LESSON 3

PROJECT NETWORK

LESSON OUTLINE

- The key concepts
- Construction of project network diagram

LEARNING OBJECTIVES

After reading this lesson you should be able to

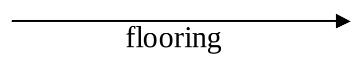
- understand the definitions of important terms
- understand the development of project network diagram
- work out numerical problems

KEY CONCEPTS

Certain key concepts pertaining to a project network are described below:

1. Activity

An activity means a work. A project consists of several activities. An activity takes time. It is represented by an arrow in a diagram of the network. For example, an activity in house construction can be flooring. This is represented as follows:



Construction of a house involves various activities. Flooring is an activity in this project. We can say that a project is completed only when all the activities in the project are completed.

2. Event

It is the beginning or the end of an activity. Events are represented by circles in a project network diagram. The events in a network are called the **nodes**. **Example:**

Starting a punching machine is an activity. Stopping the punching machine is another activity.

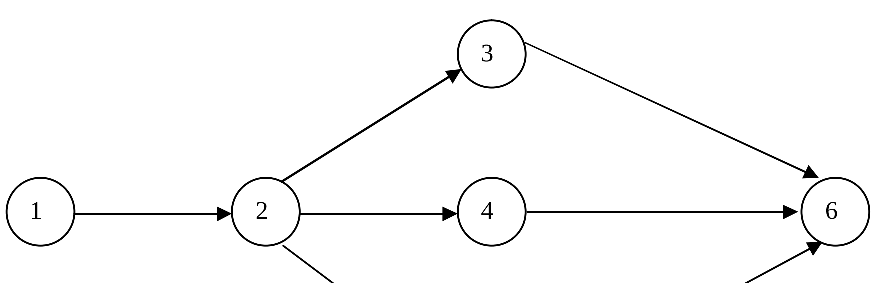
3. Predecessor Event

The event just before another event is called the predecessor event.

4. Successor Event

The event just following another event is called the successor event.

Example: Consider the following.



In this diagram, event 1 is predecessor for the event 2.

Event 2 is successor to event 1.

Event 2 is predecessor for the events 3, 4 and 5.

Event 4 is predecessor for the event 6.

Event 6 is successor to events 3, 4 and 5.

5. Network

A network is a series of related activities and events which result in an end product or service. The activities shall follow a prescribed sequence. For example, while constructing a house, laying the foundation should take place before the construction of walls. Fitting water tapes will be done towards the completion of the construction. Such a sequence cannot be altered.

6. Dummy Activity

A dummy activity is an activity which does not consume any time. Sometimes, it may be necessary to introduce a dummy activity in order to provide connectivity to a network or for the preservation of the logical sequence of the nodes and edges.

7. Construction of a Project Network

A project network consists of a finite number of events and activities, by adhering to a certain specified sequence. There shall be a **start event** and an **end event (or stop event)**. All the other events shall be between the start and the end events. The activities shall be marked by directed arrows. An activity takes the project from one event to another event.

An event takes place at a point of time whereas an activity takes place from one point of time to another point of time.

CONSTRUCTION OF PROJECT NETWORK DIAGRAMS

Problem 1:

Construct the network diagram for a project with the following activities**:**

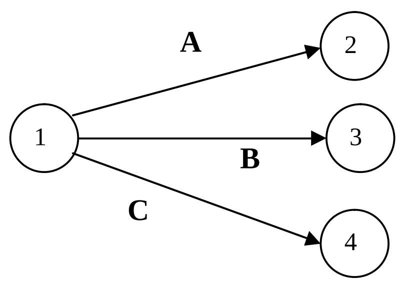
Activity	Name of	Immediate

Event→Event	Activity	Predecessor Activity
1→2	A	-
1→3	B	-
1→4	C	-
2→5	D	A
3→6	E	B
4→6	F	C
5→6	G	D

Solution:

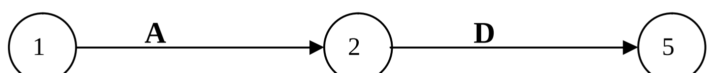
The start event is node 1.

The activities A, B, C start from node 1 and none of them has a predecessor activity. A joins nodes1 and 2; B joins nodes 1 and 3; C joins nodes 1 and 4. So we get the following:

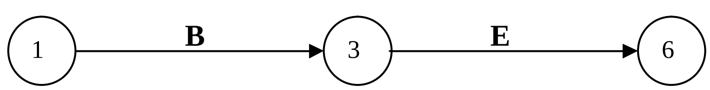


This is a part of the network diagram that is being constructed.

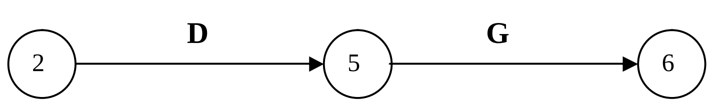
Next, activity D has A as the predecessor activity. D joins nodes 2 and 5. So we get



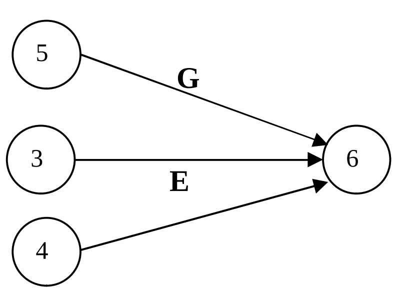
Next, activity E has B as the predecessor activity. E joins nodes 3 and 6. So we get



Next, activity G has D as the predecessor activity. G joins nodes 5 and 6. Thus we obtain



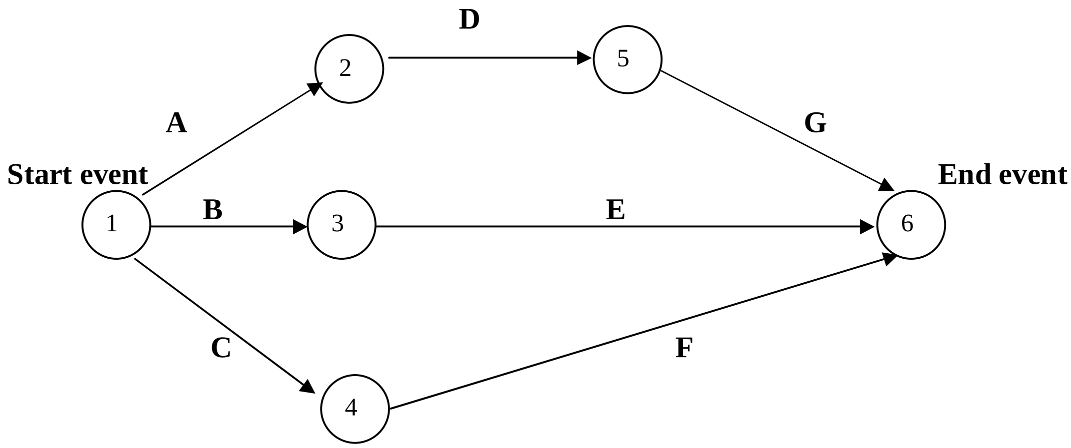
Since activities E, F, G terminate in node 6, we get



F

6 is the end event.

Combining all the pieces together, the following network diagram is obtained for the given project:



We validate the diagram by checking with the given data.

Problem 2:

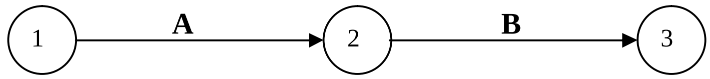
Develop a network diagram for the project specified below:

Activity	Immediate Predecessor Activity
A	-
B	A
C, D	B
E	C
F	D
G	E, F

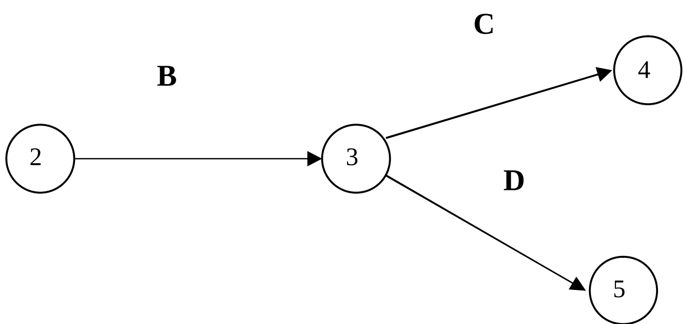
Solution:

Activity A has no predecessor activity. i.e., It is the first activity. Let us suppose that activity A takes the project from event 1 to event 2. Then we have the following representation for A:

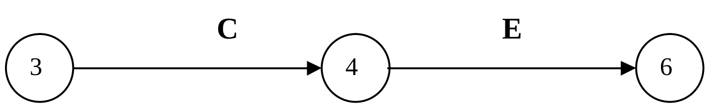
For activity B, the predecessor activity is A. Let us suppose that B joins nodes 2 and 3. Thus we get



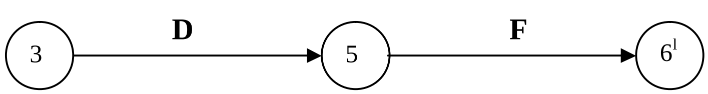
Activities C and D have B as the predecessor activity. Therefore we obtain the following:



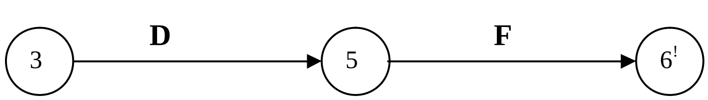
Activity E has D as the predecessor activity. So we get



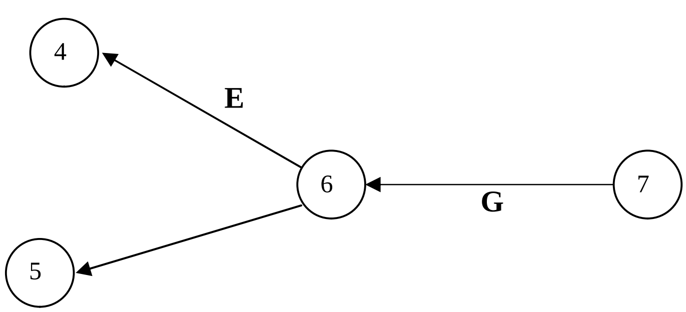
Activity F has D as the predecessor activity. So we get



Activity G has E and F as predecessor activities. This is possible only if nodes 6 and 6^{l} are one and the same. So, rename node 6^{l} as node 6. Then we get



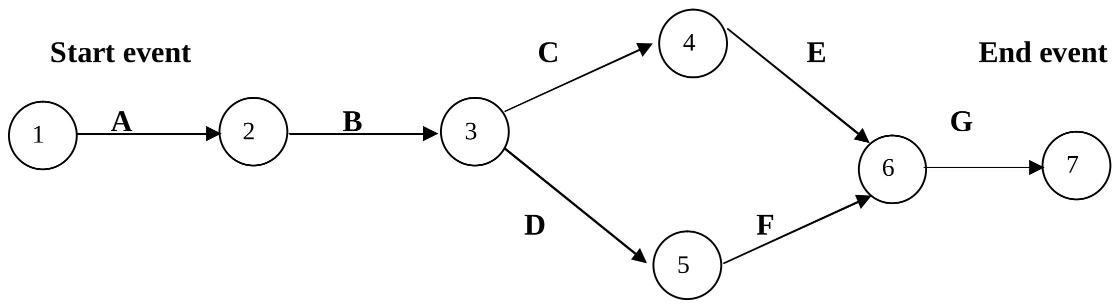
and



F

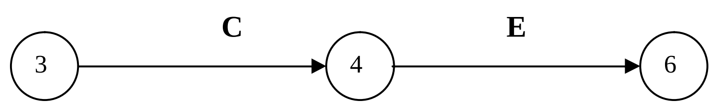
G is the last activity.

Putting all the pieces together, we obtain the following diagram the project network:

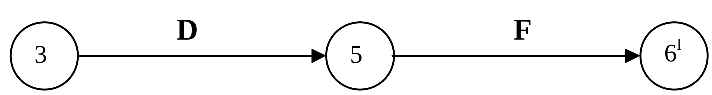


The diagram is validated by referring to the given data.

