Operations Research –

Operations Research is the systematic, method-oriented study of the basis structure, characteristics, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making.

Linear Programming Problem (LPP) –

A linear form is meant a mathematical expression of the type $a_1x_1 + a_2x_2 + \cdots\cdots + a_nx_n$, where $a_1, a_2, \cdots a_n$ are constants and $x_1, x_2, \cdots, x_n$ are variables. The term 'programming' refers to the process of determining a particular programme or plan of action.

Linear programming deals with the optimization (Maximization/Minimization) of a function of variables known as objective function, subject to a set of linear equations and/or inequalities known as constraints or restrictions.

General Form of Linear Programming Problem (LPP) –

Definition:– The general LPP calls for optimizing (Maximizing or minimizing) a linear function of variables called the **'objective function'** subject to a set of linear equations and/or inequalities called the **'constraints'** or **'Restrictions'**.

General Form:

Objective Function (Max or Min) $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to contraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq \text{ or } \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq \text{ or } \geq)\ b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq \text{ or } \geq)\ b_m$$

and $x_1, x_2, \ldots x_n \geq 0$

Formulation of LP problems –

Eg: A firm manufactures two types of products A and B sells them at a profit of Rs.2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as LPP.

Solⁿ: x_1 be the number of products of type A

x_2 be the number of products of type B

	Type of products (minutes)		
Machine	Type A (x_1 units)	Type B (x_2 units)	Available time (minutes)
G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	

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G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	
	$2x_1$	$3x_2$	

$2x_1 + 3x_2$ (objective Function)

M

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profit per unit	Rs. 2	Rs. 3	
	$2x_1$	$3x_2$	

$$\text{Max } Z = 2x_1 + 3x_2 \quad \text{(objective Function)}$$

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	Type of products (minutes)		
Machine	Type A (x_1 units)	Type B (x_2 units)	Available time (minutes)
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H	2	1	600
profit per unit	Rs. 2	Rs. 3	

$2x_1$ $3x_2$

$$\text{Max } Z = 2x_1 + 3x_2 \quad \text{(objective Function)}$$

$$\left.\begin{aligned} x_1 + x_2 &\le 400 \\ 2x_1 + x_2 &\le 600 \end{aligned}\right\} \text{constraints}$$

non-negative constraints are

$$x_1 \ge 0, \quad x_2 \ge 0$$

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profit per unit	Rs. 2	Rs. 3	

$2x_1$ $3x_2$

$$\text{Max } Z = 2x_1 + 3x_2 \quad \text{(objective Function)}$$

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LPP

$$\text{Max } Z = 2x_1 + 3x_2$$

subject to constraints

$$x_1 + x_2 \le 400$$
$$2x_1 + x_2 \le 600$$

and $x_1, x_2 \ge 0$

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Graphical Solution of LP problems :-

Q. Max $Z = 3x_1 + 4x_2$

subject to constraints

$$x_1 + x_2 \leq 450 \quad \text{---(1)}$$

$$2x_1 + x_2 \leq 600 \quad \text{---(2)}$$

and $x_1, x_2 \geq 0$

Solⁿ. from constraints (1)

$$x_1 + x_2 = 450$$

put $x_1 = 0 \Rightarrow 0 + x_2 = 450$

$\Rightarrow x_2 = 450$

put $x_2 = 0 \Rightarrow x_1 = 450$

from constraints (2)

$$2x_1 + x_2 = 600$$

put $x_1 = 0 \Rightarrow 0 + x_2 = 600$

$\Rightarrow x_2 = 600$

put $x_2 = 0 \Rightarrow 2x_1 = 600$

$\Rightarrow x_1 = 300$

... solution of LP problems :-

Q. Max $Z = 3x_1 + 4x_2$

Subject to constraints

$x_1 + x_2 \leq 450$ —(1)

$2x_1 + x_2 \leq 600$ —(2)

and $x_1, x_2 \geq 0$

Solⁿ: from constraints (1)

$x_1 + x_2 = 450$

put $x_1 = 0 \Rightarrow 0 + x_2 = 450$
$\Rightarrow x_2 = 450$

put $x_2 = 0 \Rightarrow x_1 = 450$

from constraints (2)

$2x_1 + x_2 = 600$

put $x_1 = 0 \Rightarrow 0 + x_2 = 600$
$\Rightarrow x_2 = 600$

put $x_2 = 0 \Rightarrow 2x_1 = 600$
$\Rightarrow x_1 = 300$

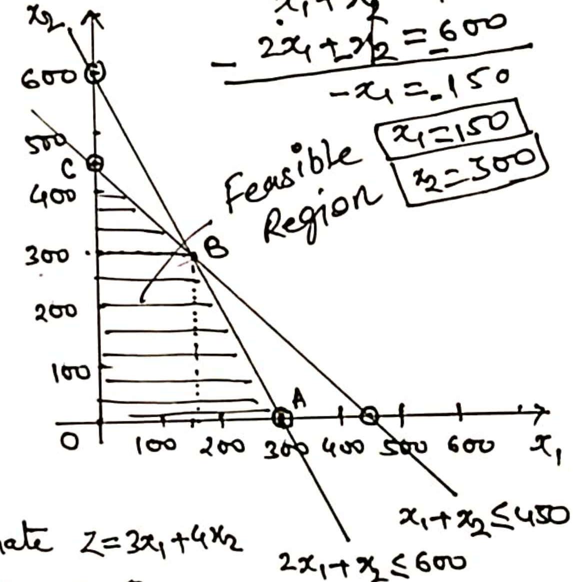
constraints (1)

x_1	x_2
0	450
450	0

constraints (2)

x_1	x_2
0	600
300	0

$$\begin{aligned} x_1 + x_2 &= 450 \\ -\; 2x_1 + x_2 &= 600 \\ \hline -x_1 &= -150 \end{aligned}$$



points	coordinate	$Z = 3x_1 + 4x_2$
O	(0, 0)	= 0
A	(300, 0)	= 900
B	(150, 300)	= 1650
C	(0, 450)	= 1800

optimal solution <u>Max $Z = 1800$</u> at $x_1 = 0$
$x_2 = 450$

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Graphical Solution using Objective Function Line :-

$$\text{Min } Z = 20x_1 + 10x_2$$

Subject to constraints

$$x_1 + 2x_2 \leq 40 \quad \text{---} (1)$$
$$3x_1 + x_2 \geq 30 \quad \text{---} (2)$$
$$4x_1 + 3x_2 \geq 60 \quad \text{---} (3)$$

and $x_1, x_2 \geq 0$

Sol^n: The first constraint written in form of equation

$$x_1 + 2x_2 = 40$$

put $x_1 = 0 \Rightarrow 0 + 2x_2 = 40$
$\Rightarrow x_2 = 20$

put $x_2 = 0 \Rightarrow x_1 + 0 = 40$
$\Rightarrow x_1 = 40$

The second constraint written in form of equation

$$3x_1 + x_2 = 30$$

put $x_1 = 0 \Rightarrow 0 + x_2 = 30$
$\Rightarrow x_2 = 30$

put $x_2 = 0 \Rightarrow 3x_1 + 0 = 30$
$\Rightarrow x_1 = 10$

The third constraints written in form of equation

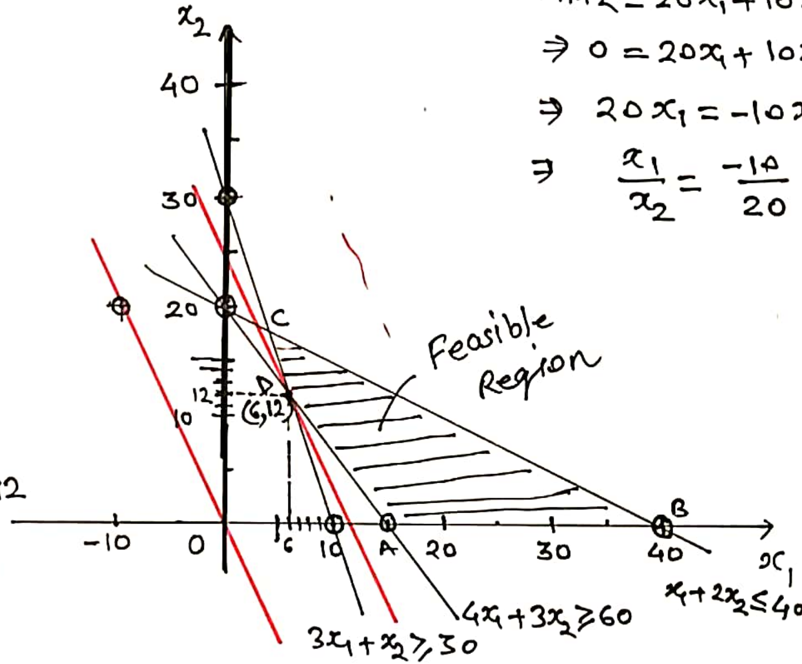
$$4x_1 + 3x_2 = 60$$

put $x_1 = 0 \Rightarrow 0 + 3x_2 = 60$
$\Rightarrow x_2 = 20$

put $x_2 = 0 \Rightarrow 4x_1 + 0 = 60$
$\Rightarrow x_1 = 15$

Constraints	x_1	x_2
1. $x_1+2x_2 \leq 40$	0	20
	40	0
2. $3x_1+x_2 \geq 30$	0	30
	10	0
3. $4x_1+3x_2 \geq 60$	0	20
	15	0

$$\text{Min } Z = 20x_1 + 10x_2$$
$$\Rightarrow 0 = 20x_1 + 10x_2$$
$$\Rightarrow 20x_1 = -10x_2$$
$$\Rightarrow \frac{x_1}{x_2} = \frac{-10}{20}$$



optimal solution

$$\text{Min } Z = 20 \times 6 + 10 \times 12$$
$$= 120 + 120$$
$$= \underline{\underline{240}}$$

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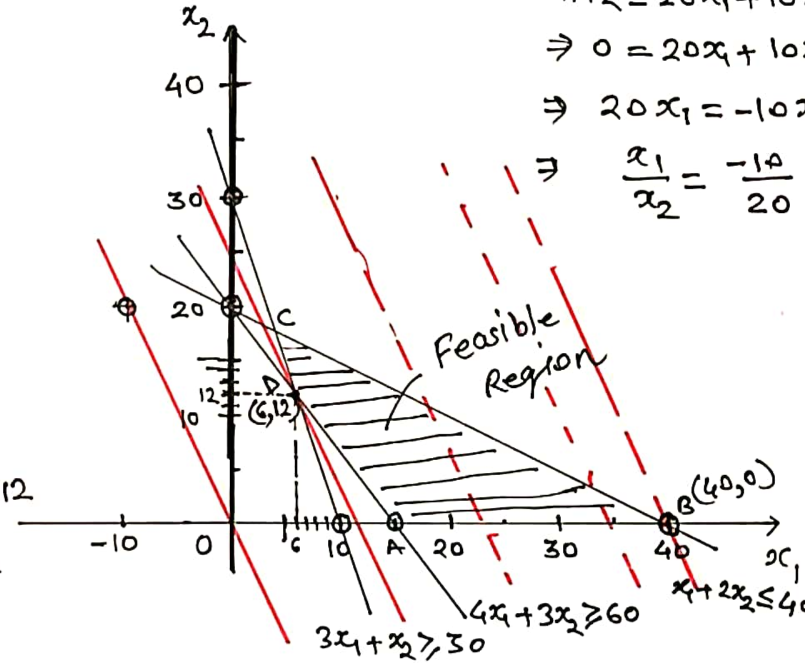
Constraints	x_1	x_2
1. $x_1+2x_2\le 40$	0	20
	40	0
2. $3x_1+x_2\geq 30$	0	30
	10	0
3. $4x_1+3x_2\geq 60$	0	20
	15	0

