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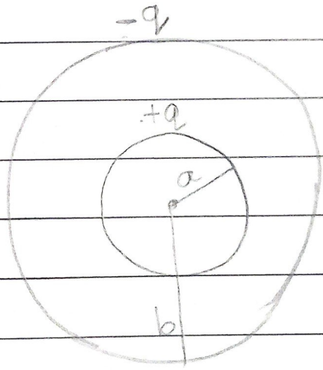
Batch 5

Electrostatic Energy & Superposition Principle

Q. Consider two concentric spherical shells of radii a and b. Suppose the inner one carries a charge q, and the outer one a charge $-q$ (both of them uniformly distributed over the surface). Calculate the energy of this configuration.



$$W_{tot} = \frac{\epsilon_0}{2}\int E^2 \, d\tau$$

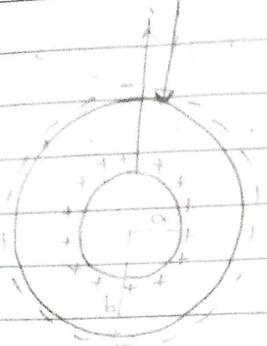
Because electrostatic energy is quadratic in the fields, it does not obey a superposition principle.

Thus the energy of the compound system is not the sum of energies of its parts considered seperately.

$$\therefore \; W_{tot} = \frac{\epsilon_0}{2}\int E_a^2 \, d\tau + \frac{\epsilon_0}{2}\int E_b^2 \, d\tau$$

But $W_{tot} = \frac{\epsilon_0}{2}\int (\bar{E}_a + \bar{E}_b) \, d\tau$

$$\therefore W_{tot} = \frac{\epsilon_0}{2}\int E_a^2 \cdot d\tau + \frac{\epsilon_0}{2}\int E_b^2\, d\tau + \epsilon_0 \int E_a \cdot E_b \cdot d\tau$$



1] Energy for shell 'a'.

$E_a = 0 \quad (r<a)$ & $E_a \neq 0 \,(a \leq r < \infty)$

$$\therefore W_a = \frac{\epsilon_0}{2}\int_a^\infty \left(\frac{kq}{r^2}\right)^2 \cdot 4\pi r^2 dr$$

$$\therefore W_a = \frac{1}{8\pi\epsilon_0}\frac{q^2}{a}$$

2]. Energy for shell 'b'.

$E_b = 0 \quad (r<b)$ and $E_b \neq 0 \quad (b \leq r < \infty)$

$$\therefore W_b = \frac{\epsilon_0}{2}\int_b^\infty \left(\frac{-kq}{r^2}\right)^2 \cdot 4\pi r^2 dr.$$

$$\therefore W_b = \frac{1}{8\pi\epsilon_0}\frac{q^2}{b}$$

3] To find $\epsilon_0 \int E_a E_b \, d\tau$.

$\because E_b = 0 \quad (a \leq r < b)$

$\therefore \bar{E}_a \cdot \bar{E}_b$ exists for $(r \geq b)$

Limits for integration are b to ∞

$$W_{ab} = \epsilon_0 \int_b^\infty \left(\frac{kq}{r^2}\right)\left(\frac{-kq}{r^2}\right) \cdot 4\pi r^2 dr$$

$$\therefore W_{ab} = \frac{-q^2}{4\pi\epsilon_0 b}$$

$$\therefore W_{total} = W_a + W_b + W_{ab}$$

$$\therefore W_{total} = \frac{q^2}{8\pi\epsilon_0 a} + \frac{q^2}{8\pi\epsilon_0 b} - \frac{q^2}{4\pi\epsilon_0 b}$$

$$\therefore W_{total} = \frac{q^2}{8\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$