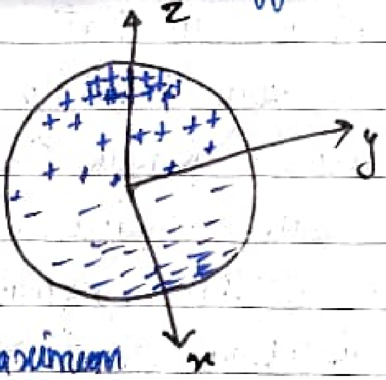
Q→ We have a sphere on whose surface, charge is distributed with surface charge density $\sigma = \sigma_0 \cos\theta$. θ corresponds to the spherical coordinate system. Find the electric field inside and outside ~~of~~ the sphere.

By principle this question can be solved by solving the integral

↳ $\left(\int \frac{\sigma(\vec{r}')\, da' (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \right)$

But this is not very easy to solve in this case. So, I would try to do this using a different method where I would convert this case into a different problem.

Given sphere →



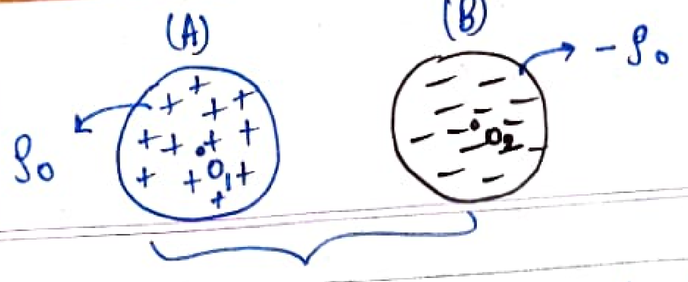
The charges vary according to $\sigma_0 \cos\theta$ and hence are the maximum concentrated at $\theta = 0$ & π.

Our Method →

⊕ We would look inside & outside separately.

So, let us imagine two spheres with ~~same~~ uniform volume charge densities in bulk which are same in magnitude for both but opposite in charge.
Let their centers be called O_1 and O_2

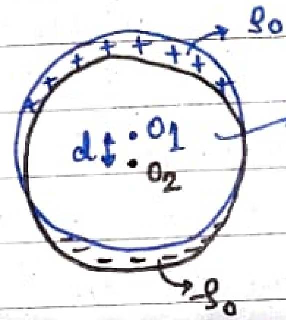
PAGE NO.:
DATE: / /



Now, let us merge them together (one over another) such that their origins coincide and hence all the charges cancel out.

↓

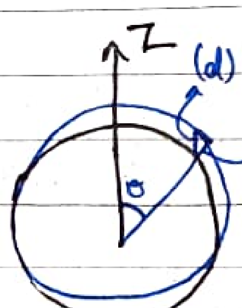
After this if we move (A) slightly upward being merged with (B) such that there is a separation between the origins (centers) O_1 and O_2. Now it would look like this;



(still all charges cancel out in the overlapping region.)

Let us say that $(d \to 0)$ and this movement happened very slightly. Then the outer leftout charges would behave as the surface charge of a sphere of same radius.

So if this movement has been along z-axis, then we observe that the thickness of the charged region would depend on θ {spherical coordinate}



(d) {shift along z-direction}

$(d\cos\theta)$ shift along θ angle from z-axis

PAGE NO.:
DATE: / /

so, if we look at infinitesimal charges along different θ, we get

$$dq = (\rho_0)\underbrace{(d\cos\theta)}_{\text{thickness}}\underbrace{(da)}_{\text{small area}}$$

$$dq = (\rho_0 d)(\cos\theta)(da)$$

which resembles, $(\sigma_0 \cos\theta\, da) \rightarrow$ infinitesimal charge on the surface of given sphere.

Hence we showed that, $\boxed{\sigma_0 = \rho_0 d}$ in our approximation.

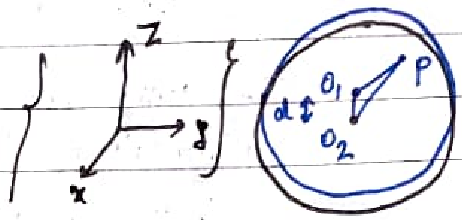
Calculating Electric Fields →

Inside →

Electric Field inside a uniformly charged sphere →

By Gauss Law → $$\boxed{\vec{E}(\vec{r}) = \frac{\rho \vec{r}}{3\epsilon_0}}$$

So here,



$$\vec{E_1} = \frac{\rho_0 (\vec{O_1P})}{3\epsilon_0}$$

$$\vec{E_2} = \frac{-\rho_0 (\vec{O_2P})}{3\epsilon_0}$$

Hence $\vec{E}_{\text{Resultant}} = \frac{\rho_0}{3\epsilon_0}\left(\vec{O_1P} - \vec{O_2P}\right) = \frac{\rho_0 (\vec{O_1O_2})}{3\epsilon_0}$

and $(|\vec{O_1O_2}| = d)$ whereas direction is $-\hat{k}$

so, $\left(\vec{E} = -\frac{\rho_0 d}{3\epsilon_0}\hat{k}\right) \rightarrow$ (for the given sphere) with $(\sigma = \sigma_0 \cos\theta) \rightarrow$ PTO

Here we also show that the Electric Field inside the sphere will be constant at all points in this surface charge arrangement.

Here, $(\rho_0 d = \sigma_0)$ as shown earlier

Hence $\boxed{\vec{E}_{in} = -\dfrac{\sigma_0 \hat{k}}{3\epsilon_0}}$ {Uniform Electric Field}

- <u>Outside</u> →

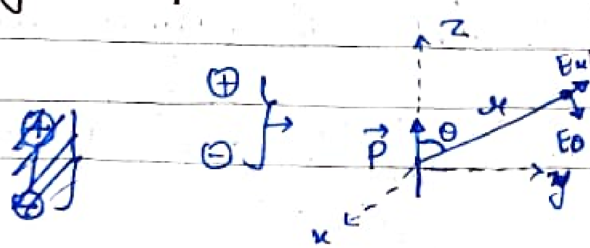
The given sphere is overall neutral and hence at very far distances we would observe that the monopole term is cancelled and the dipole term dominates the multipole expansion. If we calculate the dipole moment, then we get the below mentioned expression.

- With this method the problem simplifies to considering the upper and lower spheres as point charges at a distance 'd' apart and hence a dipole.

$$\vec{P} = qd = \left(\frac{4}{3}\pi R^3\right)(\rho_0) d\,\hat{k} = \frac{4}{3}\pi R^3 \sigma_0 \hat{k}$$

(The same expression we get by calculating dipole moment using the given case.)

- Hence the Electric Field can be calculated using the dipole moment as below →



$$\boxed{\vec{E}(r,\theta) = \frac{k\,\vec{P}}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$