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Suppose the electric field in some region is found to be $E = kr^3\hat{r}$ in spherical coordinates. Find the charge density ρ and the total charge contained in a sphere of radius R centered at the origin.

Soln:- Given $E = kr^3\hat{r}$ – directed in one coordinate

$$\vec{E} = E_r(r)\,\hat{r}$$

From gauss law :-

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$

using divergence : ⇒ $\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

Charge density :-

$$\rho = \epsilon_0(\nabla\cdot\vec{E})$$

$$= \epsilon_0\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi}\right]$$

θ & $\phi = 0$

$$= \epsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}kr^5 \Rightarrow 5k\epsilon_0 r^2 \Rightarrow \rho(r)$$

$$\boxed{\rho = 5k\epsilon_0 r^2}$$

Now, the total charge enclosed by the gaussian surface

$$Q = \epsilon_0\oint\vec{E}\cdot d\vec{a}$$

$= \epsilon_0\, E\cdot A$ ($\because$ gaussian surface is symmetrical to the area element)

$= \epsilon_0 kr^3(4\pi R^2)$

$$\boxed{Q = 4\pi\epsilon_0 kr^5}$$