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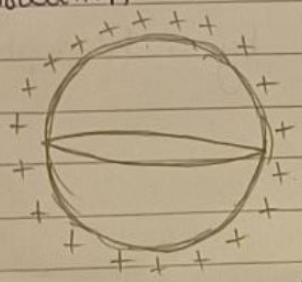
Physics Presentation

Problem

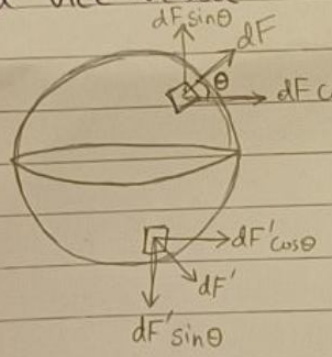
A metal sphere of radius R carries a total charge Q
What is the force of repulsion between 'northern' and 'southern' hemisphere?

Solution



When Q amount of charge is confined to a metal sphere, charges'll be distributed across

The charges in northern hemispheres will experience a force due to charges in southern hemisphere and vice versa



Force per unit area

$$F = \frac{1}{2\varepsilon_0}\sigma^2 A$$

$$\frac{F}{A} = \frac{\epsilon q}{A}$$

$$= \frac{\sigma^2 A^2}{2\varepsilon_0 A}$$

∴ Net repulsive force experienced by an area element in a hemisphere $= \int dF \cdot \cos\theta$

$$\int dF\cos\theta = \int \frac{1}{2\varepsilon_0}\sigma^2 dA\cos\theta$$

$$= \int \frac{1}{2\varepsilon_0}\left(\frac{Q}{4\pi R^2}\right)^2 \times \left(R^2 \sin\theta\, d\theta\, d\phi\right)\cos\theta$$

[spherical coordinates $dA = R^2\sin\theta\, d\theta\, d\phi$]

$$= \frac{1}{2\varepsilon_0} \frac{Q^2}{16\pi^2 R^4} \times R^2 \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \times \int_0^{2\pi} d\phi$$

$$= \frac{Q^2}{32\pi^2 \varepsilon_0 R^2} \times \frac{1}{2} \int_0^{\pi/2} \sin 2\theta \, d\theta \times [2\pi]$$

$$= \frac{Q^2}{32\pi\varepsilon_0 R^2} \left(-\frac{\cos 2\theta}{2}\right)_0^{\pi/2}$$

Net repulsive force experienced by a hemisphere,

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

Physical Significance