Game Theory -

Sandeep Kumar Gour

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results.

Like & Subscribe

Game Theory -

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results.

game

Game is defined as an activity between two or more persons involving activities by each person according to a set of rules, at the end of which each person receives some benefit or satisfaction or suffers loss (negative benefit)

The set of rules defines the game. Going through the set of rules once by the participants defines a play.

Like & Subscribe

Characteristics of Game Theory –

Sandeep Kumar Gour

There can be various types of games. They can be classified on the basis of the following characteristics –

(i) Chance of strategy –

If in a game, activities are determined by skill, it is said to be a game of strategy, if they are determined by chance, it is a game of chance.

In general, a game may involve game of strategy as well as a game of chance.

Like & Subscribe

classified on the basis of the following characteristics –

Sandeep Kumar Gour

(i) Chance of strategy –

If in a game, activities are determined by skill, it is said to be a game of strategy, if they are determined by chance, it is a game of chance.

In general, a game may involve game of strategy as well as a game of chance.

(ii) Number of persons –

A game is called an n-person game if the number of persons playing is n. The person means an individual or a group aiming at a particular objective.

Like & Subscribe

(iii) Number of activities -

These may be finite or infinite.

(iv) Number of alternatives (choices) available to each person in a particular activity may also be finite or infinite. A finite gar
has a finite number of activities, each involving a finite number of alternatives, otherwise the game is said to be infinite.

(v) Information to the players about the past activities of othe
players is completely available, partly available or not availab
at all.

(vi) Payoff -

A quantitative measure of satisfaction a person gets

Like & Subscribe

(iv) Number of alternatives (choices) available to each person in a particular activity may also be finite or infinite. A finite game has a finite number of activities, each involving a finite number of alternatives, otherwise the game is said to be infinite.

(v) Information to the players about the past activities of other players is completely available, partly available or not available at all.

(vi) Payoff –

A quantitative measure of satisfaction a person gets at the end of each play is called a payoff. It is a real-valued function of variables in the game.

Like & Subscribe

Basic Definitions -

1. Competitive Game -

A competitive situation is called a competitive game if it has the following four properties -

(i) There are finite number (n) of competitors (called players).

(ii) Each player has a list of finite number of possible activitie

(iii) A play is said to occur when each player chooses one of his activities.

(iv) Every combination of activities determines an outcome.

Like & Subscribe

2. Zero-sum and Non-zero sum game -

Sandeep Kumar Gour

Competitive games are classified according to the number of player involved, i,e as a two-person game, three-person game etc.

If the players make payments only to each other, i,e the loss of one is the gain of others and nothing comes from outside, the competitive game is said to be zero-sum

A game which is not zero sum is called a Non-zero sum game.

B

Player A +10

Like & Subscribe

number of player involved, i,e as a two-person game, ...
Sandeep Kumar Gour
person game etc.

If the players make payments only to each other, i,e the loss of one is the gain of others and nothing comes from outside, the competitive game is said to be zero-sum.

A game which is not zero sum is called a Non-zero sum game.

B

player A +10

n players , payoff

$v_1 \quad v_2 \quad \cdots \quad v_n$

P_1

P_2

$\vdots$

P_n

$$\boxed{\sum_{i=1}^{n} v_i = 0}$$

Like & Subscribe

3. **Strategy** -

Sandeep Kumar Gour

A strategy for a given player is a set of rules that specifies which of the available course of action he should make at each play. This strategy may be of two kinds

(i) Pure strategy -

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

(ii) Mixed strategy -

If a player is guessing as to which activity is to be selected by the other on any particular occassion, a probabilistic

Like & Subscribe

(i) Pure strategy –

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

(ii) Mixed strategy –

If a player is guessing as to which activity is to be selected by the other on any particular occassion, a probabilistic situation is obtained and objctive function is to maximise the expected gain.

Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

Like & Subscribe

4. Two person, zero sum (Rectangular) Games -

A game with only two players (say, player A and player B) is called a 'two person zero sum game' if the losses of one player are equivalent to the gains of the other, so that the sum of their net gains is zero.

Two person, zero sum games are also called rectangular games as these are usually represented by a payoff matrix in rectangular form.

Like & Subscribe

5. Payoff matrix -

Suppose the player A has m activities and the player B has n activities. Then a payoff matrix can be formed by adopting the following rules.

(i) Row designations for each matrix are activities available to player A.

(ii) Column d nations for each matrix are activities available to player

(iii) Cell entr v_{ij}' is the payment to player A in A's payoff matrix A chooses the activity i and B chooses the activity

(iv) With a sum, two person game', the cell entry in the player B's off matrix will be negative of the

Like & Subscribe

5. Payoff matrix -

Suppose the player A has m activities and the player B has n activities. Then a payoff matrix can be formed by adopting the following rules.

(i) Row designations for each matrix are activities available to player A.

(ii) Column designations for each matrix are activities available to player B.

(iii) Cell entry 'V_{ij}' is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j

(iv) With a 'zero sum, two person game', the cell entry in the player B's payoff matrix will be negative of the

Like & Subscribe

player B has n activities. Then a payoff matrix can be formed by adopting the following rules.

(i) Row designations for each matrix are activities available to player A.

(ii) Column designations for each matrix are activities available to player B.

