Dual Simplex method -

1. First convert the minimization LPP into maximization form, if it is given in the minimization form.
2. Convert the '$\geq$' type inequalities into '$\leq$' type by multiplying -1.
3. Introduce slack variables in the constraints of the given problem and obtain an initial basic solution.
4. Test the nature of Δ_j in the starting table
   (i) If all Δ_j and X_B are non-negative, then an optimal basic feasible solution has been attained.
   (ii) If all Δ_j are non-negative and atleast one basic variable X_B is negative, then goto step (5)
   (iii) If atleast one Δ_j is negative, then method is not appropriate.
5. Select the most negative X_B, $\therefore$ Outgoing vector (leaving vector) $[X_r]$
6. Test the nature of X_r
   (i) If all X_r are non-negative, then there does not exist any feasible s
   (ii) If atleast one X_r is negative, then compute $\text{Max}\left[\Delta_j / X_r, X_r < 0\right]$

3. Introduce slac
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4. Test the natu
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solution has b
(ii) If all Δ_j are
is negative, th
(iii) If atleast on
5. Select the mo
6. Test the natur
(i) If all X_r a
(ii) If atleast on
to determine
7. Test the new

Example:

$\text{Min } Z = 3x_1 + x_2$

Subject to

$x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

and $x_1, x_2 \geq 0$

Sol^n: $\text{Max } Z' = -3x_1 - x_2$

Subject to

$-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } Z' = -3x_1 - x_2$

s.t.

$-x_1 - x_2 + s_1 = -1$

$-2x_1 - 3x_2 + s_2 = -2$

and $x_1, x_2, s_1, s_2 \geq 0$

4. Test the nature of Δ_j in the starting table

(i) If all Δ_j and X_B are non-negative, then an optimal basic feasible solution has been attained.

(ii) If all Δ_j are non-negative and at least one basic variable X_B is negative, then goto step (5)

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Subject to

$-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

<u>Starting table</u>

		C_j	-3	-1	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-1	-1	-1	1	0
S_2	0	-2	-2	-3	0	1
$Z'=0$		Δ_j	3	1	0	0

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5. Select the most negative X_B, : Outgoing vector (leaving vector) $[X_r]$
6. Test the nature of X_r
   (i) If all X_r are non-negative, then there does not exist any feasible so
   (ii) If atleast one X_r is negative, then compute $\text{Max}\left[\Delta_j / X_r,\ X_r < 0\right]$ to determine incoming vector.
7. Test the new iterated dual simplex table for optimality.

Example:

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Subject to

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Starting table

		C_j	-3	-1	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-1	-1	-1	1	0
S_2	0	-2	[illegible]	-3	0	1
$Z' = 0$		Δ_j	3	[illegible]	0	0

5. Select the most negative X_B, : Outgoing vector (leaving vector) $[X_r]$

6. Test the nature of X_r

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Subject to

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$-2x_1 - 3x_2 \leq -2$

Starting table

		C_j	-3	-1	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-1	-1	-1	1	0
(X_r) outgoing vector ← S_2	0	-2	-2	-3	0	1
$Z' = 0$		Δ_j	3	1 ↑ incoming	0	0

$\max\left[\frac{\Delta_j}{X_r}, X_r < 0\right] = \left[\frac{3}{-2}, \frac{1}{-3}\right] = \frac{1}{-3}$

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5. Select the most negative X_B, : Outgoing vector (leaving vector) $[X_r]$

6. Test the nature of X_r

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Starting table

		C_j	-3	-1	0	0	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	
S_1	0	-1	-1	-1	1	0	
(X_r) outgoing vector ← S_2	0	-2	-2	$\boxed{-3}$	0	1	
$Z' = 0$		Δ_j	3	1 ↑ incoming	0	0	
S_1	0	$-1/3$	$-1/3$	0	1	$-1/3$	$R_1 = R_1 + R_2$ (New
x_2	-1	$2/3$	$2/3$	1	0	$-1/3$	$R_2 / -3$
$Z' = -2/3$		Δ_j	$7/3$	0	0	$1/3$	

Example:

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Subject to

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$2x_1 + 3x_2 \geq 2$

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SLPP

Max $Z' = -3x_1 - x_2$

s.t.