$\text{Min } Z = 20x_1+10x_2$

$\Rightarrow 0 = 20x_1+10x_2$

$\Rightarrow 20x_1 = -10x_2$

$\Rightarrow \dfrac{x_1}{x_2} = \dfrac{-10}{20}$



optimal solution

$\text{Min } Z = 20\times 6+10\times 12$

$= 120+120$

$= 240$

$\text{Max } Z = 20\times 40+10\times 0$

$= 800$

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Multiple Optimal Solution in Graphical Method -

Max $Z = 4x_1 + 3x_2$

subject to constraints

$$4x_1 + 3x_2 \leq 24 \quad \text{--- (1)}$$

$$x_1 \leq 4.5 \quad \text{--- (2)}$$

$$x_2 \leq 6 \quad \text{--- (3)}$$

and $x_1, x_2 \geq 0$

Solⁿ: The first constraint written in a form of equation

$$4x_1 + 3x_2 = 24$$

put $x_1 = 0 \Rightarrow 0 + 3x_2 = 24$

$\Rightarrow x_2 = 8$

put $x_2 = 0 \Rightarrow 4x_1 + 0 = 24$

$\Rightarrow x_1 = 6$

The second constraint written in a form of equation

$x_1 = 4.5$, $x_2 = 0$

The third constraint written in a form of equation

$x_2 = 6$, $x_1 = 0$

constraints	x_1	x_2
1. $4x_1 + 3x_2 \leq 24$	0	8
	6	0
2. $x_1 \leq 4.5$	4.5	0
3. $x_2 \leq 6$	0	6

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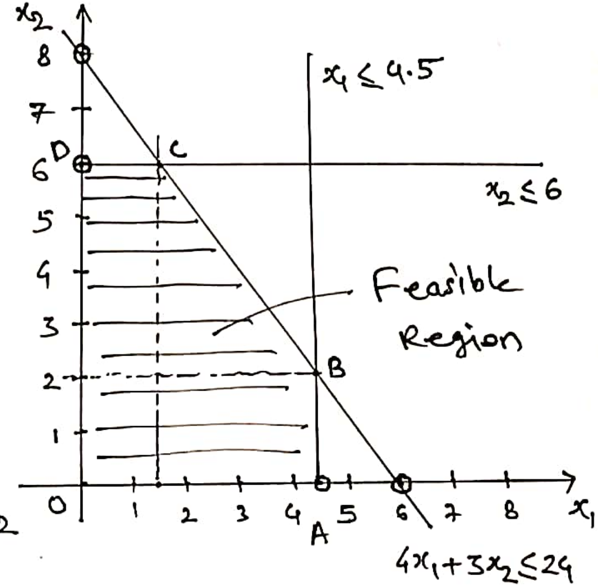
constraints	x_1	x_2
1. $4x_1+3x_2 \le 24$	0	8
	6	0
2. $x_1 \le 4.5$	4.5	0
3. $x_2 \le 6$	0	6



points	coordinate	Max $Z=4x_1+3x_2$
0	(0,0)	= 0
A	(4.5,0)	= 18
B	(4.5,2)	= 24
C	(1.5,6)	= 24
D	(0,6)	= 18

optimal solution

Max $Z = 24$

at point B(4.5, 2)

point C(1.5, 6).

No optimal Solution in Graphical Method–

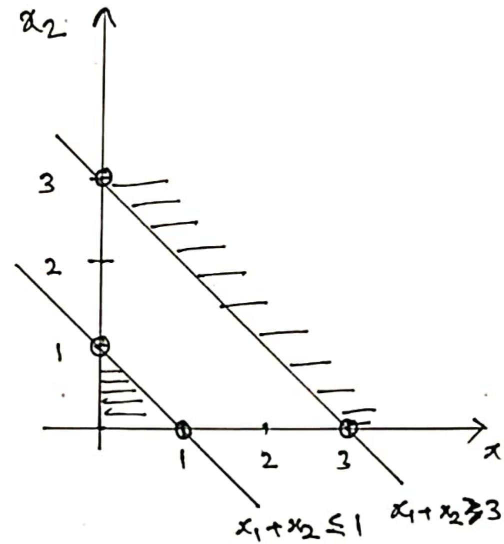
Qu: Max $z = 3x_1 + 2x_2$

Subject to constraints

$x_1 + x_2 \leq 1$ —(1)

$x_1 + x_2 \geq 3$ —(2)

and $x_1, x_2 \geq 0$



Solⁿ: from constraint (1)

$x_1 + x_2 = 1$

put $x_1 = 0 \Rightarrow x_2 = 1$ $(0, 1)$

put $x_2 = 0 \Rightarrow x_1 = 1$ $(1, 0)$

from constraint (2)

$x_1 + x_2 = 3$

put $x_1 = 0 \Rightarrow x_2 = 3$ $(0, 3)$

put $x_2 = 0 \Rightarrow x_1 = 3$ $(3, 0)$

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No optimal Solution in Graphical Method –

Qu: Max $Z = 3x_1 + 2x_2$

Subject to constraints

$x_1 + x_2 \leq 1$ —(1)

$x_1 + x_2 \geq 3$ —(2)

and $x_1, x_2 \geq 0$

Solⁿ: from constraint (1)

$x_1 + x_2 = 1$

put $x_1 = 0 \Rightarrow x_2 = 1$ $(0, 1)$

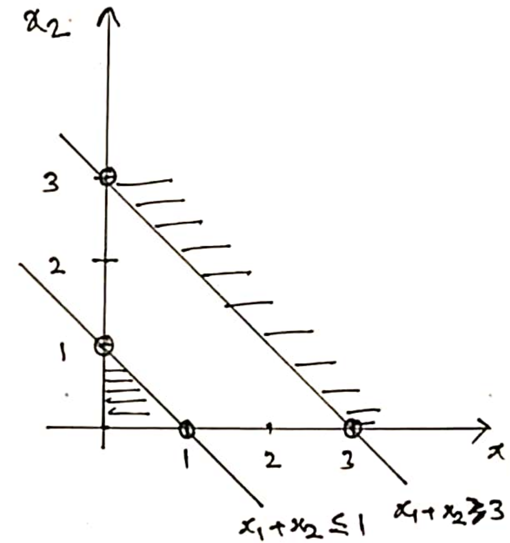
put $x_2 = 0 \Rightarrow x_1 = 1$ $(1, 0)$

from constraint (2)

$x_1 + x_2 = 3$

put $x_1 = 0 \Rightarrow x_2 = 3$ $(0, 3)$

put $x_2 = 0 \Rightarrow x_1 = 3$ $(3, 0)$



There is no feasible Region.

Hence there is 'No optimal' Solution.

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Unbounded solution in Graphical Method –

Q: Max $Z = 3x_1 + 2x_2$

Subject to constraints

$$x_1 - x_2 \leq 1 \quad \text{—} (1)$$

$$x_1 + x_2 \geq 3 \quad \text{—} (2)$$

and $x_1, x_2 \geq 0$

Solⁿ: from constraint (1)

$$x_1 - x_2 = 1$$

put $x_1 = 0 \Rightarrow x_2 = -1$ $(0, -1)$

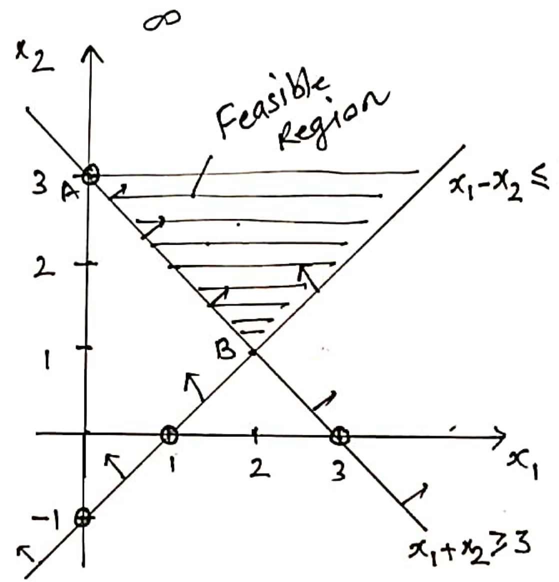
put $x_2 = 0 \Rightarrow x_1 = 1$ $(1, 0)$

from constraint (2)

$$x_1 + x_2 = 3$$

put $x_1 = 0 \Rightarrow x_2 = 3$ $(0, 3)$

put $x_2 = 0 \Rightarrow x_1 = 3$ $(3, 0)$



The maximum value of the objective function occurs at a point at ∞.

Hence the given problem has **Unbounded** solution

General LPP to Standard LPP –

1. Write the objective function in the maximization form.

2. Convert all inequalities as equations.

3. The right side element of each constraint should be made non-negative

4. All variables must have non-negative values.

General LPP to Standard LPP –

1. Write the objective function in the maximization form.

$\text{Max } Z \leftarrow \text{Min } Z$ $\boxed{\text{Max } Z' = \text{Min}(-Z)}$ Eg: $\text{Min } Z = -3x_1 + x_2$

$\text{Max } Z' = 3x_1 - x_2$

2. Convert all inequalities as equations.

$\leq$ + slack | Eg: $x_1 + 2x_2 \leq 12$ & $2x_1 + x_2 \geq 15$

$\geq$ – surplus | $x_1 + 2x_2 + \underline{S_1} = 12$ $2x_1 + x_2 - \underline{S_2} = 15$

3. The right side element of each constraint should be made non-negative

Eg: $2x_1 + x_2 - S_2 = -15$ (Multiplying by -1)

$\Rightarrow -2x_1 - x_2 + S_2 = 15$

4. All variables must have non-negative values.

Eg: $x_1 + x_2 \leq 3$, $x_1 \geq 0$, x_2 is <u>unrestricted</u> in sign

$x_1 + (x_2' - x_2'') \leq 3$ $x_1, x_2', x_2'' \geq 0$

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General LPP to Standard LPP –

1. Write the objective function in the maximization form.

Max Z ← Min Z $\boxed{\text{Max } Z' = \text{Min}(-Z)}$ Eg: $\text{Min } Z = -3x_1 + x_2$

$\text{Max } Z' = 3x_1 - x_2$

2. Convert all inequalities as equations.

$\leq$ + slack | Eg: $x_1 + 2x_2 \leq 12$ & $2x_1 + x_2 \geq 15$

$\geq$ − surplus | $x_1 + 2x_2 + \underline{S_1} = 12$ $\quad 2x_1 + x_2 - \underline{S_2} = 15$

