Transportation Problem

Destination (warehouse)

Source (Factory)

	D_1	D_2	.....	D_n		supply (a_i)
S_1	c_{11}	c_{12}	...	c_{1n}	0	a_1
S_2	c_{21}	c_{22}	...	c_{2n}	0	a_2
⋮	⋮	⋮	⋮	⋮		⋮
S_m	c_{m1}	c_{m2}	...	c_{mn}	0	a_m
	0	0		0		a_{m+1}
demand (b_j)	b_1	b_2	....	b_n	b_{n+1}	

$$\boxed{\sum b_j = \sum a_i}$$

$\boxed{TP}$

- Balanced
  - demand = supply
- Unbalanced
  1. supply < demand (dummy row)
  2. supply > demand (dummy col)

Balanced

demand = supply

Unbalanced

1. supply < demand (dummy row)
2. supply > demand (dummy col)

Solution of the Transportation Problem

1. Find an Initial Basic Feasible Solution
   - (i) North-west Corner Rule
   - (ii) Row minima method
   - (iii) column minima method
   - (iv) Lowest cost entry method (Matrix minima)
   - (v) Vogel's Approximation method (VAM) or Unit cost penalty method

2. Perform an optimality test and iterating towards optimal solution is obtained.
   - (i) MODI method (modified distribution) or UV
   - (ii) stepping stone method.

TP

		W_1	W_2	W_3	W_4	capacity
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
warehouse requirement		5	8	7	14	$\boxed{34}$

$\underline{\text{Sol}^n}$: Initial Basic feasible Solution by $\underline{\text{NWCR}}$

19 (5)	30 (2)	50	10	~~7~~ ~~2~~ 0
70	30 (6)	40 (3)	60	~~9~~ ~~3~~ 0
40	8	70	20	18
~~5~~ 0	~~8~~ ~~6~~ 0	~~7~~ 4	14	

f_3	40	8	70	20	18
warehouse requirement	5	8	7	14	$\boxed{34}$

$\underline{Sol^n}$: Initial Basic feasible Solution by $\underline{NWCR}$

19 (5)	30 (2)	50	10	$\cancel{7}$ $\cancel{2}$ 0
70	30 (6)	40 (3)	60	$\cancel{9}$ $\cancel{3}$ 0
40	8	70 (4)	20 (14)	$\cancel{18}$ $\cancel{14}$ 0
$\cancel{5}$ 0	$\cancel{8}$ $\cancel{6}$ 0	$\cancel{7}$ $\cancel{4}$ 0	$\cancel{14}$ 0	

$$\text{Total cost} = 19\times5 + 30\times2 + 30\times6 + 40\times3 + 70\times4 + 20\times14$$

$$= 1015$$

Row Minima Method

		To D1	D2	D3	D4	supply
From	S_1	6	3	5	4	22
	S_2	5	9	2	7	15
	S_3	5	7	8	6	8
Demand		7	12	17	9	

Row Minima Method

To

		D_1	D_2	D_3	D_4	supply
	S_1	6	3	5	4	22
From	S_2	5	9	2	7	15
	S_3	5	7	8	6	8
Demand		7	12	17	9	$\boxed{45}$

Balanced TP

Sol^n: Initial Basic feasible Solution

					supply
	6	3 (12)	5	4	~~22~~ 10
	5	9	2	7	15
	5	7	8	6	8
Demand	7	~~12~~ 0	17	9	

Row Minima Method

To

		D_1	D_2	D_3	D_4	supply
From	S_1	6	3	5	4	22
	S_2	5	9	2	7	15
	S_3	5	7	8	6	8
Demand		7	12	17	9	$\boxed{45}$

Balanced TP

Sol: Initial Basic feasible Solution

					supply
	6	3 (12)	5	4 (9)	~~22~~ ~~10~~ 1
	5	9	2	7	15
	5	7	8	6	8
Demand	7	~~12~~ 0	17	~~9~~ 0	

Row Minima Method

		To				
		D_1	D_2	D_3	D_4	supply
From	S_1	6	3	5	4	22
	S_2	5	9	2	7	15
	S_3	5	7	8	6	8
	Demand	7	12	17	9	$\boxed{45}$

Balanced TP

Sol^n^: Initial Basic feasible Solution

					supply
	6	3 (12)	5 (1)	4 (9)	~~22~~ ~~10~~ ~~1~~ 0
	5	9	2	7	15
	5	7	8	6	8
Demand	7	~~12~~ 0	~~17~~ 16	~~9~~ 0	

		D_1	D_2	D_3	D_4	Supply
	S_1	6	3	5	4	22
From	S_2	5	9	2	7	15
	S_3	5	7	8	6	8
Demand		7	12	17	9	$\boxed{45}$

Balanced TP

Sol^n: Initial Basic feasible Solution

				supply
6	3 (12)	5 (1)	4 (9)	$\cancel{22}$ $\cancel{10}$ $\cancel{1}$ 0
5	9	2 (15)	7	$\cancel{15}$ 0
5 (7)	7	8 (1)	6	$\cancel{8}$ $\cancel{1}$ 0
Demand: $\cancel{7}$ 0	$\cancel{12}$ 0	$\cancel{17}$ $\cancel{16}$ $\cancel{1}$ 0	$\cancel{9}$ 0	

Total cost $= 3\times12 + 5\times1 + 4\times9 + 2\times15$
$+ 5\times7 + 8\times1$
$= 150.$

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Column Minima Method

		To		Available
From	10	13	6	10
	16	7	13	12
	8	22	2	8
Requirement	6	11	13	

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Column Minima Method

	To			Available
From	10	13	6	10
	16	7	13	12
	8	22	2	8
Requirement	6	11	13	$\boxed{30}$

Balanced

Sol^n: Initial Basic Feasible Solution

				Available
	10	13	6	10
	16	7 (11)	13	~~12~~ 1
	8 (6)	22	2	~~8~~ 2
Requirement	~~6~~ 0	~~11~~ 0	13	

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From				
	10	13	6	10
	16	7	13	12
	8	22	2	8
Requirement	6	11	13	$\boxed{30}$

