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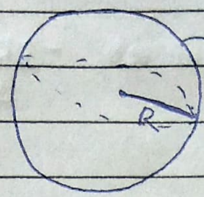
The potential at the surface of a hollow sphere (radius R) is given by $V_0 = k\cos 3\theta$, where k is a constant. Find the potential inside the sphere. (Assume there is no charge inside (or) outside the sphere)

Sol:



Using the solutions of Laplacian Equations for spherical coordinates we try to express the given potential as a function of Legendre Polynomials.

It is known that the solution comes out to be

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

$V(\theta) = k\cos 3\theta$

$= k(4\cos^3\theta - 3\cos\theta)$

$$\boxed{V(\theta) = k(\alpha P_3(\cos\theta) + \beta P_1(\cos\theta))}$$

$P_1(\cos\theta) = \cos\theta$

$P_3(\cos\theta) = \frac{5\cos^3\theta - 3\cos\theta}{2}$

$\Rightarrow 4\cos^3\theta - 3\cos\theta = \alpha\left(\frac{5\cos^3\theta - 3\cos\theta}{2}\right) + \beta\cos\theta$

$\Leftrightarrow \boxed{\alpha = \frac{8}{5}}$; $\boxed{\beta = \frac{-3}{5}}$ $\Rightarrow \boxed{V(\theta) = k\left[\frac{8}{5}P_3(\cos\theta) - \frac{3}{5}P_1(\cos\theta)\right]}$

$\because V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$ for $r<R$

where $\boxed{A_l = \frac{2l+1}{2R^l}\int_0^{\pi} V_0(\theta) P_l(\cos\theta)\sin\theta\, d\theta}$

↳ which comes from Fourier's trick

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where $\int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta \, d\theta = \begin{cases} 0 & \text{for } l \neq l' \\ \dfrac{2}{2l+1} & \text{if } l = l' \end{cases}$

$$\Rightarrow A_l = \frac{2l+1}{2R^l} \left[\frac{8K}{5} \int_0^{\pi} P_3(\cos\theta) \, P_l(\cos\theta) \sin\theta \, d\theta - \frac{3K}{5} \int_0^{\pi} 3P_1(\cos\theta) P_l(\cos\theta) \sin\theta \right.$$

Put $l = 3$ and $l = 1$ respectively

$$\Rightarrow A_l = \frac{2l+1}{2R^l} \left[\frac{8K}{5} - \frac{3K}{5} \left(\frac{2}{2l+1} (\delta_l) \right) \right]$$

$$\Rightarrow A_l = \frac{K}{5R^l} \left[8\delta(u) - 3\delta(l) \right]$$

$$A_l = \frac{K}{5R^3}(8) - \frac{3K}{5K}$$

$$\boxed{\therefore V(r,\theta) = \frac{8K}{5R^3} r^3 P_3(\cos\theta) - \frac{3K}{5R} r P_1(\cos\theta)}$$