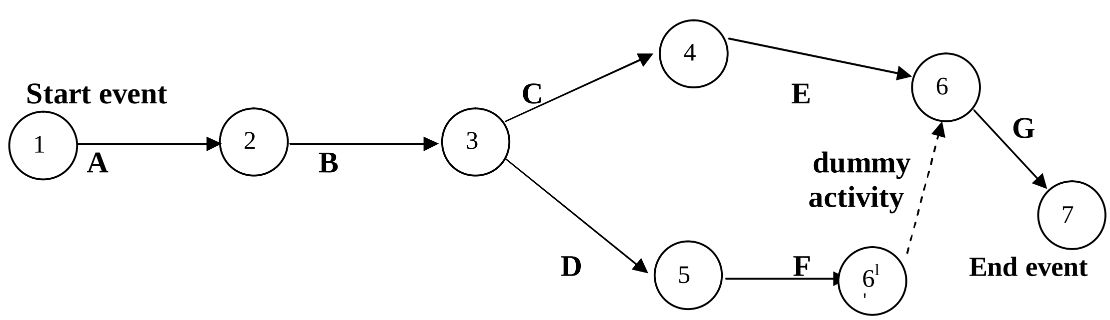
Note: An important point may be observed for the above diagram. Consider the following parts in the diagram



and



We took nodes 6 and 6^l as one and the same. Instead, we can retain them as different nodes. Then, in order to provide connectivity to the network, we join nodes 6^l and 6 by a dummy activity. Then we arrive at the following diagram for the project network:



QUESTIONS:

1. Explain the terms: event, predecessor event, successor event, activity, dummy activity, network.
2. Construct the network diagram for the following project:

Activity	Immediate Predecessor Activity
A	-
B	-
C	A
D	B
E	A
F	C, D
G	E
H	E
I	F, G
J	H, I

LESSON 4

CRITICAL PATH METHOD (CPM)

LESSON OUTLINE

- The concepts of critical path and critical activities
- Location of the critical path
- Evaluation of the project completion time

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the definitions of critical path and critical activities
- identify critical path and critical activities
- determine the project completion time

INTRODUCTION

The critical path method (CPM) aims at the determination of the time to complete a project and the important activities on which a manager shall focus attention.

ASSUMPTION FOR CPM

In CPM, it is assumed that precise time estimate is available for each activity.

PROJECT COMPLETION TIME

From the start event to the end event, the time required to complete all the activities of the project in the specified sequence is known as the project completion time.

PATH IN A PROJECT

A continuous sequence, consisting of nodes and activities alternatively, beginning with the start event and stopping at the end event of a network is called a path in the network.

CRITICAL PATH AND CRTICAL ACTIVITIES

Consider all the paths in a project, beginning with the start event and stopping at the end event. For each path, calculate the time of execution, by adding the time for the individual activities in that path.

The path with the largest time is called the **critical path** and the activities along this path are called the **critical activities** or **bottleneck activities**. The activities are called critical because they cannot be delayed. However, a non-critical activity may be delayed to a certain extent. Any delay in a critical activity will delay the completion of the whole project. However, a certain permissible delay in a non –critical activity will not delay the completion of the whole project. It shall be noted that delay in a non-critical activity beyond a limit would certainly delay the completion the whole project. Sometimes, there may be several critical paths for a project. A project manager shall pay special attention to critical activities.

Problem 1:

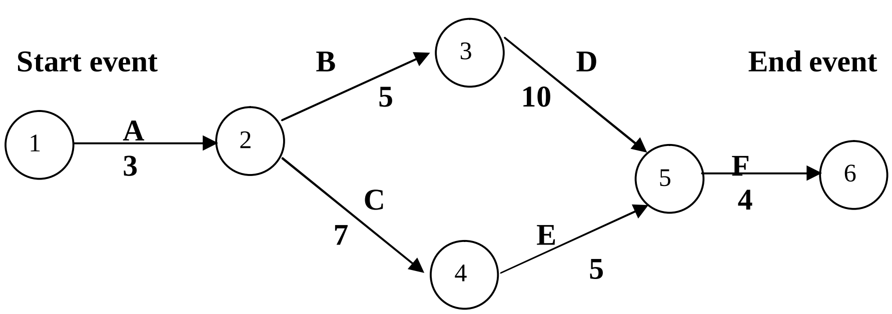
The following details are available regarding a project:

Activity	Predecessor Activity	Duration (Weeks)
A	-	3
B	A	5
C	A	7
D	B	10
E	C	5
F	D,E	4

Determine the critical path, the critical activities and the project completion time.

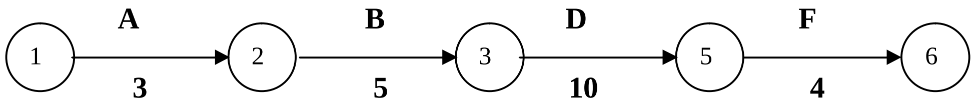
Solution:

First let us construct the network diagram for the given project. We mark the time estimates along the arrows representing the activities. We obtain the following diagram:



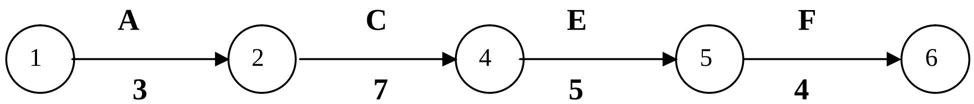
Consider the paths, beginning with the start node and stopping with the end node. There are two such paths for the given project. They are as follows:

Path I



with a time of 3 + 5 + 10 + 4 = 22 weeks.

Path II



with a time of 3 + 7 + 5 + 4 = 19 weeks.

Compare the times for the two paths. Maximum of {22,19} = 22. We see that path I has the maximum time of 22 weeks. Therefore, path I is the critical path. The critical activities are A, B, D and F. The project completion time is 22 weeks.

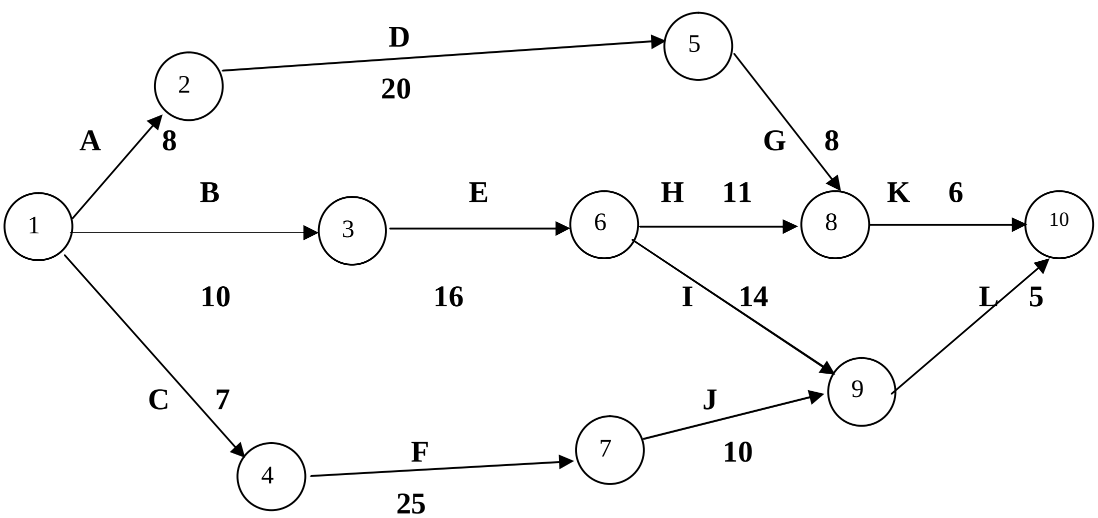
We notice that C and E are non- critical activities.

Time for path I - Time for path II = 22- 19 = 3 weeks.

Therefore, together the non- critical activities can be delayed upto a maximum of 3 weeks, without delaying the completion of the whole project.

Problem 2:

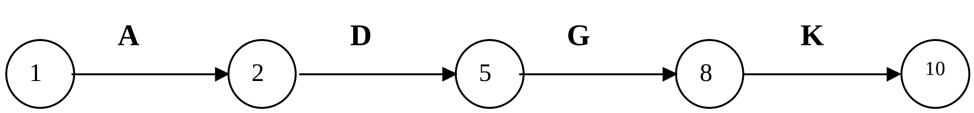
Find out the completion time and the critical activities for the following project:



Solution:

In all, we identify 4 paths, beginning with the start node of 1 and terminating at the end node of 10. They are as follows:

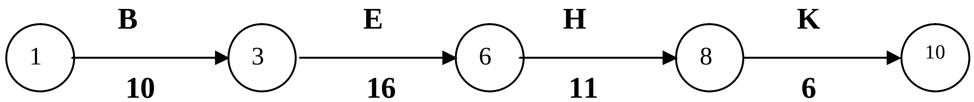
Path I



8 **20** **8** **6**

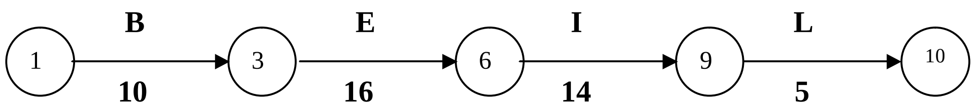
Time for the path **=** 8 + 20 + 8 + 6 = 42 units of time.

Path II



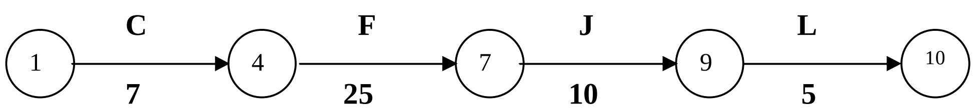
Time for the path **=** 10 + 16 + 11 + 6 = 43 units of time.

Path III



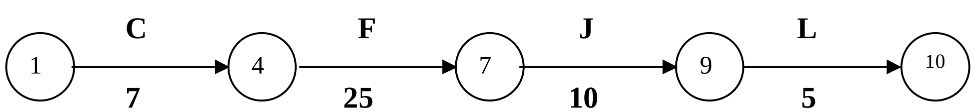
Time for the path **=** 10 + 16 + 14 + 5 = 45 units of time.

Path IV



Time for the path **=** 7 + 25 + 10 + 5 = 47 units of time.

Compare the times for the four paths. Maximum of {42, 43, 45, 47} = 47. We see that the following path has the maximum time and so it is the critical path:



The critical activities are C, F, J and L. The non-critical activities are A, B, D, E, G, H, I and K. The project completion time is 47 units of time.

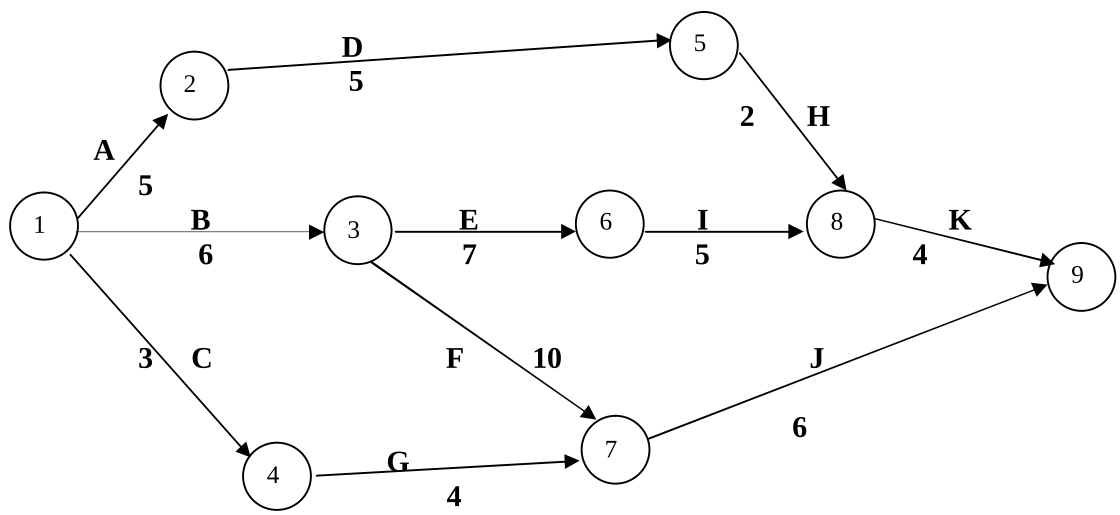
Problem 3:

Draw the network diagram and determine the critical path for the following project:

Activity	Time estimate (Weeks)
1- 2	5
1- 3	6

1- 4	3
2 -5	5
3 -6	7
3 -7	10
4 -7	4
5 -8	2
6 -8	5
7 -9	6
8 -9	4

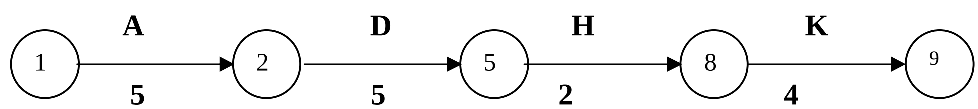
Solution: We have the following network diagram for the project:



Solution:

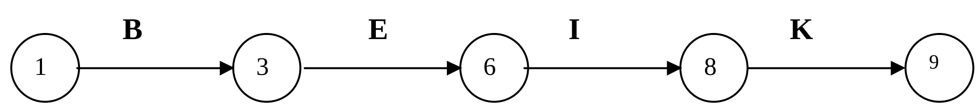
We assert that there are 4 paths, beginning with the start node of 1 and terminating at the end node of 9. They are as follows:

Path I



Time for the path = 5 + 5 + 2 + 4 = 16 weeks.