$-x_1 - x_2 + s_1 = -1$

$-2x_1 - 3x_2 + s_2 = -2$

and $x_1, x_2, s_1, s_2 \geq 0$

IBFS

starting table

		C_j	-3	-1	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	-1	-1	-1	1	0
← S_2	0	-2	-2	$\boxed{-3}$	0	1
$Z' = 0$		Δ_j	3	1 ↑ incoming	0	0
← S_1	0	$-1/3$	$-1/3$	0	1	$-1/3$
x_2	-1	$2/3$	$2/3$	1	0	$-1/3$
$Z' = -2/3$		Δ_j	$7/3$	0	0	$1/3$ ↑

(X_n) outgoing vector

outgoing (X_n)

$R_1 = R_1 + R_2$ (New

$R_2/-3$

$$\max\left[\frac{\Delta_j}{X_n}\right] = \left[\frac{7/3}{-1/3}, \frac{1/3}{-1/3}\right]$$

$$= [-7, -1] = -1$$

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Example:

Min $Z = 3x_1 + x_2$

Subject to

$x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

and $x_1, x_2 \geq 0$

Solⁿ Max $Z' = -3x_1 - x_2$

Subject to

$-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

and $x_1, x_2 \geq 0$

SLPP

Max $Z' = -3x_1 - x_2$

s.t.

$-x_1 - x_2 + s_1 = -1$

$-2x_1 - 3x_2 + s_2 = -2$

and $x_1, x_2, s_1, s_2 \geq 0$

IBFS

starting table

		C_j	-3	-1	0	0	
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	
s_1	0	-1	-1	-1	1	0	
(x_r) outgoing vector $\leftarrow s_2$	0	-2	-2	-3	0	1	
$Z' = 0$		Δ_j	3	$1 \uparrow$ incoming	0	0	
outgoing (x_r) $\leftarrow s_1$	0	$-1/3$	$-1/3$	0	1	$-1/3$	$R_1 = R_1 + R_2$ (New
x_2	-1	$2/3$	$2/3$	1	0	$-1/3$	$R_2/-3$
$Z' = -2/3$		Δ_j	$7/3$	0	0	$1/3 \uparrow$ incoming	
s_2	0	1	1	0	-3	1	$R_1/-1/3$
x_2	-1	1	1	1	-1	0	$R_2 = R_2 + \frac{1}{3}R_1$
$Z' = -1$		Δ_j	2	0	1	0	

Example:

Min $Z = 3x_1 + x_2$

Subject to

$x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

and $x_1, x_2 \geq 0$

Sol^n Max $Z' = -3x_1 - x_2$

Subject to

$-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

and $x_1, x_2 \geq 0$

SLPP

Max $Z' = -3x_1 - x_2$

s.t.

$-x_1 - x_2 + s_1 = -1$

$-2x_1 - 3x_2 + s_2 = -2$

and $x_1, x_2, s_1, s_2 \geq 0$

IBFS

Starting table

				C_j	-3	-1	0	0	
		Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	
		S_1	0	-1	-1	-1	1	0	
(X_r) outgoing vector	←	S_2	0	-2	-2	-3	0	1	
		$Z'=0$		Δ_j	3	1 ↑ incoming	0	0	
outgoing (X_r)	←	S_1	0	$-1/3$	$-1/3$	0	1	$-1/3$	$R_1 = R_1 + R_2$ (Ne
		x_2	-1	$2/3$	$2/3$	1	0	$-1/3$	$R_2/-3$
		$Z' = -2/3$		Δ_j	$7/3$	0	0	$1/3$ ↑ incoming	
		S_2	0	1	1	0	-3	1	$R_1/-1/3$
		x_2	-1	1	1	1	-1	0	$R_2 = R_2 + \frac{1}{3}R_1$
		$Z' = -1$		Δ_j	2	0	1	0	

$\because$ all $\Delta_j \geq 0$ and $X_B \geq 0$,

Therefore optimal solution is

Max $Z' = -1$, Min $Z = 1$, $x_1 = 0$, $x_2 = 1$.