(iii) Cell entry 'v_{ij}' is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j

(iv) With a 'zero sum, two person game', the cell entry in the player B's payoff matrix will be negative of the correspond cell entry 'v_{ij}' in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

Like & Subscribe

Player A's payoff matrix

player B

		1	2	3	----	n
ayer A	1	v_{11}	v_{12}	--		v_{1n}
	2	v_{21}	v_{22}	---		v_{2n}
	3	$\vdots$				
	$\vdots$	$\vdots$				
	m	v_{m1}	v_{m2}		---	v_{mn}

player B's payoff mat

player B

		1	2	---	n
player A	1	$-v_{11}$	$-v_{12}$	----	$-v_{1n}$
	2	$-v_{21}$	$-v_{22}$	----	$-v_{2n}$
	$\vdots$	$\vdots$			
	m	$-v_{m1}$	$-v_{m2}$	---	$-v_{mn}$

Like & Subscribe

player A	1	2	3	....	n
1	V_{11}	V_{12}	..		V_{1n}
2	V_{21}	V_{22}	...		V_{2n}
3	⋮				
⋮	⋮				
m	V_{m1}	V_{m2}	...		V_{mn}

player B

player A	1	2	---	n
1	$-V_{11}$	$-V_{12}$	----	$-V_{1n}$
2	$-V_{21}$	$-V_{22}$	----	$-V_{2n}$
⋮	⋮			
m	$-V_{m1}$	$-V_{m2}$	----	$-V_{mn}$

Eg: 2-coin <u>HH, TT</u>

player B

player A	H	T	
H	+1	-1	= 0
T	-1	+1	= 0

Like & Subscribe

Maximin Criteria -

The player who is maximising his outcome or payoff finds out his minimum gains from each strategy (course of action) and selects the maximum value out of these minimum gains.

Minimax Criteria -

In this criterion the minimising player determines the maximum loss from each strategy and then selects with minimum loss out of the maximum loss list.

Saddle point -

A saddle point of a payoff matrix is the position of such an element in the payoff matrix which is minimum in its row and maximum in its column.

Mathematicall...

Like & Subscribe

Minimax criteria -

In this criterion the minimising player determines the maximum loss from each strategy and then selects with minimum loss out of the maximum loss list.

Saddle point -

A saddle point of a payoff matrix is the position of such an element in the payoff matrix which is minimum in its row and maximum in its column.

Mathematically, if a payoff matrix $\{v_{ij}\}$ is such that

$$\max_i \left[\min_j \{v_{ij}\}\right] = \min_j \left[\max_i \{v_{ij}\}\right] = v_{rs} \text{ (say)}$$

then the matrix is said to have a saddle point (r, s)

Sandeep Kumar Gour

Like & Subscribe

Saddle point -

Sandeep Kumar Gour

A saddle point of a payoff matrix is the position of such an element in the payoff matrix which is minimum in its row and maximum in its column.

Rules for determining a saddle point -

1. Select the minimum element of each row of the payoff matrix and mark them by '○'
2. Select the greatest element of each column of the payoff matrix and mark them by '□'
3. If there appears an element in the payoff matrix marked by '○' and '□' both, the position of that element is a saddle point of the payoff matrix.

Like & Subscribe

matrix and mark them by $\square$

3. If there appears an element in the payoff matrix marked by '0' and '$\square$' both, the position of that element is a saddle point of the payoff matrix.

Optimal strategy -

The course of action which maximizes the profit of a player or minimizes his loss is called an optimal strategy.

If the payoff matrix $\{v_{ij}\}$ has the saddle point (r,s) then the players (A and B) are said to have r^{th} and s^{th} optimal strategies, respectively.

Like & Subscribe

Value of Game -

The payoff (V_{rs}) at the saddle point (r, s) is called the value of game and it is obviously equal to the maximin ($\underline{V}$) and minimax value ($\overline{V}$) of the game.

A game is said to be a fair game if $\overline{V} = \underline{V} = 0$.

A game is said to be strictly determinable if $\overline{V} = V = \underline{V}$

Like & Subscribe

to the player A. Find the optimal strategy, if any

		Player B		
		I	II	III
Player A	I	-3	-2	6
	II	2	0	2

A → maximising
B → minimising

Sol^n:-

		Player B			
		I	II	III	Row minimum
	I	-3	-2	$\boxed{6}$	-3
Player A	II	2	$\boxed{0}$	2	0 maximin value ($\underline{V}$)
	III	$\boxed{5}$	-2	-4	-4
column maximum		5	0	6	

Like & Subscribe

Eg (2)

Player B

		I	II	III	Row minimum
Player A	I	3	-4	$\boxed{8}$	-4 $(\underline{v})$ maximin value
	II	-8	$\boxed{5}$	-6	-8
	III	$\boxed{6}$	-7	6	-7
Column maximum		6	$\boxed{5}$	8	
			minimax value $(\bar{v})$		

Such game are said to be the games without saddle point.

Like & Subscribe

(i) Analytical method -

A 2×2 payoff matrix where there is no saddle point can be solved by Analytical method.