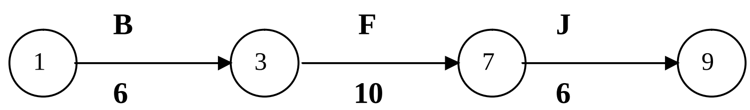
Path II



6 7 5 4

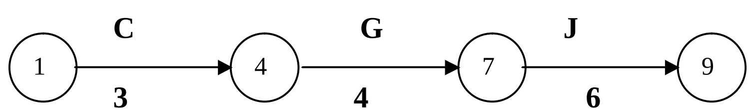
Time for the path = 6 + 7 + 5 + 4 = 22 weeks.

Path III



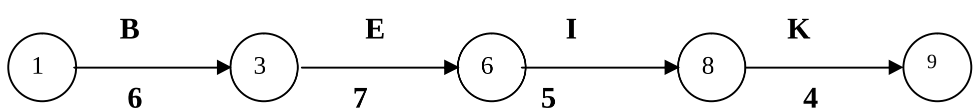
Time for the path = 6 + 10 + 6 = 16 weeks.

Path IV



Time for the path = 3 + 4 + 6 = 13 weeks.

Compare the times for the four paths. Maximum of {16, 22, 16, 13} = 22. We see that the following path has the maximum time and so it is the critical path:



The critical activities are B, E, I and K. The non-critical activities are A, C, D, F, G, H and J. The project completion time is 22 weeks.

QUESTIONS:

1. Explain the terms: critical path, critical activities.
2. The following are the time estimates and the precedence relationships of the activities in a project network:

Activity	IMMEDIATE Predecessor Activity	time estimate (weeks)
A	-	4
B	-	7
C	-	3
D	A	6
E	B	4
F	B	7
G	C	6
H	E	10
I	D	3
J	F, G	4
K	H, I	2

Draw the project network diagram. Determine the critical path and the project completion time.

LESSON 5

PERT

LESSON OUTLINE

- The concept of PERT
- Estimates of the time of an activity
- Determination of critical path
- Probability estimates

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the importance of PERT
- locate the critical path
- determine the project completion time
- find out the probability of completion of a project before a stipulated time

INTRODUCTION

Programme Evaluation and Review Technique (PERT) is a tool that would help a project manager in project planning and control. It would enable him in continuously monitoring a project and taking corrective measures wherever necessary. This technique involves statistical methods.

ASSUMPTIONS FOR PERT

Note that in CPM, the assumption is that precise time estimate is available for each activity in a project. However, one finds most of the times that this is not practically possible.

In PERT, we assume that it is not possible to have precise time estimate for each activity and instead, probabilistic estimates of time alone are possible. A multiple time estimate approach is followed here. In probabilistic time estimate, the following 3 types of estimate are possible:

1. Pessimistic time estimate (t_p)
2. Optimistic time estimate (t_o)
3. Most likely time estimate (t_m)

The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period. On the other hand, a pessimistic estimate is made on the assumption that there would be unexpected problems during the execution of an activity and hence it would consume more time. The most likely time estimate is made in between the optimistic and the pessimistic estimates of time. Thus the three estimates of time have the relationship

$t_o \le t_m \le t_p$.

Practically speaking, neither the pessimistic nor the optimistic estimate may hold in reality and it is the most likely time estimate that is expected to prevail in almost all cases. Therefore, it is preferable to give more weight to the most likely time estimate.

We give a weight of 4 to most likely time estimate and a weight of 1 each to the pessimistic and optimistic time estimates. We arrive at a time estimate (t_e) as the weighted average of these estimates as follows:

$$t_e = \frac{t_o + 4\, t_m + t_p}{6}$$

Since we have taken 6 units (1 for t_p, 4 for t_m and 1 for t_o), we divide the sum by 6. With this time estimate, we can determine the project completion time as applicable for CPM.

Since PERT involves the average of three estimates of time for each activity, this method is very practical and the results from PERT will be have a reasonable amount of reliability.

MEASURE OF CERTAINTY

The 3 estimates of time are such that

$$t_o \le t_m \le t_p .$$

Therefore the range for the time estimate is $t_p - t_o$.

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

i.e., The standard deviation = $\sigma = \dfrac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

The certainty of the time estimate of an activity can be analysed with the help of the variance. The greater the variance, the more uncertainty in the time estimate of an activity.

Problem 1:

Two experts A and B examined an activity and arrived at the following time estimates.

Expert	Time Estimate

	t_o	t_m	t_p
A	4	6	8
B	4	7	10

Determine which expert is more certain about his estimates of time:

Solution:

$$\text{Variance } (\sigma^2) \text{ in time estimates} = \left(\frac{t_p - t_o}{6}\right)^2$$

In the case of expert A, the variance = $\left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$

As regards expert B, the variance = $\left(\frac{10-4}{6}\right)^2 = 1$

So, the variance is less in the case of A. Hence, it is concluded that the expert A is more certain about his estimates of time.

Determination of Project Completion Time in PERT

Problem 2:

Find out the time required to complete the following project and the critical activities:

Activity	Predecessor Activity	Optimistic time estimate (t_o days)	Most likely time estimate (t_m days)	Pessimistic time estimate (t_p days)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D, E	16	21	26
G	D, E	19	22	25
H	F	2	5	8
I	G	1	3	5

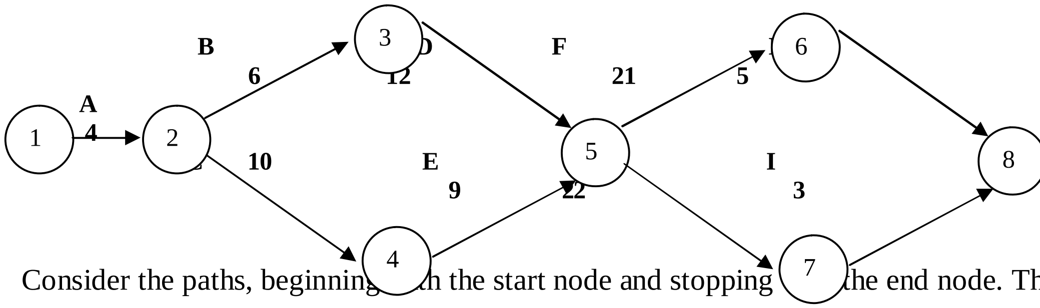
Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. We obtain the following table:

Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	$t_o + 4t_m + t_p$	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
A	2	16	6	24	4
B	3	24	9	36	6
C	8	40	12	60	10
D	9	48	15	72	12
E	8	36	10	54	9
F	16	84	26	126	21

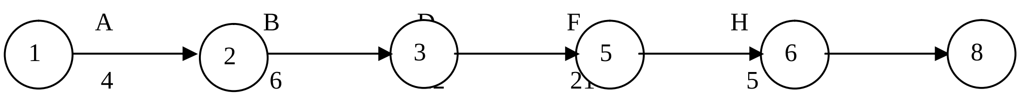
G	19	88	25	132	22
H	2	20	8	30	5
I	1	12	5	18	3

Using the single time estimates of the activities, we get the following network diagram for the project.



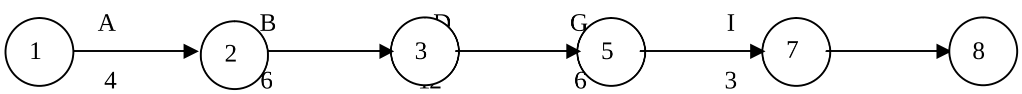
Consider the paths, beginning with the start node and stopping with the end node. There are four such paths for the given project. They are as follows:

Path I



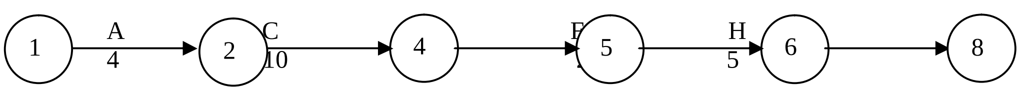
Time for the path: 4+6+12+21+5 = 48 days.

Path II



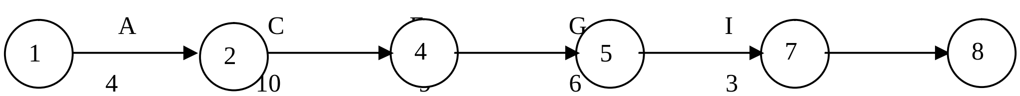
Time for the path: 4+6+12+ 6+3 = 31 days.

Path III



Time for the path: 4+10+9+ 21+5 = 49 days.

Path IV



Time for the path: 4+10+9+ 6+3 = 32 days.

Compare the times for the four paths.
Maximum of {48, 31, 49, 32} = 49.
We see that Path III has the maximum time.
Therefore the critical path is Path III. i.e., $1 \to 2 \to 4 \to 5 \to 6 \to 8$.
The critical activities are A, C, E, F and H.
The non-critical activities are B, D, G and I.
Project time (Also called project length) = 49 days.

Problem 3:

Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

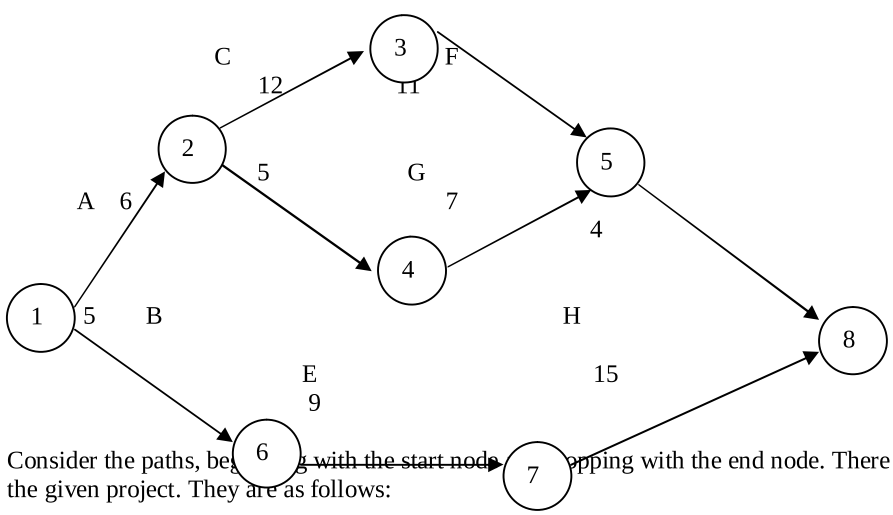
Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)
1-2	3	6	9
1-6	2	5	8
2-3	6	12	18
2-4	4	5	6
3-5	8	11	14
4-5	3	7	11
6-7	3	9	15
5-8	2	4	6
7-8	8	16	18

Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. We obtain the following table:

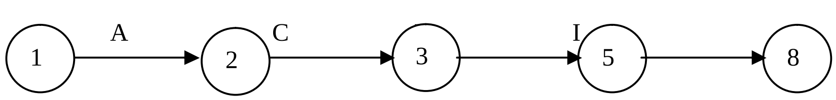
Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	$t_o + 4t_m + t_p$	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
1-2	3	24	9	36	6
1-6	2	20	8	30	5
2-3	6	48	18	72	12
2-4	4	20	6	30	5
3-5	8	44	14	66	11
4-5	3	28	11	42	7
6-7	3	36	15	54	9
5-8	2	16	6	24	4
7-8	8	64	18	90	15

With the single time estimates of the activities, we get the following network diagram for the project.



