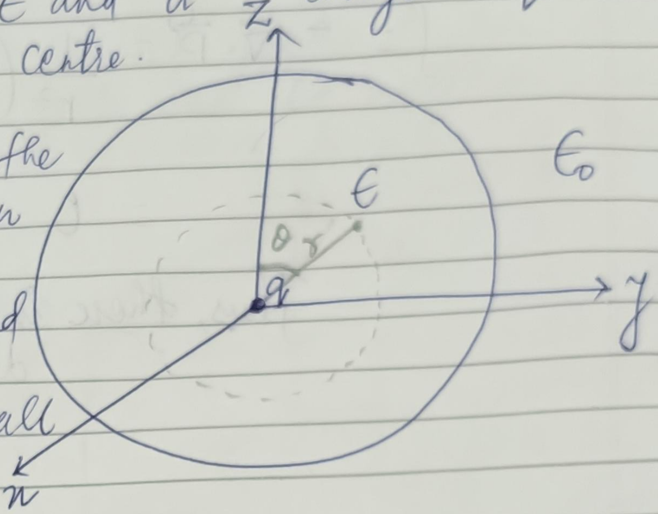
Name :- Subham Kumar Mahato
Rollno :- 20221265

Q. A point charge 'q' is placed at the center of a spherical Tank of water with permittivity 'ε'. The spherical Tank of water is placed in free space. Find the induced surface and volume charge densities.

Let us take a spherical tank of water of radius 'a' of permittivity ε and a charge 'q' is placed at its centre.



Using Gauss law to get the electric displacement in water. The electric displacement is radial and independent of angle.
Let us assume a small spherical shell centred on the charge.

$$\therefore \oint \vec{D}\cdot d\vec{A} = Q_{free}$$

$$\Rightarrow \vec{D} = \frac{1}{4\pi}\left(\frac{q}{r^2}\right)\hat{r} \quad \text{—} (1)$$

$$\because \vec{D} = \epsilon \vec{E}$$

$\therefore$ polarisation is,

$$\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$$
$$= \epsilon_0(\epsilon_r - 1)\left(\frac{1}{4\pi\epsilon}\right)\frac{q}{r^2}\hat{r}$$

∵ The surface charge density at $r = a$ is,

$$\sigma = \vec{P}\cdot\hat{r} = \epsilon_0(\epsilon_r - 1)\vec{E} = \epsilon_0(\epsilon_r - 1)\frac{1}{(4\pi\epsilon)}\left(\frac{q}{a^2}\right)$$

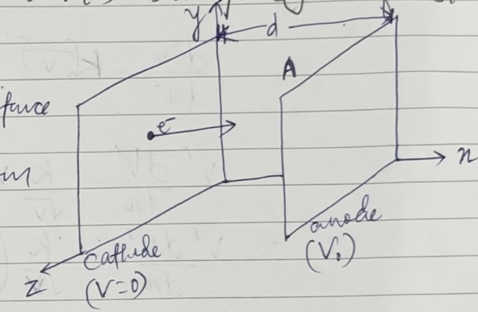
∵ Volume charge density ρ;

$$\rho = -\vec{\nabla}\cdot\vec{P} = \frac{1}{r^2}\left(\frac{\partial r^2 P_r}{\partial r}\right)E$$

$$= 0$$

Thus, there is no volume charge density.

Q. In a vacuum diode, electrons are 'boiled' off a hot cathode at potential zero and accelerated across a gap to the anode, held at positive potential V_0. The cloud of moving electrons within the gap quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then, a steady current I flows in between the plates. Suppose $A \gg d^2$ so that edge effects can be neglected. Find the potential $V(x)$ as a function of distance x.



Let,
$v(x)$ = speed of e^- at a distance x from the cathode,
$V(x)$ = potential at 'x' from cathode

$$\therefore eV(x) = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2e}{mV}} \quad \text{—(1)}$$

a small charge dq in vacuum;

$$dq = \rho A\,dx$$

$$\therefore I = \frac{dq}{dt} = \frac{\rho A\,dx}{dt}$$

$$I = \rho A v \quad \text{—(2)}$$

now, from Poisson's equation, we have

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\Rightarrow \rho = -\epsilon_0 \left(\frac{d^2V}{dx^2}\right) \quad \text{③}$$

putting equ ① & ③ in equ ②, we get,

$$I = -\epsilon_0 A\sqrt{V}\sqrt{\frac{2e}{m}}\frac{d^2V}{dx^2}$$

$$\Rightarrow \frac{-I}{\epsilon_0 A}\sqrt{\frac{m}{2e}} = \sqrt{V}\,\frac{d^2V}{dx^2}$$

$$\Rightarrow \frac{d^2V}{dx^2} = K(\sqrt{V})^{-1}$$

where, $K = \frac{-I}{\epsilon_0 A}\sqrt{\frac{m}{2e}}$

$$\Rightarrow V'\frac{dV'}{dx} = \frac{K}{\sqrt{V}}V'$$

$$\Rightarrow V'\frac{dV'}{dx} = \frac{K}{\sqrt{V}}\left(\frac{dV}{dx}\right) \quad \left\{\because V' = \frac{dV}{dx}\right\}$$

$$\Rightarrow \int V'dV' = \int \frac{K}{\sqrt{V}}\,dV$$

$$\Rightarrow \frac{V'^2}{2} = 2K\sqrt{V}$$

$$\Rightarrow V' = 2\sqrt{K}\left(V^{\frac{1}{4}}\right)$$

$$\Rightarrow \int \frac{dV}{V^{\frac{1}{4}}} = \int 2\sqrt{K}\,dx$$

$$\Rightarrow \frac{4}{3}V^{\frac{3}{4}} = 2\sqrt{K}\,x$$

Name:- Subham Kumar Mahato
Rollno:- 20221265

classmate
Date
Page

$$\Rightarrow V^{\frac{3}{4}} = \frac{3}{2}\sqrt{K_1}\, x$$

$$\Rightarrow V = \left(\frac{3}{2}\sqrt{K_1}\, x\right)^{\frac{4}{3}}$$

$$\boxed{V = \left(\frac{81 I^2 m}{32 \epsilon_0^2 e A^2}\right)^{\frac{1}{3}} x^{\frac{4}{3}}}$$

$\therefore$ potential is a function of 'x'.