Dual Simplex method-1. First convert the minimization LPP into maximization form, if it is given in the minimization towns. ?. Convert the '?' type inequalities into 'E' type by multiplying -1. 3. Introduce slack variables in the constraints of the given problem and obtain an initial basic solution.

- 4. Test the nature of Dj in the starting table (1) If all A; and XB are non-negative, then an optimal basic feasible
- solution has been attained. Un If all Di are non-negative and at least one basic variable XB
- is negative, then goto step (5) (iii) It atleast one Aj is negative, then method is not appropriate.
- 5. Select the most negative Xa,: Outgoing vector (leaving vector) [Xx] 6. Test the nature of Xx

i) It all X1 are non-negative, then there does not exist any feasibles (1) 2+ atleast one Xx is negative, then compute Max [4]/xx, Xx(0]

P ------

3. Introduce slac' and obtain an 4. Test the nativ i) It all A; and solution has be in If all Di are (iii) If atleast on 5. Select the mo 6. Test the natur i) It all X a (i) 2+ atleast on to determine 7. Pest the new

Example: Min Z= 3x1+x2 Subject to 21+20 21 2x1+3x2 >2 and 21, 227,0 is negative, th = Max z'=-3x1-x2 Subject to -x1-22 <-1 - 2x, -3x2 5-2 and 24, 2530 SLPP max 2 = - 32(1- X2 -x1-22+31=-1 -2×1-324+2=-2 and 26, 25, 51, 527,0 4. Test the nature of Dj in the starting table

(i) If all Dj and XB are non-negative, then an optimal basic feasible solution has been attained.

is negative, then goto step (5)

(iii) It atleast one At is negative, then method is not appropriate.

Example: Min $Z = 3x_1 + x_2$ Subject to $x_1 + x_2 \ge 1$ $2x_1 + 3x_2 \ge 2$ and $x_1, x_2 \ge 0$ Sol Max $Z^1 = -3x_1 - x_2$ Storking table $C_j - 3 - 1 \quad 0$ Basic Co $x_3 \quad x_1 \quad x_2 \quad S_1$ $S_1 \quad 0 - 1 \quad -1 \quad -1 \quad 1$ $S_2 \quad 0 - 2 \quad -2 \quad -3 \quad 0$ $Z^1 = 0 \quad A_j \quad 3 \quad 1 \quad 0$

Subject to $-x_1-x_2 \le -1$ $-2x_1-3x_2 \le -2$

5. Select the most negative xa,: Outgoing vector (leaving vector) [xn]
6. Test the nature of xn
i) It all x are non-negative, then there does not exist any feasible so
ii) It at least one x is negative, then compute Max [Aj/x, xxco]
to determine incoming vector.
7. Test the new iterated dual simplex table for optimality.

starting table Example: Min Z= 3x1+x2 XB Subject to CB x, x2 variable $x_1 + x_2 \ge 1$ 2x1+3x2 >2 -2 and 21, 227,0 Z'=0 A: Sol Max Z'=-321-X2 Subject to -x1-22 <-1

 $-2x_1-3x_2 \leq -2$

Subscri

5. Select the most negative xa; Outgoing vector (leaving vector) [xn]
6. Test the nature of xn
i) It all x are non-negative, then there does not exist any feasible so

(i) It at least one X is negative, then compute Max [Ai/x, Xx<o] to determine incoming vector.

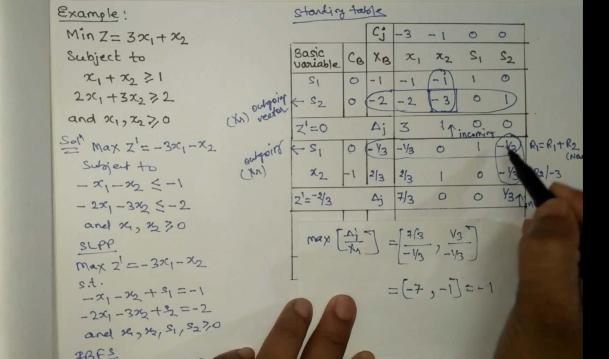
7. Pest the new "terated dual simplex table for optimality.

Subject to $-x_1-n_2 \leq -1$ $-2x_1-3x_2 \leq -2$ $\max \left[\frac{A_1}{M}, x_1 \leq 0\right] = \left[\frac{3}{-2}, \frac{1}{-3}\right] = \frac{1}{-3}$

Subsc.