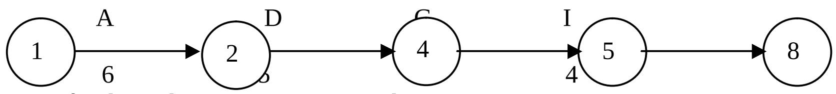
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



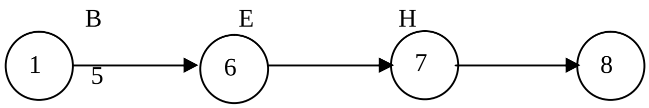
6 12 11 4
Time for the path: 6+12+11+4 = 33 weeks.

Path II



Time for the path: 6+5+7+ 4= 22 weeks.

Path III



Time for the path: 5+9+15 = 29 weeks.

Compare the times for the three paths.
Maximum of {33, 22, 29} = 33.
It is noticed that Path I has the maximum time.
Therefore the critical path is Path I. i.e., $1 \to 2 \to 3 \to 5 \to 8$
The critical activities are A, C, F and I.
The non-critical activities are B, D, G and H.
Project time = 33 weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)	Range (t_p - t_o)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: $1 \to 2$	3	6	9	6	1	1
C: $2 \to 3$	6	12	18	12	2	4
F: $3 \to 5$	8	11	14	6	1	1
I: $5 \to 8$	2	4	6	4	2/3	4/9

Variance of project time (Also called Variance of project length) =
Sum of the variances for the critical activities = 1+4+1+ 4/9 = 58/9 Weeks.
Standard deviation of project time = $\sqrt{}$Variance = $\sqrt{}$58/9 = 2.54 weeks.

Problem 4
A project consists of seven activities with the following time estimates. Find the probability that the project will be completed in 30 weeks or less.

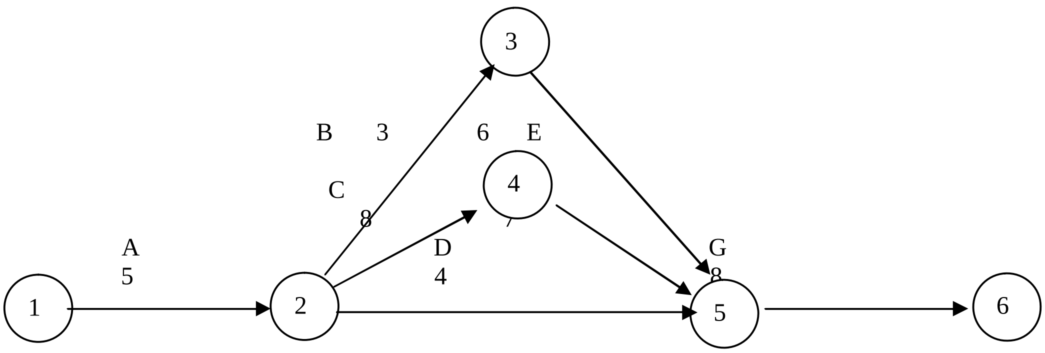
Activity	Predecessor Activity	Optimistic time estimate (t_o days)	Most likely time estimate (t_m days)	Pessimistic time estimate (t_p days)
A	-	2	5	8
B	A	2	3	4
C	A	6	8	10
D	A	2	4	6
E	B	2	6	10
F	C	6	7	8
G	D, E, F	6	8	10

Solution:

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. The results are furnished in the following table:

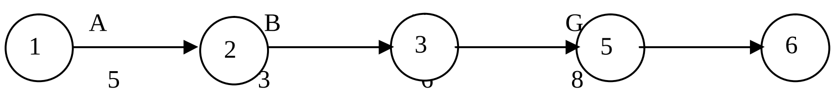
Activity	Optimistic time estimate (t_o)	4 x Most likely time estimate	Pessimistic time estimate (t_p)	t_o+ 4t_m+ t_p	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
A	2	20	8	30	5
B	2	12	4	18	3
C	6	32	10	48	8
D	2	16	6	24	4
E	2	24	10	36	6
F	6	28	8	42	7
G	6	32	10	48	8

With the single time estimates of the activities, the following network diagram is constructed for the project.



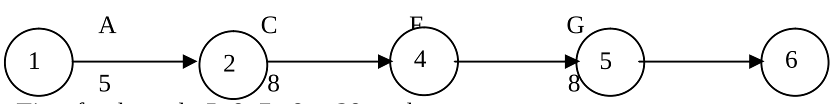
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



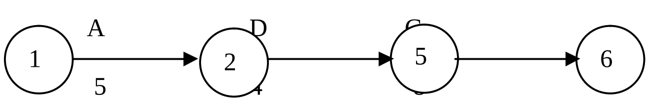
Time for the path: 5+3+6+8 = 22 weeks.

Path II



Time for the path: 5+8+7+ 8 = 28 weeks.

Path III



Time for the path: 5+4+8 = 17 weeks.

Compare the times for the three paths.
Maximum of {22, 28, 17} = 28.
It is noticed that Path II has the maximum time.

Therefore the critical path is Path II. i.e., $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$.
The critical activities are A, C, F and G.
The non-critical activities are B, D and E.
Project time = 28 weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (t_o)	Most likely time estimate (t_m)	Pessimistic time estimate (t_p)	Range (t_p - t_o)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: $1 \rightarrow 2$	2	5	8	6	1	1
C: $2 \rightarrow 4$	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$
F: $4 \rightarrow 5$	6	7	8	2	$\frac{1}{3}$	$\frac{1}{9}$
G: $5 \rightarrow 6$	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$

Standard deviation of the critical path = $\sqrt{2}$ = 1.414

The standard normal variate is given by the formula

$$Z = \frac{\textit{Given value of } t - \textit{Expected value of } t \text{ in } \textit{the critical path}}{\textit{SD for the critical path}}$$

So we get $Z = \frac{30 - 28}{1.414}$ = 1.414

We refer to the Normal Probability Distribution Table.
Corresponding to Z = 1.414, we obtain the value of 0.4207
We get 0.5 + 0.4207 = 0. 9207
Therefore the required probability is 0.92
i.e., There is 92% chance that the project will be completed before 30 weeks. In other words, the chance that it will be delayed beyond 30 weeks is 8%

QUESTIONS:

1. Explain how time of an activity is estimated in PERT.
2. Explain the measure of certainty in PERT.
3. The estimates of time in weeks of the activities of a project are as follows:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	2	4	6
B	A	8	11	20
C	A	10	15	20
D	B	12	18	24

E	C	8	13	24
F	C	4	7	16
G	D,F	14	18	28
H	E	10	12	14
I	G,H	7	10	19

Determine the critical activities and the project completion time.

4. Draw the network diagram for the following project. Determine the time, variance and standard deviation of the project.:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	12	14	22
B	-	16	17	24
C	A	14	15	16
D	A	13	18	23
E	B	16	18	20
F	D,E	13	14	21
G	C,F	6	8	10

5. Consider the following project with the estimates of time in weeks:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	2	4	6
B	-	3	5	7
C	A	5	6	13
D	A	4	8	12
E	B,C	5	6	13
F	D,E	6	8	14

Find the probability that the project will be completed in 27 weeks.

NORMAL DISTRIBUTION TABLE

Area Under Standard Normal Distribution

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

LESSON 6

EARLIEST AND LATEST TIMES

LESSON OUTLINE

- The concepts of earliest and latest times
- The concept of slack
- Numerical problems

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concepts of earliest and latest times
- understand the concept of slack
- calculate the earliest and latest times
- find out the slacks
- identify the critical activities
- carry out numerical problems

INTRODUCTION

A project manager has the responsibility to see that a project is completed by the stipulated date, without delay. Attention is focused on this aspect in what follows.

Key concepts

Certain key concepts are introduced below.

EARLIEST TIMES OF AN ACTIVITY

We can consider (i) Earliest Start Time of an activity and (ii) Earliest Finish Time of an activity.

Earliest Start Time of an activity is the earliest possible time of starting that activity on the condition that all the other activities preceding to it were began at the earliest possible times.

Earliest Finish Time of an activity is the earliest possible time of completing that activity. It is given by the formula.

The Earliest Finish Time of an activity = The Earliest Start Time of the activity + The estimated duration to carry out that activity.

LATEST TIMES OF AN ACTIVITY

We can consider (i) Latest Finish Time of an activity and (ii) Latest Start Time of an activity.

Latest Finish Time of an activity is the latest possible time of completing that activity on the condition that all the other activities succeeding it are carried out as per the plan of the management and without delaying the project beyond the stipulated time.

Latest Start Time of an activity is the latest possible time of beginning that activity. It is given by the formula

Latest Start Time of an activity = The Latest Finish Time of the activity - The estimated duration to carry out that activity.

TOTAL FLOAT OF AN ACTIVITY

Float seeks to measure how much delay is acceptable. It sets up a control limit for delay.

The total float of an activity is the time by which that activity can be delayed without delaying the whole project. It is given by the formula

Total Float of an Activity = Latest Finish Time of the activity - Earliest Finish Time of that activity.

It is also given by the formula

Total Float of an Activity = Latest Start Time of the activity - Earliest Start Time of that activity.

Since a delay in a critical activity will delay the execution of the whole project, the total float of a critical activity must be zero.

EXPECTED TIMES OF AN EVENT

An event occurs at a point of time. We can consider (i) Earliest Expected Time of Occurrence of an event and (ii) Latest Allowable Time of Occurrence an event.

The Earliest Expected Time of Occurrence of an event is the earliest possible time of expecting that event to happen on the condition that all the preceding activities have been completed.

The Latest Allowable Time of Occurrence of an event is the latest possible time of expecting that event to happen without delaying the project beyond the stipulated time.

PROCUDURE TO FIND THE EARLIEST EXPECTED TIME OF AN EVENT

Step 1. Take the Earliest Expected Time of Occurrence of the Start Event as zero.

Step 2. For an event other than the Start Event, find out all paths in the network which connect the Start node with the node representing the event under consideration.

Step 3. In the **"Forward Pass"** (i.e., movement in the network from left to right), find out the sum of the time durations of the activities in each path identified in Step 2.

Step 4. The path with the longest time in Step 3 gives the Earliest Expected Time of Occurrence of the event

Working Rule for finding the earliest expected time of an event:

For an event under consideration, locate all the predecessor events and identify their earliest expected times. With the earliest expected time of each event, add the time duration of the activity connecting that event to the event under consideration. The maximum among all these values gives the Earliest Expected Time of Occurrence of the event.

PROCUDURE TO FIND THE LATEST ALLOWABLE TIME OF AN EVENT

We consider the **“Backward Pass”** (i.e., movement in the network from right to left).

The latest allowable time of occurrence of the End Node must be the time of completion of the project. Therefore it shall be equal to the time of the critical path of the project.

Step 1. Identify the latest allowable time of occurrence of the End Node.

Step 2. For an event other than the End Event, find out all paths in the network which connect the End node with the node representing the event under consideration.

Step 3. In the **“Backward Pass”** (i.e., movement in the network from right to left), subtract the time durations of the activities along each such path.

Step 4. The Latest Allowable Time of Occurrence of the event is determined by the path with the longest time in Step 3. In other words, the smallest value of time obtained in Step 3 gives the Latest Allowable Time of Occurrence of the event.

Working Rule for finding the latest allowable time of an event:

For an event under consideration, locate all the successor events and identify their latest allowable times. From the latest allowable time of each successor event, subtract the time duration of the activity that begins with the event under consideration. The minimum among all these values gives the Latest Allowable Time of Occurrence of the event.

SLACK OF AN EVENT

The allowable time gap for the occurrence of an event is known as the slack of that event. It is given by the formula

Slack of an event = Latest Allowable Time of Occurrence of the event - Earliest Expected Time of Occurrence of that event.

SLACK OF AN ACTIVITY

The slack of an activity is the float of the activity.

