Name: NIGEL RICFIELD KURBAH

Registration Number: 20221173

Batch: 3

Course Name: Electricity and Magnetism.

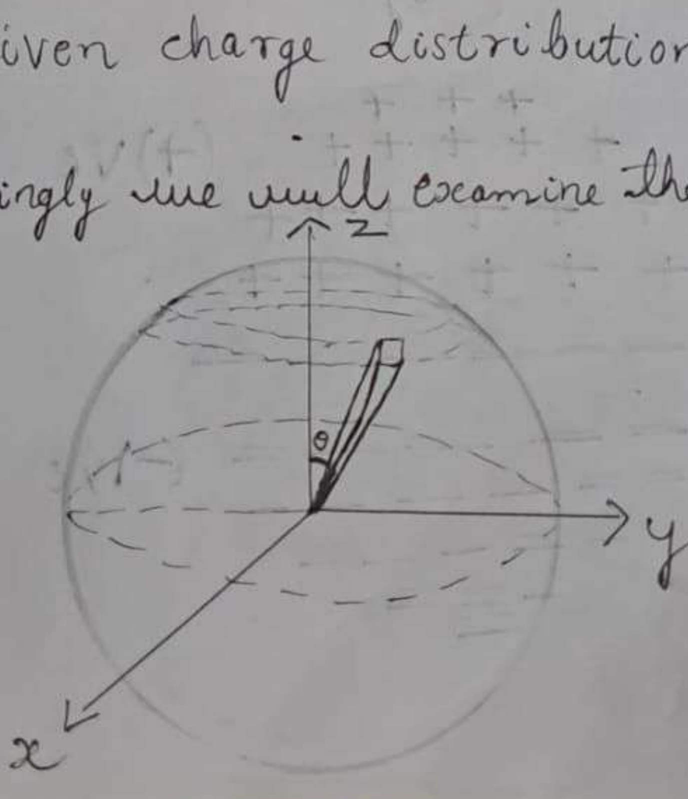
Course Code: PH1213

———————— xxx ————————

Q. Charge is distributed on the surface of a sphere with surface charge density $\sigma = \sigma_0 \cos\theta$ in spherical coordinates with origin at the centre. Find the electric field due to this distribution inside the sphere.

Soln: In this problem,

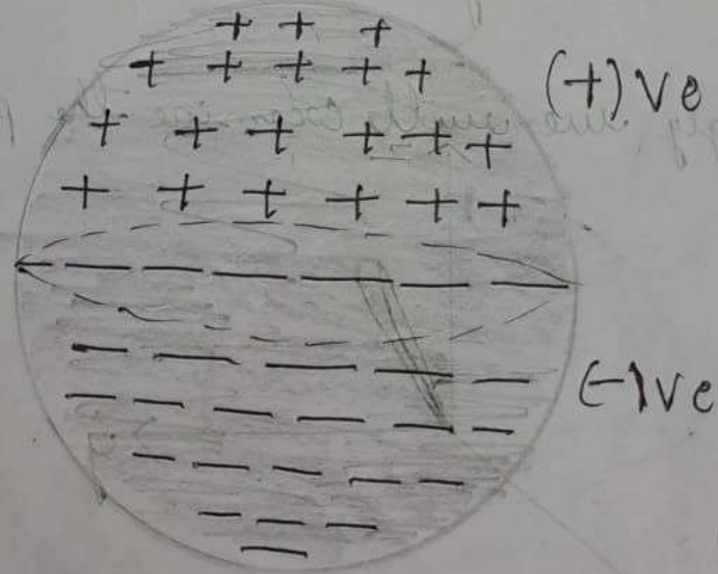
The given charge distribution is unsymmetric accordingly we will examine the problem.



For one particullar theta (θ), we will get a particudar charge distribution, i.e, for a selected θ we can trace out a ring on the sphere with different ϕ angles and on this ring the charge distribution will be $\sigma_0 \cos\theta$. It is evident from our observation that the charge density is maximum at the poles. and the charge density is minimum at the equator. i.e, at the poles it is σ_0 and at the equator it is zero. $\sigma_{pole} = \sigma_0 \cos 0^\circ = \sigma_0$.

