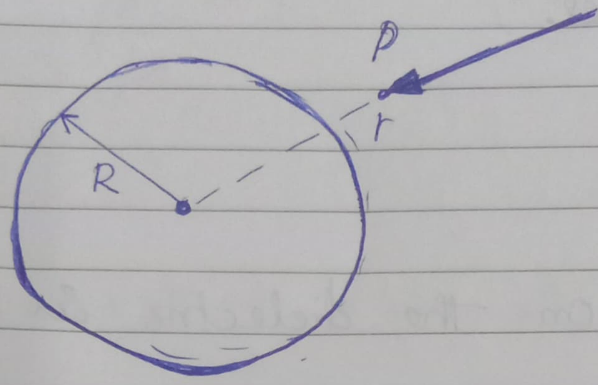
Find the potential inside and outside a spherical shell of radius R, which carries a uniform surface charge. set the reference at ∞



From Gauss's law, the feild outside is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r},$$

q = total charge on the sphere.
Also, feild inside is zero.

For points outside the sphere $(r > R)$,

$$V(r) = -\int_{\mathcal{O}}^{r} E \cdot dl = \frac{-1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} dr'$$

$$-\int_{0}^{r} E \cdot dl = \frac{-1}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{q}{r^2} dr'$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r'} \Big|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

Now, to find the potential inside the sphere we must break integral into two sections

$$V(r) = \frac{-1}{4\pi\varepsilon_0} \int_{\infty}^{R} \frac{q}{r^2} dr' - \int_{R}^{r} (0)\, dr' = \frac{1}{4\pi\varepsilon_0} \frac{q}{r'} \Big|_{\infty}^{R} + 0$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

$\therefore$ Potential is not zero inside the shell even though feild is equal to zero.
V is a constant in this region, to be sur
so that $\nabla V = 0$ $\longrightarrow$ that's what matters.