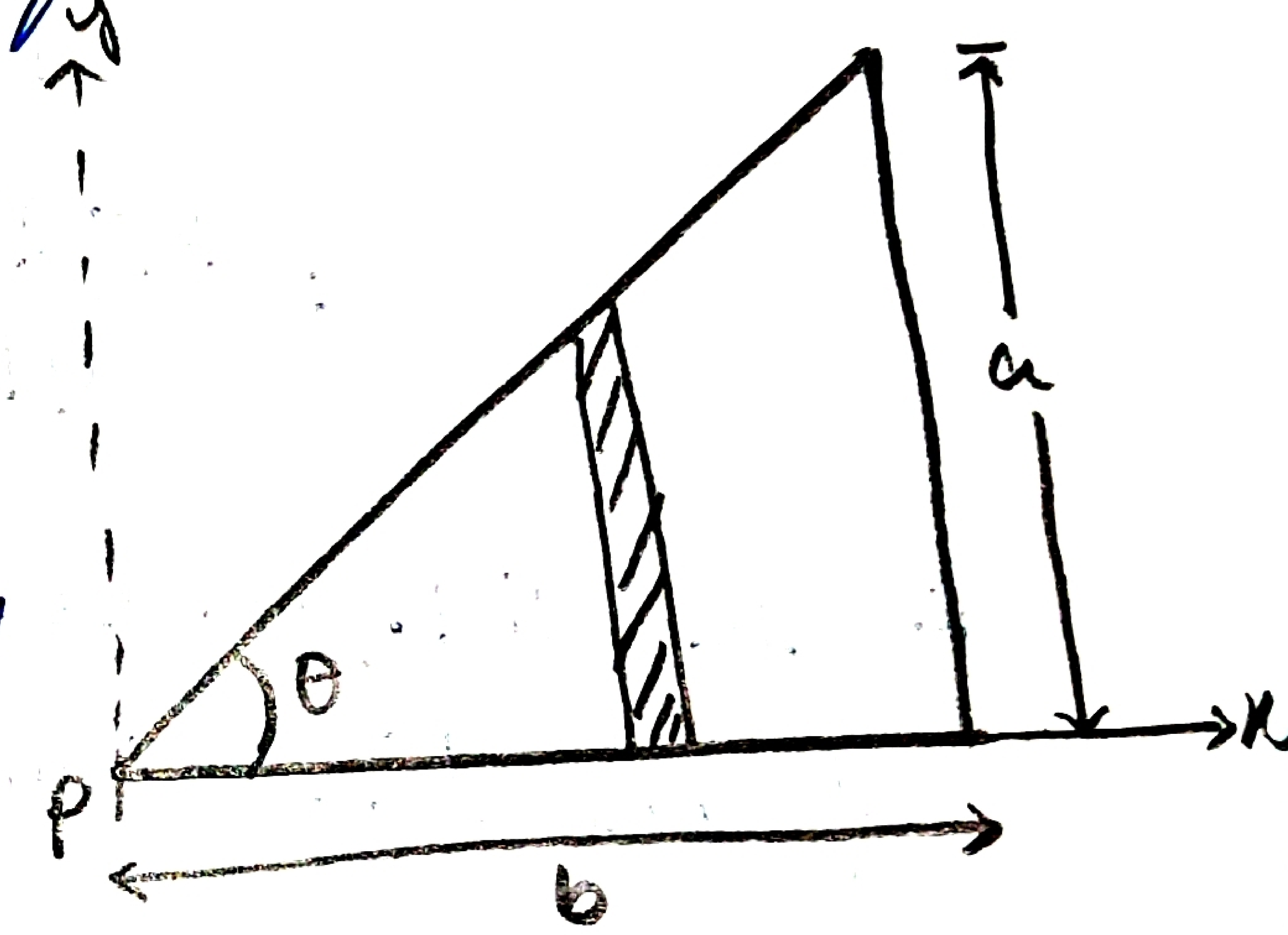
Naorem Sovinity Devi
Regd. No. 20221167

Q. If a square with surface density σ and side s has the same potential at its centre as a disk with the same surface charge density and diameter d, what must be the ratio s/d?
Is your answer reasonable?

Ans:- To find the potential of square at its centre:

In general, let us take a right triangle with base 'b' and height 'a'.
Let P be the vertex where potential is to be found and $\angle P = \theta$.



Then, Potential at P due to the triangle,

$$V_p = \int \frac{\sigma da}{4\pi\varepsilon_0 r}$$

$$\Rightarrow V_p = \frac{\sigma}{4\pi\varepsilon_0}\int_0^b dx \int_0^{ax/b} \frac{dy}{\sqrt{x^2+y^2}}$$

$$\Rightarrow V_p = \frac{\sigma}{4\pi\varepsilon_0} b \left[\ln\left(\sqrt{x^2+y^2}+y\right)\right]_0^{ax/b}$$

$$\Rightarrow V_p = \frac{\sigma b}{4\pi\varepsilon_0} \ln\left(\sqrt{1+\frac{a^2}{b^2}}+\frac{a}{b}\right)$$

$$\Rightarrow V_p = \frac{\sigma b}{4\pi\varepsilon_0} \ln\left(\frac{\sqrt{a^2+b^2}+a}{b}\right) \qquad \left[\because \frac{\sqrt{a^2+b^2}}{b} = \frac{1}{\cos\theta},\ \frac{a}{b} = \tan\theta\right]$$

$$\Rightarrow V_p = \frac{\sigma b}{4\pi\varepsilon_0} \ln\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

P.T.O.

For the square, $b = \frac{s}{2}$ and $\theta = 45°$. $\left[\because a = b = \frac{s}{2}\right]$

Here, P is the centre of the square and the triangle is $\frac{1}{8}$ of the square.

$\therefore$ potential of square at centre,

$$V_s = 8 \times \frac{\sigma\left(\frac{s}{2}\right)}{4\pi\varepsilon_0} \ln\left(1 + \frac{\sin 45°}{\cos 45°}\right)$$

$$= \frac{3.525\,\sigma s}{4\pi\varepsilon_0} \quad \text{——— (I)}$$

For the disk, we can consider it as the integral of concentric rings.

potential of disk at centre, $\left[\because \text{Potential of ring at centre} = \text{Potential of point charge}\right]$

$$V_d = \frac{1}{4\pi\varepsilon_0} \int_0^{d/2} \frac{dq}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{d/2} \frac{2\pi r \sigma\, dr}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \times 2\pi\sigma \times \frac{d}{2}$$

$$= \frac{\pi\sigma d}{4\pi\varepsilon_0} \quad \text{——— (II)}$$

P.T.O.

Since $V_s = V_d$,

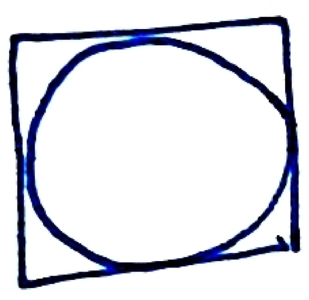
from ① and ⓘⓘ,

$$3.525\, s = \pi d$$

$$\Rightarrow \frac{d}{s} = \frac{3.525}{\pi} = 1.122 > 1 .$$

~~If $d=s$, the disc is larger than circle and if their centres coincide, the disc inscribes the circle. i.e.~~

If $d = s$, the square can inscribe the disc.

i.e. 

They have same surface charge and due to superposition principle, the square will have more potential at the centre.

Therefore, the disk should be larger than the inscribed circle.

i.e. $\frac{d}{s} > 1$ is reasonable.