Assignment Problem (Hungarian method)

		Subordinates I	II	III	IV
Tasks	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

How the tasks should be allocated to each person so as to minimize the total man-hours?

Sol ① subtracting the smallest element in each row from every element of that row.

Row Reduction

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

② subtracting the smallest element in each column from every element of that column

Column Reduction

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

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Tasks	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How the tasks should be allocated to each person so as to minimize the total man-hours?

Sol: ① subtracting the smallest element in each row from every element of that row.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Row Reduction

② Subtracting the smallest element in each column from every element of that column

[0]	14	9	3
9	20	[0]	22
23	[0]	3	~~0~~
9	12	14	[0]

Column Reduction

(Assignment ☐ X)

[Reduced matrix]

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Tasks	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How the tasks should be allocated to each person so as to minimize the total man-hours?

Sol^n ① subtracting the smallest element in each row from every element of that row.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Row Reduction

② subtracting the smallest element in each column from every element of that column

	I	II	III	IV
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~0~~
D	9	12	14	[0]

Column Reduction

Assignment ☐ ✗

[Reduced matrix]

4×4 = 4 = 4

Optimal Assignment A-I B-III C-II D-IV

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Tasks	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How the tasks should be allocated to each person so as to minimize the total man-hours?

Sol: ① subtracting the smallest element in each row from every element of that row.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Row Reduction

② Subtracting the smallest element in each column from every element of that column

	I	II	III	IV
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~0~~
D	9	12	14	[0]

Column Reduction

(Assignment ☐ ✗)

[Reduced matrix]

4×4 = 4 ± 4

Optimal Assignment A-I B-III C-II D-IV

Hours 8 4 19 10 (41 hours)

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Tasks \ Men	a	b	c	d
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as minimize the total man hours.

Soln ① Row Reduction

7	15	6	0
6	15	1	13
23	4	3	0
9	16	14	0

② Column Reduction → Reduced matrix

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

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D	19	26	24	10

man hours.

Soln ① Row Reduction

7	15	6	0
6	15	1	13
23	4	3	0
9	16	14	0

② Column Reduction → Reduced matrix

7	11	5	[0]
[0]	11	~~0~~	13
23	[0]	2	~~0~~
9	12	13	~~0~~

∴ row 4 and col. 3, do not have any assignment. so it is not optimal assignment. therefore we improve matrix.

7	11	5	[0]	✓③
[0]	11	~~0~~	13	
23	[0]	2	~~0~~	
9	12	13	~~0~~	✓①
			②	

rows → unticked
cols → ticked

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① Row Reduction

7	15	6	0
6	15	1	13
23	4	3	0
9	16	14	0

② Column Reduction → Reduced matrix

7	11	5	[0]
[0]	11	~~0~~	13
23	[0]	2	~~0~~
9	12	13	~~0~~

∵ row 4 and col. 3, do not have any assignment. so it is not optimal assignment. Therefore we improve matrix.

7	11	5	[0] ✓③
[0]	11	~~0~~	13
23	[0]	2	~~0~~
9	12	13	~~0~~ ✓①
			②

uncovered smallest element $x = 5$

rows → unticked
cols → ticked

2	6	[0]	~~0~~
[0]	11	~~0~~	18
23	[0]	2	5
4	7	8	[0]

element
- covered
  - line (same)
  - intersection + x
- uncovered − x

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7	15	6	0
6	15	1	13
23	4	3	0
9	16	14	0

② Column Reduction → Reduced matrix

7	11	5	[0]
[0]	11	⊗	13
23	[0]	2	⊗
9	12	13	⊗

∵ row 4 and col. 3, do not have any assignment. so it is not optimal assignment. Therefore we improve matrix.

7	11	5	[0] ✓③
[0]	11	⊗	13
23	[0]	2	⊗
9	12	13	⊗
			↓②

uncovered smallest element $x = 5$

rows → unticked
cols → ticked

	a	b	c	d
A	2	6	[0]	⊗
B	[0]	11	⊗	18
C	23	[0]	2	5
D	4	7	8	[0]

✓①

element
covered — uncovered − x
line (same) — intersection + x

optimal assign. A-c B-a C-b D-d
hours. 17 12 19 10 (59)

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7	15	6	0
6	15	1	13
23	4	3	0
9	16	14	0

② Column Reduction → Reduced matrix

7	11	5	0
0	11	⊗	13
23	0	2	⊗
9	12	13	⊗

∵ row 4 and col. 3, do not have any assignment. so it is not optimal assignment. Therefore we improve matrix.

7	11	5	0
0	11	⊗	13
23	0	2	⊗
9	12	13	⊗

✓ ③ uncovered smallest element x=5 → rows → unticked, cols → ticked

✓ ①

	a	b	c	d
A	2	6	0	⊗
B	0	11	⊗	18
C	23	0	2	5
D	4	7	8	0

② element = covered + uncovered − x

line (same) + intersection + x

optimal assign. A-c B-a C-b D-d

hours. 17 13 19 10 (59 hrs.)