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Eg: $2x_1 + x_2 - S_2 = -15$ (Multiplying by −1)

$\Rightarrow -2x_1 - x_2 + S_2 = 15$

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Eg: $x_1 + x_2 \leq 3$, $x_1 \geq 0$, x_2 is <u>unrestricted</u> in sign

$x_1 + (x_2' - x_2'') \leq 3 \quad x_1, x_2', x_2'' \geq 0$

$x_1 + x_2' - x_2'' + s_1 = 3$

Examples: GLPP to SLPP -

① $\text{Max } z = 3x_1 + x_2$

Subject to

$$2x_1 + x_2 \overset{+s_1}{\leq} 2$$

$$3x_1 + 4x_2 \underset{-s_2}{\geq} 12$$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } z = 3x_1 + x_2$

s.t.

$$2x_1 + x_2 + s_1 = 2$$

$$3x_1 + 4x_2 - s_2 = 12$$

and $x_1, x_2, s_1, s_2 \geq 0$

② $\text{Min } z = 4x_1 + 2x_2$

subject to

$$3x_1 + x_2 \geq 2$$

$$x_1 + x_2 \geq 21$$

$$x_1 + 2x_2 \geq 30$$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } z' = -4x_1 - 2x_2$

subject to

$$3x_1 + x_2 - s_1 = 2$$

$$x_1 + x_2 - s_2 = 21$$

$$x_1 + 2x_2 - s_3 = 30$$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

③ $\text{Min } z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 + 3x_2 + 3x_3 \geq -4 \quad \to \leq$$

$$3x_1 + 5x_2 + 2x_3 \leq 7$$

and $x_1, x_2 \geq 0$,

x_3 is unrestricted in sign

SLPP

$\text{Max } z' = -x_1 - 2x_2 - 3x_3$

Subject to

$$-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$$

$$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$$

and $x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$

Step 4: Calculation of Z and Δ_j (Net Evaluation) and test the basic feasible solution for optimality by the rules given.

$\boxed{Z = C_B X_B}$ and $\boxed{\Delta_j = Z_j - C_j = C_B X_j - C_j}$

Rule 1 – If all $\Delta_j \geq 0$, the solution under the test will be optimal.

Rule 2 – If atleast one Δ_j is negative, the solution is not optimal and then proceeds to improve the solution in the next step.

Rule 3 – If corresponding to any negative Δ_j, all elements of the column X_j are negative or zero, then the solution under test will be unbounded.

Simplex Method –

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

and $x_1, x_2 \geq 0$

Sol^n: step① SLPP

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
S_1	0	40	4	2	1	0	
S_2	0	32	2	4	0	1	

$Z = 0 \times 40 + 0 \times 32$
$= 0$

$\Delta_1 = C_B X_1 - C_1$
$= (0,0)(4,2) - 80$
$= 0 + 0 - 80 = -80$

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \le 40$$

$$2x_1 + 4x_2 \le 32$$

and $x_1, x_2 \ge 0$

Soln: step① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$$

Subject to

$$4x_1 + 2x_2 + S_1 = 40$$

$$2x_1 + 4x_2 + S_2 = 32$$

and $x_1, x_2, S_1, S_2 \ge 0$

step② Represent in matrix form

$$AX = B$$

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \end{array}$$
$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

step③ construct starting simplex table

		$c_j \rightarrow$	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
S_1	0	40	4	2	1	0	
S_2	0	32	2	4	0	1	

Step 4: Calculation of Z and Δ_j (Net Evaluation) and test the basic feasible solution for optimality by the rules given.

$\boxed{Z = C_B X_B}$ and $\boxed{\Delta_j = Z_j - C_j = C_B X_j - C_j}$

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Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \le 40$

$2x_1 + 4x_2 \le 32$

and $x_1,$

Sol^n: step ①

Max Z =

Subject t

$4x_1+$

$2x_1+$

and

step

step ③ construct starting simplex table

		$C_j \to$	80	55	0	0
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2
s_1	0	40	4	2	1	0
s_2	0	32	2	4	0	1
Z = 0		$\Delta_j \to$	-80	-55	0	0

$Z = 0 \times 40 + 0 \times 32$
$= 0$

$\Delta_1 = C_B X_1 - C_1$
$= (0,0)(4,2) - 80$
$= 0 + 0 - 80 = -80$

$\Delta_2 = C_B X_2 - C_2$
$= (0,0)(2,4) -$
$= 0 + 0 - 55 = -$

$\Delta_3 = C_B X_3 - C_3$
$= (0,0)(1,0) - 0$
$= 0 + 0 - 0 = 0$

$\Delta_4 = C_B X_4 - C_4$
$= (0,0)(0,1) - 0$
$= 0 + 0 - 0 = 0$

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Step 5: To improve the basic feasible solution, the incoming and outgoing vectors are determined.

(i) Incoming (Entering) Vector (x_k) :- most negative value of Δ_j

(ii) Outgoing (Leaving) Vector :- $\min\left[x_B / x_k \, , \, x_k > 0 \right]$

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

and $x_1, x_2 \geq 0$

Sol^n: Step ① SLPP

Max $Z = 80x_1 +$ ... $0S_2$

Sub... to

... $+ 2x_2$

Step ② construct starting simplex table

		$C_j \rightarrow$	80	55	0	0
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2
S_1	0	40	4	2	1	0
S_2	0	32	2	4	0	1
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0

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Step 5: To improve the basic feasible solution, the incoming and outgoing vectors are determined.

(i) Incoming (Entering) Vector (x_k) :- most negative value of Δ_j

(ii) Outgoing (Leaving) Vector :- $\min\left[x_B/x_k , x_k > 0 \right]$

Simplex Method –

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

and $x_1, x_2 \geq 0$

Sol^n: step ① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$$

Subject to

$$4x_1 + 2x_2 + S_1 = 40$$

step ③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R.
Basic Variable	C_B	X_B	x_1	x_2	S_1	S_2	X_B/X_k
$\leftarrow S_1$	0	40	4	2	1	0	$40/4 = 10$
S_2	0	32	2	4	0	1	$32/2 = 16$
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0	

outgoing vector

$(x_k)\uparrow$ incoming vector

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Step 6: Mark the key element (pivot element) at the intersection of outgoing vector and incoming vector.

(i) Key element should be one.

(ii) Remaining values of X_K (column) should be zero.

Step 7: Now repeat step 4 to step 6 until an optimal solution is obtained.

Simplex Method –

Q. Solve the following LPP

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

and $x_1, x_2 \geq 0$

Sol^n: step ① SLPP

Max $Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$

Subject to

step ③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R.
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
← S_1	0	40	4	2	1	0	$40/4 = 10$
S_2	0	32	2	4	0	1	$32/2 = 16$
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0	

outgoing vector

(x_1) incoming vector

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(column)

Step 7: Now repeat step 4 to step 6 until an optimal solution is obtained.

Simplex Method –

Q. Solve the following LPP

$\text{Max } Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \le 40$

$2x_1 + 4x_2 \le 32$

and $x_1, x_2 \ge 0$

Sol^n: step① SLPP

$\text{Max } Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$

Subject to

$4x_1 + 2x_2 + S_1 = 40$

$2x_1 + 4x_2 + S_2 = 32$

and $x_1, x_2, S_1, \ldots 0$

step② Represent ... form

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R.
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
$\leftarrow S_1$	0	40	4	2	1	0	$40/4=10$
S_2	0	32	2	4	0	1	$32/2=16$
$Z=0$		$\Delta_j \rightarrow$	-80	-55	0	0	

outgoing vector

(x_K) incoming vector.

		C_j	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
x_1	80	10	1	$1/2$	$1/4$	0	
S_2	0	12	0	3	$-1/2$	1	

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Simplex Method –

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \le 40$$

$$2x_1 + 4x_2 \le 32$$

and $x_1, x_2 \ge 0$

Sol^n: step① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

and $x_1, x_2, s_1, s_2 \ge 0$

step② Represent in matrix form

$$AX = B$$

x_1	x_2	s_1	s_2
4	2	1	0
2	[illegible]	0	[illegible]

step③ construct starting simplex table

		$c_j \rightarrow$	80	55	0	0	Min R.
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
outgoing vector ← s_1	0	40	4	2	1	0	$40/4 = 10$
s_2	0	32	2	4	0	1	$32/2 = 16$
$Z = 0$		$\Delta_j \rightarrow$	−80	−55	0	0	

(x_1) incoming vector.

		c_j	80	55	0	0	min R
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
x_1	80	10	1	$1/2$	$1/4$	0	20
outgoing vector ← s_2	0	12	0	3	$-1/2$	1	4 ←
$Z = 800$		Δ_j	0	−15	20	0	

incoming

$2x_1 + 4x_2 = 32$

and $x_1, x_2 \geq 0$

Solⁿ: step ① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

and $x_1, x_2, s_1, s_2 \geq 0$

step ② Represent in matrix form

$$AX = B$$

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \end{array}$$
$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

(x_2) incoming vector.

		c_j	80	55	0	0	min R
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
x_1	80	10	1	1/2	1/4	0	20
← s_2	0	12	0	$\boxed{3}$	-1/2	1	4 ←
$Z = 800$		Δ_j	0	-15 ↑	20	0	

outgoing vector

incoming

		c_j	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
x_1	80						
x_2	55	4	0	1	-1/6	1/3	

$$R_1 = R_1 - \tfrac{1}{2}R_2$$

$$\begin{array}{rrrrrr} & 10 & 1 & \frac{1}{2} & \frac{1}{4} & 0 \\ - & 2 & 0 & \frac{1}{2} & +\frac{1}{12} & \frac{1}{6} \\ & - & - & - & & - \\ \hline & 8 & 1 & 0 & \frac{1}{3} & -\frac{1}{6} \end{array}$$

$$\frac{1}{4} + \frac{1}{12}$$

$$\frac{3+1}{12} = \frac{4}{12}$$

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Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \le 40$$

$$2x_1 + 4x_2 \le 32$$

and $x_1, x_2 \ge 0$

Sol^n: step① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

and $x_1, x_2, s_1, s_2 \ge 0$

step② Represent in matrix form

$$AX = B$$

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \end{array}$$
$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R.
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
outgoing vector $\leftarrow s_1$	0	40	$\boxed{4}$	2	1	0	$40/4 = 10$
s_2	0	32	2	4	0	1	$32/2 = 16$
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0	

$(x_K)\uparrow$ incoming vector

		C_j	80	55	0	0	Min R
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
x_1	80	10	1	1/2	1/4	0	20
outgoing vector $\leftarrow s_2$	0	12	0	$\boxed{3}$	-1/2	1	$4 \leftarrow$
$Z = 800$		Δ_j	0	-15	20	0	

$\uparrow$ incoming

		C_j	80	55	0	0
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2
x_1	80	8	1	0	1/3	-1/6
x_2	55	4	0	1	-1/6	1/3
$Z = 860$		Δ_j	0	0	35/2	5

and $x_1, x_2 \geq 0$

Sol^n: step① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

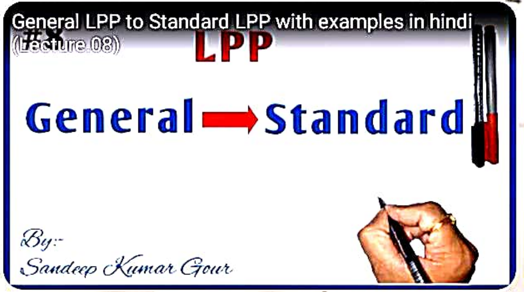
$$2x_1 + 4x_2 + s_2 = 32$$

and $x_1, x_2, s_1, s_2 \geq 0$

step② Represent in matrix form

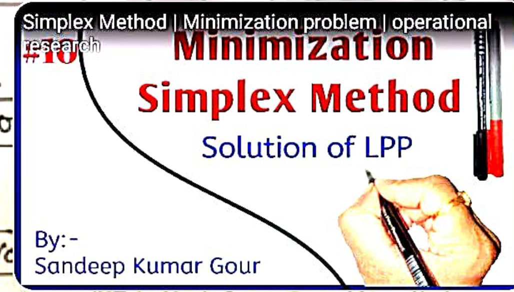
$$AX = B$$

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \\ & & 1 & 0 \\ & & 0 & 1 \end{array} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$



		C_j	80	55	0	0	min R
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	X_B/X_K
x_1							
s_2							
Z = 80							

outgoing vector



Basic Va						
x_1	80	8	1	0	1/3	-1/6
x_2	55	4	0	1	-1/6	1/3
Z = 860		Δ_j	0	0	35/2	5

Since all $\Delta_j \geq 0$, optimal basic feasible sol^n is obtained.

