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# PH1213 Presentation

Q. An intense energetic electron beam can pass normally through a grounded metal foil. The beam is switched on at $t = 0$ at a current $I = 3 \times 10^{6}$ amp & a cross-sectional area $A = 1000\ cm^2$. After the beam has run for $10^{-8}$ sec, calculate the electric field at the point P on the output face on the foil & near the beam axis due to space charge of the beam.

Soln

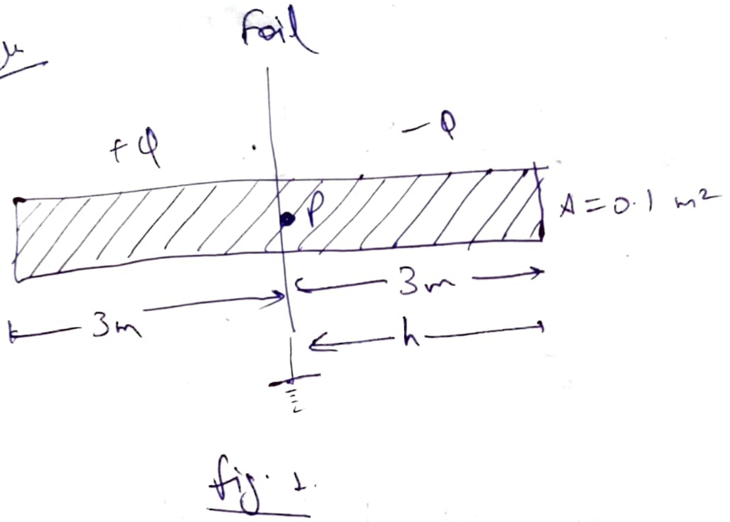



fig. 1

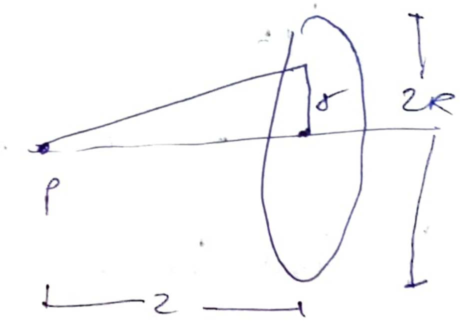



Fig. 2

Assuming, electron beam travels c

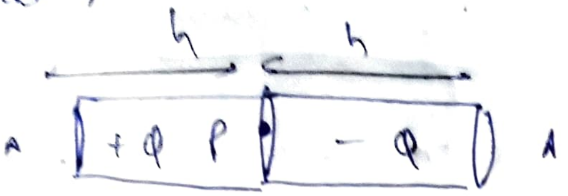
$$\therefore \quad h = ct = 3 \times 10^{8} \times 10^{-8} = 3m$$

$$\text{Also,} \quad Q = -I \times t = -3 \times 10^{6} \times 10^{-8} = -3 \times 10^{-2}\ C.$$

As charge on the left side of the foil does not contribute to the electric field at point P (due to shielding effect). The foil is at net voltage 0, so the action of the grounded metal foil can be ~~separated~~ replaced by an image charged cylinder. The image cylinder & the real cylinder are symmetrical w.r.t to the metal foil & their charges are opp. in sign in fig 1.

So, the problem is now ~~as~~ ⇒

Electric field at P




We first calculate the electric field at point P on the axis of a uniformly charged disc of surface charge density $\sigma$ as shown in fig 2. The V is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma\, 2\pi r\,dr}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma\pi\,dr^2}{\sqrt{z^2+r^2}}$$

$$= \frac{\pi\sigma}{4\pi\varepsilon_0}\, 2\sqrt{z^2+r^2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2}-z\right]$$

So the field Intensity is:

$$E_P = -\frac{\partial \cancel{\phi}}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2+R^2}}-1\right)$$

from fig. 1.62, the field at the point P produced by the right charge cylinder is:

$$E_P = \frac{1}{2\varepsilon_0}\int_0^h \left(\frac{Q}{h\pi R^2}dz\right)\left(\frac{z}{\sqrt{z^2+R^2}}-1\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 hR^2}\int_0^h\left(\frac{\frac{1}{2}dz^2}{\sqrt{z^2+R^2}}-dz\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 hR^2}\int_0^h \frac{\frac{1}{2}dz^2}{\sqrt{z^2+R^2}} - dz$$

$$= \frac{Q}{2\pi\varepsilon_0 hR^2}\left[\sqrt{z^2+R^2}\Big|_0^h - h\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 hR^2}\left[\sqrt{h^2+R^2}-R-h\right]$$

$$\therefore\ E_{net} \text{ at } P = 2 \times E_P$$

$$= \frac{-Q}{\pi \varepsilon_0 h R^2}\left[R + h - \sqrt{R^2 + h^2}\right]$$

$$= \frac{-3 \times 10^{-2}}{\pi \times 8.85 \times 10^{-12} \times 3 \times \frac{0.1}{\pi}} \times \left[3 + \sqrt{\frac{0.1}{\pi}} - \sqrt{3^2 + \frac{0.1}{\pi}}\right]$$

$$= -1.42 \times 10^{9}\ V/m$$

The minus sign indicates that field intensity points to the right.

Motivation: The physical importance of this phenomenon lies in the ability ~~the~~ to study the properties of material at a very small scale. Electron microscopy uses this phenomenon to produce highly detailed images of the structure of materials. By analyzing the behaviour of electrons as they interact with a material, scientists can gain insights into the material property & behaviour.

Other application include using electron beam to manipulate & modify materials at a small scale, such as in manufacturing of computer chips.