Presentation Problem (Date - 06/04/2023)

Name - Sattwik Maji
Roll - 20221239
Batch - 06

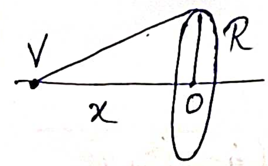
Q A conical surface (an empty ice-cream cone) carries a uniform surface charge density of σ. Height of the cone is h, as is the radius of the top. Find out the potential difference between points A (the vertex) & B (the center of the top).

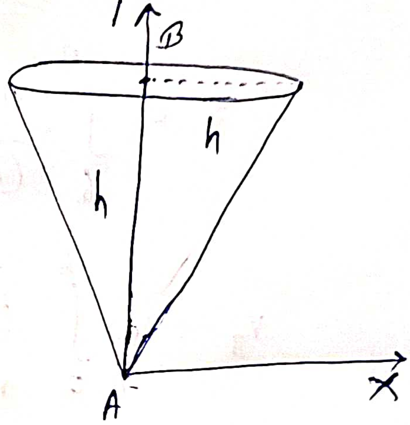
$\Rightarrow$

we know,

the potential ~~difference~~ of a uniformly charged ring Q at a distance x is

$$\boxed{V = k\frac{Q}{\sqrt{R^2+x^2}}}$$

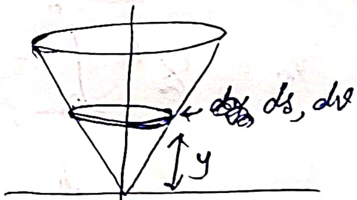




To calculate the potential at point A take ~~an en~~ a ring of length ~~ds~~ ds at a distance of y from the origin.

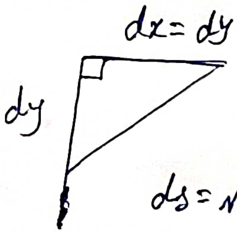
$\therefore$ potential for that ring at origin is

$$dV = k\frac{dq}{\sqrt{y^2+y^2}} = \frac{k\,dq}{\sqrt{2}\,y}$$



charge of that ring $dq = \cancel{2\pi y}\; 2\pi y \cdot \sigma \cdot ds$

$$= 2\sqrt{2}\pi\sigma y \,\cancel{ds}\, dy$$



$$ds = \sqrt{dx^2+dy^2} = \sqrt{2\,dy^2} = \sqrt{2}\,dy$$

$$\therefore \quad V_A = \int dV = \int_0^h \frac{k 2\sqrt{2}\pi\sigma y\, dy}{\sqrt{2}\, y}$$

$$= k2\pi\sigma \int_0^h dy$$

$$\therefore \boxed{V_A = k2\pi\sigma h}$$

Similarly,

for potential at point B —

$$dV = k\frac{dq}{\sqrt{(h-y)^2+y^2}} = \frac{k 2\sqrt{2}\pi\sigma y\, dy}{\sqrt{y^2+(h-y)^2}}$$

$$V_B = k\int_0^h \frac{2\sqrt{2}\,\pi\sigma y\,dy}{\sqrt{y^2+(h-y)^2}}$$

$$= 2\sqrt{2}\,\sigma k\pi \int_0^h \frac{y}{\sqrt{y^2+(h-y)^2}}\,dy \qquad [\text{take, } 2\sqrt{2}\,\sigma k\pi = c]$$

$$\therefore V_B = c\int_0^h \frac{y}{\sqrt{y^2+(h-y)^2}}\,dy \quad \text{------ (i)}$$

$$= c\int_h^0 \frac{h-x}{\sqrt{(h-x)^2+x^2}}\,(-dx) \qquad \Big|\; x = h-y \quad \therefore dx = -dy$$

$$V_B = c\int_0^h \frac{h-y}{\sqrt{(h-y)^2+y^2}}\,dy \qquad \cancel{[\text{put, } x = y \text{ (change of variable)}]}$$

$$\text{------ (ii)}$$

adding (i) & (ii) —

$$V_B = \frac{c}{2}h\int_0^h \frac{1}{\sqrt{y^2+(h-y)^2}}\,dy$$

$$= \frac{c}{2}h\int_0^h \frac{dy}{\sqrt{(\sqrt{2}y-h/\sqrt{2})^2+(h/\sqrt{2})^2}}$$

$$= \frac{ch}{2\sqrt{2}}\ln\left|\frac{h/\sqrt{2}+\sqrt{\frac{h^2}{2}+\frac{h^2}{2}}}{-h/\sqrt{2}+\sqrt{\frac{h^2}{2}+\frac{h^2}{2}}}\right|$$

$$= \frac{ch}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}h+h}{\sqrt{2}h-h}\right|$$

$$= \frac{ch}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| = \cancel{2\sqrt{2}}\,\sigma\pi k\,\frac{h}{\cancel{2\sqrt{2}}}\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$$

$$\therefore V_B = k\sigma\pi h\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$$

$$\therefore \boxed{\Delta V = k\pi\sigma h\left(\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| - 2\right)}$$