Name: SAKSHAM JAIN

Roll no.: 20221235

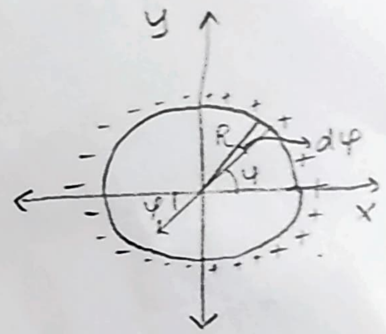
PHY - TUTORIAL PROBLEM

Q: A thin nonconducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\varphi$ where φ is the azimuthal angle. Find the electric field strength:

a) At the centre of the ring

b) On the axis of the ring a distance x away from the centre.

A: Using the sign of $\cos\varphi$ we can roughly estimate what the charge distribution would look like.



Since the right and left halves are similar in charge distribution, just opposite in sign, we can find E of one half and double it.

$$dq = \lambda (R\,d\varphi)$$
$$= \lambda_0 R \cos\varphi \, d\varphi$$

$dE = \frac{k\,dq}{R^2} \cdot \cos\varphi$ [Since the top and bottom halves are symmetric we can conclude that net electric field will be in the $-x$ direction]

$$\therefore E = 2\int_{-\pi/2}^{\pi/2} \frac{k\,dq}{R^2} \cdot \cos\varphi$$

$$= \frac{2k}{R^2}\int_{-\pi/2}^{\pi/2} \lambda_0 R \cos^2\varphi \, d\varphi$$

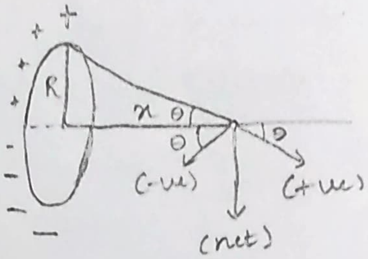
$$= \frac{2k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \cos^2\varphi \, d\varphi$$

$$= \frac{2k\lambda_0}{R}\int_{-\pi/2}^{\pi/2} \frac{\cos(2\varphi)+1}{2} \, d\varphi \quad \left[\cos\theta = \frac{\cos 2\theta + 1}{2}\right]$$

$$= \frac{k\lambda_0}{R}\left(\int_{-\pi/2}^{\pi/2} \cos(2\varphi)\, d\varphi + \int_{-\pi/2}^{\pi/2} d\varphi\right)$$

$$= \frac{k\lambda_0}{R}\left(\left[\cancel{\frac{\sin(2\varphi)}{2}}\right]_{-\pi/2}^{\pi/2}{}^{0} + \left[\varphi\right]_{-\pi/2}^{\pi/2}\right)$$

$$= \frac{k\lambda_0 \pi}{R} = \underline{\underline{\frac{\lambda_0}{4R\varepsilon_0}}} \quad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$



B: The electric field at this point is also similar to that of the previous problem except the dist is now $\sqrt{x^2+R^2}$ and there will be an extra $\sin\theta$ term to account for the shift.

$$dq = \lambda_0 R \cos\varphi \, d\varphi$$

$$dE = \frac{k\,dq}{(\sqrt{x^2+R^2})^2} \cdot \cos\varphi \cdot \sin\theta$$

$$E = 2\int_{-\pi/2}^{\pi/2} \frac{k\lambda_0 R}{(\sqrt{x^2+R^2})^2} \cdot \sin\theta \cdot \cos^2\varphi \, d\varphi \quad \left[\text{Equivalent contribution of +ve + -ve}\right]$$

$$* \tan\theta = \frac{R}{x}, \quad \cos\theta = \frac{x}{\sqrt{x^2+R^2}}, \quad \sin\theta = \frac{R}{\sqrt{x^2+R^2}}$$

$$\therefore E = \frac{2k\lambda_0 R^2}{(\sqrt{x^2+R^2})^3} \underbrace{\int_{-\pi/2}^{\pi/2} \cos^2\varphi \, d\varphi}$$

$$= \frac{2k\lambda_0 R^2}{(\sqrt{x^2+R^2})^3}\left(\frac{\pi}{2}\right) \quad \left[\text{The integration was done in the prev part}\right]$$

$$= \underline{\underline{\frac{\lambda_0 R^2}{4\varepsilon_0 (x^2+R^2)^{3/2}}}} \quad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$