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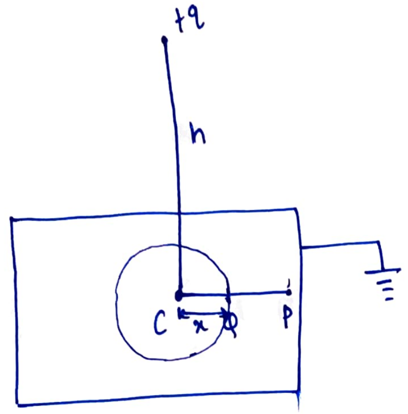
Q: A charge q is placed at height h from a grounded, large, conducting plate. Find the radius of the circular area that contains charge $\left(\frac{-q}{2}\right)$ and has its centre at the foot of perpendicular from the charge to the plate.

Soln:



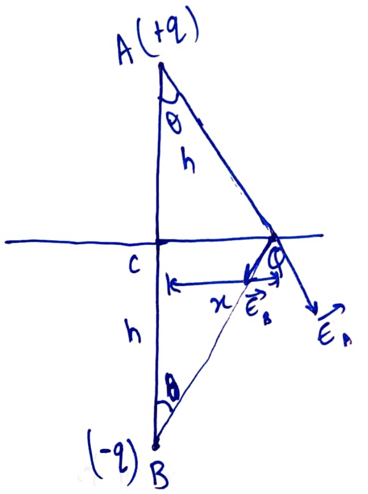
Due to the presence of +q there is an induced -q on the conducting plate.

Let A be the point (at height h from the conducting plate) carrying a charge +q.

Since, centre is the foot of perpendicular circular symmetry can be observed.

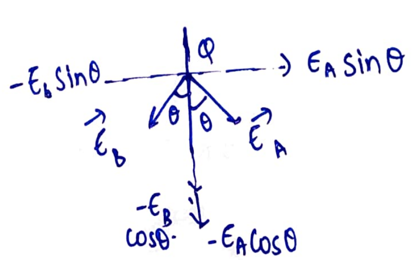
Let there be a circle of radius x passing through Q with C as its center.

Surface charge density at all of its points on the circle would be same since radius is same. This conducting plate can be removed by a point -q charge (image charge) placed at a distance 2h from +q.



Since -q satisfies the boundary conditions i.e., V is constant i.e., 0 at all points on the circle of radius x passing through P and Electric field at ∞ due to this charge is zero.

Electric field at Q:



$$E_x = E_a \sin\theta - E_b \sin\theta$$

$$= \frac{q}{4\pi\epsilon_0} \frac{x}{(h^2+x^2)^{3/2}} - \frac{qx}{4\pi\epsilon_0 (h^2+x^2)^{3/2}}$$

$$= 0$$

$$E_y = -E_a\cos\theta - E_b\cos\theta$$

$$= \frac{-q}{4\pi\epsilon_0}\frac{h}{(h^2+x^2)^{3/2}} - \frac{qh}{4\pi\epsilon_0(h^2+x^2)^{3/2}}$$

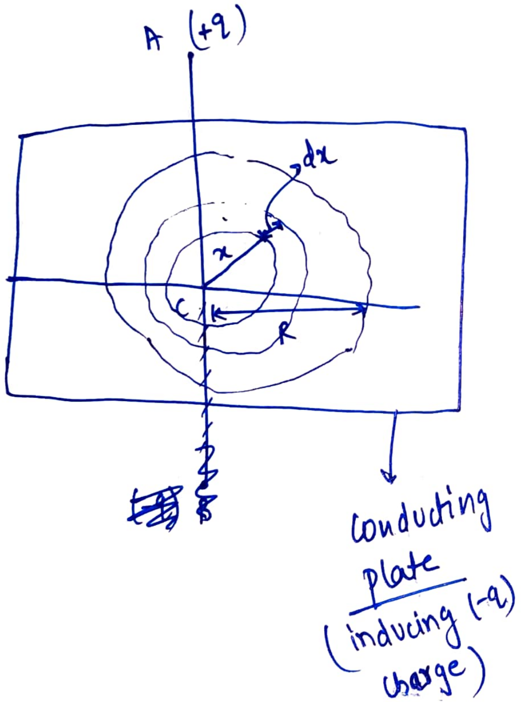
$$= \frac{-2qh}{4\pi\epsilon_0(h^2+x^2)^{3/2}}$$

Electric field at P $= E_x + E_y = \frac{-2qh}{4\pi\epsilon_0(h^2+x^2)^{3/2}}$

Through electric field at P we can find surface charge density at P through -

$$E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = E\epsilon_0 = \epsilon_0\left(\frac{-2qh}{4\pi\epsilon_0(h^2+x^2)^{3/2}}\right) = \frac{-2qh}{4\pi(h^2+x^2)^{3/2}}$$



Let R be the radius of circle consisting $-\frac{q}{2}$ charge inside the conductor. Let there be a ring of radius x and thickness dx. $(x<R)$
Charge on ring =

$$\sigma \times \text{Area of ring}$$

$$= \sigma \times 2\pi x\,dx$$

$$= \frac{-2qh}{4\pi(h^2+x^2)^{3/2}} \times 2\pi x\,dx$$

$$= \frac{-qh\,x\,dx}{(h^2+x^2)^{3/2}}$$

Charge inside the circle of radius R should be $\left(\frac{-q}{2}\right)$ according to our assumption. So,

$$\int_0^R \frac{-qhx\,dx}{(h^2+x^2)^{3/2}} = -\frac{q}{2}$$

Let $t = h^2 + x^2$

$$\frac{dt}{2} = x\,dx$$

If $x=0$ then $t = h^2$

$x = R$ then $t = h^2 + R^2$

$$\int_{h^2}^{h^2+R^2} \frac{-qh\,dt}{2\,(t)^{3/2}} = -\frac{q}{2}$$

$$\frac{-qh}{2} \int_{h^2}^{h^2+R^2} \frac{dt}{t^{3/2}} = -\frac{q}{2}$$

$$h \left. \frac{-2}{\sqrt{t}} \right|_{h^2}^{h^2+R^2} = +1$$

$$-2h \left(\frac{1}{\sqrt{h^2+R^2}} - \frac{1}{\sqrt{h^2}} \right) = 1$$

$$h \left(\frac{h - \sqrt{h^2+R^2}}{h\sqrt{h^2+R^2}} \right) = \frac{-1}{2}$$

$$\frac{h - \sqrt{h^2+R^2}}{\sqrt{h^2+R^2}} = \frac{-1}{2}$$

$$h - \sqrt{h^2+R^2} = -\frac{\sqrt{h^2+R^2}}{2}$$

$$h = \frac{\sqrt{h^2+R^2}}{2}$$

$$2h = \sqrt{h^2+R^2}$$

On squaring both sides $\Rightarrow 4h^2 = h^2 + R^2 \Rightarrow R^2 = 3h^2 \Rightarrow R = \sqrt{3}\,h$ Ans