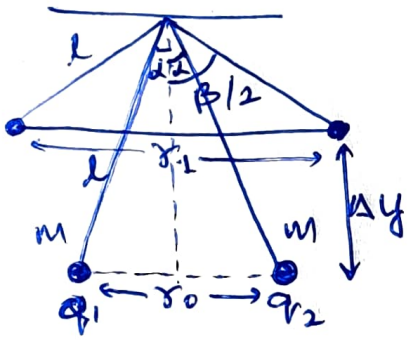
EG: Two small metal balls of mass $m = 0.1g$ are suspended at same point by insulating threads of length $l = 30cm$ one of balls is loaded with twice as much as charge as other. Pushing balls towards each other we move them to a position where both threads make an angle of $\alpha = 20°$ with vertical plane and threads remain in common vertical plane. After Releasing two balls at same time angle between two Threads Reaches longest value of $\beta = 84°$ determine charges.

Solution. since both charges experience same forces and have same mass they Reach extreme point together.



~~using the work-Energy Theorem,~~

~~$K_i = K_f = 0$ (since $v = 0$ at both instants)~~

using conservation of mechanical energy,

$E_i = K(q_1)(q_2)/r_0$ [electrostatic P.E]

$E_i = K(q_1)(q_2)/(2l\sin\alpha)$

and $E_f = K(q_1 q_2)/r_2 + 2mg(\Delta y)$

$E_f = Kq_1q_2/(2l\sin\beta) + 2mgl(\cos\alpha - \cos(\frac{\beta}{2}))$

Equating, $E_i = E_f$

$$\frac{Kq_1q_2}{2l\sin\alpha} = \frac{Kq_1q_2}{2l\sin\beta} + 2mgl(\cos\alpha - \cos\beta/2)$$

$$Kq_1q_2\left(\frac{1}{\sin\alpha} - \frac{1}{\sin(\beta/2)}\right) = 2mgl(\cos\alpha - \cos\frac{\beta}{2})$$

$$q_1q_2 = \frac{4mgl^2}{K}\left(\frac{\cos\alpha - \cos\beta/2}{\sin\beta/2 - \sin\alpha}\right)\sin\alpha\sin\beta/2$$

in the end, given $q_1 = Q$ and $q_2 = 2Q$.

$$Q(2Q) = 4\pi\varepsilon_0 l^2 \left(\frac{1}{k}\right)\left(\frac{\cos\alpha - \cos\beta/2}{\sin\beta/2 - \sin\alpha}\right)\sin\alpha \sin\beta/2$$

$$Q^2 \approx \sqrt{2\pi\varepsilon_0 l^2 \left(\frac{1}{k}\right)\left(\frac{\cos\alpha - \cos\beta/2}{\sin\beta/2 - \sin\alpha}\right)\sin\alpha \sin\beta/2}$$

$\cos(\alpha) = \cos(20°) \approx 0.94$; $\sin 20° = 0.34$

$\cos(\beta/2) = \cos(42°) \approx 0.74$ $\sin 42° = 0.67$

$$Q \approx \sqrt{\frac{2\times 10^{-8} \times (3\times 10^{-2})^2}{9\times 10^9}\left(\frac{\cos 20° - \cos 42°}{\sin 42 - \sin 20}\right)\sin 20 \sin 42}$$

$$Q \approx \left(2\times 10^{-3}\times 10^{-4}\times 10^{-9}\left(\frac{0.94 - 0.74}{0.67 - 0.34}\right) 0.67 \times 0.34\right)^{1/2}$$

$$Q \approx \left(2\times 10^{-14} \times 0.60 \times 0.67 \times 0.34\right)^{1/2}$$

$$Q \approx \left(0.27 \times 10^{-14}\right)^{1/2} \approx 0.519 \times 10^{-7}$$

or $\boxed{Q \approx 5.2 \times 10^{-8} \text{ Coulomb}}$ ✓

and $\boxed{2Q \approx 10.4 \times 10^{-8} \text{ C}}$ ✓