Balanced

Sol^n: Initial Basic Feasible Solution

				Available
	10	13	6 (10)	~~10~~ 0
	16	7 (11)	13 (1)	~~12~~ ~~1~~ 0
	8 (6)	22	2 (2)	~~8~~ ~~2~~ 0
Requirement	~~6~~ 0	~~11~~ 0	~~13~~ ~~11~~ ~~1~~ 0	

$$\text{Total cost} = 8\times6 + 7\times11 + 6\times10 + 13\times1 + 2\times2$$

$$= 202.$$

Lowest-cost Entry method
(Matrix Minima method)

		warehouse				
		W_1	W_2	W_3	W_4	capacity
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	[illegible]	7	14	$\boxed{34}$

Lowest-cost Entry method
(Matrix Minima method)

warehouse

		W_1	W_2	W_3	W_4	capacity
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

Sol^n: Initial Basic Feasible Solution

					capacity
	19	30	50	10	7
	70	30	40	60	9
	40	8 (8)	70	20	~~18~~ 10
requirement	5	~~8~~ 0	7	14	

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Lowest-cost Entry method
(Matrix Minima method)

		warehouse W_1	W_2	W_3	W_4	capacity
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

Sol^n: Initial Basic Feasible Solution

					capacity
	19	30	50	10 (7)	~~7~~ 0
	70	30	40	60	9
	40	8 (8)	70	20 (7)	~~18~~ ~~10~~ 3
requirement	5	~~8~~ 0	7	~~14~~ ~~7~~ 0	

Lowest-cost Entry method
(Matrix Minima method)

		warehouse				
		W_1	W_2	W_3	W_4	capacity
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

Sol^n: Initial Basic Feasible Solution

				capacity
19	30	50	10 (7)	~~7~~ 0
70	30	40 (7)	60	~~9~~ 2
40 (3)	8 (8)	70	20 (7)	~~18~~ ~~10~~ ~~3~~ 0
requirement: ~~5~~ 2	~~8~~	~~7~~ 0	~~14~~ ~~7~~ 0	

	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

Sol^n: Initial Basic Feasible Solution

					capacity
	19	30	50	10 (7)	~~7~~ 0
	70 (2)	30	40 (7)	60	~~9~~ ~~2~~ 0
	40 (3)	8 (8)	70	20 (7)	~~18~~ ~~10~~ ~~3~~ 0
requirement	~~5~~ ~~2~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~7~~ 0	

$$\text{Total cost} = 10\times7 + 70\times2 + 40\times7 + 40\times3 + 8\times8 + 20\times7$$
$$= 814.$$

Vogel's Approximation Method
(Unit Cost Penalty Method)

Warehouse

	W_1	W_2	W_3	W_4	capacity
F_1	19	30	50	10	7
factory F_2	70	30	40	60	9
F_3	40	8	70	20	18
requirement	5	8	7	14	

(Unit Cost Penalty Method)

		Warehouse				
		W_1	W_2	W_3	W_4	capacity
factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

step

① penalty = sec.min − min

Sol^n: Initial Basic Feasible Solution

					capacity	P.
	19	30	50	10	7	9
	70	30	40	60	9	10
	40	8	70	20	18	12
req.	5	8	7	14		

(Unit Cost Penalty Method)

	Warehouse W_1	W_2	W_3	W_4	capacity
F_1	19	30	50	10	7
factory F_2	70	30	40	60	9
F_3	40	8	70	20	18
requirement	5	8	7	14	$\boxed{34}$

step
1. penalty = sec.min − min
2. max penalty

Sol^n: Initial Basic Feasible Solution

					capacity	P.
	19	30	50	10	7	9
	70	30	40	60	9	10
	40	8	70	20	18	12
req.	5	8	7	14		
P	21	22	10	10		

(Unit Cost Penalty Method)

Warehouse

		W_1	W_2	W_3	W_4	capacity
factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
requirement		5	8	7	14	$\boxed{34}$

step

① penalty = sec.min − min

② max penalty

<u>Sol</u>

Sol^n: Initial Basic Feasible Solution

					capacity	P.
	19	30	50	10	7	9
	70	30	40	60	9	10
	40	8 (8)	[illegible]	20	~~18~~ 10	12
req.	5	~~8~~ 0	7	[illegible]		
P	21	22	10	[illegible]		

F_1	19	30	50	10	7
factory F_2	70	30	40	60	9
F_3	40	8	70	20	18
requirement	5	8	7	14	$\boxed{34}$

① penalty = sec.min − min

② max penalty

Sol^n: Initial Basic Feasible Solution

				capacity	P.
19(5)	30	50	10	$\cancel{7}$ 2	$\cancel{9}$ $\cancel{9}$ 40
70	30	40	60	9	$\cancel{10}$ $\cancel{20}$ 20
40	8(8)	70	20(10)	$\cancel{18}$ $\cancel{10}$ 0	12 $\cancel{20}$ 50
req. $\cancel{5}$ 0	$\cancel{8}$ 0	7	$\cancel{14}$ 4		
P 21	22	10	10		
21	–	10	10		
–	–	10	10		

Sol^n: Initial Basic Feasible Solution

				capacity	P.
19 (5)	30	50	10 (2)	~~7~~ ~~2~~ 0	~~9~~ ~~9~~ 40 40 –
70	30	40 (7)	60 (2)	~~9~~ ~~2~~ 0	~~10~~ ~~20~~ ~~20~~ ~~20~~ 20
40	8 (8)	70	20 (10)	~~18~~ ~~10~~ 0	12 ~~20~~ 50 ~~–~~ –
req. ~~5~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~4~~ ~~2~~ 0		

P			
21	22	~~10~~	~~10~~
21	–	~~10~~	10
–	–	~~10~~	10
–	–	~~10~~	50
–	–	.	.

$$\text{Total cost} = 19\times5 + 10\times2 + 40\times7 + 60\times2 + 8\times8 + 20\times10$$

$$= 779.$$

MODI (Modified Distribution) Method

Qu: Find an optimal solution

		Warehouse W_1	W_2	W_3	W_4	Availability
Factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	

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MODI (Modified Distribution) Method