5. Select the most negative Xa, : Outgoing vector (leaving vector) [Xn] 6. Test the nature of Xx i) It all Xx are non-negative, then there does not exist any feasible so di) 24 at least one Xx is negative, then compute Max[Ai/xx, Xx<o] to determine incoming vector. 7. Pest the new "terated dual simplex table for optimality. starting table Example: Min Z= 3x1+x2 Subject to CBI XB x1+x2 >1 221 +32 >2 and 21, 227,0 Z'=0 Sol Max Z'=-3x1-x2 0 - 1/3 -1/3 Subject to 22 - 43 R2/-3 -x1-22 <-1 2=-43 - 2x1-3x2 5-2

like St Oscribe



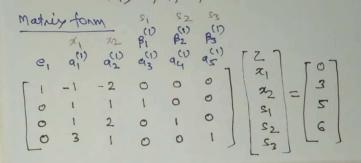
Example: starting table $Min Z = 3x_1 + x_2$ -3 Subject to Basic Ca XB x, Sz 22 variable x1+x2 >1 0 - 1 2x1+3x2 >2 (th) outgoing 0 -2 and 21, 227,0 Z'=0 A; 1 1 incoming Soll Max Z = -321, -x2 outgoird -13) R=R1+R2 - Y3 - Y3 0 0 Subject to (Kn) 22 2/3 2/2 R2/-3 -x1-22 <-1 Y31 noming 2'=-43 7/3 Ai - 2x1 -3x2 5-2 and 24, 2530 -3 S2 0 R1/-1/3 SLPP 22 R2=R2+3R1 1 -1 -1 max 2 = - 321 - x2 21=-1 0 Aj -x1-22+31=-1 -24-32+2=-2 and 26, 25, 51, 527,0 2BFS

Like St Subscrib

Example: starting table $Min Z = 3x_1 + x_2$ Subject to Basic CB XB 22 variable 21+22 21 12 0 -1 0 2x1+3x2 > 2 (Xn) outgoint - S2 0 and 21, 22 7,0 Z'=0 A; 1 pincoming Sol Max Z'=-320,-X2 outgoing - V3 - V3 - 1/3) R=R1+R2 0 Subject to (An) X2 2/3 R2/-3 -x1-x2 <-1 Y31 noming 2'=-43 - 22, -32 5-2 7/3 Ai and x, 230 S2 0 R1/-1/3 SLPP R2=R2+3R1 ! Max 2 = - 3201 - 2 2 =-1 Aj s.t. -x1-x2+31=-1 : all Aj >0 and \$6 >0. -2×1-3×2+2=-2 Therefore optimal solution is and 26, 25, 51, 527,0 Max 2 =- 1, Min 2=1, 4=0, 2=1. 2BFS

Revised Simplex method

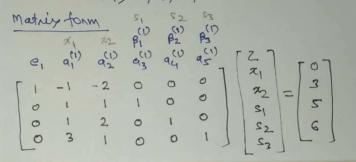
Max
$$Z = x_1 + 2x_2$$
 SLPP Max $Z = x_1 + 2x_2$
Subject to Subject to
 $x_1 + x_2 \le 3$ $x_1 + x_2 + s_1 = 3$
 $x_1 + 2x_2 \le 5$ $x_1 + 2x_2 + s_2 = 5$
 $3x_1 + x_2 \le 6$ $3x_1 + x_2 + s_3 = 6$
and $x_1, x_2 \ge 0$ and $x_1, x_2, s_1, s_2, s_3 \ge 0$
Standard form-I (slack variable only)
 $Z - x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 = 0$
 $x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 = 0$
 $x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = s_3$
 $3x_1 + x_2 + 0s_1 + s_2 + s_3 = 6$
and $x_1, x_2, s_1, s_2, s_3 \ge 0$



Revised Simples table (Iteration-I)

Additional table

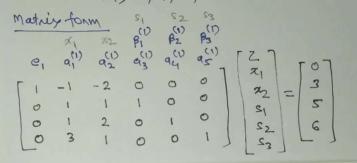




Revised Simples table (Iteration-I)

