```mathematica
In[1]:= (*///////////////This section of the code
            computes the optimal prices and investments////////////*)

(*//////Demand functions of green and brown products//////*)
Dg = αg - βg * pg + γg * pb + η * S;
Db = αb - βb * pb + γb * pg - δ * S;

(*//////STRUCTURE OF VALUE FUNCTION//////*)
V = A1 * S^2 + A2 * λ^2 + A3 * S * λ + A4 * S + A5 * λ + A6;

(*////HJB equation////*)

HJB = Dg * pg + Db * pb - (μ / 2) * Z^2 + σ * λ + ∂S V * (k1 * Z + k2 * λ - ϵ * S) + ∂λ V * (ϕ * Dg - ω * λ);
FullSimplify[Solve[∂pg HJB == 0 && ∂pb HJB == 0, {pg, pb}]]
Solve[∂Z HJB == 0 , Z]

(*Solutions from the above are*)
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$$Out[5]= \left\{\left\{p_g \to \frac{2\,\beta_b\left(S\,\eta + \alpha_g - \phi\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\beta_g\right) + \left(\gamma_b + \gamma_g\right)\left(-S\,\delta + \alpha_b + \phi\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\gamma_g\right)}{4\,\beta_b\,\beta_g - \left(\gamma_b + \gamma_g\right)^2},\right.\right.$$

$$\left.\left. p_b \to \frac{\left(S\,\eta + \alpha_g\right)\left(\gamma_b + \gamma_g\right) + \beta_g\left(-2\,S\,\delta + 2\,\alpha_b + \phi\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\left(-\gamma_b + \gamma_g\right)\right)}{4\,\beta_b\,\beta_g - \left(\gamma_b + \gamma_g\right)^2}\right\}\right\}$$

$$Out[6]= \left\{\left\{Z \to \frac{\left(2\,S\,A_1 + \lambda\,A_3 + A_4\right)k_1}{\mu}\right\}\right\}$$

$In[\cdot]:=$ (*//////////In this section of the code we take the optimal price
and investments from the above section and plug in the parameter
values to obtain the value function coefficients ////////*)
(*Parameter Values*)

(*Control variables outputs from above*)

$$p_g = \frac{2\,\beta_b\,\left(S\,\eta + \alpha_g - \phi\,\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\,\beta_g\right) + \left(\gamma_b + \gamma_g\right)\,\left(-S\,\delta + \alpha_b + \phi\,\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\,\gamma_g\right)}{4\,\beta_b\,\beta_g - \left(\gamma_b + \gamma_g\right)^2};$$

$$p_b = \frac{\left(S\,\eta + \alpha_g\right)\,\left(\gamma_b + \gamma_g\right) + \beta_g\,\left(-2\,S\,\delta + 2\,\alpha_b + \phi\,\left(2\,\lambda\,A_2 + S\,A_3 + A_5\right)\,\left(-\gamma_b + \gamma_g\right)\right)}{4\,\beta_b\,\beta_g - \left(\gamma_b + \gamma_g\right)^2};$$

$$Z = \frac{\left(2\,S\,A_1 + \lambda\,A_3 + A_4\right)\,k_1}{\mu};$$

$D_g = \alpha_g - \beta_g * p_g + \gamma_g * p_b + \eta * S;$
$D_b = \alpha_b - \beta_b * p_b + \gamma_b * p_g - \delta * S;$

(*STRUCTURE OF VALUE FUNCTION*)

$V = A_1 * S\^2 + A_2 * \lambda\^2 + A_3 * S * \lambda + A_4 * S + A_5 * \lambda + A_6;$

(*HJB equation*)

HJB1[S_, λ_] =
$D_g * p_g + D_b * p_b - \left(\mu / 2\right) * Z\^2 + \sigma * \lambda + \partial_S V * \left(k_1 * Z + k_2 * \lambda - \epsilon * S\right) + \partial_\lambda V * \left(\phi * D_g - \omega * \lambda\right);$

(*/////HJB coeeficients//////*)
(*Important Note - Note that there are multiplicative association os S and λ. Therefore,
Coefficient[S] and Coefficient[λ] will include terms multiplied with S or λ.
