# NUMERICAL METHODS



**NOTE:**

MAKAUT course structure and syllabus of $6^{th}$ semester has been changed from 2021. **NUMERICAL METHODS** has been introduced as a new subject in present curriculum. The syllabus of this subject is almost same as **Numerical Methods [M(CS) 401]**. Taking special care of this matter we are providing the relevant MAKAUT university solutions of **Numerical Methods [M(CS) 401]**, so that students can get an idea about university questions patterns.

# APPROXIMATION IN NUMERICAL COMPUTATION

## Multiple Choice Type Questions

1. The number 3.4506531 when rounded off to 4 places of decimal will give [WBUT 2003, 2016(EVEN)]

a) 3.4506 b) 3.4507 c) 3.451 d) none of these

**Answer:** (b)

2. The significant digit of 0.0001234 is [WBUT 2010, 2013(ODD), 2016(ODD)]

a) 7 b) 4 c) 8 d) 6

**Answer:** (b)

3. The number 9.6506531 when round-off to 4 places of decimal will give [WBUT 2012(ODD), 2013(EVEN), 2013(ODD)]

a) 9.6506 b) 9.6507 c) 9.6505 d) none of these

**Answer:** (a)

4. Rounding off the number 0.03709157 correct upto 5 significant figure is [WBUT 2014(EVEN), 2014(ODD)]

a) 0.03709 b) 0.037091 c) 0.037092 d) 0.0370

**Answer:** (c)

5. The total number of significant digits in 500000 is [WBUT 2015(ODD)]

a) 2 b) 1 c) 0 d) none of these

**Answer:** (b)

6. The number of significant digits in 1.00340 is [WBUT 2017(EVEN)]

a) 3 b) 4 c) 5 d) 6

**Answer:** (d)

7. The number of significant digit of .0063945 is [WBUT 2018(EVEN)]

a) 5 b) 3 c) 7 d) None of these

**Answer:** (a)

8. Which of the following digits are not significant of the number 0.025? [WBUT 2018(ODD)]

a) 0 b) 2 c) 5 d) None of these

**Answer:** (a)

## Short Answer Type Questions

**1. Determine the absolute error $E_A$ of the following approximate number given their relative error $x_A = 67.84,\ E_R = 1\%$.** **[WBUT 2013(ODD)]**

**Answer:**

We know, $E_R = \dfrac{E_A}{x_A}$

or, $\quad 0.01 = \dfrac{E_A}{67.84}$

or, $\quad E_A = 67.84 \times 0.01 = 0.6784$

**2. Round off 35.7218 to four significant figures.** **[WBUT 2013(ODD)]**

**Answer:**

$35.7218 \approx 35.72$ (Approximated upto 4 significant figures).

# INTERPOLATION

## Multiple Choice Type Questions

**1. Which of the following relations is true?**

**[WBUT 2007, 2009, 2012(EVEN), 2016(EVEN)]**

a) $E = 1 + \Delta$ b) $E = 1 - \Delta$ c) $E = 1/\Delta$ d) None of these

**Answer:** (a)

**2. In Newton's forward interpolation, the intervals should be**

**[WBUT 2008, 2014(EVEN), 2014(ODD), 2018(EVEN)]**

a) equally spaced b) not equally spaced
c) may be equally spaced d) both (a) and (b)

**Answer:** (a)

**3. Which of the following is not true (the notations have their usual meaning)?**

**[WBUT 2008, 2014(ODD), 2018(EVEN)]**

a) $\Delta = E - 1$ b) $\Delta.\nabla = \Delta - \nabla$ c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\nabla = 1 - E^{-1}$

**Answer:** (c)

**4. $\Delta^3(Y_0)$ may be expressed as which of the following terms?**

**[WBUT 2008, 2013(EVEN), 2016(EVEN), 2016(ODD)]**

a) $y_3 - 3y_2 + 3y_1 - y_0$ b) $y_2 - 2y_1 + y_0$
c) $y_3 - 3y_2 + 3y_1 + y_0$ d) none of these

**Answer:** (a)

**5. If the interval of differencing is unity and $f(x) = ax^2$ ('$a$' is a constant) which of the following choices is wrong?**

**[WBUT 2009, 2010, 2013(ODD), 2014(EVEN), 2014(ODD)]**

a) $\Delta f(x) = a(2x+1)$ b) $\Delta^2 f(x) = 2a$ c) $\Delta^3 f(x) = 2$ d) $\Delta^4 f(x) = 0$

**Answer:** (a)

**6. If $f(0) = 12, f(3) = 6$ and $f(4) = 8$, then the linear interpolation function $f(x)$ is**

**[WBUT 2010, 2013(EVEN), 2017(ODD), 2018(EVEN)]**

a) $x^2 - 3x + 12$ b) $x^2 - 5x$ c) $x^3 - x^2 + 5x$ d) $x^2 - 5x + 12$

**Answer:** (d)

7. If $f(x)=\frac{1}{x^2}$, then the dividend difference $f(a, b)$ is: [WBUT 2010, 2012(EVEN), 2013(ODD)]

a) $\frac{a+b}{(ab)^2}$ b) $-\frac{a+b}{(ab)^2}$ c) $\frac{1}{a^2-b^2}$ d) $\frac{1}{a^2}-\frac{1}{b^2}$

**Answer:** (b)

8. Which of the following relations is true? [WBUT 2011(ODD), 2015(ODD)]

a) $E=1-\Delta,\ \Delta-\nabla=\Delta\nabla$ b) $E=1-\Delta,\ \Delta+\nabla=\Delta\nabla$

c) $E=1+\Delta,\ \Delta+\nabla=\Delta\nabla$ d) $E=1+\Delta,\ \Delta-\nabla=\Delta\nabla$

**Answer:** (d)

9. If $f(3)=a+\Delta f(1)+\Delta^2 f(1)$ then $a=$ [WBUT 2012(ODD), 2014(EVEN)]

a) $f(0)$ b) $f(1)$ c) $f(2)$ d) $f(3)$

**Answer:** (c)

10. The relation between shift operator 'E' and forward difference operator '$\Delta$' is given by [WBUT 2014(EVEN)]

a) $\Delta=1+E$ b) $E=1+\Delta$ c) $E=\Delta$ d) $E=\Delta+2$

**Answer:** (b)

11. Geometrically, the Lagrange's interpolation formula for two points of interpolation represents a [WBUT 2015(EVEN)]

a) parabola b) circle c) straight line d) none of these

**Answer:** (c)

12. If $f(x)=\frac{1}{x}$, the divided difference $f[a, b, c]$ is [WBUT 2015(EVEN)]

a) $1/(a+b+c)$ b) $1/abc$ c) $1/(a^2+b^2)$ d) $1/(a+b+c)$

**Answer:** (b)

13. The value of $(1+\Delta)(1-\nabla)$ is [WBUT 2015(EVEN)]

a) 0 b) 1 c) 2 d) 3

**Answer:** (b)

14. Lagrange's interpolation can be used for [WBUT 2015(ODD), 2016(EVEN), 2017(ODD)]

a) only equi-spaced nodes b) only unequi-spaced nodes

c) for both cases of (a) and (b) d) none of these

**Answer:** (c)

15. $(\Delta - \nabla)x^2$ is equal to (the notations have their usual meanings) [WBUT 2015(ODD)]

a) $h^2$ b) $2h^2$ c) $-2h^2$ d) none of these

**Answer:** (b)

16. Newton's divided difference interpolation formula is used for [WBUT 2015(ODD)]

a) equispaced arguments only
b) unequispaced only
c) both equispaced and unequispaced arguments
d) none of these

**Answer:** (c)

17. In the Newton's Forward Interpolation formula, the value of $u = \frac{x - x_0}{h}$ lies between [WBUT 2015(ODD), 2016(EVEN)]

a) 0 and 1 b) –1 and 0 c) –1 and 1 d) 5 and 0

**Answer:** (a)

18. The $n$-th order divided difference of a polynomial of degree $n$ is [WBUT 2016 (EVEN), 2018(ODD)]

a) 0 b) constant c) 1 d) –1

**Answer:** (b)

19. The $n^{th}$ order forward difference of $n^{th}$ degree polynomial is [WBUT 2017(EVEN)]

a) $n!$ b) $(n+1)!$ c) 0 d) none of these

**Answer:** (d)

20. If $f(x) = \frac{1}{x}$, then divided difference $f(a,b)$ is [WBUT 2018(EVEN)]

a) $\frac{a+b}{(ab)^2}$ b) $-\frac{a+b}{(ab)^2}$ c) $\frac{1}{a^2 - b^2}$ d) $\frac{1}{a^2} - \frac{1}{b^2}$

**Answer:** (a)

21. The value of $\Delta^2(ax^2 + bx + c)$ is [WBUT 2018(EVEN)]

a) $2ah + b$ b) $2ah$ c) $2ah^2$ d) $2a$

**Answer:** (c)

22. If $f(x) = be^{ax}$ then $\Delta f(x)$ is [WBUT 2018(EVEN)]

a) $be^{ax}(e^{ah} - 1)$ b) $be^{ax(a-1)}$ c) $be^{ax}(1 - e^{ax})$ d) None of these

**Answer:** (a)

23. If $f(x)=\frac{1}{x}$, then the divided differences of $f(a,b)$ is [WBUT 2018(ODD)]

a) $\frac{a+b}{(ab)}$ b) $-\frac{a+b}{(ab)}$ c) $\frac{1}{a-b}$ d) None of these

Answer: (d)

24. In finite-difference method, the central-difference formula for replacing $\left(\frac{dy}{dx}\right)_{x=x_i}$ is [WBUT 2018(ODD)]

a) $\frac{y_{i-1}-y_{i+1}}{2h}$ b) $\frac{y_{i+1}-y_i}{2h}$ c) $\frac{y_{i+1}-y_{i-1}}{2h}$ d) None of these

Answer: (c)

25. In Lagrange's interpolation, the intervals should be [WBUT 2018(ODD)]

a) equally spaced b) not equally spaced
c) may be equally spaced d) Both (a) and (b)

Answer: (d)

## Short Answer Type Questions

1. a) Prove that $f(4)=f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)$. [WBUT 2008, 2014(EVEN)]

**Answer:**

Here we have to prove that $f(4)=f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)$,
where $\Delta f(x)=f(x+h)-f(x)$ and $h$ is the step length.

$\because \Delta^3 f(1)=\Delta^2 f(2)-\Delta^2 f(1)$

$\Rightarrow \Delta^2 f(1)+\Delta^3 f(1)=\Delta^2 f(2)=\Delta f(3)-\Delta f(2)$

$\Rightarrow \Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)=\Delta f(3)=f(4)-f(3)$

$\Rightarrow f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)=f(4)$

$\therefore f(4)=f(3)+\Delta f(2)+\Delta^2 f(1)+\Delta^3 f(1)$.

b) Evaluate $\Delta^2 \cos 2x$. [WBUT 2008, 2014(EVEN)]

**Answer:**

$$\Delta^2 \cos 2x=\Delta(\cos 2x)=\Delta(\cos 2(x+h)-\cos 2x)$$
$$=\Delta\left(2\sin\frac{2x+2h+2x}{2}\cdot\sin\frac{2x-2x-2h}{2}\right)$$
$$=\Delta(2\sin(2x+h)\sin(-h))=-2\sin h\,\Delta\sin(2x+h)$$
$$=-2\sin h\left(\sin(2(x+h)+h)-\sin(2x+h)\right)$$

**2. a) What is the difference between interpolation and extrapolation? Give suitable examples.** **[WBUT 2008, 2009, 2015(ODD)]**

**OR,**

**What is interpolation? What are the differences between interpolation and extrapolation? Explain with suitable examples.** **[WBUT 2017(ODD)]**

**Answer:**

**Interpolation** is the process of obtaining a value from a graph or table that is located between major points given, or between data points plotted. A ratio process is usually used to obtain the value.

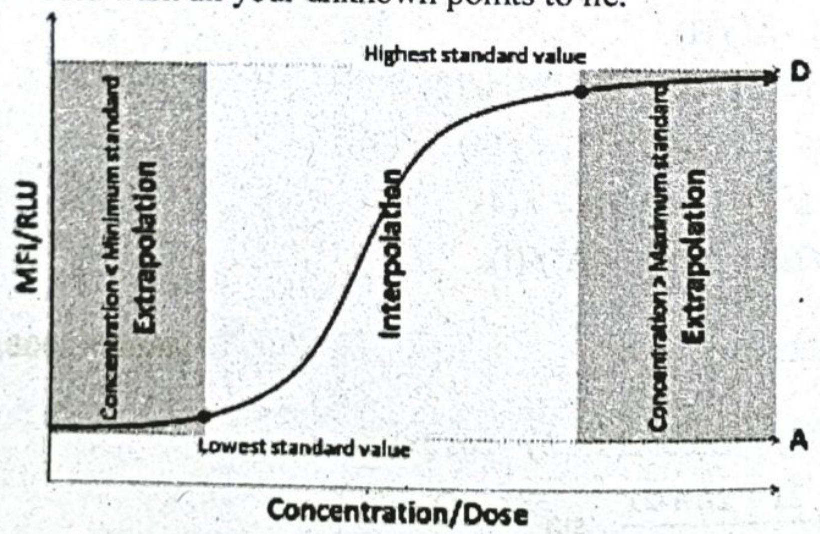
**Extrapolation** is the process of obtaining a value from a chart or graph that extends beyond the given data. The "trend" of the data is extended past the last point given and an estimate made of the value.

**Example of Interpolation:** drawing a line (fitting a curve) from the first data point you have to the last allows you to estimate data points between those two extremes (or between any data points that you have). i.e. 'filling in between'.

**Example of Extrapolation:** fitting a curve to the data that you have using an equation, then extending that line beyond the first and last points enables you to estimate values (or extrapolate them) beyond the measured data.

Extrapolation occurs when you are inferring or estimating concentrations for points that are within calculable limits (A<x<D) but are outside of the range of our standard curve. This occurs when the calculated Concentration< Minimum Standard Concentration or when the calculated Concentration > Maximum Standard Concentration. For typical bioassay standard curves, extrapolating can be very dangerous and quite often be misleading. The reason being that minute changes in MFI values on the flat parts of the standard curve can lead to huge changes in concentration or dose.

Interpolation occurs when your MFI/RLU values are within standard range or Minimum Standard Concentration $< x <$ Maximum Standard Concentration. Ideally, this is the range where you would want all your unknown points to lie.



***Extrapolation vs. Interpolation***

The only difference between interpolation and extrapolation is that first calculation is made for arguments inside of area covered with experimental data and extrapolation for outside range of tested values and meant to be used for prediction.

**b) Find the polynomial $f(x)$ & hence calculate $f(5.5)$ for the given data:**
**[WBUT 2008, 2013(ODD), 2015(ODD), 2016(EVEN)]**

| x: | 0 | 2 | 3 | 5 | 7 |
|---|---|---|---|---|---|
| f(x): | 1 | 47 | 97 | 251 | 477 |

**Answer:**

| $x$ | $f(x)$ |
|---|---|
| 0 | 1 |
| 2 | 47 |
| 3 | 97 |
| 5 | 251 |
| 7 | 477 |

Since the values of $x$ are not equally spaced, we will use Lagrange's interpolation polynomial.
The formula is

$$f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}.f(x_0)+\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}f(x_1)$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}f(x_2)+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}f(x_3)$$
$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}f(x_4)$$

Here $x_0=0,\ x_1=2, x_2=3,\ x_3=5,\ x_4=7$

Putting the value of $x=5.5$, the approximate value of $f(5.5)$ is

$$f(x)=\frac{(x-2)(x-3)(x-5)(x-7)}{(0-2)(0-3)(0-5)(0-7)}.1+\frac{(x-0)(x-3)(x-5)(x-7)}{(2-0)(2-3)(2-5)(2-7)}.47$$
$$+\frac{(x-0)(x-2)(x-5)(x-7)}{(3-0)(3-2)(3-5)(3-7)}.97+\frac{(x-0)(x-2)(x-3)(x-7)}{(5-0)(5-2)(5-3)(5-7)}.257$$
$$+\frac{(x-0)(x-2)(x-3)(x-5)}{(7-0)(7-2)(7-3)(7-5)}.477$$

$$=\frac{(x-2)(x-3)(x-5)(x-7)}{210}+\frac{x(x-3)(x-5)(x-7)}{(-30)}47$$
$$+\frac{x(x-2)(x-5)(x-7)}{(30)}97+\frac{x(x-2)(x-3)(x-7)}{(-60)}257+\frac{x(x-2)(x-3)(x-5)}{280}477$$

$$=\frac{(x-2)(x-3)(x-5)(x-7)}{210}+\frac{x(x-5)(x-7)}{30}[97x-194-47x-141]$$
$$+\frac{x(x-2)(x-3)(x-7)}{(-60)}257+\frac{x(x-2)(x-3)(x-5)}{280}477$$

**3. Use Newton's divided difference formula to find $f(5)$ from the following data:**
**[WBUT 2009, 2012(ODD), 2016(EVEN), 2017(ODD),2018(EVEN)]**

| X | 0 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| f(x) | 4 | 26 | 58 | 112 | 466 | 668 |

**Answer:**
The divided difference table for the given data is:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0 | 4 | | | |
| | | $\frac{26-4}{2-0}=11$ | | |
| 2 | 26 | | $\frac{32-11}{3-0}=7$ | |
| | | $\frac{58-26}{3-2}=32$ | | $\frac{11-7}{4-0}=1$ |
| 3 | 58 | | $\frac{54-32}{4-2}=11$ | |
| | | $\frac{112-58}{4-3}=54$ | | $\frac{16-11}{7-2}=1$ |
| 4 | 112 | | $\frac{118-54}{7-3}=16$ | |
| | | $\frac{466-112}{7-4}=118$ | | $\frac{21-16}{8-5}=1$ |
| 7 | 466 | | $\frac{202-118}{8-4}=21$ | |
| | | $\frac{668-466}{8-7}=202$ | | |
| 8 | 668 | | | |

Using Newton's divided difference formula,

$$f(x)=f(x_0)+(x-x_0)\Delta f(x_0)+(x-x_0)(x-x_1)\Delta^2 f(x_0)+\ldots.$$
$$f(x)=4+(x-0)11+(x-0)(x-2)7+(x-0)(x-2)(x-3)1$$
$$\Rightarrow f(5)=4+5.11+5.3.7+5.3.2=4+55+105+30=194$$

**4. Apply Lagrange's interpolation formula to find $f(x)$ if $f(1)=2,\ f(2)=4,\ f(3)=8,\ f(4)=16$ and $f(7)=128$.** **[WBUT 2010, 2013(ODD), 2016(ODD)]**

**Answer:**

Now according to Lagrange's interpolation formula we get,

$$f(x)=\frac{(x-2)(x-3)(x-4)(x-7)}{(1-2)(1-3)(1-4)(1-7)}\times 2+\frac{(x-2)(x-3)(x-4)(x-7)}{(2-1)(2-3)(2-4)(2-7)}\times 4$$
$$+\frac{(x-1)(x-2)(x-4)(x-7)}{(3-1)(3-2)(3-4)(3-4)}\times 8+\frac{(x-1)(x-2)(x-3)(x-7)}{(4-1)(4-2)(4-3)(4-7)}\times 16$$
$$+\frac{(x-1)(x-2)(x-3)(x-4)}{(7-1)(7-2)(7-3)(7-4)}\times 128$$
$$=\frac{1}{18}\left[\left(x^2-5x+6\right)\left(x^2-11x+28\right)\right]-\frac{2}{5}\left[\left(x^2-4x+3\right)\left(x^2-11x+28\right)\right]$$
$$+\left(x^2-3x+2\right)\left(x^2-11x+28\right)-\frac{8}{9}\left[\left(x^2-3x+2\right)\left(x^2-10x+21\right)\right]$$
$$+\frac{16}{45}\left[\left(x^2-3x+2\right)\left(x^2-7x+12\right)\right]$$
$$=1.222x^4-0.889x^3+3.278x^2-3.444x+2.933$$

**5. Given the function $y=\dfrac{1}{x}$, show that the divided difference of $n^{\text{th}}$ order**

$$y[x_0,x_1,x_2,\cdots,x_n]=(-1)^n/(x_0x_1x_2\cdots x_n)$$ **[WBUT 2011(ODD), 2015(EVEN)]**

**Answer:**

$$y(x_0,x_1)=\frac{\frac{1}{x_0}-\frac{1}{x_1}}{x_0-x_1}=\frac{(x_1-x_0)}{x_0x_1(x_0-x_1)}=(-1)\frac{1}{x_0x_1}$$

Similarly, $y(x_1,x_2)=(-1)\dfrac{1}{x_1x_2}$

Then, $y(x_0,x_1,x_2)=\dfrac{1}{x_0-x_2}\left[\dfrac{(-1)}{x_0x_1}-\dfrac{(-1)}{x_1x_2}\right]=\dfrac{(-1)}{x_0-x_2}\left[\dfrac{x_2-x_0}{x_0x_1x_2}\right]=\dfrac{(-1)^2}{x_0x_1x_2}$

That is the result holds for $n=1,\ 2$. Let the result hold for a particular value $m$ of $n$.

Then, $y(x_0,x_1,x_2,\ldots\ldots,x_m)=\dfrac{(-1)^m}{x_0x_1x_2\ldots\ldots x_m}$

Now, $y(x_0,x_1,x_2,\ldots\ldots x_{m+1})=\dfrac{1}{x_0-x_{m+1}}\left[\dfrac{(-1)^m}{x_0x_1\ldots\ldots x_m}-\dfrac{(-1)^m}{x_1x_2\ldots\ldots x_{m+1}}\right]$

$$=\frac{(-1)^m}{x_0-x_{m+1}}\left[\frac{x_{m+1}-x_0}{x_0x_1x_2......x_{m+1}}\right]=\frac{(-1)^{m+1}}{x_0.x_1x_2.....x_{m+1}}$$

So the result is true for $n=m+1$. Hence the result holds for all values of $n$, that is

$$y(x_0,x_1,x_2,....,x_n)=\frac{(-1)^n}{x_0x_1......x_n} \quad \text{(Proved)}$$

**6. Fit a polynomial to the following table of values using Lagrange interpolation formula:** **[WBUT 2011(ODD), 2015(EVEN)]**

| $x$: | 0 | 1 | 3 | 4 |
|---|---|---|---|---|
| $y$: | –12 | 0 | 6 | 12 |

**Find the value of *y* when;**
**a)** $x=2$; **b)** $x=3.5$

**Answer:**

According to Lagrange's interpolation formula the required interpolating polynomial is

$$L_n(x)=\frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}\times(-12)+\frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}\times 0$$

$$+\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}\times 6+\frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}\times 12$$

$$=(x-1)(x-3)(x-4)-(x-0)(x-1)(x-4)+(x-0)(x-1)(x-3)$$

$$=(x-1)(x-3)(x-4)-x(x-1)[x-4-x+3]$$

$$=(x-1)[(x-3)(x-4)+x] \qquad =(x-1)(x^2-7x+12+x)$$

$$=x^3-6x^2+12x-x^2+7x-12 \quad =x^3-7x^2+19x-12$$

Now, $y(2)=(2)^3-7(2)^2+19(2)-12=6$

$y(3.5)=(3.5)^3-7(3.5)^2+19(3.5)-12=11.625$

**7. Show that $(1+\Delta)(1-\nabla)\equiv 1$**

**[WBUT 2012(ODD), 2015(ODD), 2016(EVEN), 2017(ODD)]**

**Answer:**

L. H. S. $=(1+\Delta)(1-\nabla)f(x)$

$=(1+\Delta)[f(x)-\nabla f(x)]$

$=(1+\Delta)[f(x)-f(x)+f(x-h)]$

$=Ef(x-h), \quad \because 1+\Delta\equiv E$

$=f(x)$

$=1\cdot f(x)$

Hence, $(1+\Delta)(1-\nabla)\equiv 1$

8. Show that if $\Delta$ operates on $n$, then $\Delta\binom{n}{x+1}=\binom{n}{x}$ and hence $\sum_{n=1}^{N}\binom{n}{x}=\binom{n+1}{x+1}-\binom{1}{x+1}$. **[WBUT 2013(ODD)]**

**Answer:**

**1st Part:**

$$\text{LHS}=\Delta\binom{n}{x+1}=\binom{n+1}{x+1}-\binom{n}{x+1}={}^{n+1}C_{x+1}-{}^{n}C_{x+1}$$

$$=\frac{(n+1)!}{\lfloor x+1\lfloor n-x}-\frac{n!}{\lfloor x+1\lfloor n-x-1}=\frac{n!}{\lfloor nx+1\lfloor n-x-1}\left[\frac{n+1}{n-x}-1\right]$$

$$=\frac{n!}{\lfloor x+1\lfloor n-x-1}\left[\frac{n+1-(n-x)}{n-x}\right]=\frac{n!}{\lfloor x+1\lfloor n-x-1}\frac{x+1}{n-x}$$

$$=\frac{n!}{\lfloor x\lfloor n-x}={}^{n}C_x=\binom{n}{x}=\text{R.H.S (proved)}$$

**2nd Part:**

$$\sum_{n=1}^{N}\binom{n}{x}=\sum_{n=1}^{N}\Delta\binom{n}{x+1} \quad \text{[using the previous result]}$$

$$\text{R.H.S.}=\Delta\binom{N}{x+1}+\Delta\binom{N-1}{x+1}+\Delta\binom{N-2}{x+1}+\ldots\ldots\Delta\binom{1}{x+1}$$

$$=\binom{N+1}{x+1}-\binom{N}{x+1}+\binom{N}{x+1}-\binom{N-1}{x+1}+\binom{N-1}{x+1}-\binom{N-2}{x+1}+\ldots\ldots+\binom{2}{x+1}-\binom{1}{x+1}$$

$$=\binom{N+1}{x+1}-\binom{1}{x+1}=\sum_{n=1}^{N}\binom{n}{x}=\text{L.H.S.}$$

9. **Use Lagrange's Interpolation formula to find the value of $y=f(x)$ for x = 1, given in the following table:** **[WBUT 2014(EVEN)]**

| | | | | |
|---|---|---|---|---|
| **x** | **0** | **2** | **3** | **5** |
| **y** | **0** | **8** | **15** | **35** |

**Answer:**

From the definition $L_n(x)=\sum_{i=0}^{n}\frac{w(x)f(x_i)}{(x-x_i)w'(x_i)}=\sum_{i=0}^{n}w(x)\frac{f(x_i)}{D_i}$

where $w(x)=\prod_{i=0}^{n}(x-x_i),\ D_i=(x-x_i)w'(x_i)$

$$\therefore w(x)=(1-0)(1-2)(1-3)(1-5)=-8$$

| $x_0 = 0$ | $x_1 = 2$ | $x_2 = 3$ | $x_3 = 5$ | $D_i$ | $y_i = f(x_i)$ | $\frac{f(x_i)}{D_i}$ |
|---|---|---|---|---|---|---|
| $\underline{1-0=1}$ | $0-2=-2$ | $0-3=-3$ | $0-5=-5$ | $-30$ | $0$ | $\frac{0}{-30}$ |
| $2-0=2$ | $\underline{1-2=-1}$ | $2-3=-1$ | $2-5=-3$ | $-6$ | $8$ | $\frac{8}{-6}$ |
| $3-0=3$ | $3-2=1$ | $\underline{1-3=-2}$ | $3-5=-2$ | $12$ | $15$ | $\frac{15}{12}$ |
| $5-0=5$ | $5-2=3$ | $\underline{5-3=2}$ | $\underline{1-5=-4}$ | $-120$ | $35$ | $\frac{35}{-120}$ |

$\therefore \sum_{i=0}^{3} \frac{f(x_i)}{D_i} = -0.375$ $\qquad\qquad \therefore y(x=1) = -8 \times (-0.375) = 3.$

**10. Prove that $\Delta.\nabla = \Delta - \nabla$.** **[WBUT 2014(EVEN), 2014(ODD), 2018(EVEN)]**

**Answer:**

$\therefore \Delta f(x) = f(x+h) - f(x) \quad \& \quad \nabla f(x) = f(x) - f(x-h)$,

where $h$ is the step length, therefore,

$$\therefore (\Delta - \nabla) f(x) = \Delta f(x) - \nabla f(x)$$
$$= f(x+h) - f(x) - f(x) + f(x-h)$$
$$= f(x+h) - 2 * f(x) + f(x-h)$$

$$\& \ \Delta\nabla f(x) = \Delta(f(x) - f(x-h))$$
$$= \Delta f(x) - \Delta f(x-h)$$
$$= f(x+h) - f(x) - f(x) + f(x-h)$$
$$= f(x+h) - 2 * f(x) + f(x-h)$$

$\therefore$ From above two,

$(\Delta - \nabla) f(x) = \Delta\nabla f(x), \forall f(x)$

$\therefore \Delta - \nabla = \Delta\nabla$

**11. The function $y = \sin x$ is tabulated as given below:** **[WBUT 2014(EVEN)]**

| | | | |
|---|---|---|---|
| $x$ : | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ |
| $\sin x$: | 0 | 0.70711 | 1.0 |

**Find the value of $\sin\left(\frac{\pi}{3}\right)$ using Newton Backward interpolation.**

**Answer:**

$y = \sin x$

$x \ \ 0 \ \ \pi/4 \ \ \pi/2$

$y$ 0 0.70711 1.0

$x_0 = 0,\ x_1 = \pi/4,\ x_2 = \pi/2$

$y_0 = 0,\ y_1 = 0.70711,\ y_2 = 1.0$

By Lagrange's Interpolation formula, the corresponding polynomial is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{\left(x-\frac{\pi}{4}\right)\left(x-\frac{\pi}{2}\right)}{\left(0-\frac{\pi}{4}\right)\left(0-\frac{\pi}{2}\right)} \cdot 0 + \frac{(x-0)\left(x-\frac{\pi}{2}\right)}{\left(\frac{\pi}{4}-0\right)\left(\frac{\pi}{4}-\frac{\pi}{2}\right)} \cdot (0.70711) + \frac{(x-0)\left(x-\frac{\pi}{4}\right)}{\left(\frac{\pi}{2}-0\right)\left(\frac{\pi}{2}-\frac{\pi}{4}\right)} \cdot (1.0)$$

$$= -x\left(x-\frac{\pi}{2}\right) \cdot \frac{0.70711}{\frac{\pi}{4}\cdot\frac{\pi}{4}} + \frac{x\left(x-\frac{\pi}{4}\right)}{\frac{\pi}{2}\cdot\frac{\pi}{4}} \cdot 1 = \frac{8x}{\pi^2}\left[\left(x-\frac{\pi}{4}\right) - 2\times 0.70711\left(x-\frac{\pi}{2}\right)\right]$$

$$= \frac{8x}{\pi^2}(-0.41422x + 0.45711\pi)$$

$$\therefore \sin\frac{\pi}{3} = -\frac{\pi}{3}\left(\frac{\pi}{3}-\frac{\pi}{2}\right)\frac{16}{\pi^2}(0.70711) + \frac{\pi}{3}\cdot\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\cdot\frac{8}{\pi^2}\cdot 1$$

$$= +\frac{1}{3}\times\frac{1}{6}\times 16\times 0.70711 + \frac{1}{3}\times\frac{1}{12}\times 8\times 1$$

$$\approx 0.850764$$

**12. Find the polynomial of least degree which attains the prescribed values of the given points:** **[WBUT 2014(EVEN)]**

| | | | | |
|---|---|---|---|---|
| **x :** | **0** | **1** | **2** | **3** |
| **y :** | **3** | **6** | **11** | **18** |

**Hence find y for x = 1.1**

**Answer:**

To determine the least degree polynomial which satisfy the prescribed data we use Lagrange's interpolation formula:

$$L_n(x) = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}\times 3 + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}\times 6$$

$$+ \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}\times 11 + \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}\times 18$$

$$= \frac{-1}{2}(x-1)(x-2)(x-3) + x(x-2)(x-3) - \frac{11}{2}x(x-1)(x-3) + 3x(x-1)(x-2)$$

$$= \frac{-1}{2}\left(x^3-6x^2+11x-6\right) + (x^3-5x^2+6x) - \frac{11}{2}(x^3-4x^2+3x) + 3(x^3-3x^2+2x)$$

$$=\left(\frac{-1}{2}+1-\frac{11}{2}+3\right)x^3+(+3-5+22-9)x^2+\left(\frac{-11}{2}+6-\frac{33}{2}+6\right)x+3$$
$$=-2x^3+11x^2-10x+3$$

So the required least degree polynomial is $-2x^3+11x^2-10x+3$.

$\therefore y(x=1.1)=2.648$

**13. Find the missing value from the following table:** **[WBUT 2014(EVEN)]**

| | | | | | |
|---|---|---|---|---|---|
| $x$: | 1 | 2 | 3 | 4 | 5 |
| $y$: | 7 | - | 27 | 40 | 55 |

**Answer:**

Let $y(x=2)=a$. Now we construct Newton's Forward interpolation table from the given data:

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1 | 7 | | | |
| | | $a-7$ | | |
| 2 | $a$ | | $34-2a$ | |
| | | $27-a$ | | $3a-48$ |
| 3 | 27 | | $a-14$ | |
| | | 13 | | $6-a$ |
| 4 | 40 | | $-8$ | |
| | | 5 | | |
| 5 | 45 | | | |

Since 4 values of $y$ are given, so 3rd order difference will be constant.

$$3a-48=6-a$$
$$\text{or,}\quad 4a=54$$
$$\text{or,}\quad a=13.5$$
$$\therefore y(x=2)=13.5$$

**14. Use Lagrange's Interpolation formula to find the value of $y=f(x)$ for $x=1$, given in the following table:** **[WBUT 2014(ODD)]**

| $x$ | 0 | 2 | 3 | 3 |
|---|---|---|---|---|
| $y$ | 0 | 8 | 15 | 35 |

**Answer:**

The question 2. is wrongly stated because "for a given value of $x=3$ there are two values of $y$ given in the question". If the last '3' is '4', then the answer can be obtained.

**15. Evaluate $\Delta^2 \tan 2x$.** **[WBUT 2014(ODD)]**

**Answer:**

$\Delta^2 \tan x=(E-1)^2 \tan x=(E^2-2E+1)\tan x=\tan(x+2h)-2\tan(x+h)+\tan x$,

where $h$ is the step length.

16. $\Delta^m\left(\frac{1}{x}\right)=\frac{(-1)^m n!h^m}{x(x+h)(x+2h)\ldots\ldots(x+mh)}$. **[WBUT 2014(ODD)]**

**Answer:**

$$\Delta\left(\frac{1}{x}\right)=\frac{1}{x+h}-\frac{1}{x}=\frac{x-x-h}{x(x+h)}=\frac{-h}{x(x+h)}=\frac{(-1)h}{x(x+h)}$$

So, the statement is true for $m=1$

Now we consider it is true for $m=k$

$$\therefore\quad \Delta^K\left(\frac{1}{x}\right)=\frac{(-1)^k k!h^K}{x(x+h)(x+2h)\ldots\ldots.(x+kh)}$$

$$\therefore\quad \Delta^{k+1}\left(\frac{1}{x}\right)=\Delta\left(\Delta^k\left(\frac{1}{x}\right)\right)=\Delta^k\left(\frac{1}{x+h}\right)-\Delta^k\left(\frac{1}{x}\right)$$

$$=\frac{(-1)^k k!\,h^k}{(x+h)(x+2h)(x+3h)\ldots..(x+(k+1)h}-\frac{(-1)^k k!\,h^k}{x(x+h)(x+2h)\ldots..(x+kh)}$$

$$=\frac{(-1)^k\ k!h^k}{(x+h)(x+2h)\ldots\ldots(x+kh)}\left[\frac{1}{x+(k+1)h}-\frac{1}{x}\right]$$

$$=\frac{(-1)^k\ k!h^k}{(x+h)(x+2h)\ldots..(x+kh)}\left[\frac{x-x-(k+1)h}{x(x+(k+1)h)}\right]$$

$$=\frac{(-1)^{k+1}(k+1)\,!\,h^{k+1}}{x(x+h)(x+2h)\ldots\ldots.(x+kh)(x+(k+1)h)}$$

$\therefore$ The statement is true for $m=(k+1)$.

Hence by the principle of mathematical induction we can conclude that the statement is true for all values of $m$.

**17. Find the fourth degree curve $y=f(x)$ passing through the points (2, 3), (4, 43), (7, 778) and (8, 1515) using Newton's divided difference formula. [WBUT 2014(ODD)]**

**Answer:**

| $x$ | $y=f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 2 | 3 | | | |
| | | 20 | | |
| 4 | 43 | | | |
| | | | 45 | |
| | | 245 | | 13 |
| 7 | 778 | | | |
| | | 737 | 123 | |
| 8 | 1515 | | | |

From Newton's divided difference formula, we have

$$f(x)\approx f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$= 3 + (x-2)\times 20 + (x-2)(x-4)\times 45 + (x-2)(x-4)(x-7)\times 13$$
$$= 3 + 20x - 40 + 45(x^2 - 6x + 8) + 13(x^3 - 13x^2 + 50x - 56)$$
$$= 13x^3 - 124x^2 + 400x - 405$$

**18. Prove the following operator relation:**

**$\mu^2 = 1 + \frac{1}{4}\delta^2$, where the notations have their usual meanings.** **[WBUT 2015(ODD)]**

**Answer:**

We know,

$$S = E^{\frac{1}{2}} - E^{-\frac{1}{2}} \quad \text{and} \quad \mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)$$

Now, $S^2 = E + E^{-1} - 2$

or, $E + E^{-1} = S^2 + 2$ .... (1)

and $\mu^2 = \frac{1}{4}\left(E + E^{-1} + 2\right)$

$= \frac{1}{4}\left(S^2 + 2 + 2\right)$ [using (1)]

$= \frac{1}{4}\left(S^2 + 4\right) = 1 + \frac{1}{4}S^2$

**19. Evaluate the missing terms in the following table:** **[WBUT 2015(ODD)]**

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$: | 0 | - | 8 | 15 | - | 35 |

**Answer:**

Let $f(1) = a$

and $f(4) = b$

Now we construct the difference table:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0 | 0 | | | |
| | | $a$ | | |
| 1 | $a$ | | $8-2a$ | |
| | | $8-a$ | | $3a-9$ |
| 2 | 8 | | $a-1$ | |
| | | 7 | | $b-a-21$ |
| 3 | 15 | | $b-22$ | |
| | | $b-15$ | | $72-3b$ |
| 4 | $b$ | | $50-2b$ | |
| | | $35-b$ | | |
| 5 | 35 | | | |

Since four values of $f(x)$ are given so 3rd order difference must be constant.

$\therefore \quad b-a-21=3a-9 \quad \Rightarrow \quad 4a-b+12=0 \quad \ldots.(1)$

and $\quad 72-3b=b-a-21 \quad \Rightarrow \quad a-4b+93=0 \quad \ldots.(2)$

Solving we get, $15a=45 \quad \Rightarrow \quad a=3$

and $\quad 24=b \quad \Rightarrow \quad b=24$

$\therefore f(1)=3$ and $f(4)=24$

**20. Use Lagrange's inverse interpolation formula to find the value of $x$, when $y=0.143$ from the following data:** **[WBUT 2016(EVEN)]**

| $x$ | 1 | 2 | 4 | 5 | 8 |
|---|---|---|---|---|---|
| $f(x)$ | 1.000 | 0.500 | 0.250 | 0.200 | 0.125 |

**Answer:**

Let $\quad x_0=1,\ x_1=2,\ x_2=4,\ x_3=5,\ x_4=8$

and $\quad y_0=1,\ y_1=0.5,\ y_2=0.25,\ y_3=0.2,\ y_4=0.125$

Now Lagrange's computational table:

| | | | | | $D$ | $x/D$ |
|---|---|---|---|---|---|---|
| $y-y_0$ | $y_0-y_1$ | $y_0-y_2$ | $y_0-y_3$ | $y_0-y_4$ | $D_0$ | $x_0/D$ |
| $y_1-y_0$ | $y-y_1$ | $y_1-y_2$ | $y_1-y_3$ | $y_1-y_4$ | $D_1$ | $x_1/D$ |
| $y_2-y_0$ | $y_2-y_1$ | $y-y_2$ | $y_2-y_3$ | $y_2-y_4$ | $D_2$ | $x_2/D$ |
| $y_3-y_0$ | $y_3-y_1$ | $y_3-y_2$ | $y-y_3$ | $y_3-y_4$ | $D_3$ | $x_3/D$ |
| $y_4-y_0$ | $y_4-y_1$ | $y_4-y_2$ | $y_4-y_3$ | $y-y_4$ | $D_4$ | $x_4/D$ |

i.e.,

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.875 | 0.5 | 0.75 | 0.8 | +0.875 | +0.2249625 | +4.45 |
| −0.5 | −0.357 | 0.25 | 0.3 | 0.375 | 0.00502 | 398.38 |
| −0.75 | −0.25 | −0.107 | 0.05 | 0.125 | 0.000125 | 31900.3 |
| −0.8 | −0.3 | −0.05 | −0.057 | 0.075 | 0.000051 | 97465.9 |
| −0.875 | −0.375 | −0.125 | −0.075 | 0.018 | 0.000055 | 144479 |

$\sum\frac{x_i}{D}=274248.7$

$\therefore \quad L_n(y)=\sum\frac{x_i}{D}\times 0.000034=9.4048\approx 9.4$

$\therefore \quad x=9.4$ at $y=0.143$

**21. From the following table, find the value of $f(1.5)$ by Newton forward interpolation formula:** **[WBUT 2017(EVEN)]**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 10 | 15 | 20 | 25 | 30 | 35 |

**Answer:**

First we construct the difference table with the given data as follows:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ |
|---|---|---|---|
| 1 | 10 | | |
| | | 5 | |
| 2 | 15 | | 0 |
| | | 5 | |
| 3 | 20 | | 0 |
| | | 5 | |
| 4 | 25 | | 0 |
| | | 5 | |
| 5 | 30 | | 0 |
| | | 5 | |
| 6 | 35 | | |

Here we consider $x_0 = 1,\ x = 1.5, h = 1$

$$\text{So } u = \frac{x - x_0}{h} = 0.5$$

Now according to Newton's forward interpolation formula :

$$f(1.5) \approx 10 + \frac{0.5}{1} \times 5 = 10 + 2.5 = 12.5$$

**22. If $y(10) = 35.3,\ y(15) = 32.4,\ y(20) = 29.2,\ y(25) = 26.1,\ y(30) = 23.2$ and $y(35) = 20.5$ find $y(12)$ using Newton's forward interpolation formula.**

**[WBUT 2017(ODD)]**

**Answer:**

At first we construct the forward difference table from the following table:

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 10 | 35.3 | | | | |
| | | –2.9 | | | |
| 15 | 32.4 | | –0.3 | | |
| | | –3.2 | | + 0.4 | |
| 20 | 29.2 | | 0.1 | | –0.3 |
| | | –3.1 | | 0.1 | |
| 25 | 26.1 | | 0.2 | | |
| | | –2.9 | | 0.0 | |
| 30 | 23.2 | | 0.2 | | |
| | | –2.7 | | | |
| 35 | 20.5 | | | | |

Now let $x_0 = 1,\ h = 5 \quad \therefore u = \frac{x - x_0}{h} = 0.4$

According to Newton's Forward interpolation formula we get,

$$y(x)=y(x_0)+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$
$$+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0+.....$$

$$=35.3+0.4\times(-2.9)+\frac{0.4(0.4-1)}{2}\times(-0.3)$$
$$+\frac{0.4(0.4-1)(0.4-2)}{6}\times 0.4+\frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24}\times(-0.3)$$

$$=34.26$$

**23. Find the missing value in the following table:** **[WBUT 2018(EVEN)]**

| **X:** | **2** | **4** | **6** | **8** | **10** |
|---|---|---|---|---|---|
| **Y:** | **5.6** | **8.6** | **13.9** | **-** | **35.6** |

**Answer:**

Let $y(x=8)=a$

Now we construct the difference table as follows:

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 2 | 5.6 | | | |
| | | 3 | | |
| 4 | 8.6 | | 2.3 | |
| | | 5.3 | | $a-21.5$ |
| 6 | 13.9 | | $a-19.2$ | |
| | | $a-13.9$ | | $68.7-3a$ |
| 8 | $a$ | | $49.5-2a$ | |
| | | $35.6-a$ | | |
| 10 | 35.6 | | | |

As given no. of $x$-values is 5, so the approximate polynomial will be 4[th] degree and consequently 3[rd] order difference will be constant.

$\therefore \quad a-21.5=68.7-3a$

or, $\quad 4a=90.4$

or, $\quad a=22.6$

**24. Find $f(1.02)$ having given:** **[WBUT 2018(EVEN)]**

| $x$: | **1.00** | **1.10** | **1.20** | **1.30** |
|---|---|---|---|---|
| $f(x)$: | **0.8415** | **0.8912** | **0.9320** | **0.9636** |

**Answer:**

At first we construct the difference table as follows:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 1 | 0.8415 | | | |
| | | 0.0497 | | |
| 1.1 | 0.8912 | | −0.0089 | |
| | | 0.0408 | | +0.0003 |
| 1.2 | 0.9320 | | −0.0092 | |
| | | 0.0316 | | |
| 1.3 | 0.9636 | | | |

Let $x = 1.02,\ x_0 = 1.00,\ h = 0.1$

$$\therefore \quad u = \frac{x - x_0}{h} = \frac{1.02 - 1}{0.1} = 0.2$$

Now by Newton's forward interpolation formula we get,

$$f(x) \approx 0.8415 + 0.2 \times 0.0497 + \frac{0.2(0.2-1)}{2!} \times (-0.0089) + \frac{0.2(0.2-1)(0.2-2)}{3!} \times 0.0003$$
$$= 0.85217$$

**25. Evaluate the missing terms in the following table:** **[WBUT 2018(ODD)]**

| $x$: | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| $f(x)$: | 6 | 10 | ? | 17 | ? | 31 |

**Answer:**

Let $f(10) = a$ and $f(20) = b$

Now according to Newton's forward interpolation technique
We obtain:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0 | 6 | | | |
| | | 4 | | |
| 5 | 10 | | $a-14$ | |
| | | $a-10$ | | $41-3a$ |
| 10 | $a$ | | $27-2a$ | |
| | | $17-a$ | | $3a+b-61$ |
| 15 | 17 | | $-34+a+b$ | |
| | | $b-17$ | | $82-a-3b$ |
| 20 | $b$ | | $-2b+48$ | |
| | | $31-b$ | | |
| 25 | 31 | | | |

Since, four values of $f(x)$ have been given, so $3^{rd}$ order difference must be constant.

$\therefore \quad 41 - 3a = 3a + b - 61 \quad \Rightarrow \quad 6a + b = 102 \quad \ldots.(1)$

and $\quad 3a + b - 61 = 82 - a - 3b \quad \Rightarrow \quad 4a + 4b = 143 \quad \ldots.(2)$

Solving for $a$ and $b$ we get, $a = 13\frac{17}{23}$, $b = 19\frac{13}{23}$

$$\therefore \quad f(10) = 13\frac{17}{23},\ f(20) = 19\frac{13}{23}$$

**26. A table of data is given below for the equation of**

| $x$ | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 8 | 14 | 22 | 32 |

**Find the second order Newton Interpolating polynomial, and then find the function value at $x = 6$.** **[WBUT 2019(EVEN)]**

**Answer:**

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 1 | 4 | | | | |
| | | 1.33 | | | |
| 4 | 8 | | −0.13 | | |
| | | 1.2 | | 0.07 | |
| 9 | 14 | | −0.06 | | −0.04 |
| | | 1.14 | | 0.03 | |
| 16 | 22 | | −0.03 | | |
| | | 1.11 | | | |
| 25 | 32 | | | | |

$$\therefore \quad f(6) = 4 + (6-1)\times 1.33 + (6-1)(6-4)\times(-0.13) = 4 + 6.65 - 1.3 = 9.35$$

**27. Use the Newton's Divided Difference Formula to approximate $f(0.05)$ from the following table:** **[WBUT 2019(EVEN)]**

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|---|
| $f(x)$ | 1.00000 | 1.22140 | 1.49182 | 1.82212 | 2.22554 |

**Answer:**

| $x$ | : | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|---|---|
| $f(x)$ | : | 1.00000 | 1.22140 | 1.49182 | 1.82212 | 2.22554 |

Using Newton's divided difference formula, we can get,

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0.0 | 1.00000 | $\frac{1.22140-1.00000}{0.2-0.0}=1.107$ | $\frac{1.3521-1.107}{0.4-0.0}=0.61275$ | $\frac{0.7585-0.61275}{0.6-0.0}=0.22625$ |
| 0.2 | 1.22140 | $\frac{1.49182-1.22140}{0.4-0.2}=1.3521$ | $\frac{1.6515-1.3521}{0.6-0.2}=0.7485$ | $\frac{0.914-0.7485}{0.8-0.2}=0.275833$ |
| 0.4 | 1.49182 | $\frac{1.82212-1.49182}{0.6-0.4}=1.6515$ | $\frac{2.0171-1.6515}{0.8-0.4}=0.914$ | — |
| 0.6 | 1.82212 | $\frac{2.22554-1.82212}{0.8-0.6}=2.0171$ | — | |
| 0.8 | 2.22554 | — | | |

Therefore,

$$f(x)=f(x_0)+(x-x_0)\Delta f(x_0)+(x-x_0)(x-x_1)\Delta^2 f(x_0)+(x-x_0)(x-x_1)$$
$$(x-x_2)\Delta^3 f(x_0)$$
$$=1.00000+(x-0.0)\times 1.107+(x-0.0)(x-0.2)\times 0.61275+(x-0.0)$$
$$(x-0.2)(x-0.4)\times 0.22625$$
$$=0.22625x^3+0.61275x^2+1.107x-0.12255x-0.04525x-$$
$$0.0905x+1.00000$$
$$=0.22625x^3+0.8487x+0.61275x^2+1.00000$$

Putting the value $f(0.05)$, we can get,

$$=0.22625(0.05)^3+0.61275(0.05)^2+0.8487(0.05)+1.00000$$
$$=1.043958$$
$$f(0.05)=1.043958$$

## Long Answer Type Questions

**1. Derive Lagrange interpolation formula.** **[WBUT 2011(ODD), 2016(ODD)]**

**Answer:**

If $f(x_0)$, $f(x_1)$, ……, $f(x_n)$ be the values of a function $y = f(x)$ corresponding to the arguments $x_0$, $x_1$, ……, $x_n$ which are not necessarily equally spaced, then the Lagrange's interpolation polynomial is

$$L_n(x)=\sum_{r=0}^{n}\frac{\omega(x)f(x_r)}{(x-x_r)\omega'(x_r)}, \text{ ......(1) where}$$

$$\omega(x)=\prod_{r=0}^{n}(x-x_r), \text{ ......(1a) and}$$

$$\omega'(x_r)=(x_r-x_0)(x_r-x_1)......(x_r-x_{r-1})\times$$
$$(x_r-x_{r+1})......(x_r-x_n) \text{ ......(1b)}$$

***Proof***

Let us form the fundamental Lagrangian function

$$\omega_r(x)=\frac{(x-x_0)(x-x_1)......(x-x_{r-1})(x-x_{r+1})......(x-x_n)}{(x_r-x_0)(x_r-x_1)......(x_r-x_{r-1})(x_r-x_{r+1})......(x_r-x_n)} \text{ ......(2)}$$

$$=\frac{\omega(x)}{(x-x_r)\omega'(x_r)} \text{ ......(2a)}$$

The Lagrangian function $\omega_r(x)$ has the properties that

$$\omega_r(x_s)\begin{cases}=0, & if\, r\neq s\\ =1, & if\, r=s\end{cases} \text{ ......(2b)}$$

Let, $L_n(x) = \sum_r \omega_r(x) f(x_r)$ which is a polynomial of degree n such that

$L_n(x_s) = \sum_r \omega_r(x_s) f(x_r) = f(x_s)$

satisfying the condition ($I$) of interpolation.

Therefore, $L_n(x)$ is an interpolation polynomial which is unique.

**2. Define interpolation and extrapolation. Deduce the Newton's forward interpolation formula.** **[WBUT 2012(ODD)]**

**OR,**

**Derive the expression of Newton's forward interpolation formula where the function $f(x)$ is known for $n+1$ distinct equispaced arguments.**

**[WBUT 2017(EVEN)]**

**OR,**

**Derive Newton's forward interpolation formula.** **[WBUT 2017(ODD)]**

**OR,**

**What is Interpolation? Deduce the Forward Interpolation formula for $(n+1)$ given data points.** **[WBUT 2018(ODD)]**

**Answer:**

**1st Part:**

**Interpolation** is the method of finding an unknown intermediate value of a function from a given set of observed values of the said function.

If the unknown is not an intermediate value then the method by which we can estimate the value of the function is called **extrapolation.**

**2nd Part:**

***Statement:***

If $y_0, y_1, y_2, \ldots\ldots, y_n$ be the values of $y = f(x)$ corresponding to equidistant values of $x = x_0, x_1, \ldots\ldots, x_n$ where $x_r - x_{r-1} = h$, $r = 1(1)n$ (or, $x_r = x_0 + rh$), then

$$L_n(x) = f(x_0) + \frac{u}{1!}\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \ldots\ldots + \frac{u(u-1)\ldots\ldots(u-\overline{n-1})}{n!}\Delta^n f(x_0)$$

where $u = \dfrac{x - x_0}{n}$.

***Proof***

Let

$$L_n(x) = c_0 + c_1(x-x_0) + c_2(x-x_0)(x-x_1) + \ldots\ldots$$
$$+ c_n(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1}) \ \ldots\ldots(1)$$

When $x = x_0$, $y = f(x_0)$ and $L_n(x_0) = c_0$

From (1),

$$\Delta L_n(x) = c_1 + c_2.2h(x-x_0) + \ldots\ldots + c_n nh(x-x_0)\ldots\ldots(x-x_{n-2})$$

$$\Delta^2 L_n(x) = c_2.2!h^2 + \ldots\ldots + c_n n(n-1)h^2(x-x_0)\ldots\ldots(x-x_{n-3}).$$

............

$$\Delta^n L_n(x) = c_n n!h^n$$

Putting $x = x_0$ in the above $(n+1)$ equations and noting that $L_n(x_r) = f(x_r)$, we have

$$c_0 = L_n(x_0) = f(x_0),\ c_1 = \frac{\Delta L_n(x_0)}{1!h} = \frac{\Delta f(x_0)}{1!h}$$

$$c_2 = \frac{\Delta^2 L_n(x_0)}{2!h^2} = \frac{\Delta^2 f(x_0)}{2!h^2}, \ldots\ldots,\ c_n = \frac{\Delta^n L_n(x_0)}{n!h^n} = \frac{\Delta^n f(x_0)}{n!h^n}$$

$$\therefore L_n(x) = f(x_0) + \frac{1}{1!}\left(\frac{x-x_0}{h}\right)\Delta f(x_0) + \frac{1}{2!}\left(\frac{x-x_0}{h}\right)\left(\frac{x-x_1}{h}\right)\Delta^2 f(x_0)$$

$$+\ldots\ldots+\frac{1}{n!}\left(\frac{x-x_0}{h}\right)\left(\frac{x-x_1}{h}\right)\ldots\ldots\left(\frac{x-x_{n-1}}{h}\right)\Delta^n f(x_0) \ \ldots\ldots(2)$$

Now, $u = \dfrac{x-x_0}{h}$ and $\dfrac{x-x_r}{h} = u - r$ ......(3)

Then

$$L_n(x) = f(x_0) + \frac{u}{1!}\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0)$$

$$+\ldots\ldots+\frac{u(u-1)\ldots.(u-n+1)}{n!}\Delta^n f(x_0) \qquad \ldots.(4)$$

**3. Obtain the Newton's Divided Difference interpolating polynomial, hence find $f(3)$:** **[WBUT 2012(ODD), 2016(ODD)]**

| $x$: | 0 | 1 | 2 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1 | 14 | 15 | 5 | 6 | 19 |

**Answer:**

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ |
|---|---|---|---|---|---|---|
| 0 | 1 | | | | | |
| | | 13 | | | | |
| 1 | 14 | | $-\frac{13}{2}$ | | | |
| | | 1 | | $\frac{9}{8}$ | | |
| 2 | 15 | | $-2$ | | $\frac{1}{8}$ | |
| | | $-5$ | | $\frac{1}{2}$ | | $\frac{9}{40}$ |
| 4 | 5 | | 0 | | $\frac{7}{20}$ | |
| | | 1 | | $\frac{9}{4}$ | | |
| 5 | 6 | | 9 | | | |
| | | 13 | | | | |
| 6 | 19 | | | | | |

Using Newton's divided difference formula we get,

$$f(x)=1+(x-0)13+(x-0)(x-1)\left(-\frac{13}{2}\right)+(x-0)(x-1)(x-2)\frac{9}{8}$$

$$+(x-0)(x-1)(x-2)(x-4)\frac{1}{8}+(x-0)(x-1)(x-2)(x-4)(x-5)\frac{9}{40}$$

$$=1+13x+\left(\frac{-13}{2}\right)(x^2-x)+\frac{9}{8}(x^3-3x^2+2x)+\frac{1}{8}(x^3-3x^2+2x)(x-4)$$

$$+\frac{9}{40}(x^3-3x^2+2x)(x^2-9x+20)$$

$$=1+13x+\left(\frac{-13}{2}\right)(x^2-x)+\frac{9}{8}(x^3-3x^2+2x)$$

$$+\frac{1}{8}(x^4-7x^3+14x^2-8x)+\frac{9}{40}(x^5-12x^4+51x^3-78x^2+40x)$$

$$=\frac{9}{40}x^5-\frac{103}{40}x^4+\frac{469}{40}x^3-\frac{1027}{40}x^2+\frac{119}{4}x+1$$

$$\therefore f(3)=\frac{9}{40}(3)^5-\frac{103}{40}(3)^4+\frac{469}{40}(3)^3-\frac{1027}{40}(3)^2+\frac{119}{4}(3)+1=21.85$$

**4. What do you mean by interpolation?** **[WBUT 2013(EVEN)]**
**Derive Newton's backward interpolation formula. [WBUT 2013(EVEN), 2018(EVEN)]**
**Can you apply this formula for unequispaced interpolating points?** **[WBUT 2013(EVEN)]**

**Answer:**
**Interpolation:** Let the analytical formula representing the function $f(x)$ is unknown but the values of $f(x)$ are known for a given set of $n+1$ distinct values of $x$: $x_0, x_1, ......, x_n$. Suppose our problem is to find the value of $f(x)$ for a given value of $x$ in the vicinity of the above values of the argument. Since, the analytical form of $f(x)$ is not known, the precise value of $f(x)$ can't be obtained and we try to find an approximate value of the same by a function $\phi(x)$ of relatively simple known form which leads itself to ready calculation such that $f(x_i) = \phi(x_i)$, $i = 0, 1, 2, ......n$ and the computed value of $\phi(x)$ provides the required approximate value of $f(x)$ for the given value of $x$.

**Newton's Backward Interpolation Formula:**
Let the analytical formula of $f(x)$ is not known but the values of $f(x)$ are known for a given set of $n+1$ distinct values of $x: x_0, x_1, x_2 ......, x_n$, (where $x_{i+1} = x_i + h$).
Let us consider the interpolating polynomial is of the form

$$L_n(x) = C_0 + C_1(x - x_n) + C_2(x - x_n)(x - x_{n-1}) + ............ + C_n(x - x_n)(x - x_{n-1}).....(x - x_1)$$

$$\therefore \Delta L_n(x) = C_1 h + C_2 2h(x - x_{n-1}) + ....... + C_n\, nh(x - x_{n-1})(x - x_{n-2})......(x - x_1)$$

$$\Delta^2 L_n(x) = C_2 2!h^2 + C_3 3.2\, h^2(x - x_{n-2}) + .......$$
$$+ C_n\, n(n-1)h^2(x - x_{n-2})(x - x_{n-3}).........(x - x_1)$$

$$\vdots$$

$$\vdots$$

$$\Delta^n L_n(x) = C_n\, n!\, h^n$$

Since we know $L_n(x_i) = f(x_i)$, $i = 0, 1, 2, ........n$.
So substituting $x = x_n, x_{n-1}, ......x_1, x_0$ in the above relations we get,

$$C_0 = L_n(x_n) = f(x_n)$$

$$C_1 = \frac{\Delta L_n(x_{n-1})}{h} = \frac{\Delta f(x_{n-1})}{h}$$

$$C_2 = \frac{\Delta^2 L_n(x_{n-2})}{2!\, h^2} = \frac{\Delta^2 f(x_{n-2})}{2!\, h^2}$$

$$\vdots$$

$$\vdots$$

$$C_n = \frac{\Delta^n L_n(x_n)}{n!\, h^n} = \frac{\Delta^n f(x_0)}{n!\, h^n}$$

$$\therefore L_n(x) = f(x_n) + \frac{x - x_n}{h} \Delta f(x_{n-1}) + \frac{(x-x_n)(x-x_{n-1})}{2!\,h^2} \Delta^2 f(x_{n-2})$$

$$+\ldots\ldots + \frac{(x-x_n)(x-x_{n-1})\ldots\ldots(x-x_1)}{n!\,h^n} \Delta^n f(x_0)$$

Now if the consider $u = \dfrac{x - x_n}{h}$ and noting $u + i = \dfrac{x - x_{n-i}}{h}$

We get, $L_n(x) = f(x_n) + \dfrac{u}{1!} \Delta f(x_{n-1}) + \dfrac{u(u+1)}{2!} \Delta^2 f(x_{n-2}) + \ldots\ldots$

$$\ldots\ldots\ldots + \frac{u(u+1)(u+2)\ldots\ldots(u+n-1)}{n!} \Delta^n f(x_0)$$

which is the Newton's Backward interpolating polynomial for equispaced points.
No, this formula can't be applicable for unequispaced interpolating points.

**5. Compute $f(0.23)$ and $f(0.29)$ using suitable formula from the table given below:**

| $x$: | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |

**[WBUT 2013(ODD), 2015(ODD), 2016(EVEN), 2017(EVEN)]**

**Answer:**

Since all the $x$-values are equidistant, so to determine the values of $f(0.23)$ and $f(0.29)$ we apply Newton's forward and backward interpolation formulas respectively.
At first we construct the Newton's forward difference table as follows:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ |
|---|---|---|---|---|---|---|
| 0.20 | 1.6596 | | | | | |
| | | 0.0102 | | | | |
| 0.22 | 1.6698 | | 0.0004 | | | |
| | | 0.0106 | | $-0.0002$ | | |
| 0.24 | 1.6804 | | 0.0002 | | 0.0004 | |
| | | 0.0108 | | 0.0002 | | $-0.007$ |
| 0.26 | 1.6912 | | 0.0004 | | $-0.0003$ | |
| | | 0.0112 | | $-0.0001$ | | |
| 0.28 | 1.7024 | | 0.0003 | | | |
| | | 0.0115 | | | | |
| 0.30 | 1.7139 | | | | | |

Now, for $f(0.23)$;

Let $x_0 = 0.22$, $x = 0.23$, $h = 0.02$

$$\therefore u = \frac{x - x_0}{h} = \frac{0.23 - 0.22}{0.02} = 0.5$$

According to Newton's forward interpolation formula:

$$f(0.23) \approx f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(x_0) + .....$$

$$= 1.6698 + 0.5\times 0.0106 + \frac{0.5(0.5-1)}{2}\times 0.0002$$

$$+ \frac{0.5(0.5-1)(0.5-2)}{6}\times 0.0002 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24}\times(-0.0003)$$

$$= 1.6751 \quad \text{(Correct upto 4 decimal places)}$$

For $f(0.29)$: Let $x_n = 0.30,\quad x = 0.29,\quad h = 0.02$

$$\therefore u = \frac{x - x_n}{h} = \frac{0.29 - 0.30}{0.02} = -0.5$$

According to Newton's backward interpolation formula:

$$f(0.29) \approx f(x_n) + u\Delta f(x_{n-1}) + \frac{u(u+1)}{2!}\Delta^2 f(x_{n-2}) + \frac{u(u+1)(u+2)}{3!}\Delta^3 f(x_{n-3})$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 f(x_{n-4}) + ........$$

$$= 1.7139 + (-0.5)\times 0.0115 + \frac{(-0.5)(-0.5+1)}{2}\times 0.0003 + \frac{(-0.5)(-0.5+1)(-0.5+2)}{6}$$

$$\times(-0.001) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{24}\times(-0.0003)$$

$$= 1.7081 \quad \text{(Correct upto 4 decimal places)}$$

**6. a) What is interpolation? Prove that** **[WBUT 2013(ODD)]**

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + ...... + \frac{u(u-1)...(u-n+1)}{n!}\Delta^n y_0$$

**Answer:**

**Interpolation:** In the field of Numerical Analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points.

In some cases, it tries to approximate a function from a set of given data points. This is known as curve fitting or regression analysis.

Let us consider $y_0, y_1, \ldots y_n$ be the values of a function f(x) with respect to the values of x within equal intervals $(x_0, x_1, \ldots x_n)$ where $x_r - x_{r-1} = h$.

Let $L_n(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + .......c_n(x - x_0)(x - x_1).....(x - x_{n-1})$

where $x = x_0$, $y = f(x_0)$ and $L_n(x_0) = c_0$

From the above equation,

Putting $x=x_0$ in the above equations and considering that $L_n(x_r)=f(x_r)$, we get,

$$c_0 = L_n(x_0) = f(x_0), c_1 = \frac{\Delta L_n(x_0)}{1!h} = \frac{\Delta f(x_0)}{1!h}$$

$$c_n = \frac{\Delta^n L_n(x_0)}{n!hn} = \frac{\Delta^n f(x_0)}{n!hn}$$

$$\therefore L^n(x) = f(x_0) + \frac{1}{1!}(\frac{x-x_0}{h})\Delta f(x_0) + \frac{1}{2!}(\frac{x-x_0}{h})(\frac{x-x_1}{h})\Delta^2 f(x_0)$$

$$+ .... + \frac{1}{n!}(\frac{x-x_0}{h})(\frac{x-x_1}{h})...(\frac{x-x_n}{h})\Delta^n f(x0)$$

Now, $u=\frac{x-x0}{h}$ and $\frac{x-xr}{h}=u-r$

Then,

$$L_n(x) = f(x_0) + \frac{u\Delta f(x_0)}{1!} + \frac{u(u-1)\Delta^2 f(x_0)}{2!} + ...... + \frac{u(u-1)...(u-n+1)\Delta^n f(x_0)}{n!}$$

Or equivalently,

$$y = y_0 + u.\Delta y_0/1! + u(u-1)\Delta^2 y_0/2! + ......... + \{u(u-1)...(u-n+1)\}\Delta^{n-1} y_0/n!$$

This is called Newton-Gregory Forward Interpolation Formula.

**[WBUT 2013(ODD)]**

**b) Prove that** $\nabla^r y_k = \nabla^r y_{k+r}$.

**Answer:**

We know that $\Delta^k = E^k\nabla^k = \nabla^k E^k \forall k$.

Therefore,

$$\Delta^r y_k = \nabla^r E^r (yx)$$
$$= \nabla^r (E^r yk)$$
$$= \nabla^r y_{k+r}$$

**(proved)**

**[WBUT 2013(ODD)]**

**c) Find the missing term:**

| $x$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $f(x)$: | 2 | 4 | 8 | ? | 32 | 64 | 128 |

**Answer:**

$$\Delta^6 f(x) = 0$$
$$\Rightarrow (E-1)^6 f(x) = 0$$
$$\Rightarrow ({}^6C_0E^6 - {}^6C_1E^5 + {}^6C_2E^4 - {}^6C_3E^3 + {}^6C_4E^2 - {}^6C_5E + {}^6C_6.1) f(x) = 0$$
$$\Rightarrow (E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) f(x) = 0$$
$$\Rightarrow f(7) - 6f(6) + 15f(5) - 20f(4) + 15f(3) - 6f(2) + f(1) = 0$$
$$\Rightarrow 128 - 6\times64 + 15\times32 - 20f(4) + 15\times8 - 6\times4 + 2 = 0$$

$\Rightarrow 322 = 20f(4)$

$\Rightarrow f(4) = \frac{322}{20} = 16.1$

**7. What is the lowest degree polynomial which takes the following values?**

| $x$ | 0 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| $f(x)$ | 4 | 26 | 58 | 112 | 466 | 668 |

**Hence find $f(5)$.** **[WBUT 2014(ODD)]**

**Answer:**

Here we use divided difference interpolation formula

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0 | 4 | | | |
| | | 11 | | |
| 2 | 26 | | 7 | |
| | | 32 | | |
| 3 | 58 | | 11 | 1 |
| | | 54 | | |
| 4 | 112 | | 16 | 1 |
| | | 118 | | |
| 7 | 466 | | 21 | 1 |
| | | 202 | | |
| 8 | 668 | | | |

$$\therefore f(x) \approx f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$
$$+(x - x_0)(x - x_1)(x - x_2)f(x_0, x_1, x_2, x_3) + \ldots$$
$$= 4 + (x-0)\times 11 + (x-0)(x-2)\times 7 + (x-0)(x-2)(x-3)\times 1$$
$$= 4 + 11x + 7x(x-2) + x(x^2 - 5x + 6) = 4 + 11x + 7x^2 - 14x + x^3 - 5x^2 + 6x$$
$$= x^3 + 2x^2 + 3x + 4$$

which is the required lowest degree polynomial.

Now $f(5) = 194$.

**8. a) The following data represent the function $f(x) = e^x$.**

| $x$ | 1 | 1.5 | 2 | 2.5 |
|---|---|---|---|---|
| $y$ | 2.7183 | 4.4817 | 7.3891 | 12.1825 |

**Estimate the value of $f(1.25)$ using (i) Newton's forward difference interpolation (ii) Newton's backward difference interpolation.** **[WBUT 2014(ODD)]**

**Answer:**

At first we construct the Newton forward difference table:

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1 | 2.7183 | | | |
| | | 1.7634 | | |
| 1.5 | 4.4817 | | 1.144 | |
| | | 2.9074 | | 0.742 |
| 2 | 7.3891 | | 1.886 | |
| | | 4.7934 | | |
| 2.5 | 12.1825 | | | |

(i) $x=1.25,\ x_0=1 \qquad \therefore u=\dfrac{x-x_0}{0.5}=\dfrac{0.25}{.5}=0.5$

$$\therefore f(1.25)=2.7183+0.5\times1.7634+\frac{0.5(0.5-1)}{2}\times1.144+\frac{0.5(0.5-1)(0.5-2)}{6}\times0.742$$

$$=3.5034.$$

(ii) $x=1.25,\ x_n=2.5 \qquad u=\dfrac{x-x_n}{0.5}=\dfrac{1.25-2.5}{0.5}=-2.5$

$$\therefore f(1.25)=12.1825+(-2.5)\times4.7934+\frac{(-2.5)(-2.5+1)}{2}\times1.886$$

$$+\frac{(-2.5)(-2.5+1)(-2.5+2)}{6}\times0.742$$

$$=3.5034$$

**b) Prove the given relation: $\delta=\nabla\cdot E^{0.5}$.** **[WBUT 2014(ODD)]**

**Answer:**

We know, $\delta\equiv E^{1/2}-E^{-1/2}\equiv E^{1/2}(1-E^{-1})\equiv E^{1/2}\cdot\nabla\equiv\nabla\cdot E^{1/2}$ **(Proved)**

**9. a) Find the Lagrangian interpolation polynomial which agrees with the following data:** **[WBUT 2015(EVEN)]**

| $x$ (in radian) | 1.0 | 1.1 | 1.2 |
|---|---|---|---|
| $y=\cos x$ | 0.5403 | 0.4536 | 0.3624 |

**Answer:**

According to Lagrange's interpolation formula:

| | | | $D$ | $Y$ | $Y/D$ |
|---|---|---|---|---|---|
| $\underline{x-1}$ | $1.0-1.1$ | $1.0-1.2$ | $0.02(x-1)$ | 0.5403 | $\dfrac{27.015}{x-1}$ |
| $1.1-1.0$ | $\underline{x-1.1}$ | $1.1-1.2$ | $-0.01(x-1.1)$ | 0.4536 | $\dfrac{-45.36}{x-1.1}$ |
| $1.2-1.0$ | $1.2-1.1$ | $\underline{x-1.2}$ | $0.02(x-1.2)$ | 0.3624 | $\dfrac{18.12}{x-1.2}$ |

So, the required polonomial is

$$\sum \frac{Y_i}{D_i}(x-1)(x-1.2)(x-1.1) = 27.015(x-1.2)(x-1.1)$$

$$-45.36(x-1)(x-1.2)+18.12(x-1)(x-1.1)$$

**b) From the following table, find the interpolation polynomial by Newton forward interpolation formula:** **[WBUT 2015(EVEN)]**

| $x$: | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1 | 2 | 3 | 4 | 5 | 6 |

**Answer:**

At first construct the Newton's forward difference table as follows:

| | $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ |
|---|---|---|---|---|
| $x_0 \rightarrow$ | 1 | $\underline{1}$ | | |
| | | | $\underline{1}$ | |
| | 2 | 2 | | $\underline{0}$ |
| | | | 1 | |
| | 3 | 3 | | 0 |
| | | | 1 | |
| | 4 | 4 | | 0 |
| | | | 1 | |
| | 5 | 5 | | 0 |
| | | | 1 | |
| | 6 | 6 | | 0 |

According to Newton's forward interpolation formula:

$$f(x) \approx 1 + \frac{x-1}{1} \times 1 = 1 + (x-1) = x$$

**10. Using the Divided difference formula find $f(0.72)$ from the following table:**

| $x$: | 0.62 | 0.68 | 0.70 | 0.73 | 0.75 |
|---|---|---|---|---|---|
| $f(x)$: | 0.6604918 | 0.73363074 | 0.7585837 | 0.7965858 | 0.8223167 |

**[WBUT 2015(ODD)]**

**Answer:**

At first we construct the divided difference table:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 0.62 | 0.6604918 | | | | |
| | | 1.218982333 | | | |
| 0.68 | 0.73363074 | | 0.3583208375 | | |
| | | 1.247648 | | 0.2132045682 | |
| 0.70 | 0.7585837 | | 0.38177334 | | −1696.601209 |
| | | 1.266736667 | | −220.3449526 | |
| 0.73 | 0.7965858 | | −15.04237334 | | |
| | | 0.514618 | | | |
| 0.75 | 0.8223167 | | | | |

Now by divided difference interpolation formula:

$$
\begin{aligned}
f(0.72) &= 0.6604918+(0.72-0.62)\times 1.218982333\\
&+(0.72-0.62)(0.72-0.68)\times 0.3583208375\\
&+(0.72-0.62)(0.72-0.68)(0.72-0.70)\times 0.2132045682\\
&+(0.72-0.62)(0.72-0.68)(0.72-0.70)(0.72-0.73)\times(-1696.601209)\\
&=0.78519765
\end{aligned}
$$

**11. Find $f(0.9)$ using Newton's forward interpolation formula. [WBUT 2016(ODD)]**

| | | | | | | |
|---|---|---|---|---|---|---|
| **0** | **1** | **2** | **3** | **4** | **5** | **6** |
| **0.2536** | **1.0245** | **2.0145** | **0.2547** | **3.0145** | **1.0125** | **2.01245** |

**Answer:**

At first we construct Newton's forward interpolation table:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ | $\Delta^5 f(x)$ | $\Delta^6 f(x)$ |
|---|---|---|---|---|---|---|---|
| 0 | <u>0.2536</u> | | | | | | |
| | | <u>0.7709</u> | | | | | |
| 1 | 1.0245 | | <u>0.2191</u> | | | | |
| | | 0.99 | | <u>−2.9689</u> | | | |
| 2 | 2.0145 | | −2.7498 | | <u>10.2383</u> | | |
| | | −1.7598 | | 7.2694 | | <u>−26.7891</u> | |
| 3 | 0.2547 | | 4.5196 | | −16.5508 | | <u>60.38505</u> |
| | | 2.7598 | | −9.2814 | | 33.59595 | |
| 4 | 3.0145 | | −4.7618 | | 17.04515 | | |
| | | −2.002 | | 7.76375 | | | |
| 5 | 1.0125 | | 3.00195 | | | | |
| | | 0.99995 | | | | | |
| 6 | 2.01245 | | | | | | |

Now according to Newton's forward interpolation method $x=0.9$, $x_0=0$ and $u=\dfrac{x-x_0}{h}=\dfrac{0.9}{1}=0.9$

$$
\begin{aligned}
\therefore\quad f(0.9) &\approx 0.2536+0.9\times 0.7709+\frac{0.9(0.9-1)}{2}\times 0.2191\\
&+\frac{0.9(0.9-1)(0.9-2)}{6}\times(-2.9689)+\frac{0.9(0.9-1)(0.9-2)(0.9-3)}{24}\times(10.2383)\\
&+\frac{0.9(0.9-1)(0.9-2)(0.9-3)(0.9-4)(0.9-5)}{720}\times 60.38505\\
&=0.4344
\end{aligned}
$$

**12. Find $f(8)$ using Newton's divided difference formula given that [WBUT 2017(EVEN)]**

| $x$: | **4** | **5** | **7** | **10** | **11** | **13** |
|---|---|---|---|---|---|---|
| $f(x)$: | **48** | **100** | **294** | **900** | **1210** | **2028** |

**Answer:**

Divided difference table:

| | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 4 | 48 | | | |
| | | 52 | | |
| 5 | 100 | | 15 | |
| | | 97 | | 1 |
| 7 | 294 | | 21 | |
| | | 202 | | 1 |
| 10 | 900 | | 27 | |
| | | 310 | | 1 |
| 11 | 1210 | | 33 | |
| | | 409 | | |
| 13 | 2028 | | | |

Now according to the divided difference interpolation formula:

$$f(8) \approx 48 + (8-4)\times 52 + (8-4)(8-5)\times 15 + (8-4)(8-5)(8-7)\times 1 = 448$$

**13. Form the interpolation polynomial for the function $y = f(x)$ given by the table:**

**[WBUT 2017(ODD)]**

| $x$ | $-1$ | 1 | 4 | 6 |
|---|---|---|---|---|
| $y$ | 1 | $-3$ | 21 | 127 |

**Answer:**

To construct the interpolating polynomial, have we apply Lagrange's interpolation method as follows:

$$f(x) \approx \frac{(x-1)(x-4)(x-6)}{(-1-1)(-1-4)(-1-6)}\times 1 + \frac{(x+1)(x-4)(x-6)}{(1+1)(1-4)(1-6)}\times(-3)$$

$$+\frac{(x+1)(x-1)(x-6)}{(4+1)(4-1)(4-6)}\times 21 + \frac{(x+1)(x-1)(x-4)}{(6+1)(6-1)(6-4)}\times 127$$

$$= \frac{-1}{70}(x-1)(x^2-10x+24) - \frac{1}{10}(x+1)(x^2-10x+24)$$

$$-\frac{21}{30}(x+1)(x^2-7x+6) + \frac{127}{70}(x^2-1)(x-4)$$

$$= \frac{-1}{70}(x^3-11x^2+34x-24) - \frac{1}{10}(x^3-9x^2+14x+24)$$

$$-\frac{7}{10}(x^3-6x^2-x+6) + \frac{127}{70}(x^3-4x^2-x+4)$$

$$= x^3-2x^2-3x+1$$

**14. Find the value of $x$ corresponding $y=12$ using Lagrange's technique from the table:** **[WBUT 2018(EVEN)]**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 1.2 | 2.1 | 2.8 | 4.1 | 4.9 | 6.2 |
| $y$ | 4.2 | 6.8 | 9.8 | 13.4 | 15.5 | 19.6 |

**Answer:**
According to Lagrange's interpolation technique

$$x(y=12)=\frac{(12-6.8)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-15.5)(4.2-19.6)}\times 1.2$$
$$+\frac{(12-4.2)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-15.5)(6.8-19.6)}\times 2.1$$
$$+\frac{(12-4.2)(12-6.8)(12-13.4)(12-15.5)(12-19.6)}{(9.8-4.2)(9.8-6.8)(9.8-13.4)(9.8-15.5)(9.8-19.6)}\times 2.8$$
$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-15.5)(12-19.6)}{(13.4-4.2)(13.4-6.8)(13.4-9.8)(13.4-15.5)(13.4-19.6)}\times 4.1$$
$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-19.6)}{(15.5-4.2)(15.5-6.8)(15.5-9.8)(15.5-13.4)(15.5-19.6)}\times 4.9$$
$$+\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-15.5)}{(19.6-4.2)(19.6-6.8)(19.6-9.8)(19.6-13.4)(19.6-15.5)}\times 6.2$$
$$=0.021932-0.2341+1.25185+3.41933-0.964234+0.55205$$
$$=4.046828$$

**15. Fit a polynomial to the following table of values using Lagrange interpolation:** **[WBUT 2018(EVEN)]**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 3 | 4 |
| $y$ | –12 | 0 | 6 | 12 |

**Find the value of $y$ when**

(i) $x=2$
(ii) $x=3.5$

**Answer:**
According to Lagrange's interpolation formula

$$f(x)=\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}\times(-12)+\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}\times 0$$
$$+\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}\times 6+\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}\times 12$$
$$=2(x-1)(x^2-5x+6)-3x(x^2-4x+3)+6x(x^2-3x+2)$$
$$=5x^3-18x^2+25x-12$$

$$\therefore \quad f(2)=5(2)^3-18(2)^2+25(2)-12=6$$
$$f(3.5)=5(3.5)^3-18(3.5)^2+25(3.5)-12=69.375$$

**16. Write short note on Lagrange's Interpolation.** **[WBUT 2015(EVEN)]**

**Answer:**

**Lagrange's Interpolation:**

Let $y=f(x)$ be defined in a given interval $[x_0, x_n]$ and known only $(n+1)$ distinct points $x_0<x_1<x_2......<x_n$ which are not equispaced in general.

Now we assume $L_n(x)$ be the corresponding interpolation polynomial such that

$$L_n(x)=\sum_{j=0}^{n} L_j(x)y(x_j) \quad .... (1)$$

where, $y(x_j)=f(x_j)$ in which $L_j(x)$ are called the Lagrangian coefficients and they will be at most $n^{th}$ degree polynomials.

Since $L_j(x_j)=y(x_j)$ .... (2)

$\therefore \quad L_j(x_i)=\delta_{ij}$

Let $\quad L_j(x)=C_j(x-x_0)(x-x_1)(x-x_2)......(x-x_n)$

Now using condition (2) we get,

$$C_j=\frac{1}{\omega'(x_j)} \quad \text{where} \quad \omega(x)=(x-x_0)(x-x_1)......(x-x_n)$$

$$\therefore \quad L_j(x)=\frac{\omega(x)}{(x-x_j)\omega'(x_j)}$$

So, Lagrange's interpolating polynomial is

$$L_n(x)=\sum_{j=0}^{n}\frac{\omega(x)}{(x-x_j)\omega'(x_j)}y(x_j).$$

**17. The following data give the melting point of an alloy of zinc and lead. $\theta$ is the temperature and $x$ is the percentage of lead. Using a suitable interpolation formula, find $\theta$ when $x=48$.** **[WBUT 2018(ODD)]**

| $x$ | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| $\theta$ | 184 | 204 | 226 | 250 | 276 | 304 |

**Answer:**

Newton's forward difference table:

| $x$ | $\theta$ | $\Delta\theta$ | $\Delta^2\theta$ |
|---|---|---|---|
| 40 | 184 | | |
| | | 20 | |
| 50 | 204 | | 2 |
| | | 22 | |
| 60 | 226 | | 2 |
| | | 24 | |
| 70 | 250 | | 2 |
| | | 26 | |
| 80 | 276 | | 2 |
| | | 28 | |
| 90 | 304 | | |

Here we adopt Newton's forward interpolation formula taking

$$x_0 = 40,\ h = 10,\ x = 48,\ u = \frac{x - x_0}{h} = \frac{48-40}{10} = 0.8$$

$$\therefore\ \theta\Big|_{\text{at } x=48} = 184 + 0.8 \times 20 + \frac{0.8(0.8-1)}{2} \times 2 = 199.84$$

**18. Given the following data, find $\log_{10} 656$.** **[WBUT 2018(ODD)]**

| $x$ | 654 | 658 | 659 |
|---|---|---|---|
| $\log_{10} x$ | 2.8156 | 2.8182 | 2.8189 |

**Answer:**

Let $f(x) = \log_{10} x$

Using Lagrange's interpolation method we get;

$$f(656) = \frac{(656-658)(656-659)}{(654-658)(654-659)} \times 2.8156 + \frac{(656-654)(656-659)}{(658-654)(658-659)} \times 2.8182 + \frac{(656-654)(656-658)}{(659-654)(659-658)} \times 2.8189$$

$$= 2.81688$$

**19. Find $f(2)$ using Newton's divided difference interpolation method, having given:** **[WBUT 2018(ODD)]**

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

**Answer:**

Newton's Divided difference table:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 4 | 48 | | | |
| | | 52 | | |
| 5 | 100 | | 15 | |
| | | 97 | | 1 |
| 7 | 294 | | 21 | |
| | | 202 | | 1 |
| 10 | 900 | | 27 | |
| | | 310 | | 1 |
| 11 | 1210 | | 33 | |
| | | 409 | | |
| 13 | 2028 | | | |

$$\therefore \quad f(2) = 48 + (2-4)\times 52 + (2-4)\times(2-5)\times 15 + (2-4)\times(2-5)\times(2-7)\times 1$$
$$= 48 - 2\times 52 + 2\times 3\times 15 - 2\times 3\times 5$$
$$= 4$$

**20. Using Newton divided difference formula, find $\log_{10} x$ from the following table:** **[WBUT 2018(ODD)]**

| $x$ | 300 | 304 | 305 | 307 |
|---|---|---|---|---|
| $f(x)$ | 2.4771 | 2.4829 | 2.4843 | 2.4871 |

**Answer:**

Newton's Divided difference table:

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 300 | 2.4771 | | | |
| | | 0.00145 | | |
| 304 | 2.4829 | | –0.00001 | |
| | | 0.00140 | | 0.000001428 |
| 305 | 2.4843 | | 0 | |
| | | 0.00140 | | |
| 307 | 2.4871 | | | |

$$\therefore \quad f(x) \approx 2.4771 + (x-300)\times 0.00145 + (x-300)(x-304)\times(-0.00001)$$
$$+ (x-300)(x-304)(x-305)\times 0.000001428$$

**21. A curve passes through the points as given in the following table. Find the area bounded by the curve $x$-axis, $x=1$ and $x=3$.** **[WBUT 2018(ODD)]**

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $Y$ | 0.2 | 0.7 | 1 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 |

**Answer:**
At first we find the interpolating polynomial passing through the given points by using Newton's forward interpolation formula as follows:

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ |
|---|---|---|---|---|---|---|
| 1 | 0.2 | | | | | |
| | | 0.5 | | | | |
| | | | –0.2 | | | |
| 2 | 0.7 | | | | | |
| | | 0.3 | | 0.2 | | |
| | | | 0 | | –0.3 | |
| 3 | 1 | | | | | 0.5 |
| | | 0.3 | | –0.1 | | |
| | | | –0.1 | | 0.2 | |
| 4 | 1.3 | | | | | –0.3 |
| | | 0.2 | | 0.1 | | |
| | | | 0 | | –0.1 | |
| 5 | 1.5 | | | | | 0.1 |
| | | 0.2 | | 0 | | |
| | | | 0 | | 0 | |
| 6 | 1.7 | | | | | 0 |
| | | 0.2 | | 0 | | |
| | | | 0 | | 0 | |
| 7 | 1.9 | | | | | |
| | | 0.2 | | 0 Noise Level | | |
| | | | 0 | | | |
| 8 | 2.1 | | | | | |
| | | 0.2 | | | | |
| 9 | 2.3 | | | | | |

$$\therefore \quad y = 0.2 + (x-1)\times 0.5 + \frac{(x-1)(x-2)}{2}\times(-0.2)$$

$$= 0.2 + \frac{(x-1)}{2} - \frac{x^2-3x+2}{10} = -\frac{x^2}{10} + \frac{8x}{10} - \frac{1}{2}$$

$$\therefore \int_1^1 \left(-\frac{x^2}{10}+\frac{8x}{10}-\frac{1}{2}\right)dx = \left[-\frac{x^3}{30}-\frac{8x^2}{20}-\frac{x}{2}\right]_0^1 = -\frac{1}{30}+\frac{8}{20}-\frac{1}{2} = \frac{-1+40-15}{30} = \frac{24}{30} \text{ sq. unit}$$

Verify by Simpson's $\frac{1}{3}$rd rule:

$$\int_1^3 y\,dx = \frac{1}{3}[0.2+1+4\times 0.7] = \frac{1}{3}[0.2+1+2.8] = \frac{4}{3} \text{ sq. unit}$$

**22. The table below gives the values of $\tan x$ for $0.10 \le x \le 0.30$;**

| $x$ | $\tan x$ |
|---|---|
| **0.10** | **0.1003** |
| **0.15** | **0.1511** |
| **0.20** | **0.2027** |
| **0.25** | **0.2553** |
| **0.30** | **0.3093** |

Find $\tan 0.12$ **and** $\tan 0.26$. **[WBUT 2019(EVEN)]**

**Answer:**

$$u=\frac{x-x_0}{h}=\frac{0.12-0.10}{0.05}=+0.4$$

| $x$ | $y=\tan x$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 0.10 | 0.1003 | | | | |
| | | 0.0508 | | | |
| 0.15 | 0.1511 | | 0.0008 | | |
| | | 0.0516 | | 0.0002 | |
| 0.20 | 0.2027 | | 0.001 | | 0.0002 |
| | | 0.0526 | | 0.0004 | |
| 0.25 | 0.2553 | | 0.0014 | | |
| | | 0.054 | | | |
| 0.30 | 0.3093 | | | | |

$$y(0.12)=0.1003+(0.4)(0.0508)+\frac{0.4(0.4-1)}{2}\times(0.0008)$$
$$+\frac{0.4(0.4-1)(0.4-2)}{6}\times(0.02)+\frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24}\times(0.0002)$$
$$=0.1205$$

$$v=\frac{x-x_n}{h}=\frac{0.26-0.30}{0.05}=-0.4$$

$$y(0.26)=0.3093+(-0.4)(0.054)+\frac{(-0.4)(-0.4+1)}{2}\times 0.0014$$
$$+\frac{(-0.4)(-0.4+1)(-0.4+2)}{6}\times(0.0004)$$
$$+\frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{24}\times(0.0002)$$
$$=0.2876$$

**23. a) Find the value of $Y$ at $X=21$ using Lagrange Interpolating Polynomial from the following table:** **[WBUT 2019(EVEN)]**

| $X$ | 20 | 23 | 26 | 29 |
|---|---|---|---|---|
| $Y$ | 0.3420 | 0.3907 | 0.4384 | 0.4848 |

**Answer:**

By Lagrange's interpolation formula, we can get

$$X(21)=\left[\frac{(21-23)(21-26)(21-29)}{(20-23)(20-26)(20-29)}\times 0.3420\right]+\left[\frac{(21-20)(21-26)(21-29)}{(23-20)(23-26)(23-29)}\times 0.3907\right]$$
$$+\left[\frac{(21-20)(21-23)(21-29)}{(26-20)(26-23)(26-29)}\times 0.4384\right]+\left[\frac{(21-20)(21-23)(21-26)}{(29-20)(29-23)(29-26)}\times 0.4848\right]$$

$= 0.1688 + 0.2894 - 0.12989 + 0.02992$
$= 0.35823$

$\therefore$ The value of $X(21)$ is 0.35823

b) Show that $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$ **[WBUT 2019(EVEN)]**

Answer: ***Refer to Question No. 1(a) of Short Answer Type Questions.***

c) Prove that $(1+\nabla)(1-\Delta) = 1$ **[WBUT 2019(EVEN)]**

Answer: ***Refer to Question No. 7 of Short Answer Type Questions.***

# NUMERICAL INTEGRATION

## Multiple Choice Type Questions

1. The inherent error for Weddle's rule of integration is as (the notations have their usual meanings) [WBUT 2008, 2015(ODD)]

a) $-\frac{nh^5}{180}f^{iv}(x_0)$ b) $-\frac{nh^7}{180}f^{vi}(x_0)$ c) $-\frac{nh^7}{140}f^{vi}(x_0)$ d) none of these

**Answer:** (c)

2. The degree of precision of Simpson's one-third rule is
[WBUT 2009, 2012(ODD), 2014(EVEN), 2014(ODD), 2017(ODD), 2018(EVEN)]

a) 1 b) 2 c) 3 d) 5

**Answer:** (c)

3. In Simpson's 1/3 rule of finding $\int_a^b f(x)dx$, $f(x)$ is approximated by
[WBUT 2010, 2013(ODD)]

a) line segment b) parabola c) circular sector d) part of ellipse

**Answer:** (b)

4. In Trapezoidal rule, the portion of curve is replaced by
[WBUT 2013(EVEN), 2015(ODD)]

a) straight line b) circular path c) parabolic path d) none of these

**Answer:** (a)

5. Simpson's one-third rule is applicable only when the number of sub-intervals is
[WBUT 2013(ODD), 2016(EVEN)]

a) even b) odd c) both even & odd d) none of these

**Answer:** (a)

6. Simpson's $1/3^{rd}$ formula always requires [WBUT 2015(EVEN), 2019(EVEN)]

a) even number of ordinates
b) odd number of ordinates
c) even or odd number of ordinates
d) none of these

**Answer:** (b)

7. The error in the Simpson's $1/3^{rd}$ rule is of order [WBUT 2015(ODD)]

a) $h$ b) $h^2$ c) $h^3$ d) $h^4$

**Answer:** (d)

8. Trapezoidal rule can be applied if the number of equal sub-intervals of the intervals of integration is [WBUT 2016(ODD)]

a) odd
b) even
c) both (a) and (b)
d) none of these

**Answer:** (c)

9. The degree of precision of Simpson's 1/3rd rule is [WBUT 2017(EVEN)]
a) 1 b) 2 c) 3 d) none of these

Answer: (c)

10. In Trapezoidal rule, the portion of curve is replaced by [WBUT 2017(EVEN)]
a) straight line b) parabolic path
c) circular path d) none of these

Answer: (a)

11. The error in the Weddle's Rule is of order [WBUT 2018(EVEN)]
a) $h^4$ b) $h^5$ c) $h^6$ d) $h^7$

Answer: (c)

12. In evaluating $\int_a^b f(x)\,dx$, the error in Simpson's one-third rule is of order [WBUT 2018(ODD)]
a) $h^2$ b) $h^3$ c) $h^4$ d) None of these

Answer: (c)

13. The degree of precision of trapezoidal rule is [WBUT 2018(ODD)]
a) 1 b) 2 c) 3 d) None of these

Answer: (a)

## Short Answer Type Questions

1. What do you mean by geometrical interpretation of Simpson's $\frac{1}{3}$ rd rule? [WBUT 2008, 2014(EVEN)]

OR,

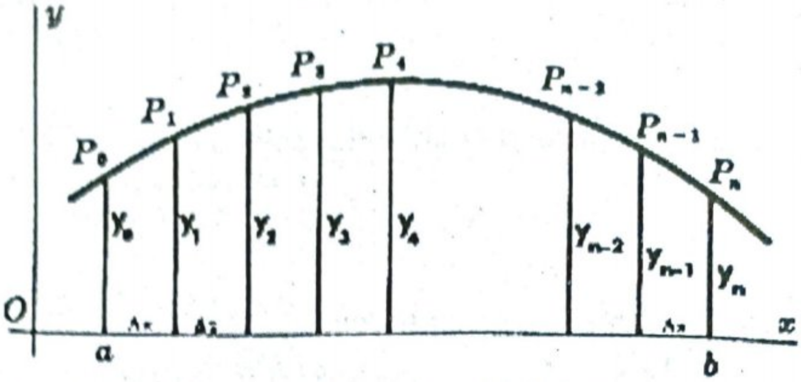
Describe geometric significance of Simpson's $\frac{1}{3}$ rd rule. [WBUT 2013(ODD), 2015(ODD)]

Answer:

Divide the interval from $x = a$ to $x = b$ into n equal subintervals of length $\Delta x = (b-a)/n$ where n must be an even number, 2. Erect ordinates $y_0, y_1, y_2, \ldots, y_n$ as shown in Fig. Simpson's Rule states

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + \ldots) + 2(y_2 + y_4 + y_6 + \ldots) + y_n\right]$$

where the number of intervals must be even.




Simpson's Rule is obtained by using the prismoidal formula to approximate the area under each of the arcs $P_0P_1P_2$, $P_2P_3P_4$, ...., $P_{n-2}P_{n-1}P_n$ and then adding together the results.

Thus $A_s = \frac{\Delta x}{3}\left[(y_0+4y_1+y_2)+(y_2+4y_3+y_4)+\ldots+(y_{n-2}+4y_{n-1}+y_n)\right]$

or, $A_s = \frac{\Delta x}{3}\left[y_0+4y_1+2y_2+4y_3+2y_4+\ldots+4y_{n-1}+y_n\right]$

From previous considerations it is clear that this formula will give the value of the integral exactly if $f(x)$ is a polynomial in x of degree not higher than 3.

**2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$rd rule taking $n=6$. Hence find the value of $\pi$**

**[WBUT 2013(EVEN)]**

**OR,**

**Calculate by Simpson's 1/3rd Rule the value of $\int_{1.2}^{1.6}\left(x+\frac{1}{x}\right)dx$, correct up to two significant figures, taking four intervals.** **[WBUT 2017(ODD)]**

**Answer:**

Let $f(x)=\frac{1}{1+x^2}$

If the given integration is of the form $\int_a^b f(x)dx$, then $a=0,\ b=1$

$\therefore\ h=\frac{b-a}{n}=\frac{1-0}{6}=\frac{1}{6}$

| $x$ | 0 | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{5}{6}$ | 1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.973 | 0.9 | 0.8 | 0.692 | 0.590 | 0.5 |

Now according to Simpson's $\frac{1}{3}$ rd rule

$\int_0^1 f(x)dx=\frac{h}{3}[1+0.5+4(0.973+0.8+0.59)+2(0.9+0.692)]=0.785$

Again, $\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_{x=0}^1 = \frac{\pi}{4}$

$\therefore \frac{\pi}{4} = 0.785$ or, $\pi = 3.140$ (Approx)

**3. Evaluate $\int_0^1 \cos x dx$, taking five equal intervals. Explain the reason behind your choice of integration formula used.** **[WBUT 2013(ODD)]**

**Answer:**

Since the number of intervals is 5, which is an odd number, so we use Trapezoidal rule to evaluate the given integral numerically.

Let, $f(x) = \cos x$

Here, $n = 5$ (number of intervals)

So, $h = \frac{1-0}{5} = 0.2$ (step length)

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.98007 | 0.92106 | 0.82534 | 0.69671 | 0.54030 |

Hence by Trapezoidal rule:

$$\int_0^1 \cos x dx = \frac{h}{2}[f(0) + f(1) + 2\{f(0.2) + f(0.4) + f(0.6) + f(0.8)\}]$$

$$= \frac{0.2}{2}[1 + 0.54030 + 2(0.98007 + 0.92106 + 0.82534 + 0.69671)]$$

$= 0.8387$ (correct up to 4 decimal places)

**4. Evaluate $\int \frac{x}{\sin x} dx$, where the interval is (0, 2) by using Trapezoidal rule taking $n = 8$.** **[WBUT 2014(ODD)]**

**Answer:**

Let, $f(x) = \frac{x}{\sin x}$

| $x$ | 0 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 1.0105 | 1.0429 | 1.10029 | 1.1884 | 1.3172 | 1.5038 | 1.7785 | 2.1995 |

So, according to Trapezoidal Rule:

$$\int_0^2 f(x)dx = \frac{0.25}{2}[1 + 2.1995 + 2\{1.0105 + 1.0429 + 1.1003 + 1.1884 + 1.3172 + 1.5038 + 1.7785]$$

$= 2.6353$ (Correct up to 4 decimal places)

**5. Find the area bounded by the curve and x-axis from $x = 7.47$ to $x = 7.52$ giving reason and the rule of integration applicable in this case.** **[WBUT 2015(EVEN)]**

| $x$: | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.93 | 1.95 | 1.98 | 2.01 | 2.03 | 2.06 |

**Answer:**

The points are taken in equal intervals and even no. of points are taken. Hence we can apply Trapezoidal rule to find the area. To evaluate the integration for the bounded area, we apply Trapezoidal rule as follows:

$$\int_{7.47}^{7.52} f(x)dx = \frac{0.01}{2}\left[f(7.47)+f(7.52)+2\{f(7.48)+f(7.49)+f(7.50)+f(7.51)\}\right]$$

$$= \frac{0.01}{2}\left[1.93+2.06+2(1.95+1.98+2.01+2.03)\right] = 0.09965 \approx 0.10$$

**6. Evaluate $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's 1/3rd rule with $n = 4$.** **[WBUT 2015(EVEN)]**

**Answer:**

$$f(x) = \frac{x^2}{1+x^3}, h = 0.25$$

$$\begin{aligned} \therefore x_0 &= 0 & f(x_0) &= 0 \\ x_1 &= 0.25 & f(x_1) &= 0.06154 \\ x_2 &= 0.50 & f(x_2) &= 0.22222 \\ x_3 &= 0.75 & f(x_3) &= 0.395604 \\ x_4 &= 1.00 & f(x_4) &= 0.500000 \end{aligned}$$

$$S_{\frac{1}{3}} = \frac{h}{3}\left[\{f(x_0)+f(x_4)\}+4\{f(x_1)+f(x_3)\}+2.f(x_2)\right]$$

$$= \frac{0.25}{3}\left[0.50000+4(0.06154+0.395604)+2\times 0.22222\right]$$

$= 0.23108$ correct upto five places of decimal.

Now,

$$\int_0^1 \frac{x^2dx}{1+x^3} = \frac{1}{3}\log(1+x^3)\Big|_0^1 = \frac{1}{3}\log 2$$

$$\therefore S_{\frac{1}{3}} = \log 2^{\frac{1}{3}} \approx 0.23108$$

**7. Compute the value of $\pi$ from the formula $\frac{\pi}{4}=\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3rd rule with 10 sub-intervals.** **[WBUT 2015(ODD)]**

**Answer:**

Let $f(x)=\frac{1}{1+x^2}$; $a=0, b=1, n=10$

$\therefore \quad h=\frac{b-a}{n}=\frac{1}{10}=0.1$

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.990 | 0.962 | 0.917 | 0.862 | 0.8 | 0.735 | 0.671 | 0.610 | 0.552 | 0.5 |

Now according to Simpson's 1/3rd rule

$$\int_0^1 f(x)\,dx=\frac{0.1}{3}\left[1+0.5+4\{0.99+0.917+0.8+0.671+0.552\}\right.$$
$$\left.+2\{0.962+0.862+0.735+0.610\}\right]$$
$=0.785$ (Correct up to 3 decimal places)

$\therefore \quad \pi=4\times 0.785=3.141$

**8. Calculate by Simpson's one third rule the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$ corrected up to three significant figures.** **[WBUT 2016(EVEN), 2019(EVEN)]**

**Answer:**

Let, $f(x)=\frac{x}{1+x}$. Here $a=0,\ b=1,\ h=\frac{b-a}{n}$

Without any loss of generality, we consider $n=10$

$\therefore \quad h=\frac{1}{10}=0.1$

Now,

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | $\frac{1}{11}$ | $\frac{1}{6}$ | $\frac{3}{13}$ | $\frac{2}{7}$ | $\frac{1}{3}$ | $\frac{3}{8}$ | $\frac{7}{17}$ | $\frac{4}{9}$ | $\frac{9}{19}$ | $\frac{1}{2}$ |

So, according to Simpson's $\frac{1}{3}$rd rule:

$$\int_0^1 f(x)\,dx=\frac{0.1}{3}\left[0+\frac{1}{2}+4\left(\frac{1}{11}+\frac{3}{13}+\frac{1}{3}+\frac{7}{17}+\frac{9}{19}\right)+2\left(\frac{1}{6}+\frac{2}{7}+\frac{3}{8}+\frac{4}{9}\right)\right]=0.30685$$

$\approx 0.307$ (Correct upto 3 significant figures)

**9. Use Simpson's 1/3 rd rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal sub-intervals of [0, 6], correct to three decimal places.** **[WBUT 2016(ODD)]**

**Answer:**

Let $f(x)=\frac{1}{(1+x)^2}$. Length of each subinterval is $h=\frac{6-0}{6}=1$.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | $\frac{1}{4}$ | $\frac{1}{9}$ | $\frac{1}{16}$ | $\frac{1}{25}$ | $\frac{1}{36}$ | $\frac{1}{49}$ |

Now according to Simpson's $\frac{1}{3}$rd rule

$$\int_0^6 f(x)\,dx=\frac{1}{3}\left[1+\frac{1}{49}+4\left(\frac{1}{4}+\frac{1}{16}+\frac{1}{36}\right)+2\left(\frac{1}{9}+\frac{1}{25}\right)\right]$$
$$=0.895 \quad \text{(Correct up to 3 decimal places)}$$

**10. Evaluate the approximate value of $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ rd rule for 4 sub-interval, correct up to 4 decimal places.** **[WBUT 2017(EVEN)]**

**Answer:**

Let $f(x)=\frac{1}{1+x^2}$ ;

$$h=\frac{1-0}{4}=0.25$$

$\therefore$

| $x$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | $\frac{16}{17}$ | $\frac{4}{5}$ | $\frac{16}{25}$ | $\frac{1}{2}$ |

So by Simpson's $\frac{1}{3}$rd rule we obtain;

$$\int_0^1 f(x)\,dx=\frac{0.25}{3}\left[1+\frac{1}{2}+4\left(\frac{16}{17}+\frac{16}{25}\right)+2\times\frac{4}{5}\right]=0.2618 \text{ (Correct up to 4 decimal places).}$$

**11. Evaluate approximately, by Trapezoidal rule, the integral $\int_0^1 (4x-3x^2)\,dx$, by taking $n=10$.** **[WBUT 2017(ODD)]**

**OR,**

**Evaluate $\int_0^1 (4x-3x^2)\,dx$ by Trapezoidal rule taking 10 subintervals. Compute the exact value and find the absolute and relative errors in your result.** **[WBUT 2019(EVEN)]**

**Answer:**

Here the length of the interval is $h=\frac{1-0}{10}=0.1$

Let $f(x)=4x-3x^2$

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | $\frac{37}{100}$ | $\frac{17}{25}$ | $\frac{93}{100}$ | $\frac{28}{25}$ | $\frac{5}{4}$ | $\frac{33}{25}$ | $\frac{133}{25}$ | $\frac{32}{25}$ | $\frac{117}{100}$ | 1 |

Now by Trapezoidal Method

$$\int_0^1 f(x)dx=\frac{0.1}{2}\left[0+1+2\left\{\frac{37}{100}+\frac{17}{25}+\frac{93}{100}+\frac{28}{25}+\frac{5}{4}+\frac{33}{25}+\frac{133}{100}+\frac{32}{25}+\frac{117}{100}\right\}\right]$$
$$=0.995$$

$\therefore$ Approximate value of $f(x)=0.995$

$\therefore$ The true solution is founded by integration

$$\int_0^1\left(4x-3x^2\right)dx=\left[4\frac{x^2}{2}-3\frac{x^3}{3}\right]_0^1=\left[2x^2-x^3\right]_0^1=2-1-0=1$$

$\therefore$ Absolute error $=(1-0.995)=0.005$

$\therefore$ Relative error $=\left|\frac{1-0.995}{1}\right|=0.005=\frac{1}{200}$

**12. Evaluate $\int xe^x dx$ where the interval $(-1,0)$ by using Trapezoidal rule taking $n=6$, correct up to four decimal places.** **[WBUT 2018(EVEN)]**

**Answer:**

Let $f(x)=xe^x$, $h=\frac{1}{6}$

| $x$ | $-1$ | $\frac{-5}{2}$ | $\frac{-4}{6}$ | $\frac{-3}{6}$ | $\frac{-2}{6}$ | $\frac{-1}{6}$ | 0 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-0.36788$ | $-0.36217$ | $-0.34228$ | $-0.30327$ | $-0.23884$ | $-0.14108$ | 0 |

So by Trapezoidal method:

$$\int_{-1}^{0} f(x)dx=\frac{1}{12}\left[-0.36788+0-2(0.36217+0.34228+0.30327+0.23884+0.14108)\right]$$
$$=-0.2619 \quad \text{(Correct up to 4 decimal places)}$$

**13. Find the approximate value of $\int_0^1\frac{x}{1+x^2}dx$ up to four decimal places by Simpson's one third rule, taking 6 equal subintervals and find the approximate value of log 2.** **[WBUT 2018(ODD)]**

**Answer:**

Let $f(x)=\frac{x}{1+x^2}$; $a=0$, $b=1$, $n=6$

$\therefore$ $h=\frac{1}{6}$

| $x$ | 0 | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{6}$ | $\frac{4}{6}$ | $\frac{5}{6}$ | 1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | $\frac{6}{37}$ | $\frac{12}{40}$ | $\frac{18}{45}$ | $\frac{24}{54}$ | $\frac{30}{61}$ | $\frac{1}{2}$ |

According to the Simpson's $\frac{1}{3}$rd rule:

$$\int_0^1 f(x)dx=\frac{h}{3}\left[f(0)+f(1)+4\left(f\left(\frac{1}{6}\right)+f\left(\frac{3}{6}\right)+f\left(\frac{5}{6}\right)\right)+2\left(f\left(\frac{2}{6}\right)+f\left(\frac{4}{6}\right)\right)\right]$$

$$=\frac{1}{18}\left[0+\frac{1}{2}+4\times\left(\frac{6}{37}+\frac{18}{45}+\frac{30}{61}\right)+2\times\left(\frac{12}{40}+\frac{24}{54}\right)\right]$$

$$=0.3447$$

Now, $\int_0^1\frac{x}{1+x^2}dx=\frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^1=\frac{1}{2}\log 2=\log\sqrt{2}$

$\therefore$ $\log 2=0.6894$ (Correct upto 4 decimal places)

## Long Answer Type Questions

**1. Using Trapezoidal and Simpson's $\frac{1}{3}$rd rule compute $\int_4^{5.2}\log_e x\,dx$ by taking seven ordinates correct up to four decimal places.** **[WBUT 2013(EVEN), 2016(ODD)]**

**Answer:**

Let $f(x)=\log_e x$

If the given integration is of the form $\int_a^b f(x)dx$ then $a=4$, $b=5.2$

$\therefore h=\frac{b-a}{6}=\frac{5.2-4}{6}=.2$

| $x$ | 4 | 4.2 | 4.4 | 4.6 | 4.8 | 5 | 5.2 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.3863 | 1.4351 | 1.4816 | 1.5261 | 1.5686 | 1.6094 | 1.6487 |

Now by Trapezoidal Rule:

$$\int_4^{5.2} f(x)dx=\frac{h}{2}[1.3863+1.6487+2(1.4351+1.4816+1.5261+1.5686+1.6094)]=1.8277$$

By Simpson's $\frac{1}{3}$ Rule:

$$\int \log_e x\,dx = \frac{h}{3}[1.3863 + 1.6487 + 4(1.4351 + 1.5261 + 1.6094) + 2(1.4861 + 1.5686)]$$
$$= 1.8285$$

**2. a) Deduce Simpson's $\frac{1}{3}$rd rule (from Newton-Cote's quadrature formula).**

**[WBUT 2014(ODD)]**

**Answer:**

The trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial over interval of integration. Simpson's 1/3 rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

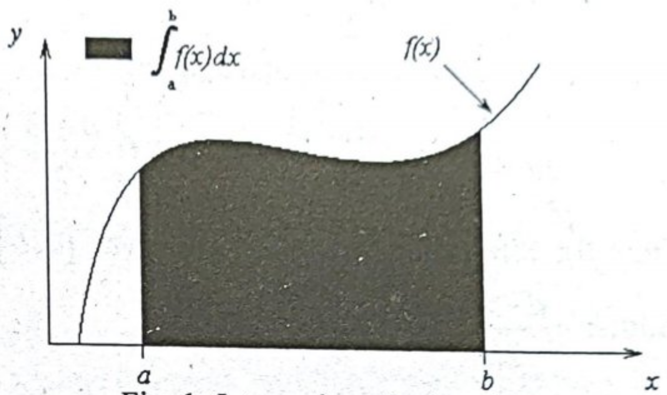



Fig. 1: Integration of a function

***Method I:***

Hence

$$I = \int_a^b f(x)dx \approx \int_a^b f_2(x)dx$$

where $f_2(x)$ is a second order polynomial given by $f_2(x) = a_0 + a_1x + a_2x^2$

Choose

$$(a, f(a)), \left(\frac{a+b}{2}, f\left(\frac{a+b}{2}\right)\right), \text{and } (b, f(b))$$

as the three points of the function to evaluate $a_0, a_1$ and $a_2$.

$$f(a) = f_2(a) = a_0 + a_1a + a_2a^2$$

$$f\left(\frac{a+b}{2}\right) = f_2\left(\frac{a+b}{2}\right) = a_0 + a_1\left(\frac{a+b}{2}\right) + a_2\left(\frac{a+b}{2}\right)^2$$

$$f(b) = f_2(b) = a_0 + a_1b + a_2b^2$$

Solving the above three equations for unknowns, $a_0$, $a_1$ and $a_2$ give

$$a_0 = \frac{a^2 f(b) + abf(b) - 4abf\left(\frac{a+b}{2}\right) + abf(a) + b^2 f(a)}{a^2 - 2ab + b^2}$$

$$a_1 = -\frac{af(a) - 4af\left(\frac{a+b}{2}\right) + 3af(b) + 3bf(a) - 4bf\left(\frac{a+b}{2}\right) + bf(b)}{a^2 - 2ab + b^2}$$

$$a_2 = \frac{2\left(f(a) - 2f\left(\frac{a+b}{2}\right) + f(b)\right)}{a^2 - 2ab + b^2}$$

Then

$$I \approx \int_a^b f_2(x)dx = \int_a^b \left(a_0 + a_1 x + a_2 x^2\right)dx = \left[a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3}\right]_a^b$$

$$= a_0(b-a) + a_1 \frac{b^2 - a^2}{2} + a_2 \frac{b^3 - a^3}{3}$$

Substituting values of $a_0, a_1$ and $a_2$ give

$$\int_a^b f_2(x)dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Since for Simpson 1/3 rule, the interval $[a,b]$ is broken into 2 segments, the segment width $h = \frac{b-a}{2}$

Hence the Simpson's 1/3 rule is given by $\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

Since the above form has 1/3 in its formula, it is called Simpson's 1/3 rule.

**b) What is the geometrical interpretation of Trapezoidal rule? [WBUT 2014(ODD)]**

**Answer:**

In trapezoidal method, the curve $y = f(x)$ is replaced by the straight line passing through the points $(a, f(a))$ and $(b, f(b))$ and the integral $\int_a^b f(x)dx$ is approximated by the area of the trapezium bounded by the straight line, the ordinates at $x = a, b$ and hence the name is trapezoidal rule.

**3. Find the value of $\int_1^5 \log_{10} x\,dx$ taking eight equal sub-intervals correct up to 4-decimal places by (i) Simpson $\frac{1}{3}$rd rule (ii) Trapezoidal rule and then compare the result. [WBUT 2016(EVEN)]**

**Answer:**

Let $f(x) = \log_{10} x,\ a = 1,\ b = 5,\ n = 8$

$\therefore \quad h = \frac{5-1}{8} = \frac{4}{8} = 0.5$

Now,

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.17609 | 0.30103 | 0.39794 | 0.47712 | 0.54407 | 0.60206 | 0.65321 | 0.69897 |

So, by Simpson's $\frac{1}{3}$ rule:

$$\int_1^5 f(x)\,dx = \frac{0.5}{3}\Big[0+0.69897+4(0.17609+0.39794+0.54407+0.65321)$$
$$+2(0.30103+0.47712+0.60206)\Big]$$
$$=1.75744$$
$$\approx 1.7574 \quad \text{(Correct upto 4 decimal places and by Trapezoidal method)}$$

$$\int_1^5 f(x)\,dx = \frac{0.5}{2}\Big[0+0.69897+2(0.17609+0.30103+0.39794+0.47712$$
$$+0.54407+0.60206+0.65321)\Big]$$
$$=1.7505025$$
$$\approx 1.7505 \quad \text{(Correct upto 4 decimal places).}$$

If we truncate those results then both the results are equal upto 2 decimal places

**4. Compute the value of $\int_{1.2}^{1.6}\left(x+\frac{1}{x}\right)dx$, taking $h=0.05$ correct up to five decimal places by using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$rd rule. [WBUT 2017(EVEN)]**

**Answer:**

Let $f(x)=x+\frac{1}{x}$

| $x$ | 1.2 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.55 | 1.60 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $2\frac{1}{30}$ | $2\frac{1}{20}$ | $2\frac{9}{130}$ | $2\frac{49}{540}$ | $2\frac{4}{35}$ | $2\frac{81}{580}$ | $2\frac{1}{6}$ | $2\frac{121}{620}$ | $2\frac{9}{40}$ |

Now by Trapezoidal Rule:

$$\int_{1.2}^{1.6} f(x)\,dx = \frac{0.05}{2}\left[2\frac{1}{30}+2\frac{9}{40}+2\left(2\frac{1}{20}+2\frac{9}{130}+2\frac{49}{540}+2\frac{4}{35}+2\frac{81}{580}+2\frac{1}{6}+2\frac{121}{620}\right)\right]$$
$$=0.84775 \text{ (Correct up to 5 decimal places)}$$

By Simpson's $\frac{1}{3}rd$ rule:

$$\int_{1.2}^{1.6} f(x)dx = \frac{0.05}{3}\left[2\frac{1}{30}+2\frac{9}{40}+4\left(2\frac{1}{20}+2\frac{49}{540}+2\frac{81}{580}+2\frac{121}{620}\right)+2\left(2\frac{9}{130}+2\frac{4}{35}+2\frac{1}{6}\right)\right]$$
$$=0.84768 \text{ (Correct up to 5 decimal places)}$$

**5. a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.** **[WBUT 2018(EVEN)]**

**b) Evaluate the same using Simpson's $\frac{1}{3}$ rule.**

**c) What is the difference between the results (a) and (b) compare in terms of error (absolute), error % and relative error.**

**Answer:**

a) Let $f(x)=\frac{1}{1+x^2}$

Here $a=0,\ b=0,\ n=6$ (let)

$\therefore \quad h=\frac{b-a}{n}=\frac{6}{6}=1$

Now,

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.5 | 0.2 | 0.1 | $\frac{1}{17}$ | $\frac{1}{26}$ | $\frac{1}{37}$ |

According to Trapezoidal rule:

$$\int_0^6 f(x)\,dx=\frac{h}{2}\left[1+\frac{1}{37}+2\left(0.5+0.2+0.1+\frac{1}{17}+\frac{1}{26}\right)\right]=1.410798581$$

b) According to Simpson's $\frac{1}{3}$rd rule;

$$\int_0^6 f(x)\,dx=\frac{h}{3}\left[1+\frac{1}{37}+4\left(0.5+0.1+\frac{1}{26}\right)+2\left(0.2+\frac{1}{17}\right)\right]=1.366173413$$

c) $\int_0^6 \frac{dx}{1+x^2}=\left[\tan^{-1}x\right]_0^6=1.405647649$

| Error | Trapezoidal Method | Simpson's $\frac{1}{3}$rd Rule |
|---|---|---|
| 1. Absolute error | 0.00510931 | 0.039474236 |
| 2. Relative error | 0.003664454 | 0.0280825666 |
| 3. Percentage error | 0.3664450% | 2.80825666% |

# NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS

## Multiple Choice Type Questions

1. When Gauss Elimination method is used to solve $AX = B$, $A$ is transformed to a [WBUT 2007, 2009, 2012(ODD), 2015(EVEN)]

a) null matrix
b) upper triangular matrix
c) identity matrix
d) diagonally dominant matrix

**Answer:** (b)

2. Which of the following is an iterative method? [WBUT 2013(EVEN), 2013(ODD), 2015(EVEN)]

a) Gauss Elimination Method
b) Gauss Jordan Method
c) LU decomposition Method
d) Gauss-Seidel Method

**Answer:** (d)

3. In LU-factorization method, the given system of equations represented by $AX = B$ is converted to another system $LUX = B$ where $U$ is [WBUT 2013(ODD)]

a) lower triangular matrix
b) upper triangular matrix
c) identity matrix
d) null matrix

**Answer:** (b)

4. Gauss elimination method does not fail, even if one of the pivotal elements is equal to zero: [WBUT 2014(ODD)]

a) true
b) false
c) neither true nor false
d) none of these

**Answer:** (a)

5. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 2 & 4 \\ 9 & 8 & 7 \end{bmatrix}$ [WBUT 2016(EVEN)]

Consider the following two statements:
S1 : LU decomposition for the matrix A is possible
S2 : LU decomposition for the matrix B is not possible

a) Both S1 and S2 are true
b) only S1 is true
c) only S2 is true
d) neither S1 nor S2 is true

**Answer:** (b)

6. Which one of the following is a direct method? [WBUT 2016(ODD)]

a) Gauss-elimination
b) Gauss-Jordan
c) Gauss-Seidel
d) none of these

**Answer:** (a)

7. The convergence condition for Gauss-Seidel iterative method for solving a system of linear equation is [WBUT 2016(ODD)]

a) the co-efficient matrix is singular
b) the co-efficient matrix has rank zero
c) the co-efficient matrix must be strictly diagonally dominant
d) none of these

**Answer:** (c)

8. Gauss elimination method does not fail even if one of the pivot elements is equal to zero. The above statement is [WBUT 2017(EVEN)]

a) True
b) False

**Answer:** (a)

9. Which one of the following is an iterative method? [WBUT 2017(EVEN)]

a) Gauss-elimination
b) Gauss-Jordon
c) Gauss-Seidel
d) none of these

**Answer:** (c)

10. Diagonal dominance is must for [WBUT 2018(EVEN)]

a) Gauss-Seidel method
b) Gauss Elimination method
c) LU factorisation method
d) All of these

**Answer:** (a)

11. A square matrix $[A]_{n\times n}$ is diagonally dominant, if [WBUT 2018(ODD)]

a) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|, i = 1, 2, \cdots\cdots\cdots, n$

b) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|, i = 1, 2, \cdots\cdots\cdots, n$ and $|a_{ii}| > \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}|$, for any $i = 1, 2, \cdots\cdots\cdots, n$

c) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, i = 1, 2, \cdots\cdots\cdots, n$ and $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$, for any $i = 1, 2, \cdots\cdots\cdots, n$

d) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, i = 1, 2, \cdots\cdots\cdots, n$

**Answer:** (a)

12. For a set of '$m$' linear equations and '$n$' no. of unknowns the system is under determined, if [WBUT 2019(EVEN)]

a) $m > n$
b) $m < n$
c) $m = n$
d) any one of these

**Answer:** (d)

## Short Answer Type Questions

**1. Using Gauss-Seidel method find the solution of the following system of linear equations correct up to two decimal places:** **[WBUT 2013(EVEN), 2013(ODD), 2015(ODD)]**

$$3x+y+5z=13$$
$$5x-2y+z=4$$
$$x+6y-2z=-1$$

**Answer:**

First we make the system of equations diagonally dominant, and rewrite them as,

$$5x-2y+z=4$$
$$x+6y-2z=-1$$
$$3x+y+5z=13$$

Now, the Gauss-Seidel Formula is given by,

$$x^{(K+1)}=\frac{1}{5}\left[4+2y^{(K)}-z^{(K)}\right]$$

$$y^{(K+1)}=\frac{1}{6}\left[-1-x^{(K+1)}+2z^{(K)}\right]$$

$$z^{(K+1)}=\frac{1}{5}\left[13-3x^{(K+1)}-y^{(K+1)}\right]$$

Let, the initial approximation to the root be,

$$x^{(0)}=0,\ y^{(0)}=0,\ z^{(0)}=0$$

*Computational table*

| K | $x^{(K)}$ | $y^{(K)}$ | $z^{(K)}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0.8 | –0.3 | 2.18 |
| 2 | 0.244 | 0.519 | 2.3497 |
| 3 | 0.5377 | 0.527 | 2.172 |
| 4 | 0.576 | 0.461 | 2.162 |
| 5 | 0.552 | 0.462 | 2.176 |
| 6 | 0.5496 | 0.467 | 2.177 |
| 7 | 0.551 | 0.467 | 2.176 |
| 8 | 0.552 | 0.467 | 2.175 |

Hence, the roots correct upto two decimal places are,

$x=0.55$

$y=0.47$

$z=2.18$

**2. Solve the following system of equations by $L-U$ decomposition method:**

$$x+y-z=2,\ 2x+3y+5z=-3,\ 3x+2y-3z=6.$$

**[WBUT 2013(ODD), 2015(ODD)]**

**Answer:**

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$
$$A = LU$$
$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 7 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$
$$AX = B$$
$$(LU)X = B$$
$$L(UX) = B$$

Let, $(UX) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$\Rightarrow \quad y_1 = 2 \quad \Rightarrow y_1 = 2$$
$$2y_1 + y_2 = -3 \quad \Rightarrow y_2 = -7$$
$$3y_1 - y_2 + 7y_3 = 6 \quad \Rightarrow y_3 = -1$$
$$UX = \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}$$
$$\Rightarrow \quad x_1 + x_2 - x_3 = 2$$
$$x_2 + 7x_3 = -7$$
$$x_3 = -1$$
$$\left.\begin{aligned} & x_3 = (-1) \\ \Rightarrow \quad & x_2 = 0 \\ \Rightarrow \quad & x_1 = 1 \end{aligned}\right\}$$

3. **Solve:** $x + y + z = 1$ **[WBUT 2014(EVEN)]**

$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

**by Gauss Elimination method.**

**Answer:**

Coefficient matrix of the given system of equations is $A=\begin{bmatrix}1 & 1 & 1\\ 2 & -3 & 4\\ 3 & 4 & 5\end{bmatrix}$

Augmented matrix is $[A|B]=\begin{bmatrix}1 & 1 & 1 & 1\\ 2 & -3 & 4 & 13\\ 3 & 4 & 5 & 40\end{bmatrix}$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1}\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & -5 & 2 & 11\\ 0 & 1 & 2 & 37\end{bmatrix}\xrightarrow{-\frac{1}{5}R_2}\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & \frac{-2}{5} & \frac{-11}{5}\\ 0 & 1 & 2 & 37\end{bmatrix}\xrightarrow{R_3-R_2}\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & \frac{-2}{5} & \frac{-11}{5}\\ 0 & 0 & \frac{12}{5} & \frac{196}{5}\end{bmatrix}$$

$$\therefore z=\frac{\frac{196}{5}}{\frac{12}{5}}=\frac{49}{3};\ y=\frac{-11}{5}+\frac{2}{5}\times\frac{49}{3}=\frac{13}{3};\ x=1-y-z=1-\frac{49}{3}-\frac{13}{3}=-\frac{59}{3}$$

**4. Find inverse of the following matrix by Gauss-Jacobi method $\begin{pmatrix}2 & 1 & 1\\ 3 & 2 & 3\\ 1 & 4 & 9\end{pmatrix}$.**

**[WBUT 2014(ODD)]**

**Answer:**

According to Gauss-Jacobi method,

Let, $A=\begin{bmatrix}2 & 1 & 1\\ 3 & 2 & 3\\ 1 & 4 & 4\end{bmatrix}$

Now, $[A/I_3]=\begin{bmatrix}2 & 1 & 1 & 1 & 0 & 0\\ 3 & 2 & 3 & 0 & 1 & 0\\ 1 & 4 & 9 & 0 & 0 & 1\end{bmatrix}$

$$\xrightarrow{\frac{1}{2}R_1}\begin{bmatrix}1 & 1/2 & 1/2 & 1/2 & 0 & 0\\ 3 & 2 & 3 & 0 & 1 & 0\\ 1 & 4 & 9 & 0 & 0 & 1\end{bmatrix}\xrightarrow[R_3-R_1]{R_2-3R_1}\begin{bmatrix}1 & 1/2 & 1/2 & 1/2 & 0 & 0\\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0\\ 0 & 7/2 & 17/2 & -1/2 & 0 & 1\end{bmatrix}$$

$$\xrightarrow[R_3-7R_2]{R_1-R_2}\begin{bmatrix}1 & 0 & -1 & 2 & -1 & 0\\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0\\ 0 & 0 & -2 & 10 & -7 & 1\end{bmatrix}\xrightarrow{2R_2}\begin{bmatrix}1 & 0 & -1 & 2 & -1 & 0\\ 0 & 1 & 3 & -3 & 2 & 0\\ 0 & 0 & -2 & 10 & -7 & 1\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_3}\left|\begin{array}{cccccc}1 & 0 & -1 & 2 & -1 & 0\\ 0 & 1 & 3 & -3 & 2 & 0\\ 0 & 0 & 1 & -5 & 7/2 & -1/2\end{array}\right|$$

$$\xrightarrow[R_2-3R_3]{R_1+R_3}\begin{bmatrix}1 & 0 & 0 & -3 & 5/2 & -1/2\\ 0 & 1 & 0 & 12 & -17/2 & 3/2\\ 0 & 0 & 1 & -5 & 7/2 & -1/2\end{bmatrix}$$

$$\therefore A^{-1}=\begin{bmatrix}-3 & 5/2 & -1/2\\ 12 & -17/2 & 3/2\\ -5 & 7/2 & -1/2\end{bmatrix}$$

**5. Solve by Gauss elimination method:** **[WBUT 2014(ODD), 2015(EVEN)]**

$$x+2y+z=0$$
$$2x+2y+3z=3$$
$$-x-3y=2$$

**Answer:**

Given system of equations can be written in the following matrix from $AX=B$

where $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 2 & 3\\ -1 & -3 & 0\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}0\\ 3\\ 2\end{bmatrix}$

Now, $[A/B]=\begin{bmatrix}1 & 2 & 1 & 0\\ 2 & 2 & 3 & 3\\ -1 & -3 & 0 & 2\end{bmatrix}$

$$\xrightarrow[R_3+R_1]{R_2-2R_1}\begin{bmatrix}1 & 2 & 1 & 0\\ 0 & -2 & 1 & 3\\ 0 & -1 & 1 & 2\end{bmatrix}\xrightarrow{R_3-\frac{1}{2}R_2}\begin{bmatrix}1 & 2 & 1 & 0\\ 0 & -2 & 1 & 3\\ 0 & 0 & 1/2 & 1/2\end{bmatrix}$$

$\therefore \frac{1}{2}z=\frac{1}{2}$ or, $z=1$,

$-2y+z=3$ or, $y=-1$

$x+2y+z=0$ or, $x=1$

Hence the required solution is $X=\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$

**6. Consider the following system of equations:**

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \\ 8 \end{pmatrix}$$

**Solve the above system of equations by Gauss-Seidel method taking initial guess $x=[0 \ \ 0 \ \ 0]^T$. Perform only two iterations.** **[WBUT 2015(EVEN), 2019(EVEN)]**

**Answer:**

According to Gauss-Seidel method; the iterative scheme of the given system of equations is

$$x_1^{(k+1)} = \frac{1}{2}\left[5 - x_2^{(k)} - x_3^{(k)}\right]$$

$$x_2^{(k+1)} = \frac{1}{5}\left[15 - 2x_3^{(k)} - 3x_1^{(k+1)}\right]$$

$$x_3^{(k+1)} = \frac{1}{4}\left[8 - 2x_1^{(k+1)} - x_2^{(k+1)}\right]$$

| $k$ | $x_1^{(k)}$ | $x_2^{(k)}$ | $x_3^{(k)}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2.5 | 1.5 | 0.375 |
| 2 | 1.5625 | 1.9125 | 0.740625 |
| 3 | 1.1734 | 1.9997 | 0.9134 |

**7. What is the order of operation needs for L-U decomposition method?** **[WBUT 2015(ODD)]**

**Answer:**

In L-U decomposition method, for a system of equations

$AX = B$ .... (1)

we have to decompose the coefficient matrix A as a product to two matrices one of which should be upper triangular matrix and other is lower triangular matrix.

$A = L \quad U$

↓ ↓

Lower triangular matrix    Upper triangular matrix

Therefore the system of equations becomes

$LUX = B$ .... (2)

Now we have to consider $UX = Y$ .... (3)

where $Y$ is an unknown vector.

So, the Eqn. (2) becomes $LY = B$ .... (4)

Then we are going to solve Eqn. (4) for $Y$ and finally using the result of $Y$, from Eqn. (3) we obtain $X$.

**8. Solve by Gauss Elimination method:** **[WBUT 2016(EVEN)]**

$$x-2y+9z=8$$
$$3x+y-z=3$$
$$2x-8y+z=-5$$

**Answer:**

Given system of equations can be written in the following matrix form:

$$AX=B$$

where $A=\begin{bmatrix}1 & -2 & 9\\ 3 & 1 & -1\\ 2 & -8 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}8\\ 3\\ -5\end{bmatrix}$

Now, $[A|B]=\begin{bmatrix}1 & -2 & 9 & : & 8\\ 3 & 1 & -1 & : & 3\\ 2 & -8 & 1 & : & -5\end{bmatrix} \xrightarrow[R_1-2R_1]{R_2-3R_1} \begin{bmatrix}1 & -2 & 9 & : & 8\\ 0 & 7 & -28 & : & -21\\ 0 & -4 & -17 & : & -21\end{bmatrix}$

$\xrightarrow{R_3+\frac{4}{7}R_2} \begin{bmatrix}1 & -2 & 9 & : & 8\\ 0 & 7 & -28 & : & -21\\ 0 & 0 & -33 & : & -33\end{bmatrix}$

$\therefore \quad z=1$

$7y-28z=-21 \Rightarrow \quad y=1$

$x-2y+9z=8 \Rightarrow \quad x=1$

Hence, $X=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$

**9. Solve the following system by matrix inversion method** $2x+y+z=10$, $3x+2y+3z=18$, $x+4y+9z=16$ **[WBUT 2016(ODD)]**

**Answer:**

Given system of equations can be written in the following form:

$$AX=B;$$

where $A=\begin{bmatrix}2 & 1 & 1\\ 3 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}10\\ 18\\ 16\end{bmatrix}$

Now, $A=\left[\frac{A}{I_3}\right]=\begin{bmatrix}2 & 1 & 1 & : & 1 & 0 & 0\\ 3 & 2 & 3 & : & 0 & 1 & 0\\ 1 & 4 & 9 & : & 0 & 0 & 1\end{bmatrix}$

$$\xrightarrow{\frac{1}{2}R_1}\left[\begin{array}{ccc:ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3-R_1]{R_2-3R_1}\left[\begin{array}{ccc:ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & \frac{7}{2} & \frac{17}{2} & \frac{-1}{2} & 0 & 1 \end{array}\right]$$

$$\xrightarrow{2R_2}\left[\begin{array}{ccc:ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & \frac{7}{2} & \frac{17}{2} & \frac{-1}{2} & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3-\frac{7}{2}R_2]{R_1-\frac{1}{2}R_2}\left[\begin{array}{ccc:ccc} 1 & 0 & -1 & \frac{4}{2} & -1 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -2 & 10 & 7 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_3}\left[\begin{array}{ccc:ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & 1 & -5 & \frac{-7}{2} & -\frac{1}{2} \end{array}\right]$$

$$\xrightarrow[R_2-3R_1]{R_1+R_3}\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -3 & \frac{-9}{2} & \frac{-1}{2} \\ 0 & 1 & 0 & 12 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -5 & \frac{-7}{2} & \frac{-1}{2} \end{array}\right]$$

$$\therefore \qquad A^{-1}=\begin{bmatrix} -3 & -\frac{9}{2} & -\frac{1}{2} \\ 12 & \frac{25}{2} & \frac{3}{2} \\ -5 & \frac{-7}{2} & \frac{-1}{2} \end{bmatrix}$$

Now $$X=A^{-1}B=\begin{bmatrix} -3 & \frac{-9}{2} & \frac{-1}{2} \\ 12 & \frac{25}{2} & \frac{3}{2} \\ -5 & \frac{-7}{2} & \frac{-1}{2} \end{bmatrix}\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}=\begin{bmatrix} -119 \\ 319 \\ -121 \end{bmatrix}.$$

**10. Solve the following system by Matrix Inversion Method:** **[WBUT 2017(EVEN)]**

$$2x+y+z=4$$
$$3x+2y+3z=8$$
$$x+4y+9z=14$$

**Answer:**

Given system of equations can be written in the following matrix form:

$$AX=B$$

Where $A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix}$

Now we apply elementary row operation on the matrix A as follows;

$$[A_1 I_3]=\begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3-R_1]{R_2-3R_1}\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & \frac{7}{2} & \frac{17}{2} & -\frac{1}{2} & 0 & 1 \end{bmatrix}\xrightarrow[R_3-7R_2]{R_1-R_2}\begin{bmatrix} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{bmatrix}$$

$$\xrightarrow{2R_2} \begin{bmatrix} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_3} \begin{bmatrix} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & 1 & -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\xrightarrow[R_2-3R_3]{R_1+R_3} \begin{bmatrix} 1 & 0 & 0 & -3 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -12 & -\frac{17}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -3 & \frac{5}{2} & -\frac{1}{2} \\ 12 & -\frac{17}{2} & -\frac{3}{2} \\ -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} -3 & \frac{5}{2} & -\frac{1}{2} \\ 12 & -\frac{17}{2} & \frac{3}{2} \\ -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

**11. Solve the following system of linear equations by matrix inversion method:**

$x+y+z=4$ **[WBUT 2018(EVEN)]**

$2x-y+3z=1$

$3x+2y-z=1$

**Answer:**

$$[A|I_3] = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & -2 & 1 & 0 \\ 0 & -1 & -4 & -3 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{3}R_2}\begin{bmatrix}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} & 0\\ 0 & -1 & -4 & -3 & 0 & 1\end{bmatrix}$$

$$\xrightarrow[R_3+R_2]{R_1-R_2}\begin{bmatrix}1 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{1}{3} & 0\\ 0 & 1 & \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} & 0\\ 0 & 0 & \frac{-13}{3} & \frac{-7}{3} & \frac{-1}{3} & 1\end{bmatrix}$$

$$\xrightarrow{\frac{-3}{13}R_3}\begin{bmatrix}1 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{1}{3} & 0\\ 0 & 1 & \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} & 0\\ 0 & 0 & 1 & \frac{7}{13} & \frac{1}{13} & \frac{-3}{13}\end{bmatrix}$$

$$\xrightarrow[R_2+\frac{1}{3}R_3]{R_1-\frac{4}{3}R_3}\begin{bmatrix}1 & 0 & 0 & \frac{-5}{13} & \frac{3}{13} & \frac{4}{13}\\ 0 & 1 & 0 & \frac{11}{13} & \frac{-4}{13} & \frac{-1}{13}\\ 0 & 0 & 1 & \frac{7}{13} & \frac{1}{13} & \frac{-3}{13}\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{1}{13}\begin{bmatrix}-05 & 3 & 4\\ 11 & -04 & -1\\ 7 & 1 & -3\end{bmatrix}$$

$$\therefore \quad \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{13}\begin{bmatrix}-5 & 3 & 4\\ 11 & -4 & -1\\ 7 & 1 & -3\end{bmatrix}\begin{bmatrix}4\\ 1\\ 1\end{bmatrix}=\frac{1}{13}\begin{bmatrix}-13\\ 39\\ 26\end{bmatrix}=\begin{bmatrix}-1\\ 3\\ 2\end{bmatrix}$$

## Long Answer Type Questions

**1. Solve the following system of equations by LU factorization method:**

$$2x-6y+8z=24$$
$$5x+4y-3z=2$$
$$3x+y+2z=16$$

**[WBUT 2009, 2012(ODD), 2014(ODD), 2016(EVEN), 2017(EVEN)]**

Answer:
The system is given as

$2x-6y+8z=24$

$5x+4y-3z=2$

$3x+y+2z=16$

$$\text{Let } A=\begin{bmatrix}2 & -6 & 8\\ 5 & 4 & -3\\ 3 & 1 & 2\end{bmatrix} \quad \text{.... (i)}$$

Let A = LU

$$\text{When } L=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix} \text{ and } U=\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$$\text{i.e. } \begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}=\begin{bmatrix}2 & -6 & 8\\ 5 & 4 & -3\\ 3 & 1 & 2\end{bmatrix} \quad \text{.... (ii)}$$

Clearly $u_{11}=2,\ u_{12}=-6,\ u_{13}=8$

$l_{21}u_{11}=5$, so that $l_{21}=\dfrac{5}{2}$

$l_{31}u_{11}=3$, so that $l_{31}=\dfrac{3}{2}$

Again $l_{21}u_{12}+u_{22}=4$ so that $\dfrac{5}{2}\times(-6)+u_{22}=4 \Rightarrow u_{22}=4+15=19$

and $l_{21}u_{13}+u_{23}=-3$

$\dfrac{5}{2}.8+u_{23}=-3 \Rightarrow u_{23}=-3-20=-23$

Finally $l_{31}u_{12}+l_{32}u_{22}=1 \Rightarrow \dfrac{3}{2}(-6)+l_{32}.19=1$

$19l_{32}=1+9=10$

$l_{32}=\dfrac{10}{19}$ and $l_{31}u_{13}+l_{32}u_{23}+u_{33}=2 \qquad \Rightarrow \dfrac{3}{2}.8+\dfrac{11}{19}(-27)+u_{33}=2$

$u_{33}\dfrac{-187}{19}=2-12=-10$

$u_{33}=\dfrac{40}{19}$

It follows $A=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ \frac{5}{2} & 1 & 0\\ \frac{3}{2} & \frac{10}{19} & 1\end{bmatrix}\begin{bmatrix}2 & -6 & 8\\ 0 & 19 & -23\\ 0 & 0 & \frac{40}{19}\end{bmatrix}$ .... (iii)

And have the given system can be written as

$$\begin{bmatrix}1 & 0 & 0\\ \frac{5}{2} & 1 & 0\\ \frac{3}{2} & \frac{10}{19} & 1\end{bmatrix}\begin{bmatrix}2 & -6 & 8\\ 0 & 19 & -23\\ 0 & 0 & \frac{40}{19}\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}24\\ 2\\ 16\end{bmatrix} \quad \text{.... (iv)}$$

or as $\begin{bmatrix}1 & 0 & 0\\ \frac{5}{2} & 1 & 0\\ \frac{3}{2} & \frac{10}{19} & 1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}24\\ 2\\ 16\end{bmatrix}$ .... (v)

where $\begin{bmatrix}2 & -6 & 8\\ 0 & 19 & -23\\ 0 & 0 & 24/19\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}$ .... (vi)

Solving (v),

$$y_1=24$$

$$\frac{5}{2}y_1+y_2=2$$

$$\frac{3}{2}y_1+\frac{10}{19}y_2+y_3=16$$

$$\frac{5}{2}.24+y_2=2$$

$$y_2=2-60=-58$$

$$\frac{3}{2}.24+\frac{10}{19}(-58)+y_3=16$$

$$36-\frac{580}{19}+y_3=16$$

$$\frac{-580}{19}+y_3=-20$$

$$y_3 = -20 + \frac{580}{19} = \frac{-380+580}{19} = +\frac{200}{19}$$

$y_1 = 24$
$y_2 = -58$
$y_3 = \frac{+200}{19}$

Pulling the values in equation (vi), we have

$2x - 6y + 8z = 24$
$19y - 23z = -58$
$\frac{40}{19}z = \frac{+200}{19}$
$\Rightarrow \boxed{z = +5}$
$19y - 115 = -58 \Rightarrow 19y = 57 \Rightarrow \boxed{y = 3}$
$2x - 6.3 + 8.5 = 24$
$2x - 18 + 40 = 24$
$2x + 22 = 24 \Rightarrow 2x = 2 \Rightarrow \boxed{x = 1}$

So, $x = 1,\ y = 3,\ z = 5$

**2. Solve by Gauss-Seidel iterative method:** **[WBUT 2016(EVEN)]**

$$3y - 2z = 3$$
$$2x - y + 4z = 27$$
$$4x - y - 3z = 3$$

**correct up to four significant figures.**

**Answer:**

The iteration scheme for Gauss-Seidel iterative method corresponding to the given system of equations is (after pivoting):

$$x^{(n+1)} = \frac{1}{4}\left[3 + y^{(n)} + 3z^{(n)}\right]$$
$$y^{(n+1)} = \frac{1}{3}\left[3 + 2z^{(n)}\right]$$
$$z^{(n+1)} = \frac{1}{4}\left[27 - 2x^{(n+1)} + y^{(n+1)}\right]$$

Here we proceed the iteration process after considering $x^{(0)} = y^{(0)} = z^{(0)} = 0$ as the initial guesses.

| $n$ | $x^{(n)}$ | $y^{(n)}$ | $z^{(n)}$ |
|---|---|---|---|
| 1 | 0.75 | 1 | 6.625 |
| 2 | 5.96875 | 5.41667 | 5.11979 |
| 3 | 5.94401 | 4.41319 | 4.88129 |
| 4 | 5.51427 | 4.25419 | 5.05641 |
| 5 | 5.60586 | 4.37094 | 5.039805 |
| 6 | 5.62259 | 4.35987 | 5.02867 |
| 7 | 5.61147 | 4.35245 | 5.03238 |
| 8 | 5.61240 | 4.35492 | 5.03253 |
| 9 | 5.613128 | 4.35502 | 5.032191 |
| 10 | 5.612898 | 4.35479 | 5.03225 |

$\therefore \quad x = 5.613,\ y = 4.355,\ z = 5.032$ (Correct upto 4-significant figures)

**3. Solve the system of equations by LU factorization method:** **[WBUT 2016(ODD)]**

$$8x_1 - 3x_2 + 2x_1 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$6x_1 + 3x_2 + 12x_3 = 36$$

**Answer:**

Let, $A = \begin{bmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 6 & 3 & 12 \end{bmatrix}$ and the given system of equation is $AX = B$.

Now we consider $A = LU$;

where $L = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$ and $U = \begin{bmatrix} 1 & h_{12} & h_{13} \\ 0 & 1 & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad \ell_{11} = 8,\ h_{12} = \dfrac{-3}{8},\ h_{13} = \dfrac{2}{8}$

$\ell_{21} = 4,\ \ell_{22} = \dfrac{25}{2},\ h_{23} = \dfrac{-4}{25}$

$\ell_{31} = 6,\ \ell_{32} = \dfrac{21}{4},\ \ell_{33} = \dfrac{1209}{100}$

To solve $LUX = B$, we set $UX = Y$, therefore the original system of equation reduces to $LY = B$

or,
$$\begin{bmatrix} 8 & 0 & 0 \\ 4 & \dfrac{25}{2} & 0 \\ 6 & \dfrac{21}{4} & \dfrac{1209}{100} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 33 \\ 36 \end{bmatrix}$$

or, $y_1 = \frac{20}{8} = \frac{5}{2}$

$4y_1 + \frac{25}{2}y_2 = 33 \Rightarrow y_2 = \frac{46}{25}$

$6y_1 + \frac{21}{4}y_2 + \frac{1209}{100}y_3 = 36 \Rightarrow y_3 = \frac{378}{403}$

Now, $UX = Y$ becomes;

$$\begin{bmatrix} 1 & \frac{-3}{8} & \frac{2}{8} \\ 0 & 1 & \frac{-4}{25} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{46}{25} \\ \frac{378}{403} \end{bmatrix}$$

or, $z = \frac{378}{403}$

$y - \frac{4z}{25} = \frac{46}{25} \Rightarrow y = \frac{46}{25} + \frac{4}{25} \times \frac{378}{403} = \frac{2}{25}\left(\frac{23}{1} + \frac{756}{403}\right) = \frac{802}{403}$

$x - \frac{3}{8}y + \frac{2}{8}z = \frac{5}{2} \Rightarrow x = 3\frac{19}{1612}$

**4. Solve the system of equations by Gauss-Seidel method correct to two decimal places:** **[WBUT 2016(ODD)]**

$10x - 5y - 2z = 3$

$4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

**Answer:**

Gauss-Seidel iterative scheme for the given problem is

$$x^{(n+1)} = \frac{1}{10}\left[3 + 5y^{(n)} + 2z^{(n)}\right]$$

$$y^{(n+1)} = \frac{+1}{10}\left[3 + 3z^{(n)} + 4x^{(n+1)}\right]$$

$$z^{(n+1)} = \frac{1}{10}\left[-3 - x^{(n+1)} - 6y^{(n+1)}\right]$$

Here we consider $x^{(0)} = y^{(0)} = z^{(0)} = 1$

| $n$ | $x^{(n)}$ | $y^{(n)}$ | $z^{(n)}$ |
|---|---|---|---|
| 1 | 1 | 1 | −1 |
| 2 | 0.6 | 0.24 | −0.504 |
| 3 | 0.3192 | 0.27648 | −0.4978 |
| 4 | 0.33868 | 0.28613 | −0.50555 |
| 5 | 0.34196 | 0.28512 | −0.50527 |
| 6 | 0.34151 | 0.28502 | −0.50516 |

$\therefore$ The required solutions are $x=0.34$, $y=0.29$, $z=-0.51$ (Correct up to 2 decimal places).

**5. Solve by Gauss-Seidel iterative method:**

$$3x+9y-2z=11$$
$$4x+2y+13z=24$$
$$4x-2y+z=-8$$

**Correct up to four significant figure.** **[WBUT 2017(EVEN)]**

**Answer:**

We rearrange the given set of equations as follows:

$$4x-2y+z=-8$$
$$3x+9y-2z=11$$
$$4x+2y+13z=24$$

So by Gauss – Seidel iterative method, the interactive scheme for the given set of equation will be

$$x^{(n+1)}=\frac{1}{4}\left[-8+2y^{(n)}-z^{(n)}\right]$$

$$y^{(n+1)}=\frac{1}{9}\left[11+2z^{(n)}-3x^{(n+1)}\right]$$

$$z^{(n+1)}=\frac{1}{13}\left[24-4x^{(n+1)}-2y^{(n+1)}\right]$$

Now we proceed for the interaction by taking $x^{(0)}=y^{(0)}=z^{(0)}=1$ as the initial guess:

| $n$ | $x^{(n+1)}$ | $y^{(n+1)}$ | $z^{(n+1)}$ |
|---|---|---|---|
| 0 | −1.75 | 2.02778 | 2.07265 |
| 1 | −0.98611 | 2.01151 | 1.84011 |
| 2 | −1.4495 | 2.11429 | 1.96688 |
| 3 | −1.43457 | 2.13750 | 1.95871 |
| 4 | −1.42093 | 2.13113 | 1.95550 |
| 5 | −1.42331 | 2.13121 | 1.95622 |
| 6 | −1.42345 | 2.13142 | 1.95623 |

Therefore, the required solutions are $x=-1.423, y=2.131$ and $z=1.956$ (Correct up to 4 significant figures)

**6. Solve the matrix inversion method:**

$$2x-3y+10z=3$$
$$-x+4y+2z=20$$
$$5x+2y+z=-12$$

**[WBUT 2017(EVEN)]**

**Answer:**

Given system of equations can be written in the following matrix form:

$$AX=B$$

where $A=\begin{bmatrix}2 & -3 & 10\\ -1 & 4 & 2\\ 5 & 2 & 1\end{bmatrix}\quad =X\begin{bmatrix}x\\ y\\ z\end{bmatrix}\quad B=\begin{bmatrix}3\\ 20\\ -12\end{bmatrix}$

Now we apply elementary row operation on A as follows:

$$[A|I_3]=\begin{bmatrix}2 & -3 & 10 & 1 & 0 & 0\\ -1 & 4 & 2 & 0 & 1 & 0\\ 5 & 2 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_1}\begin{bmatrix}1 & -\frac{3}{2} & 5 & \frac{1}{2} & 0 & 0\\ -1 & 4 & 2 & 0 & 1 & 0\\ 5 & 2 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$\xrightarrow[R_3-5R_1]{R_2+R_1}\begin{bmatrix}1 & -\frac{3}{2} & 5 & \frac{1}{2} & 0 & 0\\ 0 & \frac{5}{2} & 7 & \frac{1}{2} & 1 & 0\\ 0 & \frac{19}{2} & -24 & -\frac{5}{2} & 0 & 1\end{bmatrix}$$

$$\xrightarrow{\frac{2}{5}R_2}\begin{bmatrix}1 & -\frac{3}{2} & 5 & \frac{1}{2} & 0 & 0\\ 0 & 1 & \frac{14}{5} & \frac{1}{5} & \frac{2}{5} & 0\\ 0 & \frac{19}{2} & -24 & -\frac{5}{2} & 0 & 1\end{bmatrix}$$

$$\xrightarrow[R_3-\frac{19}{2}R_2]{R_1+\frac{3}{2}R_2}\begin{bmatrix}1 & 0 & \frac{46}{5} & \frac{8}{10} & \frac{3}{5} & 0\\ 0 & 1 & \frac{14}{5} & \frac{1}{5} & \frac{2}{5} & 0\\ 0 & 0 & -\frac{253}{5} & -\frac{44}{10} & -\frac{19}{5} & 1\end{bmatrix}$$

$$\xrightarrow{\frac{-5}{253}R_3}\begin{bmatrix} 1 & 0 & \frac{46}{5} & \frac{8}{10} & \frac{3}{5} & 0 \\ 0 & 1 & \frac{14}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 0 & 1 & \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix}$$

$$\xrightarrow[R_2-\frac{14}{5}R_3]{R_1-\frac{46}{5}R_3}\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{1}{11} & \frac{2}{11} \\ 0 & 1 & 0 & -\frac{1}{23} & \frac{48}{253} & \frac{14}{253} \\ 0 & 0 & 1 & \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 0 & -\frac{1}{11} & \frac{2}{11} \\ -\frac{1}{23} & \frac{48}{253} & \frac{14}{253} \\ \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} 0 & -\frac{1}{11} & \frac{2}{11} \\ -\frac{1}{23} & \frac{48}{253} & \frac{14}{253} \\ \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix}\begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$$

**7. Solve the given system of equations by Gauss Elimination method:**
$x+y+z=9,\ 2x-3y+4z=13,\ 3x+4y+5z=40$. **[WBUT 2017(ODD)]**

**Answer:**

Given system of equations can be expressed in the following matrix form:

$$AX = B$$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$

Now $[A|B]$ $\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{bmatrix}$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1}\begin{bmatrix}1 & 1 & 1 & 9\\0 & -5 & 2 & -5\\0 & 1 & 2 & 13\end{bmatrix}$$

$$\xrightarrow{-\frac{1}{5}R_2}\begin{bmatrix}1 & 1 & 1 & 9\\0 & 1 & -2/5 & 1\\0 & 1 & 2 & 13\end{bmatrix}$$

$$\xrightarrow{R_3-R_2}\begin{bmatrix}1 & 1 & 1 & 9\\0 & 1 & -2/5 & 1\\0 & 0 & \frac{12}{5} & 12\end{bmatrix}$$

$$\therefore \quad \frac{12}{5}z=12 \qquad \Rightarrow z=5$$

$$y-\frac{2}{5}z=1 \qquad \Rightarrow y=3$$

$$x+y+z=9 \qquad \Rightarrow x=1$$

8. **Using L-U factorisation method, solve the given system of equations:** $2x-3y+10z=3,\ -x+4y+2z=20,\ 5x+2y+z=-12$. **[WBUT 2017(ODD)]**

**Answer:**

Given system of equations can be written in the following matrix form:

$$AX=B \qquad \text{..... (1)}$$

Where $A=\begin{bmatrix}5 & 4 & 1\\-1 & 2 & 2\\2 & -3 & 10\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}-12\\20\\3\end{bmatrix}$,

Now we consider $A=LU$, with

$$L=\begin{bmatrix}\ell_{11} & 0 & 0\\\ell_{21} & \ell_{22} & 0\\\ell_{31} & \ell_{32} & \ell_{33}\end{bmatrix} \text{ and } U=\begin{bmatrix}1 & h_{12} & h_{13}\\0 & 1 & h_{23}\\0 & 0 & 1\end{bmatrix}$$

$\therefore$ From equation (1) we get,

$$\ell_{11}=5, \quad h_{12}=\frac{2}{5}, \quad h_{13}=\frac{1}{5}$$

$$\ell_{21}=-1, \quad \ell_{22}=\frac{22}{5}, \quad h_{23}=\frac{7}{22}$$

$$\ell_{31}=2, \quad \ell_{32}=\frac{-19}{5}, \quad \ell_{33}=10\frac{89}{110}$$

Let $UX=Y$, so from equation we obtain LY = B i.e., $\begin{bmatrix} 5 & 0 & 0 \\ -1 & \frac{22}{5} & 0 \\ 2 & \frac{-19}{5} & 10\frac{89}{110} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 20 \\ 3 \end{bmatrix}$

or, $y_1 = \dfrac{-12}{5}$ , $y_2 = 5\dfrac{1}{11}$ , $y_3 = 2\dfrac{608}{1189}$

Now from UX = Y we get,

$$\begin{bmatrix} 1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 1 & \frac{7}{22} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-12}{5} \\ 5\frac{1}{11} \\ 2\frac{608}{1189} \end{bmatrix}$$

$\therefore z = 2\dfrac{608}{1189},\quad y = 4\dfrac{347}{1189},\quad x = -4\dfrac{736}{1189}$

**9. Verify whether the given systems of equations are diagonally dominant. Hence solve them by Gauss-Siedel method:**
$-2x + 3y + 10z = 22,\ x + 10y - z = -22,\ 10x + 2y + z = 9$.