Given the matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Value of the game is

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

with the coordinates

$$x_1 = \frac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}, \quad x_2 = \frac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

Like & Subscribe

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

value of the game is

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

with the coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}, \quad x_2 = \frac{a_{11} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}, \quad y_2 = \frac{a_{11} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

Like & Subscribe

70%

Q: solve the payoff matrix.

$$\begin{array}{cc} & \text{player B} \\ \text{player A} & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

Sol^n :-

Like & Subscribe

Sol^n:- value of the game

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{5\times 4 - 3\times 1}{(5+4) - (1+3)} = \frac{17}{5}$$

with coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{4-3}{5} = \frac{1}{5}$$

$$x_2 = \frac{a_{11} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{5-1}{5} = \frac{4}{5}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{4-1}{5} = \frac{3}{5}$$

$$y_2 = \frac{a_{11} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{5-3}{5} = \frac{2}{5}$$

Like & Subscribe

$\therefore$ value of ...

$$V = \frac{a_{11} a_{22} - a_{21} a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{\quad}{(5+4) - (1+3)}$$

with coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{4-3}{5} = \frac{1}{5}$$

$$x_2 = \frac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{5-1}{5} = \frac{4}{5}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{4-1}{5} = \frac{3}{5}$$

$$y_2 = \frac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{5-3}{5} = \frac{2}{5}$$

optimal strategy of player $A = (1/5, 4/5)$

$B = (3/5, 2/5)$

Like & Subscribe

~~Alter~~ Alternate procedure

Qu:

$$
\begin{array}{c c c c}
 & \text{Player B} & & \\
\text{player A} & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & \begin{matrix} 1 \\ 4 \end{matrix} & \begin{matrix} 1/5 \\ 4/5 \end{matrix} \\
 & \begin{matrix} 3 & 2 \end{matrix} & & \\
 & \begin{matrix} 3/5 & 2/5 \end{matrix} & &
\end{array}
$$

value of game

$$
\begin{aligned}
V &= 5 \times \frac{1}{5} + 3 \times \frac{4}{5} \\
&= 1 + \frac{12}{5} \\
&= \frac{17}{5}
\end{aligned}
$$

or

$$
\begin{aligned}
&= 5 \times \frac{3}{5} + 1 \times \frac{2}{5} \\
&= 3 + \frac{2}{5} \\
&= \frac{17}{5}
\end{aligned}
$$

Like & Subscribe

$3 \quad 2$

$3/5 \quad 2/5$

value of game

$$V = 5\times\frac{1}{5} + 3\times\frac{4}{5} = 1+\frac{12}{5} = \frac{17}{5}$$

or

$$= 5\times\frac{3}{5} + 1\times\frac{2}{5} = 3+\frac{2}{5} = \frac{17}{5}$$

optimal strategy of player $A = \left(\frac{1}{5}, \frac{4}{5}\right)$

$B = \left(\frac{3}{5}, \frac{2}{5}\right)$

Like & Subscribe

Graphical method -

The graphical method is used to solve the games whose payoff matrix has

(1) Two rows and n columns $(2 \times n)$

(2) m rows and 2 columns $(m \times 2)$

(I) Algorithm for solving $2 \times n$ matrix games -

① Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$,

② Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second row in the payoff matrix on the vertical lines $x_1 = 0$

③ The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for $j = 1, 2, \cdots n$ and determine the highest point of the lower envelope obtained. This

Like & Subscribe

(2) m rows and 2 columns $(m \times 2)$

① Algorithm for solving $2 \times n$ matrix games -

① Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$

② Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second row in the payoff matrix on the vertical lines $x_1 = 0$

③ The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straigh lines for $j = 1, 2, \cdots n$ and determine the highest point of the lower envelope obtained. This will be the maximin point.

④ The two or more lines passing through the maximin point determines the required 2×2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Like & Subscribe

Eg: solve by graphical method

$$\begin{array}{cc} & \text{player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{player A} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix} \end{array}$$

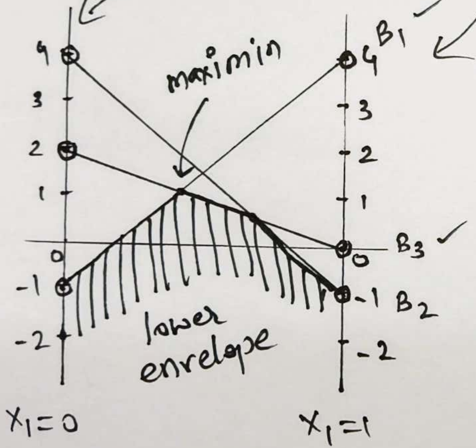
$2 \times n$

2×3

$$\longrightarrow \quad \begin{array}{cc} & \begin{array}{cc} B_1 & B_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \end{array}$$

2×2

Sol^n^:-



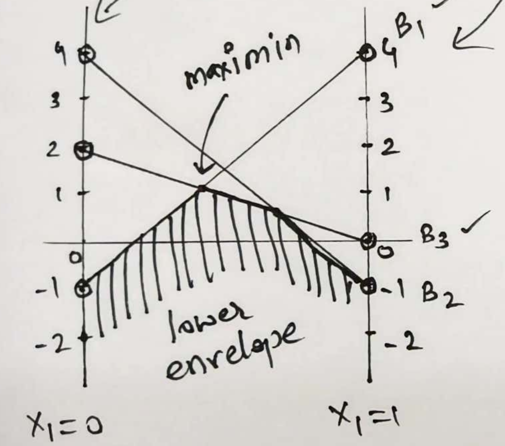
player B

$2 \times n$

2×3

$$
\text{player A}\quad
\begin{array}{c}
 \\ A_1 \\ A_2
\end{array}
\begin{array}{ccc}
B_1 & B_2 & B_3 \\
\end{array}
$$

