Physics Presentation

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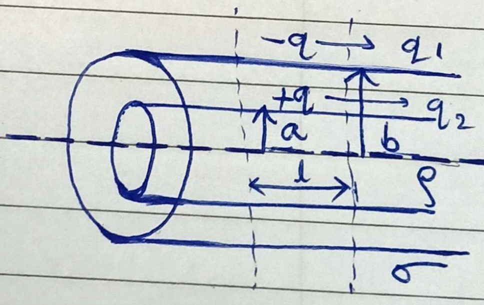
(Q) A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform ~~su~~ surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s<a$), (ii) between the cylinders ($a<s<b$), (iii) outside the cable ($s>b$). Plot $|E|$ as a function of s.

Soln:

$$q_1+q_2=0$$



$$\sigma = \frac{q_1}{2\pi b l} \therefore q_1 = \sigma 2\pi b l$$

$$\rho = \frac{q_2}{\pi a^2 l} \therefore q_2 = \rho\pi a^2 l$$

3 cases.

$s<a$
$b>s>a$
$s>b$

electric field = ?

Now,

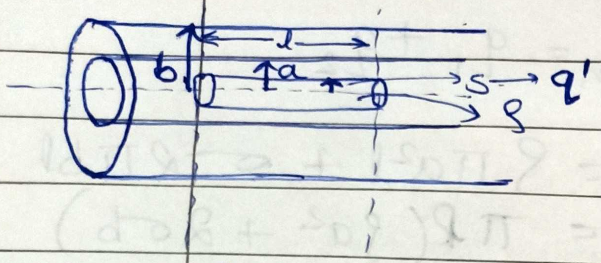
$$\sigma 2\pi b l + \rho\pi a^2 l = 0$$

$$\pi l(\sigma 2b + \rho a^2) = 0$$

$$\boxed{\sigma 2b = -\rho a^2} \quad \text{--- (1)}$$

For,

$s < a$.



$$\boxed{\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}}$$

$q' = \rho \times \pi s^2 l$

$A = 2\pi s l$

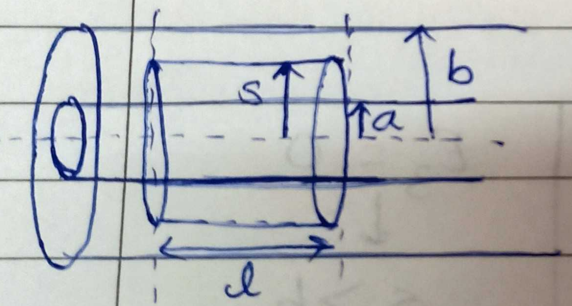
$$E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$EA = \frac{q_{in}}{\varepsilon_0}$$

$$E \cdot 2\cancel{\pi}\cancel{s}\cancel{l} = \frac{\rho \cancel{\pi} s^{\cancel{2}} \cancel{l}}{\varepsilon_0}$$

$$\boxed{E_1 = \frac{\rho s}{2\varepsilon_0}} \rightarrow s < a.$$

For, $a < s < b$

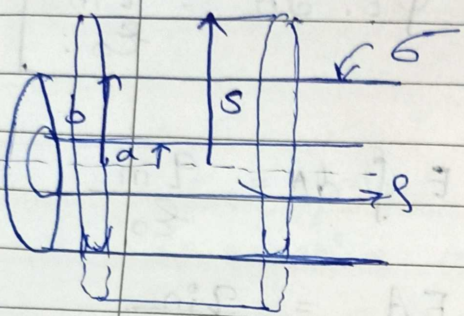


$$E \cdot A = \frac{q_{in}}{\varepsilon_0}$$

$$E \cdot 2\cancel{\pi} s \cancel{l} = \frac{\rho \cancel{\pi} a^2 \cancel{l}}{\varepsilon_0}$$

$$\boxed{E_2 = \frac{\rho a^2}{2\varepsilon_0 s}}$$

For ~~b~~ $s > b$



Here,

$q_{in} = q_1 + q_2$

$= \rho \pi a^2 l + \sigma 2\pi b l$

$= \pi l(\rho a^2 + 2\sigma b)$

$= \pi l(-2\sigma b + 2\sigma b)$ ↳ (by eq^n ①)

$= 0$

$\therefore EA = \frac{q_{in}}{\varepsilon_0} = \frac{0}{\varepsilon_0}$

$\therefore E \cdot 2\pi s l = \frac{0}{\varepsilon_0}$

$\boxed{E_3 = 0}$

$E_1 = \frac{\rho s}{2\varepsilon_0}$	$E_2 = \frac{\rho a^2}{2\varepsilon_0 s}$	$E_3 = 0$
$\therefore E_1 \propto s$	$E_2 \propto \frac{1}{s}$	↓
↓ $s < a$	$a < s < b$	$s > b$

At, $s = a$, $E_a = \frac{\rho a}{2\varepsilon_0}$

At, $s = b$, $E_b = \frac{\rho a^2}{2\varepsilon_0 b} \neq 0$

Graph: