Consider two normal distributions in one dimension N(μ_1, σ_1^2) and (μ_2, σ_2^2). Imagine that we choose two random samples X and Y, one from each of the normal distributions and calculate their sum Z = X + Y. What is the distribution of Z?

We find the distribution using moment-generating functions (MGFs).

The MGF of a random variable X is defined as:

$$M_X(t) = E[e^{tX}]$$

The MGF of the sum of independent random variables is the product of their MGFs. So, for X + Y, the MGF is

$$M_Z(t) = E[e^{tX} * e^{tY}] = E[e^{tX}] * E[e^{tY}] = M_X(t) * M_Y(t)$$

Now, let's calculate the MGF of X + Y using the MGFs of X and Y

$$\mathrm{M_X(t)} = \mathrm{e}^{(\mu_1(t)+0.5\sigma_1^2 t^2)}$$

$$\mathrm{M_Y(t)} = \mathrm{e}^{(\mu_2(t)+0.5\sigma_2^2 t^2)}$$

So,

$$\mathrm{M_Z(t)} = \mathrm{M_X(t)} * \mathrm{M_Y(t)} = \mathrm{e}^{((\mu_1+\mu_2)t + 0.5(\sigma_1^2+\sigma_2^2)t^2)}$$

But this is the MGF of a normal distribution with mean (μ_1 + μ_2) and variance (σ_1^2+ σ_2^2).

Since the MGF uniquely determines the probability distribution, we have shown that the sum of two independent normal distributions is also a normal distribution with mean (μ_1 + μ_2) and variance (σ_1^2 + σ_2^2).