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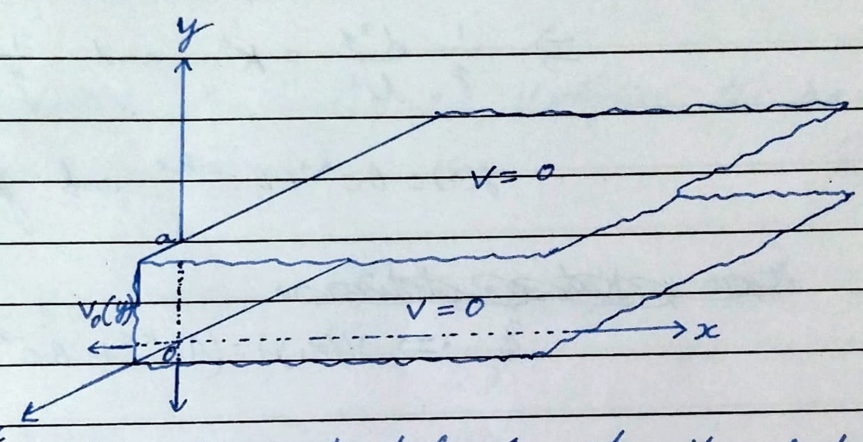
Physics Presentation

Q 2 infinite plane grounded conductors $\parallel^{l}$ to xz - plane are kept at $y = 0$ and $y = a$ with left end cut off by an infinite strip insulated from other two plates, maintained at $V = V_0(y)$ as below =



So then, what is the potential inside the slot?

See, what if $V(0, y) = V_0$ (a constant)?

Ans = Part-1: Thus, as no charge in there inside $\rho = 0$ for $x > 0$, $0 < y < a$. Thus, the Poisson's equa^n for electrostatics turns to Laplace's equa^n (Thus, V is independent of z $\Rightarrow \frac{\partial^2 V}{\partial z^2} = 0$) =

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Now, turning to see the Boundary conditions in the problem =

i) $V = 0$; $y = 0, a$

ii) $V = V_0(y)$; $x = 0$

iii) $V \to \infty$; $x \to \infty$ (By convention)

Now, lets look at sol^{n}s of form =

$$V(x, y) = f(x)\, g(y)$$

$$\Rightarrow g \frac{d^2 f}{dx^2} + f \frac{d^2 g}{dy^2} = 0$$

$$\therefore \underbrace{\frac{1}{f} \frac{d^2 f}{dx^2}}_{x \text{ dependent}} = \underbrace{-\frac{1}{g} \frac{d^2 g}{dy^2}}_{y \text{ dependent}}$$

$$\Rightarrow \frac{1}{f} \frac{d^2 f}{dx^2} = k^2 \quad \text{and} \quad \frac{1}{g} \frac{d^2 g}{dy^2} = -k^2 y$$

$$\therefore f(x) = Ae^{kx} + Be^{-kx} \quad \text{and} \quad g(y) = C\sin(ky) + D\cos(ky)$$

~~Now (but conditions~~

$$\Rightarrow V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin(ky) + D\cos(ky))$$

Thus, for condition iii) to be true, $A = 0 \Rightarrow$

$$V(x,y) = e^{-kx}(A_1 \sin(ky) + A_2 \cos(ky))$$

where $A_1 = BC$ and $A_2 = BD$. Now, $V(0,0) = 0 \Rightarrow$

$$V(0,0) = A_2 = 0$$

$$\Rightarrow V(x,y) = Ae^{-kx} \sin(ky)$$

where $A = A_1$. Now, $V(0,a) = 0 \Rightarrow$

$$\sin(ky) = 0 \Rightarrow k = \frac{n\pi}{a} \; ; \; n \in \mathbb{Z} \text{ (integers)}$$

Thus, our sol^n =

$$V_n(x,y) = Ae^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$$

So, the general ~~sol~~ sol^n (as if y_1, y_2 solve $\nabla^2 y = 0 \Rightarrow \nabla^2(c_1 y_1 + c_2 y_2) = 0$) =

$$V(x,y) = \sum_{1 \le n} c_n \sin\left(\frac{n\pi y}{a}\right) e^{-\frac{n\pi x}{a}}$$

where ~~satisfies~~ C_i are s.t., V satisfies the boundary conditions. Thus =

$$V(0,y) = \sum_{n \ge 1} c_n \sin\left(\frac{n\pi y}{a}\right) = V_0(y)$$

Thus as $\forall$ $V_i(0,y)$, $V_j(0,y)$ are orthogonal on $[0, a]$ as

$$\int_0^a \sin\left(\frac{i\pi y}{a}\right) \sin\left(\frac{j\pi y}{a}\right) dy = \begin{cases} 0 & ; i \ne j \\ \frac{a}{2} & ; i = j \end{cases}$$

. Thus =

$$\sum_{n \ge 1} c_n \int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{m\pi y}{a}\right) dy = \int_0^a V_0(y) \sin\left(\frac{m\pi y}{a}\right) dy$$

$$\Rightarrow c_n = \frac{2}{a} \int_0^a V_0(y) \sin\left(\frac{n\pi y}{a}\right) dy$$

Thus, the sol^n to our problem =

$$V(x,y) = \sum_{i \ge 1} c_n e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$$

$$\text{where, } c_n = \frac{2}{a} \int_0^a V_0(y) \sin\left(\frac{n\pi y}{a}\right) dy$$

Part-2 = Now, if $V_0(y) = V_0$, this $\Rightarrow$

$$c_n = \frac{2V_0}{a} \int_0^a \sin\left(\frac{n\pi y}{a}\right) dy = \frac{2V_0}{n\pi}(1 - \cos(n\pi)) = \begin{cases} 0 & ; n \text{ is even} \\ \frac{4V_0}{n\pi} & ; n \text{ is odd} \end{cases}$$

Thus, we get =

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n \geq 1} \frac{1}{2n-1} e^{-\frac{(2n-1)\pi x}{a}} \sin\left(\frac{(2n-1)\pi y}{a}\right)$$

$$= \frac{2V_0}{\pi} \arctan\left(\frac{\sin\left(\frac{\pi y}{a}\right)}{\sinh\left(\frac{\pi x}{a}\right)}\right)$$