Max $Z = 860$, $x_1 = 8$, $x_2 = 4$.

Simplex Method - Minimization Problem

Qu: GLPP: $\text{Min } Z = x_1 - 3x_2 + 2x_3$

subject to

$$3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

and $x_1, x_2, x_3 \geq 0$

Solⁿ: SLPP: $\text{Max } Z' = -x_1 + 3x_2 - 2x_3$

subject to

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex Method - Minimization Problem

Qu: GLPP: Min $Z = x_1 - 3x_2 + 2x_3$

subject to

$$3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

and $x_1, x_2, x_3 \geq 0$

Solⁿ: SLPP: Max $Z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

subject to

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Sol: SLPP: Max $Z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to

$$3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Representation in Matrix form

$$AX = B$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{array}$$
$$\begin{bmatrix} 3 & -1 & 3 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix}$$

$x_1 = x_2 = x_3 = 0$

$s_1 = 7, \; s_2 = 12, \; s_3 = 10$ Initial Basic feasible solⁿ

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construct starting simplex table

		C_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
S_1	0	7	3	-1	3	1	0	0
S_2	0	12	-2	4	0	0	1	0
S_3	0	10	-4	3	8	0	0	1
$Z=0$		Δ_j	1	-3	2	0	0	0

construct starting simplex table

		C_j	-1	3	-2	0	0	0	Min Ratio
Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$X_B/X_K, X_K>0$
S_1	0	7	3	-1	3	1	0	0	
S_2	0	12	-2	4	0	0	1	0	$12/4=3$ ←
S_3	0	10	-4	3	8	0	0	1	$10/3=3.3..$
$Z=0$		Δ_j	1	-3 ↑	2	0	0	0	

outgoing vector

incoming vector

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construct starting simplex table

		C_j	-1	3	-2	0	0	0	
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Min Ratio X_B/X_K, $X_K>0$
s_1	0	7	3	-1	3	1	0	0	
← s_2 outgoing vector	0	12	-2	$\boxed{4}$	0	0	1	0	$12/4=3$ ←
s_3	0	10	-4	3	8	0	0	1	$10/3=3.3..$
$Z=0$		Δ_j	1	-3 ↑ incoming vector	2	0	0	0	

		C_j						
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3

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Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
S_1	0	7	3	-1	3	1	0	0	
S_2	0	12	-2	4	0	0	1	0	$12/4 = 3 \leftarrow$
S_3	0	10	-4	3	8	0	0	1	$10/3 = 3.3..$

outgoing vector

$Z = 0$ $\quad \Delta_j \quad 1 \quad -3\uparrow \quad 2 \quad 0 \quad 0 \quad 0$

incoming vector

		C_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
S_1	0							
x_2	3	3	$-1/2$	1	0	0	$1/4$	0
S_3	0							

$R_1 = R_1 + \quad 7 \quad 3 \quad -1 \quad 3 \quad 1 \quad 0 \quad 0$

$-1/2 \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0$

$3 \quad 1 \quad 1/4 \quad 0$

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S_1	0	7	3	-1	3	1	0	0	
outgoing vector S_2	0	12	-2	$\boxed{4}$	0	0	1	0	$12/4 = 3 \leftarrow$
S_3	0	10	-4	3	8	0	0	1	$10/3 = 3.3..$

$Z = 0$ $\quad \Delta_j \quad 1 \quad -3\uparrow \quad 2 \quad 0 \quad 0 \quad 0$

incoming vector

	c_j		-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
S_1	0	10	5/2	0	3	1	1/4	0
x_2	3	3	-1/2	1	0	0	1/4	0
S_3	0							

$$R_1 = R_1 + R_2 \quad \begin{array}{ccccccc} 7 & 3 & -1 & 3 & 1 & 0 & 0 \\ 3 & -1/2 & 1 & 0 & 0 & 1/4 & 0 \\ \hline 10 & 5/2 & 0 & 3 & 1 & 1/4 & 0 \end{array}$$

variable									
S_1	0	7	3	-1	3	1	0	0	
outgoing vector S_2	0	12	-2	4	0	0	1	0	$12/4 = 3 \leftarrow$
S_3	0	10	-4	3	8	0	0	1	$10/3 = 3.3..$
$Z = 0$		Δ_j	1	$-3 \uparrow$	2	0	0	0	
				incoming vector					

		C_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
S_1	0	10	$5/2$	0	3	1	$1/4$	0
x_2	3	3	$-1/2$	1	0	0	$1/4$	0
S_3	0	1	$-5/2$	0	8	0	$-3/4$	1
$Z = 9$		Δ_j						

Basic variable									
S_1	0	7	3	-1	3	1	0	0	
outgoing vector ← S_2	0	12	-2	$\boxed{4}$	0	0	1	0	$12/4 = 3$ ←
S_3	0	10	-4	3	8	0	0	1	$10/3 = 3.3..$
$Z' = 0$		Δ_j	1	-3 ↑	2	0	0	0	
				incoming vector					

		C_j	-1	3	-2	0	0	0	Min Ratio:
Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$X_B/X_K,\ X_K>0$
out vector ← S_1	0	10	$\boxed{5/2}$	0	3	1	1/4	0	$10/\tfrac{5}{2} = 4$ ←
x_2	3	3	$-1/2$	1	0	0	1/4	0	
S_3	0	1	$-5/2$	0	8	0	$-3/4$	1	
$Z' = 9$		Δ_j	$-1/2$ ↑	0	2	0	3/4	0	
			incoming vector						

		C_j	-1	3	-2	0	0	0	Min Ratio:
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	$X_B/X_K,\ X_K>0$
out vector ← s_1	0	10	$5/2$	0	3	1	$1/4$	0	$10/\frac{5}{2}=4$ ←
x_2	3	3	$-1/2$	1	0	0	$1/4$	0	
s_3	0	1	$-5/2$	0	8	0	$-3/4$	1	
$Z'=9$		Δ_j	$-1/2$ ↑ incoming vector	0	2	0	$3/4$	0	

		C_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
x_1	-1	4	1	0	$6/5$	$2/5$	$1/10$	0
x_2	3	5	0	1	$3/5$	$1/5$	$3/10$	0
s_3	0	11	0	0	11	1	$-1/2$	1

$R_3 = R_3 + \frac{5}{2}R_1$

$$\begin{array}{ccccccc} 1 & -5/2 & 0 & 8 & 0 & -3/4 & 1 \\ 10 & 5/2 & 0 & 3 & 1 & 1/4 & 0 \\ \hline 11 & 0 & 0 & 11 & 1 & -1/2 & 1 \end{array}$$

x_2	3	3	$-1/2$	1	0	0	$1/4$	0
S_3	0	1	$-5/2$	0	8	0	$-3/4$	1
$Z' = 9$		Δ_j	$-1/2 \uparrow$	0	2	0	$3/4$	0

incoming vector

Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
		C_j	-1	3	-2	0	0	0
x_1	-1	4	1	0	$6/5$	$2/5$	$1/10$	0
x_2	3	5	0	1	$3/5$	$1/5$	$3/10$	0
S_3	0	11	0	0	11	1	$-1/2$	1
$Z' = 11$		Δ_j	0	0	$13/5$	$1/5$	$4/5$	0

vector

x_2	3	3	$-1/2$	1	0	0	$1/4$	0
S_3	0	1	$-5/2$	0	8	0	$-3/4$	1
$Z' = 9$		Δ_j	$-1/2$	0	2	0	$3/4$	0

incoming vector

		c_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
x_1	-1	4	1	0	$6/5$	$2/5$	$1/10$	0
x_2	3	5	0	1	$3/5$	$1/5$	$3/10$	0
S_3	0	11	0	0	11	1	$-1/2$	1
$Z' = 11$		Δ_j	0	0	$13/5$	$1/5$	$4/5$	0

$\because$ all $\Delta_j \geq 0$ optimal sol^n is obtained.

optimal sol^n is Min $Z = -11$, $x_1 = 4$, $x_2 = 5$.

x_2	3	3	$-1/2$	1	0	0	$1/4$	0	
S_3	0	1	$-5/2$	0	8	0	$-3/4$	1	
$Z' = 9$		Δ_j	$-1/2$ ↑	0	2	0	$3/4$	0	

incoming vector

		c_j	-1	3	-2	0	0	0
Basic variable	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
x_1	-1	4	1	0	$6/5$	$2/5$	$1/10$	0
x_2	3	5	0	1	$3/5$	$1/5$	$3/10$	0
S_3	0	11	0	0	11	1	$-1/2$	1
$Z' = 11$		Δ_j	0	0	$13/5$	$1/5$	$4/5$	0

$\because$ all $\Delta_j \geq 0$ optimal sol^n is obtained.

optimal sol^n is $\text{Min } Z = -11$, $x_1 = 4$, $x_2 = 5$, $x_3 = 0$.

Simplex Method- Unbounded Solution

$$\text{Max } z = 3x_1 + 4x_2$$

Subject to

$$x_1 - x_2 \le 1$$

$$-x_1 + x_2 \le 2$$

and $x_1, x_2 \ge 0$

Soln: SLPP

$$\text{Max } z = 3x_1 + 4x_2 + 0S_1 + 0S_2$$

Subject to

$$x_1 - x_2 + S_1 = 1$$

$$-x_1 + x_2 + S_2 = 2$$

and $x_1, x_2, S_1, S_2 \ge 0$

Initial Basic feasible solution

$x_1 = x_2 = 0$ $\boxed{S_1 = 1, S_2 = 2}$

Matrix Form $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Simplex Method - Unbounded Solution

$$\text{Max } Z = 3x_1 + 4x_2$$

Subject to

$$x_1 - x_2 \leq 1$$

$$-x_1 + x_2 \leq 2$$

and $x_1, x_2 \geq 0$.

