In[ ]:= (*///////////////This section of the code computes the optimal prices and investments///////////*)

(*//////Demand functions of green and brown products//////*)

$$D_g = \alpha_g - \beta_g * p_g + \gamma_g * p_b + \eta * S;$$
$$D_b = \alpha_b - \beta_b * p_b + \gamma_b * p_g - \delta * S;$$

(*//////STRUCTURE OF VALUE FUNCTION//////*)

$$V = A_1 * S\text{^}2 + A_2 * \lambda\text{^}2 + A_3 * S * \lambda + A_4 * S + A_5 * \lambda + A_6;$$

(*////HJB equation////*)

$$HJB = D_g * p_g + D_b * p_b - \left(\mu / 2\right) * Z\text{^}2 + \sigma * \lambda + \partial_S V * \left(k_1 * Z + k_2 * \lambda - \epsilon * S\right) + \partial_\lambda V * (\phi * D_g - \omega * \lambda);$$
$$\text{FullSimplify}[\text{Solve}[\partial_{p_g} HJB == 0 \;\&\&\; \partial_{p_b} HJB == 0, \{p_g, p_b\}]]$$
$$\text{Solve}[\partial_Z HJB == 0\ , Z]$$

(*Solutions from the above are*)

$$(*p_g \to \frac{2\ \beta_b\ (S\ \eta+\alpha_g-\phi\ (2\ \lambda\ A_2+S\ A_3+A_5)\ \beta_g)+(\gamma_b+\gamma_g)\ (-S\ \delta+\alpha_b+\phi\ (2\ \lambda\ A_2+S\ A_3+A_5)\ \gamma_g)}{4\ \beta_b\ \beta_g-(\gamma_b+\gamma_g)^2},$$
$$p_b \to \frac{(S\ \eta+\alpha_g)\ (\gamma_b+\gamma_g)+\beta_g\ (-2\ S\ \delta+2\ \alpha_b+\phi\ (2\ \lambda\ A_2+S\ A_3+A_5)\ (-\gamma_b+\gamma_g))}{4\ \beta_b\ \beta_g-(\gamma_b+\gamma_g)^2}$$
$$Z \to \frac{(2\ S\ A_1+\lambda\ A_3+A_4)\ k_1}{\mu} *)$$

Out[ ]= $$\{\{p_g \to 5.83333 + 0.0116667\, S - 0.01\, \lambda\, A_2 - 0.005\, S\, A_3 - 0.005\, A_5,\ p_b \to 6.66667 - 0.00166667\, S + 3.46945 \times 10^{-18}\, \lambda\, A_2 + 1.73472 \times 10^{-18}\, S\, A_3 + 1.73472 \times 10^{-18}\, A_5\}\}$$

Out[ ]= $$\left\{\left\{Z \to \frac{1}{10}\left(2\, S\, A_1 + \lambda\, A_3 + A_4\right)\right\}\right\}$$

In[ ]:=
```
(*//////////In this section of the code we take the optimal price
      and investments from the above section and plug in the parameter
      values to obtain the value function coefficients ////////*)
(*Parameter Values*)

(*Control variables outputs from above*)
```

$$p_g = \frac{2\,\beta_b\left(S\,\eta+\alpha_g-\phi\left(2\,\lambda\,A_2+S\,A_3+A_5\right)\beta_g\right)+(\gamma_b+\gamma_g)\left(-S\,\delta+\alpha_b+\phi\left(2\,\lambda\,A_2+S\,A_3+A_5\right)\gamma_g\right)}{4\,\beta_b\,\beta_g-(\gamma_b+\gamma_g)^2};$$

$$p_b = \frac{\left(S\,\eta+\alpha_g\right)(\gamma_b+\gamma_g)+\beta_g\left(-2\,S\,\delta+2\,\alpha_b+\phi\left(2\,\lambda\,A_2+S\,A_3+A_5\right)(-\gamma_b+\gamma_g)\right)}{4\,\beta_b\,\beta_g-(\gamma_b+\gamma_g)^2};$$

$$Z = \frac{\left(2\,S\,A_1+\lambda\,A_3+A_4\right)k_1}{\mu};$$

$$D_g = \alpha_g - \beta_g * p_g + \gamma_g * p_b + \eta * S;$$

$$D_b = \alpha_b - \beta_b * p_b + \gamma_b * p_g - \delta * S;$$

```
(*STRUCTURE OF VALUE FUNCTION*)
```

$$V = A_1 * S\text{^}2 + A_2 * \lambda\text{^}2 + A_3 * S * \lambda + A_4 * S + A_5 * \lambda + A_6;$$

```
(*HJB equation*)
```

$$\text{HJB1}[S\_,\lambda\_] = D_g * p_g + D_b * p_b - \left(\mu/2\right) * Z\text{^}2 + \sigma * \lambda + \partial_S V * \left(k_1 * Z + k_2 * \lambda - \epsilon * S\right) + \partial_\lambda V * (\phi * D_g - \omega * \lambda);$$

```
(*/////HJB coeeficients//////*)
(*Important Note - Note that there are multiplicative association os S and λ. Therefore,
Coefficient[S] and Coefficient[λ] will include terms multiplied with S or λ.
     Need to filter the appropriate terms
    for each set of parameter values. Coefficient[HJB1[S,0],S]
    will give us the coefficient of S only and free from the
    multiplicative association of λ /////*)
(*The following equations give the solutions of the VF coefficients*)

(* The equation here gives the solution of the coefficients ---
 Solve[{Coefficient[HJB1[S,λ],S^2]==r*A1 && Coefficient[HJB1[S,λ],λ^2]==r*A2 &&
      Coefficient[HJB1[S,λ],S*λ]==r*A3 && Coefficient[HJB1[S,0],S]==r*A4
      && Coefficient[HJB1[0,λ],S]==r*A5 && HJB1[0,0]=r*A6},{A1,A2,A3,A4,A5,A6}];
We illustrate this by an example below*)
```