$\sigma_{equator} = \sigma_0 \cos \frac{\pi}{2} = 0$.

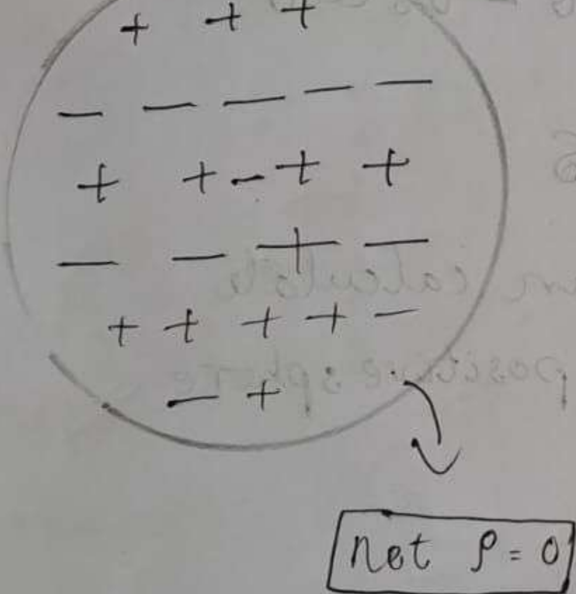
Consider the upper hemisphere to have positive charge density and lower negative charge density.

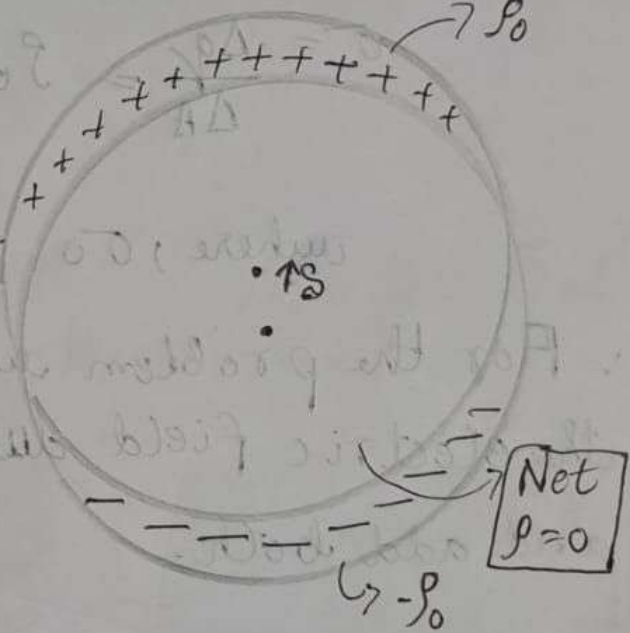


Consider 2 spheres, one with uniform positive charge distribution ρ_0,

another with uniform negative charge distribution $-\rho_0$, superimpose on each other.

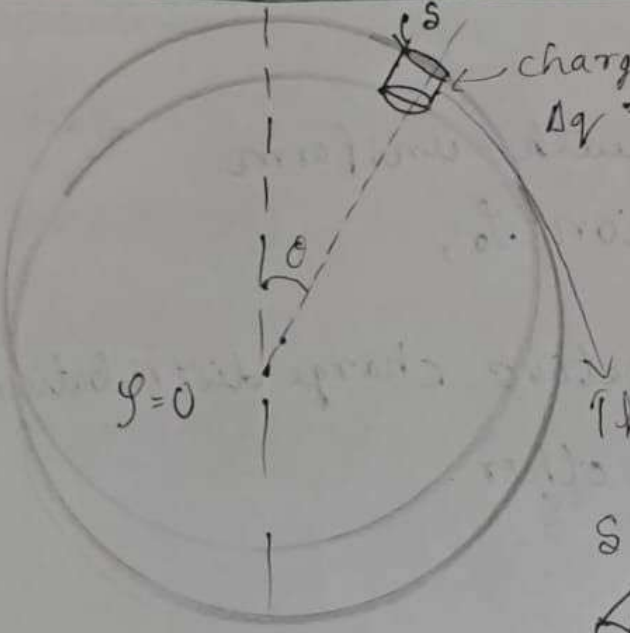
$\rho_0, -\rho_0 \rightarrow$ Volume charge densities and both spheres are identical.





