Consider the three dimensional normal distribution with

$\mu = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \\ 0 & 2 & 5 \end{pmatrix}$. Find the probability density at the point x_0 = (0.8 0.2 2)^t

We have $f(x) = \frac{1}{(2\pi)^{3/2}\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2}(x - \mu)^t \Sigma^{-1}(x - \mu)\right)$

where $|\Sigma|$ is the determinant of the covariance matrix Σ and μ is the mean vector.

First we need to calculate Σ^{-1} and $|\Sigma|$.

$\Sigma^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/5 & -2/25 \\ 0 & -2/25 & 9/25 \end{pmatrix}$ and $|\Sigma|$ = 1* (1/5) * (9/25) = 9/125.

Now, putting the values in the formula

$$f(x) = \frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{\frac{9}{125}}} \exp\left(-\frac{1}{2}\begin{pmatrix} -0.2 \\ -1.8 \\ 0 \end{pmatrix}^t \begin{pmatrix} -0.2 \\ -1.8 \\ 0 \end{pmatrix}\right)$$

$$f(x) = \frac{1}{(2\pi)^{3/2}\sqrt{9/125}} \exp(-\frac{1}{2} * \frac{399}{125})$$

$$f(x) = \frac{1}{(2\pi)^{3/2}\sqrt{9/125}} \exp(-\frac{399}{250})$$

$$f(x) \approx 0.00149$$

So, the probability density at x_0 = (0.8 0.2 2) is 0.00149.