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Physics Presentation

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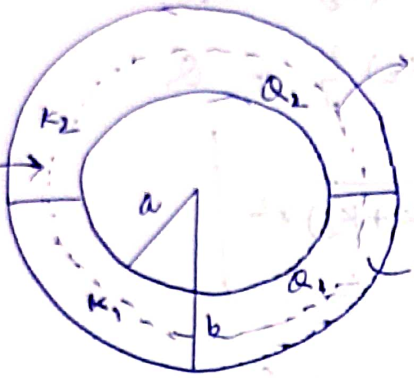
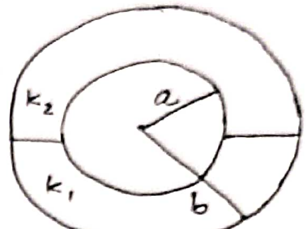
Ques. Half of the space between two concentric electrodes of a spherical capacitor is filled with uniform isotropic dielectric with relative permittivity or dielectric constants 'K_1' & 'K_2'. The charge of the capacitor is 'Q'. Find the magnitude of the electric field strength b/w the electrodes as a fxn of distance 'r' from the curvature centre of the electrodes. and also find out the potential difference?

Ans.





⇒ let the charge on K_1 dielectric medium is 'Q_1'

" " K_2 " " " 'Q_2'

Total charge $Q = Q_1 + Q_2$

We know that

$$\boxed{\oint \vec{D} \cdot d\vec{A} = q_{free}}$$

here, $\vec{D}$ ⇒ electric displacement vector

we also know that $\boxed{\vec{D} = \varepsilon_0 K \vec{E}}$

here, ε_0 → Vacuum permittivity

K → dielectric Constant

$\vec{E}$ → electric field

⇒ If we assume that the field E still has spherical symmetry.

$$\Rightarrow \boxed{E_1 = E_2} = E$$

$$\Rightarrow \frac{D_1}{\varepsilon_0 K_1} = \frac{D_2}{\varepsilon_0 K_2}$$

$$\Rightarrow D_1 = \frac{K_1}{K_2} D_2 \quad \Rightarrow \boxed{D_1 \neq D_2}$$

Take as Gaussian Surface a Concentric spherical Surface of radius r $(a < r < b)$.

From $$\oint D \cdot dA = Q$$

$\because q_{free} = Q = Q_1 + Q_2$

$$\Rightarrow D_1 2\pi r^2 + D_2 2\pi r^2 = Q$$

$$\Rightarrow 2\pi r^2 (D_1 + D_2) = Q$$

$\because D_1 = \varepsilon_0 k_1 E_1$
$D_2 = \varepsilon_0 k_2 E_2$
$E_1 = E_2 = E$

$$\Rightarrow 2\pi r^2 (\varepsilon_0 k_1 E_1 + \varepsilon_0 k_2 E_2) = Q$$

$$\Rightarrow E \{2\pi \varepsilon_0 r^2 (k_1 + k_2)\} = Q$$

$$\Rightarrow \boxed{E = \frac{Q}{2\pi \varepsilon_0 (k_1 + k_2) r^2}}$$

So,

$$D_1 = \frac{\cancel{\varepsilon_0} k_1 Q}{2\pi \cancel{\varepsilon_0} (k_1 + k_2) r^2} = \frac{k_1 Q}{2\pi (k_1 + k_2) r^2}$$

$$D_2 = \frac{k_2 Q}{2\pi (k_1 + k_2) r^2}$$

Potential difference $\Delta V = V_b - V_a$

$$\Delta V = V_b - V_a = \int_a^b E \cdot dr$$

$$\Delta V = \int_a^b \frac{Q}{2\pi \varepsilon_0 (k_1 + k_2) r^2} \cdot dr$$

$$= \frac{Q}{2\pi \varepsilon_0 (k_1 + k_2)} \int_a^b \frac{1}{r^2} \cdot dr$$

$$= \frac{Q}{2\pi \varepsilon_0 (k_1 + k_2)} \left[-\frac{1}{r} \right]_a^b$$

$$\boxed{\Delta V = \frac{Q (b - a)}{2\pi \varepsilon_0 (k_1 + k_2) ab}}$$

Applications of Capacitors

Energy storage, Digital memory, pulsed power and Motor starters, Signal processing and sensing.

$\Rightarrow$ We also know the relation b/w E, P & D Vectors

$$\boxed{\varepsilon_0 E + P = D}$$