Additional table

Basic variable		XO	v	Xg/xk			
	(z)	B(1)	β ₂ (1)	B3(1)	XB	NK.	13/XK
		1					
			9				



Revised Simples table (Iteration-I) Additional table

Basic variable	B ₁					Xv	Xg/	
	(z)	P(1)	β ₂ (1)	B3(1)	100	NK	-/XK	
			1					

d	(1)	9	(1)	
1		1		1

Revised simples table (Iteration-I)

Test for optimality

Additional table

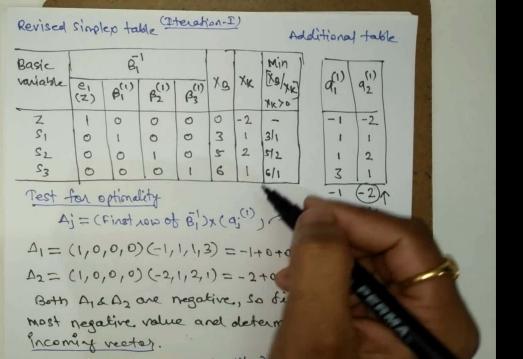
Basic		Bi	١			Y.,	Xol
vaniable	e1 (z)	B(1)	β ₂	B3(1)	N.G.	NK.	Xg/xk
Z	1	0	0	0	0		
51	10	1	0	0	3		
22	0	10	1	0	2		
53	0	10	0	1	6		
	-	_					

Aj = (First 1010 of Bi)x(Q;(1)) on Additional table

$$\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0+0+0 = -2$$

Both A16A2 are negative, so find the most negative value and determine the incoming vector.

Basic Xg/xKI β₂ β₃ χ_Δ XK variable B(1) (2) -1 Test for optimality Aj = (First 100 of Bi)x(q(1)) shadditionaltable Δ1=(1,0,0,0)(-1,1,1,3)=-1+0+0+0=-1 $\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0+0+0 = -2$ Both A1 & A2 are negative, so find the most negative value and determine the Incoming vector. compute the column rector (XK) XK = BI & di) > incoming rector



Like St Subscri Revised Simples table (Iteration-I)

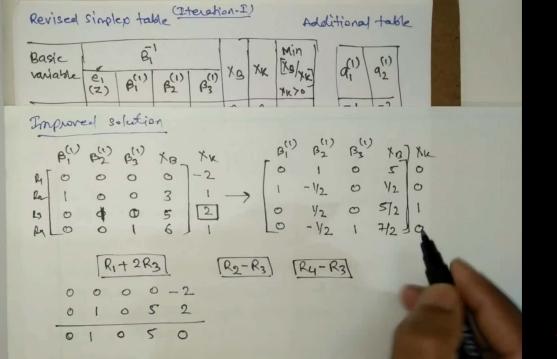
Additional table

Basic		B ₁					Min 1	10	(1)
	(z)	B(1)	β ₂ (1)	B3(1)	XB	XK	X9/XX	a(1)	92
Z	1	0	0	0	0	-2	-	-1	-2
51	0	1	0	0	3	1	311	1	1
52	0	10	1	0	2	2	5124	1	2
53	0	0	0	t	6	1	6/1	3	1
Test -	for e	ptional	lity					-1	(-2)

 $A_j = (Finst now of B_j^1) \times (a_j^{(1)}) \cap Additional tolore$ $A_1 = (1,0,0,0) (-1,1,1,3) = -1 + 0 + 0 + 0 = -1$

$$\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0+0+0=-2$$

Both A16A2 are negative, so find the most negative value and determine the Incoming vector.



Basic variable		Bi	1	XB	XK	Min Xg/xx	
variable	(Z)	B(1)	B2	B3	. 9		YKYO
7	1	0	0	0	0	-2	-
SI	10		0	0	3	1	311
£ 5;	10	0	1	0	2	2	5124
23	0	0	0	1	6	1	6/1

(di)	92
-1	-2
1	1
1	2
3	1

Ateration 2 Revised Simpless table

Basic	1	3	1				
	(2)	B(1)	B(1)	B(1)	X3	XK	Xu Xu
7	1	0	1	0	5		
5,	0	1	-1/2	0	11/2	1	
χ2	0	0	1/2	0	512		
Sa	0	0	-1/2	1	7/2	1	1

Additional table

q(1)	941
-1	0
1	0
1	1
3	0

Basic B_1 variable e_1 e_1 e_2 e_3 e_4 e_5 e_5 e_5 e_6 e_7 e_8 e_7 e_8 e_8 e

(1) | a(1) | -1 | 0 | 1 | 0 | 1 | 3 | 0

Addugonal Tarle

again test for optimality

4 = (First NOD of Bil) x (aj(1))

therefore optimal solution is
$$\max z=5, x_1=0, x_2=5/2$$
.

