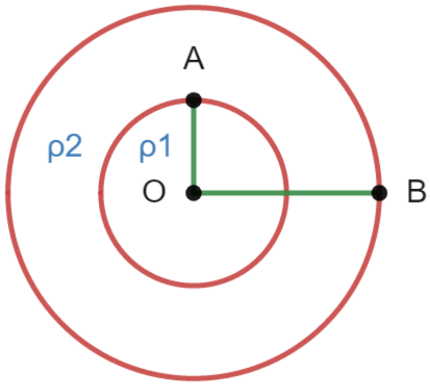
Suchet Sadekar 20221233

Question Setup:



Given is a solid sphere with uniform charge density ρ_1 for $0 \le r \le \frac{a}{2}$ and uniform charge density ρ_2 for $\frac{a}{2} < r \le a$. Also, the potential at point O is twice that at point B. Find the relation between ρ_2 and ρ_1.

Solution:

Potential at the centre of a sphere is: $\int_0^R \frac{K}{r} dq = \int_0^R \frac{K}{r} \rho 4\pi r^2 dr = 4\pi K\rho \int_o^R r\, dr = 2\pi K R^2 \rho$

Potential outside is: $V_r = \frac{KQ}{r} = \frac{K}{r} \cdot \frac{4\pi R^3 \rho}{3} = \frac{4\pi K R^3 \rho}{3r}$

Potential at the surface is: $V_r = \frac{KQ}{R} = \frac{4\pi K R^2 \rho}{3}$

The given system is equivalent to two overlapping spheres (a/2, ρ_1-ρ_2) and (a, ρ_2).

$$V_O = \frac{2}{3}\left(\frac{4\pi K\left(\frac{a}{2}\right)^2(\rho_2 - \rho_1)}{3} + \frac{4\pi K a^2 \rho_2}{3}\right) = \frac{2}{3}\left(\frac{4\pi K a^2}{3}\frac{\rho_1 + 3\rho_2}{4}\right)$$
$$= \frac{\pi K a^2(\rho_1 + 3\rho_2)}{2}$$

$$V_B = \frac{4\pi K\left(\frac{a}{2}\right)^3(\rho_1 - \rho_2)}{3a} + \frac{4\pi K a^2 \rho_2}{3} = \frac{4\pi K a^2}{3}\frac{(\rho_1 + 7\rho_2)}{8}$$
$$= \frac{\pi K a^2(\rho_1 + 7\rho_2)}{6}$$

$$V_O = 2V_B: \ \frac{\rho_1 + 3\rho_2}{2}\frac{6}{\rho_1 + 7\rho_2} = 2$$

Hence, $\rho_1 = 5\rho_2$