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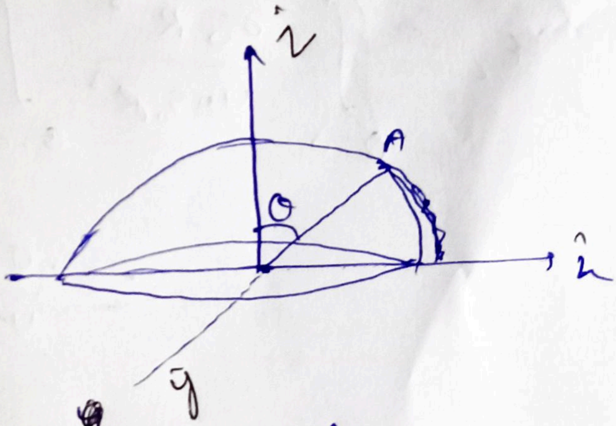
a hemisphere of Radius 'R' is placed with its centre at the origin and the rim in the x-y plane the hemisphere is on the +ve z-side. charge is distributed on its surface with surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ where θ is the angle made by the radius with the z-axis at the given position of the surface. Find the electric field at origin due to this charge distribution

→ Sol^n

Here charge density vary with 'θ' angle

$$E(r) = \frac{kq(\vec{r}-\vec{r}')}{(|r-r'|^3)}$$

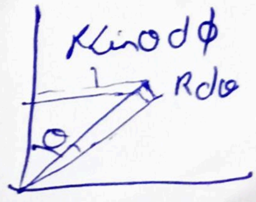


$\vec{r} = 0$

$\vec{r}' = R\cos\theta\,\hat{k} + R\sin\theta\sin\phi\,\hat{j} + R\sin\theta\cos\phi\,\hat{i}$

so we need q at A point to find E.F by this small charge.

$$\sigma = \frac{Q}{A}$$



$$\sigma = \frac{dq}{dA}$$

$dA = R\cdot d\theta \times R\sin\theta\, d\phi$

$dQ = \sigma_0 \cos\theta\, R^2 \sin\theta\, d\theta\, d\phi$

$$\therefore \quad dE_x = -\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \frac{k\sigma_0 R^2 \cos\theta \sin\theta \, d\theta \times R\sin\theta \cos\phi \, d\phi}{R^3}$$

from here

$$\int_{\phi=0}^{2\pi} \cos\phi \, d\phi$$

$= 0$

so $E_x = 0$

Similarly $E_y = 0$

$$E_z = -\frac{\sigma_0 R^3}{4\pi\varepsilon_0 R^3}\int_{\theta=0}^{\pi/2} \sin\theta \cos^2\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{-\sigma_0}{4\pi\varepsilon_0} \times 2\pi \int_{\theta=0}^{2\pi} \sin\theta \cos^2\theta \, d\theta$$

$$= +\frac{\sigma_0}{4\pi\varepsilon_0} \times 2\pi \int_0^{\pi/2} \cos^2\theta \, (d\cos\theta)$$

$$= \frac{\sigma_0}{2\varepsilon_0} \times \left| \frac{\cos^3\theta}{3} \right|_0^{\pi/2}$$

$$= \frac{\sigma_0}{2\varepsilon_0}\left[-\frac{1}{3} \right]$$

$$E = \frac{-\sigma_0}{6\varepsilon_0} \hat{k} \quad \text{(Ans)}$$