**[WBUT 2017(ODD), 2018(ODD)]**

**Answer:**
Given system of equations can be written in the following matrix form:

$$AX = B$$

Where $A = \begin{bmatrix} 10 & 2 & 4 \\ 1 & 10 & -1 \\ -2 & 3 & 10 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ -22 \\ 22 \end{bmatrix}$

Since $|a_{ii}| > \sum_j |a_{ij}|$ is satisfied for all $i = 1,2,3$. So the system of equations are diagonally dominant.

According to Gauss- Seidel Method; the interactive scheme is given by

$$x^{(n+1)} = \frac{1}{10}\left[9 - 2y^{(n)} - z^{(n)}\right]$$

$$y^{(n+1)} = \frac{1}{10}\left[-22 + z^{(n)} - x^{(n+1)}\right]$$

$$z^{(n+1)} = \frac{1}{10}\left[22 + 2x^{(n+1)} - 3y^{(n+1)}\right]$$

Take $x^{(0)} = y^{(0)} = z^{(0)} = 0$ as initial guess.

| $n$ | $x^{(n+1)}$ | $y^{(n+1)}$ | $z^{(n+1)}$ |
|---|---|---|---|
| 0 | 0.9 | – 2.29 | 3.067 |
| 1 | 1.0513 | –1.99843 | 3.009789 |
| 2 | 0.9987071 | –1.99889181 | 2.999408963 |
| 3 | 0.9998374657 | –2.00004285 | 2.999980348 |

$\therefore$ Solution is $x = 1, y = -2, z = 3$

**10. Solve the following using Gauss-Seidel iterative method correct to 4 significant digits:** **[WBUT 2018(EVEN)]**

$$5x_1 - x_2 + 2x_3 = 12$$
$$3x_1 + 8x_2 - 2x_3 = -25$$
$$x_1 + x_2 + 4x_3 = 6$$

**Answer:**

The iterative scheme for Gauss-Seidel method is given by

$$x_1^{(n+1)} = \frac{1}{5}\left[12 + x_2^{(n)} - 2x_3^{(n)}\right]$$

$$x_2^{(n+1)} = \frac{1}{8}\left[-25 + 2x_3^{(n)} - 3x_1^{(n+1)}\right]$$

$$x_3^{(n+1)} = \frac{1}{4}\left[6 - x_1^{(n+1)} - x_2^{(n+1)}\right]$$

Now we proceed to the iteration process by taking

$$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 1$$

| $n$ | $x_1^{(n)}$ | $x_2^{(n)}$ | $x_3^{(n)}$ |
|---|---|---|---|
| 1 | 2.2 | –3.7 | 1.875 |
| 2 | 0.91 | –2.9975 | 2.021875 |
| 3 | 0.99175 | –2.99144 | 1.99992 |
| 4 | 1.00174 | –3.00067 | 1.99973 |
| 5 | 0.99997 | –3.000057 | 2.00002 |
| 6 | 0.99998 | –2.99999 | 2.000002 |

**11. Solve the following set of equations using LU factorisation method:**

$$2x_1 + x_2 + 3x_3 = 17$$
$$4x_1 - x_2 + 3x_3 = 31$$
$$-2x_1 + 5x_2 + 5x_3 = -5$$

**[WBUT 2018(EVEN)]**

**Answer:**

Given system of equations can be expressed in the following matrix form:

$$AX = B \quad \text{.... (1)}$$

where $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 17 \\ 31 \\ -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now let $A = LU$, where $L = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$, $U = \begin{bmatrix} 1 & h_{12} & h_{13} \\ 0 & 1 & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$

$$\ell_{11} = 2,\ \ell_{12} = \frac{1}{2},\ h_{13} = \frac{3}{2}$$

$$\ell_{21}=4,\ \ell_{22}=-3,\ h_{23}=1$$
$$\ell_{31}=-2,\ \ell_{32}=6,\ h_{33}=2 \qquad \text{.... (2)}$$

Now, let $UX=Y$

Therefore from Eqn. (1) we get,

$$LY=B$$

or, $$\begin{bmatrix} 2 & 0 & 0 \\ 4 & -3 & 0 \\ -2 & 6 & 7 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}=\begin{bmatrix} 17 \\ 31 \\ -5 \end{bmatrix}$$

$\therefore\ y_1=\dfrac{17}{2},\ y_2=1,\ y_2=\dfrac{6}{7}$

$\therefore$ $UX=Y$ yields;

or, $$\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} \frac{17}{2} \\ 1 \\ \frac{6}{7} \end{bmatrix}$$

or, $$z=\frac{3}{7},\ y=\frac{4}{7},\ x=\frac{53}{7}$$

**12. Solve the following system of equations by Gauss-Seidel Method:**

$$10x-y-z=13$$
$$x+10y+z=36$$
$$x+y-10z=-35$$

**Compute 3 iterations.** **[WBUT 2018(ODD)]**

**Answer:**

Herative scheme for Gauss-Seidel method is as follows:

$$x^{(n+1)}=\frac{1}{10}\left[13+y^{(n)}+z^{(n)}\right]$$
$$y^{(n+1)}=\frac{1}{10}\left[36-z^{(n)}-x^{(n+1)}\right]$$
$$z^{(n+1)}=+\frac{1}{10}\left[35+x^{(n+1)}+y^{(n+1)}\right]$$

We start with the following initial guesses

$$x^{(0)}=y^{(0)}=z^{(0)}=1$$

| $n$ | $x^{(n)}$ | $y^{(n)}$ | $z^{(n)}$ |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 1.5 | 3.35 | 3.985 |
| 2 | 2.0335 | 2.99815 | 4.003165 |
| 3 | 2.0001315 | 2.99967035 | 3.999980185 |

$\therefore\ x = 2.000,\ y = 3.000,\ z = 4.000$ (Correct upto 3 decimal places)

**13. Solve the following system of equations by LU-factorisation method, correct up to one significant figure:**

$$3x + 4y + 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 2z = 10$$

**[WBUT 2018(ODD)]**

**Answer:**

Matrix representation of the given system of equations is $AX = B$

where $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 15 \\ 18 \\ 10 \end{bmatrix}$

Also, let $A = LU$

where, $L = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$ and $U = \begin{bmatrix} 1 & h_{12} & h_{13} \\ 0 & 1 & h_{23} \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore \quad \ell_{11} = 3,\ h_{12} = \frac{4}{3},\ h_{13} = \frac{2}{3}$$

$$\ell_{21} = 5,\ \ell_{22} = \frac{-14}{3},\ h_{23} = \frac{1}{2}$$

$$\ell_{31} = 2,\ \ell_{32} = \frac{1}{3},\ \ell_{33} = \frac{1}{2}$$

Again let, $UX = Y$ (where $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ is unknown)

$$\therefore \quad AX = B \Rightarrow LY = B$$

or, $$\begin{bmatrix} 3 & 0 & 0 \\ 5 & -\frac{14}{3} & 0 \\ 2 & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 18 \\ 10 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{3}{2} \\ -4 \end{bmatrix}$$

$$\therefore \quad UX = Y \quad \Rightarrow \begin{bmatrix} 1 & \frac{4}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{3}{2} \\ -4 \end{bmatrix}$$

$$\Rightarrow \quad x = 3,\ y = \frac{7}{2},\ z = -4$$

**14. Solve the linear system of equations by matrix factorisation (LU method):**

$$3x + 2y - 4z = 12$$
$$-x + 5y + 2z = 1$$
$$2x - 3y + 4z = -3$$

**[WBUT 2019(EVEN)]**

**Answer:**

Let, $L = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

The given equation can be written as

$$A_x = B \qquad \text{.... (1)}$$

where, $A = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 5 & 2 \\ 2 & -3 & 4 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ 1 \\ -3 \end{bmatrix}$

let, $A = LU$ so that (1) becomes

$$LUx = B \qquad \text{.... (2)}$$

Which gives

$$A = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 5 & 2 \\ 2 & -3 & 4 \end{bmatrix} = LU$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ \ell_{21}u_{11} & \ell_{21}u_{12} + u_{22} & \ell_{21}u_{13} + u_{23} \\ \ell_{31}u_{11} & \ell_{31}u_{12} + \ell_{32}u_{22} & \ell_{31}u_{13} + \ell_{32}u_{23} + u_{33} \end{bmatrix} \qquad \text{.... (3)}$$

Therefore, we get

$$u_{11} = 3,\ u_{12} = 2,\ u_{13} = -4$$

$$\ell_{21}u_{11} = -1 \quad \Rightarrow \quad \ell_{21} = \frac{-1}{3}$$

$$\ell_{21}u_{12}+u_{22}=5 \Rightarrow u_{22}=5-\left(-\frac{1}{3}\times 2\right)=5+\frac{2}{3}=\frac{17}{3}$$

$$\ell_{21}u_{13}+u_{23}=2 \Rightarrow u_{23}=2-\left(-\frac{1}{3}\times -4\right)=2-\frac{4}{3}=\frac{2}{3}$$

Also, $\ell_{31}u_{11}=2 \Rightarrow \ell_{31}=\dfrac{2}{3}$

$$\therefore \quad \ell_{31}u_{12}+\ell_{32}u_{22}=-3 \Rightarrow \ell_{32}=\frac{-3-\left(\frac{2}{3}\times 2\right)}{\frac{17}{3}}=\frac{-3-\frac{4}{3}}{\frac{17}{3}}=\frac{-13}{3}\times\frac{3}{17}=\frac{-13}{17}$$

$$\therefore \quad \ell_{31}u_{13}+\ell_{32}u_{23}+u_{33}=4$$

$$\Rightarrow \quad u_{33}=4-\left(\frac{2}{3}\times -4\right)-\left(\frac{-13}{17}\times\frac{2}{3}\right)=4+\frac{8}{3}+\frac{26}{51}=\frac{204+136+26}{51}=\frac{366}{51}=\frac{122}{17}$$

Now, from (2), we can write

$LY=B$, where, $UX=Y$ and $Y=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$\therefore \quad LY=B \text{ gives } \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ \frac{2}{3} & \frac{-13}{17} & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}=\begin{bmatrix} 12 \\ 1 \\ -3 \end{bmatrix}$$

$\Rightarrow \quad y_1=12,$

$$\frac{-1}{3}y_1+y_2=1 \Rightarrow y_2=1+\frac{1}{3}\times 12=5$$

$$\frac{2}{3}y_1-\frac{13}{17}y_2+y_3=-3 \Rightarrow y_3=-3-\left(\frac{2}{3}\times 12\right)+\left(\frac{13}{17}\times 5\right)$$

$$=-3-8+\frac{65}{17}=-11+\frac{65}{17}=\frac{-187+65}{17}$$

$$\therefore \quad y_3=\frac{-122}{17}$$

$$UX = Y \Rightarrow \begin{bmatrix} 3 & 2 & -4 \\ 0 & \frac{17}{3} & \frac{2}{3} \\ 0 & 0 & \frac{122}{17} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ -3 \end{bmatrix}$$

From the last row, we get

$$x_3 = \frac{-3\times 17}{122} = \frac{-51}{122} = -0.418$$

From the second row, we have

$$\frac{17}{3}x_2 + \frac{2}{3}x_3 = 1$$

$$\Rightarrow \quad x_2 = \frac{1-\frac{2}{3}\times\left(\frac{-51}{122}\right)}{\frac{17}{3}} = \frac{\left(1+\frac{17}{61}\right)}{\frac{17}{3}} = \frac{78}{61}\times\frac{3}{17} = \frac{234}{1037} = 0.266$$

$$3x_1 + 2x_2 - 4x_3 = 12$$

$$\Rightarrow \quad x_1 = \frac{\left[12+4\times\left(\frac{-51}{122}\right)-2\times\left(\frac{234}{1037}\right)\right]}{3}$$

$$= \frac{\left[12-\frac{102}{61}-\frac{468}{1037}\right]}{3} = \frac{(12-1.672-0.451)}{3} = 3.292$$

Hence, the solution is $\begin{bmatrix} -0.418 \\ 0.226 \\ 3.292 \end{bmatrix}$

# NUMERICAL SOLUTION OF ALGEBRAIC EQUATION

## Multiple Choice Type Questions

1. Regula-Falsi Method is used to [WBUT 2007, 2015(ODD)]
   a) find the root of a system of linear simultaneous equations
   b) differentiate
   c) find the root of an algebraic or transcendental equation
   d) solve linear differential equations

Answer: (c)

2. Condition for convergence of Newton-Raphson method is [WBUT 2008, 2012(EVEN), 2014(ODD), 2019(EVEN)]

a) $|f(x).f'(x)| < \{f''(x)\}^2$
b) $|f(x).f''(x)| < \{f'(x)\}^2$
c) $|f(x).f'(x)| > \{f''(x)\}^2$
d) $|f(x).f''(x)| > \{f'(x)\}^2$

Answer: (b)

3. The method of iteration formula $\phi(x)$ must satisfy [WBUT 2009, 2013(ODD)]

a) $|\phi'(x)| < 1$ b) $|\phi'(x)| > 1$ c) $|\phi'(x)| = 1$ d) $|\phi'(x)| = 2$

Answer: (a)

4. Newton-Raphson method fails when [WBUT 2011(ODD), 2016(EVEN), 2017(ODD)]

a) $f'(x) = 1$ b) $f'(x) = 0$ c) $f'(x) = -1$ d) $f''(x) = 0$

Answer: (b)

5. Regula-Falsi method is [WBUT 2012(EVEN), 2013(ODD), 2014(EVEN), 2016(EVEN), 2017(ODD)]

a) conditionally convergent
b) linearly convergent
c) divergent
d) none of these

Answer: (d)

6. Newton-Raphson method for solution of the equation $f(x) = 0$ fails when [WBUT 2013(EVEN), 2014(EVEN)]

a) $f'(x) = 1$ b) $f'(x) = 0$ c) $f'(x) = -1$ d) none of these

Answer: (b)

7. Fixed point iteration method is [WBUT 2014(ODD)]

a) conditionally convergent
b) divergent
c) linearly convergent
d) none of these

**Answer:** (d)

8. For an equation like $x^2 = 0$, a root exists at $x = 0$. The bisection method cannot be adopted to solve this equation in spite of the root existing at $x = 0$, because the function $f(x) = x^2$ [WBUT 2015(EVEN), 2019(EVEN)]

a) is a polynomial
b) has repeated roots at $x = 0$
c) is always non-negative
d) slope is zero at $x = 0$

**Answer:** (c)

9. The Newton-Raphson Method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1, the iteration will [WBUT 2015(EVEN)]

a) converges to $-1$
b) converges to $\sqrt{2}$
c) converges to $-\sqrt{2}$
d) not convergent

**Answer:** (c)

10. The rate of convergence of bisection method is [WBUT 2015(ODD), 2016(ODD)]

a) linear
b) quadratic
c) cubic
d) none of these

**Answer:** (a)

11. Which of the following does not always guarantee convergence? [WBUT 2016(EVEN)]

a) Bisection method
b) Newton-Raphson method
c) Regula-falsi
d) none of these

**Answer:** (b)

12. The order of convergence of Newton-Raphson method is [WBUT 2016(ODD), 2018(EVEN)]

a) 3
b) 2
c) 1
d) none of these

**Answer:** (b)

13. One root of the equation $x^2 + 2x - 2 = 0$ lies between [WBUT 2017(EVEN)]

a) 1 and 2
b) 0 and 0.5
c) 0.5 and 1
d) none of these

**Answer:** (c)

14. The accuracy attainable with Newton-Raphson method does not depend upon the value of the derivative $f(x)$. The above statement is [WBUT 2017(EVEN)]

a) True
b) False

**Answer:** (a)

15. The Newton-Raphson iterative formula for finding the square root of a real number $R$ is ($i$ th iteration) [WBUT 2018(EVEN)]

a) $x_{i+1} = \dfrac{x_i}{2}$ b) $x_{i+1} = \dfrac{3x_i}{2}$ c) $x_{i+1} = \dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$ d) None of these

**Answer:** (c)

16. If for a real continuous function $f(x)$, $f(a)f(b)<0$ then in the range of $[a, b]$ for $f(x)=0$, there is / are [WBUT 2019(EVEN)]

a) one root
b) undeterminable number of roots
c) no root
d) at least one root

**Answer:** (d)

## Short Answer Type Questions

1. Evaluate $\sqrt{12}$ to three places of decimals by Newton-Raphson method. [WBUT 2007, 2015(ODD)]

**Answer:**

Let,

$\sqrt{12} = x \Rightarrow x^2 = 12$ and $f(x) = x^2 - 12$, say; $f'(x) = 2x$

We have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{\ 2} - 12}{2x_n}$$

$$= \frac{x_n^{\ 2} + 12}{2x_n} = \frac{x_n}{2} + \frac{6}{x_n}.$$

Here $f(3) = 9 - 12 = -3 < 0$ and $f(4) = 16 - 12 = 4 > 0$.

Hence one root of $f(x) = 0$ lies in the interval (3, 4).

Let $x_0 = 3$, then

$$x_1 = \frac{x_0}{2} + \frac{6}{x_0} = \frac{3}{2} + \frac{6}{3} = 1.5 + 2 = 3.5$$

$$x_2 = \frac{3.5}{2} + \frac{6}{3.5} = 3.46429$$

$$x_3 = \frac{3.46429}{2} + \frac{6}{3.46429} = 3.46411$$

$$x_4 = \frac{3.46411}{2} + \frac{6}{3.46411} = 3.46411$$

Hence the calculated value of $\sqrt{12} = 3.4641$ upto 4 places of decimal by Newton-Raphson Method.

The true value is 3.4641 upto 4 places of decimal.

**2. Derive the order of convergence for Newton-Raphson method.** **[WBUT 2011(ODD)]**

OR,

**Show that rate of convergence in Newton-Rephson method is quadratic.** **[WBUT 2015(EVEN)]**

**Answer:**

According to Newton-Raphson method the iteration is produced by the following consideration.

Let at the nth iteration $\varepsilon_n$ be the error of the equation $f(x)=0$ then

$$f(x_n+\varepsilon_n)=0 \qquad \text{.... (1)}$$

or, $f(x_n)+\varepsilon_n f'(x_n)+\frac{1}{2}\varepsilon_n^2 f''(\xi_n)=0$

where $\min\{a,x_n\}<\xi_n<\max\{a,x_n\}$

Neglecting the term containing $\varepsilon_n^2$, we get

$$f(x_n)+\varepsilon_n f'(x_n)=0$$

Again, $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$ .... (2)

Now from equations (1) and (2) we obtain,

$$\varepsilon_{n+1}=-\frac{1}{2}\varepsilon_n^2\frac{f''(\xi_n)}{f'(x_n)} \qquad \text{.... (3)}$$

This is the error equation.

If the iteration converges, $x_n,\ \xi_n \to a$, so that $\underset{n\to\infty}{Lt}\left|\dfrac{\varepsilon_{n+1}}{\varepsilon_n^2}\right|=\dfrac{1}{2}\dfrac{f''(a)}{f'(a)}$

which shows that the Newton-Raphson methods is a second-order iteration process with the asymptotic error constant $=\dfrac{f''(a)}{2f'(a)}$

**3. Find the root of the equation $x\tan x=1.28$, that lies in the interval (0, 1), correct to four places of decimal, using bisection method.** **[WBUT 2012(ODD), 2015(EVEN), 2015(ODD)]**

**Answer:**

The given equation is

$x\tan x=1.28$

*i.e.* $x\tan x-1.28=0$ ... (1)

$\therefore f(x)=x\tan x-1.28$

$\because f(0)*f(1)<0$

Hence the root lies in the interval [0, 1], where the function $f$ is continuous.

The computations are given in the following table:

| $x=\frac{(a+b)}{2}$ | $f(x)=f(\frac{(a+b)}{2})$ |
|---|---|
| x=0.500000 | f(x)=-1.006849 |
| x=0.750000 | f(x)=-0.581303 |
| x=0.875000 | f(x)=-0.232256 |
| x=0.937500 | f(x)=-0.003058 |
| x=0.968750 | f(x)= 0.129819 |
| x=0.953125 | f(x)= 0.061675 |
| x=0.945312 | f(x)= 0.028898 |
| x=0.941406 | f(x)= 0.012819 |
| x=0.939453 | f(x)= 0.004856 |
| x=0.938477 | f(x)= 0.000893 |
| x=0.937988 | f(x)=-0.001084 |
| x=0.938232 | f(x)=-0.000096 |
| x=0.938354 | f(x)= 0.000398 |
| x=0.938293 | f(x)= 0.000151 |
| x=0.938263 | f(x)= 0.000028 |
| x=0.938248 | f(x)=-0.000034 |
| x=0.938255 | f(x)=-0.000003 |

Therefore, the required root is 0.93826 correct upto 5 places of decimal.

**4. Explain the geometric interpretation of Bisection method for finding a real root of an equation.** **[WBUT 2014(EVEN)]**

**Answer:**

Let $f(x)=0$ has one root in between $x_0$ and $x_1$ i.e. $f(x_0)=0=f(x_1)$


Now we bisect the segment $(x_0,x_1)$ by the point $x=x_2$ and we have to check whether the root lie within $(x_0,x_2)$ or $(x_2,x_1)$ by the process $f(x_0)f(x_2)<0$ or $f(x_2)f(x_1)<0$ respectively. Then take that segment in which the root lie and continue the process until the length of the segment is less than the desired accuracy.

**5. Find the value of $\sqrt{2}$ using Newton-Raphson method correct up to three decimal places.** **[WBUT 2014(EVEN)]**

**Answer:**

Let $x=\sqrt{2}$

or, $x^2-2=0$

Again, we consider $f(x)=x^2-2$ $\qquad f'(x)=2x$

Now, $f(0)=-2,\quad f(1)=-1,\quad f(2)=2$

$\therefore$ The root lies in the interval (1, 2).

By Newton – Raphson method: $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)},\ (n=0,1,2,\ldots\ldots),\ f'(x_n)\neq 0$

Here we assume $x_0 = +1$ as the initial guess.

| $n$ | $x_n$ | $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n}$ |
|---|---|---|
| 0 | 1 | 1.5 |
| 1 | 1.5 | 1.4167 |
| 2 | 1.4167 | 1.4142 |
| 3 | 1.4142 | 1.4142 |

Hence the required root is $x = 1.414$ (correct up to 3 decimal places)

$$\therefore \sqrt{2} = 1.414 \text{ (up to 3 decimal places)}$$

**6. Find a real root of the equation $xe^x - 2 = 0$, correct to three decimal places using Newton-Raphson method.** **[WBUT 2016(EVEN)]**

**Answer:**

Let $f(x) = xe^x - 2$ $\quad f'(x) = e^x + xe^x = (1+x)e^x$

Now, $f(0) = -2,\ f(1) = 0.718$

$\because f(0) < 0$ and $f(1) > 0$

So, $f(x) = 0$ has at least one root lies in the interval (0, 1).

Here we start the N-R iteration process with the initial guess

$x_0 = 1$

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n = \frac{-f(x_n)}{f'(x_n)}$ |
|---|---|---|---|---|
| 0 | 1 | 0.71828 | 5.4366 | −0.1321 |
| 1 | 0.86788 | 0.06716 | 4.4490 | −0.015095 |
| 2 | 0.85278 | 0.00078 | 4.34692 | −0.000179 |
| 3 | 0.85260 | | | |

$\therefore \quad x = 0.853$ (Correct upto 3 decimal places)

**7. Using Bisection method obtain a root between 1 and 2 of the equation $e^x - 3x = 0$.** **[WBUT 2016(EVEN)]**

**Answer:**

Let $f(x) = e^x - 3x,\ a_0 = 1,\ b_0 = 2$

| $n$ | $a_n$ | $b_n$ | $x_{n+1} = \frac{a_n + b_n}{2}$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 1.5 | − 0.02 |
| 1 | 1.5 | 2 | 1.75 | + 0.504 |
| 2 | 1.5 | 1.75 | 1.625 | + 0.203 |
| 3 | 1.5 | 1.625 | 1.5625 | + 0.08 |
| 4 | 1.5 | 1.5625 | 1.53125 | + 0.03 |
| 5 | 1.5 | 1.53125 | 1.515625 | + 0.005 |
| 6 | 1.5 | 1.515625 | 1.508125 | − 0.006 |
| 7 | 1.508125 | 1.515625 | | |

$\therefore \quad x = 1.5$ (Correct upto 2 significant figures) is a root of $f(x) = 0$ in (1, 2)

**8. Find the positive real root of $x^3 - x^2 - 1 = 0$ using the bisection method of 4 iterations.** **[WBUT 2016(ODD), 2017(ODD)]**

**Answer:**

Let $f(x) = x^3 - x^2 - 1$

Now $f(0) = -1,\ f(1) = -1,\ f(2) = 3$

So $f(x) = 0$ has at least one root is $(1, 2)$.

Here we apply bisection method for $f(x) = 0$ is the interval (1, 2) as follows:

| $n$ | $a_n$ | $b_n$ | $x_{n+1} = \dfrac{a_n + b_n}{2}$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 1.5 | $+\dfrac{1}{8}$ |
| 1 | 1 | 1.5 | 1.25 | $-\dfrac{39}{64}$ |
| 2 | 1.25 | 1.5 | 1.375 | $-\dfrac{149}{512}$ |
| 3 | 1.375 | 1.5 | 1.4375 | $-\dfrac{393}{4096}$ |
| 4 | 1.4375 | 1.5 | 1.46875 | +0.011 |

Therefore, the required root is $x = 1.4$

**9. Find out the root of the following equation using Regula Falsi method: $xe^x - \cos x = 0$, that lies between 0 and 1 (correct to four decimal places).** **[WBUT 2017(EVEN), 2018(EVEN)]**

**Answer:**

Let $f(x) = x e^x - \cos x$

Let $a_0 = 0$ and $b_0 = 1$ as the initial guess.

Now $f(a_0) = -1,\quad f(b_0) = 2.17798$

Now according to Regula-Falsi method:

| $n$ | $a_n$ | $b_n$ | $f(a_n)$ | $f(b_n)$ | $x_{n+1} = a_n - \dfrac{a_n - b_n}{f(a_n) - f(b_n)} f(a_n)$ | $f(x_{n+1})$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | −1 | 2.17798 | 0.314665 | −0.519871 |
| 1 | 0.314665 | 1 | −0.519871 | 2.17798 | 0.446728 | −0.203546 |
| 2 | 0.446728 | 1 | −0.203546 | 2.17798 | 0.494015 | −0.070802 |
| 3 | 0.494015 | 1 | −0.070802 | 2.17798 | 0.509946 | −0.023609 |
| 4 | 0.509946 | 1 | −0.023609 | 2.17798 | 0.515201 | −0.007760 |
| 5 | 0.515201 | 1 | −0.007760 | 2.17798 | 0.516922 | −0.002539 |
| 6 | 0.516922 | 1 | −0.002539 | 2.17798 | 0.517484 | −0.000830 |
| 7 | 0.517484 | 1 | −0.000830 | 2.17798 | 0.517668 | −0.000272 |
| 8 | 0.517668 | 1 | −0.000272 | 2.17798 | 0.517728 | −0.000089 |
| 9 | 0.517728 | 1 | −0.000089 | 2.17798 | 0.517748 | −0.000029 |
| 10 | 0.517748 | 1 | −0.000029 | 2.17798 | | |

$\therefore x = 0.5177$ correct up to `decimal places

**10. Evaluate $\sqrt[5]{3}$ up to 5 significant figures by Newton-Raphson method.** **[WBUT 2018(EVEN)]**

**Answer:**

Let $\quad x = \sqrt[5]{3}$

or, $\quad x^5 - 3 = 0$

We consider

$$f(x) = x^5 - 3$$

$$\therefore \quad f'(x) = 5x^4$$

Now, $\quad f(0) = -3,\ f(1) = -2,\ f(2) = 29$

So $f(x) = 0$ has at least one root is (1, 2) and

$$f'(x) \neq 0\ \forall x \in (1, 2)$$

Now by N-R method (taking $x_0 = 1$) we obtain

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.4$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.276185$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.247150$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.245734$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.245731$$

$\therefore \quad x = 1.24573 \quad$ (Correct up to 5 significant figures)

**11. Using Newton-Raphson method, determine a real root of the equation $x^3 - x - 1 = 0$, correct to four significant figures.** **[WBUT 2018(ODD)]**

**Answer:**

Let $\quad f(x) = x^3 - x - 1$

$$f(0) = -1,\ f(1) = 1$$

So, $f(x) = 0$ has at least one root in the interval (0, 1)

Now, $\quad f'(x) = 3x^2 - 1 \neq \forall x \in (0, 1)$

| $n$ | $x_n$ | $h_n = +\frac{f(x_n)}{f'(x_n)}$ | $x_{n+1} = x_n - h_n$ |
|---|---|---|---|
| 1 | 0 | 1 | −1 |
| 2 | −1 | −0.5 | −0.5 |
| 3 | −0.5 | 2.5 | −3 |
| 4 | −3 | −0.96154 | −2.03846 |
| 5 | −2.03846 | −0.64818 | −2.39028 |
| 6 | −1.39028 | −0.47867 | −0.91161 |
| 7 | −0.91161 | −0.56658 | −0.34503 |
| 8 | −0.34503 | 1.08273 | −1.42776 |
| 9 | −1.42776 | −0.48533 | −0.94243 |
| 10 | −0.94243 | −0.53745 | −0.404978 |
| 11 | −0.404978 | 1.302105 | −1.707083 |
| 12 | −1.707083 | −0.551197 | −1.155886 |
| 13 | −1.155886 | −0.461557 | −0.694329 |
| 14 | −0.694329 | −1.434984 | −0.74065 |

$x = -0.6943$ (upto 4 significant figures)

**12. Evaluate the square root of 5 using the equation $x^2 - 5 = 0$ by applying the Newton Raphson method.** **[WBUT 2019(EVEN)]**

**Answer:**

$$x^2 - 5 = 0$$

We consider,

$$f(x) = x^2 - 5$$

$$f'(x) = 2x$$

Now, $f(0) = -5$, $f(1) = 1 - 5 = (-4)$, $f(2) = 4 - 5 = (-1)$, $f(3) = 9 - 5 = 4$

$\therefore$ The root lies in the interval (2, 3).

By Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{where, } [n = 0, 1, 2, \ldots\ldots]$$

$$f'(x_n) \neq 0$$

Assume,

$$x_0 = 2$$

| $n$ | $x_n$ | $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 5}{2x_n}$ |
|---|---|---|
| 0 | 2 | 2.25 |
| 1 | 2.25 | 2.236 |
| 2 | 2.236 | −2.236 |
| 3 | −2.236 | 2.236 |

Hence, the calculate value is 2.236 upto 3 decimal places by Newton Raphson method.

13. Find the root of the equation $4e^{-x}\sin x = 1$, near 0.2 by Newton-Raphson method correct to three decimal places. [WBUT 2019(EVEN)]

**Answer:**

$$f(x) = 4e^{-x}\sin x - 1 = 0$$

i.e., $f(x) = 0$, where, $f(x) = 4e^{-x}\sin x - 1$

From Newton-Raphson method, it follows that the iterative scheme is –

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\forall_n = 0, 1, 2, 3$$

$$f'(x_n) = 4\left(e^{-x}\cos x + \sin x - e^{-x}\right)$$

$$= 4e^{-x}\cos x - 4e^{-x}\sin x$$

$$= 4e^{-x}\left(\cos x - \sin x\right)$$

$$f(0) = -1$$

$$f(1) = 4e^{-1}\sin 1 - 1 = 0.238$$

So the value of $x_0$ must be between 0 and 1 i.e., 0.5

Taking $x_0 = 0.5$, we compute the successive approximations as follows:

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n = \frac{-f(x_n)}{f'(x_n)}$ | $x_{n+1} = x_n + h_n$ |
|---|---|---|---|---|---|
| 0 | 0.5 | 0.1631 | 2.425 | –0.067 | 0.433 |
| 1 | 0.433 | 0.0885 | 1.2662 | –0.069 | 0.364 |

As 0.364 is close to 0.2

$\therefore$ The answer is 0.364

## Long Answer Type Questions

**1. Prove the convergence of Newton-Raphson method. Hence find the cube root of 10 up to 5 significant figures by Newton-Raphson method. [WBUT 2013(ODD)]**

**Answer:**

**1st Part:**

Let $f(x) = 0$ be the given equation. This equation can be written in $x = \phi(x)$.

In the iterative method condition of convergence is $|\phi'(x)| < 1$. $\forall x \in (a,b)$ in which the root lies.

Now in Newton – Raphson method the iterative scheme is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

i.e., in Newton – Raphson method the equation $f(x) = 0$ is written in the following form:

$$x = \phi(x) = x - \frac{f(x)}{f'(x_n)}$$

Hence the condition of convergence is $|\phi'(x)|<1$

i.e. $|f(x)\,f''(x)|<|f'(x)|^2$. $\forall x\in(a_0,b_0)$

**2nd Part:**

Let $x=(10)^{2/3}$

or, $x^3=10$

or, $x^3-10=0$

Let $f(x)=x^3-10$

$f'(x)=3x^2$

Now, $f(2)=-2$, $f(3)=17$. So $f(x)=0$ has at least one in between 2 and 3.

Let $x_0=2$ (as an initial guess).

According to Newton – Raphson method:

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n=-\dfrac{f(x_n)}{f'(x_n)}$ |
|---|---|---|---|---|
| 0 | 2 | –2 | 12 | 0.166667 |
| 1 | 2.166667 | 0.171300 | 14.083338 | –0.012163 |
| 2 | 2.154504 | 0.00096 | 13.92566 | –0.000069 |
| 3 | 2.154435 | $5.187\times10^{-6}$ | 13.92477 | $-0.000373\times10^{-3}$ |
| 4 | 2.154435 | | | |

$\therefore x=2.1544$ (Correct upto 5 significant figures)

**2. a) Write down the advantages and disadvantages of Newton-Raphson method. Derive the order of convergence of N – R method. [WBUT 2014(EVEN)]**

**Answer:**

**1st Part:**

***Advantage:***

The convergence of Newton-Raphson method is faster as the rate of convergence is quadratic.

***Disadvantage:***

(i) In N-R method, knowledge of derivative is needed.

(ii) The method is fails when the derivative $f'(x)=0$ or is small in the nbd. of the root.

**2nd Part:** ***Refer to Question No. 2 of Short Answer Type Questions.***

**b) Find an approximate value of the root of the equation $x^2+x-1=0$ near $x=1$, using Regula – Falsi method. [WBUT 2014(EVEN)]**

**Answer:**

Let $f(x)=x^3+x-1$

$f(0)=-1,\ f(1)=1$

So, one root lie within the interval (0, 1). Take, $x_0=0,\ x_1=1$

According to Regula-Falsi method the iterative scheme is

$$x_i = x_{i-1} - \frac{f(x_{i-1})}{f(x_i)-f(x_{i-1})}(x_i - x_{i-1})\ ,\ (i=1,\ 2,\ldots\ldots)$$

| $n$ | $x_{n-1}$ | $x_n$ | $f(x_{n-1})$ | $f(x_n)$ | $x_{n+1}$ | $f(x_{n+1})$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | –1 | 1 | +0.5 | –0.375 |
| 2 | 0.5 | 1 | -0.375 | 1 | 0.636 | –0.106 |
| 3 | 0.636 | 1 | –0.106 | 1 | 0.671 | –0.027 |
| 4 | 0.671 | 1 | –0.027 | 1 | 0.680 | –0.006 |
| 5 | 0.680 | 1 | –0.006 | 1 | 0.6819 | –0.001 |
| 6 | 0.6819 | 1 | –0.001 | 1 | 0.6823 | –0.00002 |

$\therefore x = 0.682$ (correct up to 3 decimal places) is the root of the equation $f(x)=0$

**3. a) Find the root of the equation $x^3-3x-5=0$, that lies between 1 and 2, correct upto 3 decimal places using the Bisection Method. State the advantages and disadvantages of this method.** **[WBUT 2014(ODD)]**

**Answer:**

Let $f(x)=x^3-3x-5$, $\quad f'(x)=3x^2-3, \quad f''(x)=6x$

Now, $f(0)=-5$

$f(1)=-7$

$f(2)=-3$

$f(3)=13$

In the interval [1,2]

$f''(x)>0$, so $f'(x)$ is an increasing function.

Hence $f(x)=0$ has no solution in the interval [1,2],as $f(1)$ and $f(2)$ are of same sign.

***Advantages of bisection method:***

1. Bisection method is always convergent
2. In this method we have to need only signs of the functional values instead of the exact values.

***Disadvantage:***

1. Rate of convergence of this method is very slow.

**b) Find the root of the equation $3x^3-11x^2+7x+2=0$, correct to 4 decimal places, using Newton-Raphson method.** **[WBUT 2014(ODD)]**

**Answer:**

Let $f(x)=3x^3-11x^2+7x+2$

$\therefore f'(x)=9x^2-22x+7$

According to the Newton – Raphson method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n = -\dfrac{f(x_n)}{f'(x_n)}$ |
|---|---|---|---|---|
| 0 | 0 | 2 | 7 | –0.28571 |
| 1 | –0.28571 | –0.96793 | 14.02041 | 0.06904 |
| 2 | –0.21667 | –0.06364 | 12.18925 | 0.00522 |
| 3 | –0.211449 | –0.00032 | 12.05430 | 0.00003 |
| 4 | –0.211423 | | | |

Hence the required root is $x = -0.2142$ (Correct up to 4 decimal places).

**4. Find a +ve root of $x + \ln x - 2 = 0$ by Newton-Raphson method correct upto six significant figures. [WBUT 2015(EVEN), 2016(EVEN), 2017(EVEN)]**

**Answer:**

Let $f(x) = x + \ln x - 2$

$\therefore \quad f'(x) = 1 + \dfrac{1}{x}.$

Here we consider $x_0 = 0.01$, as an initial guess.

Now by Newton-Raphson method.

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h = -\dfrac{f(x_n)}{f'(x_n)}$ |
|---|---|---|---|---|
| 1 | 0.01 | –6.595 | 101 | 0.0653 |
| 2 | 0.0753 | –4.5110 | 14.2802 | 0.3159 |
| 3 | 0.3912 | –2.5473 | 3.5562 | 0.7163 |
| 4 | 1.1075 | –0.7904 | 1.9029 | 0.4154 |
| 5 | 1.5229 | –0.0565 | 1.6566 | 0.0341 |
| 6 | 1.557 | –0.0002 | 1.6423 | 0.0001 |
| 7 | 1.5571 | | | |

So, the required root is $x = 1557$ (correct upto 3 decimal places).

