

Derive Clausius Mossotti equation

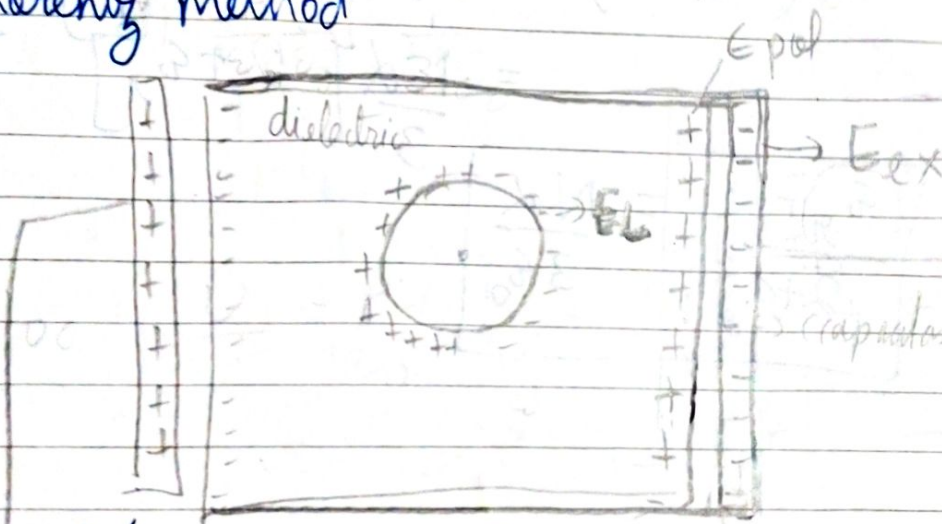
→ relation between ϵ_r [relative permittivity] and polarizability α of molecules constituting dielectric

$$P = \alpha E_{\text{local or internal}}$$

[dipole moment]

$$P = \epsilon_0 (\text{polarizability}) E_{\text{macroscopic}} = \epsilon_0 [\epsilon_r - 1] E_{\text{macroscopic}} = Np$$

Lorentz method



$$E_{\text{ex}} = \frac{V}{d}$$

$$E_{\text{near}} = 0$$

$$E_{\text{pol}} = -\frac{P}{\epsilon_0}$$

$$E$$

$$E_L = N_p \frac{P}{\epsilon_r \epsilon_0} = \frac{N_p P}{\epsilon_0} = \frac{1}{3} \frac{P}{\epsilon_0} \quad [\text{for sphere}]$$

$$E_{\text{local}} = E_{\text{ex}} + E_{\text{pol}} + E_L + E_{\text{near}}$$

$$= \frac{V}{d} - \frac{P}{\epsilon_0} + \frac{P}{3\epsilon_0} = E_{\text{macroscopic}} + \frac{P}{3\epsilon_0}$$

$$E_{\text{local}} = E_{\text{macroscopic}} \left[1 + \frac{\epsilon_0 (\epsilon_r - 1) N}{3 \epsilon_0} \right]$$

$$E_{\text{local}} = E_{\text{macro}} \left[1 + \frac{\epsilon_r - 1}{3} \right]$$

$$P = \alpha E_{\text{local}} = \alpha E_{\text{macro}} \left[1 + \frac{\epsilon_r - 1}{3} \right]$$

Using $P = Np$

$$\epsilon_0 (\epsilon_r - 1) E_{\text{macro}} = N \alpha E_{\text{macro}} \left[1 + \frac{\epsilon_r - 1}{3} \right]$$

$$\begin{aligned} \epsilon_0 (\epsilon_r - 1) &= N \alpha \left[1 + \frac{\epsilon_r - 1}{3} \right] \\ &= \frac{N \alpha}{3} [2 + \epsilon_r] \end{aligned}$$

$$\boxed{\frac{\epsilon_r - 1}{2 + \epsilon_r} = \frac{N \alpha}{3 \epsilon_0}}$$