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Maximal Assignment Problem

—by—
Sandeep Kumar Gour

machines \ Jobs	A	B	C	D	E
1	5	11	10	12	4
2	2	4	6	3	5
3	3	12	5	14	6
4	6	14	4	11	7
5	7	9	8	12	5

Assign the five jobs to the five machine so as to maximize the total expected profit.

Sol: ① Convert maximization into minimization
→ Subtract all the element from the highest element [highest element = 14]

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

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7	3	6	2	7

③ Row Reduction

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	3	7
5	3	4	0	7

③ column Reduction

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

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③ column Reduction (Reduced matrix)

3	1	2	0	7
0	2	~~0~~	3	~~0~~
7	2	9	~~0~~	7
4	0	10	3	6
1	3	4	~~0~~	6

I modified matrix

no. of assignment = 3 < 5

uncovered smallest element = 1

uncovered - 1, intersect + 1

II modified matrix

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	4	6
0	2	3	0	5

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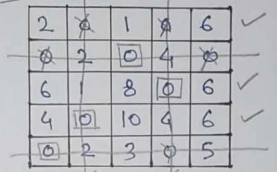
1	3	4	~~0~~	6

= 1

uncovered -1, intersect +1

II modified matrix



2	~~0~~	1	~~0~~	6
~~0~~	2	0	4	~~0~~
6	1	8	0	6
4	0	10	4	6
0	2	3	0	5

no. of assignment = 4 < 5

x = 1

III modified matrix

	A	B	C	D	E
1	1	~~0~~	0	~~0~~	5
2	~~0~~	3	~~0~~	5	0
3	5	1	7	0	5
4	3	0	9	4	5
5	0	3	3	1	5

no. of assignment = 5 = 5

optimal assignment = 1-C 2-E 3-D 4-B 5-A

Max profit = 10 5 14 14 7 = 50

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Unbalanced Assignment problem
(non-square matrix)

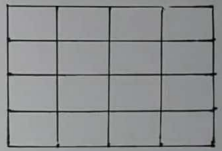
— by —
Sandeep Kumar Gour

(I)

Jobs \ Machine	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

(II)

Jobs \ Machine	M_1	M_2	M_3
1	9	26	15
2	13	27	6
3	35	20	15
4	18	30	20



How the jobs should be assigned to each machine so as to minimize the total hours?

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Unbalanced Assignment problem
(non-square matrix)

— by —
Sandeep Kumar Gour

(I)

Jobs \ Machine	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

Balanced

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22
4	0	0	0	0

(II)

jobs \ Machine	M_1	M_2	M_3
1	9	26	15
2	13	27	6
3	35	20	15
4	18	30	20

	M_1	M_2	M_3	M_4
1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

How the jobs should be assigned to each machine so as to minimize the total hours?

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(I)

Jobs \ Machine	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

↓ Balanceed

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22
4	0	0	0	0

How the jobs should be assigned to each machine so as to minimize the total hours?

(II)

jobs	M_1	M_2	M_3
1	9	26	15
2	13	27	6
3	35	20	15
4	18	30	20

① convert into balanced A.P.

	M_1	M_2	M_3	M_4
1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

Sol: ② Row Reduction

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

③ Column Reduction

0	6	9	0
4	7	0	0
26	0	9	0
9	10	14	0

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jobs	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

Balanced

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22
4	0	0	0	0

How the jobs should be assigned to each machine so as to minimize the total hours?

jobs				
1	9	26	15	
2	13	27	6	
3	35	20	15	
4	18	30	20	

1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

Sol: ② Row Reduction

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

③ Column Reduction

	M_1	M_2	M_3	M_4
1	[0]	6	9	~~0~~
2	4	7	[0]	~~0~~
3	26	[0]	9	~~0~~
4	9	10	14	[0]

no. of Assignment = 4 = 4

optimal Ass.

1-M_1, 2-M_3, 3-M_2, 4-M_4

9 6 20 0

= 35 hours.

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Assignment Problem
(Alternative Solution)

—by—
Sandeep Kumar Gour

Jobs \ Machine	A	B	C
1	2	5	4
2	4	3	5
3	5	0	6
4	2	2	4

How the jobs should be assigned to each machine so as to minimize the total hours?

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Jobs	1	2	3
1	2	5	4
2	4	3	5
3	5	0	6
4	2	2	4

How the jobs should be assigned to each machine so as to minimize the total hours?

Soln: ① convert into square matrix

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

② Row Reduction

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

③ Column Reduction

0	5	0	0
2	3	1	0
3	0	2	0
0	2	0	0

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	1	2	5	4
Jobs	2	4	3	5
	3	5	0	6
	4	2	2	4

How the jobs should be assigned to each machine so as to minimize the total hours?

Soln: ① convert into square matrix

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

② Row Reduction

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

③ Column Reduction

0	5	0	~~0~~
2	3	1	[0]
3	[0]	2	~~0~~
0	2	0	~~0~~

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4	3	5	0
5	0	6	0
2	2	4	0

② Row Reduction

2	5	4	0
4	3	5	0
5	0	6	0
2	2	4	0

③ Column Reduction

0	5	0	~~0~~
2	3	1	[0]
3	[0]	2	~~0~~
0	2	0	~~0~~

	A	B	C	D
1	[0]	5	~~0~~	~~0~~
2	2	3	1	[0]
3	3	[0]	2	~~0~~
4	~~0~~	2	[0]	~~0~~

opt. Ass: 1-A 2-D 3-B 4-C

hours 2 0 0 4

= 6 hours.

