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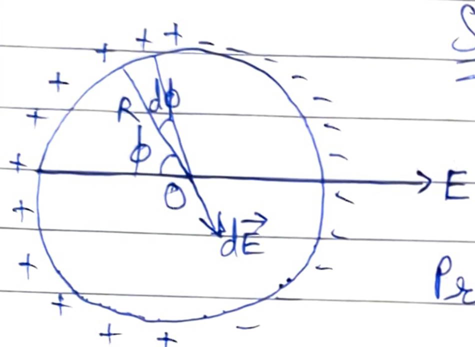
PH1213 Presentation

Batch-4

Q) A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\phi$, where λ_0 is a constant, ϕ is azimuthal angle. Find magnitude of electric field strength:

a) at centre of the ring

b) on axis of ring as a function of the distance x from its centre. Investigate obtained function at $x >> R$



Sol:-

a) Clearly, due to λ being a function of $\cos\phi$, $\vec{E}$ points to right.

Projection of $d\vec{E}$ on $\vec{E}$:

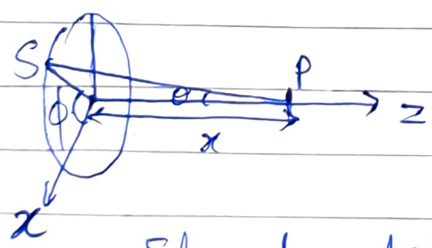
$$dE\cos\phi = \frac{k\,dq}{R^2}\cos\phi$$

$$dq = \lambda R\,d\phi = \lambda_0 R\cos\phi\,d\phi$$

$$\Rightarrow E = \frac{\lambda_0}{4\pi\varepsilon_0 R}\int_0^{2\pi}\cos^2\phi\,d\phi$$

$$= \frac{\lambda_0}{4\varepsilon_0 R}$$

b)



Take an element S at ϕ from x-axis, subtending angle $d\phi$ at centre

Elementary field at P due to element is $\frac{\lambda_0\cos\phi\,d\phi\,R}{4\pi\varepsilon_0(x^2+R^2)}$ along SP with components

$$= \frac{\lambda_0\cos\phi\,d\phi\,R}{4\pi\varepsilon_0(x^2+R^2)} \times \{\cos\theta \text{ along } OP,\ \sin\theta \text{ along } OS\}$$

Component along OP vanishes on integration as $\int_0^{2\pi}\cos\phi\,d\phi = 0$

Component along OS can be broken in components along OX and OY as $\dfrac{\lambda_0 R^2 \cos\phi \, d\phi}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}} \times \{\cos\phi \text{ along } OX, \sin\phi \text{ along } OY\}$

On integration, part along OY vanishes.

Finally, $E = E_x = \dfrac{\lambda_0 R^2}{4\varepsilon_0 (x^2+R^2)^{3/2}}$

For $x >> R$,

$$E = E_x = \frac{p}{4\pi\varepsilon_0 x^3}, \quad p = \lambda_0 \pi R^2$$