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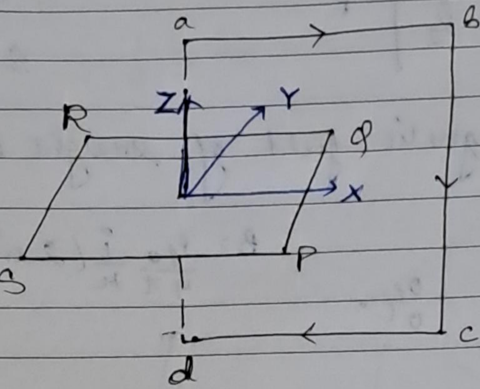
Q. Current I_0 is flowing through a bent wire $a \to b \to c \to d$ in z-x plane as shown in figure, then find $\oint \vec{B} \cdot \vec{dl}$ over the loop PQRS lying in the X-Y plane as shown in figure, due to the bent wire abcd.



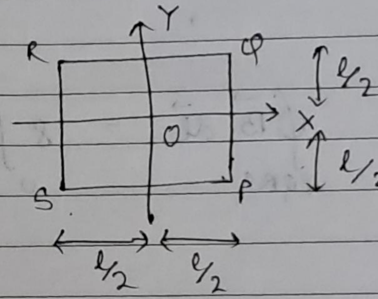
According to Ampere's law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

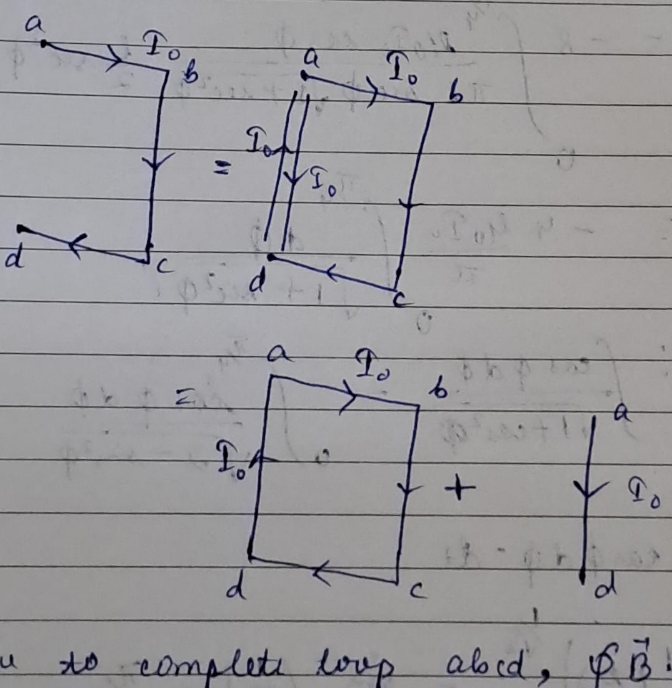
I = current in infinite wire piercing the plane of closed loop

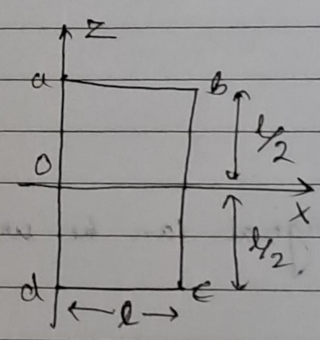
OR

current in a closed circuit piercing the plane of closed loop

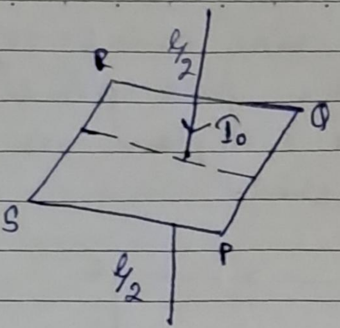


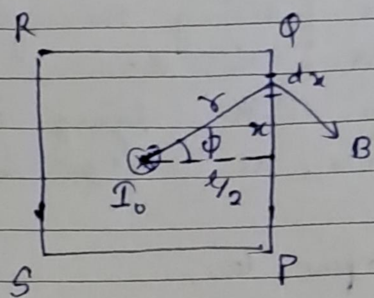
Given situation:





Due to complete loop abcd, $\oint \vec{B} \cdot \vec{dl}$ over PQRS is $\mu_0 I_0$.
Now we need to find due to finite straight wire ad.
$\oint \vec{B} \cdot \vec{dl}$ for ad will be -ve.



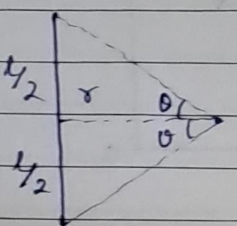


$x = \frac{l}{2}\tan\phi$

$r = \frac{l}{2}\sec\phi$

$dx = \frac{l}{2}\sec^2\phi\, d\phi$

Magnetic field of straight wire



$$B = \frac{\mu_0}{4\pi}\frac{i}{r}(2\sin\theta) = \frac{\mu_0 I_0}{2\pi \frac{l}{2}\sec\phi}\,\frac{l/2}{\sqrt{\frac{l^2}{4}+r^2}}$$

$$= \frac{\mu_0 I_0}{\pi l \sec\phi\sqrt{1+\sec^2\phi}}$$

$$\oint_{PQRS} \vec{B}\cdot d\vec{l} = 8\int_0^Q \vec{B}\cdot d\vec{l} = -8\int B\cos\phi\, dx$$

$$= -8\int_0^{\pi/4} \frac{\mu_0 I_0 \cos\phi}{\pi l \sec\phi\sqrt{1+\sec^2\phi}}\,\frac{l}{2}\sec^2\phi\, d\phi$$

$$= -4\frac{\mu_0 I_0}{\pi}\int_0^{\pi/4}\frac{d\phi}{\sqrt{1+\sec^2\phi}}$$

Integral can be written as: $\displaystyle\int \frac{\cos\phi\, d\phi}{\sqrt{1+\cos^2\phi}} = \int_0^{\pi/4}\frac{\cos\phi\, d\phi}{\sqrt{2-\sin^2\phi}}$

take $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

Now,

$$\int_0^{1/\sqrt{2}} \frac{dt}{\sqrt{2-t^2}} = \left(\sin^{-1}\left(\frac{t}{\sqrt{2}}\right)\right)_0^{1/\sqrt{2}} = \sin^{-1}\left(\frac{1}{2}\right) = \pi/6$$

So, $\oint \vec{B}\cdot d\vec{l} = -4 \cdot \frac{\mu_0 I_0}{\pi} \cdot \frac{\pi}{6} = -\frac{2}{3}\mu_0 I_0$

$\therefore$ $\oint \vec{B}\cdot d\vec{l}$ due to wire abcd $= \mu_0 I - \frac{2}{3}\mu_0 I_0$

$= \frac{1}{3}\mu_0 I_0$

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