$$
\begin{array}{c|ccc}
 & B_1 & B_2 & B_3 \\
\hline
A_1 & 4 & -1 & 0 \\
A_2 & -1 & 4 & 2
\end{array}
\longrightarrow
\begin{array}{c|cc|cc}
 & B_1 & B_3 & & \\
\hline
A_1 & 4 & 0 & 3 & 3/7 \\
A_2 & -1 & 2 & 4 & 4/7 \\
\hline
 & 2 & 5 & & \\
 & 2/7 & 5/7 & &
\end{array}
$$



value of the game

$$V = 4 \times \frac{3}{7} + (-1) \times \frac{4}{7}$$

$$= \frac{12}{7} - \frac{4}{7}$$

$$= 8/7$$

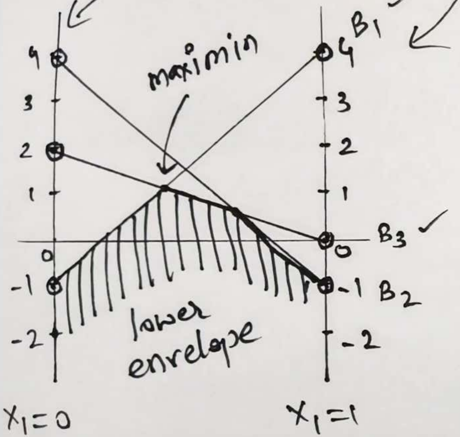
optimal strategy of player A =

yer A $\begin{array}{c} A_1 \\ A_2 \end{array} \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}$

$\rightarrow \quad A_2 \begin{bmatrix} -1 & 2 \end{bmatrix} \quad 4 \quad 4/7$

$2 \quad 5$

$2/7 \quad 5/7$



value of the game

$$v = 4 \times \frac{3}{7} + (-1) \times \frac{4}{7}$$

$$= \frac{12}{7} - \frac{4}{7}$$

$$= 8/7$$

optimal strategy of player A $= \left(3/7, \frac{4}{7}\right)$

B $= (2/7, 0, 5/7)$

(2) Algorithm for solving $m \times 2$ matrix games-

(1) Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$, $x_1 = 1$

(2) Take the points of the first column in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second column in the payoff matrix on the vertical line $x_1 = 0$

(3) The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for $j = 1, 2, \cdots n$ and determine the lowest point of the upper envelope obtained. This will be the minimax point.

(4) The two or more lines passing through the minimax point determines the required 2×2 payoff matrix

(1) Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$, $x_1 = 1$

(2) Take the points of the first column in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second column in the payoff matrix on the vertical line $x_1 = 0$

(3) The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for $j = 1, 2, \cdots n$ and determine the lowest point of the upper envelope obtained. This will be the minimax point.

(4) The two or more lines passing through the minimax point determines the required 2x2 payoff matrix. This in turn gives the optimum solution by

$x_1 = 0$, $x_1 = 1$

(2) Take the points of the first column in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second column in the payoff matrix on the vertical line $x_1 = 0$

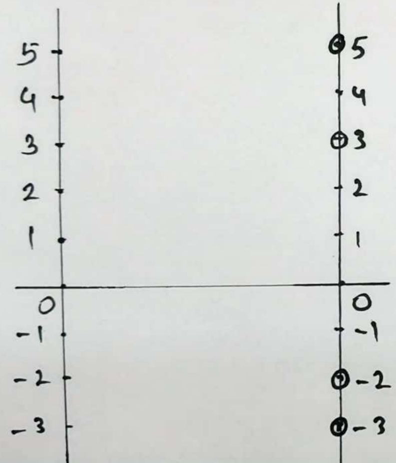
(3) The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for $j = 1, 2, \cdots n$ and determine the lowest point of the upper envelope obtained. This will be the minimax point.

(4) The two or more lines passing through the minimax point determines the required 2×2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Like & Subscribe

$$\text{player A} \quad \begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -2 & 0 \\ A_2 & 3 & -1 \\ A_3 & -3 & 2 \\ A_4 & 5 & -4 \end{array}$$

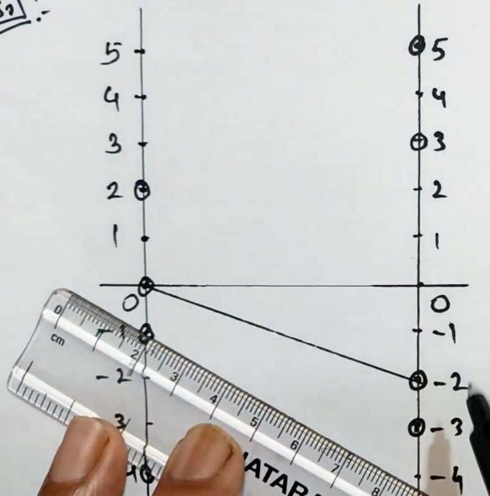
Solⁿ :-



Like & Subscribe

$$
\text{player } A \quad \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix}
$$