Solⁿ: SLPP

$$\text{Max } Z = 3x_1 + 4x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 - x_2 + s_1 = 1$$

$$-x_1 + x_2 + s_2 = 2$$

and $x_1, x_2, s_1, s_2 \geq 0$

Initial Basic feasible soluti

$x_1 = x_2 = 0$ $\boxed{s_1 = 1, s_2 = 2}$

Matrix Form

$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

construct starting simplex table

		c_j	3	4	0	0
Basic variable	c_B	x_B	x_1	x_2	s_1	s_2
s_1	0	1	1	-1	1	0
s_2	0	2	-1	1	0	1
$Z=0$		Δ_j	-3	-4	0	0

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Simplex Method - Unbounded Solution

$\text{Max } Z = 3x_1 + 4x_2$

Subject to

$x_1 - x_2 \leq 1$

$-x_1 + x_2 \leq 2$

and $x_1, x_2 \geq 0$

Sol^n: SLPP

$\text{Max } Z = 3x_1 + 4x_2 + 0s_1 + 0s_2$

subject to

$x_1 - x_2 + s_1 = 1$

$-x_1 + x_2 + s_2 = 2$

and $x_1, x_2, s_1, s_2 \geq 0$

Initial Basic feasible solu

$x_1 = x_2 = 0$ $\boxed{s_1 = 1, s_2 =}$

Matrix form

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

construct starting simplex table

		c_j	3	4	0	0	Min Ratio
Basic variable	c_B	X_B	x_1	x_2	s_1	s_2	$X_B/X_K, X_K > 0$
s_1	0	1	1	-1	1	0	-
outgoing vector ← s_2	0	2	-1	$\boxed{1}$	0	1	2/1 ←
$Z = 0$		Δ_j	-3	-4 ↑	0	0	

(x_K) incoming vector

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Simplex Method - Unbounded Solution

Max $z = 3x_1 + 4x_2$

Subject to

$x_1 - x_2 \le 1$

$-x_1 + x_2 \le 2$

and $x_1, x_2 \ge 0$.

Solⁿ: SLPP

Max $z = 3x_1 + 4x_2 + 0S_1 + 0S_2$

Subject to

$x_1 - x_2 + S_1 = 1$

$-x_1 + x_2 + S_2 = 2$

and $x_1, x_2, S_1, S_2 \ge 0$

Initial Basic feasible solution

$x_1 = x_2 = 0$ $\boxed{S_1 = 1, S_2}$

Matrix Form $Ax =$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ \end{bmatrix}$$

construct starting simplex table

		c_j	3	4	0	0	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	Min Ratio $X_B/X_K, X_K > 0$
S_1	0	1	1	-1	1	0	-
S_2	0	2	-1	$\boxed{1}$	0	1	2/1 ←
$Z=0$		Δ_j	-3	-4 ↑	0	0	

outgoing vector

(X_K) incoming vector

		c_j	3	4	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	3	0	0	1	1
x_2	4	2	-1	1	0	1

$R_1 = R_1 + R_2(\text{New})$

	1	1	-1	1	0
+	2	-1	1	0	1
R_1(New)	3	0	0	1	1

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Simplex Method - Unbounded Solution

Max $Z = 3x_1 + 4x_2$

Subject to

$$x_1 - x_2 \le 1$$

$$-x_1 + x_2 \le 2$$

and $x_1, x_2 \ge 0$

Sol^n. SLPP

Max $Z = 3x_1 + 4x_2 + 0S_1 + 0S_2$

Subject to

$$x_1 - x_2 + S_1 = 1$$

$$-x_1 + x_2 + S_2 = 2$$

and $x_1, x_2, S_1, S_2 \ge 0$

Initial Basic feasible solution

$x_1 = x_2 = 0$ $\boxed{S_1 = 1, S_2 = 2}$

Matrix form $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

construct starting simplex table

		c_j	3	4	0	0	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	Min Ratio X_B/X_k, $X_k > 0$
S_1	0	1	1	-1	1	0	–
S_2	0	2	-1	$\boxed{1}$	0	1	2/1 ←
$Z = 0$		Δ_j	-3	-4 ↑	0	0	

outgoing vector ← S_2

(x_k) incoming vector ↑ x_2

		c_j	3	4	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	3	0	0	1	1
x_2	4	2	-1	1	0	[illegible]
$Z = 8$		Δ_j	-7 ↑	0	0	4

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Simplex Method- Unbounded Solution

Max $Z = 3x_1 + 4x_2$

Subject to

$$x_1 - x_2 \le 1$$
$$-x_1 + x_2 \le 2$$
and $x_1, x_2 \ge 0$

Sol^n: SLPP

Max $Z = 3x_1 + 4x_2 + 0S_1 + 0S_2$

Subject to

$$x_1 - x_2 + S_1 = 1$$
$$-x_1 + x_2 + S_2 = 2$$
and $x_1, x_2, S_1, S_2 \ge 0$

Initial Basic feasible solution

$x_1 = x_2 = 0$ $\boxed{S_1 = 1, S_2 = 2}$

Matrix form $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

construct starting simplex table

		c_j	3	4	0	0	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	Min Ratio $X_B/X_K, X_K>0$
S_1	0	1	1	-1	1	0	–
S_2	0	2	-1	$\boxed{1}$	0	1	$2/1 \leftarrow$
$Z=0$		Δ_j	-3	$-4\uparrow$	0	0	

outgoing vector

(X_K) incoming vector

		c_j	3	4	0	0
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2
S_1	0	3	0	0	1	1
x_2	4	2	-1	1	0	1
$Z=8$		Δ_j	$-7\uparrow$	0	0	4

$\because$ all element of the column x_1 are negative or zero then the solution is Unbounded.

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Degeneracy in Simplex method
(Tie for minimum Ratio) -

Q. Given LPP: $\text{Max } Z = 2x_1 + x_2$

subject to
$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + x_2 \leq 8$$
$$4x_1 - x_2 \leq 8$$
and $x_1, x_2 \geq 0$

Sol^n SLPP: $\text{Max } Z = 2x_1 + x_2 + 0s_1 + 0s_2 + 0s_3$

subject to
$$4x_1 + 3x_2 + s_1 = 12$$
$$4x_1 + x_2 + s_2 = 8$$
$$4x_1 - x_2 + s_3 = 8$$
and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Initial Basic Feasible solution (IBFS)

$x_1 = x_2 = 0$, $s_1 = 12$, $s_2 = 8$, $s_3 = 8$, $Z = 0$

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subject to $4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$4x_1 - x_2 \leq 8$

and $x_1, x_2 \geq 0$

Soln SLPP: Max $Z = 2x_1 + x_2 + 0s_1 + 0s_2 + 0s_3$

subject to $4x_1 + 3x_2 + s_1 = 12$

$4x_1 + x_2 + s_2 = 8$

$4x_1 - x_2 + s_3 = 8$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Initial Basic Feasible solution (IBFS)

$x_1 = x_2 = 0$, $s_1 = 12$, $s_2 = 8$, $s_3 = 8$, $Z = 0$

Matrix Form $AX = B$

$$\begin{bmatrix} 4 & 3 & 1 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 \\ 4 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix}$$

		C_j	2	1	0	0	0	Min Ratio		
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	S_3	X_B/X_K	S_1/X_K	S_2/X_K
S_1	0	12	4	3	1	0	0	$12/4=3$	—	—
S_2	0	8	4	1	0	1	0	$8/4=2$	$0/4=0$	$1/4=\frac{1}{4}$
← S_3	0	8	$\boxed{4}$	-1	0	0	1	$8/4=2$	$0/4=0$	$0/4=0$ ←
$Z=0$		Δ_j	$-2\uparrow$ (X_K)	-1	0	0	0			
S_1	0	4	0	4	1	0	-1	$4/4=1$	$R_1=R_1-4R_3$ (New)	
← S_2	0	0	0	$\boxed{2}$	0	1	-1	$0/2=0$ ←	$R_2=R_2-4R_3$ (New)	
x_1	2	2	1	$-1/4$	0	0	$1/4$	—		
$Z=4$		Δ_j	0	$-3/2\uparrow$	0	0	$1/2$			
← S_1	0	4	0	0	1	-2	$\boxed{1}$	$4/1=4$ ←	$R_1=R_1-4R_2$ (New)	
✓ x_2	1	0	0	1	0	$1/2$	$-1/2$			
x_1	2	2	1	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$2/1/8=16$	$R_3=R_3+\frac{1}{4}R_2$	
$Z=4$		Δ_j	0	0	0	$3/4$	$-1/4\uparrow$			
S_3	0	4	0	0	1	-2	1			
x_2	1	2	0	1	$1/2$	$-1/2$	0		$R_2=R_2+\frac{1}{2}R_1$	
x_1	2	$3/2$	1	0	$-1/8$	$3/8$	0		$R_3=R_3-\frac{1}{8}R_1$	
$Z=5$		Δ_j	0	0	$1/4$	$1/4$	0			

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← S_1	0	12	4	3	1	0	0	$12/4=3$	—	—
S_2	0	8	4	1	0	1	0	$8/4=2$	$0/4=0$	$1/4=\frac{1}{4}$
← S_3	0	8	$\boxed{4}$	-1	0	0	1	$8/4=2$	$0/4=0$	$0/4=0$ ←
$Z=0$		Δ_j	$-2\uparrow$ (x_k)	-1	0	0	0			
S_1	0	4	0	4	1	0	-1	$4/4=1$	$R_1=R_1-4R_3$ (New)	
← S_2	0	0	0	$\boxed{2}$	0	1	-1	$0/2=0$ ←	$R_2=R_2-4R_3$ (New)	
x_1	2	2	1	$-1/4$	0	0	$1/4$	—		
$Z=4$		Δ_j	0	$-3/2\uparrow$	0	0	$1/2$			
← S_1	0	4	0	0	1	-2	$\boxed{1}$	$4/1=4$ ←	$R_1=R_1-4R_2$ (New)	
x_2	1	0	0	1	0	$1/2$	$-1/2$			
x_1	2	2	1	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$2/\frac{1}{8}=16$	$R_3=R_3+\frac{1}{4}R_2$	
$Z=4$		Δ_j	0	0	0	$3/4$	$-1/4\uparrow$			
S_3	0	4	0	0	1	-2	1			
x_2	1	2	0	1	$1/2$	$-1/2$	0		$R_2=R_2+\frac{1}{2}R_1$	
x_1	2	$3/2$	1	0	$-1/8$	$3/8$	0		$R_3=R_3-\frac{1}{8}R_1$	
$Z=5$		Δ_j	0	0	$1/4$	$1/4$	0			

$\because$ all $\Delta_j \geq 0$, optimal solⁿ is obtained.

Optimal solⁿ Max $Z=5$, $x_1=3/2$, $x_2=2$.