**5. Find a real root of the equation $x^3 = 2$ within (1, 2) by Regula Falsi method, correct up to 4 places of decimals. [WBUT 2015(ODD)]**

**Answer:**

Let $f(x) = x^3 - 2$

According to Regula-Falsi method the iterative scheme is given by,

$$x_{n+1} = a_n - f(a_n)\frac{b_n - a_n}{f(b_n) - f(a_n)}$$

Here we consider $a_0 = 1,\ b_0 = 2$.

| $n$ | $a_n$ | $b_n$ | $f(a_n)$ | $f(b_n)$ | $x_{x+1}$ |
|---|---|---|---|---|---|
| 0 | 1 | 2 | –1 | 6 | 1.14286 |
| 1 | 1.14286 | 2 | –0.50729 | 6 | 1.20968 |
| 2 | 1.20968 | 2 | –0.22986 | 6 | 1.23874 |
| 3 | 1.23874 | 2 | –0.09919 | 6 | 1.25112 |
| 4 | 1.25112 | 2 | –0.04162 | 6 | 1.25628 |
| 5 | 1.25628 | 2 | –0.01729 | 6 | 1.25915 |
| 6 | 1.25915 | 2 | –0.00365 | 6 | 1.25960 |
| 7 | 1.25960 | 2 | –0.00152 | 6 | 1.25979 |
| 8 | 1.25979 | 2 | –0.00063 | 6 | 1.25987 |
| 9 | 1.25987 | 2 | –0.00025 | 6 | 1.25990 |
| 10 | 1.25990 | 2 | –0.00009 | | |

Hence the required root $x = 1.2599$ (Correct up to 4 decimal places).

**6. Find by the method of Regula-Falsi, a positive root of $x^2 + 2x - 2 = 0$ correct up to three decimal places.** **[WBUT 2016(EVEN)]**

**Answer:**

Let $\quad f(x) = x^2 + 2x - 2 = (x+1)^2 - 3$

Now $\quad f(0) = -2,\ f(1) = 1$

So $f(x) = 0$ has at least one root in the interval (0, 1).

Now according to Regula-Falsi method (Taking $a_0 = 0$, $b_0 = 1$ as the initial guesses).

| $n$ | $a_n$ | $b_n$ | $f(a_n)$ | $f(b_n)$ | $x_{n+1} = a_n - \dfrac{a_n - b_n}{f(a_n) - f(b_n)} f(a_n)$ | $f(x_{n+1})$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | –2 | 1 | 0.66667 | –0.22222 |
| 1 | 0.66667 | 1 | –0.22222 | 1 | 0.727275 | –0.016521 |
| 2 | 0.727275 | 1 | –0.016521 | 1 | 0.731708 | –0.001187 |
| 3 | 0.731708 | 1 | –0.001189 | 1 | 0.732027 | –0.000084 |
| 4 | 0.732027 | 1 | –0.000084 | 1 | 0.732049 | –0.000005 |
| 5 | 0.732049 | 1 | –0.000005 | | | |

So, the required root is x = 0.732 (correct upto 3 decimal places).

**7. Compute one root of $x + \ln x - 2 = 0$, correct to two decimal places by using Regula falsi method.** **[WBUT 2016(ODD)]**

**Answer:**

Let $\quad f(x) = x + \ln x - 2$

Now $\quad f(0) \to -\infty,\ f(1) = -1,\ f(2) = 0.693$

So $f(x) = 0$ has at least one root in (1, 2).

The iteration scheme for Regula-Falsi method is

$$x_{n+1} = x_n - \frac{x_{n-1} - x_n}{f(x_{n-1}) - f(x_n)} f(x_n);\ n = 1, 2, 3, \ldots\ldots$$

Here we consider $x_0 = 1,\ x_1 = 2$

$$\therefore \quad x_2 = 2 - \frac{1-2}{-1-0.693} \times 0.693 = 1.591$$

$$x_3 = 1.591 - \frac{1-1.591}{-1-0.0554} \times 0.0554 = 1.560$$

$$x_4 = 1.560 - \frac{1-1.560}{-1-0.0046} \times 0.0046 = 1.557$$

$$x_5 = 1.560 - \frac{1.557-1.560}{-0.886-0.0046} \times 0.0046 = 1.560$$

Hence the required root is $x = 1.56$ (correct upto 2 decimal places).

**8. Find the root of the equation $10^x + x - 4 = 0$ where the initial value is given as 1 using Newton-Raphson Method.** **[WBUT 2016(ODD)]**

**Answer:**

Let $\quad f(x) = 10^x + x - 4$

So, $\quad f'(x) = 10^x \log_e 10 + 1$

Now $\quad f(0) = -3,\ f(1) = 7$

So $f(x) = 0$ has at least one root in (0, 1)

The iterative scheme for N–R method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)};\ n = 0, 1, 2, \ldots\ldots$$

Here we consider $x_0 = 1$.

$$\therefore \quad x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{7}{24.026} = 0.7086$$

$$x_2 = 0.7086 - \frac{f(0.7086)}{f'(7086)} = 0.7086 - \frac{1.8213}{12.7711} = 0.5655$$

$$x_3 = 0.5655 - \frac{f(0.5655)}{f'(0.5655)} = 0.5655 - \frac{0.2427}{9.4667} = 0.5399$$

$$x_4 = 0.5399 - \frac{f(0.5399)}{f'(0.5399)} = 0.5399 - \frac{0.0061}{8.9821} = 0.5392$$

So the required root is $x = 0.54$ (Correct up to 2 decimal places).

**9. Apply Newton-Raphson method to evaluate $\sqrt[3]{13}$ correct up to three places of decimal.** **[WBUT 2017(ODD)]**

**Answer:**

Let $x = \sqrt[3]{13}$

$or, x^3 - 13 = 0$

Let $f(x) = x^3 - 13$ *and* $f'(x) = 3x^2$

Let us consider $x_0 = 2$ as the initial guess for the equation $f(x) = 0$

Now by N – R method:

| $n$ | $x_n$ | $h_n = \frac{f(x_n)}{f'(x_n)}$ | $x_{n+1} = x_n + h_n$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 2 | 0.41667 | 2.41667 | 1.11400 |
| 1 | 2.41667 | −0.063582 | 2.353088 | 0.0291103 |
| 2 | 2.353088 | −0.001752 | 2.351336 | 0.0000140 |
| 3 | 2.351336 | −0.000000845 | 2.3513352 | 0.00000774 |

So the required root is $x = 2.351$ (Correct upto three decimal places)

**10. Find the smallest positive root of the given equation $3x - \cos x - 1 = 0$, correct up to three decimal places by Regula Falsi method.** **[WBUT 2017(ODD)]**

**Answer:**

Let $f(x) = 3x - \cos x - 1$

Now $f(0) = -2,\ \ f(1) = 1.459698$

So $f(x) = 0$ has at least one root lies $\dot{m}$ $(0, 1)$

Let $a_0 = 0$ *and* $b_0 = 1$ as the initial guess of Regula-Falsi Method.

| $n$ | $a_n$ | $b_n$ | $x_{m1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0.578085 | −0.103255 |
| 1 | 0.578085 | 1 | 0.605958 | −0.0004082 |
| 2 | 0.605958 | 1 | 0.607057 | −0.00016 |
| 3 | 0.607057 | 1 | 0.607100 | −0.0000047 |

Hence the required root is $x = 0.607$ (Correct upto three decimal places)

**11. Solve $x^3 - 5x = 7$ using Regula-Falsi method correct to 3 places of decimal.** **[WBUT 2018(EVEN)]**

**Answer:**

Let $f(x) = x^3 - 5x - 7$

$f(0) = -7,\ f(1) = -11,\ f(2) = -9,\ f(3) = 5$

So $f(x) = 0$ has at least one roof in (2, 3)

According to Regula-Falsi method is; we consider $a_0 = 2,\ b_0 = 3$

| $n$ | $a_n$ | $b_n$ | $x_{n+1}=\dfrac{b_n f(a_n)-a_n f(b_n)}{f(a_n)-f(b_n)}$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 2 | 3 | 2.6429 | –1.7501 |
| 1 | 2.6429 | 3 | 2.7354 | –0.20812 |
| 2 | 2.7354 | 3 | 2.74597 | –0.02424 |
| 3 | 2.74597 | 3 | 2.7472 | –0.002674 |
| 4 | 2.7472 | 3 | 2.7473 | –0.000202 |

$\therefore$ $x = 2.747$ (Correct up to 3 decimal places)

**12. Discuss advantages and disadvantages of Newton-Raphson method for finding a real root of an algebraic equation.** **[WBUT 2018(ODD)]**

**Answer:**

***Advantages:***

1. This method is the fastest convergence to the root.
2. It converges quadratic on the root.
3. This method leads to basically 'polish' a root from the other convergence technique.

***Disadvantages:***

1. This requires that we must know the derivative function $f'(x)$ of $f(x)$ in $f(x)=0$.
2. $f'(x)$ should be non-zero in the vicinity of the root.

**13. Find the positive real root of $x\log_{10}x = 1.15$, using the bisection method (perform 4 iterations).** **[WBUT 2018(ODD)]**

**Answer:**

Let $f(x) = x\log_{10}x - 1.15$

Now, $f(1) = -1.15$, $f(2) = -0.54$, $f(3) = 0.2814$

So, $f(x) = 0$ has at least one root in (2, 3).

According to Bisection method we get, (Taking $a_0 = 2$ and $b_0 = 3$):

| $n$ | $a_n$ | $b_n$ | $x_{n+1}=\frac{1}{2}(a_n+b_n)$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 2 | 3 | 2.5 | –0.155 |
| 1 | 2.5 | 3 | 2.75 | +0.058 |
| 2 | 2.5 | 2.75 | 2.625 | –0.050 |
| 3 | 2.625 | 2.75 | 2.6875 | +0.004 |
| 4 | 2.625 | 2.6875 | 2.65625 | –0.023 |

$\therefore$ $x = 2.7$ (Correct upto 2 significant figures

**14. a) Find a root of the equation $x^x + 2x - 6 = 0$, by method of bisection correct to two decimal places.** **[WBUT 2019(EVEN)]**

**Answer:**

$x^x + 2x - 6 = 0$

$f(x) = x^x + 2x - 6$

$f(0) = (-6),\ f(1) = (-3),\ f(2) = 2$

$f(x) = 0$ has at least one root is $(1, 2)$.

Here we apply for bisection method for $f(x) = 0$ is the interval $(1, 2)$ as follows:

| $n$ | $a_n$ | $b_n$ | $x_{n+1} = \dfrac{a_n + b_n}{2}$ | $f(x_{n+1})$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 1.5 | –1.1628 |
| 1 | 1.5 | 2 | 1.75 | 0.16266 |
| 2 | 1.5 | 1.75 | 1.625 | –0.5489 |
| 3 | 1.625 | 1.75 | 1.6875 | –0.2069 |
| 4 | 1.6875 | 1.75 | 1.71 | –0.0257 |
| 5 | 1.71 | 1.75 | 1.73 | 0.0411 |
| 6 | 1.71 | 1.73 | 1.72 | –0.018 |
| 7 | 1.72 | 1.72 | 1.73 | 0.011 |

Therefore, the roots correct upto two decimal places are → 1.73

**b) Discuss the advantage and disadvantage of Regula Falsi method.** **[WBUT 2019(EVEN)]**

**Answer:**

The advantages of Regula-Falsi methods are —

i) It does not require the derivative calculations

ii) This method has first order rate of convergence i.e., it is linearly convergent.

The disadvantages of Regula-Falsi method are —

i) It is used to calculate only a single unknown in the equation.

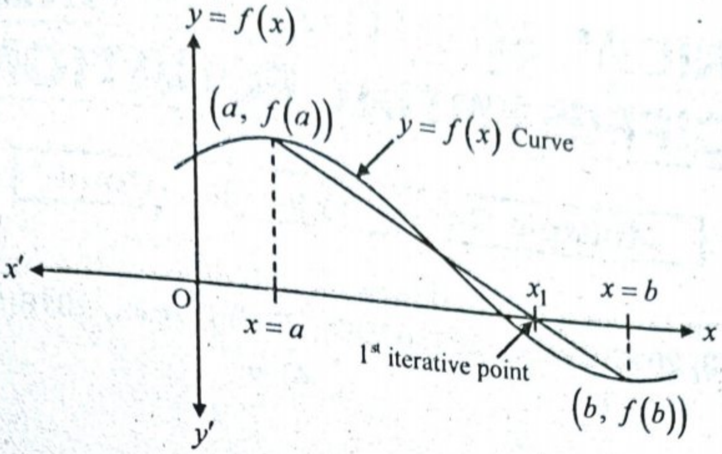
ii) As it is trial and error method in some cases it may take large time span to calculate the correct root and thereby the process make slow.

**15. Write short note on Regula-Falsi Method.** **[WBUT 2015(EVEN)]**

**Answer:**

It is one of the oldest methods to determine the roots of the algebraic or transcendental equation. In this method, at first we have to determine an interval in which the root lies, $(a, b)$, say, then join the two points whose corresponding abcisae are '$a$' and '$b$' by a straight line. The point of intersection of that straight line with the x-axis, for the equation $f(x) = 0$, will be considered as the next iterative point for the root.

**Geometrical Interpretation:**




When $|x_{n+1} - x_n| < \varepsilon$, then $x_n$ or $x_{n+1}$ will be considered as the root of the equation $f(x) = 0$ with tolerance lable $\varepsilon$.

# NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION

## Multiple Choice Type Questions

1. Runge-Kutta formula has a truncation error, which is of the order of
[WBUT 2010, 2013(EVEN), 2013(ODD), 2016(EVEN), 2016(ODD), 2019(EVEN)]

a) $h^2$ b) $h^4$ c) $h^5$ d) none of these

**Answer:** (b)

2. The finite difference method is used to solve
[WBUT 2012(ODD), 2016(ODD), 2017(ODD)]

a) a system of ordinary differential equation
b) a Boundary Value Problem
c) a partial differential equation
d) a system of transcendental equation

**Answer:** (b)

3. Runge-Kutta method is used to solve
[WBUT 2014(EVEN), 2014(ODD), 2015(EVEN)]

a) An algebraic equation
b) A first order ordinary differential equation
c) A first order partial differential equation
d) None of these

**Answer:** (b)

4. The truncation error of Euler's method is [WBUT 2014(EVEN), 2014(ODD)]

a) $O(h)$ b) $O(h^3)$ c) $O(h^2)$ d) $O(h^4)$

**Answer:** (c)

5. Which of the following are predictor-corrector methods? [WBUT 2015(EVEN)]

a) Milne's method
b) Adams Bashforth method
c) Both (a) and (b)
d) Newton's formulas

**Answer:** (c)

6. Which of the following is a multistep method? [WBUT 2016(ODD)]

a) Euler's Method
b) Predictor-corrector method
c) Taylor's series method
d) None of these

**Answer:** (b)

7. Error in 4th order Runge-Kutta method is of the order of [WBUT 2017(EVEN)]

a) $h^2$ b) $h^3$ c) $h^4$ d) none of these

**Answer:** (c)

8. Which of the following is a multistep method? [WBUT 2017(EVEN)]
a) Euler's method
b) Predictor-corrector method
c) Taylor's series method
d) None of these

**Answer:** (b)

9. To solve the ordinary differential equation [WBUT 2017(EVEN)]

$$3\frac{dy}{dx}+xy^2=\sin x,\ y(0)=5,$$

by Runge-Kutta 2nd order method, you need to rewrite the equation as

a) $\frac{dy}{dx}=\sin x-xy^2,\ y(0)=5$
b) $\frac{dy}{dx}=\frac{1}{3}\left(\sin x-xy^2\right),\ y(0)=5$
c) $\frac{dy}{dx}=\frac{1}{3}\left(-\cos x-\frac{xy^3}{3}\right),\ y(0)=5$
d) None of these

**Answer:** (b)

10. Modified Euler's method has a truncation error of the order of [WBUT 2017(ODD)]
a) $h$
b) $h^2$
c) $h^3$
d) $h^4$

**Answer:** (b)

11. Solve by using Euler's method the following differential equation for $x=1$ by taking $h=0.2,\ \frac{dy}{dx}=xy,\ y=1$ when $x=0$: [WBUT 2017(ODD)]
a) 1.5896
b) 1.4593
c) 1.3495
d) 0.4593

**Answer:** (b)

12. By Runge-Kutta method with $h=0.1$, the value of $y(0.1)$ for given $\frac{dy}{dx}=x+y,\ y=1$ when $x=0$, is [WBUT 2017(ODD)]
a) 1.11034
b) 1.15034
c) 1.22034
d) 1.23034

**Answer:** (a)

13. In Euler's method, given initial value problem $\frac{dy}{dx}=f(x,y)$ with $y(x_0)=y_0$, if $h$ is the step length, then the approximation is given by [WBUT 2018(ODD)]
a) $y_{n+1}=y_n+hf\left(x_{n-1},\ y_{n-1}\right)$
b) $y_{n+1}=y_n+hf\left(x_n,\ y_n\right)$
c) $y_{n+1}=y_n+hf\left(x_{n-1},\ y_n\right)$
d) None of these

**Answer:** (b)

14. If $\frac{dy}{dx}=\frac{y-x}{y+x}$, $y(0)=1$, $h=0.02$, then by Euler method $y(0.02)$ is [WBUT 2018(ODD)]

a) 1
b) 1.02
c) 0
d) None of these

**Answer:** (b)

15. Which one is the explicit method? [WBUT 2019(EVEN)]

a) Euler's method
b) Gauss Jacobi method
c) Gauss Seidel method
d) None of these

**Answer:** (a)

16. In Newton Cotes rules the sampling points are [WBUT 2019(EVEN)]

a) equally spaced
b) not equally spaced
c) mixed of equal spaced and not equal spaced
d) none of these

**Answer:** (a)

17. The Predictor corrector method is [WBUT 2019(EVEN)]

a) Euler's method
b) R-K $4^{th}$ order method
c) Modified Euler's method
d) none of these

**Answer:** (c)

## Short Answer Type Questions

1. Find the solution of the following differential equation by Euler's method for $x=1$ by taking $h=0.2, dy/dx=x+y$ with $y=1$ when $x=0$.

[WBUT 2012(ODD), 2013(ODD)]

**Answer:**

Here $\frac{dy}{dx}=x+y,\ x_0=0,\ y_0=1,\ h=0.2$

$$f(x,y)=\frac{dy}{dx}=y'(x)=x+y$$

$$\therefore\quad y(0.2)=y_0+hy'(0)=y_0+hf(x_0,y_0)=1+0.2(0+1)=1.2$$

$$y(0.4)=y(0.2)+hy'(0.2)=1.2+0.2\times(0.2+1.2)=1.48$$

$$y(0.6)=y(0.4)+hy'(0.4)=1.48+0.2(0.4+1.48)=1.856$$

$$y(0.8)=y(0.6)+hy'(0.6)=1.856+0.2(0.6+1.856)=2.3472$$

$$y(1)=y(0.8)+hy'(0.8)=2.3472+0.2(0.8+2.3472)=2.97664$$

2. Using Runge-Kutta method of $4^{th}$ order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$.

[WBUT 2013(EVEN), 2016(ODD)]

**Answer:**

Let $f(x,y)=\dfrac{y^2-x^2}{y^2+x^2}$

Here we consider the step length $h=0.1$

| $x$ | $y$ | $f(x,y)$ | $hf$ | |
|---|---|---|---|---|
| 0 | 1 | 1 | 0.1 | $(=k_1)$ |
| 0.05 | 1.05 | 1.1 | 0.11 | $(=k_2)$ |
| 0.05 | 1.055 | 1.111 | 0.1111 | $(=k_3)$ |
| 0.1 | 1.111 | 1.226 | 0.1226 | $(=k_4)$ |
| | | | 0.1108 | $(=k)$ |
| 0.1 | 1.1108 | 1.2257 | 0.1226 | $(=k_1)$ |
| 0.15 | 1.1721 | 1.3577 | 0.1358 | $(=k_2)$ |
| 0.15 | 1.1787 | 1.3734 | 0.1373 | $(=k_3)$ |
| 0.2 | 1.2481 | 1.5327 | 0.1533 | $(=k_4)$ |
| | | | 0.1370 | |

$\therefore y(0.2)=0.1370$

**3. Find $y(1.1)$ using Runge Kutta method of 4th order given as [WBUT 2014(EVEN)]**

$$\frac{dY}{dX}=X^2+XY,\ Y(1)=1.h=0.1$$

**Answer:**

Let, $f(x,y)=x^2+xy=x(x+y)$ and $x_0=1,\ y_0=y(x=x_0)=1$.

Now according to Runge-Kutta method:

$k_1=hf(x_0,y_0)=0.1\times1\times(1+1)=0.2$

$k_2=hf\left(x_0+\frac{h}{2},y_0+\frac{k_1}{2}\right)=0.1\times1.05\times(1.05+1.1)=0.22575$

$k_3=hf\left(x_0+\frac{h}{2},y_0+\frac{k_2}{2}\right)=0.1\times1.05\times(1.05+1.112875)=0.227101875$

$k_4=hf(x_0+h,y_0+k_3)=0.1\times1.05\times(1.1+1.227101875)=0.255981206$

$\therefore k=\frac{1}{6}(k_1+2k_2+2k_3+k_4)=0.226947492$

Hence, $y_1=y(x=1.1)=y_0+k=1.2269$ (correct up to four decimal places).

**4. Solve the equation $\dfrac{dy}{dx}=x+y$ with initial condition, y(0) = 1, taking step length 0.1 to find y(0.2) by Predictor-corrector method. [WBUT 2014(EVEN)]**

**Answer:**

Let $y_0 = 1$ at $x = x_0 = 0,\ h = 0.1$ $\qquad y_0 = y(x = x_0)$

We have to evaluate $y_2$ i.e. $y(x = x_2 = 0.2)$, with $h = 0.1$

Now to determine the value of $y_1$, we use Euler's method:

According to Euler's method: $y_{m+1} = y_m + hf(x_m, y_m)$;

$$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.1(0+1) = 1.1$$

Thus, $y_1 = 1.1$.

According to predictor-corrector method:

$y_{m+1}^{(0)} = y_{m-1} + 2hf(x_m, y_m)$

Here $f(x, y) = x + y$.

Therefore, $y_2^{(0)} = y_0 + 2hf(x_1, y_1) = 1 + 2\times 0.1 \times (0.1 + 1.1) = 1 + 0.24 = 1.24$

Now, from corrector formula: $y_{m+1}^{(i)} = y_{m-1} + \frac{h}{2}\left[f(x_m, y_m) + f\left(x_{m+1}, y_{m+1}^{(i-1)}\right)\right]$

| $i$ | $y_1^{(i)}$ |
|---|---|
| 0 | 1.24 |
| 1 | 1.132 |
| 2 | 1.1266 |
| 3 | 1.12633 |
| 4 | 1.1263165 |

Therefore, $y_1 = 1.1263$ (correct up to four decimal places).

**5. Solve by Euler's method, the equation $dy/dx = x + y, y(0) = 0$. Choose $h = 0.2$ and compute $y(0.6)$.** **[WBUT 2014(EVEN)]**

**Answer:**

Let $f(x, y) = x + y$

$\therefore y(x = x_1) = hf(x_0, y_0),\quad x_0 = 0,\ y_0 = 0$

$y(0.2) = 0.2 \times (0 + 0) = 0$

$\therefore y(0.4) = 0.2 f(x_1, y_1) = 0.2 \times 0.2 = .04$

$y(0.6) = 0.2 f(x_2, y_2) = 0.2 \times 0.04 = .008$

**6. Solve by using Modified Euler method the following differential equation for $x = 1$ by taking $h = 0.1$: $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$.** **[WBUT 2014(ODD)]**

**Answer:**

Let $f(x, y) = x + y = \frac{dy}{dx}$

Now according to modified Euler's method we get,

| $x$ | $y$ | $f(x,y)$ | $hf$ |
|---|---|---|---|
| 0 | 1 | 1 | $0.1\ (=K_1)$ |
| 0.1 | 1.1 | 1.2 | $0.12 (=K_2)$ |
| | | | 0.11 |
| 0.1 | 1.11 | 1.21 | 0.121 |
| 0.2 | 1.231 | 1.431 | 0.1431 |
| | | | 0.13205 |
| 0.2 | 1.24205 | 1.44205 | 0.144205 |
| 0.3 | 1.386255 | 1.686255 | 0.1686255 |
| | | | 0.15641525 |
| 0.3 | 1.39846 | 1.69846 | 0.169846 |
| 0.4 | 1.56831 | 1.96831 | 0.196831 |
| | | | 0.18334 |
| 0.4 | 1.58180 | 1.98180 | 0.198180 |
| 0.5 | 1.77998 | 2.27998 | 0.227998 |
| | | | 0.21309 |
| 0.5 | 1.79889 | 2.294889 | 0.2294889 |
| 0.6 | 2.02838 | 2.62838 | 0.262838 |
| | | | 0.24616 |
| 0.6 | 2.04505 | 2.64505 | 0.264505 |
| 0.7 | 2.30956 | 3.00956 | 0.300956 |
| | | | 0.32292 |
| 0.8 | 2.65070 | 3.45070 | 0.345070 |
| 0.9 | 2.99577 | 3.89577 | 0.389577 |
| | | | 0.36732 |
| 0.9 | 3.01802 | 3.91802 | 0.391802 |
| 1.0 | 3.40982 | 4.40982 | 0.440982 |
| | | | 0.416392 |
| 1.0 | 3.43441 | | |

$\therefore y(x=1.0)=3.4344$ (Correct up to 4 decimal places)

**7. Solve the equation $\dfrac{dy}{dx}=\dfrac{1}{xy}$, $y=1$ when $x=0$, for $y(0.1), y(0.2)$ and $y(0.3)$ using Runge-Kutta method of the fourth order.** **[WBUT 2014(ODD)]**

**Answer:**

Let, $f(x,y)=\dfrac{1}{xy}$. It is an ill-posed problem.

According to Runge-Kutta method:

| $x$ | $y$ | $f(x,y)$ | $hf$ |
|---|---|---|---|
| 0 | 1 | Undefined | |

Given initial condition $y(x=0)=1$ is not compatible with the given diff. equation.

8. **Compute $y(0.5)$, by Runge-Kutta method of $4^{th}$ order from the equation $\frac{dy}{dx}=\frac{1}{x+y}$, $y(0)=1$** **[WBUT 2015(EVEN)]**

**Answer:**

We know, according to Runge-Kutta method:

$$y(x_0+h)=y(x_0)+k\text{, where } k=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

and $$k_1=hf(x_0,y_0),\ k_2=hf\left(x_0+\frac{1}{2}h,\ y_0+\frac{1}{2}k_1\right)$$

$$k_3=hf\left(x_0+\frac{1}{2}h,\ y_0+\frac{1}{2}k_2\right),\ k_4=hf(x_0+h,\ y_0+h)$$

Here $f(x,y)=\frac{1}{x+y}$, $x_0=0$, $y_0=1$, take $h=0.5$

$\therefore$ $k_1=0.5\times f(0,1)=0.5$

$k_2=0.5\times f(0.25,\ 1.25)=0.3333$

$k_3=0.5\times f(0.25,\ 1.1667)=0.3529$

$k_4=0.5\times f(0.5,\ 1.3529)=0.2698$

$\therefore$ $y(0.5)=1+\frac{1}{6}(0.5+0.3333+0.3529+0.2698)=1.2427$ (upto 4-decimal places).

9. **Evaluate $y(0.02)$, given $\frac{dy}{dx}=x^2+y$, $y(0)=1$ by modified Euler's method.** **[WBUT 2016(EVEN)]**

**OR,**

**Use modified Euler's method to find the value of $y(0.02)$ by taking $h=0.01$ of the differential equation $\frac{dy}{dx}=x^2+y$, given that $y(0)=1$.** **[WBUT 2017(EVEN)]**

**OR,**

**Solve by Euler's modified method the following differential equation for $x=0.02$ by taking step length $h=0.01$, (correct to 4 decimal places)** **[WBUT 2019(EVEN)]**

$y'=x^2+y,\ y(0)=1$.

**Answer:**

Let $f(x,y)=x^2+y$, $x_0=0$, $y_0=1$, $h=0.01$

According to modified Euler's method

$$y_{n+1}=y_n+\frac{h}{2}\left[f(x_n,y_n)+f(x_n+h,\ y_n+hy_n)\right]$$

| $n$ | $y_n$ | $f(x_n, y_n)$ | $f(x_n+h, y_n+hy_n)$ |
|---|---|---|---|
| 0 | 1 | 1 | 1.0101 |
| 1 | 1.0100505 | 1.0101505 | 1.020551005 |
| 2 | 1.010153508 | | |

$\therefore \quad y(0.02) = 1.010153508$

**10. Using Euler's method obtain the solution of $\frac{dy}{dx} = x - y$, $y(0) = 1$ and $h = 0.1$ at $x = 0.5$.**

**[WBUT 2016(ODD), 2017(EVEN)]**

**Answer:**

Let $\quad f(x, y) = x - y$

According to Euler's method $= y_{m+1} = y_m + hf(x_m, y_m)$; $m = 0, 1, 2$

Here $\quad x_0 = 0,\ y_0 = 1,\ h = 0.1$

$$\therefore \quad y_1 = y_0 + 0.1 \times f(0,1) = 1 + (-0.1) = 0.9$$

$$y_2 = 0.9 + 0.1(0.1 - 0.9) = 0.9 - 0.08 = 0.82$$

$$y_3 = 0.82 + 0.1(0.2 - 0.82) = 0.82 - 0.062 = 0.758$$

$$y_4 = 0.758 + 0.1(0.3 - 0.758) = 0.758 - 0.0458 = 0.7022$$

$$y_5 = 0.7022 - 0.1(0.4 - 0.7022) = 0.7022 - 0.03022 = 0.67198$$

$$\therefore \quad y(x = 0.5) = 0.67198$$

**11. Evaluate $y(4.4)$, given $5xy' = 2 - y^2$, $y(4) = 1$, by Taylor Series Method.**

**[WBUT 2017(ODD)]**

**Answer:**

We have $5x\frac{dy}{dx} = 2 - y^2$

$$\text{or,} \quad \frac{dy}{dx} = \frac{2 - y^2}{5x} = f(x, y) \text{ say}$$

$$5y' + 5xy'' = -2yy' \qquad \Rightarrow y''(x) = \frac{-2yy' - 5y'}{5x}$$

and $\quad 5y'' + 5y'' + 5xy''' = -2y'^2 - 2yy'' \Rightarrow y'''(x) = \frac{-2y'^2 - 2yy'' - 10y''}{5x}$

Given $\quad y(4) = 1$

Let $\quad x_0 = 4 \text{ and } h = 0.2$

Now $\quad y(x_0 + h) = y(x_0) + h\,y'(x_0) + \frac{1}{2}h^2 y''(x_0) + \frac{h^3}{3!}y'''(x_0) + \cdots$

$$= 1 + 0.2 \times 0.05 - \frac{(0.2)^2}{2} \times 0.0175 - \frac{(0.2)^3}{6} \times 0.0090875 = 1.009637883$$

Again $y(x_1+h)=y(x_1)+h\,y'(x_1)+\frac{1}{2}h^2\,y''(x_1)+\frac{h^3}{3!}y'''(x_1)$ +.....

$$=1.009637883+0.2\times 0.4716010079-\frac{(0.2)^2}{2}\times 0.01576332155$$

$$-\frac{(0.2)^3}{6}\times 0.5393157036$$

$$=1.103570909$$

$\therefore y(4.4)=1.103570909$

**12. Solve the following system, by Gauss-Seidel method,**

$$20x+5y-2z=14$$
$$3x+10y+z=17$$
$$x-4y+10z=23$$

**[WBUT 2019(EVEN)]**

**Answer:**

$$20x+5y-2z=14$$
$$3x+10y+z=17$$
$$x-4y+10z=23$$

First we make the system of equation diagonally dominant,

Then, $20x+5y-2z=14$

$$3x+10y+z=17$$
$$x-4y+10z=23$$

Now, the Guass Seidel formula is given by,

$$x^{(k+1)}=\frac{1}{20}\left[14-5y^k+2z^k\right]$$

$$y^{(k+1)}=\frac{1}{10}\left[17-3x^{k+1}-z^{k+1}\right]$$

$$z^{(k+1)}=\frac{1}{10}\left[23-x^{k+1}+4y^{k+1}\right]$$

Let, the initial approximation to the root be, $x^{(0)}=0,\ y^{(0)}=0,\ z^{(0)}=0$

**Computational Table:**

| $k$ | $x^k$ | $y^k$ | $z^k$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0.7 | 1.49 | 2.82 |
| 2 | 0.609 | 1.60 | 2.87 |
| 3 | 0.587 | 1.23 | 2.733 |
| 4 | 0.665 | 1.227 | 2.724 |
| 5 | 0.665 | 1.228 | 2.724 |
| 6 | 0.665 | 1.228 | 2.724 |

Hence, the roots correct upto 3 decimal places are,

$$\left.\begin{aligned}x&=0.665\\y&=1.228\\z&=2.724\end{aligned}\right\}\quad \text{(Ans.)}$$

## Long Answer Type Questions

**1. Find the solution of the following differential equation by Euler's method for $x=1$, by taking $h=0.2$, $\frac{dy}{dx}=xy$, with $y=1$ when $x=0$.** **[WBUT 2007, 2013(ODD)]**

**Answer:**

$\frac{dy}{dx}=xy \quad y(0)=1, \quad h=0.2$

$y(0.2)=y(0)+0.2\times y'(0)=1$

$y(0.4)=y(0.2)+0.2y'(0.2)=1.04$

$y(0.6)=y(0.4)+0.2y'(0.4)=1.1232$

$y(0.8)=y(0.6)+0.2y'(0.6)=1.257984$

$y(1.0)=y(0.8)+0.2y'(0.8)=1.45926144\approx 1.46$ (Correct upto 2 decimal places)

**2. a) Apply Milne's method to find $y(0.8)$ for the equation $\frac{dy}{dx}=x+y^2$, given that $y(0)=0$, $y(0.2)=0.02$, $y(0.4)=0.0805$, $y(0.6)=0.1839$.** **[WBUT 2013(EVEN), 2017(ODD)]**

**Answer:**

Here we consider, $f(x)=x+y^2$

According to the problem, let us consider,

$x_0=0,\ x_1=0.2,\ x_2=0.4,\ x_3=0.6,\ x_4=0.8$

and $y_0=0,\ y_1=0.02,\ y_2=0.0805,\ y_3=0.1839,\ y_4=?$

Now from the predictor formula, we get,

$$y_{i+1}=y_{i-3}+\frac{4h}{3}\left(2f_{i-2}-f_{i-1}+2f_i\right),\ i\ge 3$$

$$\therefore y_4=y_0+\frac{4h}{3}\left(2f_1-f_2+2f_3\right)=0+\frac{4\times 0.2}{3}\left(2\times 0.2004-0.4065+2\times 0.6338\right)=0.3365$$

Now by Milne's corrector formula we obtain,

$$y_{i+1}^{(m+1)}=y_i+\frac{h}{3}\left[f_{i-1}+4f_i+f(x_{i+1},y_{i+1}^{(m)})\right]\ (m=0,1,2,\ldots\ldots)$$

Taking $y_4^{(0)}=0.3365$ we get,

$$y_4^{(1)}=y_3+\frac{0.2}{3}\left[f_2+4f_3+f(x_4,y_4^{(0)})\right]=0.4409$$

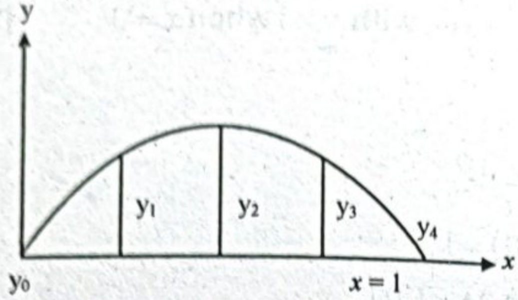
$y_4^{(2)}=0.4463$

$y_4^{(3)}=0.4463$

$\therefore y(0.8)=0.4463$ (correct upto 4 decimal places)

**b) Using finite difference method solve the boundary value problem:**
$\frac{d^2y}{dx^2}+y+1=0$ **with** $y(0)=0,\ y(1)=0$. **[WBUT 2013(EVEN), 2014(EVEN), 2016(ODD)]**

**Answer:**




We have, $\frac{d^2y}{dx^2}+y+1=0,\ y(0)=0,\ y(1)=0$

We divide the interval (0, 1) into 4 parts (i.e. $n=4$) by taking $h=0.25$

Now replacing $\frac{d^2y}{dx^2}$ by $\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$

We obtain the following difference equation

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}+y_i+1=0$$

For $x=0.25 : \frac{1}{h^2}(y_2-2y_1+y_0)+y_1+1=0$

For $x=0.5 : \frac{1}{h^2}(y_3-2y_2+y_1)+y_2+1=0$

For $x=0.75 : \frac{1}{h^2}(y_4-2y_3+y_2)+y_3+1=0$

Solving these three equations for $y_1, y_2, y_3$ we get,

$y_1=-0.7060,\ y_2=-0.4302,\ y_3=-0.1899$

**3. a) Solve the equation** $\frac{dy}{dx}=\frac{1}{x+y}, y(0)=1$, **for** $y(0.1)$ **and** $y(0.2)$, **using Runge-Kutta method of the fourth order.** **[WBUT 2013(ODD), 2015(ODD), 2016(EVEN)]**

**Answer:**

$$\frac{dy}{dx}=f(x,y)=\frac{1}{x+y}, x_0=0, y_0=1, h=0.1$$

$$k_1=hf(k_0,y_0)=0.1\times\frac{1}{0+1}=0.1$$

$$k_2=hf\left(k_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)=0.1\times f(0.05,1.05)=0.09091$$

$k_3 = 0.1 \times f(0.05, 0.04545) = 0.1 \times \dfrac{1}{0.05 + 1.04545} = 0.09129$

$k_4 = hf(k_0 + h, y_0 + k_3) = 0.1 \times f(0.1, 1.09129) = 0.08394$

To find $y(0.2)$,

$y_1 = y(0.1)$ and $x_1 = 0.1, h = 0.1$

$= 1.0914$

$k_1 = hf(x_1, y_1) = 0.1 \times f(0.1, 1.0914) = 0.08393$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.1 \times f(0.15, 1.13337) = 0.07792$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.1 \times f(0.15, 1.13036) = 0.07810$$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.1 \times f(0.2, 1.16950) = 0.07302$

$$y_2 = y(x = 0.2) = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.16957 = 1.1696$$

**b) Given $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{x^2}, y(1) = 1$. Evaluate $y(1.2)$ by modified Euler's method correct up to 4 decimal places. [WBUT 2013(ODD), 2015(EVEN), 2015(ODD), 2018(ODD)]**

**Answer:**

$\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{1}{x^2}, \; y(1) = 1$

$$f(x,y) = \frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} = \frac{1 - xy}{x^2}$$

Modified Eular's method, $y_{m+1} = y_m + h\left[\dfrac{f(x_m, y_m) + f(x_{m+1}, y_{m+1}^{(1)})}{2}\right]$

Here $x_0 = 1, y_0 = 1, h = 0.2$,

$$y_1 = y_0 + h\left[\frac{f(x_0, y_0) + f(x_1, y_1^{(1)})}{2}\right]$$

$y_1^{(1)} = y_0 + hf(x_0, y_0) = 1 + 0.2 \times (0) = 1, f(x_0, y_0) = 0$

$$y_1 = 1 + 0.2\left[\frac{0 + f(1.2, 1)}{2}\right]$$

$$f(1.2, 1) = \frac{1 - (1.2)(1)}{(1.2)^2} = -0.13889$$

$$y_1 = 1 + 0.2\left[\frac{-0.13889}{2}\right] = 0.986111$$

**4. a) Compute $y(0.3)$ by Milne's predictor-corrector method from the equation $\frac{dy}{dx} = xy + y^2$ given that $y(0) = 1,\ y(0.1) = 1.1169$, $y(0.2) = 1.2773$.**

**Answer:** **[WBUT 2015(EVEN), 2018(ODD)]**

Let $f(x,\ y) = xy + y^2$

Now according to Milne's predictor:

$$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_{i-2} - f_{i-1} + 2f_i) \quad \ldots.(1)$$

Given, $y(0) = 1,\ y(0.1) = 1.1169,\ y(0.2) = 1.2773$

We consider, $x_0 = 0,\ x_1 = 0.1,\ x_2 = 0.2$ and $x_3 = 0.3$

$$\therefore \quad y_3 = y_{-1} + \frac{4h}{3}(2f_0 - f_1 + 2f_2)$$

$$= -0.1 + \frac{4 \times 0.1}{3}\left(2f(x_0,\ y_0) + f(x_1,\ y_1) + 2f(x_2,\ y_2)\right)$$

According to Milne's method (order 4) four prior values are needed, but only three point values are given. Now to calculate $y_{-1}$ i.e., $y(x = -0.1)$ we use Euler's method as follows:

$$y(-0.1) = -hf(x_0,\ y_0) = -0.1 \times f(0,\ 1) = -0.1 \times 1 = -0.1$$

$$= -0.1 + \frac{0.4}{3}(2 \times 1 + 1.3592 + 2 \times 1.8870) = 0.5995$$

Now by corrector formula:

$$y_{i+1}^{(m+1)} = y_i + \frac{h}{3}\left[f_{i-1} + 4f_i + f\left(x_{i+1},\ y_{i+1}^{(m)}\right)\right]$$

| | $m$ | $y_{i+1}^{(m)}$ | $y_{i+1}^{(m+1)}$ |
|---|---|---|---|
| $\boxed{i=2}$ | 0 | 0.5995 | 1.60539 |
| | 1 | 1.60539 | 1.68938 |
| | 2 | 1.68938 | 1.69945 |
| | 3 | 1.69945 | 1.70068 |
| | 4 | 1.70068 | 1.70083 |
| | 5 | 1.70083 | 1.70085 |

Hence $y(0.3) = 1.7008$ (correct upto 4 decimal places).

