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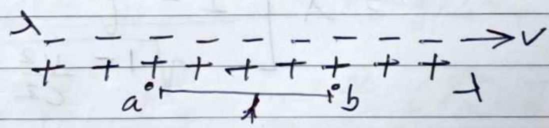
Q. If there's a wire carrying current I and at distant r there a charge q moving with vel. v in same direction as of I. If we observe the ~~po~~ charge q from a frame moving with vel. v in the same direction as of charge q. What kind of force equation we'll get?

→ Assume there a positive charge line with linear charge density λ. there's also a line of -ive charge superimposed on +ive charge line with charge density λ and moving with vel. v in +tive x-axis.



S-frame

Electric field for this system is 0.

Consider a S'-frame moving with vel. v in positive x-axis.
Consider any 2 points on +ive charge line a & b with dist. l.

Charge between a & b = λ l

Seeing the system from S'-frame the length between a & b i.e. l will contract. But since the total charge b/w a & b is constant. Linear charge density will increase.

S'-frame : $\lambda'_+ = \dfrac{\lambda}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Similarly, since the length l for -ive charge line increases. The linear charge density decrease.

S'-frame : $\lambda'_- = \lambda\sqrt{1-\dfrac{v^2}{c^2}}$

Net charge on ~~char~~ system for S'-frame:-

$$\lambda' = \lambda'_+ - \lambda'_-$$

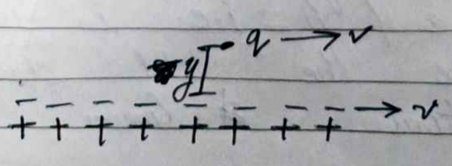
$$= \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda\sqrt{1-\frac{v^2}{c^2}}$$

$$= \lambda\left[\frac{1-\left(1-\frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$\cancel{= \frac{\lambda v^2}{c^2\sqrt{1-\frac{v^2}{c^2}}}}$$

$$= \frac{\lambda v^2}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

~~ch~~ Consider a point charge q at distance y from the system moving with vel. v in positive x-axis.

S-frame :- 

In S'-frame :-

$\cancel{\vec{F}_x = 0}$

Force on at charge q in S'-frame :-

$$\vec{F}_x' = 0$$

$$\vec{F}_y' = q\vec{E}_y' = \frac{q\lambda'}{2\pi\varepsilon_0 y}\hat{j}$$

$$\vec{F}_y' = \frac{q\lambda v^2/c^2}{2\pi\varepsilon_0 y\sqrt{1-\frac{v^2}{c^2}}}$$

Using Lorentz transformation :-

$$\vec{F}_x' = \frac{\vec{F}_x - v/c^2(\vec{F}\cdot\vec{u})}{1-\frac{vu_x}{c^2}}$$

$$F_y' = \frac{F_y\sqrt{1-v^2/c^2}}{1-vu_y/c^2}$$

$$F_x = \frac{F_x' - \frac{v}{c^2}(\vec{F}_x'\cdot\vec{u}')}{1+\frac{vu_x'}{c^2}}$$

$$F_y = \frac{F_y'\sqrt{1-v^2/c^2}}{1+\frac{vu_y'}{c^2}}$$

Since, $u_x' = 0$

$$\therefore F_x = F_x'$$

$$F_y = F_y'\sqrt{1-\frac{v^2}{c^2}}$$

Since, $F_x' = 0 \Rightarrow F_x = 0$

And $F_y = q\dfrac{\lambda v^2/c^2}{2\pi\varepsilon_0 y}$

We know : $I = \lambda v$

$$\therefore \; F_y = \frac{q I v}{2 c^2 \pi \varepsilon_0 y}$$

$$\mu_0 = \frac{1}{c^2 \varepsilon_0}$$

$$F_y = q \frac{\mu_0}{2\pi} \frac{I}{y} v$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{y}$$

$$\therefore \; F_y = q \cancel{B} v B$$

If $\vec{v} = v\hat{i}$ & $B = -B\hat{k}$

$$\cancel{B} \; \underset{+}{\vec{F}} = q(\vec{v} \times \vec{B})$$