First Order Differential Equation and its Implications

Mathematics-III (Differential Calculus)

Code: BSC-301

Bachelor of Technology
Computer Science and Engineering

Submitted By

ARKAPRATIM GHOSH (13000121058)

August, 2022

Techno Main Salt Lake
EM-4/1, Sector-V, Salt Lake
Kolkata- 700091
West Bengal
India

TABLE OF CONTENTS

1. Introduction

A differential equation of first order and n-th degree is of the form

pn +P1pn+P2pn-2+...+Pn-1p+Pn=0 ….(1), where p=dy/dx and P1, P2, ...Pn are functions of x and y. Since it is a differential equation of the first order ,its general solution will contain only one arbitrary constant.

2 . Body

Equations solvable for p.

Let the equation (1) can be put in the form {p-f1(x,y)}{p-f2(x,y)}...{p-fn(x,y)}=0

which is equivalent to p-f1(x,y0=0, p-f2(x,y)=0,...p-fn(x,y)=0. ...i.e.

dy/dx-f1(x,y0=0, dy/dx-f2(x,y)=0,…,dy/dx-fn(x,y)=0.

Each of these equations is of the first order and first degree and can be solved by methods .Let the solution be F1(x,y,c)=0, F2(x,y,c)=0,…,Fn(x,y,cn)=0 …

where c1,c2,…,cn are arbitrary constants. Hence the general solution of (1) is given by

F1(x,y,c) F29x,y,c) … Fn(x,y,c)=0

where c1=c2=...=cn=c is any arbitrary constant .We should have one arbitrary constant because this is a first order differential equation.

Example:

The equation from Newton's law of cooling,y˙=k(M−y)y˙=k(M−y)is a first order differential equation

;F(t,y,y˙)=k(M−y)−y˙F(t,y,y˙)=k(M−y)−y˙.
y˙=t2+1y˙=t2+1 is a first order differential equation
;F(t,y,y˙)=y˙−t2−1F(t,y,y˙)=y˙−t2−1. All solutions to this equation are of the form t3/3+t+Ct3/3+t+C.
A first order initial value problem is a system of equations of the form F(t,y,y˙)=0F(t,y,y˙)=0,y(t0)=y0y(t0)=y0. Here t0t0 is a fixed time and y0y0 is a number.

A solution of an initial value problem is a solution f(t)f(t) of the differential equation that also satisfies the initial condition f(t0)=y0f(t0)=y0.

The general first order equation is rather too general, that is, we can't describe methods that will work on them all, or even a large portion of them. We can make progress with specific kinds of first order differential equations. For example, much can be said about equations of the form y˙=ϕ(t,y)y˙=ϕ(t,y) where ϕϕ is a function of the two variables tt and yy. Under reasonable conditions on ϕϕ, such an equation has a solution and the corresponding initial value problem has a unique solution. However, in general, these equations can be very difficult or impossible to solve explicitly. Consider this specific example of an initial value problem for Newton's law of cooling:y˙=2(25−y)y˙=2(25−y),y(0)=40y(0)=40. We first note that if y(t0)=25y(t0)=25, the right hand side of the differential equation is zero, and so the constant function y(t)=25y(t)=25 is a solution to the differential equation. It is not a solution to the initial value problem, since y(0)≠40y(0)≠40. (The physical interpretation of this constant solution is that if a liquid is at the same temperature as its surroundings, then the liquid will stay at that temperature.) So long as yy is not 25, we can rewrite the differential equation as

(17.1.1)dydt125−y=2125−ydy=2dt,dydt125−y125−ydy=2=2dt,

so

(17.1.2)∫125−ydy=∫2dt,∫125−ydy=∫2dt,

that is, the two anti-derivatives must be the same except for a constant difference. We can calculate these anti-derivatives and rearrange the results:

(17.1.3)∫125−ydy=∫2dt(−1)ln|25−y|=2t+C0ln|25−y|=−2t−C0=−2t+C|25−y|
=e−2t+C=e−2teCy−25=±eCe−2ty=25±eCe−2t=25+Ae−2t.∫125−ydy(−1)ln|25−y|ln|25−y||25−
∫2dt=2t+C0=−2t−C0=−2t+C=e−2t+

Here A=±eC=±e−C0A=±eC=±e−C0 is some non-zero constant. Since we want y(0)=40y(0)=40, we substitute and solve for AA:

(17.1.4)40=25+Ae015=A,4015=25+Ae0=A,

and so y=25+15e−2ty=25+15e−2t is a solution to the initial value problem. Note that yy is never 25, so this makes sense for all values of tt. However, if we allow A=0A=0 we get the solution y=25y=25 to the differential equation, which would be the solution to the

initial value problem if we were to requirey(0)=25y(0)=25. Thus,y=25+Ae−2ty=25+Ae−2t describes all solutions to the differential equation y˙=2(25−y)y˙=2(25−y), and all solutions to the associated initial value problems.

2. Conclusion

A first order differential equation is separable if it can be written in the form y˙=f(t)g(y)y˙=f(t)g(y). a differential equation typically has an infinite number of solutions. Ideally, but certainly not always, a corresponding initial value problem will have just one solution. A solution in which there are no unknown constants remaining is called a particular solution.The general approach to separable equations is this: Suppose we wish to solve y˙=f(t)g(y)y˙=f(t)g(y) where ff and gg are continuous functions. If g(a)=0g(a)=0 for some aa then y(t)=ay(t)=a is a constant solution of the equation, since in this case y˙=0=f(t)g(a)y˙=0=f(t)g(a). For example, y˙=y2−1y˙=y2−1 has constant solutions y(t)=1y(t)=1 and y(t)=−1y(t)=−1.To find the nonconstant solutions, we note that the function 1/g(y)1/g(y) is continuous where g≠0g≠0, so 1/g1/g has an antiderivative GG. Let FF be an antiderivative of ff. Now we write

(17.1.8)G(y)=∫1g(y)dy=∫f(t)dt=F(t)+CG(y)=∫1g(y)dy=∫f(t)dt=F(t)+C,

so G(y)=F(t)+CG(y)=F(t)+C. Now we solve this equation for yy. Of course, there are a few places this ideal description could go wrong: we need to be able to find the anti derivatives GG and FF, and we need to solve the final equation for yy. The upshot is that the solutions to the original differential equation are the constant solutions, if any, and all functions yy that satisfy G(y)=F(t)+CG(y)=F(t)+C.

3. References

The information in this report has been taken from the following;-

Book:

ENGINEERING MATHEMATICS VOLUME 3A BY BK PAL AND K DAS

Link:

https://math.libretexts.org/Bookshelves/Calculus