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Revised Simplex method

$\text{Max } Z = x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 3$

$x_1 + 2x_2 \leq 5$

$3x_1 + x_2 \leq 6$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } \boxed{Z = x_1 + 2x_2}$

subject to

$x_1 + x_2 + s_1 = 3$

$x_1 + 2x_2 + s_2 = 5$

$3x_1 + x_2 + s_3 = 6$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Standard form-I (slack variable only)

$$Z - x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 = 0$$

$$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$$

$$x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 5$$

$$3x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Matrix form

$$
\begin{array}{cccccc}
 & & & s_1 & s_2 & s_3 \\
 & & & \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} \\
 & x_1 & x_2 & & & \\
e_1 & a_1^{(1)} & a_2^{(1)} & a_3^{(1)} & a_4^{(1)} & a_5^{(1)}
\end{array}
$$

$$
\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}
$$

Revised simplex table (Iteration-I)

Additional table

Bas

Matrix form

$$
\begin{array}{cccccc}
 & x_1 & x_2 & s_1 & s_2 & s_3 \\
 & & & \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} \\
e_1 & a_1^{(1)} & a_2^{(1)} & a_3^{(1)} & a_4^{(1)} & a_5^{(1)}
\end{array}
$$

$$
\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}
$$

Revised simplex table (Iteration-I)

Additional table

Basic variable	B_1^{-1}				X_B	X_K	X_B/X_K
	e_1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			

Matrix form

$$
\begin{array}{cccccc}
 & x_1 & x_2 & s_1 & s_2 & s_3 \\
 & & & \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} \\
e_1 & a_1^{(1)} & a_2^{(1)} & a_3^{(1)} & a_4^{(1)} & a_5^{(1)}
\end{array}
$$

$$
\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}
$$

Revised simplex table (Iteration-I)

Basic variable	B_1^{-1}				X_B	X_K	X_B/X_K
	e_1 (z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			

Additional table

$a_1^{(1)}$	$a_2^{(1)}$

Revised simplex table (Iteration-I) Additional table

Basic variable	B_1^{-1}				X_B	X_K	X_B/X_K
	e_1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
Z	1	0	0	0	0		
S_1	0	1	0	0	3		
S_2	0	0	1	0	5		
S_3	0	0	0	1	6		

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1

Test for optimality

$\Delta_j = (\text{First row of } B_1^{-1}) \times (a_j^{(1)})$ → Additional table

$\Delta_1 = (1,0,0,0)(-1,1,1,3) = -1+0+0+0 = -1$

$\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0+0+0 = -2$

Both Δ_1 & Δ_2 are negative, so find the most negative value and determine the incoming vector.

Basic variable	B_1^{-1}				X_B	X_K	X_B/X_K
	e_1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
Z	1	0	0	0	0		
S_1	0	1	0	0	3		
S_2	0	0	1	0	5		
S_3	0	0	0	1	6		

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1
-1	-2 ↑

Test for optimality

$\Delta_j = (\text{First row of } B_1^{-1}) \times (a_j^{(1)})$ → Additional table

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Both Δ_1 & Δ_2 are negative, so find the most negative value and determine the incoming vector.

compute the column vector (X_K)

$X_K = B_1^{-1} \times a_2^{(1)}$ → incoming vector

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Revised simplex table (Iteration-I)

