S.V. Ananthakrishna
20221230

PHY : Tutorial Presentation

[From Griffiths - Electrodynamics - P7.11]

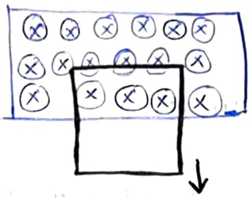
Problem: A square aluminium loop is placed so that the top portion is in a uniform magnetic field B, and is allowed to fall under gravity.

(a) If the magnetic field is 1T, find the terminal velocity of loop (in m/s).

(b) Find the velocity of the loop as a function of time

(c) How long does it take (in s) to reach 90% of the terminal velocity?

(d) What would happen if you cut a tiny slit in the loop, breaking the circuit?



Some data

$\rho = 2.8 \times 10^{-8}\ \Omega m$

$g = 9.8\ m\,s^{-2}$

$\eta = 2.7 \times 10^{3}\ kg/m^3$

$B = 1T$

Soln: The downward force due to gravity is the external force in this system. So and emf $\mathcal{E}$ is produced to oppose it.

$$\mathcal{E} = Blv = IR \qquad \text{(From Ohm's Law)}$$

Here the square has side length l and velocity v. Let it have mass 'm'

$$I = \frac{Blv}{R}$$

$$\text{Upward magnetic force} = BIl = B\left(\frac{Blv}{R}\right)l = \frac{B^2l^2v}{R}$$

Thus, we have

$$mg - \frac{B^2l^2v}{R} = m\frac{dv}{dt}$$

write $\frac{B^2l^2}{mR}$ as α for simplicity.

$$m(g - \alpha v) = m\frac{dv}{dt}$$

$$g - \alpha v_t = 0$$

$$v_t = g/\alpha$$

$$\boxed{v_t = \frac{mgR}{B^2l^2}} \quad (a)$$

$$\frac{dv}{dt} = g - \alpha v$$

$$\int_0^v \frac{dv}{g - \alpha v} = \int_0^t dt$$

$$-\frac{1}{\alpha}\ln(g - \alpha v) = t + c$$

$$g - \alpha v = Ae^{-\alpha t}$$

Now, at $t=0, v=0$ so
So. $A = g$

$$-\alpha v = ge^{-\alpha t} - g$$

$$\alpha v = g(1 - e^{-\alpha t})$$

$$v = \frac{g}{\alpha}(1 - e^{-\alpha t}) \Rightarrow \boxed{v = v_t(1 - e^{-\alpha t})}$$

(b)

At 90% of v_t, $\frac{v}{v_t} = 0.9$

$$\Rightarrow 0.9 = 1 - e^{-\alpha t}$$

$$\Rightarrow e^{-\alpha t} = 0.1$$

$$\ln(0.1) = -\alpha t \quad \text{or} \quad \ln 10 = \alpha t$$

$$\text{So } t = \frac{1}{\alpha}\ln 10 \qquad \text{But } \frac{g}{\alpha} = v_t$$

$$\boxed{t_{90\%} = \frac{v_t \ln 10}{g}} \text{ (c)}$$

Now, we can easily find the values for m, v_t etc.

$m = 4(\eta A l)$ where A is c.s area, l is length, η is mass density

Resistance: $R = \frac{\rho L}{A} = \frac{4l}{A\sigma}$

$$v_t = \frac{mgR}{B^2 l^2}$$

$$v_t = \frac{(4\eta A l)\, g\, (4l/A\sigma)}{B^2 l^2}$$

$$v_t = \cancel{16m}\cdot \frac{16\eta g}{B^2\sigma}$$

$$v_t = \frac{16\eta g \rho}{B^2}$$

$$v_t = \frac{16(2.7\times10^3)(9.8)(2.8\times10^{-8})}{1^2}$$

$$\boxed{v_t = 1.2\times10^{-2}\ m/s} \Rightarrow \boxed{t_{90} = \frac{1.2\times10^{-2}}{9.8}\ln 10 = 2.8\times10^{-3}\ s}$$

(d) If the loop was cut, there would be no emf and it would fall with acceleration g.