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Thursday

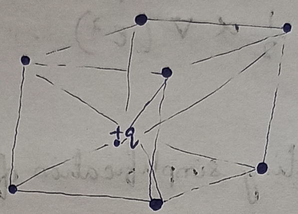
E E : An Application...

↓
Not Electronics Engineering.

Earnshaw's theorem for Extended bodies.

Earnshaw's theorem : No charge configuation can trap a charged particle in an electrostatic field and keep it in stable equilibrium.

Consider a cube with point charges +q at all 8 corners.



Now say you keep a charge of +q in the centre.

It is acted on by 8 repulsive forces, but the configur~n~ is such that the forces

It is ∴, apparently, in stable equilibrium. Any disturbance will shift it out of the stable eqbm state, creating a reaction that will bring it back.

This is illusion, though. Poisson's equation says :

$$-\nabla^2 V_{(\vec{r})} = \frac{\rho_{(\vec{r})}}{\varepsilon_0}$$

The second I displace the point charge, it is no longer in the centre. Recalling that point charges have densities written as Dirac - Delta functions,

$\rho_{(centre)}$ is now $= 0$.

∴, the result just becomes Laplace's equation.

If the double derivative is 0, this is a saddle point, not a minima.

(Recall that minimas have +ve double derivatives).

∴ this is not a minima; this is not a stable eqbm.

There actually is a leakage of flux, the box "leaks" through the centre of each face.

<u>Problem</u>/<u>Question</u>: Okay, so Earnshaw's theorem says you can't trap a charged particle in an electrostat field.

Can you trap a neutral (but polarisable) atom in one?

<u>Ans</u>:

Step (a) Let's try to show that the force on the atom is

$$F = \frac{1}{2}\alpha \nabla (E^2).$$

<u>Why?</u>

This yields an interesting simplification of the problem.

<u>How?</u>

Experimentally, (1) $\vec{p} = \alpha \vec{E}$ for a neutral atom once p ($\vec{p}$ = dipole moment)

and

(2) $\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$ (for very short dipoles)

How? Force on a dipole in a non-uniform field:

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ + \vec{E}_-)$$
$$= q(\Delta\vec{E})$$

For short dipoles, can approximate

Putting (1) in (2),

$$\vec{F} = \alpha(\vec{E}\cdot\vec{\nabla})\vec{E}.$$

From the product rule

$$\nabla E^2 = \nabla(\vec{E}\cdot\vec{E}) = 2\vec{E}\times(\vec{\nabla}\times\vec{E}) + 2(\vec{E}\cdot\vec{\nabla})\vec{E}.$$

But in electrostatics

$$\vec{\nabla}\times\vec{E} = 0$$

$$\Rightarrow (\vec{E}\cdot\vec{\nabla})\vec{E} = \frac{1}{2}\nabla(E^2)$$

$$\therefore \vec{F} = \frac{1}{2}\alpha\vec{\nabla}(E^2)$$

<u>Note:</u> Can't write (2) as, $\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E})$, $\longrightarrow$ (3)
factually incorrect
bcoz $\vec{p}$ may vary with posn, and (3) means that too must be differentiated, which is not true.

→ Note here that $E = \overset{\text{mag of}}{\text{field}}$, not energy.

→ Now for stable eqbm, all I need to do is maximise E^2 in a charge-free region.

→ Then the force would push any displaced charge back to its equilibrium position.

<u>Proof that E^2 cannot be maximised</u>: (Proof by contradiction)

Suppose E^2 has a maximum at P (arbitrary point)

Then there is a sphere (of radius R) about P s.t.

$E^2(P') < E^2(P)$

$\Rightarrow \quad |E(P')| < |E(P)| \ \forall$ all points on the surface.

The ~~point~~ sphere then behaves like a point charge, where avg. field equals field at the center:

$$\frac{1}{4\pi R^2} \int \vec{E}\, da = \vec{E}(P)$$

Choosing the z-axis to lie along $\vec{E}(P)$,

$$\frac{1}{4\pi R^2} \int E_z\, da = E(P)$$

But if E^2 has a maximum at P, then

$$\int E_z\, da \le \int |E|\, da < \int |E(P)|\, da = 4\pi R^2\, E(P)$$

$\Rightarrow E(P) < E(P)$.

This is a contradiction.

$\therefore$, E^2 cannot have a maximum.

(It can have a minimum, however; at the midpoint b/w two equal charges field = 0. This is a minimum.)