Basic variable	B_1^{-1}				X_B	X_K	Min $[X_B/X_K]$ $X_K>0$
	e_1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
Z	1	0	0	0	0	-2	-
S_1	0	1	0	0	3	1	3/1
S_2	0	0	1	0	5	2	5/2
S_3	0	0	0	1	6	1	6/1

Additional table

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1
-1	-2 ↑

Test for optimality

$$\Delta_j = (\text{First row of } B_1^{-1}) \times (a_j^{(1)})$$

$$\Delta_1 = (1,0,0,0)(-1,1,1,3) = -1+0+0$$

$$\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0$$

Both Δ_1 & Δ_2 are negative, so fi

most negative value and determ

incoming vector.

Revised simplex table (Iteration-I)

Basic variable	B_1^{-1}				X_B	X_K	Min $[X_B/X_K]$, $X_K>0$
	e_1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
Z	1	0	0	0	0	-2	-
S_1	0	1	0	0	3	1	3/1
← S_2	0	0	1	0	5	2	5/2 ←
S_3	0	0	0	1	6	1	6/1

Additional table

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1
-1	-2 ↑

Test for optimality

$\Delta_j = (\text{First row of } B_1^{-1}) \times (a_j^{(1)})$ → Additional table

$\Delta_1 = (1,0,0,0)(-1,1,1,3) = -1+0+0+0 = -1$

$\Delta_2 = (1,0,0,0)(-2,1,2,1) = -2+0+0+0 = -2$

Both Δ_1 & Δ_2 are negative, so find the most negative value and determine the incoming vector.

Revised simplex table (Iteration-I)

Basic variable	B_1^{-1}				X_B	X_K	Min $[X_B/X_K]$, $X_K>0$
	e_1 (z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			

Additional table

$a_1^{(1)}$	$a_2^{(1)}$

Improved solution

$$
\begin{array}{c|c}
 & \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & X_B \end{array} \\
\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \end{bmatrix}
\end{array}
\begin{array}{c} X_K \\ -2 \\ 1 \\ \boxed{2} \\ 1 \end{array}
\longrightarrow
\begin{array}{c} \begin{array}{cccc} \beta_1^{(1)} & \beta_2^{(1)} & \beta_3^{(1)} & X_B \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & -1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 & 5/2 \\ 0 & -1/2 & 1 & 7/2 \end{bmatrix} \end{array}
\begin{array}{c} X_K \\ 0 \\ 0 \\ 1 \\ 0 \end{array}
$$

$\boxed{R_1 + 2R_3}$ $\boxed{R_2 - R_3}$ $\boxed{R_4 - R_3}$

$$
\begin{array}{ccccc}
0 & 0 & 0 & 0 & -2 \\
0 & 1 & 0 & 5 & 2 \\
\hline
0 & 1 & 0 & 5 & 0
\end{array}
$$

Basic variable	B_1^{-1}				X_B	X_K	Min $[X_B/X_K]$ $X_K>0$
	e_1 (z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
z	1	0	0	0	0	-2	-
s_1	0	1	0	0	3	1	3/1
← s_2	0	0	1	0	5	2	5/2 ←
s_3	0	0	0	1	6	1	6/1

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1

Iteration 2

Revised simplex table

Basic variable	B_1^{-1}				X_B	X_K	$\frac{X_B}{X_K}$
	e_1 (z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
z	1	0	1	0	5		
s_1	0	1	-1/2	0	1/2		
x_2	0	0	1/2	0	5/2		
s_3	0	0	-1/2	1	7/2		

Additional table

$a_1^{(1)}$	$a_4^{(1)}$
-1	0
1	0
1	1
3	0

Additional table

Basic variable	B_1^{-1}				X_B	X_k	$\frac{X_B}{X_k}$
	e_1 (2)	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$			
Z	1	0	1	0	5		
s_1	0	1	$-1/2$	0	$1/2$		
x_2	0	0	$1/2$	0	$5/2$		
s_3	0	0	$-1/2$	1	$7/2$		

