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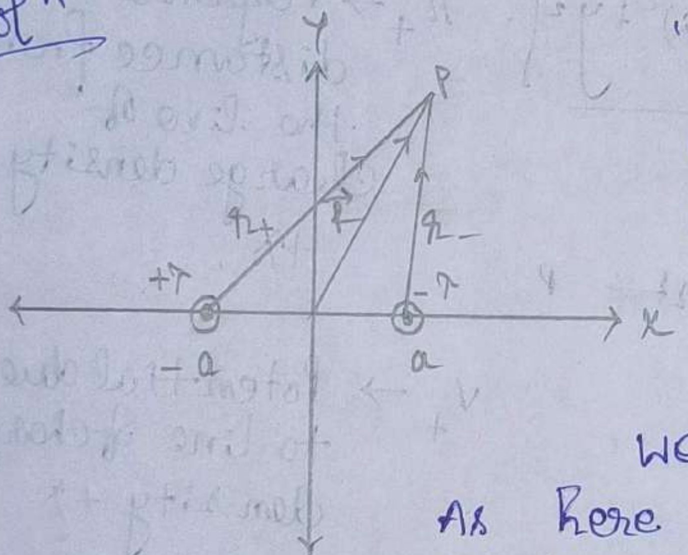
Q. Two infinite line have linear charge densities $-\tau$ and $+\tau$. They are parallel to z axis, passing through x-axis at points $x=-a$ and $x=a$ respectively. Show that the equipotential surface having potential $\frac{\tau \ln(2)}{4\pi\varepsilon_0}$ is a cylinder having radius $2\sqrt{2}\,a$.

Solⁿ



Electric field at distance 'r' from an infinite line charge of charge density τ is

$$\varepsilon = \frac{\tau}{2\pi\varepsilon_0 r}$$

We know $dv = -\vec{\varepsilon}\cdot\vec{dr}$

As here $\vec{\varepsilon}$ and $\vec{dr}$ is in the same direction $dv = -\varepsilon\, dr$

Let's take potential to be zero at $r = r_0$.

$$\int_0^v dv = -\int_{r_0}^{r} \varepsilon\, dr$$

$$\Rightarrow v = -\int_{r_0}^{r} \frac{\tau}{2\pi\varepsilon_0 r}\, dr$$

$$= -\frac{\tau}{2\pi\varepsilon_0}\left[\ln r\right]_{r_0}^{r}$$

$$= -\frac{\tau}{2\pi\varepsilon_0}\ln r + \underbrace{\frac{\tau}{2\pi\varepsilon_0}\ln r_0}_{\text{constant}}$$

$$= -\frac{\tau}{2\pi\varepsilon_0}\ln r + C$$

If the line charge has negative charge density the constant will be $-C$. [$\because$ depending on λ]

Now let P (x,y,z) is a point with position vector $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

perpendicular distance of point P from the line of charge density $-\lambda$ is $\boxed{r_- = \sqrt{(x+a)^2 + y^2}}$

and $\boxed{r_+ = \sqrt{(x-a)^2 + y^2}}$. $r_+ \rightarrow$ Perpendicular distance from the line of charge density $+\lambda$.

Now potential at point P

$$V_p = V_+ + V_-$$

$V_+ \rightarrow$ Potential due to line of charge density $+\lambda$

$V_- \rightarrow$ Potential due to line of charge density $-\lambda$.

$$= \left[-\frac{\lambda}{2\pi\varepsilon_0}\ln(r_+) + C\right] + \left[\frac{\lambda}{2\pi\varepsilon_0}\ln r_- - C\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_-}{r_+}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\sqrt{\frac{(x+a)^2+y^2}{(x-a)^2+y^2}}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

Now $V_p = \dfrac{\lambda \ln 2}{4\pi\varepsilon_0}$

Comparing these two we get

$$\frac{(x+a)^2+y^2}{(x-a)^2+y^2} = 2$$

$$\Rightarrow x^2+y^2+2ax+a^2 = 2x^2+2y^2-4ax+2a^2$$

$$\Rightarrow x^2+y^2-6ax+a^2=0$$

$$\Rightarrow (x-3a)^2+y^2 = 8a^2$$

$\therefore$ Locus of the ~~the~~ point 'p' is a circle ~~which~~ whose equation is $(x-3a)^2+y^2 = 8a^2$ and radius is '$2\sqrt{2}a$'.

$\therefore$ In 3d it will form a cylinder (varying z component) of radius $2\sqrt{2}a$.