Name - Prachi Roll no - 20221191

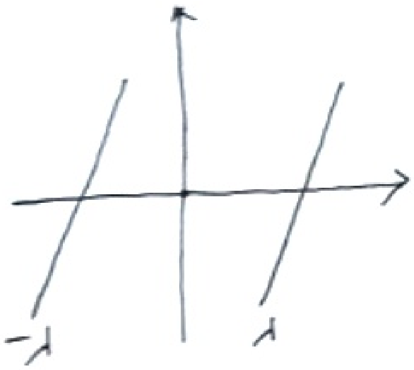
Q- Two infinitely long wires running parallel to the x-axis carry uniform charge densities $+\lambda$ and $-\lambda$.

(a) Find the potential at any point $(x, y, z)$ using origin as reference.

(b) Show that the equitarial surfaces are circular cylinders and locate the axis and radius of the cylinder.

We know that, $V(r) = -\int_0^r \vec{E} \cdot d\vec{l}$

Sol- (a)




From Gauss Law, $\oint E \cdot da = \frac{Q_{in}}{\varepsilon_0}$

$$|E| (2\pi s l) = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi s \varepsilon_0} \hat{s}$$

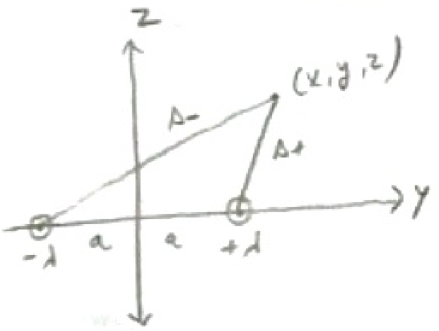
$$\Rightarrow V(r) = \int_{a}^{r} \frac{\lambda}{2\pi s \varepsilon_0} \, ds = \frac{-\lambda}{2\pi \varepsilon_0} \ln\left(\frac{r}{a}\right)$$

So, potential of $+\lambda$ is $V_+ = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_+}{a}\right)$

potential of $-\lambda$ is $V_- = \frac{+\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_-}{a}\right)$

$$\therefore \text{Total } V = \frac{\lambda}{2\pi\varepsilon_0}\left[-\ln\left(\frac{s_+}{a}\right) + \ln\left(\frac{s_-}{a}\right)\right] = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{a}{s_+}\right) + \ln\left(\frac{s_-}{a}\right)\right]$$

$$\Rightarrow \boxed{V = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{s_-}{s_+}\right]}$$




$$s_+ = \sqrt{(y-a)^2 + z^2}$$

$$s_- = \sqrt{(y+a)^2 + z^2}$$

$$\text{So, } V(x,y,z) = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{(y+a)^2+z^2}}{\sqrt{(y-a)^2+z^2}}\right)$$

$$\Rightarrow V_{(x,y,z)} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{(y+a)^2+z^2}{(y-a)^2+z^2}\right]$$

(b). Equipotentials are given by $\frac{(y+a)^2+z^2}{(y-a)^2+z^2} = e^{(4\pi\varepsilon_0 V/\lambda)} = k$

$$(y+a)^2 + z^2 = k\left[(y-a)^2 + z^2\right]$$

$$y^2+a^2+2ay+z^2 = K\left[y^2+a^2-2ay+z^2\right]$$

$$y^2(K-1)+z^2(K-1)+a^2(K-1)-2ay(K+1)=0$$

$$y^2+z^2+a^2-2ay\left(\frac{K+1}{K-1}\right)=0 \longrightarrow (I)$$

The equation for a circle with centre at $(y_0, 0)$ and radius $R$ is,

$$(y-y_0)^2+z^2=R^2$$

$$\Rightarrow y^2+y_0^2-2yy_0+z^2-R^2=0$$

$$\Rightarrow y^2+z^2+(y_0^2-R^2)-2yy_0=0 \longrightarrow (II)$$

Comparing (I) & (II), we get

$$\rightarrow y_0 = a\left(\frac{K+1}{K-1}\right)$$

and

$$\rightarrow a^2 = y_0^2-R^2$$

$$R^2 = y_0^2-a^2 = a^2\left(\frac{K+1}{K-1}\right)^2-a^2$$

$$= a^2\left(\frac{K^2+2K+1-K^2+2K-1}{(K-1)^2}\right)$$

$$= \frac{a^2 4K}{(K-1)^2}$$

$$\Rightarrow R = \frac{2a\sqrt{K}}{|K-1|}$$

Hence, the equipotentials are circles with $\left(y_0 = a\left(\frac{K+1}{K-1}\right), 0\right)$ as centre and radius $= \frac{2a\sqrt{K}}{|K-1|}$