$a_1^{(1)}$	$a_4^{(1)}$
-1	0
1	0
1	1
3	0

again test for optimality

$$\Delta_j = (\text{First row of } B_1^{-1}) \cdot (a_j^{(1)})$$

$$\Delta_1 = (1,0,1,0) \cdot (-1,1,1,3) = -1+0+1+0 = 0$$

$$\Delta_4 = (1,0,1,0) \cdot (0,0,1,0) = 0+0+1+0 = 1$$

$\because$ Both Δ_1 and Δ_4 are positive.

therefore optimal solution is

$$\text{Max } Z = 5,\ x_1 = 0,\ x_2 = 5/2.$$

Revised Simplex method (Standard form-II)

Min $Z = x_1 + 2x_2$ <u>SLPP</u> Max $Z' = -x_1 - 2x_2$

Subject to

$$2x_1 + 5x_2 \geq 6$$

$$x_1 + x_2 \geq 2$$

and $x_1, x_2 \geq 0$

<u>Standard form-II</u>

$$z' + x_1 + 2x_2 = 0$$

$$-3x_1 - 6x_2 + x_3 + x_4 + x_5 = -8 \quad | \; x_5 = -(x_6 + x_7)$$

$$\left.\begin{aligned} 2x_1 + 5x_2 - x_3 + x_6 &= 6 \\ x_1 + x_2 - x_4 + x_7 &= 2 \end{aligned}\right]$$

and $x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

Revised Simplex method (Standard form-II)

Min $Z = x_1 + 2x_2$ $\underline{\text{SLPP}}$ Max $Z' = -x_1 - 2x_2$

Subject to

$$2x_1 + 5x_2 \geq 6$$
$$x_1 + x_2 \geq 2$$

and $x_1, x_2 \geq 0$

<u>Standard form-II</u>

$$Z' + x_1 + 2x_2 = 0$$
$$-3x_1 - 6x_2 + x_3 + x_4 + x_5 = -8 \quad | \; x_5 = -(x_6 + x_7)$$
$$\left.\begin{aligned} 2x_1 + 5x_2 - x_3 + x_6 &= 6 \\ x_1 + x_2 - x_4 + x_7 &= 2 \end{aligned}\right]$$

and $x_1, x_2, \underbrace{x_3, x_4}_{\text{surplus}}, \underbrace{x_5, x_6, x_7}_{\text{artificial}} \geq 0$

$$z' + x_1 + 2x_2 = 0$$

$$-3x_1 - 6x_2 + x_3 + x_4 + x_5 = -8 \quad | \; x_5 = -(x_6 + x_7)$$

$$2x_1 + 5x_2 - x_3 + x_6 = 6$$

$$x_1 + x_2 - x_4 + x_7 = 2$$

and $x_1, x_2, \underbrace{x_3, x_4}_{\text{surplus}}, \underbrace{x_5, x_6, x_7}_{\text{artificial}} \geq 0$

Matrix form

$$\begin{array}{cccccccc} a_0^{(2)} & a_1^{(2)} & a_2^{(2)} & a_3^{(2)} & a_4^{(2)} & a_5^{(2)} & a_6^{(2)} & a_7^{(2)} \\ (e_1) & & & & & (e_2) & \beta_1^{(2)} & \beta_2^{(2)} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -6 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z' \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{array}{cccccccc} Z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array}$$

Phase I

Basic variable	B_2^{-1} e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	Min X_B/X_k
z'	1	0	0	0	0		
x_5	0	1	0	0	-8		
x_6	0	0	1	0	6		
x_7	0	0	0	1	2		

Additional table

$a_1^{(2)}$	$a_2^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	2	0	0
-3	-6	1	1
2	5	-1	0
1	1	0	-1