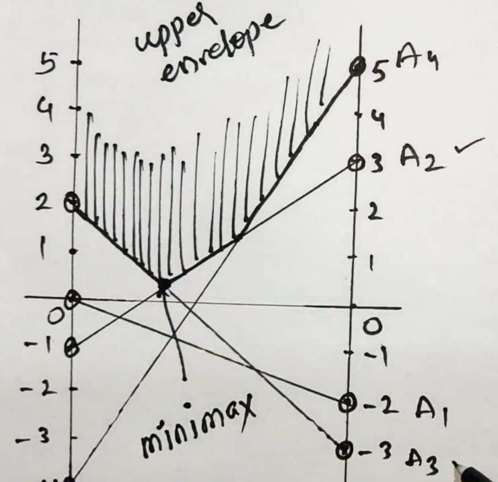
Sol^n:-



Like & Subscribe

$$
\begin{array}{cc}
 & \begin{array}{cc} B_1 & B_2 \end{array} \\
\text{player } A \quad \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix}
\end{array}
$$

Sol^n :-



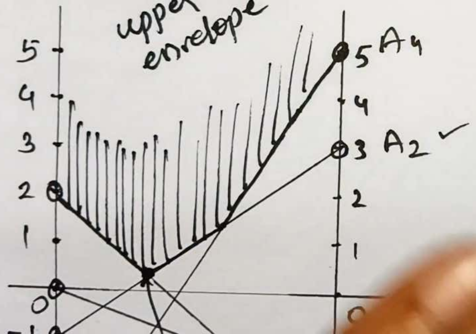
Like & Subscribe

Eg:- solve by graphical method

$m \times 2$

$$\begin{array}{c|c} & \text{player B} \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix} \end{array} \longrightarrow \begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_2 \\ A_3 \end{array} & \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix} \end{array}$$

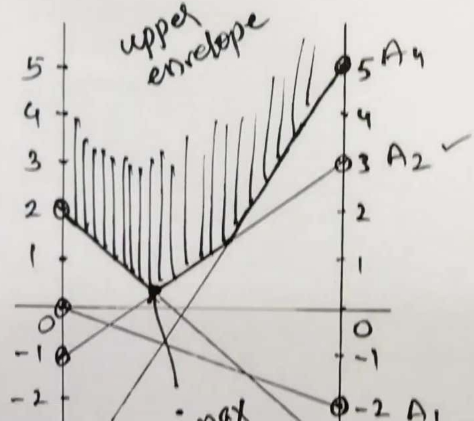
Solⁿ:-



$m \times 2$

$$\begin{array}{c c} & \text{player B} \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix} \end{array} \longrightarrow \begin{array}{c c c c} & \begin{array}{cc} B_1 & B_2 \end{array} & & \\ \begin{array}{c} A_2 \\ A_3 \end{array} & \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix} & \begin{array}{c} 5 \\ 4 \end{array} & \begin{array}{c} 5/9 \\ 4/9 \end{array} \\ & \begin{array}{cc} 3 & 6 \end{array} & & \\ & \begin{array}{cc} 3/9 & 6/9 \end{array} & & \end{array}$$

Sol^n:-



value of the game

$$V = \cancel{3} \times \frac{5}{\cancel{9}_3} + (-\cancel{3}) \times \frac{4}{\cancel{9}_3}$$

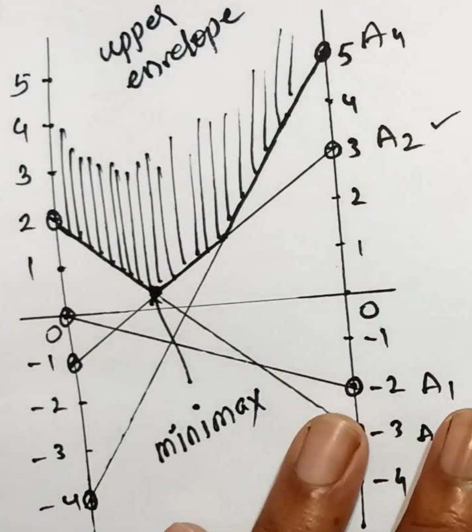
$$= \frac{5}{3} - \frac{4}{3}$$

$$= 1/3$$

$$\text{player A}\quad \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} -2 & \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix} \longrightarrow A_3 \; [\,-3 \quad\quad]$$

3 6

3/9 6/9



value of the game

$$V = \cancel{3} \times \frac{5}{\cancel{9}_3} + (-\cancel{3}) \times \frac{4}{\cancel{9}_3}$$

$$= \frac{5}{3} - \frac{4}{3}$$

$$= 1/3$$

optimal strategy of player A

$= (0, 5/9, 4/9, 0)$

player B

$= (3/9, 6/9)$

Like & Subscribe

Game Theory

Simplex method - [m×n games]

Sandeep Kumar Gour

LPP

Qu:- Solve by Simplex method

$$A \begin{array}{c} B \\ \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} \end{array}$$

Sol^n: Given matrix

$$A \begin{array}{cc} B & \\ \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} & \begin{array}{c} -2 \\ -1 \\ 2 \end{array} \\ \begin{array}{ccc} 3 & 4 & 6 \end{array} & \end{array}$$

maximin

minimax

$V > 0$

$\boxed{2 - 3}$

Like & Subscribe

$$A \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

Sandeep Kumar Gour

Sol^n: Given matrix

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} \begin{array}{c} -2 \\ -1 \\ 2 \end{array} \\ & \begin{array}{ccc} 3 & 4 & 6 \end{array} \end{array}$$

maximin

minimax

$v < 0$
$v = 0$ → constant

$v > 0$

$\boxed{2 - 3}$

We can infer that $2 \le v \le 3$. Hence it can be concluded that the value of the game lies between 2 and 3 and $v > 0$

