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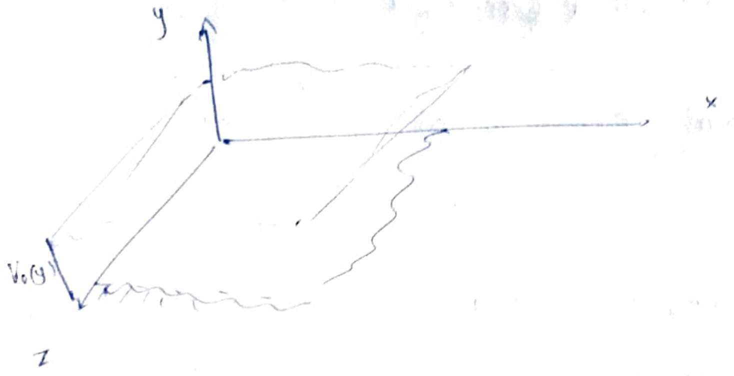
Two infinite grounded metal plates lie parallel to xz plane one at $y=0$ and the other at $y=a$. The left end is closed off with infinite strip insulated from the two plates, and maintained at a specific potential $V_0(y)$. Find the potential inside the slot



- Configuration independent of 'z'
- We must therefore solve Laplace's equation.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

- Boundary Conditions,
  - $V = 0$, at $y = 0$
  - $V = 0$ at $y = a$
  - $V = V_0(y)$, at $x = 0$
  - $V \to 0$ as $x \to \infty$.

Looking for solution in form of products,
$V(x,y) = X(x)\,Y(y)$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{f(x)} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{g(y)} = 0$$

Both functions have to be constant.

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \quad \& \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

$C_1 + C_2 = 0$

By Fourier trick

$$\frac{d^2X}{dx^2} = k^2 X \qquad \cancel{Y^2}\ \frac{d^2Y}{dy^2} = -k^2 Y$$

$$X(x) = Ae^{kx} + Be^{-kx}$$

$$Y(y) = C\sin ky + D\cos ky$$

$$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$$

As ~~x → ∞~~ $V \to 0$. $x \to \infty$.

$\therefore A = 0$.

$$V(x,y) = Be^{-kx}(C\sin ky + D\cos ky)$$

Absorbing B into C & D.

$$V(x,y) = e^{-kx}(C\sin ky + D\cos ky)$$

As $y=0$ $V=0$.

$\therefore D = 0$

$$\therefore V(x,y) = C\cdot e^{-kx}\cdot \sin ky.$$

at $y = a$, then, $k = \frac{n\pi}{a}$ for $V = 0$.

now $$V(x,y) = \sum_{n=1}^{\infty} C_n e^{-n\pi x/a} \sin(n\pi y/a)$$

~~for~~ It Satisfies 3 Boundary Condn

for.

$$V(0,y) = \sum C_n \cancel{e^{0}} \sin(n\pi y/a) = V_0(y)$$

By Fourier trick

$$C_n = \frac{2}{a} \int_0^a V_0(y) \sin\left(\frac{n\pi y}{a}\right) dy$$

$$C_n = \frac{2V_0}{a} \int_0^a \sin\left(\frac{n\pi y}{a}\right) dy$$

$$= \frac{2V_0}{n\pi} (1 - \cos n\pi)$$

$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4V_0}{n\pi} & \text{if } n \text{ is odd} \end{cases}$$

$$\therefore V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5\ldots} \frac{1}{n} e^{-n\pi x/a} \sin\left(\frac{n\pi y}{a}\right)$$