$\because x_5 = -8$

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Phase I

Basic variable	B_2^{-1}				X_B	X_k	min X_B/X_k
	e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$			
z'	1	0	0	0	0		
x_5	0	1	0	0	-8		
x_6	0	0	1	0	6		
x_7	0	0	0	1	2		

Additional table

$a_1^{(2)}$	$a_2^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	2	0	0
-3	-6	1	1
2	5	-1	0
1	1	0	-1

$\because x_5 = -8,$ (Iteration-I)

Computation of Δ_j

$$\Delta_j = (\text{second row of } B_2^{-1}) \cdot (a_j^{(2)})$$

$$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$$

$$\Delta_2 = (0,1,0,0)(2,-6,5,1) = -6$$

$$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$$

$$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$$

$\therefore x_5 = -8$, (Iteration-I)

Computation of Δ_j

$\Delta_j = (\text{second row of } B_2^{-1}) \cdot (a_j^{(2)})$

$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$

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$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$

$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$

$\uparrow$ (a_k) incoming vector

Entering vector $\Delta_2 = -6$

compute X_k for leaving vector

$$X_k = B_2^{-1} \cdot a_2^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$$

$\therefore x_5 = -8$, (Iteration–I)

Computation of Δ_j'

$\Delta_j = (\text{second row of } B_2^{-1}) \cdot (a_j^{(2)})$

(a_k) 'incoming vector'

$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$

$\Delta_2 = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$

$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$

$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$

Entering vector $\Delta_2 = -6$

compute X_k for leaving vector

$$X_k = B_2^{-1} \cdot a_2^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$$

Basic variable	B_2^{-1}				X_B	X_k	min X_B/X_k
	e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$			
z'	1	0	0	0	0	2	-
x_5	0	1	0	0	-8	-6	-
← x_6	0	0	1	0	6	$\boxed{5}$	6/5 ←
x_7	0	0	0	1	2	1	2/1

$a_1^{(2)}$	$a_2^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	2	0	0
-3	-6	1	1
2	5	-1	0
1	1	0	-1

↑ (a_k) 'incoming vector

$\because x_5 = -8$, (Iteration-I)

Computation of Δ_j

$$\Delta_j = (\text{second row of } B_2^{-1}) \cdot (a_j^{(2)})$$

$$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$$

$$\Delta_2 = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$$

$$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$$

$$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$$

Entering vector $\Delta_2 = -6$

$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$

$\Delta_2 = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$

$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$

$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$

Entering vector $\Delta_2 = -6$

compute X_k for leaving vector

$$X_k = B_2^{-1} \cdot a_2^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$$

improve sol^n

$$\begin{array}{cccc} \beta_1^{(2)} & \beta_2^{(2)} & X_B & X_k \\ \left[\begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix} & \left.\begin{matrix} 0 \\ -8 \\ 6 \\ 2 \end{matrix}\right] & \begin{matrix} 2 \\ -6 \\ \boxed{5} \\ 1 \end{matrix} \end{array}$$

Like & Subscribe

$\Delta_1 = (0,1,0,0)(1,-3,2,1) = -3$

$\Delta_2 = (0,1,0,0)(2,-6,5,1) = -6 \rightarrow$

$\Delta_3 = (0,1,0,0)(0,1,-1,0) = 1$

$\Delta_4 = (0,1,0,0)(0,1,0,-1) = 1$

Entering vector $\Delta_2 = -6$

compute X_k for leaving vector

$$X_k = B_2^{-1} \cdot a_2^{(2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 5 \\ 1 \end{bmatrix}$$

improve sol^n

$$\begin{array}{cccc} \beta_1^{(2)} & \beta_2^{(2)} & X_B & X_k \end{array} \qquad\qquad \begin{array}{cccc} \beta_1^{(2)} & \beta_2^{(2)} & X_B & X_k \end{array}$$

$$\left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & -8 \\ 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right] \begin{array}{c} 2 \\ -6 \\ \boxed{5} \\ 1 \end{array} \longrightarrow \left[\begin{array}{ccc} -2/5 & 0 & -12/5 \\ 6/5 & 0 & -4/5 \\ 1/5 & 0 & 6/5 \\ -1/5 & 1 & 4/5 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} \quad \begin{array}{c} R_1 - 2R_3 \\ R_2 + 6R_3 \\ R_3/5 \\ R_4 - R_3 \end{array}$$