Qu: find an optimal solution

		warehouse				
		W_1	W_2	W_3	W_4	Availability
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	$\boxed{34}$

Sol^n ① Find the initial basic feasible solution (IBFS)
(by Vogel's Approximation method)

19 (5)	30	50	10 (2)
70	30	40 (7)	60 (2)
40	8 (8)	70	20 (10)

Min. cost $= 19\times5+10\times2+40\times7+60\times2+8\times8+20\times10$
$= 779.$

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Warehouse

		W_1	W_2	W_3	W_4	Availability
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	$\boxed{34}$

Sol^n ① Find the initial basic feasible solution (IBFS)
(by Vogel's Approximation method)

$m = 3$
$n = 4$

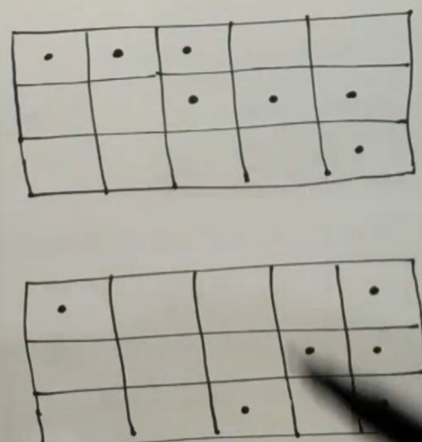
19 (5)	30	50	10 (2)
70	30	40 (7)	60 (2)
40	8 (8)	70	20 (10)

$$\text{Min. cost} = 19\times5 + 10\times2 + 40\times7 + 60\times2 + 8\times8 + 20\times10$$
$$= 779.$$

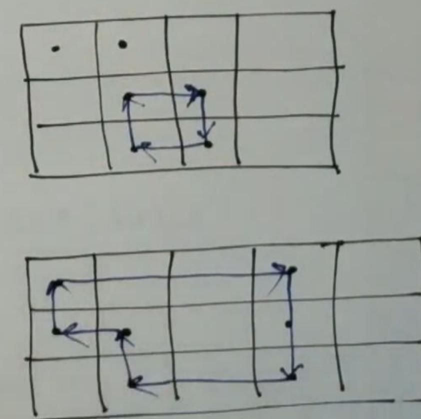
② check for Non-degeneracy
IBFS has $m+n-1 = 6$ allocations in independent positions.

		W_1	W_2	W_3	W_4	Availability
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18

Independent Positions



Non-Independent Positi
(Dependent)



		W_1	W_2	W_3	W_4	Availability
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	$\boxed{34}$

Sol^n ① Find the initial basic feasible solution (IBFS) (by Vogel's Approximation method)

$m=3$
$n=4$

19 (5)	30	50	10 (2)
70	30	40 (7)	60 (2)
40	8 (8)	70	20 (10)

•			•
		•	•
	•		•

$$\text{Min. cost} = 19\times5 + 10\times2 + 40\times7 + 60\times2 + 8\times8 + 20\times10$$
$$= 779.$$

② check for Non-degeneracy

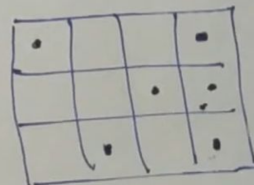
IBFS has $m+n-1 = 6$ allocations in independent positions.

	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	$\boxed{34}$

Solⁿ ① Find the initial basic feasible solution (IBFS)
(by Vogel's Approximation method)

$m=3$
$n=4$

19(5)	30	50	10(2)
70	30	40(7)	60(2)
40	8(8)	70	20(10)



$$\text{Min. cost} = 19\times5+10\times2+40\times7+60\times2+8\times8+20\times10$$
$$= 779.$$

② check for Non-degeneracy
IBFS has $m+n-1=6$ allocations in independent positions. Hence optimality test is satisfied.

is satisfied.

③ Calculation of u_i and v_j : $\boxed{u_i + v_j = c_{ij}}$
(occupied cells)

19			10	$u_1 = 0$
		40	60	u_2
	8		20	u_3
v_1	v_2	v_3	v_4	

independent positions. Hence optimality
is satisfied.

③ Calculation of u_i and v_j : $\boxed{u_i + v_j = c_{ij}}$
(occupied cells)

19			10	$u_1 = 0$
		40	60	u_2
	8		20	u_3
v_1	v_2	v_3	v_4	
19				

$$u_1 + v_1 = c_{11}$$

$$\Rightarrow 0 + v_1 = 19$$

$$\boxed{v_1 = 19}$$

independent positions. Hence

is satisfied.

③ Calculation of u_i and v_j : $\boxed{u_i + v_j = c_{ij}}$

(occupied cells)

19			10	$u_1 = 0$
		40	60	$u_2 = 50$
	8		20	u_3
v_1	v_2	v_3	v_4	
19			10	

$u_1 + v_1 = c_{11}$ $\Rightarrow 0 + v_1 = 19$ $\boxed{v_1 = 19}$

$u_1 + v_4 = c_{14}$ $0 + v_4 = 10$ $v_4 = 10$

$u_2 + v_4 = c_{24}$ $u_2 + 10 = 60$ $u_2 = 50$

independent positions. Hence optimality is satisfied.

③ Calculation of u_i and v_j : $\boxed{u_i + v_j = c_{ij}}$ (occupied cells)

19			10	$u_1 = 0$
		40	60	$u_2 = 50$
	8		20	$u_3 = 10$
v_1	v_2	v_3	v_4	
19	-2	-10	10	

$u_3 + v_4 = c_{34}$

$\Rightarrow u_3 + 10 = 20$

$\Rightarrow u_3 = 10$

$u_3 + v_2 = c_{32}$

$10 + v_2 = 8$

$v_2 = -2$

$u_2 + v_3 = c_{23}$

$50 + v_3 = 40$

$v_3 = -10$

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Sol^n ① Find the initial basic feasible solution (IBFS)
(by Vogel's Approximation method)

$m=3$
$n=4$

19 (5)	30	50	10 (2)
70	30	40 (7)	60 (2)
40	8 (8)	70	20 (10)