	A	B	C	D
1	~~0~~	5	[0]	~~0~~
2	2	3	1	[0]
3	3	[0]	2	~~0~~
4	[0]	2	~~0~~	~~0~~

1-C 2-D 3-B 4-A

4 0 0 2

= 6 hours.

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Restrictions on Assignment

Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 can not be placed at C and M_3 can not be placed at A. C_{ij} the assignment cost of machine i to place j in rupees is shown below. Find the optimal assignment schedule.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3

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Restrictions on Assignment

by Sandeep Kumar Goel

Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 can not be placed at C and M_3 can not be placed at A. C_{ij} the assignment cost of machine i to place j in rupees is shown below. Find the optimal assignment schedule.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	-	5	4
M_3	-	6	9	6	2
M_4	9	3	7	2	3

Sol^n ① prepare a square matrix

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
M_5	0	0	0	0	0

② Row Reduction

0	2	6	1	2
3	0	∞	1	0
∞	4	7	4	0
7	1	5	0	1
0	0	0	0	0

③ Column Reduction

	A	B	C	D	E
M_1					
M_2					
M_3					
M_4					

optimal Assignment | Total cost

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M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 can not be placed at C and M_3 can not be placed at A. C_{ij} the assignment cost of machine i to place j in rupees is shown below. Find the optimal assignment schedule.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3

Soln ① prepare a square matrix

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
M_5	0	0	0	0	0

② Row Reduction

0	2	6	1	2
3	0	∞	1	0
∞	4	7	4	0
7	1	5	0	1
0	0	0	0	0

No. of Assignment = 5

③ Column Reduction

	A	B	C	D	E
M_1	[0]	2	6	1	2
M_2	3	[0]	∞	1	⊗
M_3	∞	4	7	4	[0]
M_4	7	1	5	[0]	1
M_5	⊗	⊗	[0]	⊗	⊗

& modified matrix

optimal Assignment	Total cost
M_1 – A	4
M_2 – B	4
M_3 – E	2
M_4 – D	2
M_5 – C	0
	12 rupees

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Travelling Salesman Problem

By - Sandeep Kumar Gour

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

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Travelling Salesman Problem

By - Sandeep kumar Gour

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

Two additional constraints

① No city is to be visited twice before the tour of all cities is completed.

② Going from city i to i is not permitted.

Sol^n: ① Row Reduction

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

② Column Reduction

modified matrix

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	0
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	0	7	∞

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Travelling Salesman Problem

By - Sandeep Kumar Gaur

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

Two additional constraints

① No city is to be visited twice before the tour of all cities is completed.

② Going from city i to j is not permitted.

Sol^{n}: ① Row Reduction

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

② Column Reduction

modified matrix

	A	B	C	D	E
A	∞	1	3	6	[0]
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	[0]	1	∞	1
E	[0]	2	~~0~~	7	∞

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Travelling Salesman Problem

By - Sandeep Kumar Gour

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

Two additional constraints

① No city is to be visited twice before the tour of all cities is completed.

② Going from city i to i is not permitted.

Sol^n: ① Row Reduction

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

② Column Reduction

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	~~0~~
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	~~0~~	7	∞

modified matrix

E − A

A − E , B, C, D ✗

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Travelling Salesman Problem

By - Sandeep Kumar Gour

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

Two additional constraints:

① No city is to be visited twice before the tour of all cities is completed.

② Going from city i to i is not permitted.

Sol^{n}: ① Row Reduction

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

② Column Reduction

	A	B	C	D	E
A	∞	1	3	6	[0]
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	[0]	1	∞	1
E	[0]	2	~~0~~	7	∞

modified matrix

E - A

A - E, B, C, D ✗

1. reference (1) (next highest)

	A	B	C	D	E
A	∞	[1]	3	6	~~0~~
B	4	∞	[0]	6	~~0~~
C	4	3	∞	[0]	3
D	8	~~0~~	1	∞	[1]
E	[0]	2	~~0~~	7	∞

A-B, B-C, C-D, D-E, E-A ✓

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Travelling Salesman Problem

By - Sandeep Kumar Gour

city	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Find the least cost route?

Two additional constraints

① No city is to be visited twice before the tour of all cities is completed.

② Going from city i to i is not permitted.

Sol^{n}: ① Row Reduction

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	∞
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	∞	7	∞

② Column Reduction

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	∞
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	∞	7	∞

modified matrix

A–E, E–A

B, C, D ✗

1. reference (↓) (next highest)

	A	B	C	D	E
A	∞	1	3	6	∞
B	4	∞	0	6	∞
C	4	3	∞	0	3
D	8	∞	1	∞	1
E	0	2	∞	7	∞

A–B, B–C, C–D, D–E, E–A

2 3 4 5 1

= 15

Maximization Assignment Problem in Hindi (Lecture.34)