Basic variable	e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	min $\frac{X_B}{X_k}$
z'	1	0	$-2/5$	0	$-12/5$		
x_5	0	1	$6/5$	0	$-4/5$		
x_2	0	0	$1/5$	0	$6/5$		
x_7	0	0	$-1/5$	1	$4/5$		

$a_1^{(2)}$	$a_6^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	0	0	0
-3	0	1	1
2	1	-1	0
1	0	0	-1

$\because x_5 = -4/5$, Now enter the iteration 2.

Computation of Δ_j = (second row of B_2^{-1}) $\cdot$ $(a_j^{(2)})$

$\Delta_1 = (0,1,6/5,0)\cdot(1,-3,2,1) = -3/5$

$\Delta_6 = (0,1,6/5,0)(0,0,1,0) = 6/5$

$\Delta_3 = (0,1,6/5,0)(0,1,-1,0) = -1/5$

$\Delta_4 = (0,1,6/5,0)(0,1,0,-1) = 1$

Basic variable	e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	Min $\frac{X_B}{X_k}$
z'	1	0	$-2/5$	0	$-12/5$		
x_5	0	1	$6/5$	0	$-4/5$		
x_2	0	0	$1/5$	0	$6/5$		
x_7	0	0	$-1/5$	1	$4/5$		

$a_1^{(2)}$	$a_6^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	0	0	0
-3	0	1	1
2	1	-1	0
1	0	0	-1

$\because x_5 = -4/5$, Now enter the iteration 2.

Computation of Δ_j = (second row of B_2^{-1}) $\cdot$ ($a_j^{(2)}$)

$\Delta_1 = (0, 1, 6/5, 0) \cdot (1, -3, 2, 1) = -3/5 \leftarrow$

$\Delta_6 = (0, 1, 6/5, 0)(0, 0, 1, 0) = 6/5$

$\Delta_3 = (0, 1, 6/5, 0)(0, 1, -1, 0) = -1/5$

$\Delta_4 = (0, 1, 6/5, 0)(0, 1, 0, -1) = 1$

Entering vector $\Delta_1 = -3/5$

x_2	0	0	1/5	0	6/5				2	1	-1	0
x_7	0	0	-1/5	1	4/5				1	0	0	-1

$\because x_5 = -4/5$, Now enter the iteration 2.

Computation of Δ_j = (second row of B_2^{-1}) $\cdot$ ($a_j^{(2)}$)

$\Delta_1 = (0,1,6/5,0)\cdot(1,-3,2,1) = -3/5 \leftarrow$

$\Delta_6 = (0,1,6/5,0)(0,0,1,0) = 6/5$

$\Delta_3 = (0,1,6/5,0)(0,1,-1,0) = -1/5$

$\Delta_4 = (0,1,6/5,0)(0,1,0,-1) = 1$

Entering vector $\Delta_1 = -3/5 \Rightarrow a_1^{(2)}$

Compute X_k

$$X_k = B_2^{-1} \cdot a_1^{(2)} = \begin{bmatrix} 1 & 0 & -2/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & 1/5 & 0 \\ 0 & 0 & -1/5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/5 \\ -3/5 \\ 2/5 \\ 3/5 \end{bmatrix}$$

Basic variable	β_2^{-1}: e_1	e_2	$\beta_1^{(2)}$	$\beta_2^{(2)}$	X_B	X_k	min $\frac{X_B}{X_k}$
z'	1	0	$-2/5$	0	$-12/5$	$1/5$	-
x_5	0	1	$6/5$	0	$-4/5$	$-3/5$	-
x_2	0	0	$1/5$	0	$6/5$	$2/5$	3
$\leftarrow x_7$	0	0	$-1/5$	1	$4/5$	$3/5$	$4/3 \leftarrow$

additional table

$a_1^{(2)}$	$a_6^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
1	0	0	0
-3	0	1	1
2	1	-1	0
1	0	0	-1
$\uparrow$			

$\because x_5 = -4/5$, Now enter the iteration 2.