**b) Using Runge-Kutta method of order 4 to obtain the solution of $\frac{dy}{dx} = 2x + y^2$, $y(0) = 1$ and $h = 0.1$ at $x = 0.2$.** **[WBUT 2015(EVEN)]**

**Answer:**

Let $f(x,\ y) = 2x + y^2;\ x_0 = 0,\ y_0 = 1,\ h = 0.1$

According to Runge-Kutta method:

| $x$ | $y$ | $f(x, y)$ | $hf$ |
|---|---|---|---|
| 0 | 1 | 1 | 0.1 $(=k_1)$ |
| 0.05 | 1.05 | 1.2025 | 0.12025 $(=k_2)$ |
| 0.05 | 1.060125 | 1.223865 | 0.1223865 $(=k_3)$ |
| 0.1 | 1.1223865 | 1.459751 | 0.1459751 $(=k_4)$ |
| | | | 0.7312481 $(=k)$ |
| 0.1 | 1.121875 | 1.458603 | 0.1458603 |
| 0.15 | 1.194805 | 1.727560 | 0.1727560 |
| 0.15 | 1.208253 | 1.759875 | 0.1759875 |
| 0.2 | 1.297862 | 2.084447 | 0.2084447 |
| 0.2 | 1.297174 | | 1.051792 |

$\therefore y(0.2) = 1.297175 \approx 1.2972$ (upto 4 decimal places)

**5. Solve by method of finite difference for $h = 0.25$.** **[WBUT 2015(ODD)]**

$\frac{d^2y}{dx^2} + y = 0,\ y(0) = 0,\ y(1) = 1$

**OR,**

**Using finite different method, solve the boundary value problem $\frac{d^2y}{dx^2} + y = 0$ with $y(0) = 0,\ y(1) = 1$ taking $h = 0.25$.** **[WBUT 2017(ODD)]**

**Answer:**

$\frac{d^2y}{dx^2} + y = 0 \qquad y(0) = 0,\quad y(1) = 1$

According to finite difference method $\frac{d^2y}{dx^2}$ will be replaced by

$$\frac{y_{i+1} - 2y_i - y_{i-1}}{h^2} \qquad \ldots (1)$$

Therefore, from equation (1) we get,

$$y_{i+1} = 2y_i - y_{i-1} + h^2 y_i = 0 \qquad \ldots (2)$$

At $i = 1$

$x = 0.25$ $\quad y_2 - 2y_1 - y_0 + 0.0625\ y_1 = 0$

$or,\ y_2 - (2 - 0.0625)y_1 - y_0 = 0$

$or,\ y_2 = 1.9375\, y_1\ [\because y_0 = 0]$

At $i = 2$

$x = 0.5$

$y_3 - 2y_2 - y_1 + 0.0625\ y_2 = 0$

$or,\ y_3 = (2 - 0.0625)y_2 + y_1$

$= (3.7539 + 1)y_1 \quad = 4.7539\, y_1$

$At\ i=3$

$x=0.75 \qquad y_4-2y_3-y_2+0.0625\,y_3=0$

$$or,\ y_4=(2-0.0625)y_3+y_2$$

$$=(7.2732+4.7539)y_1 \quad =12.0271y_1$$

Now, $y_4=1 \quad y_1=0.0831$

$y_2=0.1611$

$y_3=0.3950$

**6. Apply Finite difference method to solve the equation $\frac{d^2y}{dx^2}=3x+4y$, subject to the conditions $y(0)=0,\ y(1)=1$ by taking the mesh length $h=\frac{1}{4}$.**

**[WBUT 2016(EVEN)]**

**Answer:**

We have $\frac{d^2y}{dx^2}-4y=3x$ .... (1)

Now, to replace the derivative in (1) by its finite difference expressions, rewrite eqn. (1) at $x=x_i$ as $y_i-4y_i=3x_i$ .... (2)

Substitution of the difference ratio in the above equation yields the following equation:

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}-4y_i=3x_i \qquad .... (3)$$

for $\quad i=1,\ 2,\ 3$

and $\quad x_0=0,\ x_i=x_0+ih,\ h=0.25$

For $\quad i=1;$

$$\frac{y_2-2y_1-y_0}{(0.15)^2}-4y_1=0$$

or, $\quad 16y_2-28y_1-16y_0=0$

or, $\quad 4y_2-7y_1-4y_0=0$ .... (4)

or, $\quad 4y_2=7y_1 \qquad [\because\ y_0=0]$

For $\quad i=2;$

$$\frac{y_3-2y_2+y_1}{(0.25)^2}-4y_2=3\times0.25$$

or, $\quad 64y_3-144y_2+64y_1=3$

or, $\quad 64y_3-252y_1+64y_1=3$

or, $\quad 64y_3-188y_1=3$ .... (5)

For $\quad i=3;$

$$\frac{y_4-2y_3+y_2}{(0.25)^2}-4y_3=3\times 0.5$$

$$32y_4-72y_3+32y_2=3 \qquad [\because y_4=1]$$

or, $\quad -72y_3+56y_1=-29$

or, $\quad 72y_3-56y_1=29 \qquad \text{.... (6)}$

or,

Solving Eqns. (5) and (6) we get, $y_3=\dfrac{1321}{2488}, y_1=\dfrac{22960}{139328}$

Thus, $\quad y_0=0,\ y_1=0.1648,\ y_3=0.5309,\ y_4=1$

**7. Compute $y(0.8)$, by Runge-Kutta method correct up to five decimal places from the equation $\dfrac{dy}{dx}=xy,\ y(0)=2$ taking $h=0.2$. [WBUT 2017(EVEN), 2019(EVEN)]**

**Answer:**

Let $f(x,y)=xy \quad ; \quad x_0=0, \quad y_0=2,\ h=0.2$

Now according to Runge – Kutta method (4$^{th}$ order) we obtain:

$$K_1=h\,f(x_0,y_0)=0$$

$$K_2=h\,f\left(x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right)=0.04$$

$$K_3=h\,f\left(x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right)=0.0404$$

$$K_4=h\,f(x_0+h,\ y_0+k_3)=0.081616$$

$$\therefore y_1=y_0+\frac{1}{6}(k_1+2k_2+2k_3+k_4)=2.040536$$

Again for $y_2$; *i.e.* $\quad y(x_0+2h)$:

$$K_1=h\,f(x_1,y_1)=0.408107$$

$$K_2=h\,f\left(x_1+\frac{h}{2},\ y_1+\frac{k_1}{2}\right)=0.134676$$

$$K_3=h\,f\left(x_1+\frac{h}{2},\ y_1+\frac{k_2}{2}\right)=0.126472$$

$$K_4=h\,f\left(x_1+h,\ y_1+k_3\right)=0.173361)$$

$$\therefore y_2=y_1+\frac{1}{6}(K_1+2K_2+2K_3+K_4)=2.224497$$

For $y_3$:

$$K_1=h\,f(x_2,y_2)=0.177960$$

$$K_2 = h\,f\left(x_2+\frac{h}{2},\, y_2+\frac{K_1}{2}\right)=0.231348$$

$$K_3 = h\,f\left(x_2+\frac{h}{2},\; y_2+\frac{K_2}{2}\right)=0.234017$$

$$K_4 = h\,f\left(x_2+h,\; y_2+K_3\right)=0.295022$$

$$\therefore y_3 = y_2+\frac{1}{6}(K_1+2K_2+2K_3+K_4)=2.926553$$

For $y_4$ :

$$K_1 = h\,f(x_3, y_3)\; = 0.351162$$

$$K_2 = h\,f\left(x_3+\frac{h}{2},\, y_3+\frac{K_1}{2}\right)=0.434271$$

$$K_3 = h\,f\left(x_3+\frac{h}{2},\, y_3+\frac{K_2}{2}\right)=0.440088$$

$$K_4 = h\,f(x_3+h,\, y_3+K_3)=0.538631$$

$$\therefore y_4 = y_3+\frac{1}{6}(K_1+2K_2+2K_3+K_4)=3.36610477$$

$\therefore\;\; y(0.8)=3.36610\quad$ (Correct up to 5 decimal places).

**8. Using** $R.K.$ **4th order method, solve the differential equation to find** $y(0.2)$**:**
$\frac{dy}{dx}=xy,\; y(0)=1$ **taking** $h=0.1$ **[WBUT 2017(ODD), 2018(EVEN)]**

**Answer:**

Let $\quad f(x,y)=xy,\quad x_0=0,\;\; y_0=1,\quad h=0.1$

Now, $\quad k_1 = h\; f(x_0,y_0)=0$

$$k_2 = h\,f\left(x_0+\frac{h}{2},\, y_0+\frac{k_1}{2}\right)=0.005$$

$$k_3 = h\,f\left(x_0+\frac{h}{2},\, y_0+\frac{k_2}{2}\right)=0.0050125$$

$$k_4 = h\,f(x_0+h,\; y_0+k_3)=0.010050125$$

$$\therefore y_1 = y_0+\frac{1}{6}\,(k_1+2k_2+2k_3+k_4)=1.005012521$$

Again,

$$k_1 = h\,f(x_1,y_1)=0.01005012521$$

$$k_2 = h\,f\left(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}\right)=0.01515056375$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.000264381$$
$$k_4 = hf(x_1 + h, y_1 + k_3) = 0.00020629$$
$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= 1.01186238$$

**9. Solve $\frac{dy}{dx} = 1 - y,\ y(0) = 0$ using Euler's method. Find $y$, at $x = 0.1$ and $x = 0.2$. Compare the result with result of exact solution. [Take $h = 0.1$] [WBUT 2018(ODD)]**

**Answer:**

Let $f(x, y) = 1 - y;\quad x_0 = 0,\ y_0(x = x_0) = 0$

According to Euler's method:

$$y_{n+1} = y_n + hf(x_n, y_n);\ n = 0, 1, 2, \ldots\ldots$$
$$\therefore\quad y_1 = y_0 + 0.1 \times f(0, 0) = 0 + 0.1 \times 1 = 0.1$$
$$y_2 = y_1 + 0.1 \times f(x_1, y_1) = 0.1 + 0.1 \times (1 - 0.1) = 0.19$$
$$\therefore\quad y(x = 0.1) = 0.1 \quad \text{and} \quad y(x = 0.2) = 0.19$$

**10. Write short note on Milne's Method. [WBUT 2015(EVEN)]**

**Answer:**

This is one of the multistep method for solving an ODE with initial conditions. In this method, there are two recursive formula, 1[st] formula predicts the initial value of the required value of $y$ at $x = x_3$ from the differential equation.

$$\frac{dy}{dx} = f(x, y)$$

where $y(x_{-1})$, $y(x_0)$, $y(x_1)$ and $y(x_2)$ are prescribed.

That is this method depends on four prior values, so this is called a 4[th] order formula and $y(x_3)$ will be evaluated from the following equation.

$$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_{i-2} - f_{i-1} + 2f_i),\ i \geq 2 \qquad \ldots. (1)$$

This calculated value will be treated as the 1[st] iterative value for $y(x_3)$ i.e. $y^{(0)}_{(x_3)}$

Then by another formula, called corrector formula, we try to improve that initial value.

The corrector formula for Milne's method is

$$y^{(m+1)}_{i+1} = y_i + \frac{h}{3}\left[f_{i-1} + 4f_i + \left(x_{i+1},\ y^{(m)}_{i+1}\right)\right]$$

where $h$ is the step length of $x_i$.

# ERROR

## Multiple Choice Type Questions

1. The no. of significant digits in 1.00234 is [WBUT 2007, 2015(EVEN), 2017(ODD)]

a) 4 b) 6 c) 3 d) 5

**Answer:** (b)

2. The ratio of absolute error of the true value is called [WBUT 2010, 2013(ODD)]

a) relative error b) absolute error
c) truncation error d) inherent error

**Answer:** (a)

3. The percentage error in approximating 4/3 to 1.3333 is [WBUT 2010, 2013(ODD), 2017(ODD)]

a) 0.0025% b) 25% c) 0.00025% d) 0.25%

**Answer:** (a)

4. The percentage error in approximation 5/3 to 1.6667 is [WBUT 2014(EVEN)]

a) 0.06% b) 0.006% c) 0.6% d) 6%

**Answer:** None of these

5. If $E_a$ is the absolute error in a quantity whose true and approximate value are given by $x_t$ and $x_a$, then the relative error is given by [WBUT 2014(ODD)]

a) $\left|\dfrac{E_a}{x_a}\right|$ b) $\left|\dfrac{E_a}{x_t}\right|$ c) $\left|\dfrac{E_a}{x_t - x_a}\right|$ d) $|E_a|$

**Answer:** (b)

6. The relation between true value, approximate value and error is [WBUT 2015(EVEN), 2019(EVEN)]

a) true value = approximate value + error
b) approximate value = true value + error
c) approximate value = true value /error
d) none of these

**Answer:** (a)

7. If $\dfrac{11}{7}$ is approximated as 1.5714, then absolute error is [WBUT 2015(EVEN), 2019(EVEN)]

a) 0.0000256 b) 0.0000278 c) 0.00002857 d) 0.00001234

**Answer:** (c)

**8. If $A$ be the actual value and $T$ be its estimated value, the formula for relative error is** **[WBUT 2015(EVEN), 2015(ODD), 2016(ODD)]**

a) $\frac{A}{T}$  b) $\frac{A-T}{T}$  c) $\frac{|A-T|}{A}$  d) $\frac{|A-T|}{T}$

**Answer:** (c)

**9. The percentage error if 1/3 is approximated by 0.333 is** **[WBUT 2017(EVEN)]**

a) 0.1  b) 0.11  c) 0.01  d) 1.01

**Answer:** (a)

**10. Which of the following is not a computational error?** **[WBUT 2017(ODD)]**

a) Truncation error  b) Round-off error
c) Inherent error  d) None of these

**Answer:** (c)

**11. Let $a$ be the actual value and $e$ be its estimated value, the formula for relative error is** **[WBUT 2018(EVEN)]**

a) $\frac{a}{e}$  b) $\frac{|a-e|}{e}$  c) $\frac{(a-e)}{a}$  d) $\frac{|a-e|}{a}$

**Answer:** (d)

**12. The error which is inherent in a numerical method itself, is called computational error.** **[WBUT 2018(ODD)]**

a) True  b) False  c) None of these

**Answer:** (b)

**13. In Trapezoidal Rule, the order of $h$ in the total error is** **[WBUT 2019(EVEN)]**

a) 3  b) 4  c) 2  d) none of these

**Answer:** (c)

## Short Answer Type Questions

**1. Find the maximum absolute error in computing $u=\frac{x^3y^2}{z}$, when $x=y=z=0.1$ and $\Delta x=\Delta y=\Delta z=0.002$.** **[WBUT 2014(EVEN)]**

**Answer:**

Absolute error $(E_A)=\left|\frac{(x+\Delta x)^3(y+\Delta y)^2}{z+\Delta z}-\frac{x^3y^2}{z}\right|=\left|\frac{(x+0.002)^3(y+0.002)^2}{z+0.002}-\frac{x^3y^2}{z}\right|$

Therefore, absolute error at $x=y=z=0.1$ is

$$\left|\frac{(0.1+0.002)^3(0.1+0.002)^2}{0.1+0.002}-\frac{(0.1)^3(0.1)^2}{0.1}\right|=0.000008243$$

**2. Explain the difference between Round-Off and Chopping.** **[WBUT 2014(ODD)]**

**Answer:**

If a number is correct up to q decimal places, it must be borne in mind that the digit in the qth place of decimal is not exact but affected with round-off error which lies within the limits $\pm\frac{1}{2}$

For example, the number 31.520457......rounded off to 5, 4 and 3 decimal places are respectively

31.52046, 31.5205, 31.520

But if the same number is chopped to 5, 4 and 3 decimal places are respectively

31.52045, 31.5204, 31.520

So, after chopping the digit in the qth place of decimal is exact but the error does not always lie within the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$.

**3. Determine the absolute error $E_A$ of the following approximate number given their relative error, $x_A = 67.84$, $E_R = 1\%$.** **[WBUT 2015(ODD)]**

**Answer:**

We know, $E_R = \frac{E_A}{x_A}$ or, $E_A = E_R \times x_A = 0.01 \times 67.84 = 0.6784$

**4. a) Find the truncation error in the result of the following function for $x = \frac{1}{5}$ when the first three terms used.** **[WBUT 2019(EVEN)]**

$$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + x^6/6!$$

**Answer:**

$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + x^6/6!$

This is the Maclaurin series of $f(x) = e^x$ and $n = 6$

Thus at $x = \frac{1}{5}$ the bound of the truncation error is

$$R_6 \le \left|\frac{e^x}{(6+1)!}x^{(6+1)}\right| = \left|\frac{e^{1/5}}{7!}(1/5)^7\right| \approx 0.3102 \times 10^{-8}$$

**b) State the difference between round off error and truncation error.** **[WBUT 2019(EVEN)]**

**Answer:**

**Round-off error:** If a number is correct up to $q$ decimal places, it must be borne in mind that the digit in the $q^{th}$ place of decimal is not exact but affected with round-off errors which lies within the limit $+\frac{1}{2}$.

For example, the number 31.520457 …….. rounded off to 5.4 and 3 decimal places are respectively.

31.52046, 31.5205, 31.520

**Truncation Error:** In numerical computation, when any number is truncated after a certain decimal places then the error occurs in this process is termed as truncation error. Suppose $x$ is approximated by $x^*$ then the error is defined by $\varepsilon = x - x^*$.

## Long Answer Type Questions

**1. Write short notes on any three of the following:** **[WBUT 2015(EVEN)]**
**a) Truncation, Percentage and Absolute Error**
**b) Error in Interpolating polynomial**

**Answer:**

**a) Truncation, Percentage and Absolute Error:**

- **Truncation Error:** In numerical computation, when any number is truncated after a certain decimal places then the error occurs in this process is termed as truncation error. Suppose $x$ is approximated by $x^*$ then the error is defined by $\varepsilon = x - x^*$.
- **Absolute Error:** Absolute value of the error is numerical computation i.e., the difference between the exact number and the corresponding approximated number is called the absolute error. So absolute error $= |x - x^*|$.
- **Percentage Error:** Percentage error $= \dfrac{|x - x^*|}{x} \times 100$.

**b) Error in Interpolating polynomial:**

Let $p_n(x)$ be the polynomial of degree n interpolating the data

| $x$ | $x_0$ | $x_1$ | $x_2$ | … | … | … | …. | $x_n$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $y_0$ | $y_1$ | $y_2$ | …. | …. | …. | ….. | $y_n$ |

Assume that the data are given by a function $y = f(x)$ with the property that $y_i = f(x_i)$ and $x_i \in [a, b]$ for $i = 0; 1; ……; n$. The question that we consider here is: how accurately does the polynomial $p_n(x)$ approximate the function $f(x)$ at any point $x$? Let the evaluation point be $x$ and let all the nodes $\{x_i\}_{i=0}^{n}$ lie in a closed interval $[a, b]$. Then, as we shall prove shortly, if the function $f$ has $n+1$ continuous derivatives on the interval $[a, b]$, the error expression takes the form

$$f(x) - p_n(x) = \frac{\omega_{n+1}(x)}{(n+1)!} f^{(n+1)}(\xi_x), \qquad \text{…. (1)}$$

where $\omega_{n+1}(x) \equiv (x - x_0)(x - x_1)……(x - x_n) = \prod_{j=0}^{n}(x - x_j)$, and $\xi_x$ is some (unknown) point in the interval $[a, b]$. The precise location of this point depends on $\{x_i\}_{i=0}^{n}$. Here $f^{(n+1)}(\xi_x)$ is the $(n+1)^{st}$ derivative of $f(x)$ evaluated at the point $x = \xi_x$.

To prove the desired error expression (1), note first that the result is trivially true when $x$ is any node $x_i$ since then both sides of the expression are zero. Assume that $x$ does not equal to any node and consider the function $F(t)$ where

$$F(t) = f(t) - p_n(t) - c\omega_{n+1}(t) \text{ and } c = \frac{[f(x) - p_n(x)]}{\omega_{n+1}(x)}.$$

Observe that $c$ is well defined because $\omega_{n+1}(x) \neq 0$ since $x$ is not a node. Note also that $F(x_i) = 0,\ i = 0, \ldots\ldots, n$ and $F(x) = 0$. Thus $F(t)$ has at least $n+2$ distinct zeros is $[a, b]$. Now invoke Mean Value theorem which states that between any two zeros of $F$ there must occur a zero of $F'$. Thus, $F'$ has at least $n+1$ distinct zeros. By similar reasoning, $F''$ has at least $n$ distinct zeros and so on. Finally, it can be inferred that $F^{(n+1)}$ must have at least one zero. Let $\xi_x$ be a zero of $F^{(n+1)}(t)$. Thus we have

$$0 = F^{(n+1)}(\xi_x) = f^{(n+1)}(\xi_x) - c(n+1)! = f^{(n+1)}(\xi_x) - \frac{(n+1)!}{\omega_{n+1}(x)}[f(x) - p_n(x)],$$

since $\omega_{n+1}^{(n+1)}(t) = (n+1)!$. The desired result (1) follows.

The following are some of the intrinsic properties of the interpolation error:

- For any value of $i$, the error is zero when $x = x_i$ because $\omega_{n+1}(x_i) = 0$ (the interpolating conditions).
- The error is zero when the data $f_i$ are data points from a polynomial $f(x)$ of degree less or equal to $n$ because then the $(n+1)^{st}$ derivative, $f^{(n+1)}(\xi_x)$, is identically zero. This is simply a statement of the uniqueness theorem of polynomial interpolation.

Taking absolute values in the interpolation error expression and maximizing both sides of the resulting inequality over $x \in [a, b]$, we obtain the polynomial interpolation error bound

$$\max_{x\in[a,\, b]} |f(x) - p_n(x)| \leq \max_{x\in[a,\, b]} |\omega_{n+1}(x)| \cdot \frac{\max_{x\in[a,\, b]} |f^{(n+1)}(x)|}{(n+1)!} \quad \ldots. (2)$$

Therefore, we are left with estimating the terms on the right-hand side of this inequality in order to characterize the error in polynomial interpolation.

***For equally spaced nodes:*** Consider the special case in which the points $\{x_i\}_{i=0}^{n}$ are equally-spaced on an interval $[a, b]$. Specifically, these points $x_i$ are defined by $x_i = a + \frac{i}{n}(b-a)$, $i = 0, 1, \ldots\ldots, n$.

We would like to derive an estimate for the following term appearing in (2):

$$W = \max_{x\in[a,\,b]} \left| \omega_{n+1}(x) \right|.$$

To this end, we make the change of variable $x = a + \dfrac{(b-a)s}{n}$

to rewrite $\omega_{n+1}(x)$ as $\omega_{n+1}(x) = \prod_{j=0}^{n}(x - x_j) = \left(\dfrac{b-a}{n}\right)^{n+1} \prod_{j=0}^{n}(s-j)$,

where $s \in (0, n)$. Therefore, we have the following estimate:

$$W = \left(\frac{b-a}{n}\right)^{n+1} \max_{s\in(0,\,n)} \prod_{j=0}^{n} |s-j| \qquad \text{.... (3)}$$

For any given $s \in (0, n)$, let $i$ be an integer such that $i < s < i+1$. It follows that

$$\prod_{j=0}^{n} |s-j| = \left|(s-i)(s-i-1)\right| \prod_{j=0}^{i-1} |s-j| \prod_{j=i+2}^{n} |s-j| \qquad \text{.... (4)}$$

and, since $s < i+1$,

$$\left|(s-i)(s-i-1)\right| \le \frac{1}{4}, \text{ (please show this)} \qquad \text{.... (5)}$$

$$\prod_{j=0}^{i-1} |s-j| \le \prod_{j=0}^{i-1} (i+1-j) \le (i+1)! \qquad \text{.... (6)}$$

and, since $s > i$

$$\prod_{j=i+2}^{n} |s-j| \le \prod_{j=i+2}^{n} (j-i) \le (n-i)! \qquad \text{.... (7)}$$

On substituting these three estimates, (5) – (7) into (4), we obtain

$$\prod_{j=0}^{n} |s-j| \le \frac{1}{4} n! \qquad \text{.... (8)}$$

On substituting (8) into (3), we obtain $W \le \left(\dfrac{b-a}{n}\right)^{n+1} \dfrac{1}{4} n!$,

and with this bound in (2), we obtain the following estimate:

$$\max_{x\in[a,\,b]} \left| f(x) - p_n(x) \right| \le \left(\frac{b-a}{n}\right)^{n+1} \frac{1}{4(n+1)} \max_{x\in[a,\,b]} \left| f^{(n+1)}(x) \right| \qquad \text{.... (9)}$$

Thus, if a function has ill-behaved higher derivatives, then the quality of the polynomial interpolation may actually decrease as the degree of the polynomial increases.

# QUESTION 2015 (EVEN)

## GROUP – A
(Multiple Choice Type Questions)

1. Answer any *ten* questions:

i) The relation between true value, approximate value and error is

✓a) true value = approximate value + error   b) approximate value = true value + error
c) approximate value = true value /error   d) none of these

ii) If $\frac{11}{7}$ is approximated as 1.5714, then absolute error is

a) 0.0000256   b) 0.0000278   ✓c) 0.00002857   d) 0.00001234

iii) For an equation like $x^2 = 0$, a root exists at x = 0. The bisection method cannot be adopted to solve this equation in spite of the root existing at x = 0, because the function $f(x) = x^2$

a) is a polynomial   b) has repeated roots at x = 0
✓c) is always non-negative   d) slope is zero at x = 0

iv) Simpson's $1/3^{rd}$ formula always requires

a) even number of ordinates   ✓b) odd number of ordinates
c) even or odd number of ordinates   d) none of these

v) To solve the system of equation Ax = b by Gauss elimination method, A is transformed to a

a) lower triangular matrix   ✓b) upper triangular matrix
c) diagonal matrix   d) none of these

vi) Runge-Kutta method is used to solve

a) an algebraic equation   ✓b) a first order ordinary differential equation
c) a first order partial differential equation   d) none of these

vii) Which of the following is an iterative method?

a) Gauss Elimination method   b) Gauss Jordon method
c) LU method   ✓d) Gauss-Seidel method

viii) Geometrically, the Lagrange's interpolation formula for two points of interpolation represents a

a) parabola   b) circle   ✓c) straight line   d) none of these

ix) The Newton-Raphson Method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1, the iteration will

a) converges to –1   b) converges to $\sqrt{2}$
✓c) converges to $-\sqrt{2}$   d) not convergent

x) If $f(x)=\frac{1}{x}$, the divided difference $f[a, b, c]$ is

a) $1/(a+b+c)$ ✓b) $1/abc$ c) $1/(a^2+b^2)$ d) $1/(a+b+c)$

xi) The value of $(1+\Delta)(1-\nabla)$ is

a) 0 ✓b) 1 c) 2 d) 3

xii) If $A$ be the actual value and $T$ be its estimated value, the formula for relative error is

a) $\frac{A}{T}$ b) $\frac{A-T}{T}$ ✓c) $\frac{|A-T|}{A}$ d) $\frac{|A-T|}{T}$

xiii) The no. of significant digits in 1.00234 is

a) 4 ✓b) 6 c) 5 d) none of these

xiv) Which of the following are predictor-corrector methods?

a) Milne's method b) Adams Bashforth method
✓c) Both (a) and (b) d) Newton's formulas

**GROUP – B**
**(Short Answer Type Questions)**

2. Given $y=\frac{1}{x}$, show that, $y[x_0, x_1, x_2, \ldots\ldots\ldots, x_n]=\frac{(-1)^n}{(x_0x_1x_2\ldots\ldots x_n)}$.

**See Topic: INTERPOLATION, Short Answer Type Question No. 5.**

3. Find the area bounded by the curve and x-axis from $x=7.47$ to $x=7.52$ giving reason and the rule of integration applicable in this case.

| $x$: | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.93 | 1.95 | 1.98 | 2.01 | 2.03 | 2.06 |

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 5.**

4. Consider the following system of equations:

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 5 \\ 15 \\ 8 \end{pmatrix}$$

Solve the above system of equations by Gauss-Seidel method taking initial guess $x=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$. Perform only two iterations.

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 6.**

5. Compute $y(0.5)$, by Runge-Kutta method of 4th order from the equation $\frac{dy}{dx}=\frac{1}{x+y}$, $y(0)=1$.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 8.**

6. Fit a polynomial to the following table of values using Lagrange interpolation formula:

| $x$: | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$: | –12 | 0 | 6 | 12 |

Find the value of y when $x = 2$.

**See Topic: INTERPOLATION, Short Answer Type Question No. 6.**

**GROUP – C**

**(Long Answer Type Questions)**

7. a) Find the root of equation $x\tan x = 1.28$, that lies in the interval $(0,1)$, correct to 4 decimal places, using Bisection method.

b) Given $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x^2}$, $y(1)=1$. Evaluate $y(1.2)$ by modified Euler's method correct upto 4 decimal places.

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 3.**

**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 3(b).**

8. a) Compute $y(0.3)$ by Milne's predictor-corrector method from the equation $\frac{dy}{dx}=xy+y^2$ given that $y(0)=1,\ y(0.1)=1.1169$, $y(0.2)=1.2773$

b) Show that rate of convergence in Newton-Rephson method is quadratic.

c) Evaluate $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's 1/3rd rule with $n=4$

**a) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 4(a).**

**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 2.**

**c) See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 6.**

9. a) Find the Lagrangian interpolation polynomial which agrees with the following data:

| $x$ (in radian) | 1.0 | 1.1 | 1.2 |
|---|---|---|---|
| $y=\cos x$ | 0.5403 | 0.4536 | 0.3624 |

b) From the following table, find the interpolation polynomial by Newton forward interpolation formula:

| $x$: | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1 | 2 | 3 | 4 | 5 | 6 |

c) Solve by Gauss Elimination method

$$x+2y+z=0$$
$$2x+2y+3z=3$$
$$-x-3z=2$$

a) & b) See Topic: INTERPOLATION, Long Answer Type Question No. 9(a)& b).
c) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 5.

10. a) Find a +ve root of $x+\ln x-2=0$ by Newton-Raphson method correct upto six significant figures.

b) Using Rungé-Kutta method of order 4 to obtain the solution of $\frac{dy}{dx}=2x+y^2$, $y(0)=1$ and $h=0.1$ at $x=0.2$

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 4.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 4(b).**

11. Write short notes on any three of the following:
   a) Truncation, Percentage and Absolute Error
   b) Lagrange's Interpolation
   c) Milne's Method
   d) Regula-Falsi Method
   e) Error in Interpolating polynomial

**a) See Topic: ERROR, Long Answer Type Question No. 1(a).**
**b) See Topic: INTERPOLATION, Long Answer Type Question No. 16.**
**c) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 10.**
**d) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 15.**
**e) See Topic: ERROR, Long Answer Type Question No. 1(b).**

# QUESTION 2015 (ODD)

**GROUP – A**
**(Multiple Choice Type Questions)**

1. Answer any *ten* questions:

i) Lagrange's interpolation can be used for
   a) only equi-spaced nodes
   b) only unequi-spaced nodes
   ✓c) for both cases of (a) and (b)
   d) none of these

ii) The inherent error for Trapezoidal rule of integration is as (the notations have their usual meanings)
   a) $-\frac{nh^5}{140}f''(x_0)$
   b) $-\frac{nh^5}{140}f^{iv}(x_0)$
   ✓c) $-\frac{nh^3}{140}f''(x_0)$
   d) none of these

iii) The total number of significant digits in 500000 is
   a) 2
   ✓b) 1
   c) 0
   d) none of these

iv) $(\Delta - \nabla)x^2$ is equal to (the notations have their usual meanings)

a) $h^2$ ✓b) $2h^2$ c) $-2h^2$ d) none of these

v) Newton's divided difference interpolation formula is used for

a) equispaced arguments only b) unequispaced only
✓c) both equispaced and unequispaced arguments d) none of these

vi) The rate of convergence of bisection method is

✓a) linear b) quadratic c) cubic d) none of these

vii) The error in the Simpson's 1/3rd rule is of order

a) $h$ b) $h^2$ c) $h^3$ ✓d) $h^4$

viii) In the Newton's Forward Interpolation formula, the value of $u = \frac{x - x_0}{h}$ lies between

✓a) 0 and 1 b) –1 and 0 c) –1 and 1 d) 5 and 0

ix) In Trapezoidal rule, the portion of curve is replaced by

✓a) straight line b) circular path c) parabolic path d) none of these

x) Which relations are true?

✓a) $E = 1 + \Delta,\ \Delta\nabla = \Delta - \nabla$ b) $E = 1 - \Delta,\ \Delta\nabla = \Delta + \nabla$
c) $E = 1 - \Delta,\ \Delta\nabla = \Delta - \nabla$ d) $E = 1 + \Delta,\ \Delta\nabla = \Delta + \nabla$

xi) Regula-Falsi method is used to

a) solve the differential equation of boundary value problem
✓b) solve transcendental equation numerically
c) solve a system of equations numerically
d) none of these

xii) If 'A' be the actual value and 'T' be its estimated value, the formula for relative error is

a) $A/T$ b) $(A-T)/T$ ✓c) $|A-T|/A$ d) $|A-T|/T$

**GROUP – B**
**(Short Answer Type Questions)**

2. What is the difference between interpolation and extrapolation? Give suitable examples.
**See Topic: INTERPOLATION, Short Answer Type Question No. 2(a).**

3. Compute the value of $\pi$ from the formula $\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3rd rule with 10 sub-intervals.
**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 7.**

4. Prove the following operator relation: $\mu^2 = 1 + \frac{1}{4}\delta^2$, where the notations have their usual meanings.

**See Topic: INTERPOLATION, Short Answer Type Question No. 18.**

5. Evaluate $\sqrt{12}$ to three places of decimals by Newton-Raphson Method.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 1.**

6. Evaluate the missing terms in the following table:

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$: | 0 | – | 8 | 15 | – | 35 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 19.**

## Group – C
## (Long Answer Type Questions)

7. a) Compute $f(0.23)$ and $f(0.29)$ using suitable formula from the table below:

| $x$: | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |

b) Describe Geometric Significance of Simpson's 1/3rd Rule.

c) Determine the absolute error $E_A$ of the following approximate number given their relative error, $x_A = 67.84,\ E_R = 1\%$.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 5.**
**b) See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 1.**
**c) See Topic: ERROR, Short Answer Type Question No. 3.**

8. a) Using Gauss-Seidel method, find the solution of the following system of linear equations correct up to two decimal places:

$$3x + y + 5z = 13,\ 5x - 2y + z = 4,\ x + 6y - 2z = -1.$$

b) Solve the equation $\frac{dy}{dx} = \frac{1}{x+y},\ y(0) = 1$, for $y(0.1)$ and $y(0.2)$, using Runge-Kutta method of the fourth order.

c) Show that $(1+\Delta)(1-\nabla) \equiv 1$.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 1.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 3(a).**
**c) See Topic: INTERPOLATION, Short Answer Type Question No. 7.**

9. a) Find the root of the equation $x\tan x = 1.28$, that lies in the interval (0, 1), correct to 4 decimal places, using Bisection method.

b) Given $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{x^2}$, $y(1)=1$. Evaluate $y(1.2)$ by modified Euler's method correct up to 4 decimal places.