Two-phase Method-

Given GLPP $\text{Max } Z = 3x_1 - x_2$

$$\text{Max } Z = 2x_1 + x_2$$

s.t.

$$4x_1 + 3x_2 \leq 12 \Rightarrow 4\underline{x_1} + 3\underline{x_2} + s_1 = 12$$

$$4x_1 + x_2 \leq 8 \Rightarrow 4x_1 + x_2 + s_2 = 8$$

and $x_1, x_2 \geq 0$ $\quad \underline{x_1 = x_2 = 0}$ $\quad \boxed{s_1 = 12,\ s_2 = 8}$

$$\text{Max } Z = -2x_1 - x_2$$

s.t.

$$\left.\begin{aligned} 3x_1 + x_2 = 3 &\Rightarrow 3x_1 + x_2 = 3 \\ 4x_1 + 3x_2 \geq 6 &\Rightarrow 4x_1 + 3x_2 - s_1 = 6 \\ x_1 + 2x_2 \leq 6 &\Rightarrow x_1 + 2x_2 + s_2 = 6 \end{aligned}\right\} \Rightarrow s_1 = -6$$

and $x_1, x_2 \geq 0$

and $x_1, x_2 \geq 0$

G $\quad \text{Max } Z = -2x_1 - x_2$

s.t.

$$3x_1 + x_2 = 3 \Rightarrow 3x_1 + x_2 + = 3$$
$$4x_1 + 3x_2 \geq 6 \Rightarrow 4x_1 + 3x_2 - s_1 = 6 \Rightarrow s_1 = -6$$
$$x_1 + 2x_2 \leq 6 \Rightarrow x_1 + 2x_2 + s_2 = 6$$

and $x_1, x_2 \geq 0$ artificial

$\leq$ + slack

$\geq$ − surplus + artificial

$=$ + artificial

I: eliminate

II: optimal sol^n

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Two-phase Method-

Given GLPP $\quad \text{Max } Z = 3x_1 - x_2$

Subject to the constraints

$$2x_1 + x_2 \geq 2 \quad \text{——} \quad (1)$$

$$x_1 + 3x_2 \leq 2 \quad \text{——} \quad (2)$$

$$x_2 \leq 4 \quad \text{——} \quad (3)$$

and $x_1, x_2 \geq 0$

S <u>Sol^n</u> <u>SLPP</u> $\quad \text{Max } Z = 3x_1 - x_2$

s.t.

$$2x_1 + x_2 - s_1 + a_1 = 2$$

$$x_1 + 3x_2 + s_2 = 2$$

$$x_2 + s_3 = 4$$

and $x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$.

Phase-I : Eliminating artificial variables

The simplex method is applied to a specially constructed auxiliary linear programming problem (ALPP).

Step 1: Assign a cost -1 to each artificial variable and a cost 0 to all other variables in the objective function.

Step 2: Construct the Auxiliary LPP in which the new objective function Z^* is to be maximized subject to the constraints.

Step 3: Solve the auxiliary problem by simplex method until either of the following three possibilities do arise.

(i) Max $Z^* < 0$ and atleast one artificial vector appear in the optimum basis at a positive level, then Given problem does not possess any feasible solution.

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S <u>Sol</u>

Soln SLPP

$$\text{Max } Z = 3x_1 - x_2$$

s.t.

$$2x_1 + x_2 - s_1 + a_1 = 2$$
$$x_1 + 3x_2 + s_2 = 2$$
$$x_2 + s_3 = 4$$

and $x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$.

Auxiliary linear programming problem

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - a_1$$

s.t.

$$2x_1 + x_2 - s_1 + a_1 = 2$$
$$x_1 + 3x_2 + s_2 = 2$$
$$x_2 + s_3 = 4$$

and $x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

Phase-I

		C_j	0	0	0	0	0	-1	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	S_3	a_1	Min Ratio X_B/X_K
a_1	-1	2	2	1	-1	0	0	1	
S_2	0	2	1	3	0	1	0	0	
S_3	0	4	0	1	0	0	1	0	
$Z^* = -2$		Δ_j	-2	-1					

$$Z^* = C_B X_B = (-1, 0, 0)(2, 2, 4)$$
$$= -2 + 0 + 0 = -2$$

$$\Delta_j = C_B X_j - C_j \Rightarrow \Delta_1 = (-1, 0, 0)(2, 1, 0) - 0$$
$$= -2 + 0 + 0 - 0 = -2$$

$$\Delta_2 = C_B X_2 - C_2 = (-1, 0, 0)(1, 3, 1) - 0$$
$$= -1 + 0 + 0 - 0 = -1$$

Phase-I

		C_j	0	0	0	0	0	-1	
Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	S_3	a_1	Min Ratio X_B/X_K, $X_K>0$
← a_1	-1	2	$\boxed{2}$	1	-1	0	0	1	$2/2=1$ ←
S_2	0	2	1	3	0	1	0	0	$2/1=2$
S_3	0	4	0	1	0	0	1	0	
$Z^*=-2$		Δ_j	-2 ↑ (X_K)	-1	1	0	0	0	

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Phase-I

Basic variable	C_B	X_B	C_j: 0	0	0	0	0	-1	
			x_1	x_2	S_1	S_2	S_3	a_1	Min Ratio $X_B/X_K, X_K>0$
$\xleftarrow{\text{outgoing}}$ a_1	-1	2	$\boxed{2}$	1	-1	0	0	1	$2/2=1 \leftarrow$
S_2	0	2	1	3	0	1	0	0	$2/1=2$
S_3	0	4	0	1	0	0	1	0	
$Z^*=-2$		Δ_j	-2 $\uparrow$ incoming (X_K)	-1	1	0	0	0	
x_1	0	1	1	1/2	-1/2	0	0	1/2	
S_2	0	1	0	5/2	1/2	1	0	-1/2	
S_3	0	4	0	1	0	0	1	0	
$Z^*=0$		Δ_j	0	0	0	0	0	1	

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Step 1: Assign a cost -1 to each artificial variable and a cost 0 to all other variables in the objective function.

Step 2: Construct the Auxiliary LPP in which the new objective function Z^* is to be maximized subject to the constraints.

Step 3: Solve the auxiliary problem by simplex method until either of the following three possibilities do arise.

(i) Max $Z^* < 0$ and atleast one artificial vector appear in the optimum basis at a positive level, then Given problem does not possess any feasible solution.

(ii) Max $Z^* = 0$ and at least one artificial vector appears in the optimum basis at a zero level. then proceed to phase II.

(iii) Max $Z^* = 0$ and no one artificial vector appears in the optimum basis, then proceed to phase II.

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Phase-I

Basic variable	C_B	X_B	x_1	x_2	S_1	S_2	S_3	a_1	Min Ratio $X_B/X_K, X_K>0$
		C_j	0	0	0	0	0	-1	
a_1 outgoing	-1	2	$\boxed{2}$	1	-1	0	0	1	2/2=1 ←
S_2	0	2	1	3	0	1	0	0	2/1=2
S_3	0	4	0	1	0	0	1	0	
$Z^* = -2$		Δ_j	-2 (X_K) ↑ incoming	-1	1	0	0	0	
x_1	0	1	1	1/2	-1/2	0	0	1/2	
S_2	0	1	0	5/2	1/2	1	0	-1/2	
S_3	0	4	0	1	0	0	1	0	
$Z^* = 0$		Δ_j	0	0	0	0	0	1	

subject to the constraints.

Step 3: Solve the auxiliary problem by simplex method until either of the following three possibilities do arise.

(i) Max $Z^* < 0$ and atleast one artificial vector appear in the optimum basis at a positive level, then

$\therefore$ Given problem does not possess any feasible solution.

(ii) Max $Z^* = 0$ and at least one artificial vector appears in the optimum basis at a zero level. then proceed to phase II.

(iii) Max $Z^* = 0$ and no one artificial vector appears in the optimum basis, then proceed to phase II.

phase II : Get an optimal solution.

Now assign the actual cost to the variables in the objective function and a zero cost to every artificial variable that appears in the basis at the zero level.

wo

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x_1	0	1	1	1/2	-1/2	0	0	1/2
s_2	0	1	0	5/2	1/2	1	0	-1/2
s_3	0	4	0	1	0	0	1	0
$z^* = 0$		Δ_j	0	0	0	0	0	1

$\because$ all $\Delta_j \geq 0$, Max $z^* = 0$, No artificial vector appears in the basis, we proceed to phase II.

Phase-II.

		C_j	3	-1	0	0	0	MR
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	s_3	X_B/X_k
x_1	3	1	1	1/2	-1/2	0	0	
← outgoing s_2	0	1	0	5/2	1/2	1	0	$1/{1/2} = 2$ ←
s_3	0	4	0	1	0	0	1	
$z = 3$		Δ_j	0	5/2	-3/2 ↑ incoming	0	0	

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$Z^* = 0$		Δ_j	0	0	0	0	0	1

$\because$ all $\Delta_j \geq 0$, $\text{Max } Z^* = 0$, No artificial vector appears in the basis, we proceed to phase II.

Phase-II.

		C_j	3	-1	0	0	0	MR
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	s_3	X_B/X_k
x_1	3	1	1	1/2	-1/2	0	0	
s_2 outgoing	0	1	0	5/2	1/2	1	0	$1/\frac{1}{2} = 2$ ←
s_3	0	4	0	1	0	0	1	
$Z=3$		Δ_j	0	5/2	-3/2 ↑ incoming	0	0	
x_1	3	2	1	3	0	1	0	
s_1	0	2	0	5	1	2	0	
s_3	0	4	0	1	0	0	1	
$Z=6$		Δ_j	0	10	0	3	0	

$\because$ all $\Delta_j \geq 0$, optimal solⁿ is obtained.

Optimal solⁿ Max $Z = 6$, $x_1 = 2$, $x_2 = 0$.

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Infeasible (No feasible) Solution in Two-phase method-

Min $Z = x_1 - 2x_2 - 3x_3$

Subject to constraints

$$-2x_1 + x_2 + 3x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 1$$

and $x_1, x_2, x_3 \geq 0$

Sol^n: SLPP

Max $Z' = -x_1 + 2x_2 + 3x_3$

s.t.

$$-2x_1 + x_2 + 3x_3 + a_1 = 2$$

$$2x_1 + 3x_2 + 4x_3 + a_2 = 1$$

and $x_1, x_2, x_3, a_1, a_2 \geq 0$

Auxiliary LPP

Max $Z^* = 0x_1 + 0x_2 + 0x_3 - a_1 - a_2$

s.t. $-2x_1 + x_2 + 3x_3 + a_1 = 2$

$2x_1 + 3x_2 + 4x_3 + a_2 = 1$

and $x_1, x_2, x_3, a_1, a_2 \geq 0$

		c_j	0	0	0	-1	-1	Min Ratio
Basic variable	C_B	X_B	x_1	x_2	x_3	a_1	a_2	X_B/X_K
a_1	-1	2	-2	1	3	1	0	2/3
outgoing ← a_2	-1	1	2	3	$\boxed{4}$	0	1	1/4 ←
$Z^* = -3$		Δ_j	0	-4	-7 ↑ (Y_K) incoming	0	0	

Infeasible (No feasible) Solution in Two-phase method-

$$\text{Min } Z = x_1 - 2x_2 - 3x_3$$

Subject to constraints

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$

and $x_1, x_2, x_3 \geq 0$

Sol^n: SLPP

$$\text{Max } Z' = -x_1 + 2x_2 + 3x_3$$

s.t.

$$-2x_1 + x_2 + 3x_3 + a_1 = 2$$
$$2x_1 + 3x_2 + 4x_3 + a_2 = 1$$

and $x_1, x_2, x_3, a_1, a_2 \geq 0$

Auxiliary LPP

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0x_3 - a_1 - a_2$$

s.t. $-2x_1 + x_2 + 3x_3 + a_1 = 2$

$2x_1 + 3x_2 + 4x_3 + a_2 = 1$

and $x_1, x_2, x_3, a_1, a_2 \geq 0$

		C_j	0	0	0	-1	-1	
Basic variable	C_B	X_B	x_1	x_2	x_3	a_1	a_2	Min Ratio X_B/X_k
a_1	-1	2	-2	1	3	1	0	2/3
outgoing ← a_2	-1	1	2	3	$\boxed{4}$	0	1	1/4 ←
$Z^* = -3$		Δ_j	0	-4	-7 ↑ (Y_k) incoming	0	0	
a_1	-1	5/4	-7/2	-5/4	0	1	-3/4	
x_3	0	1/4	1/2	3/4	1	0	1/4	
$Z^* = -5/4$		Δ_j	7/2	5/4	0	0	7/4	

$\because$ all $\Delta_j \geq 0$, But Max Z^* is negative and the artificial variable a_1 appear in the basis solution at a positive level. Hence the original problem does not posses any feasible solution.