•			•
		•	•
	•		•

④ calculation of cost differences for unoccupied cells $\boxed{d_{ij} = c_{ij} - (u_i + v_j)}$

c_{ij}

•	30	50	•
70	30	•	•
40	•	70	•

$u_i + v_j$

				u_i
•	-2	-10	•	0
69	48	•	•	50
29	•	0	•	10
$v_j \rightarrow$ 19	-2	-10	10	

c_{ij}

•	30	50	•
70	30	•	•
40	•	70	•

$u_i + v_j$

				u_i
•	-2	-10	•	0
69	48	•	•	50
29	•	0	•	10
$v_j \rightarrow$ 19	-2	-10	10	

$d_{ij} =$

•	32	60	•
1	-18	•	•
11	•	70	•

(5) optimality test

$d_{ij} < 0$ i.e $d_{22} = -18$

so x_{22} is entering the basis

$v_j \rightarrow$ 19   -2   -10   10

$d_{ij}'' =$

•	32	60	•
1	-18	•	•
11	•	70	•

⑤ optimality test

$d_{ij} < 0$ i,e $d_{22} = -18$

so x_{22} is entering the basis

⑥ construction of loop and allocation of unknown quantity θ

.5			2.
	$+\theta$	7.	2.
	.8.		10.

$v_j' \to$ 19   -2   -10   10

$d_{ij}' =$

•	32	60	•
1	-18	•	•
11	•	70	•

⑤ optimality test

$d_{ij} < 0$ i.e $d_{22} = -18$

So x_{22} is entering the basis

⑥ construction of loop and allocation of unknown quantity θ

.5			2.
	$+\theta$	7.	$2-\theta$
	$8-\theta$		$10+\theta$

so x_{22} is

(6) construction of loop and allocation of unknown quantity θ

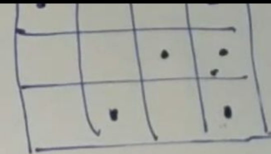
.5			2.
	$+\theta$	7	$2-\theta$
	$8-\theta$		$10+\theta$

$$\min[8-\theta, 2-\theta] = 0$$

$8-\theta=0 \Rightarrow \theta=8$ | $2-\theta=0$, $\boxed{\theta=2}$

$m=3$
$n=4$

19(5)	30	50	10(2)
70	30	40(7)	60(2)
40	8(8)	70	20(10)



$$\text{Min. cost} = 19\times5+10\times2+40\times7+60\times2+8\times8+20\times10$$
$$=779.$$

.5			2.
	$+\theta$	7	$2-\theta$
	$8-\theta$		$10+\theta$

$$\min[8-\theta,\ 2-\theta]=0$$

$8-\theta=0 \Rightarrow \theta=8$ | $2-\theta=0$, $\boxed{\theta=2}$

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

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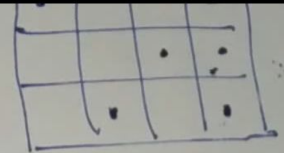
$m=3$
$n=4$

19(5)	30	50	10(2)
70	30	40(7)	60(2)
40	8(8)	70	20(10)



$$\text{Min. cost} = 19\times5+10\times2+40\times7+60\times2+8\times8+20\times10$$
$$= 779.$$

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

$$\text{Min cost} = 19\times5+10\times2+30\times2+40\times7+8\times6+20\times12$$
$$= 743.$$

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

$m+n-1 = 6$

$$\text{Min cost} = 19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$$
$$= 743.$$

7 Improved solution

19			10	u_1
	30	40		u_2
	8		20	u_3
v_1	v_2	v_3	v_4	

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

$m+n-1 = 6$

$\text{min cost} = 19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$

$= 743.$

(7) Improved solution

$\boxed{u_i + v_j = c_{ij}}$

19			10	u_1
	30	40		u_2
	8		20	$u_3 = 0$
v_1	v_2	v_3	v_4	

19(5)			10(2)
	30(2)	40(7)	
	8(6)		20(12)

$m+n-1 = 6$

$\text{min cost} = 19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$
$= 743.$

(7) Improved solution

$\boxed{u_i + v_j = c_{ij}}$

19			10	u_1
	30	40		u_2
	8		20	$u_3 = 0$
v_1	v_2	v_3	v_4	
	8		20	

$u_3 + v_2 = c_{32}$
$0 + v_2 = 8$

$u_3 + v_4 = c_{34}$
$0 + v_4 = 20$

$u_2 + v_2 = c_{22}$
$u_2 + 8 = 30$
$u_2 = 22$

Min cost = 11×...

= 743.

(7) Improved solution

$$\boxed{u_i + v_j = c_{ij}}$$

	v_1	v_2	v_3	v_4	
	19			10	$u_1 = -10$
		30	40		$u_2 = 22$
		8		20	$u_3 = 0$
	29	8	18	20	

$u_3 + v_2 = c_{32}$
$0 + v_2 = 8$

$u_3 + v_4 = c_{34}$
$0 + v_4 = 20$

$u_2 + v_2 = c_{22}$
$u_2 + 8 = 30$
$u_2 = 22$

$u_2 + v_3 = c_{23}$
$\Rightarrow 22 + v_3 = 40$
$\Rightarrow v_3 = 18$

$u_1 + v_4 = c_{14}$
$u_1 + 20 = 10$
$u_1 = -10$

$u_1 + v_1 = c_{11}$
$-10 + v_1 = 19$
$v_1 = 29$

d_i

Like & Subscribe

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$$c_{ij} \qquad \begin{array}{|c|c|c|c|} \hline \cdot & 30 & 50 & \cdot \\ \hline 70 & \cdot & \cdot & 60 \\ \hline 40 & \cdot & 70 & \cdot \\ \hline \end{array}$$

$$u_i + v_j \qquad \begin{array}{cccc|c} \cdot & -2 & 8 & \cdot & -10 \\ 51 & \cdot & \cdot & 42 & 22 \\ 29 & \cdot & 18 & \cdot & 0 \\ \hline 29 & 8 & 18 & 20 & \end{array}$$

$$d_{ij} = \begin{array}{|c|c|c|c|} \hline \cdot & 32 & 42 & \cdot \\ \hline 19 & \cdot & \cdot & 18 \\ \hline 11 & \cdot & 52 & \cdot \\ \hline \end{array}$$

$$\boxed{d_{ij} = c_{ij} - (u_i + v_j)}$$

c_{ij}

•	30	50	•
70	•	•	60
40	•	70	•

$u_i + v_j$

•	-2	8	•	-10
51	•	•	42	22
29	•	18	•	0
29	8	18	20	

$d_{ij} =$

•	32	42	•
19	•	•	18
11	•	52	•

$\because d_{ij} > 0$ an optimal solution is obtained.

