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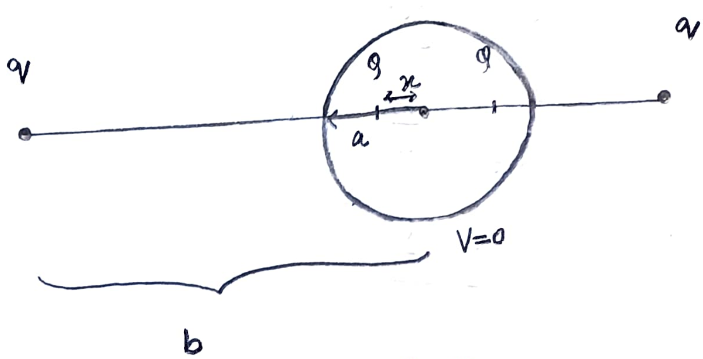
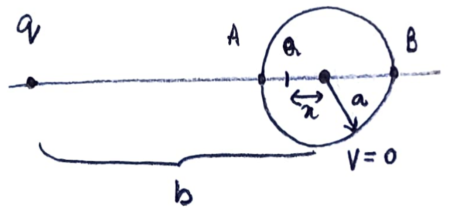
Q - a) Two similar sized charges are placed at a distance 2b apart. find, approximately, minimum radius of a grounded conducting sphere placed midway between them that would neutralize their mutual repulsion.

b) what is force on the each of the two charges if the same sphere with radius determined in part (a) is now charged to potential V?

Motivation: To understand the use of Method of Images so to solve and simplify a given system.



(1) Case for only one sphere



Here charge Q is the Image charge kept at Distance x from the centre. Since, the sphere is grounded Potential at every point at ~~surface~~ sphere is given to be 0

$V_a = V_b = 0$

$\frac{kq}{b-a} + \frac{kQ}{a-x} = 0$ For the Point A

$\frac{kq}{b-a} = \frac{-kQ}{a-x}$

$-\frac{q}{Q} = \frac{b-a}{a-x}$

Potential for Point B

$\frac{kq}{b+a} = -\frac{kQ}{a+x}$

$-\frac{q}{Q} = \frac{b+a}{a+x}$

Equating the Equations

$$-\frac{q}{g} = \frac{b-a}{a-x} \quad , \quad -\frac{q}{g} = \frac{b+a}{a+x}$$

$$\frac{b-a}{a-x} = \frac{b+a}{a+x}$$

$$\frac{(a+x)(b-a)}{(b+a)} = a-x$$

$$\frac{a(b-a)}{(b+a)} - a = -x - \frac{x(b-a)}{(b+a)}$$

$$\frac{a(b-a) - a(b+a)}{b+a} = -x\left(\frac{b+a+b-a}{(b+a)}\right)$$

$$\cancel{ab} - a^2 - \cancel{ab} - a^2 = -x(2b)$$

$$\frac{a^2}{b} = x$$

Putting $\frac{a^2}{b} = x$ in the given Equation

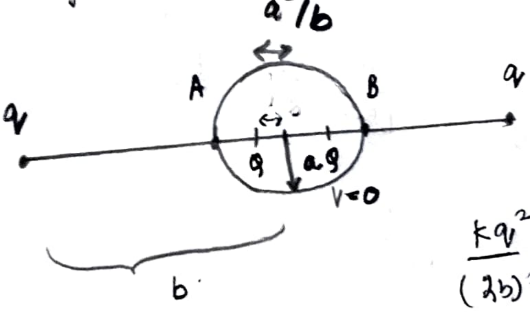
$$-\frac{q}{g} = \frac{b-a}{a-\frac{a^2}{b}}$$

$$-\frac{q}{g} = \frac{\cancel{b-a}}{\frac{a(\cancel{b-a})}{b}}$$

$$-\frac{q}{g} = \frac{b}{a}$$

$$-\frac{a}{b}q = g$$

* Note - This solution is unique can be determined by the ~~boundary conditions~~ uniqueness Theorem. Acc to question, at the sphere the Potential is 0 and at $x \to \infty$ V is also 0. Hence, in accordance with uniqueness theorem there only one charge distribution that will satisfy this which I have found.



Now Applying Equation for forces on charge q (left most)

$$\frac{kq^2}{(2b)^2} - \frac{kq\cdot q\, a/b}{\left(b-\frac{a^2}{b}\right)^2} - \frac{kq\cdot q\, a/b}{\left(b+\frac{a^2}{b}\right)^2} = 0$$

$$\frac{kq^2}{4b^2} = \frac{kq^2\cdot a/b}{\left(b-\frac{a^2}{b}\right)^2} + \frac{kq^2\cdot a/b}{\left(b+\frac{a^2}{b}\right)^2}$$

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$$\frac{1}{4b^2} = \frac{a/b}{(b - a^2/b)^2} + \frac{a/b}{(b + a^2/b)^2}$$

$$\frac{1}{4b^2} = \frac{a/b \times b^2}{(b^2 - a^2)^2} + \frac{a/b \times b^2}{(b^2 + a^2)^2}$$

$$\frac{1}{4b^2} = ab\left[\frac{1}{(b^2-a^2)^2} + \frac{1}{(b^2+a^2)^2}\right]$$

$$\frac{1}{4b^2} = ab\left[\frac{(b^2+a^2)^2 + (b^2-a^2)^2}{[(b^2-a^2)(b^2+a^2)]^2}\right]$$

$$\frac{1}{4b^2} = ab\left[\frac{b^4+a^4+\cancel{2a^2b^2}+b^4+a^4-\cancel{2a^2b^2}}{(b^4-a^4)^2}\right]$$

$$\frac{1}{4b^2} = \frac{ab \times 2 \times [b^4+a^4]}{[b^4-a^4]^2}$$

$$\frac{1}{4b^2} = \frac{ab \times 2 \times b^4[1 + a^4/b^4]}{b^8\left[1 - \frac{a^4}{b^4}\right]^2}$$

$$\frac{1}{4} = \frac{2a\,[1+a^4/b^4]}{b\left[1-\frac{a^4}{b^4}\right]^2}$$

$$\frac{b}{8} = \frac{a \times [1+a^4/b^4]}{\left[1-\frac{a^4}{b^4}\right]^2}$$

$$\Rightarrow \boxed{\frac{b}{8} = \frac{a\left[1+\frac{a^4}{b^4}\right]}{\left[1-\frac{a^4}{b^4}\right]}}$$

[Please Turn over
P.T.O.]

b) Now the Sphere is also Charged to a Potential V

~~$\Rightarrow$~~ The surrounding Potential is increased by -

$$\phi \simeq \frac{Vd}{8x}$$

(where x is distance b/w field Point and Centre of Sphere)

$$\left(\frac{kQ}{x_0} = V \right)$$

$(kQ = Vx_0)$ (x_0 = radius)

Here at a distance

$$\phi = \frac{Vx_0}{x}$$

Since we know - $\nabla \phi = -\mathcal{E}$

Hence

$$-\nabla \phi = \mathcal{E}$$

for Magnitude of $\mathcal{E}$ $\quad \mathcal{E} = \frac{Vd}{8x^2}$

Hence, when $x = d$

$$\mathcal{E} = \frac{V}{8d}$$

$$\text{force} = q\mathcal{E} = \frac{qV}{8d}$$

(Here, Just to Make this Question More clear and to get More depth of information about the system I calculated the force at a Point but we can also add other things like Potential at a far Point r_0 or $\mathcal{E}$ at a Point x_0 or add its interaction with another charge.)