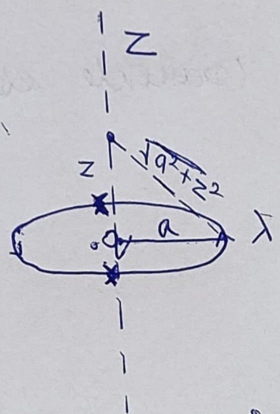
Name - Vaishnavi Dharmadhikari

Roll no. - 20221281

→ **Problem-**

consider a uniform ring having λ charge distribution and there is a current flowing. There is a charge q at the centre along z axis. what will be the motion that particle follows when little jerk is given.

radius = a



$q = \lambda \times 2\pi a$

→ since Forces are symmetric, charge will remain on z axis (its motion)

→ To find force on charge q at any value of z =

Potential $V_z(z) = \frac{1}{4\pi\varepsilon_0} \frac{2\pi a\lambda}{\sqrt{a^2+z^2}}$

Electric field $E_z(z) =$ ∵ $E = -\nabla V$

$$E_z(z) = -\frac{\partial V}{\partial z}$$

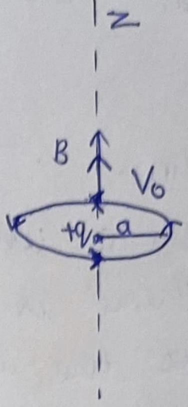
$$E_z(z) = \frac{1}{4\pi\varepsilon_0} 2\pi a\lambda \cdot \frac{\frac{1}{2}}{(a^2+z^2)^{3/2}} \cdot 2z$$

$$E_z(z) \Rightarrow \frac{2\pi a\lambda z}{4\pi\varepsilon_0 (a^2+z^2)^{3/2}}$$

If $\lambda > 0$

Two cases-

case (I)- $q > 0$



when charge q is pushed up with velocity V_0

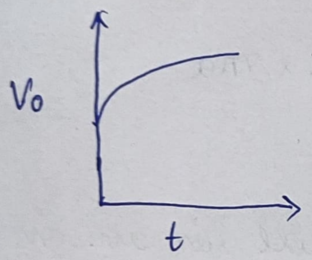
Force due to Magnetic field

$F_B = 0 \quad [\because \vec{V} \times \vec{B} = 0]$

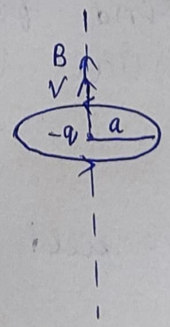
Force due to electric field

$F_E = q E_z > 0$

$m \frac{dV_z}{dt} = F_E(z) > 0$ (particle escape to infinity)



case (II) - $q < 0$



$F_B = 0$

$F_E(z) = q E_z < 0$

$m \frac{dV_z}{dt} = f_z < 0$

→ Total energy is constant-

$m \frac{dV_z}{dt} = -q \cdot \frac{2\pi a \lambda z}{4\pi\varepsilon_0 (a^2+z^2)^{3/2}}$

Charge q will oscillate (undergo SHM)

multiplying both side by V_z

$m V_z \frac{dV_z}{dt} = q E_z(z) V_z$

$m V_z \frac{dV_z}{dt} = q E_z(z) \frac{dz}{dt}$

It can be written as

$$\frac{d}{dt}\left(\frac{1}{2}m v_z^2\right) = q E_z(z)\frac{dz}{dt}$$

$$\frac{d}{dt}\left(\frac{1}{2}m v_z^2\right) = -q \frac{dv}{dz}\frac{dz}{dt}$$

$$\frac{d}{dt}\underbrace{\left(\frac{1}{2}m v_z^2\right)}_{K.E} = -\frac{d}{dt}\underbrace{(qV)}_{P.E}$$

$$\therefore \quad \frac{d}{dt}\left(\frac{1}{2}m v_z^2 + qV\right) = 0$$

$$E_{Total} = \frac{1}{2}m v_z^2 + q V_z \Rightarrow \text{constant}$$