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B3

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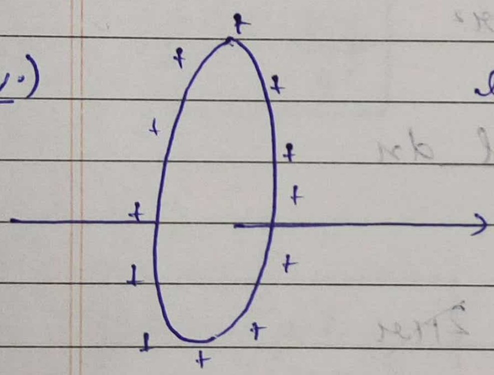
Physics Problem (Presentation)

Q.) a) A thin wire ring of radius r carries a charge q. Find the magnitude of the Electric field strength on the axis of ring as a function of distance l from its centre.

b) Investigate the obtained function at $l >> r$.

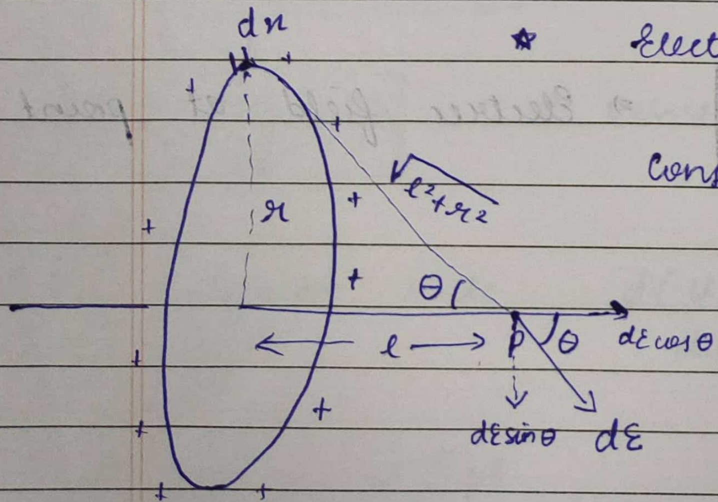
c) Find the maximum strength magnitude and corresponding distance L.

sol.)



Given = a ring in which charge q is distributed uniformly.

Total charge on ring = q.



★ Electric field at P

Consider an element of length = dx

Charge on dx = dq = linear charge density (λ) × dx

$dq = \lambda \cdot dx$

$$\boxed{dq = \frac{q}{2\pi r} \cdot dx}$$

$$\boxed{dE = \frac{Kq}{r^2}}$$

$$dE = \frac{K \cdot q\,dx}{2\pi r} \cdot \frac{1}{(\sqrt{l^2+r^2})^2}$$

$$\boxed{dE = \frac{Kq \cdot dx}{2\pi r(l^2+r^2)}}$$

$$\boxed{\cos\theta = \frac{l}{\sqrt{l^2+r^2}}}$$

For $E_{net} = \int dE \cos\theta$

$$E_{net} = \int \frac{Kq}{2\pi r(l^2+r^2)} \cdot \frac{l}{\sqrt{l^2+r^2}}\,dx$$

$$E_{net} = \int \frac{Kq}{2\pi r(l^2+r^2)^{3/2}} \cdot l\,dx$$

$$E_{net} = \oint \frac{Kq}{\cancel{2\pi r}(l^2+r^2)^{3/2}} \cdot l \cdot \cancel{2\pi r}$$

$$\boxed{E_p = \frac{Kql}{(l^2+r^2)^{3/2}}}$$ $\Rightarrow$ Electric field at point P.

Part B

b) Electric field at P = $E_p = \frac{Kql}{(l^2+r^2)^{3/2}}$ if $l >> r$

If $l >> r$ then neglect r.

$$E_p = \frac{Kql}{l^3} = \boxed{E_p = \frac{Kq}{l^2}}$$

Now for a point charge at a distance l

$$\boxed{E_p = \frac{Kq}{l^2}}$$

∴ Ring behaves like a point charge, reducing the field to the value.

© $\boxed{E_p = \frac{Kql}{(l^2+r^2)^{3/2}}}$ E_p is maximum.

$\boxed{\frac{dE_p}{dl} = 0}$ l value is maxima or minima

Division rule $\frac{d(U/V)}{dl} = \frac{V\frac{du}{dl} - U\frac{dv}{dl}}{V^2}$

$U = Kql$

$V = (l^2+r^2)^{3/2}$

$$\boxed{V\frac{du}{dl} - U\frac{dv}{dl}}$$

$$\Rightarrow (l^2+r^2)^{\cancel{3/2}\,1} \cdot \cancel{Kq} = \cancel{Kq}\,l \cdot \frac{3}{\cancel{2}} (l^2+r^2)^{\frac{1}{2}} \cdot \cancel{2}l$$

$$l^2+r^2 = 3l^2$$

$$2l^2 = r^2$$

$$\boxed{l = \pm \frac{r}{\sqrt{2}}}$$

Hence $l = +\frac{r}{\sqrt{2}}$ will give maxima

$l = -\frac{r}{\sqrt{2}}$ will give minima