[illegible]omputation of Δ_j = (second row of B_2^{-1})$\cdot$($a_j^{(2)}$)

$= (0,1,6/5,0)\cdot(1,-3,2,1) = -3/5 \leftarrow$

$= (0,1,6/5,0)(0,0,1,0) = 6/5$

[illegible] $/5,0)(0,1,-1,0) = -1/5$

[illegible] $(0,1,0,-1) = 1$

[illegible] $-3/5 \Rightarrow a_1^{(2)}$

[illegible] $0 \quad -2/5 \quad 0 \,][1] \quad [1/5]$

Like & Subscribe

$\Delta_3 = (0,1,6/5,0)(0,1,-1,0) = -1/5$

$\Delta_4 = (0,1,6/5,0)(0,1,0,-1) = 1$

Entering vector $\Delta_1 = -3/5 \Rightarrow a_1^{(2)}$

Compute X_k

$$X_k = B_2^{-1} \cdot a_1^{(2)} = \begin{bmatrix} 1 & 0 & -2/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & 1/5 & 0 \\ 0 & 0 & -1/5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/5 \\ -3/5 \\ 2/5 \\ 3/5 \end{bmatrix}$$

improve sol^n

$$\begin{array}{cccc} \beta_1^{(2)} & \beta_2^{(2)} & X_B & X_k \end{array}$$

$$\left[\begin{array}{ccc|c} -2/5 & 0 & -12/5 & 1/5 \\ 6/5 & 0 & -4/5 & -3/5 \\ 1/5 & 0 & 6/5 & 2/5 \\ -1/5 & 1 & 4/5 & \boxed{3/5} \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} -1/3 & -1/3 & -8/3 & 0 \\ 1 & 1 & 0 & 0 \\ 1/3 & -2/3 & 2/3 & 0 \\ -1/3 & 5/3 & 4/3 & 1 \end{array}\right] \begin{array}{l} R_1 - \frac{1}{5}R_4 \\ R_2 + \frac{3}{5}R_4 \\ R_3 - \frac{2}{5}R_4 \\ \end{array}$$

variable	e_1	e_2	$B_1^{(2)}$	$B_2^{(2)}$	X_B	Y_u	$\frac{X_B}{Y_u}$
z'	1	0	$-1/3$	$-1/3$	$-8/3$		
x_5	0	1	1	1	0		
x_2	0	0	$1/3$	$-2/3$	$2/3$		
x_1	0	0	$-1/3$	$5/3$	$4/3$		

$a_7^{(2)}$	$a_6^{(2)}$	$a_3^{(2)}$	$a_4^{(2)}$
0	0	0	0
0	0	1	1
0	1	-1	0
1	0	0	1
x	x		

$\because x_5 = 0$, Now enter <u>phase II</u>

<u>compute Δ_j</u>

$$\Delta_j = (\text{First row of } B_2^{-1})(a_j^{(2)}),$$

$$\Delta_3 = (1, 0, -1/3, -1/3)(0, 1, -1, 0) = 1/3$$

$$\Delta_4 = (1, 0, -1/3, -1/3)(0, 1, 0, 1) = 1/3$$

$\because \Delta_3, \Delta_4$ are positive.

$\therefore$ optimal solⁿ $z' = -8/3$

$$\Rightarrow z = 8/3,\ x_1 = 4/3,\ x_2 = 2/3$$