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(B6)

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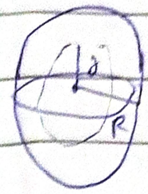
1. Find the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere?

Ans =



$$Q_{int} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q = \frac{r^3}{R^3} Q \qquad \text{so, } E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{int}}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{r^3}{R^3} Q \frac{1}{r^2}\hat{r}$$

$$\boxed{E = \frac{KQ}{R^3} r \hat{r}}$$

So, force ~~cost~~ per unit volume $= f = \rho E = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{Q}{4\pi\varepsilon_0 R^3} r\right)$

$$= \frac{3}{\varepsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r$$

So force in the z direction on $d\tau \Rightarrow$

$$dF_z = f_z\, d\tau$$

$$= \frac{3}{\varepsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r(\cos\theta)(r^2 \sin\theta\, dr\, d\theta\, d\phi)$$

The Total force on the northern hemisphere

$$F_z = \int dF_z$$

$$= \int f_z\, d\tau$$

$$= \int \frac{3}{\varepsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r\cos\theta\,(r^2 \sin\theta\, dr\, d\theta\, d\phi)$$

$$= \frac{3}{\varepsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 \int_0^R r^3\, dr \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{3}{\varepsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 \left(\frac{R^4}{4}\right)\left(\frac{\sin^2\theta}{2}\right)_0^{\pi/2} (2\pi)$$

$$= \frac{3Q^2}{64\pi\varepsilon_0 R^2}$$