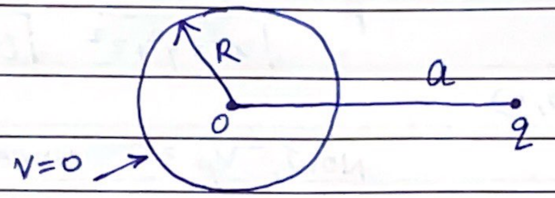
NAME- RACCHIT GUPTA ROLL NO. : 20221210

Start WRITING Below

Question



A point charge q is situated at a distance 'a' from the centre of a grounded conducting sphere of Radius R.
Find the potential outside sphere.

Solution → Boundary condition for the sphere (grounded) is that potential on surface of sphere is Zero.

∴ by method of images, we try to consider a charge q' replacing the conducting sphere, placed at some distance 'b' from centre of sphere
such that, the potential at any point on the locus of removed grounded sphere, due to both q and q' is Zero.

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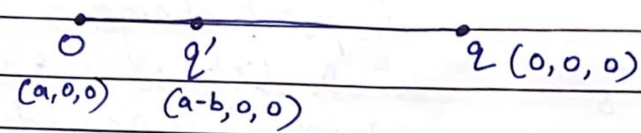
satisfies to be a point on sphere $\leftarrow P(x, y, z)$



$$V_P = \frac{kq}{\sqrt{x^2+y^2+z^2}} + \frac{kq'}{\sqrt{(x-a+b)^2+y^2+z^2}}$$

Now, $V_P = 0$ since P is a point on grounded sphere.

$$\therefore \quad \frac{kq}{\sqrt{x^2+y^2+z^2}} = -\frac{kq'}{\sqrt{(x-a+b)^2+y^2+z^2}}$$

$$\therefore \quad q^2\left[(x-a+b)^2+y^2+z^2\right] = q'^2\left[x^2+y^2+z^2\right]$$

on simplifying, above expression yields

$$(q^2-q'^2)x^2 + (q^2-q'^2)y^2 + (q^2-q'^2)z^2 + q^2(2b-2a)x = -(a-b)^2 q^2$$

Now, equation of grounded sphere : $(x-a)^2+y^2+z^2 = R^2$

$$\Rightarrow x^2+y^2+z^2-2ax = R^2-a^2$$

Now, this equation is also the locus of grounded sphere

$\therefore$ on comparing both the equations, we get: $-2a = \dfrac{q^2(2b-2a)}{q^2 - q'^2}$ — ①

& $R^2 - a^2 = -\dfrac{(a-b)^2 q^2}{q^2 - q'^2}$ — ②

Dividing eq^n ① by ②

$$\therefore \frac{\cancel{2}a}{R^2-a^2} = \frac{\cancel{2}(\cancel{b-a})}{(b-a)^{\cancel{2}}} \Rightarrow \cancel{a^3 + ab^2 - 2a^2b} \quad ab - \cancel{a^2} = R^2 - \cancel{a^2}$$

$$\therefore \boxed{b = \frac{R^2}{a}}$$

using the above b, putting in eq^n ② gives

$$\cancel{R^2 - a^2 = -\frac{(a-b)^2 q^2}{q^2}} \quad R^2 - a^2 = \frac{-q^2(b-a)^2}{q^2 - q'^2}$$

$$\Rightarrow q^2R^2 - q'^2R^2 - \cancel{a^2q^2} + a^2q'^2 = -q^2b^2 - \cancel{q^2a^2} + 2abq^2$$

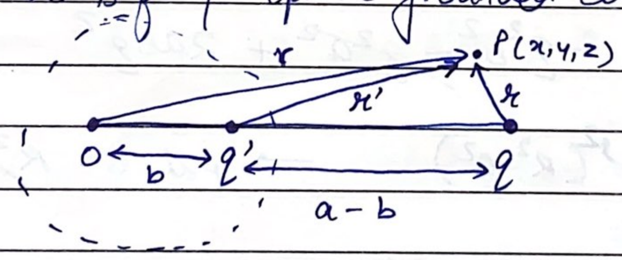
$$\Rightarrow q^2(R^2 + b^2 - 2ab) = q'^2(R^2 - a^2) \longrightarrow \text{put } b = R^2/a$$

we see, $q' = \pm \dfrac{R}{a} q$ satisfies the above equation but $q' = -\dfrac{R}{a}q$ because, q' & q has to be of opposite signs in order for the potentials due to q & q' to cancel each other.

$$\therefore \quad b = \frac{R^2}{a} \quad \& \quad q' = -\frac{R}{a} q$$

Notice, that b is less than R, so the 'image' charge q' is safely inside sphere & that it satisfies the ~~bo~~ spherical boundary condition of $V=0$ at sphere boundary. q' image charge cannot be kept in the region where we are calculating V, as it would change ρ, and we'd be solving Poisson's equation with wrong source ρ.

∴ we safely replace grounded conductor with q' image charge



$$\therefore \quad V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{q'}{r'}\right)$$

$\because q' = -\frac{R}{a} q$ placed at $b = \frac{R^2}{a}$ from centre of sphere.