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Big-M method (Charne's Penalty method)

Steps:

1. Express the problem in the standard form.
2. Assign the large negative price $-M$ to the artificial variables.
3. Use simplex method.

Qu: Given LPP

$$\text{Max } Z = -2x_1 - x_2$$

Subject to the constraints

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$

and $x_1, x_2 \geq 0$

Big-M method (Charne's Penalty method)

Steps:

1. Express the problem in the standard form.
2. Assign the large negative price $-M$ to the artificial variables.
3. Use simplex method.

Max $-M$

Min $+M$

Qu: Given LPP

$$\text{Max } Z = -2x_1 - x_2$$

Subject to the constraints

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

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$$x_1 + 2x_2 \leq 4$$

and $x_1, x_2 \geq 0$

SLPP

$$\text{Max } Z = -2x_1 - x_2 + 0s_1 + 0s_2 - Ma_1 - Ma_2$$

s.t.

$$3x_1 + x_2 + a_1 = 3$$
$$4x_1 + 3x_2 - s_1 + a_2 = 6$$
$$x_1 + 2x_2 + s_2 = 4$$

and $\underbrace{x_1, x_2}, \underbrace{s_1, s_2}, \underbrace{a_1, a_2} \geq 0$

Starting Simplex table

	C_j		-2	-1	0	0	-M	-M	Min Ratio
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	X_B/X_K
a_1	-M	3	3	1	0	0	1	0	
a_2	-M	6	4	3	-1	0	0	1	
s_2	0	4	1	2	0	1	0	0	
$Z=-9M$		Δ_j							

$$Z = C_B X_B = (-M,-M,0)(3,6,4)$$
$$= -3M - 6M + 0 = -9M$$

$$\Delta_j = C_B X_j - C_j$$
$$\Delta_1 = C_B X_1 - C_1 = (-M,-M,0)(3,4,1) - (-2)$$
$$= -3M - 4M + 0 + 2 = 2 - 7M$$

Starting Simplex table

		C_j	-2	-1	0	0	$-M$	$-M$	Min Ratio
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	X_B/X_K
a_1	$-M$	3	3	1	0	0	1	0	
a_2	$-M$	6	4	3	-1	0	0	1	
s_2	0	4	1	2	0	1	0	0	
$Z=-9M$		Δ_j	$2-7M$	$1-4M$	M	0	0	0	

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a_2	$-M$	6	4	3	-1	0	0	1	6/4
s_2	0	4	1	2	0	1	0	0	4/1
$Z=-9M$		Δ_j	$2-7M$	$1-4M$	M	0	0	0	
			↑incoming (x_k)						

$\boxed{R_2 = R_2 - 4R_1}$ $\boxed{R_3 = R_3 - R_1}$

		C_j	-2	-1	0	0	$-M$	$-M$
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2
x_1	-2	1	1	$1/3$	0	0	$1/3$	0
a_2	$-M$	2	0	$5/3$	-1	0	$-4/3$	1
s_2	0	3	0	$5/3$	0	1	$-1/3$	0

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$R_2 = R_2 - 4R_1$ $R_3 = R_3 - R_1$

		C_j	-2	-1	0	0	-M	-M	MR
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	X_B/X_K
x_1	-2	1	1	1/3	0	0	1/3	0	3
a_2 (outgoing)	-M	2	0	5/3	-1	0	-4/3	1	6/5 ←
s_2	0	3	0	5/3	0	1	-1/3	0	9/5
$Z = -2-2M$		Δ_j	0	$\frac{1-5M}{3}$ ↑	M	0	$\frac{-2+7M}{3}$	0	

(X_K) incoming

		C_j		
Basic variable	C_B	X_B		

$R_2 = R_2 - 4R_1$ $R_3 = R_3 - R_1$

		C_j	-2	-1	0	0	-M	-M	MR
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	X_B/X_K
x_1	-2	1	1	1/3	0	0	1/3	0	3
outgo. a_2	-M	2	0	5/3	-1	0	-4/3	1	6/5 ←
S_2	0	3	0	5/3	0	1	-1/3	0	9/5
$Z = -2-2M$		Δ_j	0	$\frac{1-5M}{3}$ ↑	M	0	$\frac{-2+7M}{3}$	0	

(X_K) incoming

		C_j	-2	-1	0	0	-M	-M	
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	
x_1	-2	3/5	1	0	1/5	0	3/5	-1/5	$R_1 = R_1 - \frac{1}{3}R_2$
x_2	-1	6/5	0	1	-3/5	0	-4/5	3/5	
S_2	0	1	0	0	1	1	1	-1	$R_3 = R_3 - \frac{5}{3}R_2$
$Z = \frac{-12}{5}$		Δ_j	0	0	1/5	0	$M - \frac{2}{5}$	$M - \frac{1}{5}$	

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Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	X_B/X_k
x_1	-2	1	1	$1/3$	0	0	$1/3$	0	3
outgo $\leftarrow$ a_2	$-M$	2	0	$\boxed{5/3}$	-1	0	$-4/3$	1	$6/5 \leftarrow$
s_2	0	3	0	$5/3$	0	1	$-1/3$	0	$9/5$
$Z=-2-2M$		Δ_j	0	$\frac{1-5M}{3}\uparrow$	M	0	$\frac{-2+7M}{3}$	0	

(X_k) incoming

		c_j	-2	-1	0	0	$-M$	$-M$	
Basic variable	C_B	X_B	x_1	x_2	s_1	s_2	a_1	a_2	
x_1	-2	$3/5$	1	0	$1/5$	0	$3/5$	$-1/5$	$R_1=R_1-\frac{1}{3}R_2$
x_2	-1	$6/5$	0	1	$-3/5$	0	$-4/5$	$3/5$	
s_2	0	1	0	0	1	1	1	-1	$R_3=R_3-\frac{5}{3}R_2$
$Z=\frac{-12}{5}$		Δ_j	0	0	$1/5$	0	$M-\frac{2}{5}$	$M-\frac{1}{5}$	

$\because$ all $\Delta_j \geq 0$, Hence optimal sol^n is obtained.

Optimal sol^n Max $Z=\frac{-12}{5}$, $x_1=3/5$, $x_2=6/5$.

Duality in Linear Programming

LPP

Original $\boxed{\text{primal}}$

↓

Derived $\boxed{\text{Dual}}$

① $\boxed{\text{Primal} \rightarrow \text{Dual}}$

② Dual Simplex method

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Derived | Dual |

1. $\boxed{\text{Primal} \rightarrow \text{Dual}}$

2. Dual simplex method

im.

concept 1:

LPP Primal → Standard primal → Dual

Min / Max   Max ✓   Min

$\leq$ ✓ $\geq$

concept 2:

primal → Standard primal → Dual

✓ Min → Min $\geq$ Max $\leq$

Max → Max $\leq$ Min $\geq$

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Duality in Linear Programming

Steps for a Standard primal Form :-

1. Change the objective function to Maximization Form.
2. If the constraints have an inequality sign '$\geq$' the convert into '$\leq$'
3. If the constraint has an '=' sign then replace it by two constraints involving the inequalities going in opposite directions.

   Eg:- $x_1 + 2x_2 = 4 \Rightarrow x_1 + 2x_2 \leq 4$ ✓

   $x_1 + 2x_2 \geq 4 \Rightarrow -x_1 - 2x_2 \leq -4$
4. Every unrestricted variable is replaced by the difference of two non-negative variables.
5. Standard primal form of the given LPP in which -

   (i) All constraints have '$\leq$' sign, where the objective function is of Maximization form.

Duality in Linear Programming

Steps for a Standard primal Form :-

1. Change the objective function to Maximization Form.
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   Eg:- $x_1 + 2x_2 = 4 \Rightarrow x_1 + 2x_2 \leq 4$ ✓

   $x_1 + 2x_2 \geq 4 \Rightarrow -x_1 - 2x_2 \leq -4$
4. Every unrestricted variable is replaced by the difference of two non-negative variables. $x_3 = x_3' - x_3''$
5. Standard primal form of the given LPP in which -

   (i) All constraints have '$\leq$' sign, where the objective function is of Maximization form.

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1. Change the objective function to Maximization Form.

2. If the constraints have an inequality sign '$\geq$' the convert into '$\leq$'

3. If the constraint has an '=' sign then replace it by two constraints involving the inequalities going in opposite directions.

Eg:- $x_1 + 2x_2 = 4 \Rightarrow x_1 + 2x_2 \leq 4$ ✓
$x_1 + 2x_2 \geq 4 \Rightarrow -x_1 - 2x_2 \leq -4$

4. Every unrestricted variable is replaced by the difference of two non-negative variables. $x_3 = x_3' - x_3''$

5. Standard primal form of the given LPP in which –

✓ (i) All constraints have $\boxed{\text{'}\leq\text{' sign}}$, where the objective function is of Maximization form.

(ii) All constraints have '$\geq$' sign, wher the objective function is of Minimization form.

... for converting Primal into its Dual

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Rules for converting Primal into its Dual

1. Transpose the rows and columns of the constraint co-efficient.
2. Transpose the co-efficient of the objective function and the right side constants.
3. Change the inequalities from '$\leq$' to '$\geq$' sign.
4. Minimize the objective function instead of maximizing it.

Standard primal

$$\text{Max } Z = 3x_1 + 4x_2$$

s.t.

$$2x_1 + 6x_2 \leq 16$$

$$-5x_1 - 2x_2 \leq -20$$

and $x_1, x_2 \geq 0$

Dual

primal to Dual conversion

① Max $Z = 3x_1 + 4x_2$

s.t.

$2x_1 + 6x_2 \le 16$ —— ①

$5x_1 + 2x_2 \ge 20$ —— ②

and $x_1, x_2 \ge 0$

primal

Max

$\le$

Standard primal

Max $Z = \boxed{3x_1 + 4x_2}$

s.t.

