INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE
BS - MS DUAL DEGREE / Ph.D. / Int Ph.D. / MID SEM / END SEM EXAMINATION - 20

Name of Student Dhritiraj Bastav Kalita   Invigilator's Signature :

Registration Number 20221091   Course Name PH1213

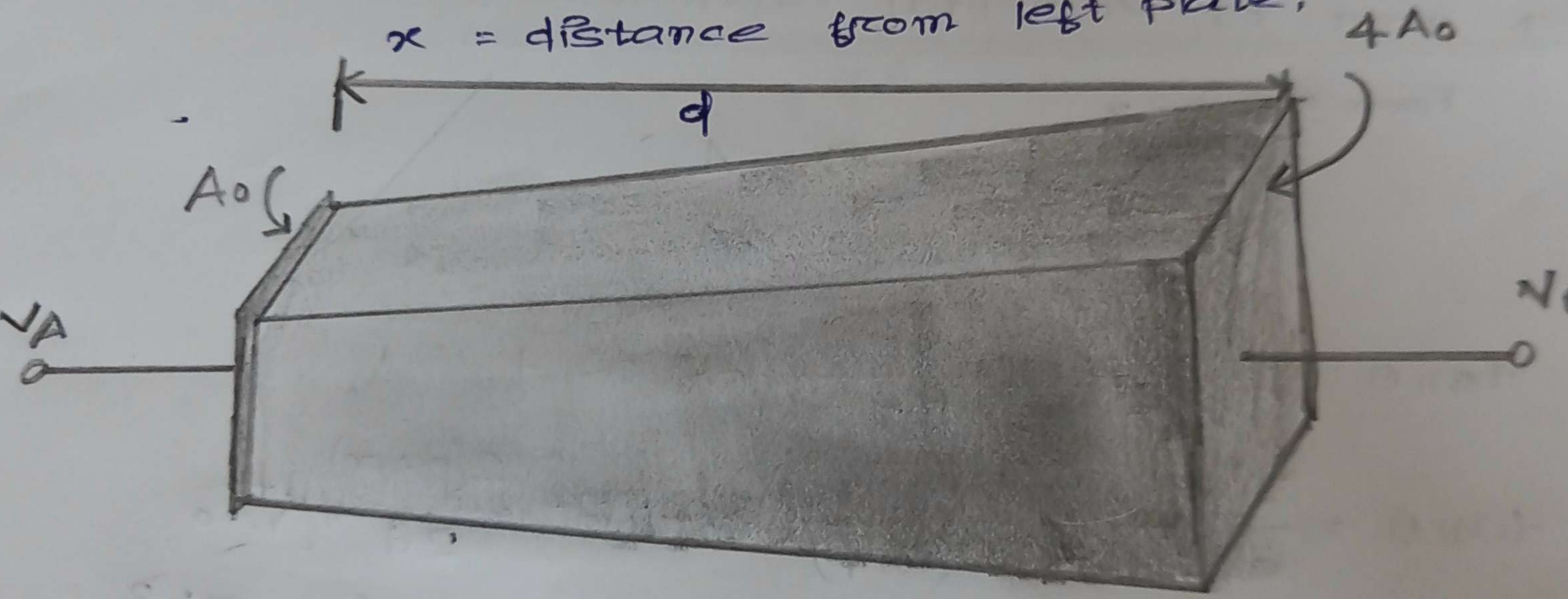
PRESENTATION

Outlining the problem :

We need a capacitor that can store a very high amount of energy. For a given voltage, we should therefore increase the capacitance. Now some engineers proposed to make a capacitor with increasing area with "Mica" inserted inside. The dielectric constant of Mica changes like –

$$K = K_0(1 + ax)$$

x = distance from left plate.



square area of plate also changes linearly.