with minimum cost Rs. 743

40	•	70	•		29	•	0	•	10

$v_j \rightarrow 19 \quad -2 \quad -10 \quad 10$

$d_{ij} =$

•	32	60	•
1	-18	•	•
11	•	70	•

most negative value

⑤ optimality test

$d_{ij} < 0$ i,e $d_{22} = -18$

so x_{22} is entering the basis

⑥ construction of loop and allocation of unknown quantity θ

.5			2.
	$+\theta$	7.	$2-\theta$
	$.8-\theta$		$10+\theta$

$\min[8-\theta, 2-\theta] = 0$

$8-\theta = 0 \Rightarrow \theta = 8$ | $2-\theta = 0$, $\boxed{\theta = 2}$

Stepping - Stone method -

Qu: Find an optimal solution

	1	2	3	supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	

Sol^n: (1) Find an initial basic feasible solution (IBFS) by Vogel's Approximation method.

				supply
	2	7	4	5
	3	3	1	8
	5	4	7	7
	1	6	2	14
demand	7	9	18	

3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

<u>Sol</u>

Solⁿ: (1) Find an initial basic feasible solution (IBFS) by Vogel's Approximation method.

			Supply	Penalty
2(3)	7(2)	4	~~5~~ ~~2~~ 0	2 2 5 5
3	3	1(8)	~~8~~ 0	2 - - -
5	4(7)	7	~~7~~ 0	1 1 1 1
1(4)	6	2(10)	~~14~~ ~~4~~ 0	1 1 5 -
demand ~~7~~ ~~3~~ 0	~~9~~ ~~2~~ 0	~~18~~ ~~10~~ 0		
penalty 1	1	1		
1	2	2		
1	2	-		
3	3	-		

Min. Transportation cost

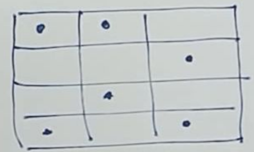
$$= 2\times3 + 7\times2 + 1\times8 + 4\times7 + 1\times4 + 2\times10$$

$$= 80$$

Demand 7 9 18 34

Sol^n: (1) Find an initial basic feasible solution (IBFS) by Vogel's Approximation method.

			Supply	Penalty			
2(3)	7(2)	4	~~5~~ ~~2~~ 0	2	2	5	5
3	3	1(8)	~~8~~ 0	2	1	1	–
5	4(7)	7	~~7~~ 0	1	1	1	1
1(4)	6	2(10)	~~14~~ ~~4~~ 0	1	1	5	–
demand: ~~7~~ ~~3~~ 0	~~9~~ ~~2~~ 0	~~18~~ ~~10~~ 0					
penalty: 1	1	1					
1	2	2					
1	2	–					
3	3	–					



Min. Transportation cost

$= 2\times3 + 7\times2 + 1\times8 + 4\times7 + 1\times4 + 2\times10$

$= 80$

(2) Check for Non-degeneracy

$m+n-1 = 6$ allocation in Independent position.

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

c(1,3) =

c(2,1) =

c(2,2) =

c(3,1) =

c(3,3) =

c(4,2) =

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) =$

$c(2,1) =$

$c(2,2) =$

$c(3,1) =$

$c(3,3) =$

$c(4,2) =$

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

c(1,3) =

c(2,1) =

c(2,

c(3,1

c(3,3

c(4,2

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(2,2) =$

$c(3,1) =$

$c(3,[illegible]) =$

$c[illegible]2) =$

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(2,2) = +3-1+2-1+2-7 = -2$

$c(3,1) =$

$c(3,3) =$

$c(4,2) =$

Like & Subscribe

③ Apply stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(2,2) = +3-1+2-1+2-7 = -2$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) =$

$c(4,2) =$

③ Apply Stepping stone method

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(2,2) = +3-1+2-1+2-7 = -2$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

Like & Subscribe

③ Apply Stepping stone method

cell evaluation / net change

→ -ve

$\boxed{+ve, 0}$

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(2,2) = +3-1+2-1+2-7 =$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 =$

$c(4,2) = +6-1+2-7 = 0$

(3) Apply Stepping stone method

cell evaluation / net change

→ -ve

→ $\boxed{\text{+ve, 0}}$

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$\boxed{c(2,2) = +3-1+2-1+2-7 = -2}$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

$\underbrace{-2,\ -8,\ -4}_{\text{lowest -ve}}$

2	7	4
3	3	1
5	4	7
1	6	2

③ Apply Stepping stone method (unoccupied)

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$\boxed{c(2,2) = +3-1+2-1+2-7 = -2}$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

→ −ve

$\boxed{+ve, 0}$

$\underbrace{-2, -8, -4}_{\text{lowest } -ve}$

2 (5)	7	4
3	3 (2)	1 (6)
5	4 (7)	7
1 (2)	6	2 (12)

③ Apply stepping stone method (unoccupied cell evaluation / net change)

→ -ve

{+ve, 0}

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$\boxed{c(2,2) = +3-1+2-1+2-7 = -2}$ ✓

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

$\underbrace{-2, -8, -4}_{\text{lowest -ve}}$

2 (5)	7	4
3	3 (2)	1 (6)
5	4 (7)	7
1 (2)	6	2 (12)

min cost $= 2\times5 + 3\times2 + 1\times6$

$+ 4\times7 + 1\times2 + 2\times12$

$= 76$

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③ Apply Stepping stone method (unoccupied)

cell evaluation / net change

2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$\boxed{c(2,2) = +3-1+2-1+2-7 = -2}$ ✓