In[∘]:=

```
(*An Example*)
αg = 10; βg = 2; γg = 1; η = .05;
αb = 15; βb = 2; γb = 1; δ = .03;
k1 = 1; k2 = 1; ϕ = .01; μ = 10;
r = .05;
ϵ = .1;
σ = .1;
ω = .1;
Expand[HJB1[S, λ]]
Coefficient[HJB1[S, λ], S^2]
 Coefficient[HJB1[S, λ], λ^2]
 Coefficient[HJB1[S, λ], S * λ]
Coefficient[HJB1[S, 0], S]
Coefficient[HJB1[0, λ], λ]
 HJB1[0, 0]
Solve[{Coefficient[HJB1[S, λ], S^2] == r * A1 && Coefficient[HJB1[S, λ], λ^2] == r * A2 &&
   Coefficient[HJB1[S, λ], S * λ] == r * A3 && Coefficient[HJB1[S, 0], S] == r * A4
   && Coefficient[HJB1[0, λ], S] == r * A5 && HJB1[0, 0] == r * A6}, {A1, A2, A3, A4, A5, A6}]
```

Out[∘]= $79.1667 + 0.0916667\,S + 0.000316667\,S^2 + 0.1\,\lambda - 0.2\,S^2\,A_1 + 2\,S\,\lambda\,A_1 + \frac{1}{5}\,S^2\,A_1^2 + 0.1\,\lambda\,A_2 + 0.0005\,S\,\lambda\,A_2 - 0.2\,\lambda^2\,A_2 + 0.0002\,\lambda^2\,A_2^2 + 0.05\,S\,A_3 + 0.00025\,S^2\,A_3 - 0.2\,S\,\lambda\,A_3 + \lambda^2\,A_3 + \frac{1}{5}\,S\,\lambda\,A_1\,A_3 + 0.0002\,S\,\lambda\,A_2\,A_3 + 0.00005\,S^2\,A_3^2 + \frac{1}{20}\,\lambda^2\,A_3^2 - 0.1\,S\,A_4 + \lambda\,A_4 + \frac{1}{5}\,S\,A_1\,A_4 + \frac{1}{10}\,\lambda\,A_3\,A_4 + \frac{A_4^2}{20} + 0.05\,A_5 + 0.00025\,S\,A_5 - 0.1\,\lambda\,A_5 + 0.0002\,\lambda\,A_2\,A_5 + 0.0001\,S\,A_3\,A_5 + 0.00005\,A_5^2$

Out[∘]= $0.000316667 - 0.2\,A_1 + \frac{A_1^2}{5} + 0.00025\,A_3 + 0.00005\,A_3^2$

Out[∘]= $-0.2\,A_2 + 0.0002\,A_2^2 + A_3 + \frac{A_3^2}{20}$

Out[∘]= $0.0000166667\,A_2 + 0.01\,A_2\left(0.0483333 + \frac{1}{6}\left(-4\left(0.05 - 0.02\,A_3\right) - 2\left(-0.03 + 0.01\,A_3\right)\right)\right) + 0.00166667\,A_2\left(4\left(0.05 - 0.02\,A_3\right) + 2\left(-0.03 + 0.01\,A_3\right)\right) + 2\,A_1\left(1 + \frac{A_3}{10}\right) + \left(-0.1 + \frac{A_1}{5}\right)A_3 - \frac{A_1\,A_3}{5} + \left(-0.1 + 0.0002\,A_2\right)A_3$

Out[∘]= $0.0916667 + 0.05\,A_3 - 0.1\,A_4 + \frac{A_1\,A_4}{5} + 0.00025\,A_5 + 0.0001\,A_3\,A_5$

Out[∘]= $0.1 + 0.1\,A_2 + A_4 + \frac{A_3\,A_4}{10} - 0.1\,A_5 + 0.0002\,A_2\,A_5$

Out[ ]= $$0. + \left(0. + \frac{A_4}{10}\right) A_4 - \frac{A_4^2}{20} + 6.66667 \left(1.66667 + \frac{1}{12} \left(4 \left(10. - 0.02 A_5\right) + 2 \left(15. + 0.01 A_5\right)\right)\right) + \frac{1}{12} \left(16.6667 + \frac{1}{6} \left(-4 \left(10. - 0.02 A_5\right) - 2 \left(15. + 0.01 A_5\right)\right)\right) \left(4 \left(10. - 0.02 A_5\right) + 2 \left(15. + 0.01 A_5\right)\right) + \left(0. + 0.01 \left(16.6667 + \frac{1}{6} \left(-4 \left(10. - 0.02 A_5\right) - 2 \left(15. + 0.01 A_5\right)\right)\right)\right) A_5$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[ ]= $$\{\{A_1 \to 0.279912, A_2 \to 1128.61, A_3 \to -35.4556, A_4 \to -17.8808, A_5 \to 0., A_6 \to 1903.06\},$$
$$\{A_1 \to 1.2301, A_2 \to 1268.58, A_3 \to -12.391, A_4 \to 5.49766, A_5 \to 0., A_6 \to 1613.56\},$$
$$\{A_1 \to 0.00127832, A_2 \to 0.0413028, A_3 \to 0.01032, A_4 \to 0.6156, A_5 \to 0., A_6 \to 1583.71\},$$
$$\{A_1 \to 0.222588, A_2 \to 868.885, A_3 \to 27.7436, A_4 \to 14.0199, A_5 \to 0., A_6 \to 1779.89\}\}$$