$= r^2\sin\theta\, d\phi\, d\theta$$

$$F = \frac{\sigma^2}{2\epsilon_0}R^2\int_0^{\pi}\sin\phi\, d\phi\int_0^{\pi}\sin^2\theta\, d\theta$$

The limit of ϕ range from 0 to π as the sphere is cut into half.

$$F = \frac{\pi \sigma^2 R^2}{2\varepsilon_0} = \frac{Q^2}{32\pi\varepsilon_0 R^2} \quad \left[\because \sigma = \frac{Q}{4\pi R^2}\right]$$

This is the force needed to hold the two halves together.