Need to filter the appropriate terms
for each set of parameter values. Coefficient[HJB1[S,0],S]
will give us the coefficient of S only and free from the
multiplicative association of λ /////*)
(*The following equations give the solutions of the VF coefficients*)

(* The equation here gives the solution of the coefficients ---
Solve[{Coefficient[HJB1[S,λ],S\^2]==r*A$_1$ && Coefficient[HJB1[S,λ],λ\^2]==r*A$_2$ &&
Coefficient[HJB1[S,λ],S*λ]==r*A$_3$ && Coefficient[HJB1[S,0],S]==r*A$_4$
&& Coefficient[HJB1[0,λ],S]==r*A$_5$ && HJB1[0,0]==r*A$_6$},{A$_1$,A$_2$,A$_3$,A$_4$,A$_5$,A$_6$}];
We illustrate this by an example below*)

In[ ]:=
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(*An Example*)
αg = 10; βg = 2; γg = 1; η = .05;
αb = 15; βb = 2; γb = 1; δ = .03;
k₁ = 1; k₂ = 1; ϕ = .01; μ = 10;
r = .05;
ϵ = .1;
σ = .1;
ω = .1;
Expand[HJB1[S, λ]]
Coefficient[HJB1[S, λ], S^2]
 Coefficient[HJB1[S, λ], λ^2]
 Coefficient[HJB1[S, λ], S * λ]
Coefficient[HJB1[S, 0], S]
Coefficient[HJB1[0, λ], λ]
 HJB1[0, 0]
Solve[{Coefficient[HJB1[S, λ], S^2] == r * A₁ && Coefficient[HJB1[S, λ], λ^2] == r * A₂ &&
    Coefficient[HJB1[S, λ], S * λ] == r * A₃ && Coefficient[HJB1[S, 0], S] == r * A₄
    && Coefficient[HJB1[0, λ], S] == r * A₅ && HJB1[0, 0] == r * A₆}, {A₁, A₂, A₃, A₄, A₅, A₆}]
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Out[ ]= $79.1667 + 0.0916667\,S + 0.000316667\,S^2 + 0.1\,\lambda - 0.2\,S^2\,A_1 + 2\,S\,\lambda\,A_1 + \frac{1}{5}\,S^2\,A_1^2 + 0.1\,\lambda\,A_2 +$

$0.0005\,S\,\lambda\,A_2 - 0.2\,\lambda^2\,A_2 + 0.0002\,\lambda^2\,A_2^2 + 0.05\,S\,A_3 + 0.00025\,S^2\,A_3 - 0.2\,S\,\lambda\,A_3 + \lambda^2\,A_3 +$

$\frac{1}{5}\,S\,\lambda\,A_1\,A_3 + 0.0002\,S\,\lambda\,A_2\,A_3 + 0.00005\,S^2\,A_3^2 + \frac{1}{20}\,\lambda^2\,A_3^2 - 0.1\,S\,A_4 + \lambda\,A_4 + \frac{1}{5}\,S\,A_1\,A_4 +$

$\frac{1}{10}\,\lambda\,A_3\,A_4 + \frac{A_4^2}{20} + 0.05\,A_5 + 0.00025\,S\,A_5 - 0.1\,\lambda\,A_5 + 0.0002\,\lambda\,A_2\,A_5 + 0.0001\,S\,A_3\,A_5 + 0.00005\,A_5^2$

Out[ ]= $0.000316667 - 0.2\,A_1 + \frac{A_1^2}{5} + 0.00025\,A_3 + 0.00005\,A_3^2$

Out[ ]= $-0.2\,A_2 + 0.0002\,A_2^2 + A_3 + \frac{A_3^2}{20}$

Out[ ]= $0.0000166667\,A_2 + 0.01\,A_2\,\left(0.0483333 + \frac{1}{6}\,\left(-4\,\left(0.05 - 0.02\,A_3\right) - 2\,\left(-0.03 + 0.01\,A_3\right)\right)\right) +$

$0.00166667\,A_2\,\left(4\,\left(0.05 - 0.02\,A_3\right) + 2\,\left(-0.03 + 0.01\,A_3\right)\right) +$

$2\,A_1\,\left(1 + \frac{A_3}{10}\right) + \left(-0.1 + \frac{A_1}{5}\right)\,A_3 - \frac{A_1\,A_3}{5} + \left(-0.1 + 0.0002\,A_2\right)\,A_3$

Out[ ]= $0.0916667 + 0.05\,A_3 - 0.1\,A_4 + \frac{A_1\,A_4}{5} + 0.00025\,A_5 + 0.0001\,A_3\,A_5$

Out[ ]= $0.1 + 0.1\,A_2 + A_4 + \frac{A_3\,A_4}{10} - 0.1\,A_5 + 0.0002\,A_2\,A_5$

Out[•]= $0. + \left(0. + \dfrac{A_4}{10}\right) A_4 - \dfrac{A_4^2}{20} + 6.66667 \left(1.66667 + \dfrac{1}{12} \left(4 \left(10. - 0.02\, A_5\right) + 2 \left(15. + 0.01\, A_5\right)\right)\right) +$

$\dfrac{1}{12} \left(16.6667 + \dfrac{1}{6} \left(-4 \left(10. - 0.02\, A_5\right) - 2 \left(15. + 0.01\, A_5\right)\right)\right) \left(4 \left(10. - 0.02\, A_5\right) + 2 \left(15. + 0.01\, A_5\right)\right) +$

$\left(0. + 0.01 \left(16.6667 + \dfrac{1}{6} \left(-4 \left(10. - 0.02\, A_5\right) - 2 \left(15. + 0.01\, A_5\right)\right)\right)\right) A_5$

⚫⚫⚫ **Solve**: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= $\{\{A_1 \to 0.279912,\ A_2 \to 1128.61,\ A_3 \to -35.4556,\ A_4 \to -17.8808,\ A_5 \to 0.,\ A_6 \to 1903.06\},$
$\{A_1 \to 1.2301,\ A_2 \to 1268.58,\ A_3 \to -12.391,\ A_4 \to 5.49766,\ A_5 \to 0.,\ A_6 \to 1613.56\},$
$\{A_1 \to 0.00127832,\ A_2 \to 0.0413028,\ A_3 \to 0.01032,\ A_4 \to 0.6156,\ A_5 \to 0.,\ A_6 \to 1583.71\},$
$\{A_1 \to 0.222588,\ A_2 \to 868.885,\ A_3 \to 27.7436,\ A_4 \to 14.0199,\ A_5 \to 0.,\ A_6 \to 1779.89\}\}$