Like & Subscribe

Simplex method - [m×n games]

Qu:- Solve by simplex method

$$A \quad \overset{B}{\begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}}$$

$V \to min$

$\frac{1}{V} \to max$

$\frac{x_1}{V} + \frac{x_2}{V} + \frac{x_3}{V} = \frac{1}{V}$

$\boxed{X_1 + X_2 + X_3 = \frac{1}{V}}$

$\frac{y_1}{V} + \frac{y_2}{V} + \frac{y_3}{V} = \frac{1}{V}$

$\boxed{Y_1 + Y_2 + Y_3 = \frac{1}{V}}$

Sol^n: Given matrix

$$A \quad \begin{array}{c|ccc|c} & y_1 & y_2 & y_3 & \\ \hline x_1 & 3 & -2 & 4 & -2 \\ x_2 & -1 & 4 & 2 & -1 \\ x_3 & 2 & 2 & 6 & 2 \\ \hline & 3 & 4 & 6 & \end{array}$$

B

gain

maximin

minimax

$V < 0$

$V = 0$

$V > 0$

$\boxed{2 - 3}$

const

We can infer that $2 \le V \le 3$. Hence it can gain

$$\begin{array}{c} x_2 \\ x_3 \end{array}\begin{bmatrix} -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix} \begin{array}{c} \\ 2 \end{array} \text{ maximin}$$

$$\begin{array}{ccc} 3 & 4 & 6 \end{array}$$

minimax

$v = 0$ → constant

$v > 0$

$\boxed{2 - 3}$

We can infer that $2 \le v \le 3$. Hence it can be concluded that the value of the game lies between 2 and 3 and $v > 0$

LPP:

$$\max \frac{1}{v} = Y_1 + Y_2 + Y_3$$

Subject to constraints

$$3Y_1 - 2Y_2 + 4Y_3 \le 1$$

$$3\frac{y_1}{v} - 2\frac{y_2}{v} + 4\frac{y_3}{v} \le \frac{v}{v}$$

Like & Subscribe

minimax

Constant

We can infer that $2 \leq v \leq 3$. Hence it can be concluded that the value of the game lies between 2 and 3 and $v > 0$

LPP:

$$\max \frac{1}{v} = Y_1 + Y_2 + Y_3$$

Subject to constraints

$$3Y_1 - 2Y_2 + 4Y_3 \leq 1$$
$$-1Y_1 + 4Y_2 + 2Y_3 \leq 1$$
$$2Y_1 + 2Y_2 + 6Y_3 \leq 1$$

and $Y_1, Y_2, Y_3 \geq 0$

$$3\frac{y_1}{v} - 2\frac{y_2}{v} + 4\frac{y_3}{v} \leq \frac{v}{v}$$

Like & Subscribe

LPP: $\max \frac{1}{v} = y_1 + y_2 + y_3$ | $3y_1 - 2y_2 + 4y_3 \leq v$

SLPP: $\max \frac{1}{v} = y_1 + y_2 + y_3 + 0S_1 + 0S_2 + 0S_3$

s.t.

$$3y_1 - 2y_2 + 4y_3 + S_1 = 1$$

$$-1y_1 + 4y_2 + 2y_3 + S_2 = 1$$

$$2y_1 + 2y_2 + 6y_3 + S_3 = 1$$

and $y_1, y_2, y_3, S_1, S_2, S_3 \geq 0$

Like & Subscribe

s.t.

$$3y_1 - 2y_2 + 4y_3 + s_1 = 1$$

$$-1y_1 + 4y_2 + 2y_3 + s_2 = 1$$

$$2y_1 + 2y_2 + 6y_3 + s_3 = 1$$

and $y_1, y_2, y_3, s_1, s_2, s_3 \geq 0$

Now the following simplex table is formed

		c_j	1	1	1	0	0	0	
Basic variable	C_B	Y_B	y_1	y_2	y_3	s_1	s_2	s_3	Min Ratio
← s_1	0	1	3	-2	4	1	0	0	1/3 ←
s_2	0	1	-1	4	2	0	1	0	–
s_3	0	1	2	2	6	0	0	1	1/2
$\frac{1}{v} = 0$		Δ_j	-1 ↑	-1	-1	0	0	0	
y_1	1	1/3	1	-2/3	4/3	1/3	0	0	–

Basic variable	C_B	Y_B	y_1	y_2	y_3	s_1	s_2	s_3	Min Ratio
← s_1	0	1	3	-2	4	1	0	0	$1/3$ ←
s_2	0	1	-1	4	2	0	1	0	–
s_3	0	1	2	2	6	0	0	1	$1/2$
$\frac{1}{v}=0$		Δ_j	-1 ↑	-1	-1	0	0	0	
y_1	1	$1/3$	1	$-2/3$	$4/3$	$1/3$	0	0	–
s_2	0	$4/3$	0	$10/3$	$10/3$	$1/3$	1	0	$2/5$
← s_3	0	$1/3$	0	$10/3$	$10/3$	$-2/3$	0	1	$1/10$ ←
$\frac{1}{v}=\frac{1}{3}$		Δ_j	0	$-5/3$ ↑	$1/3$	$1/3$	0	0	
y_1	1	$2/5$	1	0	2	$1/5$	0	$1/5$	
s_2	0	1	0	0	0	1	1	-1	
y_2	1	$1/10$	0	1	1	$-1/5$	0	$3/10$	
$\frac{1}{v}=\frac{1}{2}$		Δ_j	0	0	2	0	0	$1/2$	