Problem 1:

The following details are available regarding a project:

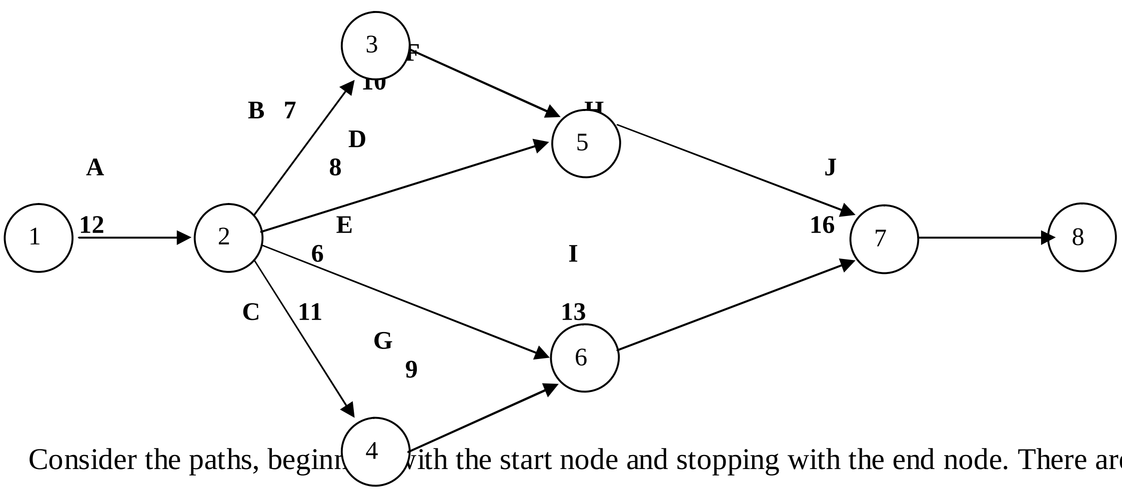
Activity	Predecessor Activity	Duration (Weeks)
A	-	12

B	A	7
C	A	11
D	A	8
E	A	6
F	B	10
G	C	9
H	D, F	14
I	E, G	13
J	H, I	16

Determine the earliest and latest times, the total float for each activity, the critical activities and the project completion time.

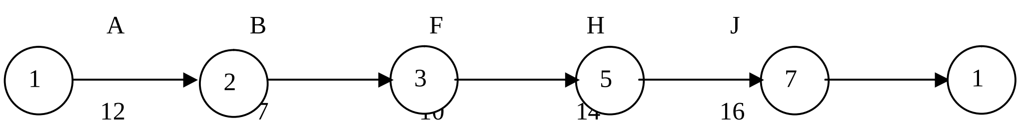
Solution:

With the given data, we construct the following network diagram for the project.



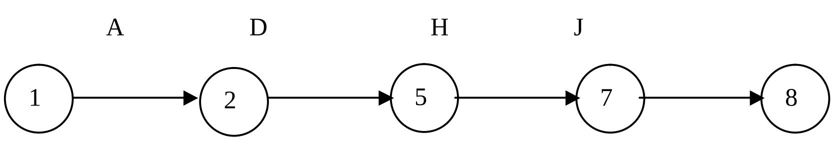
Consider the paths, beginning with the start node and stopping with the end node. There are four such paths for the given project. They are as follows:

Path I



Time of the path = 12 + 7 + 10 + 14 + 16 = 59 weeks.

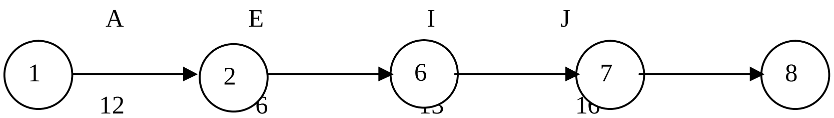
Path II



12 8 14 16

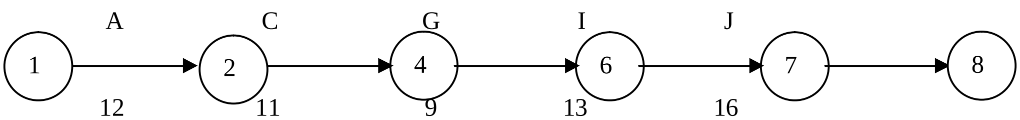
Time of the path = 12 + 8 + 14 + 16 = 50 weeks.

Path III



Time of the path = 12 + 6 + 13 + 16 = 47 weeks.

Path IV



Time of the path = 12 + 11 + 9 + 13 + 16 = 61 weeks.

Compare the times for the four paths. Maximum of {51, 50, 47, 61} = 61. We see that the maximum time of a path is 61 weeks.

Forward pass:

Calculation of Earliest Expected Time of Occurrence of Events

Node	Earliest Time of Occurrence of Node
1	0
2	Time for Node 1 + Time for Activity A = 0 + 12 = 12
3	Time for Node 2 + Time for Activity B = 12 + 7 = 19
4	Time for Node 2 + Time for Activity C = 12 + 11 = 23
5	Max {Time for Node 2 + Time for Activity D, Time for Node 3 + Time for Activity F} = Max {12 + 8, 19 + 10} = Max {20, 29} = 29
6	Max {Time for Node 2 + Time for Activity E, Time for Node 4 + Time for Activity G} = Max {12 + 6, 23 + 9} = Max {18, 32} = 32
7	Max {Time for Node 5 + Time for Activity H, Time for Node 6 + Time for Activity I} = Max {29 + 14, 32 + 13} = Max {43, 45} = 45
8	Time for Node 7 + Time for Activity J = 45 + 16 = 61

Using the above values, we obtain the Earliest Start Times of the activities as follows:

Activity	Earliest Start Time (Weeks)

A	0
B	12
C	12
D	12
E	12
F	19
G	23
H	29
I	32
J	45

Backward pass:

Calculation of Latest Allowable Time of Occurrence of Events

Node	Latest Allowable Time of Occurrence of Node
8	Maximum time of a path in the network = 61
7	Time for Node 8 - Time for Activity J = 61 -16 = 45
6	Time for Node 7 - Time for Activity I = 45 -13 = 32
5	Time for Node 7 - Time for Activity H = 45 -14 = 31
4	Time for Node 6 - Time for Activity G = 32 - 9 = 23
3	Time for Node 5 - Time for Activity F = 31- 10 = 21
2	Min {Time for Node 3 - Time for Activity B, Time for Node 4 - Time for Activity C, Time for Node 5 - Time for Activity D, Time for Node 6 - Time for Activity E} = Min {21 - 7, 23 - 11, 31 - 8, 32 - 6} = Min {14, 12, 23, 26} = 12
1	Time for Node 2 - Time for Activity A = 12- 12 = 0

Using the above values, we obtain the Latest Finish Times of the activities as follows:

Activity	Latest Finish Time (Weeks)
J	61
I	45
H	45
G	32
F	31

E	32
D	31
C	23
B	21
A	12

Calculation of Total Float for each activity:

Activity	Duration (Weeks)	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Total Float = Latest Finish Time - Earliest Finish Time
A	12	0	12	0	12	0
B	7	12	19	14	21	2
C	11	12	23	12	23	0
D	8	12	20	23	31	11
E	6	12	18	26	32	14
F	10	19	29	21	31	2
G	9	23	32	23	32	0
H	14	29	43	31	45	2
I	13	32	45	32	45	0
J	16	45	61	45	61	0

The activities with total float = 0 are A, C, G, I and J. They are the critical activities.
Project completion time = 61 weeks.

Problem 2:

The following are the details of the activities in a project:

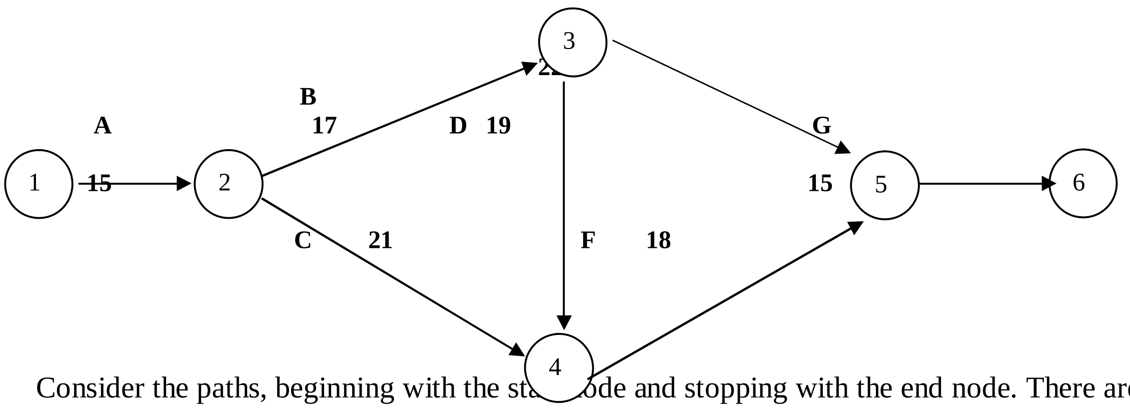
Activity	Predecessor Activity	Duration (Weeks)
A	-	15
B	A	17
C	A	21
D	B	19
E	B	22

F	C, D	18
G	E, F	15

Calculate the earliest and latest times, the total float for each activity and the project completion time.

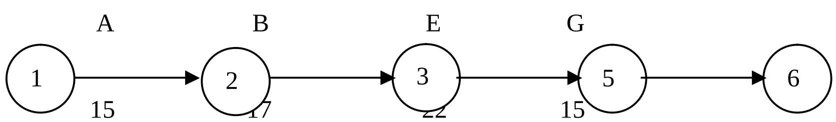
Solution:

The following network diagram is obtained for the given project.



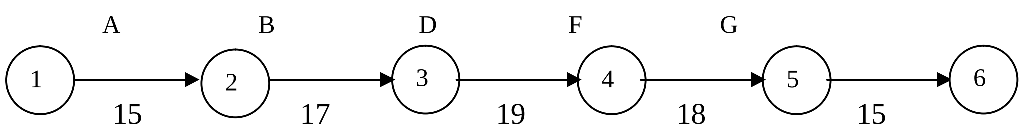
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



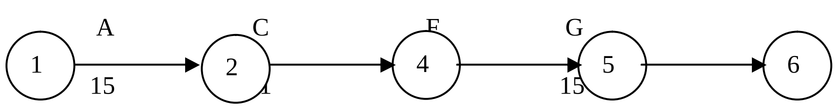
Time of the path = 15 + 17 + 22 + 15 = 69 weeks.

Path II



Time of the path = 15 + 17 + 19 + 18 + 15 = 84 weeks.

Path III



Time of the path = 15 + 21 + 18 + 15 = 69 weeks.

Compare the times for the three paths. Maximum of {69, 84, 69} = 84. We see that the maximum time of a path is 84 weeks.

Forward pass:

Calculation of Earliest Time of Occurrence of Events

Node	Earliest Time of Occurrence of Node
1	0
2	Time for Node 1 + Time for Activity A = 0 + 15 = 15
3	Time for Node 2 + Time for Activity B = 15 + 17 = 32
4	Max {Time for Node 2 + Time for Activity C, Time for Node 3 + Time for Activity D} = Max {15 + 21, 32 + 19} = Max {36, 51} = 51
5	Max {Time for Node 3 + Time for Activity E, Time for Node 4 + Time for Activity F} = Max {32 + 22, 51 + 18} = Max {54, 69} = 69
6	Time for Node 5 + Time for Activity G = 69 + 15 = 84

Calculation of Earliest Time for Activities

Activity	Earliest Start Time (Weeks)
A	0
B	15
C	15
D	32
E	32
F	51
G	69

Backward pass:

Calculation of the Latest Allowable Time of Occurrence of Events

Node	Latest Allowable Time of Occurrence of Node
6	Maximum time of a path in the network = 84
5	Time for Node 6 - Time for Activity G = 84 -15 = 69
4	Time for Node 5 - Time for Activity F = 69 - 18 = 51
3	Min {Time for Node 4 - Time for Activity D, Time for Node 5 - Time for Activity E} = Min {51 - 19, 69 - 22} = Min {32, 47} = 32
2	Min {Time for Node 3 - Time for Activity B, Time for Node 4 - Time for Activity C} = Min {32 - 17, 51 - 21} = Min {15, 30} = 15
1	Time for Node 2 - Time for Activity A = 15 - 15 = 0

Calculation of the Latest Finish Times of the activities

Activity	Latest Finish Time (Weeks)
G	84

F	69
E	69
D	51
C	51
B	32
A	15

Calculation of Total Float for each activity:

Activity	Duration (Weeks)	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Total Float = Latest Finish Time - Earliest Finish Time
A	15	0	15	0	15	0
B	17	15	32	15	32	0
C	21	15	36	30	51	15
D	19	32	51	32	51	0
E	22	32	54	47	69	15
F	18	51	69	51	69	0
G	15	69	84	69	84	0

The activities with total float = 0 are A, B, D, F and G. They are the critical activities.
Project completion time = 84 weeks.