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

→ −ve

→ +ve, 0

$\underbrace{-2,\ -8,\ -4}_{\text{lowest } -ve}$

2 (5)	7	4
3	3 (2)	1 (6)
5	4 (7)	7
1 (2)	6	2 (12)

$c(1,2) = +7-3+1-2+1-2 = 2$

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(3,1) = +5-4+3-1+2-1 = 4$

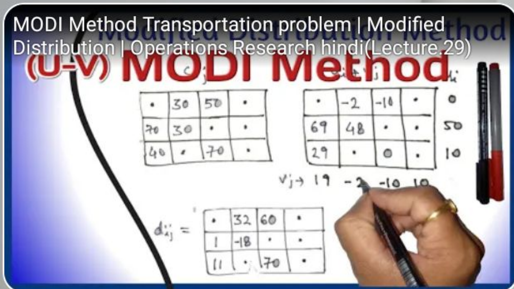
$c(3,3) = +7-1+3-4 = 5$

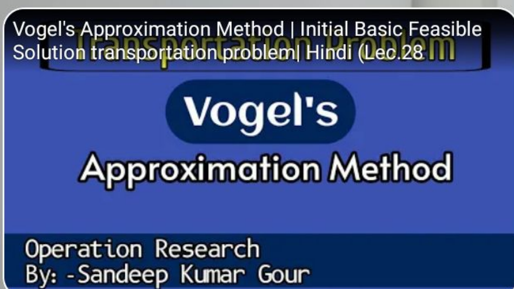
$c(4,2) = +6-2+1-3 = 2$

min cost $= 2\times5 + 3\times2 + 1\times6$

$+ 4\times7 + 1\times2 + 2\times12$

$= 76$





2 (3)	7 (2)	4
3	3	1 (8)
5	4 (7)	7
1 (4)	6	2 (10)

$c(1,3) = +4-2+1-2 = 1$ +ve, 0

$c(2,1) = +3-1+2-1 = 3$

$\boxed{c(2,2) = +3-1+2-1+2-7 = -2}$

$c(3,1) = +5-2+7-4 = 6$

$c(3,3) = +7-2+1-2+7-4 = 7$

$c(4,2) = +6-1+2-7 = 0$

$\underbrace{-2, -8, -4}_{\text{lowest -ve}}$

2 (5)	7	4
3	3 (2)	1 (6)
5	4 (7)	7
1 (2)	6	2 (12)

$c(1,2) = +7-3+1-2+1-2 = 2$

$c(1,3) = +4-2+1-2 = 1$

$c(2,1) = +3-1+2-1 = 3$

$c(3,1) = +5-4+3-1+2-1 = 4$

$c(3,3) = +7-1+3-4 = 5$

$c(4,2) = +6-2+1-3 = 2$

Min cost $= 2\times5 + 3\times2 + 1\times6 + 4\times7 + 1\times2 + 2\times12$

$= 76$

∴ all cell evaluations are +ve

therefore optimal solⁿ is obtained.

Min cost = 76.

Degeneracy in Transportation Problem

IBFS

$m+n-1 =$ no. of allocation.] Non-degeneracy

$m+n-1 >$ no. of allocation] degeneracy

① Degeneracy in the initial solution.

② Degeneracy at any intermediate stage.

① Resolution of degeneracy during the initial stage

lowest cost entry

			Available
8	7	3	60
3	8	9	70
11	3	5	80
Requirement 50	80	80	$\boxed{210}$

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Degeneracy in Transportation Problem

IBFS

$m+n-1 =$ no. of allocation.] Non-degeneracy

$m+n-1 >$ no. of allocation] degeneracy

① Degeneracy in the initial solution.

② Degeneracy at any intermediate stage.

① Resolution of degeneracy during the initial stage

lowest cost entry

			Available
8	7	3 (60)	~~60~~ 0
3 (50)	8	9 (20)	~~70~~ ~~20~~ 0
11	3 (80)	5	~~80~~ 0
~~50~~ 0	~~80~~ 0	~~80~~ ~~20~~ 0	$\boxed{210}$

Requirement: ~~50~~ 0, ~~80~~ 0, ~~80~~ ~~20~~ 0

$m+n-1 = 5 > \underline{4}$ [degeneracy]

① Degeneracy in the initial solution.

② Degeneracy at any intermediate stage.

① Resolution of degeneracy during the initial stage

lowest cost entry

			Available
8	7	3 (60)	~~60~~ 0
3 (50)	8	9 (20)	~~70~~ ~~20~~ 0
11	3 (80)	5	~~80~~ 0
Requirement ~~50~~ 0	~~80~~ 0	~~80~~ ~~20~~ 0	$\boxed{210}$

$m+n-1 = 5 > \underline{4}$ [degeneracy]

8	7	3 (60)
3 (50)	8	9 (20)
11	3 (80)	5 (Δ)

1. min cost (unoccupied cells)
2. independent position

① Degeneracy in the initial solution.

② Degeneracy at any intermediate stage.

① Resolution of degeneracy during the initial stage

lowest cost entry

			Available
~~8~~	~~7~~	3 (60)	~~60~~ 0
3 (50)	~~8~~	9 (20)	~~70~~ ~~20~~ 0
~~11~~	3 (80)	~~5~~	~~80~~ 0
Requirement ~~50~~ 0	~~80~~ 0	~~80~~ ~~20~~ 0	$\boxed{210}$

$m+n-1 = 5 > \underline{4}$ [degeneracy]

8	7	3 (60)	60
3 (50)	8	9 (20)	70
11	3 (80)	5 (Δ)	80+Δ = 80
Req. 50	80	80+Δ=80	

1. min cost (unoccupied cells)
2. independent position

$\boxed{\Delta = \text{zero}}$

$\text{min cost} = 3\times60 + 3\times50 + 9\times20 + 3\times80 + 5\times0$
$= 750$

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② Resolution of degeneracy during the solution Stage

						Available
	4	3	1	2	6	40
	5	2	3	4	5	30
	3	5	6	3	2	20
	2	4	4	5	3	10
Required	30	30	15	20	5	$\boxed{100}$

4	3	1	2	6	40
5	2	3	4	5	30
3	5	6	3	2	20
2	4	4	5	3	10
Required 30	30	15	20	5	$\boxed{100}$

Sol^n:- ① IBFS by North-West Corner Rule

4 (30)	3 (10)	1	2	6
5	2 (20)	3 (10)	4	5
3	5	6 (5)	3 (15)	2
2	4	4	5 (5)	3 (5)

Min Cost = 335

② Check for non-degeneracy

IBFS has $m+n-1 = 8$ allocations in independent positions.