← S_3	0	1/3	0	10/3	10/3	-2/3	0	1	1/10 ←
$\frac{1}{V}=\frac{1}{3}$	Δ_j	0	-5/3 ↑	1/3	1/3	0	0		
Y_1	1	2/5	1	0	2	1/5	0	1/5	
S_2	0	1	0	0	0	1	1	-1	
Y_2	1	1/10	0	1	1	-1/5	0	3/10	
$\frac{1}{V}=\frac{1}{2}$	Δ_j	0	0	2	0	0	1/2		

Thus, the solution for B's original problem is obtained as

$$y_1 = Y_1 \times V = \frac{2}{5} \times 2 = \frac{4}{5}$$

$$y_2 = Y_2 \times V = \frac{1}{10} \times 2 = \frac{1}{5}$$

$$y_3 = Y_3 \times V = 0 \times 2 = 0$$

$$Y_1 = \frac{y_1}{V}$$

$$y_1 = Y_1 \times V$$

$$\frac{1}{V} = \frac{1}{2}$$

i.e. $\boxed{V=2}$

Like & Subscribe

$y_3 = Y_3 \times v = 0 \times 2 = 0$

$\frac{1}{v} = \frac{1}{2}$

i,e $\boxed{v=2}$

The optimal strategies for the player A are obtained from the final table of the above problem. This is given by duality rules.

$X_1 = \frac{x_1}{V}$

$x_1 = X_1 \times V$

$x_1 = X_1 \times v = 0 \times 2 = 0$

$x_2 = X_2 \times v = 0 \times 2 = 0$

$x_3 = X_3 \times v = \frac{1}{2} \times 2 = 1$

Like & Subscribe

The optimal strategies for the player A are obtained from the final table of the above problem. This is given by duality rules.

$$X_1 = \frac{x_1}{V}$$

$$x_1 = X_1 \times V$$

$$x_1 = X_1 \times V = 0 \times 2 = 0$$

$$x_2 = X_2 \times V = 0 \times 2 = 0$$

$$x_3 = X_3 \times V = \frac{1}{2} \times 2 = 1$$

Therefore

optimal strategy of player $A = (0, 0, 1)$

$B = (\frac{4}{5}, \frac{1}{5}, 0)$

Like & Subscribe

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \end{array}$$

$\boxed{v=0}$ $v>0$

$-1 \quad +1$

$\underline{\text{Sol}^n}$:- Given matrix

$$\begin{array}{ccc} & B & \\ A & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} & \begin{matrix} -1 \\ -1 \\ -1 \end{matrix} \\ & \begin{matrix} 1 & 2 & 3 \end{matrix} & \end{array}$$

minimax

maximin $= -1$

we can infer that $-1 \leq v \leq 1$

since maximin value is -1, it is possible that value of the game may be negative or

$[-1 \quad 2 \quad -1]$

$-1 \quad +1$

Sol^n:- Given matrix

$$
\begin{array}{cc}
 & B \\
A & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}
\end{array}
\begin{array}{c} -1 \\ -1 \\ -1 \end{array}
\qquad \text{maximin} = -1
$$

$$1 \quad 2 \quad 3$$

minimax

we can infer that $-1 \leq v \leq 1$

since maximin value is -1, it is possible that value of the game may be negative or zero, thus the constant c

$-1 \quad +1$

$\underline{\text{Sol}^n}$:- Given matrix

$$
\begin{array}{c|ccc|c}
 & & B & & \\
A & 1 & -1 & -1 & -1 \\
 & -1 & -1 & 3 & -1 \\
 & -1 & 2 & -1 & -1 \\
\hline
 & 1 & 2 & 3 &
\end{array}
$$

maximin $= -1$

minimax

we can infer that $-1 \leq v \leq 1$

since maximin value is -1, it is possible that value of the game may be negative or zero, thus the constant c is added to all the elements of matrix which is at least equal to the negative of maximin

$$A \begin{array}{c} \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{matrix} -1 \\ -1 \\ -1 \end{matrix} \\ \begin{matrix} 1 & 2 & 3 \end{matrix} \end{array}$$

maximin $= -\underline{1}$

minimax

$c = 1, 2, \ldots$

we can infer that $-1 \leq v \leq 1$

since maximin value is -1, it is possible that value of the game may be negative or zero, thus the constant 'c' is added to all the elements of matrix which is at least equal to the negative of maximin.

let $c = 1$, add this value to all the elements of the matrix.

