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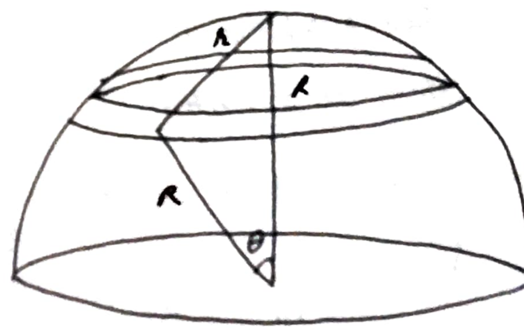
Q) An inverted hemispherical bowl of radius $R$ carries a uniform surface charge density $\sigma$. Find the potential difference between the "north pole" and the centre.

We know,

Potential $V, = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma}{r}\,da$




$\therefore\ V_{centre} = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma}{r}\,da$

$= \frac{1}{4\pi\epsilon_0}\frac{\sigma}{R}\int da$

$= \frac{1}{4\pi\epsilon_0}\frac{\sigma}{R}(2\pi R^2)$

$= \frac{\sigma R}{2\epsilon_0}$

Now, $V_{pole} = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma}{r}\,da$

Here, $da = 2\pi R^2\sin\theta\,d\theta$

& according to the law of cosines, $c = \sqrt{a^2+b^2-2ab\cos\gamma}$

In this case, $r = \sqrt{R^2+R^2-2R^2\cos\theta}$

$= \sqrt{2R^2(1-\cos\theta)}$

$= R\sqrt{2}\sqrt{1-\cos\theta}$

Substituting these values we get,

$$V_{pole} = \frac{\sigma}{4\pi\epsilon_0} \cancel{2\pi} \int \frac{da}{r}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \int_0^{\pi/2} \frac{2\pi R^2 \sin\theta \, d\theta}{R\sqrt{2}\sqrt{1-\cos\theta}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \frac{(2\pi R^2)}{R\sqrt{2}} \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\sqrt{1-\cos\theta}}$$

Let $t = 1-\cos\theta$

then $dt = \sin\theta \, d\theta$

Putting these values in the integral we get

$$V_{pole} = \frac{\sigma}{4\pi\epsilon_0} \frac{(2\pi R^2)}{R\sqrt{2}} \int_0^{\pi/2} \frac{dt}{\sqrt{t}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} (2\sqrt{t}) \Big|_0^{\pi/2}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \left(2\sqrt{1-\cos\theta}\right) \Big|_0^{\pi/2} \qquad (\text{Resubstituting } t = 1-\cos\theta)$$

$$= \frac{\sigma R}{\sqrt{2}\epsilon_0} (1-0)$$

$$= \frac{\sigma R}{\sqrt{2}\epsilon_0}$$

$$\therefore V_{pole} - V_{centre} = \frac{\sigma R}{\sqrt{2}\epsilon_0} - \frac{\sigma R}{2\epsilon_0} = \frac{\sigma R}{2\epsilon_0}(\sqrt{2}-1)$$

Thus, the potential ~~energy~~ difference between the "north pole" and the centre of the hemispherical bowl is $\boxed{\frac{\sigma R}{2\epsilon_0}(\sqrt{2}-1)}$