$\boxed{2x_1 + 6x_2} \le \boxed{16}$

$-5x_1 - 2x_2 \le \boxed{-20}$

and $x_1, x_2 \ge 0$

Dual

Min $Z = \boxed{16w_1 - 20w_2}$

s.t.

$2w_1 - 5w_2 \ge 3$

$6w_1 - 2w_2 \ge 4$

and $w_1, w_2 \ge 0$

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$$-5x_1 - 2x_2 \leq \boxed{-20}$$

and $x_1, x_2 \geq 0$

$$\boxed{6} w_1 - 2w_2 \geq \boxed{4}$$

and $w_1, w_2 \geq 0$

(2) $\text{Min } Z = 2x_2 + 5x_3$

s.t

$$x_1 + x_2 \geq 2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$

standard primal

$\text{Max } Z = -2x_2 - 5x_3$

s.t

$$-x_1 - x_2 \leq -2$$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 \leq 4$$

$$-x_1 + x_2 - 3x_3 \leq -4$$

and $x_1, x_2, x_3 \geq 0$

Dual

$\text{Min } Z = -2w_1 + 6w_2 + 4w_3 - 4w_4$

s.t.

$$-w_1 + 2w_2 + w_3 - w_4 \geq 0$$

$$-w_1 + w_2 - w_3 + w_4 \geq -2$$

$$6w_2 + 3w_3 - 3w_4 \geq -5$$

and $w_1, w_2, w_3, w_4 \geq 0$

③ Min $Z = x_1 + x_2 + x_3 \Rightarrow (-x_1 - x_2 - x_3)$

s.t.
$$x_1 - 3x_2 + 4x_3 = 5 \longrightarrow x_1 - 3x_2 + 4x_3 \leq 5$$
$$x_1 - 3x_2 + 4x_3 \geq 5$$
$$x_1 - 2x_2 \leq 3$$
$$2x_2 - x_3 \geq 4 \rightarrow 2x_2 - (x_3' - x_3'') \geq 4$$

and $x_1, x_2 \geq 0$, x_3 is unrestricted. $\mid x_3 = x_3' - x_3''$

Solⁿ: Max $Z = -x_1 - x_2 - (x_3' - x_3'')$

s.t.
$$x_1 - 3x_2 + 4(x_3' - x_3'') \leq 5$$
$$-x_1 + 3x_2 - 4(x_3' - x_3'') \leq -5$$
$$x_1 - 2x_2 \leq 3$$
$$-2x_2 + (x_3' - x_3'') \leq -4$$

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$$-2x_2 + (x_3' - x_3'') \leq -4$$

Standard primal

$$\text{Max } Z = -x_1 - x_2 - x_3' + x_3''$$

s.t.

$$x_1 - 3x_2 + 4x_3' - 4x_3'' \leq 5$$

$$-x_1 + 3x_2 - 4x_3' + 4x_3'' \leq -5$$

$$x_1 - 2x_2 \leq 3$$

$$-2x_2 + x_3' - x_3'' \leq -4$$

and $x_1, x_2, x_3', x_3'' \geq 0$

$$-2x_2 + (x_3' - x_3'') \leq -4$$

Standard primal

$$\text{Max } Z = -x_1 - x_2 - x_3' + x_3''$$

s.t.

$$x_1 - 3x_2 + 4x_3' - 4x_3'' \leq 5$$

$$-x_1 + 3x_2 - 4x_3' + 4x_3'' \leq -5$$

$$x_1 - 2x_2 \leq 3$$

$$-2x_2 + x_3' - x_3'' \leq -4$$

and $x_1, x_2, x_3', x_3'' \geq 0$

$$x_1 - 2x_2 \leq 3$$

$$-2x_2 + (x_3' - x_3'') \leq -4$$

Standard primal

$$\text{Max } Z = -x_1 - x_2 - x_3' + x_3''$$

s.t.

$$x_1 - 3x_2 + 4x_3' - 4x_3'' \leq 5$$

$$-x_1 + 3x_2 - 4x_3' + 4x_3'' \leq -5$$

$$x_1 - 2x_2 + 0x_3' + 0x_3'' \leq 3$$

$$0x_1 - 2x_2 + x_3' - x_3'' \leq -4$$

and $x_1, x_2, x_3', x_3'' \geq 0$

Dual

$$\text{Min } Z = 5w_1 - 5w_2 + 3w_3 - 4w_4$$

s.t.

$$w_1 - w_2 + w_3 \geq -1$$

$$-3w_1 + 3w_2 - 2w_3 - 2w_4 \geq -1$$

$$4w_1 - 4w_2 + w_4 \geq -1$$

$$-4w_1 + 4w_2 - w_4 \geq 1$$ and $w_1, w_2, w_3, w_4 \geq 0$

Primal to Dual Conversion.

Concept-2

Primal	Standard	Dual
Min	$\boxed{\text{Min}}$ $\geq$	Max $\leq$
Max	Max $\leq$	min $\geq$

Find the dual?

Eg(1) Given Primal form

$$\text{Max } Z = 3x_1 + 10x_2 + 2x_3$$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 = 3$$

$$\rightarrow 3x_1 - 2x_2 + 4x_3 \leq 3$$

$$\rightarrow (3x_1 - 2x_2 + 4x_3 \geq 3) \times -1$$

and $x_1, x_2, x_3 \geq 0$

standard primal

$$\text{Max } Z = 3x_1 + 10x_2 + 2x_3$$

subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 = 3 \rightarrow (3x_1 - 2x_2 + 4x_3 \geq 3) \times -1$$

and $x_1, x_2, x_3 \geq 0$

standard primal

Max $Z = 3x_1 + 10x_2 + 2x_3$

subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 \leq 3$$

$$-3x_1 + 2x_2 - 4x_3 \leq -3 \quad \text{and } x_1, x_2, x_3 \geq 0$$

Dual Form

Min $Z = 7w_1 + 3w_2 - 3w_3$

subject to

$$2w_1 + 3w_2 - 3w_3 \geq 3$$

$$3w_1 - 2w_2 + 2w_3 \geq 10$$

$$2w_1 + 4w_2 - 4w_3 \geq 2$$

and $w_1, w_2, w_3 \geq 0$

$$\text{Min } Z = 3x_1 + 5x_2 + 6x_3$$

Subject to

$$x_1 + 4x_2 + 6x_3 \le 5$$
$$2x_1 + 3x_2 + 5x_3 \ge 4$$
$$3x_1 + x_2 + 7x_3 = 3$$

and $x_1, x_2 \ge 0$,

x_3 is unrestricted.

$$\text{Min } Z = 3x_1 + 5x_2 + 6(x_3' - x_3'')$$

subject to

$$-x_1 - 4x_2 - 6(x_3' - x_3'') \ge -5$$
$$2x_1 + 3x_2 + 5(x_3' - x_3'') \ge 4$$
$$3x_1 + x_2 + 7(x_3' - x_3'') \ge 3$$
$$\boxed{3x_1 + x_2 + 7(x_3' - x_3'') \le 3} \times -1$$
$$-3x_1 - x_2 - 7(x_3' - x_3'') \ge -3$$

$$\text{Min } Z = 3x_1 + 5x_2 + 6x_3' - 6x_3''$$

s.t.

$$-x_1 - 4x_2 - 6x_3' + 6x_3'' \ge -5$$
$$2x_1 + 3x_2 + 5x_3' - 5x_3'' \ge 4$$
$$3x_1 + x_2 + 7x_3' - 7x_3'' \ge 3$$
$$-3x_1 - x_2 - 7x_3' + 7x_3'' \ge -3$$

and $x_1, x_2, x_3', x_3'' \ge 0$

$$x_1 + 4x_2 + 6x_3 \leq 5$$
$$2x_1 + 3x_2 + 5x_3 \geq 4$$
$$3x_1 + x_2 + 7x_3 = 3$$
and $x_1, x_2 \geq 0$,
x_3 is unrestricted.

$$-x_1 - 4x_2 - 6(x_3' - x_3'') \geq -5$$
$$2x_1 + 3x_2 + 5(x_3' - x_3'') \geq 4$$
$$3x_1 + x_2 + 7(x_3' - x_3'') \geq 3$$
$$\boxed{3x_1 + x_2 + 7(x_3' - x_3'') \leq 3} \times -1$$
$$-3x_1 - x_2 - 7(x_3' - x_3'') \geq -3$$

Standard Primal

$$\text{Min } Z = 3x_1 + 5x_2 + 6x_3' - 6x_3''$$

s.t.

$$-x_1 - 4x_2 - 6x_3' + 6x_3'' \geq -5$$
$$2x_1 + 3x_2 + 5x_3' - 5x_3'' \geq 4$$
$$3x_1 + x_2 + 7x_3' - 7x_3'' \geq 3$$
$$-3x_1 - x_2 - 7x_3' + 7x_3'' \geq -3$$

and $x_1, x_2, x_3', x_3'' \geq 0$

$$\text{Min } Z = \boxed{3x_1 + 5x_2 + 6x_3' - 6x_3''}$$

s.t.

$$-x_1 - 4x_2 - 6x_3' + 6x_3'' \geq -5$$

$$2x_1 + 3x_2 + 5x_3' - 5x_3'' \geq 4$$

$$3x_1 + x_2 + 7x_3' - 7x_3'' \geq 3$$

$$-3x_1 - x_2 - 7x_3' + 7x_3'' \geq -3$$

and $x_1, x_2, x_3', x_3'' \geq 0$

Dual

$$\text{Max } Z = -5w_1 + 4w_2 + 3w_3 - 3w_4$$

s.t.

$$-w_1 + 2w_2 + 3w_3 - 3w_4 \leq 3$$

$$-4w_1 + 3w_2 + w_3 - w_4 \leq 5$$

$$-6w_1 + 5w_2 + 7w_3 - 7w_4 \leq 6$$

$$6w_1 - 5w_2 - 7w_3 + 7w_4 \leq -6$$

and $w_1, w_2, w_3, w_4 \geq 0$

$$\text{Min } Z = \boxed{3x_1 + 5x_2 + 6x_3' - 6x_3''}$$

s.t.

$$-x_1 - 4x_2 - 6x_3' + 6x_3'' \geq -5$$

$$2x_1 + 3x_2 + 5x_3' - 5x_3'' \geq 4$$

$$3x_1 + x_2 + 7x_3' - 7x_3'' \geq 3$$

$$-3x_1 - x_2 - 7x_3' + 7x_3'' \geq -3$$

and $x_1, x_2, x_3', x_3'' \geq 0$

<u>Dual</u>

$$\text{Max } Z = -5w_1 + 4w_2 + 3w_3 - 3w_4$$

s.t.

$$-w_1 + 2w_2 + 3w_3 - 3w_4 \leq 3$$

$$-4w_1 + 3w_2 + w_3 - w_4 \leq 5$$

$$-6w_1 + 5w_2 + 7w_3 - 7w_4 \leq 6$$

$$6w_1 - 5w_2 - 7w_3 + 7w_4 \leq -6$$

and $w_1, w_2, w_3, w_4 \geq 0$

$w =$

$(w_3 - w_4)$

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