Problem 3:

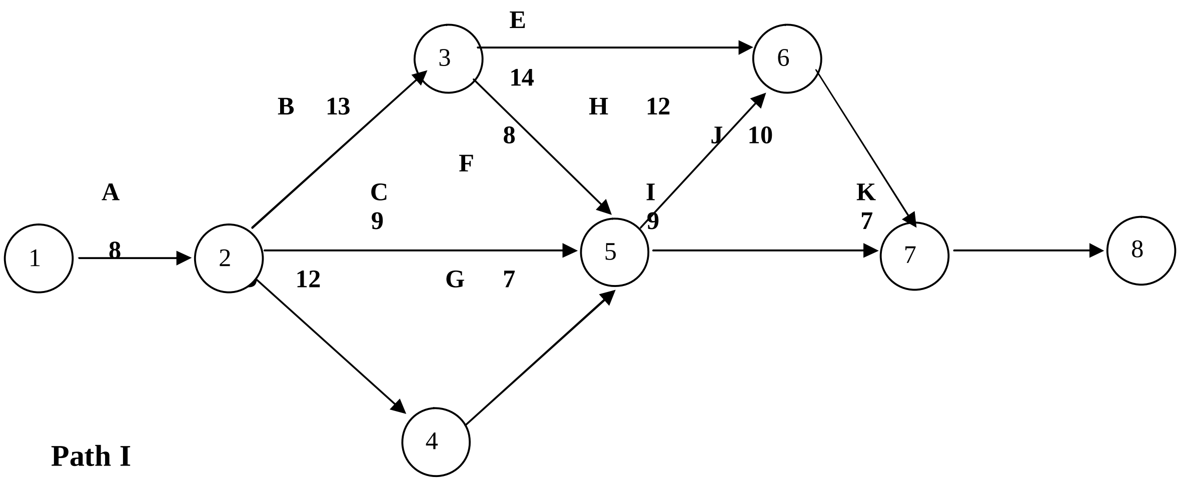
Consider a project with the following details:

Name of Activity	Predecessor Activity	Duration (Weeks)
A	-	8
B	A	13
C	A	9
D	A	12
E	B	14
F	B	8
G	D	7
H	C, F, G	12
I	C, F, G	9
J	E, H	10
K	I, J	7

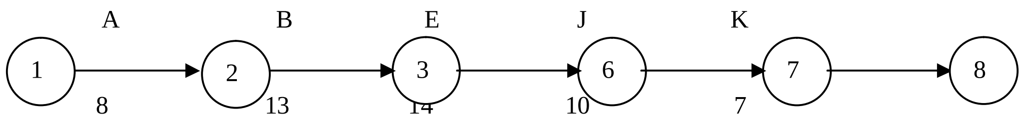
Determine the earliest and latest times, the total float for each activity, the critical activities, the slacks of the events and the project completion time.

Solution:

The following network diagram is got for the given project:

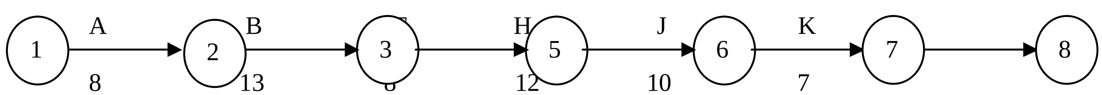


Path I



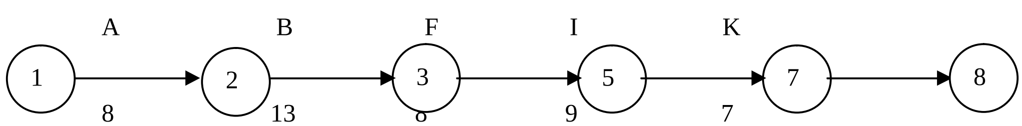
Time of the path = 8 + 13 + 14 + 10 + 7 = 52 weeks.

Path II



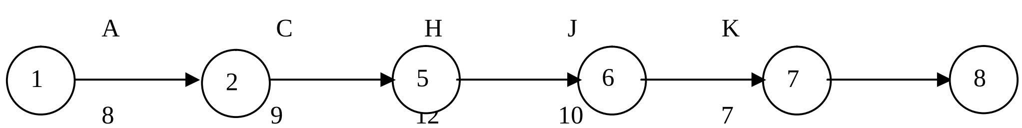
Time of the path = 8 + 13 + 8 + 12 + 10 + 7 = 58 weeks.

Path III



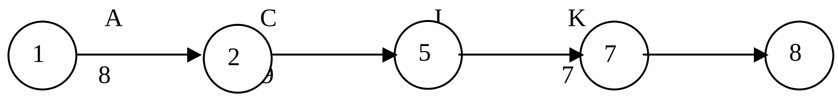
Time of the path = 8 + 13 + 8 + 9 + 7 = 45 weeks.

Path IV



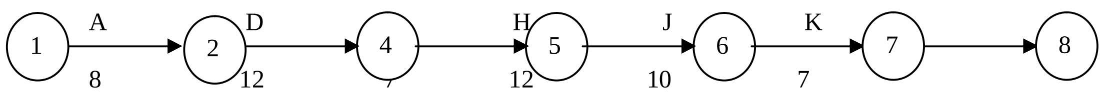
Time of the path = 8 + 9 + 12 + 10 + 7 = 46 weeks.

Path V



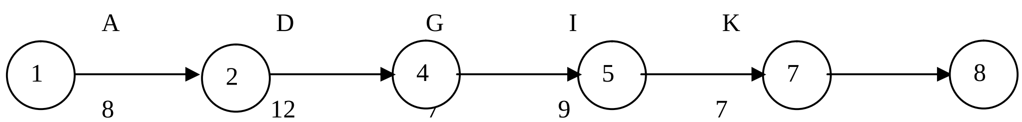
Time of the path = 8 + 9 + 9 + 7 = 33 weeks.

Path VI



Time of the path = 8 + 12 + 7 + 12 + 10 + 7 = 56 weeks.

Path VII



Time of the path = 8 + 12 + 7 + 9 + 7 = 43 weeks.

Compare the times for the three paths. Maximum of {52, 58, 45, 46, 33, 56, 43} = 58.
We see that the maximum time of a path is 58 weeks.

Forward pass:

Calculation of Earliest Time of Occurrence of Events

Node	Earliest Time of Occurrence of Node
1	0
2	Time for Node 1 + Time for Activity A = 0 + 8 = 8
3	Time for Node 2 + Time for Activity B = 8 + 13 = 21
4	Time for Node 2 + Time for Activity D = 8+ 12 = 20
5	Max {Time for Node 2 + Time for Activity C, Time for Node 3 + Time for Activity F, Time for Node 4 + Time for Activity G} = Max { 8 + 9, 21 + 8 , 20 + 7 } = Max {17, 29, 27} = 29
6	Max {Time for Node 3 + Time for Activity E, Time for Node 5 + Time for Activity H} = Max {21 + 14 , 29 + 12} = Max {35, 41} = 41
7	Max {Time for Node 5 + Time for Activity I, Time for Node 6 + Time for Activity J} = Max {29 + 9, 41 + 10} = Max {38, 51} = 51
8	Time for Node 7 + Time for Activity J = 51+ 7 = 58

Earliest Start Times of the activities

Activity	Earliest Start Time (Weeks)
A	0
B	8
C	8
D	8
E	21
F	21
G	20
H	29
I	29
J	41
K	51

Backward pass:

Calculation of Latest Allowable Time of Occurrence of Events

Node	Latest Allowable Time of Occurrence of Node
8	Maximum time of a path in the network = 58
7	Time for Node 8 - Time for Activity K = 58 -7 = 51
6	Time for Node 7 - Time for Activity J = 51 -10 = 41

5	Min {Time for Node 6 - Time for Activity H, Time for Node 7 - Time for Activity I} = Min {41 - 12, 51 - 9} = Min {29, 42} = 29
4	Time for Node 5 - Time for Activity G = 29 - 7 = 22
3	Min {Time for Node 5 - Time for Activity F, Time for Node 6 - Time for Activity E} = Min {29 - 8, 41 - 14} = Min {21, 27} = 21
2	Min {Time for Node 3 - Time for Activity B, Time for Node 4 - Time for Activity D, Time for Node 5 - Time for Activity C} = Min {21 - 13, 22 - 12, 29 - 9} = Min {8, 10, 20} = 8
1	Time for Node 2 - Time for Activity A = 8 - 8 = 0

Latest Finish Times of the activities

Activity	Latest Finish Time (Weeks)
K	58
J	51
I	51
H	41
G	29
F	29
E	41
D	22
C	29
B	21
A	8

Calculation of Total Float for each activity:

Activity	Duration (Weeks)	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Total Float = Latest Finish Time - Earliest Finish Time
A	8	0	8	0	8	0
B	13	8	21	8	21	0
C	9	8	17	20	29	12
D	12	8	20	10	22	2
E	14	21	35	27	41	6
F	8	21	29	21	29	0

G	7	20	27	22	29	2
H	12	29	41	29	41	0
I	9	29	38	42	51	13
J	10	41	51	41	51	0
K	7	51	58	51	58	0

The activities with total float = 0 are A, B, F, H, J and K. They are the critical activities.
Project completion time = 58 weeks.

Calculation of slacks of the events

Slack of an event = Latest Allowable Time of Occurrence of the event - Earliest Expected Time of Occurrence of that event.

Event (Node)	Earliest Expected Time of Occurrence of Event	Latest Allowable Time of Occurrence of Event	Slack of the Event
1	0	0	0
2	8	8	0
3	21	21	0
4	20	22	2
5	29	29	0
6	41	41	0
7	51	51	0
8	58	58	0

Interpretation:

On the basis of the slacks of the events, it is concluded that the occurrence of event 4 may be delayed upto a maximum period of 2 weeks while no other event cannot be delayed.

QUESTIONS

1. Explain the terms: The earliest and latest times of the activities of a project.
2. Explain the procedure to find the earliest expected time of an event.
3. Explain the procedure to find the latest allowable time of an event.
4. What is meant by the slack of an activity? How will you determine it?
5. Consider the project with the following details:

activity	Duration (weeks)
1→2	1
2→3	3

2→4	7
3→4	5
3→5	8
4→5	4
5→6	1

Determine the earliest and the latest times of the activities. Calculate the total float for each activity and the slacks of the events.

LESSON 7

CRASHING OF A PROJECT

LESSON OUTLINE

- The idea of crashing of a project
- The criterion of selection of an activity for crashing
- Numerical problems

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of crashing of a project
- choose an activity for crashing
- work out numerical problems

THE MEANING OF CRASHING:

The process of shortening the time to complete a project is called crashing and is usually achieved by putting into service additional labour or machines to one activity or more activities. Crashing involves more costs. A project manager would like to speed up a project by spending as minimum extra cost as possible. Project crashing seeks to minimize the extra cost for completion of a project before the stipulated time.

