Posterior Probability / Bayes' Theorem

P(A|B) = P(B|A) * P(A) / P(B),

where

P(A|B) = the probability of event A occurring provided the evidence B (posterior probability)

P(A) = The probability of event A occurring (prior probability)

P(B) = The probability of event B occurring (evidence or marginal likelihood)

P(B|A) = The probability of event B occurring, provided the evidence A (likelihood function)

Example 1:
A box containing 50% dark chocolates and 50% white chocolates. Half of the dark chocolates are wrapped in gold paper and the other half in silver paper. All the white chocolates are wrapped in silver paper. A kid picked a chocolate from the box, wrapped in silver paper. What is the probability that the picked chocolate is dark chocolate?

- D is the event that the picked chocolate is dark
- S is the event that the picked chocolate is wrapped in silver paper

According to Bayesian theory:
- Posterior probability, P(dark chocolate|silver wrapping): P(D|S), it is the probability that the chocolate picked is "dark chocolate", given the evidence of "chocolate wrapped in silver paper".
- Likelihood function, P(silver wrapping|dark chocolate):P(S|D), it is the probability of the evidence given the parameter; the probability of silver wrapping given the chocolate is a dark chocolate.
- P(D), the probability that the selected chocolate is "dark chocolate", given no prior information. P(D), here is 50%, 0.5.
- P(W), the probability that the selected chocolate is "white chocolate"
- P(S|D): the probability of chocolate wrapped in silver paper and given that it is a dark chocolate and is 0.5 as 50% of dark chocolates are wrapped golden paper and the remaining 50% are wrapped in silver paper.

- P(S|W): the probability of chocolate wrapped in silver paper and given that it is a white chocolate and is 1.0 since all white chocolates are wrapped in silver paper.
- P(S): the probability of randomly picked chocolate being wrapped in silver paper, given no prior information.

P(S) = P(S|D) * P(D) + P(S|W) * P(W) = 0.5 * 0.5 + 1 * 0.5 = 0.75

So, P(D|S) = P(S|D) * P(D) / P(S) = 0.5 * 0.5 / 0.75 =0.25 / 0.75 = 0.3333 = 33.33%

Hence, the probability of random chocolate selected is "dark chocolate", with the evidence that the chocolate wrapping is silver is derived as 33.33%.

Example 2:
Consider a group of people with 50% of them being employed P(E) = 0.5 and 40% of the people being women. Given that a selected individual is employed with the conditional probability that the selected individual is a woman is P(W|E) = 0.4 * 0.5 = 0.2, i.e., 20%.
Hence, the probability that aperson selected from the group is employed , given that the selected person is a woman is
P(E|W) = P(W|E) * P(E) /P(W) = 0.20 * 0.50 / 0.4 = 0.25, i.e., 25%.

Example 3:
If we toss two coins and look at all the different possibilities, we have the sample space as {HH, HT, TH, TT}.

- The probability of getting two heads = ¼
- The probability of getting at least one tail = ¾
- The probability of second coin being head given the first coin is tail = ½
- The probability of getting two heads given the first coin is a head = ½

The Bayes theorem gives us the conditional probability of event A, given that event B has occurred. In this case, the first toss will be B and the second coin toss A. Thus could be confusing because we have reversed the order of them and go from B to A instead of A to B.

Let us apply Bayes theorem to this situation. Here we have two coins and the first two probabilities of getting two heads ad at least one tail are computed directly from the sample space.

Now in this sample space, let A be the event that the second coin is head, and B be the event that the first coin is tail. Again, we reversed it because we want to know what the second event is going to be.

The probability of A, given B, $P(A|B) = P(B|A) * P(A) / P(B)$
= P(First coin being tail given the second coin is head) * P(Second coin being head) / P(First coin being tail)
$= (½) * (½) / (½) = ½ = 0.5$
Bayes theorem calculates the conditional probability of occurrence of an event on prior knowledge of conditions that might be related to the event.