$\rho_0, -\rho_0$

Now we displace the concentric positively charged sphere by a small distance δ upward.
If δ is very small the system looks like a sphere with charge concentrated on poles because the small volume almost acts as a surface.



charge $\Delta q = \rho_0 [(\Delta A)(\delta \cos\theta)]$

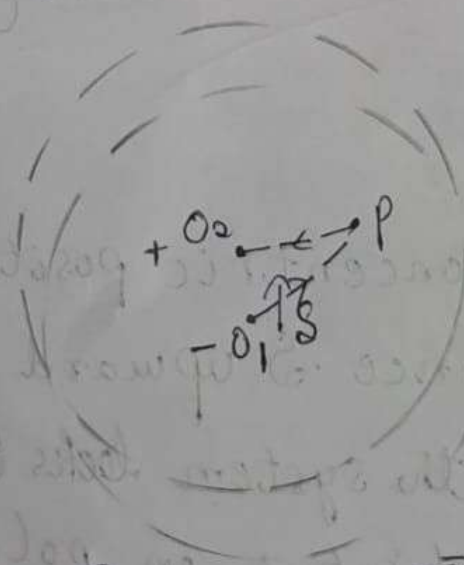
Thickness $= \delta \cos\theta$

∵ δ is small we get a surface charge density

$$\sigma = \frac{\Delta q}{\Delta A} = \rho_0 \delta \cos\theta = \sigma_0 \cos\theta$$

where, $\sigma_0 = \rho_0 \delta$

∴ For the problem we can calculate the electric field due to positive sphere and add both.



Electric field due to negative sphere: $\vec{E_1} = \dfrac{-\rho_0 \overrightarrow{O_1 P}}{3 \varepsilon_0}$

Electric field due to positive Sphere

$$\vec{E_2} = \frac{+\rho_0 \vec{O_2 P}}{3\varepsilon_0}.$$

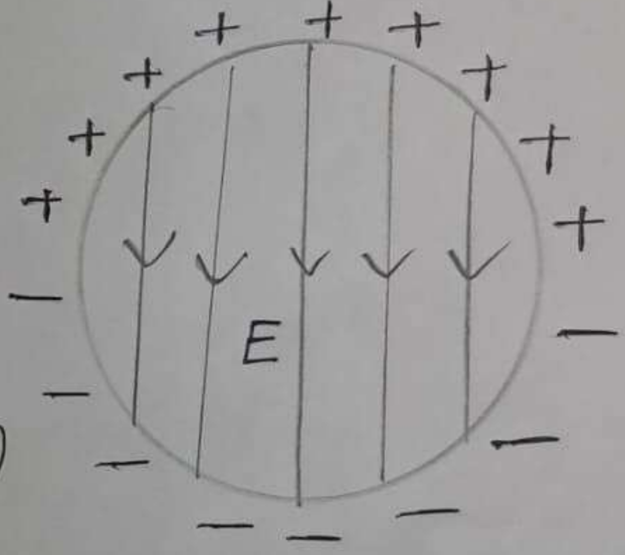
$$\therefore \vec{E_{net}} = \vec{E_1} + \vec{E_2}$$

$$= \frac{\rho_0}{3\varepsilon_0}\left[\vec{O_2 P} + \vec{P O_1}\right].$$

$$\vec{E_{net}} = \frac{\rho_0}{3\varepsilon_0}\vec{O_2 O_1} = \frac{\rho_0 \delta}{3\varepsilon_0}(-\hat{k}).$$

$\vec{P_x}$

(dipole moment)



$\therefore$ In idenical case.

$$\boxed{\vec{E} = \frac{\sigma_0}{3\varepsilon_0}(-\hat{k})}.$$