③ Calculation of u_i and v_j :- $u_i + v_j = c_{ij}$ (for occupied cells)

4 .	3 .				$u_1 = 0$
	2 .	3 .			$u_2 = -1$
		6 .	3 .		$u_3 = 2$
			5 .	3 .	$u_4 = 4$
$v_1 = 4$	$v_2 = 3$	$v_3 = 4$	$v_4 = 1$	$v_5 = -1$	

④ Calculation of cost differences for unoccupied cells

$$d_{ij} = c_{ij} - (u_i + v_j)$$

c_{ij}

•	•	1	2	6
5	•	•	4	5
3	5	•	•	2
2	4	4	•	•

−

$u_i + v_j$

•	•	4	1	−1	0
3	•	•	0	−2	−1
6	5	•	•	1	2
8	7	8	•	•	4
4	3	4	1	−1	

$d_{ij} =$

•	•	−3	1	7
2	•	•	4	7
−3	0	•	•	1
−6	−3	−4	•	•

$d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

.	.	1	2	6
5	.	.	4	5
3	5	.	.	2
2	4	4	.	.

$-$

$u_i + v_j$

.	.	4	1	-1	0
3	.	.	0	-2	-1
6	5	.	.	1	2
8	7	8	.	.	4
4	3	4	1	-1	

$d_{ij} =$

.	.	-3	1	7
2	.	.	4	7
-3	0	.	.	1
-6	-3	-4	.	.

(5) optimality test $d_{ij} < 0$

4 (30) $-\theta$	3 (10) $+\theta$			
	2 (20) $-\theta$	3 (10) $+\theta$		
		6 (5) $-\theta$	3 (15) $+\theta$	
$+\theta$			5 (5) $-\theta$	3 (5)

min $[30-\theta, 20-\theta, 5-\theta]$

$\theta = 30, 20, 5$

$\boxed{\theta = 5}$

5	.	.	4	5
3	5	.	.	2
2	4	4	.	.

$-$

3	.	.	0	-2	-1
6	5	.	.	1	2
8	7	8	.	.	4
4	3	4	1	-1	

$d_{ij} =$

.	.	-3	1	7
2	.	.	4	7
-3	0	.	.	1
-6	-3	-4	.	.

⑤ optimality test $d_{ij} < 0$

$4\,(30^{-\theta})$	$3\,(10^{+\theta})$			
	$2\,(20^{-\theta})$	$3\,(10^{+\theta})$		
		$6\,(5^{-\theta})$	$3\,(15^{+\theta})$	
$+\theta$			$5\,(5^{-\theta})$	$3\,(5)$

min $[30-\theta, 20-\theta, 5-\theta]$

$\theta = 30, 20, 5$

$\boxed{\theta = 5}$

4 (25)	3 (15)			
	2 (15)	3 (15)		
			3 (20)	
2 (5)				3 (5)

5	.	.	4	5
3	5	.	.	2
2	4	4	.	.

−

3	.	.	0	−2	−1
6	5	.	.	1	2
8	7	8	.	.	4
4	3	4	1	−1	

$d_{ij} =$

.	.	−3	1	7
2	.	.	4	7
−3	0	.	.	1
−6	−3	−4	.	.

(5) optimality test $d_{ij} < 0$

4 $(30^{-\theta})$	3 $(10^{+\theta})$			
	2 $(20^{-\theta})$	3 $(10^{+\theta})$		
		6 $(5^{-\theta})$	3 $(15^{+\theta})$	
$+\theta$			5 $(5^{-\theta})$	3 (5)

min $[30-\theta, 20-\theta, 5-\theta]$

$\theta = 30, 20, 5$

$\boxed{\theta = 5}$

4 (25)	3 (15)			
	2 (15)	3 (15)		
		*	3 (20)	
2 (5)			5 (Δ)*	3 (5)

$m+n-1 = 8$

5	.	.	4	5	−	3	.	.	0	−2	−1
3	5	.	.	2		6	5	.	.	1	2
2	4	4	.	.		8	7	8	.	.	4
						4	3	4	1	−1	

$d_{ij} =$

.	.	−3	1	7
2	.	.	4	7
−3	0	.	.	1
−6	−3	−4	.	.

⑤ optimality test $d_{ij} < 0$

4 (30) $-\theta$	3 (10) $+\theta$			
	2 (20) $-\theta$	3 (10) $+\theta$		
		6 (5) $-\theta$	3 (15) $+\theta$	
$+\theta$			5 (5) $-\theta$	3 (5)

min $[30-\theta, 20-\theta, 5-\theta]$

$\theta = 30, 20, 5$

$\boxed{\theta = 5}$

4 (25)	3 (15)			
	2 (15)	3 (15)		
			3 (20)	
2 (5)			5 (Δ)	3 (5)

$m+n-1 = 8$

min cost = 305.

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2	4	4	.	.

8	7	8	.	.	4
4	3	4	1	-1	

$d_{ij} =$

.	.	-3	1	7
2	.	.	4	7
-3	0	.	.	1
-6	-3	-4	.	.

⑤ optimality test $d_{ij} < 0$

$4\,(30)^{-\theta}$	$3\,(10)^{+\theta}$			
	$2\,(20)^{-\theta}$	$3\,(10)^{+\theta}$		
		$6\,(5)^{-\theta}$	$3\,(15)^{+\theta}$	
$+\theta$			$5\,(5)^{-\theta}$	$3\,(5)$

min $[30-\theta, 20-\theta, 5-\theta]$

$\theta = 30, 20, 5$

$\boxed{\theta = 5}$

4 (25)	3 (15)			
	2 (15)	3 (15)		
		*	3 (20)	
2 (5)			5 (Δ)*	3 (5)

$m+n-1 = 8$

min cost = 305.