Th

$$A \begin{bmatrix} -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$V = 0$

$-1 \quad +1$

Sol^n:- Given matrix

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \end{array} \begin{array}{c} -1 \\ -1 \\ -1 \end{array}$$

$$\begin{array}{ccc} 1 & 2 & 3 \end{array}$$

minimax

maximin $= -1$

$e = 1, 2, \ldots$

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 3 & 0 \end{bmatrix} \end{array}$$

Like & Subscribe

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 3 & 0 \end{bmatrix} \end{array}$$

$$Y_1 = \frac{y_1}{v}$$

LPP –

$$\text{Max } \frac{1}{v} = Y_1 + Y_2 + Y_3$$

subject to constraints

$$2Y_1 + 0Y_2 + 0Y_3 \leq 1$$

$$0Y_1 + 0Y_2 + 4Y_3 \leq 1$$

$$0Y_1 + 3Y_2 + 0Y_3 \leq 1$$

and $Y_1, Y_2, Y_3 \geq 0$

Like & Subscribe

Basic variable	C_B	Y_B	Y_1	Y_2	Y_3	S_1	S_2	S_3	Min Ratio Y_B/Y_K
		c_j	1	1	1	0	0	0	
← S_1	0	1	2	0	0	1	0	0	1/2 ←
S_2	0	1	0	0	4	0	1	0	–
S_3	0	1	0	3	0	0	0	1	–
	$\frac{1}{v}=0$	Δ_j	-1 ↑	-1	-1	0	0	0	
Y_1	1	1/2	1	0	0	1/2	0	0	–
S_2	0	1	0	0	4	0	1	0	–
← S_3	0	1	0	3	0	0	0	1	1/3 ←
	$\frac{1}{v}=1/2$	Δ_j	0	-1	-1	1/2	0	0	
Y_1	1	1/2	1	0	0	1/2	0	0	–
← S_2	0	1	0	0	4	0	1	0	1/4 ←
Y_2	1	1/3	0	1	0	0	0	1/3	–
	$\frac{1}{v}=5/6$	Δ_j	0	0	-1	1/2	0	1/3	
Y_1	1	1/2	1	0	0	1/2	0	0	

Like & Subscribe

S_3	0	1	0	3	0	0	0	1	–
	$\frac{1}{v} = 0$	Δ_j	$-1 \uparrow$	-1	-1	0	0	0	
Y_1	1	1/2	1	0	0	1/2	0	0	–
S_2	0	1	0	0	4	0	1	0	–
← S_3	0	1	0	3	0	0	0	1	1/3 ←
	$\frac{1}{v} = 1/2$	Δ_j	0	-1	-1	1/2	0	0	
Y_1	1	1/2	1	0	0	1/2	0	0	–
← S_2	0	1	0	0	4	0	1	0	1/4 ←
Y_2	1	1/3	0	1	0	0	0	1/3	–
	$\frac{1}{v} = 5/6$	Δ_j	0	0	-1	1/2	0	1/3	
Y_1	1	1/2	1	0	0	1/2	0	0	
Y_3	1	1/4	0	0	1	0	1/4	0	
Y_2	1	1/3	0	1	0	0	0	1/3	
	$\frac{1}{v} = \frac{13}{12}$	Δ_j	0	0	0	1/2	1/4	1/3	

Like & Subscribe

Y_1	1	1/2	1	0	0	1/2	0	0	
← S_2	0	1	0	0	4	0	1	0	1/4 ←
Y_2	1	1/3	0	1	0	0	0	1/3	–
	$\frac{1}{V} = 5/6$	Δ_j	0	0	-1	1/2	0	1/3	
Y_1	1	1/2	1	0	0	1/2	0	0	
Y_3	1	1/4	0	0	1	0	1/4	0	
Y_2	1	1/3	0	1	0	0	0	1/3	
	$\frac{1}{V} = \frac{13}{12}$	Δ_j	0	0	0	1/2	1/4	1/3	
						Δ_4	Δ_5	Δ_6	

Here $\frac{1}{V} = \frac{13}{12}$

i.e $V = 12/13$

$y_1 = Y_1 \times V = \frac{1}{2} \times \frac{12}{13} = \frac{6}{13}$

$y_2 = Y_2 \times V = \frac{1}{3} \times \frac{12}{13} = \frac{4}{13}$

$y_3 = Y_3 \times V = \frac{1}{4} \times \frac{12}{13} = \frac{3}{13}$

$x_1 = X_1 \times V = \frac{1}{2} \times \frac{12}{13} = \frac{6}{13}$

$x_2 = X_2 \times V = \frac{1}{4} \times \frac{12}{13} = \frac{3}{13}$

$x_3 = X_3 \times V = \frac{1}{3} \times \frac{12}{13} = \frac{4}{13}$

Like & Subscribe

Y_3	1	$1/4$	0	0	1	0	$1/4$	0
Y_2	1	$1/3$	0	1	0	0	0	$1/3$
	$\frac{1}{V} = \frac{13}{12}$	Δ_j	0	0	0	$1/2$	$1/4$	$1/3$
						Δ_4	Δ_5	Δ_6

Here $\frac{1}{V} = \frac{13}{12}$

i/e $V = 12/13$

$y_1 = Y_1 \times V = \frac{1}{2} \times \frac{12}{13} = \frac{6}{13}$

$y_2 = Y_2 \times V = \frac{1}{3} \times \frac{12}{13} = \frac{4}{13}$

$x_1 = X_1 \times V = \frac{1}{2} \times \frac{12}{13} = \frac{6}{13}$

$x_2 = X_2 \times V = \frac{1}{4} \times \frac{12}{13} = \frac{3}{13}$

optimal strategy of player $A = (6/13, 3/13, 4/13)$

$B = (6/13, 4/13, 3/13)$

value of game $= V - C = \frac{12}{13} - 1$

$= -1/13$ **Ans.**

Like & Subscribe