STEPS IN PROJECT CRASHING:

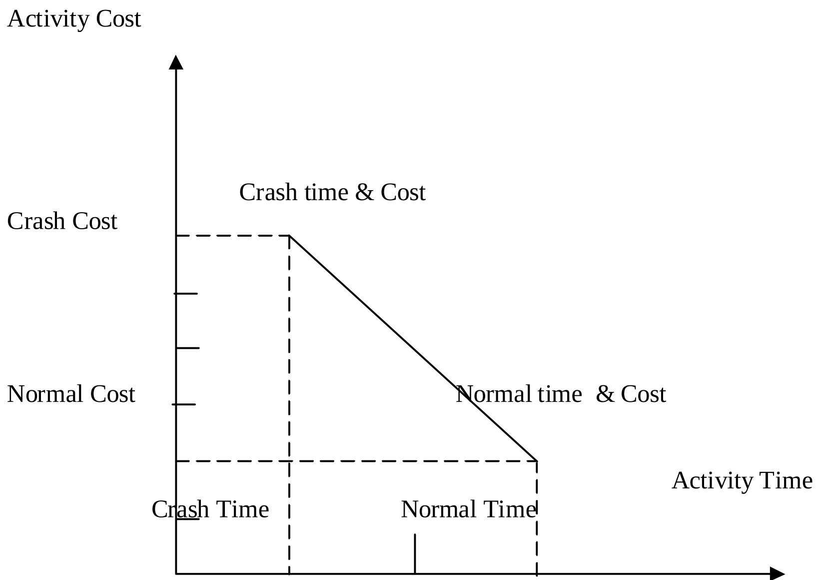
Assumption: It is assumed that there is a linear relationship between time and cost.

Let us consider project crashing by the critical path method. The following four-step procedure is adopted.

Step 1: Find the critical path with the normal times and normal costs for the activities and identify the critical activities.

Step 2: Find out the crash cost per unit time for each activity in the network. This is calculated by means of the following formula.

$$Crash\ \cos t \big/ Time\ \ period = \frac{Crash\ \cos t - Normal\ \cos t}{Normal\ time - Crash\ \ time}$$



Step 3: Select an activity for crashing. The **criteria for the selection** is as follows:
Select the activity on the critical path with the smallest crash cost per unit time. Crash this activity to the maximum units of time as may be permissible by the given data.

Crashing an activity requires extra amount to be spent. However, even if the company is prepared to spend extra money, the activity time cannot be reduced beyond a certain limit in view of several other factors.

In step 1, we have to note that reducing the time of on activity along the critical path alone will reduce the completion time of a project. Because of this reason, we select an activity along the critical path for crashing.

In step 3, we have to consider the following question:

If we want to reduce the project completion time by one unit, which critical activity will involve the least additional cost?

On the basis of the least additional cost, a critical activity is chosen for crashing. If there is a tie between two critical activities, the tie can be resolved arbitrarily.

Step 4: After crashing an activity, find out which is the critical path with the changed conditions. Sometimes, a reduction in the time of an activity in the critical path may cause a non-critical path to become critical. If the critical path with which we started is still the longest path, then go to Step 3. Otherwise, determine the new critical path and then go to Step 3.

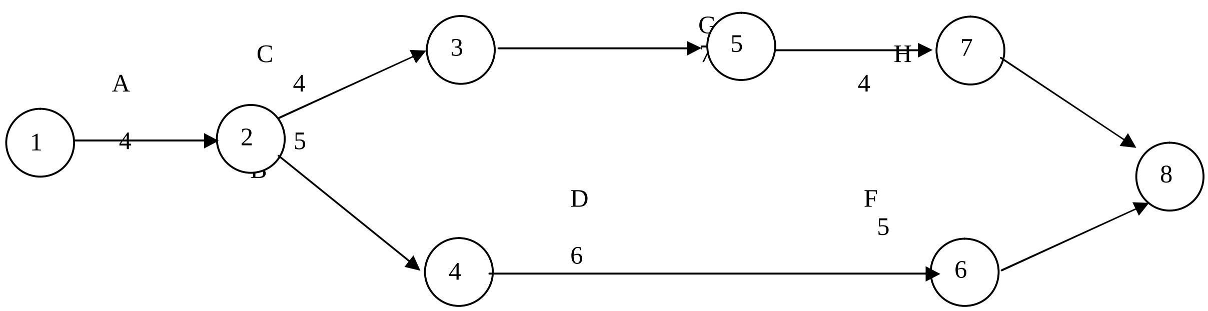
Problem 1: A project has activities with the following normal and crash times and cost:

Activity	Predecessor Activity	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost (Rs.)	Crash Cost (Rs.)
A	-	4	3	8,000	9,000
B	A	5	3	16,000	20,000
C	A	4	3	12,000	13,000
D	B	6	5	34,000	35,000
E	C	6	4	42,000	44,000
F	D	5	4	16,000	16,500
G	E	7	4	66,000	72,000
H	G	4	3	2,000	5,000

Determine a crashing scheme for the above project so that the total project time is reduced by 3 weeks.

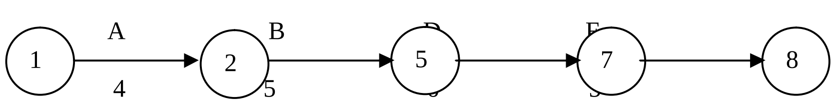
Solution:

We have the following network diagram for the given project with **normal costs**:



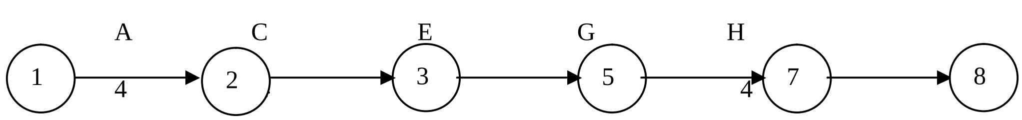
Beginning from the Start Node and terminating with the End Node, there are two paths for the network as detailed below:

Path I:



The time for the path = 4 + 5 + 6 + 5 = 20 weeks.

Path II:



The time for the path = 4 + 4 + 6 + 7 + 4 = 25 weeks.

Maximum of {20, 25} = 25.

Therefore Path II is the critical path and the critical activities are A, C, E, G and H. The non-critical activities are B, D and F.

Given that the normal time of activity A is 4 weeks while its crash time is 3 weeks. Hence the time of this activity can be reduced by one week if the management is prepared to spend an additional amount. However, the time cannot be reduced by more than one week even if the management may be prepared to spend more money. The normal cost of this activity is Rs. 8,000 whereas the crash cost is Rs. 9,000. From this, we see that crashing of activity A by one week will cost the management an extra amount of Rs. 1,000. In a similar fashion, we can work out the crash cost per unit time for the other activities also. The results are provided in the following table.

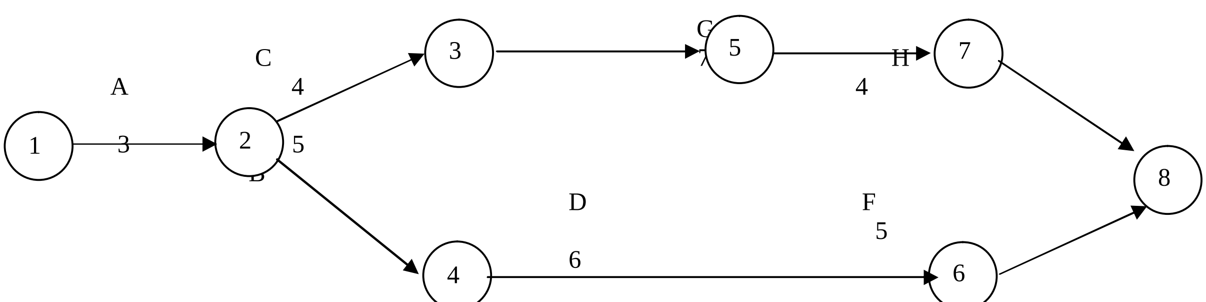
Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Crash cost - Normal Cost	Normal Time - Crash Time	Crash Cost per unit time
A	4	3	8,000	9,000	1,000	1	1,000
B	5	3	16,000	20,000	4,000	2	2,000
C	4	3	12,000	13,000	1,000	1	1,000
D	6	5	34,000	35,000	1,000	1	1,000
E	6	4	42,000	44,000	2,000	2	1,000
F	5	4	16,000	16,500	500	1	500
G	7	4	66,000	72,000	6,000	1	6,000
H	4	3	2,000	5,000	3,000	1	3,000

A non-critical activity can be delayed without delaying the execution of the whole project. But, if a critical activity is delayed, it will delay the whole project. Because of this reason, we have to select a critical activity for crashing. Here we have to choose one of the activities A, C, E, G and H The crash cost per unit time works out as follows:

Rs. 1,000 for A; Rs. 1,000 for C; Rs. 1,000 for E; Rs. 6,000 for G; Rs. 3,000 for H.

The maximum among them is Rs. 1,000. So we have to choose an activity with Rs.1,000 as the crash cost per unit time. However, there is a tie among A, C and E. The tie can be resolved arbitrarily. Let us select A for crashing. We reduce the time of A by one week by spending an extra amount of Rs. 1,000.

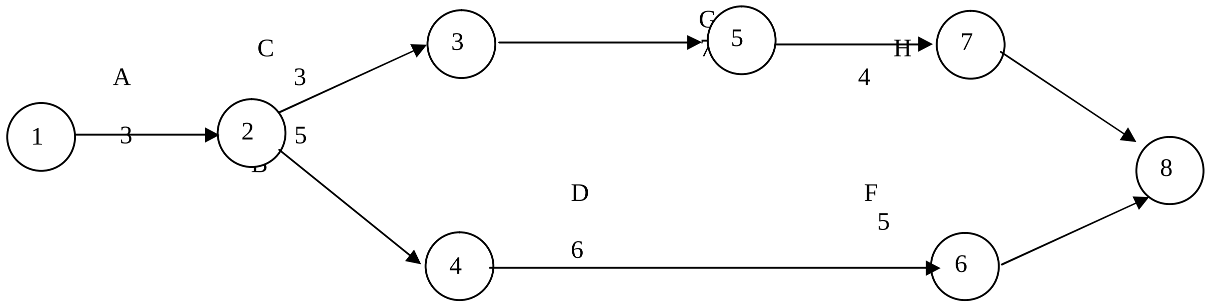
After this step, we have the following network with the revised times for the activities:



The revised time for Path I = 3 + 5 + 6 + 5 = 19 weeks.
The time for Path II = 3 + 4 + 6 + 7 + 4 = 24 weeks.
Maximum of {19, 24} = 24.

Therefore Path II is the critical path and the critical activities are A, C, E, G and H. However, the time for A cannot be reduced further. Therefore, we have to consider C, E, G and H for crashing. Among them, C and E have the least crash cost per unit time. The tie between C and E can be resolved arbitrarily. Suppose we reduce the time of C by one week with an extra cost of Rs. 1,000.

After this step, we have the following network with the revised times for the activities:



The time for Path I = 3 + 5 + 6 + 5 = 19 weeks.
The time for Path II = 3 + 3 + 6 + 7 + 4 = 23 weeks.
Maximum of {19, 23} = 23.
Therefore Path II is the critical path and the critical activities are A, C, E, G and H. Now the time for A or C cannot be reduced further. Therefore, we have to consider E, G and H for crashing. Among them, E has the least crash cost per unit time. Hence we reduce the time of E by one week with an extra cost of Rs. 1,000.

By the given condition, we have to reduce the project time by 3 weeks. Since this has been accomplished, we stop with this step.

Result: We have arrived at the following crashing scheme for the given project:

Reduce the time of A, C and E by one week each.
Project time after crashing is 22 weeks.
Extra amount required = 1,000 + 1,000 + 1,000 = Rs. 3,000.

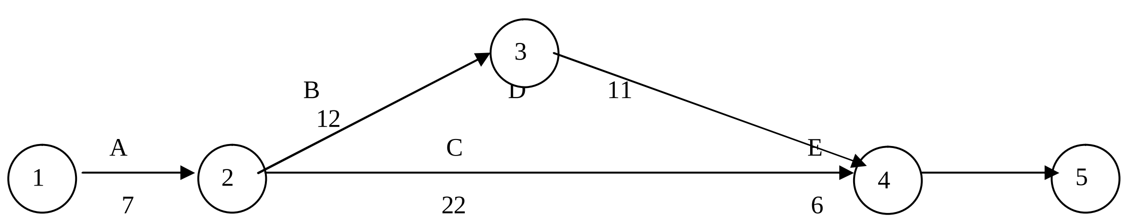
Problem 2:

The management of a company is interested in crashing of the following project by spending an additional amount not exceeding Rs. 2,000. Suggest how this can be accomplished.