Revised Simplex method (Standard form-II) Min $Z=x_1+2x_2$ SLPP Max $z'=-x_1-2x_2$ Subject to 2x,+5x236 x1+ x2 32 and x1, x2 30 Standard form-I $Z+x_1+2x_2=0$ $-3x_1 - 6x_2 + x_3 + x_4 + x_5 = -8 | x_5 = -(x_6 + x_7)$ $2x_1 + 5x_2 - x_3 + x_6 = 6$ x1+x2-x4+x2=2 and x1, x2, x3, x4, x5, x6, x2 70

Revised Simplex method (Standard form-II) Min $Z=x_1+2x_2$ SLPP Max $z'=-x_1-2x_2$ Subject to 2x1+5x236 x1+ x2 32 and x1, x2 30 Handard form-I $Z+x_1+2x_2=0$ -3x1-6x2+x3+x4+x5=-8 |x5=-(x6+x2) $2x_1 + 5x_2 - x_3 + x_6 = 6$ x1+ x2- x4+ x2=2 and x1, x2, x3, x4, x5, x6, x3 70 artificial surplus

Madrix form

$$\begin{bmatrix} (2) & (2$$

shase 1 Basic		B2			1		1 1
variable	e,	e2	B(2)	B(2)	XB	XK	YB/XW
21	1	0	0	0	0		
Xs	0	1	0	0	-8		
26	0	O	1	0	6		-
17	0	0	0	1	2		

Additional take

0 -1

-3

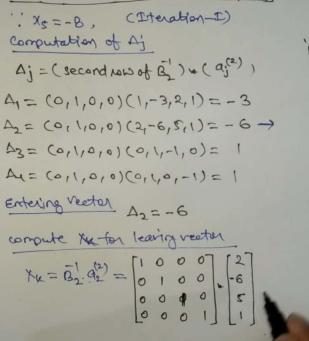
.; xz=-8



hase I		3					1
Basic variable	e,	e ₂	B(2)	B(2)	XB	XK	MIN
21	1	0	0	0	0		
Xs	0	1	0	0	-8		
x ₆	0	O	1	0	6		+
17		0	0	1	2		
Compi	= - 8 Hati	ion o	t Aj	teral			
Δj=	(se	condi	10 Ca	f B2) • (a;	2))
		0.0					

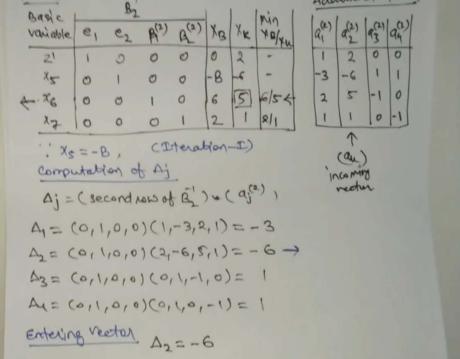
 $\Delta_2 = (0, 1, 0, 0)(2, -6, 5, 1) = -6$ $\Delta_3 = (0, 1, 0, 0)(0, 1, -1, 0) = 1$ $\Delta_4 = (0, 1, 0, 0)(0, 1, 0, -1) = 1$

Additional take



(au) incoming nector

(Iteration-I) : Xe=-8, (au) Computation of Aj Incoming rector Aj = (second now of By) ~ (a;(2)) A= (0,1,0,0)(1,-3,2,1)=-3 Az= (0,1,0,0)(2,-6,5,1)=-6 -> A3 = (0,1,0,0)(0,1,-1,0)= A(= (0,1,0,0)(0,1,0,-1)=1 Entering rector D2 = -6 compute Me for leaving rector $7x = \frac{1}{32} \cdot \frac{a_1^{(2)}}{a_2^{(2)}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$



$$A_{1} = (0,1,0,0)(1,-3,2,1) = -3$$

$$A_{2} = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$$

$$A_{3} = (0,1,0,0)(0,1,-1,0) = 1$$

$$A_{4} = (0,1,0,0)(0,1,0,-1) = 1$$
Entering Vector
$$A_{2} = -6$$
compute x_{k} for leaving vector
$$x_{k} = \frac{1}{6} \cdot \frac{9^{2}}{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 5 \end{bmatrix}$$