↓

min cost = $\underline{\underline{210}}$

Unbalanced Transportation Problem -

① supply < demand
(dummy row)

	d_1	d_2	d_3	supply
s_1	28	17	26	500
s_2	19	12	16	300
demand	250	250	500	1000 \ 800

	d_1	d_2	d_3	
s_1	28	17	26	500
s_2	19	12	16	300
s_3	0	0	0	200
	250	250	500	1000

② supply > demand
(dummy column)

	d_1	d_2	d_3	supply
s_1	4	3	2	10
s_2	2	5	0	13
s_3	3	8	6	12
demand	8	5	4	17 \ 35

	d_1	d_2	d_3	d_4	
s_1	4	3	2	0	10
s_2	2	5	0	0	13
s_3	3	8	6	0	12
	8	5	4	18	35

Initial basic feasible sol^n by vogel's method

28	17	26	500
19	12	16	300
0	0	0	200
250	250	500	

s_2	19	12	16	300
demand	250	250	500	~~800~~ 1000

s_2	2	5	0	13
s_3	3	8	6	12
demand	8	5	4	~~17~~ 35

	d_1	d_2	d_3	
s_1	28	17	26	500
s_2	19	12	16	300
s_3	0	0	0	200
	250	250	500	1000

	d_1	d_2	d_3	d_4	
s_1	4	3	2	0	10
s_2	2	5	0	0	13
s_3	3	8	6	0	12
	8	5	4	18	35

Initial basic feasible sol^n by vogel's method

28 (50)	17 (250)	26 (200)	~~500~~ ~~250~~ ~~50~~ 0	~~9~~	~~9~~	
19	12	16 (300)	~~300~~ 0	~~4~~	~~4~~	
0 (200)	0	0	~~200~~ 0	~~0~~	~~1~~	
~~250~~ ~~50~~ 0	~~250~~ 0	~~500~~ ~~200~~ 0				
19	12	16				
9	5	10				

$$\text{min cost} = 28\times50 + 17\times250 + 26\times200 + 16\times300 + 0\times200$$
$$= \underline{\underline{15,650}}\,/-$$

Initial basic feasible solⁿ by vogel's method

28 (50)	17 (250)	26 (200)	~~500~~ ~~250~~ ~~50~~ 0	9 9
19	12	16 (300)	~~300~~ 0	4 4
0 (200)	0	0	~~200~~ 0	0 7
~~250~~ 0 ~~50~~	~~250~~ 0	~~500~~ ~~200~~ 0		
19	12	16		
9	5	10		

$$\text{min cost} = 28 \times 50 + 17 \times 250 + 26 \times 200 + 16 \times 300 + 0 \times 200$$
$$= \underline{\underline{15,650}} /-$$

check for Non-degeneracy

$m+n-1 = 5$ allocations in independent positions

Apply MODI method

Initial basic feasible sol^n by vogel's method

28 (50)	17 (250)	26 (200)	~~500~~ ~~250~~ ~~50~~ 0	9	9
19	12	16 (300)	~~300~~ 0	4	4
0 (200)	0	0	~~200~~ 0	0	7
~~250~~	~~250~~	~~500~~			

check for Non-degeneracy

$m+n-1 = 5$ allocations in independent positions

Apply MODI method

1. calculate u_i & v_j for occupied cells $\boxed{c_{ij} = u_i + v_j}$

28 .	17 .	26 .	$u_1 = 0$
		16 .	$u_2 = -10$
0 .			$u_3 = -28$
v_1	v_2	v_3	
28	17	26	

$u_1 + v_1 = c_{11}$

$\Rightarrow 0 + v_1 = 28$

$u_1 + v_2 = c_{12}$

$0 + v_2 = 17$

$u_2 + v_3 = c_{23}$

$u_2 + 26 = 16$

$u_3 + v_1 = c_{31}$

$u_3 + 28 = 0$

$u_3 = -28$

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* Apply MODI method

1. calculate u_i & v_j for occupied cells $\boxed{c_{ij} = u_i + v_j}$

28 .	17 .	26 .	$u_1 = 0$
		16 .	$u_2 = -10$
0 .			$u_3 = -28$
v_1	v_2	v_3	
28	17	26	

$u_1 + v_1 = c_{11}$
$\Rightarrow 0 + v_1 = 28$
$u_1 + v_2 = c_{12}$
$0 + v_2 = 17$
$u_2 + v_3 = c_{23}$
$u_2 + 26 = 16$

$u_3 + v_1 = c_{31}$
$u_3 + 28 = 0$
$u_3 = -28$

2. calculate cost differences $d_{ij} = c_{ij} - (u_i + v_j)$ (unoccupied cells)

c_{ij}

.	.	.
19	12	.
.	0	0

−

$u_i + v_j$

.	.	.	0
18	7	.	-10
.	-11	-2	-28
28	17	26	

$d_{ij} =$

.	.	.
1	5	.
.	11	2

28 .	17 .	26 .	$u_1 = 0$
		16 .	$u_2 = -10$
0 .			$u_3 = -28$
v_1	v_2	v_3	
28	17	26	

$\Rightarrow$

$u_1 + v_1 = c_{11}$

$0 + v_1 = 28$

$u_1 + v_2 = c_{12}$

$0 + v_2 = 17$

$u_2 + v_3 = c_{23}$

$u_2 + 26 = 16$

$u_3 + v_1 = c_{31}$

$u_3 + 28 = 0$

$u_3 = -28$

2. calculate cost differences $\underline{d_{ij}} = c_{ij} - (u_i + v_j)$
(unoccupied cells)

c_{ij}

.	.	.
19	12	.
.	0	0

—

$u_i + v_j$

.	.	.	0
18	7	.	-10
.	-11	-2	-28
28	17	26	

$d_{ij} =$

.	.	.
1	5	.
.	11	2

$\because$ $d_{ij} > 0$, an optimal sol^n is obtained
with minimum cost = $\underline{\underline{15,650}}$.