Activity	Predecessor Activity	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost (Rs.)	Crash Cost (Rs.)
A	-	7	6	15,000	18,000
B	A	12	9	11,000	14,000
C	A	22	21	18,500	19,000
D	B	11	10	8,000	9,000
E	C, D	6	5	4,000	4,500

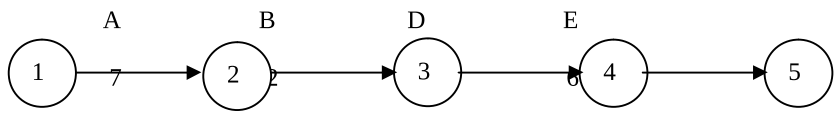
Solution:

We have the following network diagram for the given project with **normal costs**:



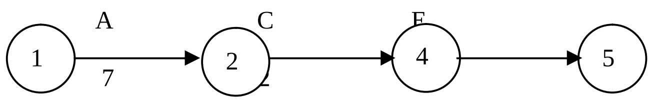
There are two paths for this project as detailed below:

Path I:



The time for the path = 7 + 12 + 11 + 6 = 36 weeks.

Path II:



The time for the path = 7 + 22 + 6 = 35 weeks.

Maximum of {36, 35} = 36.

Therefore Path I is the critical path and the critical activities are A, B, D and E. The non-critical activity is C.
The crash cost per unit time for the activities in the project are provided in the following table.

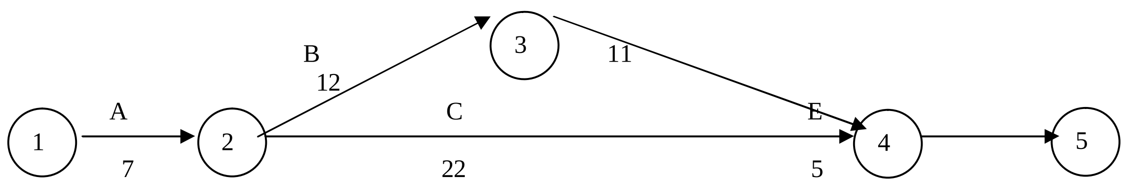
Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Crash cost - Normal Cost	Normal Time - Crash Time	Crash Cost per unit time
A	7	6	15,000	18,000	3,000	1	3,000
B	12	9	11,000	14,000	3,000	3	1,000
C	22	21	18,500	19,000	500	1	500
D	11	10	8,000	9,000	1,000	1	1,000
E	6	5	4,000	4,500	500	1	500

We have to choose one of the activities A, B, D and E for crashing. The crash cost per unit time is as follows:

Rs. 3,000 for A; Rs. 1,000 for B; Rs. 1,000 for D; Rs. 500 for E.

The least among them is Rs. 500. So we have to choose the activity E for crashing. We reduce the time of E by one week by spending an extra amount of Rs. 500.

After this step, we have the following network with the revised times for the activities:



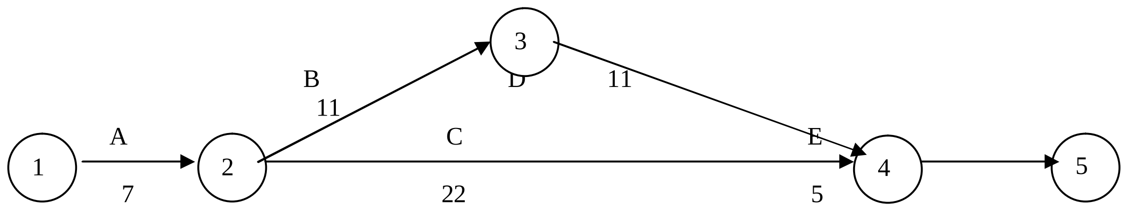
The revised time for Path I = 7 + 12 + 11 + 5 = 35 weeks.

The time for Path II = 7 + 22 + 5 = 34 weeks.

Maximum of {35, 34} = 35.

Therefore Path I is the critical path and the critical activities are A, B, D and E. The non-critical activity is C. The time of E cannot be reduced further. So we cannot select it for crashing. Next B and have the smallest crash cost per unit time. Let us select B for crashing. Let us reduce the time of E by one week at an extra cost of Rs. 1,000.

After this step, we have the following network with the revised times for the activities:



The revised time for Path I = 7 + 11 + 11 + 5 = 34 weeks.

The time for Path II = 7 + 22 + 5 = 34 weeks.

Maximum of {34, 34} = 34.

Since both paths have equal times, both are critical paths. So, we can choose an activity for crashing from either of them depending on the least crash cost per unit time. In path I, the activities are A, B, D and E. In path II, the activities are A, C and E.

The crash cost per unit time is the least for activity C. So we select C for crashing. Reduce the time of C by one week at an extra cost of Rs. 500.

By the given condition, the extra amount cannot exceed Rs. 2,000. Since this state has been met, we stop with this step.

Result: The following crashing scheme is suggested for the given project:
Reduce the time of E, B and C by one week each.

Project time after crashing is 33 weeks.

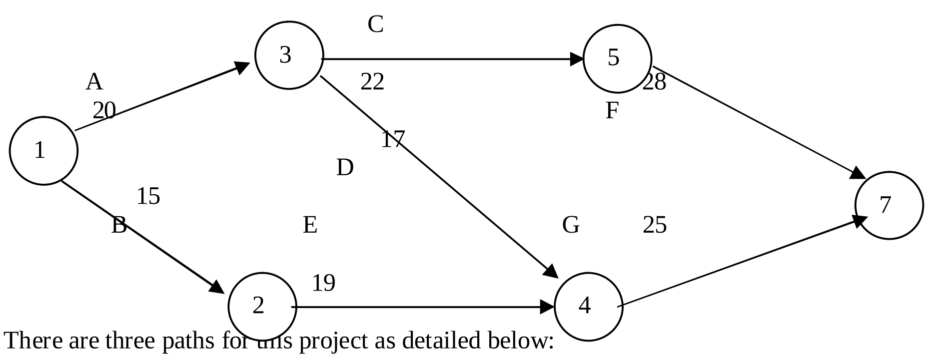
Extra amount required = 500 + 1,000 + 500 = Rs. 2,000.

Problem 3:

The manager of a company wants to apply crashing for the following project by spending an additional amount not exceeding Rs. 2,000. Offer your suggestion to the manager.

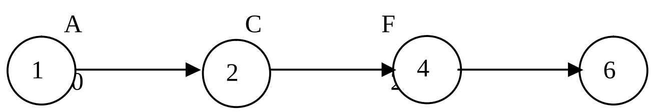
Activity	Predecessor Activity	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost (Rs.)	Crash Cost (Rs.)
A	-	20	19	8,000	10,000
B	-	15	14	16,000	19,000
C	A	22	20	13,000	14,000
D	A	17	15	7,500	9,000
E	B	19	18	4,000	5,000
F	C	28	27	3,000	4,000
G	D, E	25	24	12,000	13,000

Solution:
We have the following network diagram for the given project with **normal costs**:



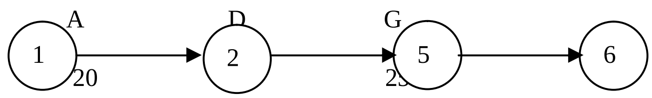
There are three paths for this project as detailed below:

Path I:



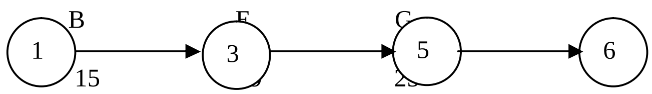
The time for the path = 20 + 22 + 28= 70 weeks.

Path II:



The time for the path = 20 + 17 + 25= 62 weeks.

Path III:



The time for the path = 15+19 +25 = 69 weeks.
Maximum of {70, 62, 69} = 70.
Therefore Path I is the critical path and the critical activities are A, C and F. The non-critical activities are B, D, E and G.
The crash cost per unit time for the activities in the project are provided in the following table

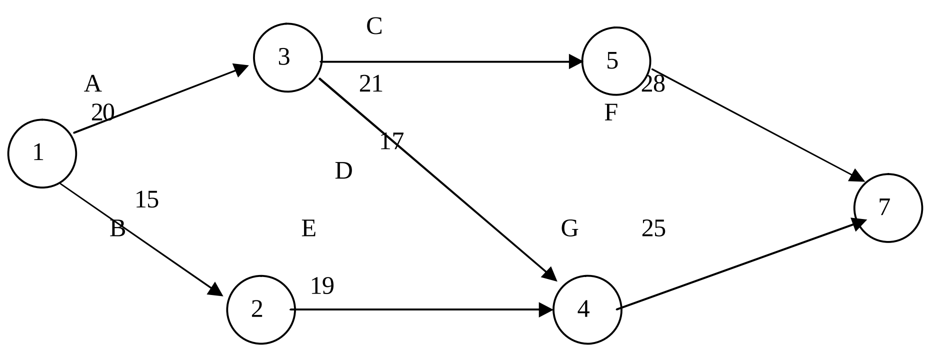
Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Crash cost - Normal Cost	Normal Time - Crash Time	Crash Cost per unit time
A	20	19	8,000	10,000	2,000	1	2,000
B	15	14	16,000	19,000	3,000	1	3,000
C	22	20	13,000	14,000	1,000	2	500
D	17	15	7,500	9,000	1,500	2	750
E	19	18	4,000	5,000	1,000	1	1,000
F	28	27	3,000	4,000	1,000	1	1,000
G	25	24	12,000	13,000	1,000	1	1,000

We have to choose one of the activities A, C and F for crashing. The crash cost per unit time is as follows:

Rs. 2,000 for A; Rs. 500 for C; Rs. 1,000 for F.

The least among them is Rs. 500. So we have to choose the activity C for crashing. We reduce the time of C by one week by spending an extra amount of Rs. 500.

After this step, we have the following network with the revised times for the activities:



The revised time for Path I = 20 + 21 + 28= 69 weeks.

The time for Path II = 20 + 17 + 25= 62 weeks.

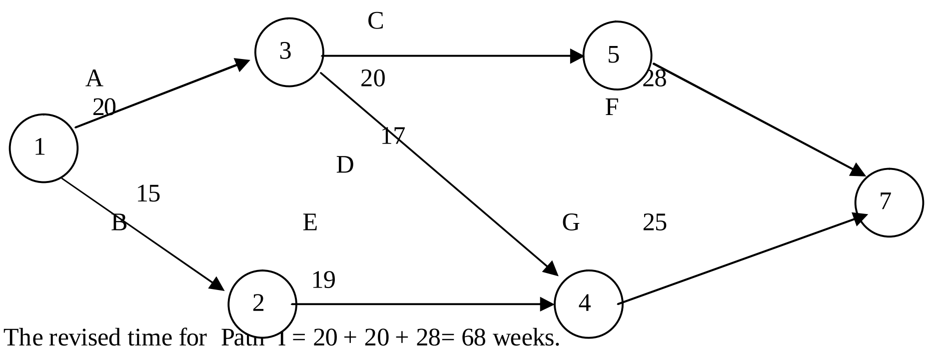
The time for Path III = 15+19 +25 = 69 weeks.
Maximum of {69, 62, 69} = 69.

Since paths I and III have equal times, both are critical paths. So, we can choose an activity for crashing from either of them depending on the least crash cost per unit time.

In path I, the activities are A, C and F. In path III, the activities are B, E and G.

The crash cost per unit time is the least for activity C. So we select C for crashing. Reduce the time of C by one week at an extra cost of Rs. 500.

After this step, we have the following network with the revised times for the activities:



The revised time for Path I = 20 + 20 + 28= 68 weeks.
The time for Path II = 20 + 17 + 25= 62 weeks.

The time for Path III = 15+19 +25 = 69 weeks.
Maximum of {68, 62, 69} = 69.

Therefore path III is the critical activities. Hence we have to select an activity from Path III for crashing. We see that the crash cost per unit time is as follows:

Rs. 3,000 for B; Rs. 1,000 for E; Rs. 1,000 for G.

The least among them is Rs. 1,000. So we can select either E or G for crashing. Let us select E for crashing. We reduce the time of E by one week by spending an extra amount of Rs. 1,000.

By the given condition, the extra amount cannot exceed Rs. 2,000. Since this condition has been reached, we stop with this step.

Result: The following crashing scheme is suggested for the given project:

Reduce the time of C by 2 weeks and that of E by one week.

Project time after crashing is 67 weeks.

Extra amount required = 2 x 500 + 1,000 = Rs. 2,000.

QUESTIONS

1. Explain the concept of crashing of a project.
2. Explain the criterion for selection of an activity for crashing.