$\underline{r} = (ct : \vec{r})$ [Position 4-vector]

$$(c\tau)^2 = (ct)^2 - (x^2+y^2+z^2)$$

$$\underline{r} * \underline{r} = (ct)^2 - (x^2+y^2+z^2)$$

$$\underline{a} * \underline{b} = c^2 t_a t_B - x_a x_B - y_a y_B - z_a z_B$$

$$\underline{v} \; \cancel{\frac{d\underline{r}}{dt}} = \frac{d\underline{r}}{d\tau} = \left(\frac{c\,dt}{d\tau} : \frac{d\vec{r}}{d\tau}\right) = (c\gamma : \gamma\vec{v}) \quad \boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

$$\underline{p} = (\gamma mc : \gamma m\vec{v}) = \left(\frac{\gamma E}{c} : \gamma\vec{p}\right)$$

$\underline{f} = \dfrac{d\underline{p}}{d\tau}$; we will consider $\underline{p} = \left(\cancel{\gamma}\dfrac{E}{c} : \cancel{\gamma}\vec{p}\right)$

$$\underline{f} = \left(\frac{1}{c}\frac{dE}{d\tau} : \frac{d\vec{p}}{d\tau}\right) = \left(\frac{\gamma}{c}\frac{dE}{dt} : \gamma\frac{d\vec{p}}{dt}\right)$$

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$$\underline{f} = \left(\frac{\gamma}{c}\phi : \gamma\vec{F}\right) = \left(\frac{\gamma}{c}\vec{F}\cdot\vec{v} : \gamma\vec{F}\right)$$

Q. Consider particle moving with velocity $\underline{u}$ facing an electric field $E_x\hat{x}$ and magnetic field $B_z\hat{z}$.

$$\underline{f_e} = \left(\frac{\gamma}{c}\vec{E_x}q\cdot\vec{u} : \gamma\vec{E_x}q\right)$$

$$\hat{t}\left(q\frac{\gamma}{c}\vec{E_x}\cdot\vec{u}\right) = \hat{t}\,\frac{q\gamma}{c}E_x u_x = \left(\frac{q\gamma E_x}{c}\right)\hat{t}\,(\hat{x} * \underline{u})(-1$$

$$q\gamma \vec{E_x} = (\gamma E_x q)\,\hat{x}\left(\frac{\hat{t} * \underline{u}}{c}\right)$$

$$\boxed{\underline{f_e} = \frac{\gamma E_x q}{c}\left[\hat{x}(\hat{t} * \underline{u}) - \hat{t}(\hat{x} * \underline{u})\right]}$$

$$\underline{f_B} = (0 : \gamma q \vec{B_z} \times \underline{u})$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & B_z \\ u_x & u_y & u_z \end{vmatrix} = \hat{x}(-B_z u_y) - \hat{y}(0 - B_z u_x)$$

$$B_z\left[(u_x \hat{y}) - (u_y \hat{x})\right]$$

$$\boxed{\underline{f_B} = B_z q\left[\hat{y}(\hat{x} * \underline{u}) - \hat{x}(\hat{y} * \underline{u})\right]}$$

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