$$\frac{1}{1}$$

$$\frac$$

$$A_{1} = (0,1,0,0)(1,-3,2,1) = -3$$

$$A_{2} = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$$

$$A_{3} = (0,1,0,0)(0,1,-1,0) = 1$$

$$A_{4} = (0,1,0,0)(0,1,0,-1) = 1$$
Entering vector
$$A_{2} = -6$$
compute x_{k} for leaving vector
$$x_{k} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$$

$$x_{k} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \times \frac$$

1598'c	12					Min		1 1			
variable	61	ez	B(2)	B2 2	XB	The	YA.	de	a(2)	9(2)	a(2)
ZI	1	0	-2/5	0	-12/5			1	0	0	0
75	0	1	6/5	0	-415			-3	0	1	1
X2	0	0	45	O	615			2	1	-1	0
97	0	0	-115	1	14/5	1	1	1	10	10	-1

1. 15 = -415, Now enter the iteration 2.

599'c.	Con I to Audi	,	7		-		MUM			1	_	-
variable	61	ez	B(2)	β ₂ ⁽²⁾	YB	The	XI XI		a(2)	a(2)	03	a(2)
21	1	0	-2/5	0	-12/5-				1	0	0	0
25	0	1	6/5	0	-415				-3	0	1	1
×2	0	0	45	O	615				2	1	-1	0
97	0	0	-115	1	14/5	1			1	10	10	-1
1. 15= -4/5, How eater the iteration 2.												
computation of A' = (seeond now of B') ~ (a(2))												
A = (0,1,6/5,0).(1,-3,2,1)=-3/5												
De=(01/6/5/0)(010/1/0)=6/5												
13=(011/6/5/0)(011/-1/0)=-1/5												
Ay=	(0	14	6/5/0)(0	,1,0	1-1.)=	1				
Entering recetor A = -3/5												

$$x_{7}$$
 0 0 -1/5 | 4/5 | 1 0 0 -1 |

i x_{7} 0 0 -1/5 | 4/5 | 1 0 0 -1 |

i x_{7} 0 0 -1/5 | 4/5 | 1 0 0 0 -1 |

Computation of Δ_{1} = (see and now of B_{2}^{-1}) = ($a_{2}^{(2)}$)

 Δ_{1} = (a_{1} , a_{1}) = (a_{2})

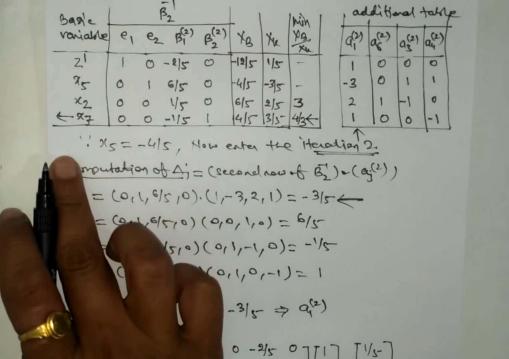
 Δ_{2} = (a_{1} , a_{1}) = (a_{1}) (a_{1}) = -3/5 |

 Δ_{3} = (a_{1} , a_{1}) = (a_{1}) (a_{1}) = -1/5 |

 Δ_{4} = (a_{1} , a_{1}) = (a_{1}) (a_{1}) = 1

Entering vector Δ_{1} = -3/5 \Rightarrow $a_{1}^{(2)}$ |

(ompute the a_{1}) = a_{1} | a_{2} | a_{1} | a_{2} | a_{2} | a_{3} | a_{4} |



$$\Delta_{3} = (0,1),6|5|0)(0,1)-1,0)=-15$$

$$\Delta_{4} = (0,1),6|5|0)(0,1)-1,0)=-15$$
Entering readon
$$\Delta_{1} = -3|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac{1}{2}|5-\frac$$

variable
$$e_1$$
 e_2 $e_1^{(2)}$ $e_2^{(2)}$ $e_2^{(2)}$ $e_3^{(2)}$ $e_4^{(2)}$ $e_4^{(2$

Az=(1,01-1/3,-1/3)(0,4-1,0)= /3

Ay = (1,0,-1/3,-1/3)(0,1,0,1)=1/3

. Az, Ay are positive.

: optimal sor z1 = -0/3 7 = 3/2, 2= 4/3, 2= 3/2