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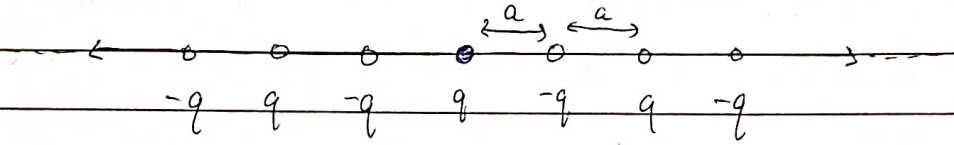
Problem

Q Consider an infinite chain of point charges $\pm q$ (with alternating signs), strung out along the x-axis, each a distance 'a' from its nearest neighbours. Find the work per particle required to assemble this system.

Soln)



Work done to assemble a configuration of point charges:

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)$$

Charges on both sides result in vanishing of $\frac{1}{2}$

Then $$W = \sum_{i=1}^{n} q_i V(r_i)$$

for ∞ charges: $$W = q\left[\frac{1}{4\pi\epsilon_0}\frac{-q}{a} + \frac{1}{4\pi\epsilon_0}\frac{q}{2a} \cdots\cdots\right]$$

$$W = \sum_{n=1}^{\infty} \frac{q^2}{4\pi\epsilon_0 a}\left[\frac{(-1)^n}{n}\right] = \frac{-q^2}{4\pi\epsilon_0 a}\left[1 - \frac{1}{2} + \frac{1}{3} \cdots\right]$$

Using expansion: $$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots\cdots$$

putting $x = 1 \Rightarrow$ $$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots\cdots$$

$$W = \frac{-q^2}{4\pi\epsilon_0 a} \cdot \ln 2$$

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In this system ln2 is known as Madelung constant which is used in determining the electrostatic potential of a single ion in a crystal by approximating the ions by point charges.