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Physics Presentation.

→ Effect of change of coordinate origin on Electric Dipole and Quadrupole moments.

Definition of dipole moment = $p_i = \int r_i dq$

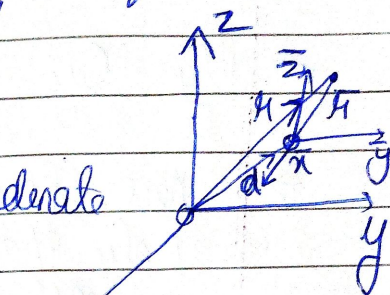
$$\vec{P} = \int \vec{r}_i dq$$

$dq = d\bar{q}$ as charge is independent of coordinate system.

using Eq (i)

$$\begin{aligned} \vec{P}_i &= \int (r_i - d_i) dq \\ &= \int r_i dq - \int d_i dq \end{aligned}$$

$$\boxed{\vec{P}_i = \vec{P}_i - d_i q} \quad \text{--- (A)}$$



So by triangular law of vector addition
 $\vec{d} + \vec{r} = \vec{r}$

$$\Rightarrow \vec{r} = \vec{r} - \vec{d}$$

$$\& \quad \vec{r}_i = \vec{r}_i - \vec{d}_i \rightarrow \text{(i)}$$

$$\vec{r}_j = \vec{r}_j - \vec{d}_j \rightarrow \text{(ii)}$$

Similarly for Quadrupole moment $Q_{ij} = \int r_i r_j dq$

$$Q_{ij} = \int r_i r_j dq$$

Using (i) & (ii)

$$Q_{ij} = \int (r_i - d_i)(r_j - d_j) dq$$

$$= \int r_i r_j dq - \int r_i d_j dq - \int r_j d_i dq + \int d_i d_j dq$$

$$\boxed{Q_{ij} = Q_{ij} - r_i d_j - r_j d_i + d_i d_j q} \quad \text{--- (B)}$$

Eg. A & B are desired result for origin dependency for dipole & Quadrupole moment.

Discussion about Result

- Both are valid for discrete distribution of charges.
- In Eqⁿ A is independent of origin only when charge distribution is neutral ($q=0$)
- $\text{If } Q_{ij}$ is origin independent only when $q=0$ & $P=0$
- These Results can be generalized as:
only the leading non vanishing electric multipole moment of an arbitrary charge distribution is origin independent.
- If $q \neq 0$, A shows that $\vec{P}=0$. It is always possible to find an origin \vec{O} such that electric dipole moment is zero.
 $d = P/q$ gives