for given, $d = \frac{1}{a}$

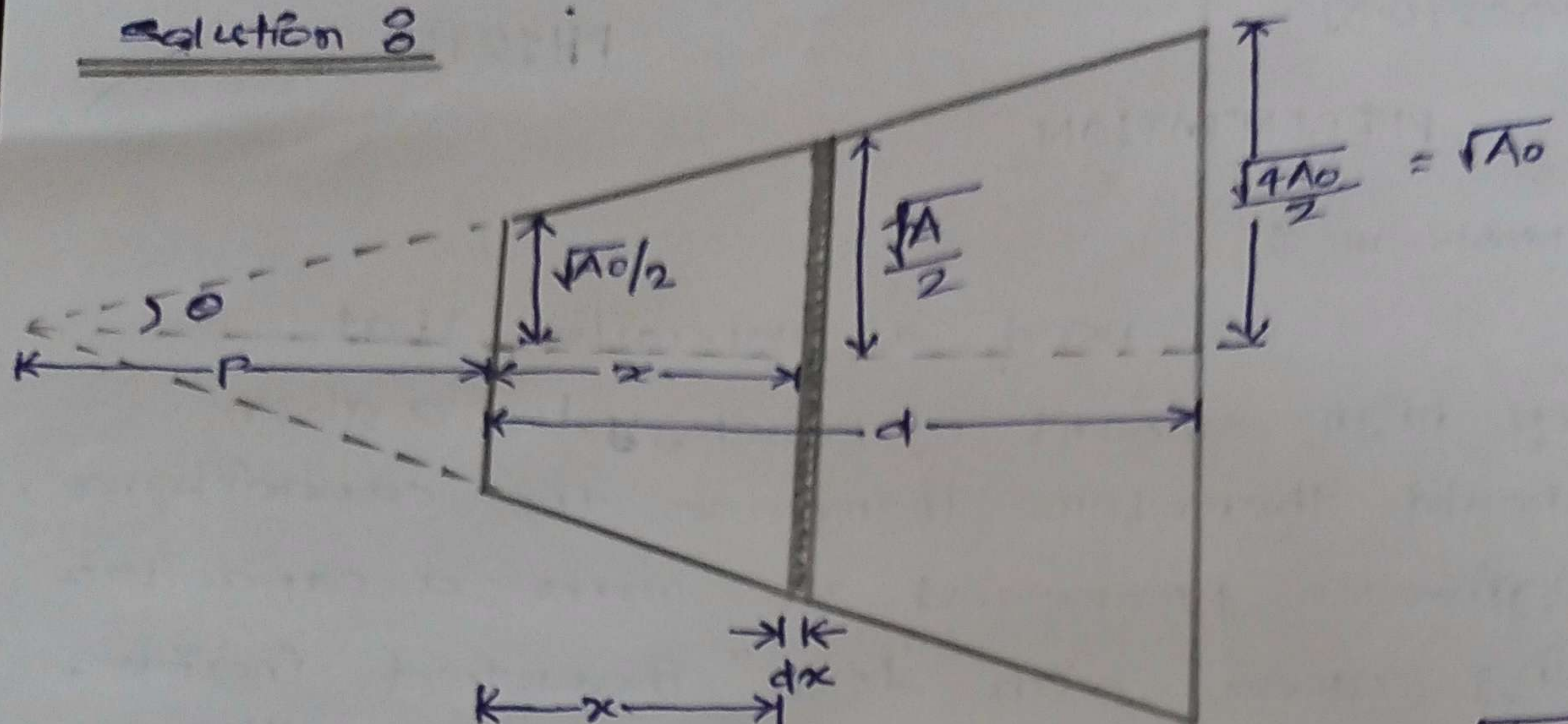
We need to find the capacitance of the system.

Outlining the solution :

We will have to consider elemental capacitors in parallel connection and integrate it all over. We will takes dx wide element at x distance from left and find its Area by linear proportion method. We then integrate these small capacitance dc to get c.

$$\frac{1}{c} = \int \frac{1}{dc}$$

Solution :



sidelengths at $x=0$ and $x=d$ are $\sqrt{A_0}$ and $\sqrt{4A_0}$

Finding area of elemental capacitor :

From figure,

Let, Area of it $= A$

$\therefore$ side length $= \sqrt{A}$

$$\therefore \quad \tan\theta = \frac{\sqrt{A_0}}{2p} = \frac{\sqrt{A_0}}{d+p} \Rightarrow p = d$$

$$\tan\theta = \frac{\sqrt{A_0}}{d+d} = \frac{\sqrt{A}}{2(x+p)} \Rightarrow \sqrt{A} = \frac{2(d+x)}{2d}\sqrt{A_0}$$

$$\Rightarrow A = A_0\left(1+\frac{x}{d}\right)^2$$

$$\therefore \quad \text{Area of element} = A_0\left(1+\frac{x}{d}\right)^2 = A_0(1+ax)^2$$

Now,

$$dC = \frac{K\epsilon_0 A}{dx} = \frac{K_0(1+ax)A_0(1+ax)^2\epsilon_0}{dx}$$

$$= \frac{K_0A_0(1+ax)^3\epsilon_0}{dx}$$

Now, we just have to integrate x from 0 to d.

∴ As for parallel capacitors, we know,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

for C_1, C_2, being very small,

$$\frac{1}{C} = \int_0^{n} \frac{1}{dC}$$

∴ Capacitance of the system here $= C$

$$\therefore \frac{1}{C} = \int_0^d \frac{dx}{\epsilon_0 K_0 A_0 (1+ax)^3}$$

$$= \frac{1}{K_0 A_0 \epsilon_0} \cdot \int_0^d \frac{dx}{(1+ax)^3}$$

$$= \frac{1}{\epsilon_0 K_0 A_0} \cdot \left[\frac{(1+ax)^{-2}}{-2} \cdot \frac{1}{a} \right]_0^d$$

$$= \frac{1}{\epsilon_0 K_0 A_0} \frac{1}{2a} \cdot \left(1 - \frac{1}{(1+ad)^2}\right)$$

$$= \frac{1}{2 K_0 A_0 \epsilon_0 a} \cdot \left(1 - \frac{1}{4}\right) \qquad (\because ad = 1)$$

$$= \frac{3}{8 K_0 A_0 \epsilon_0 a}$$

$$\therefore C = \frac{8 K_0 A_0 \epsilon_0 a}{3}$$

$$\boxed{\therefore \text{Capacitance of the system} = \frac{8 K_0 A_0 \epsilon_0 a}{3}}$$