

Suppose Box A contains 4 red and 5 blue coins and box B contains 6 red and 3 blue coins. A coin is chosen at random from the box A and placed in box B. Finally, a coin is chosen at random from among those now in box B. What is the probability a blue coin is transferred from box A to box B given that the coin chosen from box B is red?

To find the probability that a blue coin is transferred from box A to box B given that the coin chosen from box B is red, we can use conditional probability.

Let's denote the events:

- Event R: The coin chosen from box B is red.
- Event B: A blue coin is transferred from box A to box B.

We need to find $P(B|R)$, the probability of event B occurring given that event R has occurred.

We'll use Bayes' theorem, which states:

$$P(B|R) = \frac{P(R|B) * P(B)}{P(R)}$$

Here,

- $P(R|B)$ is the probability that the coin chosen from box B is red, given that a blue coin was transferred from box A to box B.
- $P(B)$ is the probability that a blue coin is transferred from box A to box B.
- $P(R)$ is the probability that the coin chosen from box B is red.

We can calculate each of these probabilities:

1. $P(R|B)$: The probability that the coin chosen from box B is red, given that a blue coin was transferred from box A to box B. Since the transferred coin must be blue, this probability is 0.
2. $P(B)$: The probability that a blue coin is transferred from box A to box B. This is the probability of selecting a blue coin from box A, which is $5/9$.

3. $P(R)$: The probability that the coin chosen from box B is red. This can happen in two ways: either a red coin was already in box B, or a blue coin was transferred from box A to box B and then a red coin was chosen from box B. So,

$$P(R) = P(R \cap R) + P(R \cap B)$$

$$\text{Or, } P(R) = P(R) * P(R|R) + P(B) * P(R|B)$$

$$\text{Or, } P(R) = P(R) + P(B) * 1$$

$$\text{Or, } P(R) = P(R) + 5/9$$

$$\text{Or, } P(R) - 5/9 = P(R)$$

$$\text{Or, } 1 - 5/9 = P(R)$$

$$\text{Or, } P(R) = 4/9$$

Now, putting these into Bayes' theorem

$$P(B|R) = \frac{P(R|B) * P(B)}{P(R)}$$

$$\text{Or, } P(B|R) = \frac{0 * 5/9}{4/9} = 0$$

So, the probability that a blue coin is transferred from box A to box B given that the coin chosen from box B is red is 0.