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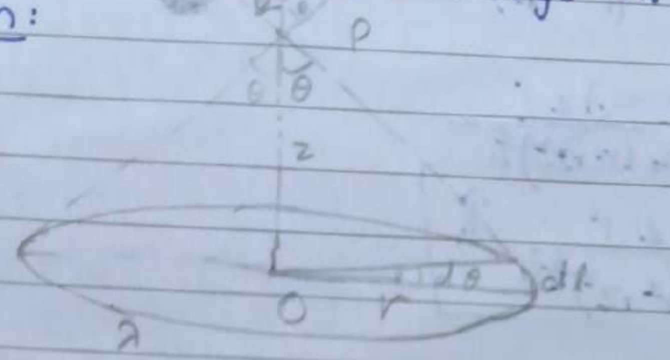
Problem: Find the electric field a distance z above the center of a circular loop of radius r that carries a uniform line charge λ.

Solution:



Net electric field due to ring at point P can be written as the vector sum of small elements of length dl of the ring.
The given charge ~~distru~~ distribution is continuous (not discrete), thus, the sum becomes an integral.
Electric field ~~has~~ due to each element has two components: along line OP and perpendicular to line OP.
Magnitude of electric field due to small element (dl)

$$\cancel{dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}} = dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\cancel{= \frac{1}{4\pi\varepsilon_0}} \qquad \Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{(r^2+z^2)}$$

Component of $\vec{dE}$ (due to one element) perpendicular to OP gets cancelled out due to radially opposite element.
Components of $\vec{dE}$ along OP all add up.

$$\therefore |\vec{E}| = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^2+z^2}\cos\theta$$

$$= \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda z}{(r^2+z^2)^{\frac{3}{2}}}\, dl$$

All quantities in the integral (except dl) are independent of dl

$$\therefore |\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z}{(r^2+z^2)^{\frac{3}{2}}} \int dl$$

$$|\vec{E}| = \frac{1}{2\cancel{4\pi}\varepsilon_0} \frac{\lambda z}{(r^2+z^2)^{\frac{3}{2}}} \cancel{2\pi} r$$

$$\therefore \cancel{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z}{(r^2+z^2)^{\frac{3}{2}}}}$$

$$\therefore \boxed{\vec{E} = \frac{\lambda z r}{2\varepsilon_0 (r^2+z^2)^{\frac{3}{2}}} \hat{k}}$$