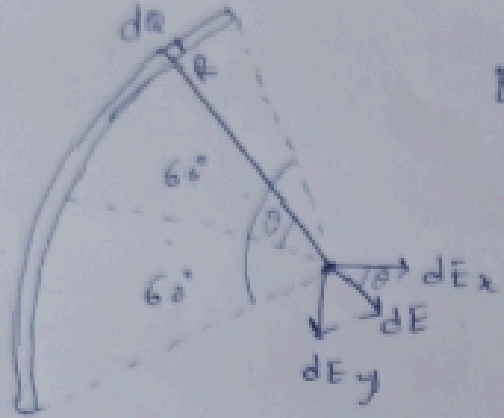
Find the electric field due to a arc of charge at the point O in the center of that arc.



$\vec{E} = ?$

$dE = \frac{k\,dQ}{R^2}$

$dE_x = dE\cos\theta$

$= \frac{k\,dq}{R^2}\cos\theta = \frac{k\lambda\,d\theta}{R}\cos\theta$

λ = Linear charge density $= \frac{Q}{\frac{1}{3}2\pi R} = \frac{3Q}{2\pi R}$

$dq = \lambda R\,d\theta$

$$E = 2\int_{0^\circ}^{60^\circ} dE_x = 2\int_{\theta=0^\circ}^{\theta=60^\circ} \frac{k\lambda\cos\theta\,d\theta}{R} \quad [k, \lambda, R = \text{constant}]$$

$$E = \frac{2k\lambda}{R}\int_{0^\circ}^{60^\circ}\cos\theta\,d\theta$$

$$= \frac{2k\lambda}{R}\left[\sin\theta\right]_{0^\circ}^{60^\circ}$$

$$= \frac{2k\lambda}{R}\cdot\frac{\sqrt{3}}{2} = \frac{1.73k\lambda}{R}$$

$$= 1.73\,k\,\frac{3Q}{2\pi R^2} \quad \left[\lambda = \frac{3Q}{2\pi R}\right]$$

$$= 0.827\frac{kQ}{R^2}.$$

$$\boxed{\vec{E} = 0.827 \frac{kQ}{R^2} \hat{x}}$$