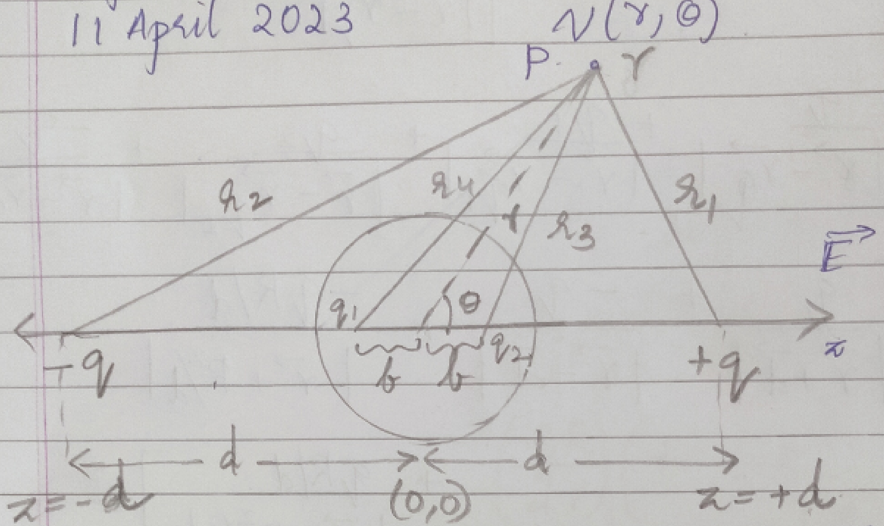
Induced Charge Density on a Spherical conducting Body Placed on an Electric Field.

20221223

Rijul B

11 April 2023



A Spherical body, conducting, is placed on an electric field created by the charges $+q$ and ~~&~~ $-q$

The Electric Field $E_0 = \dfrac{2kQ}{d^2}$ at $(0,0)$

$E = E_0 \hat{z}$ at centre (approximately)

if q remains constant & $d \to \infty$

By method of charges inside the conducting sphere,

$$q_1 = \frac{-qR}{d} \quad \text{at } z = -R^2/d$$

$$q_2 = \frac{+qR}{d} \quad \text{at } z = R^2/d$$

$$|b| = R^2/d.$$

First we find the net potential on an arbitrary point P.

$$V(r,\theta) = V(\vec{r}) = \cancel{k\left[\frac{q}{|\vec{r}-\vec{r}_q|}\right.} +$$

$$= k\left[\frac{q}{|\vec{r}-\vec{r}_q|} + \frac{q_1}{|\vec{r}-\vec{r}_{q_1}|} + \frac{q_2}{|\vec{r}-\vec{r}_{q_2}|} + \frac{-q}{|\vec{r}-\vec{r}_q|}\right]$$

$$= k\left[\frac{q}{|\vec{r}+\vec{d}|} + \frac{-q}{|\vec{r}-\vec{d}|} + \frac{-qR/d}{|\vec{r}+R^2/d|} + \frac{qR/d}{|\vec{r}-R^2/d|}\right]$$

$$= k\left[\frac{q}{(r^2+d^2+2rd\cos\theta)^{1/2}} - \frac{q}{(r^2+d^2-2rd\cos\theta)^{1/2}} - \frac{qR/d}{\left(r^2+R^4/d^2+\frac{2rR^2}{d}\cos\theta\right)^{1/2}} + \frac{qR/d}{\left(r^2+R^4/d^2-\frac{2rR^2}{d}\cos\theta\right)^{1/2}}\right]$$

If $r \ll d$

$$(r^2+d^2+2rd\cos\theta)^{-1/2} = d^{-1}\left(1+r^2/d^2+\frac{2}{d}r\cos\theta\right)^{-1/2}$$

$$\approx d^{-1}\left(1-\frac{r^2}{2d^2}-\frac{r}{d}\cos\theta \ldots\right)$$

$$\left(r^2 + \frac{R^4}{d^2} + \frac{2rR^2\cos\theta}{d}\right)^{-1/2}$$

$$= r^{-1}\left(1 + \frac{R^4}{r^2d^2} + \frac{2R^2}{rd}\cos\theta\right)^{-1/2}$$

$$\approx r^{-1}\left(1 - \frac{1}{2}\frac{R^4}{r^2d^2} - \frac{R^2}{rd}\cos\theta \cdots\right)$$

$$V(\vec{r}) = k\Bigg[\frac{q}{d}\left(1 - \frac{r^2}{2d^2} - \frac{r}{d}\cos\theta\right)$$

$$+ \left(\frac{-q}{d}\right)\left(1 - \frac{r^2}{2d^2} + \frac{r}{d}\cos\theta\right)$$

$$+ \left(\frac{-qR}{dr}\right)\left(1 - \frac{1}{2}\frac{R^4}{r^2d^2} - \frac{R^2}{rd}\cos\theta\right)$$

$$+ \left(\frac{qR}{dr}\right)\left(1 - \frac{R^4}{2r^2d^2} + \frac{R^2}{rd}\cos\theta\right)\Bigg]$$

$$= k\left[\frac{q}{d}\left[\frac{-2r\cos\theta}{d}\right] + \frac{qR}{dR}\left[\frac{2R^2\cos\theta}{rd}\right]\right]$$

$$= \left\{-k\frac{2qr\cos\theta}{d^2} + \frac{2qR^3k\cos\theta}{r^2d^2}\right\}$$

$$= -\frac{2kq}{d^2}\left\{r - \frac{R^3}{r^2}\right\}\cos\theta$$

$$= -E_0\left\{r - \frac{R^3}{r^2}\right\}\cos\theta$$

$V(r, \theta) = -E_0 r \cos\theta + E_0 \frac{R^3}{r^2} \cos\theta$

$-E_0 r\cos\theta$: Potential due to uniform electric field

$E_0 \frac{R^3}{r^2} \cos\theta$: Potential due to induced surface charge density (Image charges)

Induced surface charge density

$$\sigma = -\varepsilon_0 \frac{\partial \phi}{\partial r}\Big|_{r=R}$$

$$= -\varepsilon_0 \left[-E_0 \left(1 + \frac{2a^3}{r^3}\right) \cos\theta \right]_{r=R}$$

$$= \underline{\underline{3E_0 \varepsilon_0 \cos\theta}}$$

———— X ————