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION , Short Answer Type Question No. 3.**

**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 3(b).**

10. a) Find the polynomial $f(x)$ and hence calculate $f(5.5)$ for the given data:

| $x$: | 0 | 2 | 3 | 5 | 7 |
|---|---|---|---|---|---|
| $f(x)$: | 1 | 47 | 97 | 251 | 477 |

b) What is the order of operation needs for L-U decomposition method?

c) Solve the following system of equations by L-U decomposition method:

$$x+y-z=2,\ 2x+3y+5z=-3,\ 3x+2y-3z=6$$

**a) See Topic: INTERPOLATION, Short Answer Type Question No. 2(b).**

**b) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 7.**

**c) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 2**

11. a) Find a real root of the equation $x^3 = 2$ within (1, 2) by Regula Falsi method, correct up to 4 places of decimals.

b) Solve by method of finite difference for $h = 0.25$. $\frac{d^2y}{dx^2}+y=0,\ y(0)=0,\ y(1)=1$

c) Using the Divided difference formula find $f(0.72)$ from the following table:

| $x$: | 0.62 | 0.68 | 0.70 | 0.73 | 0.75 |
|---|---|---|---|---|---|
| $f(x)$: | 0.6604918 | 0.73363074 | 0.7585837 | 0.7965858 | 0.8223167 |

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 5**

**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 5.**

**c) See Topic: INTERPOLATION Long Answer Type Question No. 10.**

# QUESTION 2016 (EVEN)

## GROUP – A
### (Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) The number 3.4506531 when rounded off to 4 places of decimal will give
a) 3.4506 ✓b) 3.4507 c) 3.451 d) none of these

ii) Lagrange's Interpolation formula is used for
a) Equally spaced arguments
b) Unequally spaced arguments
✓c) Both equally and unequally spaced arguments
d) none of these

iii) In the Newton's Forward Interpolation formula, the value of $u = \frac{x - x_0}{h}$ lies between
✓a) 0 & 1 b) –1 & 0 c) –1 & 1 d) 5 & 0

iv) $\Delta^3(y_0)$ may be expressed as which of the following terms?
✓a) $y_3 - 3y_2 + 3y_1 - y_0$ b) $y_2 - 2y_1 + y_0$
c) $y_3 + 3y_2 + 3y_1 + y_0$ d) none of these

v) Trapezoidal rule can be applied if the number of equal subintervals of the interval of integration is
a) odd ✓b) even c) both d) none

vi) Regula Falsi Method is
a) conditionally convergent b) divergent
c) linearly convergent ✓d) none of these

vii) The $n$-th order divided difference of a polynomial of degree $n$ is
a) 0 ✓b) constant c) 1 d) –1

viii) Runge-Kutta formula has a truncation error which is of order
a) $h^2$ b) $h^3$ ✓c) $h^4$ d) $h^5$

ix) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 2 & 4 \\ 9 & 8 & 7 \end{bmatrix}$

Consider the following two statements:
S1 : LU decomposition for the matrix A is possible
S2 : LU decomposition for the matrix B is not possible

a) Both S1 and S2 are true
✓b) only S1 is true
c) only S2 is true
d) neither S1 nor S2 is true

x) Newton-Raphson method for solution of the equation $f(x)=0$ fails when

a) $f'(x)=1$ ✓b) $f'(x)=0$ c) $f'(x)=-1$ d) none of these

xi) Which of the following does not always guarantee convergence?

a) Bisection method
✓b) Newton-Raphson method
c) Regula-falsi
d) none of these

xii) Choose the correct alternative:

✓a) $E=1+\Delta$ b) $E=1-\Delta$ c) $E=1/\Delta$ d) None of these

**GROUP – B**
**(Short Answer Type Questions)**

2. Find $f(5)$ using Newton's Divide difference formula, for the following data:

| $x$ | 0 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| $f(x)$ | 4 | 26 | 58 | 112 | 466 | 668 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 3.**

3. Solve by Gauss Elimination method:

$$x-2y+9z=8$$
$$3x+y-z=3$$
$$2x-8y+z=-5$$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 8.**

4. Calculate by Simpson's one third rule the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$ corrected up to three significant figures.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 8.**

5. Find a real root of the equation $xe^x-2=0$, correct to three decimal places using Newton-Raphson method.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 6.**

6. Evaluate $y(0.02)$, given $\frac{dy}{dx}=x^2+y$, $y(0)=1$ by modified Euler's method.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 9.**

## GROUP – C
### (Long Answer Type Questions)

7. a) Find the value of $\int_1^5 \log_{10} x\,dx$ taking eight equal sub-intervals correct up to 4-decimal places by (i) Simpson $\frac{1}{3}$rd rule (ii) Trapezoidal rule and then compare the result.

b) Solve by Gauss-Seidel iterative method:

$$3y - 2z = 3$$
$$2x - y + 4z = 27$$
$$4x - y - 3z = 3$$

correct up to four significant figures.

**a) See Topic: NUMERICAL INTEGRATION, Long Answer Type Question No. 3.**
**b) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 2.**

8. a) Solve the system of linear equations by LU factorization method:

$$2x - 6y + 8z = 24;\; 5x + 4y - 3z = 2;\; 3x + y + 2z = 16$$

b) Find the polynomial $f(x)$ and hence find the $f(5.5)$ from the given data:

| $x$ | 0 | 2 | 3 | 5 | 7 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 47 | 97 | 251 | 477 |

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 1.**
**b) See Topic: INTERPOLATION, Short Answer Type Question No. 2(b).**

9. a) Compute $f(0.23)$ and $f(0.29)$ using suitable formula from the table below:

| $x$: | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |

b) Solve the equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$, for $y(0.1)$ and $y(0.2)$, using Runge-Kutta method of the fourth order.

c) Show that $(1+\Delta)(1-\nabla) \equiv 1$.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 5.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 3(a).**
**c) See Topic: INTERPOLATION, Short Answer Type Question No. 7.**

10. a) Find by the method of Regula-Falsi, a positive root of $x^2+2x-2=0$ correct up to three decimal places.

b) Apply Finite difference method to solve the equation $\frac{d^2y}{dx^2}=3x+4y$, subject to the conditions $y(0)=0,\ y(1)=1$ by taking the mesh length $h=\frac{1}{4}$.

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 6.**

**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 6.**

11. a) Use Lagrange's inverse interpolation formula to find the value of $x$, when $y=0.143$ from the following data:

| $x$ | 1 | 2 | 4 | 5 | 8 |
|---|---|---|---|---|---|
| $f(x)$ | 1.000 | 0.500 | 0.250 | 0.200 | 0.125 |

b) Find the positive real root of $x+\ln x=2$ by Newton-Raphson method, correct to six significant figures.

c) Using Bisection method obtain a root between 1 and 2 of the equation $e^x-3x=0$.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 20.**

**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 4.**

**c) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 7.**

# QUESTION 2016 (ODD)

**Group – A**
**(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any ten of the following:

i) Runge-Kutta method has a truncation error, which is of the order of

a) $h^2$ b) $h^4$ ✓c) $h^5$ d) None of these

ii) The rate of convergence of bisection method is

✓a) linear b) quadractic c) cubic d) none of these

iii) The significant digit of 0.0001234 is

a) 7 ✓b) 4 c) 8 d) none of these

iv) $\Delta^3(y_0)$ may be expressed as which of the following terms?

✓a) $(y_3 - 3y_2 + 3y_1 - y_0)$
b) $(y_2 - 2y_1 + y_0)$
c) $(y_3 - 3y_2 + 3y_1 + y_0)$
d) both (a) and (c)

v) If A be the actual value and T be its estimated value the formula for relative error is

a) $A/T$
b) $(A-T)/T$
c) $|A-T|/A$
✓d) $|A-T|/T$

vi) Which one of the following is a direct method?

✓a) Gauss-elimination
b) Gauss-Jordan
c) Gauss-Seidel
d) none of these

vii) The order of convergence of Newton-Raphson method is

a) 3
✓b) 2
c) 1
d) none of these

viii) The error for Simpson' is 1/3 rd rule of integration is

a) $\frac{-nh^5}{180} f''(x_0)$
b) $\frac{-nh^4}{140} f''(x_0)$
c) $\frac{-nh^2}{12} f''(x_0)$
d) $\frac{-nh^5}{180} f^{iv}(x_0)$

**Answer:** (none of the above) Correct answer is $-\frac{h^4}{180} f^{(iv)}(\xi),\ x_i < \xi < x_{i+1}$

ix) Trapezoidal rule can be applied if the number of equal sub-intervals of the intervals of integration is

a) odd
b) even
✓c) both (a) and (b)
d) none of these

x) The convergence condition for Gauss-Seidel iterative method for solving a system of linear equation is

a) the co-efficient matrix is singular
b) the co-efficient matrix has rank zero
✓c) the co-efficient matrix must be strictly diagonally dominant
d) none of these

xi) The finite difference method is used to solve

✓a) a boundary value problem
b) a system of ordinary differential equation
c) a partial differential equation
d) a system of transcendental equation

xii) Which of the following is a multistep method?

a) Euler's Method
✓b) Predictor-corrector method
c) Taylor's series method
d) None of these

## Group – B
### (Short Answer Type Questions)

2. Using Euler's method obtain the solution of $\frac{dy}{dx} = x - y$, $y(0) = 1$ and $h = 0.1$ at $x = 0.5$.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 10.**

3. Find the positive real root of $x^3 - x^2 - 1 = 0$ using the bisection method of 4 iterations.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 8.**

4. Apply Lagrange's interpolation formula to find $f(x)$ if $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$ and $f(7) = 128$.

**See Topic: INTERPOLATION, Short Answer Type Question No. 4.**

5. Use Simpson's 1/3 rd rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal sub-intervals of [0, 6], correct to three decimal places.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 9.**

6. Solve the following system by matrix inversion method $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 9.**

## Group – C
### (Long Answer Type Questions)

7. a) Solve the system of equations by LU factorization method:

$$8x_1 - 3x_2 + 2x_1 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$6x_1 + 3x_2 + 12x_3 = 36$$

b) Find $f(0.9)$ using Newton's forward interpolation formula.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 0.2536 | 1.0245 | 2.0145 | 0.2547 | 3.0145 | 1.0125 | 2.01245 |

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 3.**

**b) See Topic: INTERPOLATION, Long Answer Type Question No. 11.**

8. a) Compute one root of $x+\ln x-2=0$, correct to two decimal places by using Regula falsi method.
b) Solve the system of equations by Gauss-Seidel method correct to two decimal places:
$10x-5y-2z=3$
$4x-10y+3z=-3$
$x+6y+10z=-3$

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 7.**
**b) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 4.**

9. a) Derive Lagrange interpolation formula.

b) Using Trapezoidal and Simpson's $\frac{1}{3}$rd rule compute $\int_{4}^{5.2} \log x dx$ by taking seven ordinates correct up to four decimal places.

**a) See Topic: INTERPOLATION, Long Answer type Question No. 1.**
**b) See Topic: NUMERICAL INTEGRATION, Long Answer Type Question No. 1.**

10. a) Obtain the Newton's Divide Difference interpolating formula hence find $f(3)$.

| $x$: | 0 | 1 | 2 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 14 | 15 | 5 | 6 | 19 |

b) Find the root of the equation $10^x+x-4=0$ where the initial value is given as 1 using Newton-Raphson Method.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 3.**
**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 8.**

11. a) Using finite difference method solve the boundary value problem: $\frac{d^2y}{dx^2}+y+1=0$ with $y(0)=0, y(1)=0$.

b) By using RK method of 4th order solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$.

**a) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 2(b).**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 2.**

# QUESTION 2017 (EVEN)

**Group-A**
**(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any *ten* of the following:

i) The number of significant digits in 1.00340 is
a) 3 b) 4 c) 5 ✓d) 6

ii) The $n^{th}$ order forward difference of $n^{th}$ degree polynomial is
a) $n!$ b) $(n+1)!$ c) 0 ✓d) none of these

iii) Error in 4th order Runge-Kutta method is of the order of
a) $h^2$ b) $h^3$ ✓c) $h^4$ d) none of these

iv) The degree of precision of Simpson's 1/3rd rule is
a) 1 b) 2 ✓c) 3 d) none of these

v) In Trapezoidal rule, the portion of curve is replaced by
✓a) straight line b) parabolic path c) circular path d) none of these

vi) One root of the equation $x^2+2x-2=0$ lies between
a) 1 and 2 b) 0 and 0.5 ✓c) 0.5 and 1 d) none of these

vii) The accuracy attainable with Newton-Raphson method does not depend upon the value of the derivative $f(x)$. The above statement is
✓a) True b) False

viii) Gauss elimination method does not fail even if one of the pivot elements is equal to zero. The above statement is
✓a) True b) False

ix) The percentage error if 1/3 is approximated by 0.333 is
✓a) 0.1 b) 0.11 c) 0.01 d) 1.01

x) Which one of the following is an iterative method?
a) Gauss-elimination b) Gauss-Jordon
✓c) Gauss-Seidel d) none of these

xi) Which of the following is a multistep method?

a) Euler's method
✓b) Predictor-corrector method
c) Taylor's series method
d) None of these

xii) To solve the ordinary differential equation: $3\frac{dy}{dx}+xy^2=\sin x,\ y(0)=5$, by Runge-Kutta 2nd order method, you need to rewrite the equation as

a) $\frac{dy}{dx}=\sin x-xy^2,\ y(0)=5$
✓b) $\frac{dy}{dx}=\frac{1}{3}(\sin x-xy^2),\ y(0)=5$
c) $\frac{dy}{dx}=\frac{1}{3}\left(-\cos x-\frac{xy^3}{3}\right),\ y(0)=5$
d) None of these

**Group – B**
**(Short Answer Type Questions)**

2. From the following table, find the value of $f(1.5)$ by Newton forward interpolation formula:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 10 | 15 | 20 | 25 | 30 | 35 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 21**

3. Solve the following system by Matrix Inversion Method:

$$2x+y+z=4$$
$$3x+2y+3z=8$$
$$x+4y+9z=14$$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 10.**

4. Using Euler's method obtain the solution of $\frac{dy}{dx}=x-y,\ y(0)=1$ and $h=0.1$ at $x=0.5$

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 10.**

5. Find out the root of the following equation using Regula Falsi method:
$xe^x-\cos x=0$, that lies between 0 and 1 (correct to four decimal places).

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 9.**

6. Evaluate the approximate value of $\int_0^1\frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}$ rd rule for 4 sub-interval, correct up to 4 decimal places.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 10.**

## Group – C
### (Long Answer Type Questions)

7. a) Using appropriate formula find $f(0.29)$ from the following table:

| $x$: | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |
|---|---|---|---|---|---|---|
| $f(x)$: | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |

b) Compute the value of $\int_{1.2}^{1.6}\left(x+\frac{1}{x}\right)dx$, taking $h=0.05$ correct up to five decimal places by using

(i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$rd rule.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 5.**
**b) See Topic: NUMERICAL INTEGRATION, Long Answer Type Question No. 4.**

8. a) Solve by Gauss-Seidel iterative method:

$$3x+9y-2z=11$$
$$4x+2y+13z=24$$
$$4x-2y+z=-8$$

Correct up to four significant figure.

b) Find a +ve root of $x+\ln x \quad x-2=0$ by Newton-Raphson method, correct up to six significant figure.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 5.**
**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 4.**

9. a) Compute $y(0.8)$, by Runge-Kutta method correct up to five decimal places from the equation $\frac{dy}{dx}=xy,\ y(0)=2$ taking $h=0.2$.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 7.**

b) Solve by LU decomposition method:

$$2x-6y+8z=24$$
$$5x+4y-3z=2$$
$$3x+y+2z=16$$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 1.**

10. a) Find $f(8)$ using Newton's divided difference formula given that

| $x:$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x):$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

b) Derive the expression of Newton's forward interpolation formula where the function $f(x)$ is known for $n+1$ distinct equispaced arguments.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 12.**
**b) See Topic: INTERPOLATION, Long Answer Type Question No. 2.**

11. a) Solve the matrix inversion method:

$$2x-3y+10z=3$$
$$-x+4y+2z=20$$
$$5x+2y+z=-12$$

b) Use modified Euler's method to find the value of $y(0.02)$ by taking $h=0.01$ of the differential equation $\frac{dy}{dx}=x^2+y$, given that $y(0)=1$.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 6.**
**b) See Topic NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 9.**

## QUESTION 2017 (ODD)

### Group – A
### (Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) Newton-Raphson method fails when

✓a) $f'(x)=0$ b) $f''(x)=0$ c) $f'(x)=1$ d) $f'(x)=-1$

ii) Which of the following is not a computational error?

a) Truncation error b) Round-off error ✓c) Inherent error d) None of these

iii) Finite difference method is used to solve

a) a system of linear simultaneous equations
b) a system of non-linear simultaneous equations
c) partial differential equations
d) non-linear equations

**Correct Answer: a Boundary Value Problem**

iv) Regula Falsi method is
a) conditionally convergent
b) linearly convergent
c) divergent
✓d) none of these

v) Modified Euler's method has a truncation error of the order of
a) $h$
✓b) $h^2$
c) $h^3$
d) $h^4$

vi) The percentage error in approximating 4/3 to 1.3333 is
✓a) 0.0025%
b) 25%
c) 0.00025%
d) 0.25%

vii) The no. of significant digits in 1.00234 is
a) 4
✓b) 6
c) 5
d) 7

viii) Solve by using Euler's method the following differential equation for $x=1$ by taking $h=0.2,\ \frac{dy}{dx}=xy,\ y=1$ when $x=0$:
a) 1.5896
✓b) 1.4593
c) 1.3495
d) 0.4593

ix) By Runge-Kutta method with $h=0.1$, the value of $y(0.1)$ for given $\frac{dy}{dx}=x+y$, $y=1$ when $x=0$, is
✓a) 1.11034
b) 1.15034
c) 1.22034
d) 1.23034

x) If $f(0)=12,\ f(3)=6$ and $f(4)=8$ then linear interpolation function $f(x)$ is
a) $x^2-3x+12$
b) $x^2-5x$
c) $x^3-x^2-5x$
✓d) $x^2-5x+12$

xi) The degree of precision of Simpson's one third rule is
a) 1
b) 2
✓c) 3
d) 5

xii) Lagrange's interpolation formula deals with
a) Equispaced arguments only
b) Unequispaced arguments only
✓c) both (a) and (b)
d) none of these

## Group – B
### (Short Answer Type Questions)

2. Find the positive real root of $x^3-x^2-1=0$ using bisection method of 4 iterations.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 8.**

3. Evaluate $y(4.4)$, given $5xy'=2-y^2,\ y(4)=1$, by Taylor Series Method.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 11.**

4. a) What is interpolation? What are the differences between interpolation and extrapolation? Explain with suitable examples.

b) If $y(10)=35.3,\ y(15)=32.4,\ y(20)=29.2,\ y(25)=26.1,\ y(30)=23.2$ and $y(35)=20.5$ find $y(12)$ using Newton's forward interpolation formula.

**a) See Topic: INTERPOLATION, Short Answer Type Question No. 2(a).**
**b) See Topic: INTERPOLATION, Short Answer Type Question No. 22.**

5. Find $f(5)$ using Newton's Divide difference formula, for the following data:

| $X$ | 0 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| $f(x)$ | 4 | 26 | 58 | 112 | 466 | 668 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 3.**

6. Show that $(1+\Delta)(1-\nabla)=1$.

**See Topic: INTERPOLATION, Short Answer Type Question No. 7.**

**Group – C**
**(Long Answer Type Questions)**

7. a) Solve the given system of equations by Gauss Elimination method:

$$x+y+z=9,\ 2x-3y+4z=13,\ 3x+4y+5z=40.$$

b) Apply Milne's method to find $y(0.8)$ for the equation $\frac{dy}{dx}=x+y^2$; given that $y(0)=0, y(0.2)=0.02, y(0.4)=0.0805$ and $y(0.6)=0.1839$.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 7.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 2(a).**

8. a) Apply Newton-Raphson method to evaluate $\sqrt[3]{13}$ correct up to three places of decimal.

b) Form the interpolation polynomial for the function $y=f(x)$ given by the table:

| $x$ | $-1$ | 1 | 4 | 6 |
|---|---|---|---|---|
| $y$ | 1 | $-3$ | 21 | 127 |

c) Find the smallest positive root of the given equation $3x-\cos x-1=0$, correct up to three decimal places by Regula Falsi method.

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 9.**
**b) See Topic: INTERPOLATION, Long Answer Type Question No. 13.**
**c) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 10.**

9. a) Derive Newton's forward interpolation formula.
b) Using L-U factorisation method, solve the given system of equations: $2x-3y+10z=3,\ -x+4y+2z=20,\ 5x+2y+z=-12$.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 2.**
**b) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 8.**

10. a) Using $R.K.$ 4th order method, solve the differential equation to find $y(0.2)$:

$$\frac{dy}{dx}=xy,\ y(0)=1 \text{ taking } h=0.1$$

b) Using finite different method, solve the boundary value problem $\frac{d^2y}{dx^2}+y=0$ with $y(0)=0,\ y(1)=1$ taking $h=0.25$.

**a) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 8.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 5.**

11. a) Verify whether the given systems of equations are diagonally dominant. Hence solve them by Gauss-Siedel method: $-2x+3y+10z=22,\ x+10y-z=-22,\ 10x+2y+z=9$.

b) i) Calculate by Simpson's 1/3rd Rule the value of $\int_{1.2}^{1.6}\left(x+\frac{1}{x}\right)dx$, correct up to two significant figures, taking four intervals.

ii) Evaluate approximately, by Trapezoidal rule, the integral $\int_0^1\left(4x-3x^2\right)dx$, by taking $n=10$.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 9.**
**b) i) See Topic: NUMERICAL INTEGRATION, Long Answer Type Question No. 2.**
**ii) See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 11.**

# QUESTION 2018 (EVEN)

**Group – A**
**(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any *ten* of the following:
i) In Newton's forward Interpolation, the interval should be
✓a) equally spaced
b) not equally spaced
c) Both (a) and (b)
d) None of these

ii) If $f(x)=\frac{1}{x}$, then divided difference $f(a,b)$ is

✓a) $\frac{a+b}{(ab)^2}$ b) $-\frac{a+b}{(ab)^2}$ c) $\frac{1}{a^2-b^2}$ □ d) $\frac{1}{a^2}-\frac{1}{b^2}$

iii) If $f(0)=12,\ f(3)=6,\ f(4)=8$, then $f(x)$ will be

a) $x^2-3x+12$ b) $x^2-5x$ c) $x^3-x^2-5x$ ✓d) $x^2-5x+12$

iv) The number of significant digit of .0063945 is

✓a) 5 b) 3 c) 7 d) None of these

v) Let $a$ be the actual value and $e$ be its estimated value, the formula for relative error is

a) $\frac{a}{e}$ b) $\frac{|a-e|}{e}$ c) $\frac{(a-e)}{a}$ ✓d) $\frac{|a-e|}{a}$

vi) The value of $\Delta^2(ax^2+bx+c)$ is

a) $2ah+b$ b) $2ah$ ✓c) $2ah^2$ d) $2a$

vii) Which of the following is not true?

a) $\Delta=E-I$ b) $\Delta\cdot\nabla=\Delta-\nabla$ ✓c) $\frac{\Delta}{\nabla}=\Delta+\nabla$ d) $\nabla=I-E^{-1}$

viii) If $f(x)=be^{ax}$ then $\Delta f(x)$ is

✓a) $be^{ax}(e^{ah}-1)$ b) $be^{ax(a-1)}$ c) $be^{ax}(1-e^{ax})$ d) None of these

ix) The degree of precisions of Simpson's one third rule is

a) 1 b) 2 ✓c) 3 d) 5

x) The error in the Weddle's Rule is of order

a) $h^4$ b) $h^5$ ✓c) $h^6$ d) $h^7$

xi) The order of convergence of a Newton-Raphson method:

a) 3 ✓b) 2 c) 1 d) 4

xii) Diagonal dominance is must for

✓a) Gauss-Seidel method b) Gauss Elimination method

c) LU factorisation method d) All of these

## Group – B
### (Short Answer Type Questions)

2. a) Find the missing value in the following table:

| | | | | | |
|---|---|---|---|---|---|
| X: | 2 | 4 | 6 | 8 | 10 |
| Y: | 5.6 | 8.6 | 13.9 | - | 35.6 |

b) Show that $\Delta \cdot \nabla \equiv \Delta - \nabla$ .

**a) See Topic: INTERPOLATION, Short Answer Type Question No. 23.**
**b) See Topic: INTERPOLATION, Short Answer Type Question No. 10.**

3. Find $f(1.02)$ having given:

| $x$: | 1.00 | 1.10 | 1.20 | 1.30 |
|---|---|---|---|---|
| $f(x)$: | 0.8415 | 0.8912 | 0.9320 | 0.9636 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 24.**

4. Evaluate $\int xe^x dx$ where the interval $(-1, 0)$ by using Trapezoidal rule taking $n = 6$, correct up to four decimal places.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 12.**

5. Evaluate $\sqrt[5]{3}$ up to 5 significant figures by Newton-Raphson method.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 10.**

6. Solve the following system of linear equations by matrix inversion method:

$$x + y + z = 4$$
$$2x - y + 3z = 1$$
$$3x + 2y - z = 1$$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 11.**

## Group – C
### (Long Answer Type Questions)

7. a) Solve the following using Gauss-Seidel iterative method correct to 4 significant digits:

$$5x_1 - x_2 + 2x_3 = 12$$
$$3x_1 + 8x_2 - 2x_3 = -25$$
$$x_1 + x_2 + 4x_3 = 6$$

b) Solve $x^3 - 5x = 7$ using Regula-Falsi method correct to 3 places of decimal.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 10.**
**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 11.**

8. a) Solve the following set of equations using LU factorisation method:

$$2x_1 + x_2 + 3x_3 = 17$$
$$4x_1 - x_2 + 3x_3 = 31$$
$$-2x_1 + 5x_2 + 5x_3 = -5$$

b) Find the value of $x$ corresponding $y = 12$ using Lagrange's technique from the table:

| $x$ | 1.2 | 2.1 | 2.8 | 4.1 | 4.9 | 6.2 |
|---|---|---|---|---|---|---|
| $y$ | 4.2 | 6.8 | 9.8 | 13.4 | 15.5 | 19.6 |

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 11.**
**b) See Topic: INTERPOLATION, Long Answer Type Question No. 14.**

9. a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.

b) Evaluate the same using Simpson's $\frac{1}{3}$ rule.

c) What is the difference between the results (a) and (b) compare in terms of error (absolute), error % and relative error.

**See Topic: NUMERICAL INTEGRATION, Long Answer Type Question No. 5 (a), (b) & (c).**

10. a) Using Runge-Kutta method of fourth order, with $h = 0.1$, find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = -xy$, $y(0) = 1$.

b) Using Newton's divided difference formula to find $f(5)$ from the following data:

| $x$: | 0 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| $f(x)$: | 4 | 26 | 58 | 112 | 466 | 668 |

**a) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 8.**
**b) See Topic: INTERPOLATION, Short Answer Type Question No. 3.**

11. a) Fit a polynomial to the following table of values using Lagrange interpolation:

| $x$ | 0 | 1 | 3 | 4 |
|---|---|---|---|---|
| $y$ | –12 | 0 | 6 | 12 |

Find the value of $y$ when

(i) $x = 2$, (ii) $x = 3.5$

b) Find the root of $xe^x = \cos x$ using Regula-Falsi method correct up to four decimal places.

c) Derive Newton's Backward Difference Interpolation formula.

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 15.**
**b) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 9.**
**c) See Topic: INTERPOLATION, Long Answer Type Question No. 4 (2nd Part).**

# QUESTION 2018 (ODD)

## Group-A
## (Multiple Choice Type Questions)

1. Choose the correct alternatives for *any* ten of the following:

i) The $n$th order divided difference of a polynomial of degree $n$ is

a) $n$ ✓b) constant c) zero d) All of these

ii) If $f(x)=\frac{1}{x}$, then the divided differences of $f(a,b)$ is

a) $\frac{a+b}{(ab)}$ b) $-\frac{a+b}{(ab)}$ c) $\frac{1}{a-b}$ ✓d) None of these

iii) A square matrix $[A]_{n\times n}$ is diagonally dominant, if

✓a) $|a_{ii}|\geq\sum_{\substack{j=1\\i\neq j}}^{n}|a_{ij}|, i=1,2,\cdots\cdots\cdots,n$

b) $|a_{ii}|\geq\sum_{\substack{j=1\\i\neq j}}^{n}|a_{ij}|, i=1,2,\cdots\cdots\cdots,n$ and $|a_{ii}|>\sum_{\substack{j=1\\i\neq j}}^{n}|a_{ij}|$, for any $i=1,2,\cdots\cdots\cdots,n$

c) $|a_{ii}|\geq\sum_{j=1}^{n}|a_{ij}|, i=1,2,\cdots\cdots\cdots,n$ and $|a_{ii}|>\sum_{j=1}^{n}|a_{ij}|$, for any $i=1,2,\cdots\cdots\cdots,n$

d) $|a_{ii}|\geq\sum_{j=1}^{n}|a_{ij}|, i=1,2,\cdots\cdots\cdots,n$

iv) The degree of precision of trapezoidal rule is

a) 1 b) 2 c) 3 d) None of these

v) In evaluating $\int_a^b f(x)dx$, the error in Simpson's one-third rule is of order

a) $h^2$ b) $h^3$ ✓c) $h^4$ d) None of these

vi) The Newton-Raphson iterative formula for finding the square root of a real number $R$ is ($i$th iteration)

a) $x_{i+1}=\frac{x_i}{2}$ b) $x_{i+1}=\frac{3x_i}{2}$ ✓c) $x_{i+1}=\frac{1}{2}\left(x_i+\frac{R}{x_i}\right)$ d) None of these

vii) If $\frac{dy}{dx}=\frac{y-x}{y+x},\ y(0)=1,\ h=0.02$, then by Euler method $y(0.02)$ is

a) 1 ✓b) 1.02 c) 0 d) None of these

viii) In finite-difference method, the central-difference formula for replacing $\left(\frac{dy}{dx}\right)_{x=x_i}$ is

a) $\frac{y_{i-1}-y_{i+1}}{2h}$ b) $\frac{y_{i+1}-y_i}{2h}$ ✓c) $\frac{y_{i+1}-y_{i-1}}{2h}$ d) None of these

ix) In Lagrange's interpolation, the intervals should be

a) equally spaced b) not equally spaced
c) may be equally spaced ✓d) Both (a) and (b)

x) In Euler's method, given initial value problem $\frac{dy}{dx}=f(x,y)$ with $y(x_0)=y_0$, if $h$ is the step length, then the approximation is given by

a) $y_{n+1}=y_n+hf(x_{n-1},\ y_{n-1})$ ✓b) $y_{n+1}=y_n+hf(x_n,\ y_n)$
c) $y_{n+1}=y_n+hf(x_{n-1},\ y_n)$ d) None of these

xi) The error which is inherent in a numerical method itself, is called computational error.

a) True ✓b) False c) None of these

xii) Which of the following digits are not significant of the number 0.025?

✓a) 0 b) 2 c) 5 d) None of these

## Group – B
### (Short Answer Type Questions)

2. Find the approximate value of $\int_0^1 \frac{x}{1+x^2}dx$ up to four decimal places by Simpson's one third rule, taking 6 equal subintervals and find the approximate value of log 2.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 13.**

3. Evaluate the missing terms in the following table:

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$: | 0 | 5 | 10 | 15 | 20 | 25 |
| $f(x)$: | 6 | 10 | ? | 17 | ? | 31 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 25.**

4. Solve by Euler's method, the following differential equation for $x=1$ and $h=0.2$:

$\frac{dy}{dx}=xy$, given that $y(0)=1$

**Similar to, NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 4.**

5. What is Interpolation? Deduce the Forward Interpolation formula for $(n+1)$ given data points.

**See Topic: INTERPOLATION, Long Answer Type Question No. 2.**

6. Using Newton-Raphson method, determine a real root of the equation $x^3-x-1=0$, correct to four significant figures.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 11.**

**Group – C**

**(Long Answer Type Questions)**

7. a) The following data give the melting point of an alloy of zinc and lead. $\theta$ is the temperature and $x$ is the percentage of lead. Using a suitable interpolation formula, find $\theta$ when $x=48$.

| $x$ | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| $\theta$ | 184 | 204 | 226 | 250 | 276 | 304 |

b) Given the following data, find $\log_{10} 656$.

| $x$ | 654 | 658 | 659 |
|---|---|---|---|
| $\log_{10} x$ | 2.8156 | 2.8182 | 2.8189 |

**See Topic: INTERPOLATION, Long Answer Type Question No. 17 & 18.**

8. a) Check whether the following system of equations is diagonally dominant. If not, rearrange them and solve by Gauss-Seidel method, up to two places of decimals.

$$-2x+3y+10z=22$$
$$x+10y-z=-22$$
$$10x+2y+z=9$$

b) A curve passes through the points as given in the following table. Find the area bounded by the curve $x$-axis, $x=1$ and $x=3$.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $Y$ | 0.2 | 0.7 | 1 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 |

c) Discuss advantages and disadvantages of Newton-Raphson method for finding a real root of an algebraic equation.

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 9.**

**b) See Topic: INTERPOLATION, Long Answer Type Question No. 21.**

**c) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 12.**

9. a) Find the positive real root of $x\log_{10} x = 1.15$, using the bisection method (perform 4 iterations).

b) Solve $\frac{dy}{dx} = 1 - y, y(0) = 0$ using Euler's method. Find $y$, at $x = 0.1$ and $x = 0.2$. Compare the result with result of exact solution. [Take $h = 0.1$]

c) Using 4th order Runge Kutta method, evaluate the value of $y$ when $x = 1.1$, given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, y(1) = 1$. [Take $h = 0.1$]

**a) See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 13.**
**b) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 9.**
**c) See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 3(b).**

10. a) Solve the following system of equations by Gauss-Seidel Method:

$$10x - y - z = 13$$
$$x + 10y + z = 36$$
$$x + y - 10z = -35$$

Compute 3 iterations.

b) Find $f(2)$ using Newton's divided difference interpolation method, having given:

| $x$ | 4 | 5 | 7 | 10 | 11 | 13 |
|---|---|---|---|---|---|---|
| $f(x)$ | 48 | 100 | 294 | 900 | 1210 | 2028 |

**a) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 12.**
**b) See Topic: INTERPOLATION, Long Answer Type Question No. 19.**

11. a) Using Newton divided difference formula, find $\log_{10} x$ from the following table:

| $x$ | 300 | 304 | 305 | 307 |
|---|---|---|---|---|
| $f(x)$ | 2.4771 | 2.4829 | 2.4843 | 2.4871 |

b) Solve the following system of equations by LU-factorisation method, correct up to one significant figure:

$$3x + 4y + 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 2z = 10$$

**a) See Topic: INTERPOLATION, Long Answer Type Question No. 20.**
**b) See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 13.**

# QUESTION 2019 (EVEN)

## Group-A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) The relation between true value, approximate value and error is

✓a) True value = Approximate value ± error
b) Approximate value = True value + error
c) Approximate value = True value / error
d) none of these

ii) For a set of '$m$' linear equations and '$n$' no. of unknowns the system is under determined, if

a) $m > n$ b) $m < n$ c) $m = n$ ✓d) any one of these

iii) Which one is the explicit method?

✓a) Euler's method b) Gauss Jacobi method.
c) Gauss Seidel method d) None of these

iv) In Newton Cotes rules the sampling points are

✓a) equally spaced b) not equally spaced
c) mixed of equal spaced and not equal spaced d) none of these

v) If 11/7 is approximated as 1.5714 then absolute error is

a) 0.0000256 b) 0.0000278 ✓c) 0.00002857 d) none of these

vi) For an equation like $x^2 = 0$, a root exists at $x = 0$. The bisection method cannot be adopted to solve this equation in spite of the root existing at $x = 0$ because the function $f(x) = x^2$

a) is a polynomial b) has repeated root at $x = 0$
✓c) is always non-negative d) none of these

vii) If for a real continuous function $f(x)$, $f(a)f(b) < 0$ then in the range of $[a, b]$ for $f(x) = 0$, there is / are

a) one root b) undeterminable number of roots
c) no root ✓d) at least one root

viii) Simpson's 1/3rd formula always requires

a) even number of ordinates ✓b) odd number of ordinates
c) even or odd number of ordinates d) none of these

ix) Condition for convergence of Newton Raphson method is

a) $\left|f(x)\cdot f'(x)\right|<\{f''(x)\}^2$

✓b) $\left|f(x)\cdot f''(x)\right|<\{f'(x)\}^2$

c) $\left|f(x)\cdot f'(x)\right|>\{f''(x)\}^2$

d) none of these

x) $4^{th}$ order Runge-Kutta formula has a truncation error, which is of the order of

a) $h^2$ b) $h^4$ ✓c) $h^5$ d) none of these

xi) The Predictor corrector method is

a) Euler's method

b) R-K $4^{th}$ order method

✓c) Modified Euler's method

d) none of these

xii) In Trapezoidal Rule, the order of $h$ in the total error is

a) 3 b) 4 ✓c) 2 d) none of these

## Group – B
### (Short Answer Type Questions)

2. Consider the following system of equation:

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 5 \\ 15 \\ 8 \end{pmatrix}$$

Solve the above system of equations by Gauss Seidel method taking initial guess $x=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$. Perform only two iterations.

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Short Answer Type Question No. 6.**

3. a) Find the truncation error in the result of the following function for $x=\frac{1}{5}$ when the first three terms used.

$$e^x = 1+x+x^2/2!+x^3/3!+x^4/4!+x^5/5!+x^6/6!$$

b) State the difference between round off error and truncation error.

**See Topic: ERROR, Short Answer Type Question No. 4(a) & (b).**

4. Evaluate the square root of 5 using the equation $x^2-5=0$ by applying the Newton Raphson method.

**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 12.**

5. A table of data is given below for the equation of

| $x$ | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 8 | 14 | 22 | 32 |

Find the second order Newton Interpolating polynomial, and then find the function value at $x = 6$.

**See Topic: INTERPOLATION, Short Answer Type Question No. 26.**

6. Calculate by Simpson's one-third rule the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$ correct up to three significant figures.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 8.**

**Group – C**

**(Long Answer Type Questions)**

7. a) Compute $y(0.8)$, by Runge-Kutta method (4th order) correct to five decimal places, from the equation $\frac{dy}{dx} = xy,\ y(0) = 2$, taking $h = 0.2$.

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Long Answer Type Question No. 7.**

b) Solve by Euler's modified method the following differential equation for $x = 0.02$ by taking step length $h = 0.01$, (correct to 4 decimal places)

$$y' = x^2 + y,\ y(0) = 1.$$

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 9.**

8. a) Evaluate $\int_0^1 (4x - 3x^2)\,dx$ by Trapezoidal rule taking 10 subintervals. Compute the exact value and find the absolute and relative errors in your result.

**See Topic: NUMERICAL INTEGRATION, Short Answer Type Question No. 11.**

b) Solve the following system, by Gauss-Seidel method,

$$20x + 5y - 2z = 14$$
$$3x + 10y + z = 17$$
$$x - 4y + 10z = 23$$

**See Topic: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION, Short Answer Type Question No. 12.**

c) Find the root of the equation $4e^{-x}\sin x = 1$, near 0.2 by Newton-Raphson method correct to three decimal places.
**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Short Answer Type Question No. 13.**

9. a) Find a root of the equation $x^x + 2x - 6 = 0$, by method of bisection correct to two decimal places.
**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 14(a).**

b) Use the Newton's Divided Difference Formula to approximate $f(0.05)$ from the following table:

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|---|
| $f(x)$ | 1.00000 | 1.22140 | 1.49182 | 1.82212 | 2.22554 |

**See Topic: INTERPOLATION, Short Answer Type Question No. 27.**

c) Discuss the advantage and disadvantage of Regula Falsi method.
**See Topic: NUMERICAL SOLUTION OF ALGEBRAIC EQUATION, Long Answer Type Question No. 14(b).**

10. a) Solve the linear system of equations by matrix factorisation (LU method):

$$3x + 2y - 4z = 12$$
$$-x + 5y + 2z = 1$$
$$2x - 3y + 4z = -3$$

**See Topic: NUMERICAL SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, Long Answer Type Question No. 14.**

b) The table below gives the values of $\tan x$ for $0.10 \le x \le 0.30$;

| $x$ | $\tan x$ |
|---|---|
| 0.10 | 0.1003 |
| 0.15 | 0.1511 |
| 0.20 | 0.2027 |
| 0.25 | 0.2553 |
| 0.30 | 0.3093 |

Find $\tan 0.12$ and $\tan 0.26$
**See Topic: INTERPOLATION, Long Answer Type Question No. 22.**

11. a) Find the value of $Y$ at $X = 21$ using Lagrange Interpolating Polynomial from the following table:

| $X$ | 20 | 23 | 26 | 29 |
|---|---|---|---|---|
| $Y$ | 0.3420 | 0.3907 | 0.4384 | 0.4848 |

b) Show that $f(4) = f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$

c) Prove that $(1+\nabla)(1-\Delta) = 1$

**See Topic: INTERPOLATION, Long Answer Type Question No. 23(a), (b) & (c).**